Distributed Filtering for Fuzzy Time-Delay Systems With Packet Dropouts and Redundant Channels

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***Abstract*—This paper is concerned with the distributed H_∞ filtering problem for a class of discrete-time Takagi–Sugeno (T–S) fuzzy systems with time-varying delays. The data communications among sensor nodes are equipped with redundant channels subject to random packet dropouts that are modeled by mutually independent Bernoulli stochastic processes. The practical phenomenon of uncertain packet dropout rate is considered, and the norm-bounded uncertainty of the packet dropout rate is asymmetric to the nominal rate. Sufficient conditions on the existence of the desired distributed filters are established by employing the scaled small gain theorem to ensure that the closed-loop system is stochastically stable and achieves a prescribed average H_∞ performance index. Finally, an illustrative example is provided to verify the theoretical findings.**

***Index Terms*—Distributed H_∞ filtering, sensor networks, T–S fuzzy systems, time-varying delays, unreliable communication links**

I. Introduction

Sensor networks (SNs) are typically composed of a number of spatially distributed autonomous nodes that are capable of environment monitoring, data communication and signal processing. During the past decades, SNs have received significant research attention due to their broad applications in diverse areas such as health care, traffic control, intelligent buildings, surveillance, industrial automation, and so on [1]–[4]. The basic issue of distributed filtering over SNs is mainly concerned with the state estimation of the target plant from not only its own measurements but also its neighbors' according to the topology of the underlying SN [5]–[8]. With explicit physical meanings in the attenuation against energy-bounded noises, the H_∞ filtering techniques have been applied to the distributed filtering issues and some useful results have appeared for a variety of dynamic systems, see, e.g., [9]–[11].

Time delays, which cannot be ignored in many dynamic systems, constitute a great challenge in distributed filtering problem. The advances in the field of time-delay systems are mainly related to the basic stability and/or performance issues, see e.g., [12]–[17]. It has been shown in [16] that the input-output (IO) method, which benefits from the application of the scaled small gain (SSG) theorem, is fairly effective in tackling the time-varying delays. Very recently, the two-term approximation idea proposed in [16] has been demonstrated to outperform most existing methodologies in terms of a less approximation error. In addition, since most physical plants or processes are essentially nonlinear, the studies on distributed filtering of nonlinear systems with or without time delays have been carried out in the past few years, see. e.g., [18], [19]. In [20], a class of Takagi-Sugeno (T–S) fuzzy models, which is well recognized to be capable of approximating smooth nonlinear systems to any degree of accuracy, is adopted and has been shown to be quite helpful in facilitating the design of distributed filters. On the basics of T–S fuzzy modeling for nonlinear systems, and their recent advances, we refer readers to [21]–[24] for more details.

It is worthy noting that, in most of engineering situations, the data transmissions from the target plant to sensor nodes are inevitably subject to random packet dropouts, which often exert a tremendous influence on the filtering performance. So far, most of efforts have been devoted to the ways how to overcome the effect of packet dropouts on the SNs where data are transmitted over *one channel* [9], [25]–[27]. Yet, in practice, two or more channels can be simultaneously available for a concrete application of SN. Intuitively, inclusion of redundant channels would help resolve the issue of packet dropouts and even alleviate the effect of disconnection of partial channels. As such, it is of great necessity and significance to allow for the use of redundant channels in designing distributed filters via SNs. However, to the best of the authors' knowledge, no insightful investigations have been reported up to date in the area of distributed filtering with redundant channels.

With regard to the packet dropout rate of each communication channel, it should be pointed out that almost all the previous works have assumed that the packet dropout rate is exactly known, which is rarely the case in practice. The packet dropout rate is not invariant due to the complex network environment and fluctuation of power supply, and the upper and lower bounds of the packet dropout rate can be also asymmetric to the "nominal" rate. The performance of a filter designed with a fixed packet dropout rate may become worse if the actual rate is far from the fixed value. Meanwhile, for some scenarios, adequate samples for computing the mathematical expectation of the Bernoulli process are too costly or time-consuming to acquire. In other words, it is fairly difficult to determine an exact expectation. Owing to these two reasons, it makes practical sense to study the issue of uncertain packet dropout rate and account for its effects on the underscored distributed filtering performance.

