Corrigendum: Quantum steering ellipsoids, extremal physical states and monogamy (2014 *New J. Phys.* **16** 083017)

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The proof of theorem 6(a) is incorrect, although the monogamy of steering result $\sqrt{V_{A|B}} + \sqrt{V_{C|B}} \leqslant \sqrt{\frac{4\pi}{3}}$ holds as stated. Here we give a corrected proof, which reveals a remarkable new result relating the volume of Alice's steering ellipsoid to the centre of Charlie's: $V_{A|B} = \frac{4\pi}{3}c_{C|B}^2$. We are very grateful to Michael Hall for identifying the mistake, independently verifying our numerical tests and assisting with the corrected proof.

Proof. The pure three-qubit state held by Alice, Bob and Charlie is $|\phi_{ABC}\rangle$. The canonical transformation $|\phi_{ABC}\rangle \rightarrow |\tilde{\phi}_{ABC}\rangle = (\mathbb{1} \otimes \frac{1}{\sqrt{2\rho_B}} \otimes \mathbb{1})|\phi_{ABC}\rangle$ leaves $\mathcal{E}_{A|B}$ and $\mathcal{E}_{C|B}$ invariant. To prove the monogamy of steering we therefore need consider only canonical states for which $\tilde{\boldsymbol{b}} = \boldsymbol{0}$. (When ρ_B is singular and the canonical transformation cannot be performed, no steering by Bob is possible; we then have $V_{A|B} = V_{C|B} = 0$ so that the bound holds trivially.)

We begin by showing that $V_{A|B} = \frac{4\pi}{3}c_{C|B}^2$. Denote the eigenvalues of $\tilde{\rho}_{AB} = \mathrm{tr}_C\, |\tilde{\phi}_{ABC}\rangle\langle\tilde{\phi}_{ABC}|$ as $\{\lambda_i\}$. For a canonical state Charlie's Bloch vector coincides with $\boldsymbol{c}_{C|B}$, and so $\tilde{\rho}_C = \mathrm{tr}_{AB}\, |\tilde{\phi}_{ABC}\rangle\langle\tilde{\phi}_{ABC}| = \frac{1}{2}(\mathbb{1} + \boldsymbol{c}_{C|B} \cdot \boldsymbol{\sigma})$. By Schmidt decomposition we therefore have $\{\lambda_i\} = \{\frac{1}{2}(1 + c_{C|B}),\ \frac{1}{2}(1 - c_{C|B}),\ 0,\ 0\}$.

From the expression for $V_{A|B}$ given in [1] we obtain $V_{A|B} = \frac{64\pi}{3}\ |\det \tilde{\rho}_{AB}^{\mathrm{T_A}}|$ since $\det \tilde{\rho}_{AB} = 0$. Define the reduction map [2, 3] as $\Lambda(X) = \mathbb{1}\,\mathrm{tr}\, X - X$. Following [4] we note that $\det \tilde{\rho}_{AB}^{\mathrm{T_A}} = \det\left((\sigma_y \otimes \mathbb{1})\tilde{\rho}_{AB}^{\mathrm{T_A}}(\sigma_y \otimes \mathbb{1})\right)$ and that $(\sigma_y \otimes \mathbb{1})\tilde{\rho}_{AB}^{\mathrm{T_A}}(\sigma_y \otimes \mathbb{1}) = (\Lambda \otimes \mathbb{1})(\tilde{\rho}_{AB}) = \frac{1}{2}\mathbb{1} \otimes \mathbb{1} - \tilde{\rho}_{AB}$, where we have used the fact that Bob's local state is maximally mixed. Since the eigenvalues of $\frac{1}{2}\mathbb{1} \otimes \mathbb{1} - \tilde{\rho}_{AB}$ are $\{\frac{1}{2} - \lambda_i\}$ we obtain $\det \tilde{\rho}_{AB}^{\mathrm{T_A}} = \prod_i\ (\frac{1}{2} - \lambda_i)$ $= (-\frac{1}{2}c_{C|B})(\frac{1}{2}c_{C|B})(\frac{1}{2})(\frac{1}{2}) = -\frac{1}{16}c_{C|B}^2$, which gives $V_{A|B} = \frac{4\pi}{3}c_{C|B}^2$.

From theorem 3 we have $V_{C|B} \leqslant V_{c_{C|B}}^{\max} = \frac{4\pi}{3}(1 - c_{C|B})^2$. Hence $\sqrt{V_{A|B}} + \sqrt{V_{C|B}} \leqslant \sqrt{\frac{4\pi}{3}}c_{C|B} + \sqrt{\frac{4\pi}{3}}(1 - c_{C|B}) = \sqrt{\frac{4\pi}{3}}$. □

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