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Based on the aforementioned discussions, in this paper, we aim at addressing the distributed H_∞ filtering problem for a class of discrete-time T–S fuzzy systems with time-varying delays. The main contributions of this paper can be summarized as follows. *1) For the first time, the data communications among sensor nodes are allowed to be conducted along redundant channels, where the random packet dropouts in different channels are modeled by mutually independent Bernoulli stochastic processes. 2) In each communication channel, the practical phenomenon of uncertain packet dropout rate is considered where the norm-bounded uncertainty is used to describe the packet dropout rate. 3) A two-term approximation idea [16] is employed for the fuzzy systems with time-varying delays to reduce the resulting approximation error in model transformation.* The rest of this paper is outlined as follows. In Section II, some preliminaries and problem formulation are introduced. Section III is devoted to the problems of model transformation, stability analysis and distributed H_∞ filter design. The necessity of taking uncertain packet dropout rates into account in the design phase of distributed filters, as well as the performance improvements by including redundant channels, are shown via an illustrative example in Section IV. Finally, the conclusion is drawn in Section V.

Notations: The notation used throughout this paper is fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The notation $P > 0$ ($P \geq 0$) means that P is symmetric and positive (semi-positive) definite. I_n, $\mathbf{0}_n$ and $\mathbf{0}_{m\times n}$ represent, respectively, the $n \times n$ identity matrix, the $n \times n$ zero matrix and the $m \times n$ zero matrix. $\mathbf{1}_n = [1 \quad 1 \quad \cdots \quad 1]^T \in \mathbb{R}^n$. The notation $\mathrm{vec}_n\{x_p\}_p$ denotes $[x_1 \quad x_2 \quad \ldots \quad x_n]$. $\mathrm{diag}\{...\}$ stands for a block-diagonal matrix, and $\mathrm{diag}_n\{A_p\}_p$ stands for the block-diagonal matrix $\mathrm{diag}\{A_1, A_2, \cdots A_n\}$. In symmetric block matrices or complex matrix expressions, we use the symbol "$*$" as an ellipsis for the terms that are introduced by symmetry. The symbol $\otimes$ denotes the Kronecker product. $\mathbf{G}_1 \circ \mathbf{G}_2$ represents the series connection of mapping $\mathbf{G}_1$ and $\mathbf{G}_2$. The notation $\|\cdot\|$ stands for the Euclidean vector norm, $\|\cdot\|_2$ the usual $l_2[0,\infty)$ norm and $\|\cdot\|_\infty$ the l_2-induced norm of a transfer function matrix. $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of a symmetric matrix. In addition, $E\{x\}$ and $E\{x|y\}$ denote, respectively, the expectation of x and the expectation of x conditional on y.

II. Preliminaries

Consider the SN in Fig. 1, which has n intermediate sensor nodes distributed in space according to a fixed interconnection network topology described by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$, where $\mathcal{V} = \{1, 2, \ldots, n\}$ is the nonempty set of sensor nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{L} = (l_{pq})_{n\times n}$ is a weighted adjacency matrix with nonnegative adjacency element l_{pq}. An edge of $\mathcal{G}$ is denoted by ordered pair (p, q). The adjacency elements associated with the edges of the graph are positive, i.e., $l_{pq} > 0, (p, q) \in \mathcal{E}$, which means that sensor node q is one of the neighbors of sensor node p, and sensor node p can obtain information from sensor node q. Moreover, we assume that $l_{pp} = 1$ for all $p \in V$, which can be regarded as the case that the sensors are self-connected, and therefore (p, p) can

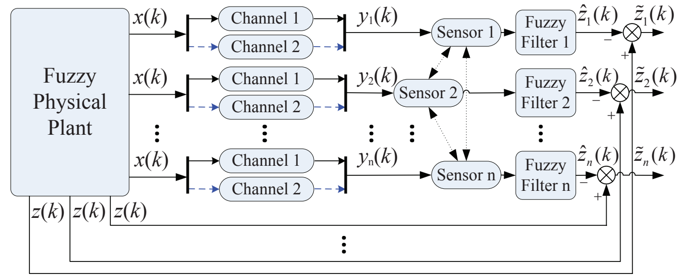

Fig. 1. An illustration of the use of two channels in distributed filtering over sensor network.

represent an additional edge. The set of neighbors of node $p \in V$ plus the node itself is denoted by $\mathcal{N}_p \triangleq \{q \in V \mid (p, q) \in \mathcal{E}\}$.

In this paper, the target plant is described by the following discrete-time T–S fuzzy model with time-varying delays:

Plant Rule i: **IF** $\theta_1(k)$ is M_{i1}, $\theta_2(k)$ is $M_{i2}, \ldots$, and $\theta_g(k)$ is M_{ig}, **THEN**

$$\begin{cases} x(k+1) = A_i x(k) + A_{di} x\left(k - d(k)\right) + E_i w(k) \\ z(k) = C_i x(k) \\ x(k) = \varphi(k), \ k = \{-d_2, -d_2+1, ..., 0\} \end{cases} \tag{1}$$

where $i \in \mathbb{S} \triangleq \{1, 2, ..., s\}$, and s is the number of IF–THEN rules; $\theta_1(k)$, $\theta_2(k), ..., \theta_g(k)$ are the premise variables, and g is the number of these premise variables; M_{ij} is the fuzzy set; $x(k) \in \mathbb{R}^{n_x}$ is the state vector which cannot be observed directly; $z(k) \in \mathbb{R}^{n_z}$ is the signal to be estimated; $w(k) \in \mathbb{R}^{n_w}$ denotes the disturbance input belonging to $l_2[0,\infty)$; $\varphi(k)$ is a real-valued initial condition sequence on $\{-d_2, -d_2+1, ..., 0\}$; A_i, A_{di}, E_i, and C_i are known matrices with appropriate dimensions, which are real and constant. In (1), $d(k)$ is a positive integer satisfying

$$d_1 \leq d(k) \leq d_2 \tag{2}$$

where d_1 and d_2 represent the minimum and maximum time delays, respectively.

The overall model of the discrete-time fuzzy systems with time delays can be expressed as follows:

$$\begin{cases} x(k+1) = \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))[A_i x(k) + A_{di} x(k - d(k)) + E_i w(k)] \\ z(k) = \sum_{i=1}^{s} h_i\left(\boldsymbol{\theta}\left(k\right)\right) C_i x(k) \end{cases} \tag{3}$$

where $\boldsymbol{\theta}(k) \triangleq \left[\theta_1(k) \quad \theta_2(k) \quad \cdots \quad \theta_g(k)\right]$, and

$$h_i\left(\boldsymbol{\theta}(k)\right) \triangleq \frac{\prod_{j=1}^{g} M_{ij}(\theta_j(k))}{\sum_{i=1}^{s} \prod_{j=1}^{g} M_{ij}(\theta_j(k))}.$$

In (3), $h_i(\boldsymbol{\theta}(k))$ is the fuzzy basis function, and $M_{ij}(\theta_j(k))$ is the grade of membership of $\theta_j(k)$ in M_{ij}. Therefore, we have $h_i\left(\boldsymbol{\theta}(k)\right) \geq 0, i \in \mathbb{S}$, and $\sum_{i=1}^{s} h_i\left(\boldsymbol{\theta}(k)\right) = 1$ for all k.

For each p ($p = 1, 2, \ldots, n$), the model of sensor node p is given as follows:

$$y_p(k) = \alpha_p(k) B_{pi} x(k) + (1 - \alpha_p(k)) \beta_p(k) D_{pi} x(k) + F_{pi} w(k) \tag{4}$$

where $y_p(k) \in \mathbb{R}^{n_y}$ is the measured output received by intermediate sensor node p; $B_{pi} \in \mathbb{R}^{n_y \times n_x}$, $D_{pi} \in \mathbb{R}^{n_y \times n_x}$, and $F_{pi} \in \mathbb{R}^{n_y \times n_w}$ are known real-valued constant matrices. In this paper, as shown in Fig. 1, two parallel communication channels, as a special case of redundant channels, are considered in the SN with a fixed topology. The stochastic variables $\alpha_p(k)$ and $\beta_p(k)$, which are mutually independent Bernoulli processes and also independent with the premise variables, are introduced to model the packet dropout phenomena in the first and the second communication channel, respectively. Moreover, it is assumed that the packet dropout rate in each channel is uncertain. More specifically, we have

$$\begin{aligned}
&\Pr\{\alpha_p(k)=1\} = E\{\alpha_p(k)\} = E\{\alpha_p^2(k)\} = \bar{\alpha}_p + \Delta\alpha_p(k)\\
&\Pr\{\alpha_p(k)=0\} = 1-\bar{\alpha}_p - \Delta\alpha_p(k)\\
&\Pr\{\beta_p(k)=1\} = E\{\beta_p(k)\} = E\{\beta_p^2(k)\} = \bar{\beta}_p + \Delta\beta_p(k)\\
&\Pr\{\beta_p(k)=0\} = 1-\bar{\beta}_p - \Delta\beta_p(k) \qquad (5)
\end{aligned}$$

where $E\{\alpha_p(k)\}$ or $E\{\beta_p(k)\}$ stands for the packet arriving rate in the corresponding channel, respectively. In (5), $\bar{\alpha}_p$ and $\bar{\beta}_p$ are the nominal expectations of packet arrivals, and $\Delta\alpha_p(k)$ and $\Delta\beta_p(k)$ are the norm-bounded uncertainties of $\bar{\alpha}_p$ and $\bar{\beta}_p$, respectively. The uncertainties are bounded as $-\eta_{p2} \leq \Delta\alpha_p(k) \leq \eta_{p1}$ and $-\delta_{p2} \leq \Delta\beta_p(k) \leq \delta_{p1}$ with $\eta_{p1}, \eta_{p2}, \delta_{p1}, \delta_{p2} \geq 0$. Therefore, it is clear that

$$\begin{aligned}
&\bar{\alpha}_p + \Delta\alpha_p(k) \leq \bar{\alpha}_p + \eta_{p1},\ 1-\bar{\alpha}_p - \Delta\alpha_p(k) \leq 1-\bar{\alpha}_p + \eta_{p2}\\
&\bar{\beta}_p + \Delta\beta_p(k) \leq \bar{\beta}_p + \delta_{p1},\ 1-\bar{\beta}_p - \Delta\beta_p(k) \leq 1-\bar{\beta}_p + \delta_{p2}.
\end{aligned}$$

Moreover, since the packet arriving rate at sensor node p can be formulated as

$$\Pr\{\mathrm{R}_p(k)\} = 1-(1-E\{\alpha_p(k)\})(1-E\{\beta_p(k)\}) \qquad (6)$$

where $\{\mathrm{R}_p(k)\}$ denotes the event that the measurement of sensor p is transmitted successfully at time k. Then it holds that

$$\begin{aligned}
\Pr\{\mathrm{R}_p(k)\} &= E\{\alpha_p(k)\} + E\{\beta_p(k)\} - E\{\alpha_p(k)\}E\{\beta_p(k)\}\\
&\geq \max\{E\{\alpha_p(k)\}, E\{\beta_p(k)\}\}
\end{aligned}$$

which clearly shows the advantage of adopting two channels.

Remark 1: The proposed measurement model in (4) provides a novel unified framework to account for the phenomenon of two-channel packet dropouts at each time instant by resorting to the random variables $\alpha_p(k)$ and $\beta_p(k)$. In particular, if $\alpha_p(k) = 1, \beta_p(k) = 1$ or 0, sensor node p in model (4) takes the first channel; and if $\alpha_p(k) = 0, \beta_p(k) = 1$, packet dropout occurs at the first channel, and sensor node p chooses the second channel. In the two aforementioned cases, the packet at time k is not dropped, while if $\alpha_p(k) = 0, \beta_p(k) = 0$, the packet dropout occurs at sensor node p. Note that the measurement model in (4) can be further extended to the multi-channel case. Also, by generalizing (6) to multi-channel case, it is straightforward that the occurrence probability of packet dropouts at each sensor node can be further reduced.

Remark 2: In practice, the packet arriving rates $E\{\alpha_p(k)\}$ and $E\{\beta_p(k)\}$ are generally related to the external environment and transport protocols. In this paper, without loss of generality, we suppose that η_{p2} is greater than η_{p1}, and δ_{p2} is greater than δ_{p1}, which means that the packet arriving rates easily tend to become smaller.

A more compact presentation of the SN in sensor node p can be given by

$$\begin{cases}
x(k+1) = A(k)x(k) + A_d(k)x(k-d(k)) + E(k)w(k)\\
y_p(k) = \alpha_p(k)B_p(k)x(k) + (1-\alpha_p(k))\beta_p(k)D_p(k)x(k)\\
\qquad\quad + F_p(k)w(k)\\
z(k) = C(k)x(k)
\end{cases} \qquad (7)$$

where

$$\begin{aligned}
&A(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))A_i,\ A_d(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))A_{di}\\
&B_p(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))B_{pi},\ C(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))C_i\\
&D_p(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))D_{pi},\ E(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))E_i\\
&F_p(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))F_{pi}. \qquad (8)
\end{aligned}$$

The key point in designing distributed filters for SNs is to fuse the information available for the filter on sensor node p both from the node itself and its neighbors. Now, we assume that the filter's premise variable on each node is the same as the plant's premise variable. The following fuzzy filter structure is adopted on sensor node p:

Filter Rule j : **IF** $\theta_1(k)$ is M_{j1}, $\theta_2(k)$ is $M_{j2}, \ldots,$ and $\theta_g(k)$ is M_{jg}, **THEN**

$$\begin{cases}
\hat{x}_p(k+1) = \sum\limits_{q\in\mathcal{N}_p} l_{pq}K_{pqj}\hat{x}_q(k) + \sum\limits_{q\in\mathcal{N}_p} l_{pq}H_{pqj}y_q(k)\\
\hat{z}_p(k) = L_{pj}\hat{x}_p(k)
\end{cases} \qquad (9)$$

where $j \in \mathbb{S} \triangleq \{1, 2, ..., s\}$; l_{pq} is the weighted adjacency scalar between node p and node q; $\hat{x}_p(k) \in \mathbb{R}^{n_x}$ and $z_p(k) \in \mathbb{R}^{n_z}$ are the estimation of $x(k)$ and $z(k)$ on sensor node p, respectively. The matrices $K_{pqj} \in \mathbb{R}^{n_x \times n_x}$, $H_{pqj} \in \mathbb{R}^{n_x \times n_y}$, and $L_{pj} \in \mathbb{R}^{n_z \times n_x}$ in (9) are the fuzzy filter parameters to be determined for node p. Moreover, the initial values of fuzzy filters are assumed to be $\hat{x}_p(0) = \mathbf{0}_{n_x \times 1}$ for all $p = 1, 2, \ldots n$. Then, the overall fuzzy filter on sensor node p can be inferred by

$$\begin{cases}
\hat{x}_p(k+1) = \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))\left[\sum_{q\in\mathcal{N}_p} l_{pq}K_{pqj}\hat{x}_q(k)\right.\\
\qquad\qquad \left.+ \sum_{q\in\mathcal{N}_p} l_{pq}H_{pqj}y_q(k)\right]\\
\hat{z}_p(k) = \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))L_{pj}\hat{x}_p(k).
\end{cases}$$

Thus, the fuzzy filter for sensor node p can be represented by the following more compact form:

$$\begin{cases}
\hat{x}_p(k+1) = \sum\limits_{q\in\mathcal{N}_p} l_{pq}K_{pq}(k)\hat{x}_q(k) + \sum\limits_{q\in\mathcal{N}_p} l_{pq}H_{pq}(k)y_q(k)\\
\hat{z}_p(k) = L_p(k)\hat{x}_p(k)
\end{cases} \qquad (10)$$

where

$$\begin{aligned}
&K_{pq}(k) \triangleq \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))K_{pqj},\ L_p(k) \triangleq \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))L_{pj}\\
&H_{pq}(k) \triangleq \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))H_{pqj}. \qquad (11)
\end{aligned}$$

Defining $e_p(k) \triangleq x(k) - \hat{x}_p(k)$ and $\tilde{z}_p(k) \triangleq z(k) - \hat{z}_p(k)$, the fuzzy filtering error dynamics for sensor p can be obtained by (7) and (10):

$$\begin{aligned} e_p(k+1) = &\Big\{A(k) - \textstyle\sum_{q\in\mathcal{N}_p} l_{pq}H_{pq}(k)\left[\alpha_q(k)B_q(k)\right. \\ &+ \left.\left(\beta_q(k) - \alpha_q(k)\beta_q(k)\right)D_q(k)\right] \\ &- \textstyle\sum_{q\in\mathcal{N}_p} l_{pq}K_{pq}(k)\Big\}x(k) + A_d(k) \\ &\times x(k-d(k)) + \textstyle\sum_{q\in N_p} l_{pq}K_{pq}(k)e_q(k) \\ &+ \Big[E(k) - \textstyle\sum_{q\in\mathcal{N}_p} l_{pq}H_{pq}(k)F_q(k)\Big]w(k) \\ \tilde{z}_p(k) = &\left[C(k) - L_p(k)\right]x(k) + L_p(k)e_p(k) \\ &p = 1,2,\ldots,n, q\in\mathcal{N}_p. \end{aligned} \tag{12}$$

For notational simplification, we denote

$$\begin{aligned} &\bar{A}_i \triangleq I_n \otimes A_i,\ \bar{A}_{di} \triangleq I_n \otimes A_{di},\ \bar{C}_i \triangleq I_n \otimes C_i \\ &\bar{E}_i \triangleq \mathbf{1}_n \otimes E_i,\ \bar{x}(k) \triangleq \mathbf{1}_n \otimes x(k),\ e(k) \triangleq \text{vec}_n\{e_p(k)\}_p \\ &\bar{F}_i \triangleq \text{vec}_n\{F_{pi}\}_p,\ \bar{z}(k) \triangleq \text{vec}_n\{\tilde{z}_p(k)\}_p \\ &\bar{B}_i \triangleq \text{diag}_n\{B_{pi}\}_p,\ \bar{D}_i \triangleq \text{diag}_n\{D_{pi}\}_p \\ &\bar{L}_j \triangleq \text{diag}_n\{L_{pj}\}_p,\ \mathcal{I}_q \triangleq \text{diag}\{\mathbf{0}_{q-1}, 1, \mathbf{0}_{n-q}\} \otimes I_{n_y} \\ &\bar{K}_j \triangleq [\breve{K}_{pqj}]_{n\times n}^{p\times q},\ \bar{H}_j \triangleq [\breve{H}_{pqj}]_{n\times n}^{p\times q} \end{aligned} \tag{13}$$

where

$$\begin{aligned} \breve{K}_{pqj} &\triangleq \begin{cases} l_{pq}K_{pqj}, & p = 1,2,\ldots n; q\in\mathcal{N}_p \\ 0, & p = 1,2,\ldots n; q\notin\mathcal{N}_p \end{cases} \\ \breve{H}_{pqj} &\triangleq \begin{cases} l_{pq}H_{pqj}, & p = 1,2,\ldots n; q\in\mathcal{N}_p \\ 0, & p = 1,2,\ldots n; q\notin\mathcal{N}_p \end{cases}. \end{aligned} \tag{14}$$

Obviously, since $l_{pq} = 0$ when $q \notin \mathcal{N}_p$. $\bar{K}_j \in \mathbb{R}^{nn_x\times nn_x}$ and $\bar{H}_j \in \mathbb{R}^{nn_x\times nn_y}$ are two sparse matrices. Moreover, combining (8), (11) and (13), we have

$$\begin{aligned} &\bar{A}(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))\bar{A}_i,\ \bar{A}_d(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))\bar{A}_{di} \\ &\bar{C}(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))\bar{C}_i,\ \bar{D}(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))\bar{D}_i \\ &\bar{E}(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))\bar{E}_i,\ \bar{F}(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))\bar{F}_i \\ &\bar{B}(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))\bar{B}_i,\ \bar{L}(k) \triangleq \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))\bar{L}_j \\ &\bar{K}(k) \triangleq \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))\bar{K}_j,\ \bar{H}(k) \triangleq \sum_{j=1}^{s} h_j(\boldsymbol{\theta}(k))\bar{H}_j. \end{aligned} \tag{15}$$

Then, the error dynamics governed by (12) can be obtained in the following compact form:

$$\begin{aligned} e(k+1) = &\Big\{\bar{A}(k) - \textstyle\sum_{q=1}^{n}\alpha_q(k)\bar{H}(k)\mathcal{I}_q\bar{B}(k) \\ &- \textstyle\sum_{q=1}^{n}\left(\beta_q(k) - \alpha_q(k)\beta_q(k)\right)\bar{H}(k)\mathcal{I}_q\bar{D}(k) \\ &-\bar{K}(k)\Big\}\bar{x}(k) + \bar{A}_d(k)\bar{x}(k-d(k)) \\ &+ \bar{K}(k)e(k) + \left[\bar{E}(k) - \bar{H}(k)\bar{F}(k)\right]w(k) \\ \bar{z}(k) = &\left[\bar{C}(k) - \bar{L}(k)\right]\bar{x}(k) + \bar{L}(k)e(k). \end{aligned}$$

By setting $\mathcal{E}(k) \triangleq \begin{bmatrix}\bar{x}^T(k) & e^T(k)\end{bmatrix}^T$, augmenting the original model to include the fuzzy filter error dynamics, the following augmented system can be obtained:

$$\begin{cases} \mathcal{E}(k+1) = \Big[\mathcal{A}(k) - \sum_{q=1}^{n}\alpha_q(k)\mathcal{H}_{\mathbf{I}q}(k) \\ \qquad - \sum_{q=1}^{n}(\beta_q(k) - \alpha_q(k)\beta_q(k))\mathcal{H}_{\mathbf{II}q}(k)\Big]\mathcal{E}(k) \\ \qquad + \mathcal{A}_d(k)\mathcal{E}(k-d(k)) + \mathfrak{E}(k)w(k) \\ \bar{z}(k) = \mathcal{C}(k)\mathcal{E}(k) \\ \mathcal{E}(k) = \boldsymbol{\phi}(k),\ k = \{-d_2, -d_2+1, ..., 0\} \end{cases} \tag{16}$$

where

$$\begin{aligned} \mathcal{A}(k) &\triangleq \begin{bmatrix} \bar{A}(k) & \mathbf{0}_{nn_x} \\ \bar{A}(k) - \bar{K}(k) & \bar{K}(k) \end{bmatrix} \\ \mathcal{A}_d(k) &\triangleq \begin{bmatrix} \bar{A}_d(k) & \mathbf{0}_{nn_x} \\ \bar{A}_d(k) & \mathbf{0}_{nn_x} \end{bmatrix},\ \mathfrak{E}(k) \triangleq \begin{bmatrix} \bar{E}(k) \\ \bar{E}(k) - \bar{H}(k)\bar{F}(k) \end{bmatrix} \\ \mathcal{C}(k) &\triangleq \begin{bmatrix} \bar{C}(k) - \bar{L}(k) & \bar{L}(k) \end{bmatrix} \\ \mathcal{H}_{\mathbf{I}q}(k) &\triangleq \begin{bmatrix} \mathbf{0}_{nn_x} & \mathbf{0}_{nn_x} \\ \bar{H}(k)\mathcal{I}_q\bar{B}(k) & \mathbf{0}_{nn_x} \end{bmatrix} \\ \mathcal{H}_{\mathbf{II}q}(k) &\triangleq \begin{bmatrix} \mathbf{0}_{nn_x} & \mathbf{0}_{nn_x} \\ \bar{H}(k)\mathcal{I}_q\bar{D}(k) & \mathbf{0}_{nn_x} \end{bmatrix}. \end{aligned} \tag{17}$$

Before presenting the main objective of this paper, we introduce the following definitions:

Definition 1: [28] The distributed fuzzy filtering error system (DFFES) in (16) is said to be stochastically stable, if in the case of $w(k) \equiv \mathbf{0}_{n_w\times 1}$, for any initial condition $\mathcal{E}(0)$, the following is satisfied:

$$E\left\{\textstyle\sum_{k=0}^{\infty}|\mathcal{E}(k)|^2 \mid \mathcal{E}(0)\right\} < \infty. \tag{18}$$

Definition 2: [29] A mapping $\mathbf{G} : u(k) \to y(k)$ is input-output stable if there exists $\varrho \geq 0$ such that

$$\|y(k)\|_2 = \|\mathbf{G}(u(k))\|_2 \leq \varrho\|u(k)\|_2.$$

Given integers $d_2 \geq d_1 \geq 1$ and a prescribed scalar $\gamma > 0$, our aim in this paper is to find the distributed filter matrices $(K_{pqj}, H_{pqj}, L_{pj})$ in (9) such that for any time-varying delay $d(k)$ satisfying (2):

1) (Stochastic stability) The DFFES in (16) is stochastically stable in the sense of (18).

2) (Average H_∞ performance index) Under zero initial condition, the filter error $\bar{z}(k)$ satisfies $\frac{1}{n}\|\bar{z}(k)\|_{E_2}^2 \leq \gamma^2\|w(k)\|_2^2$ where $\|\bar{z}(k)\|_{E_2} \triangleq E\left\{\sqrt{\sum_{k=0}^{\infty}\bar{z}^T(k)\bar{z}(k)}\right\}$.

III. Main Results

In this section, we are aiming at establishing a sufficient criterion for the error $\bar{z}(k)$ in (16) to satisfy the H_∞ performance constraint by applying the scaled small gain (SSG) theorem. Then, distributed H_∞ fuzzy filters in the form of (9) will be designed such that the DFFES in (16) is stochastically stable with a prescribed average H_∞ performance.

A. Model Transformation Approach

Consider an interconnected system consisting of two subsystems:

$$(S_1) : \vartheta(k) = \mathbf{G}v(k),\ (S_2) : v(k) = \boldsymbol{\Delta}\vartheta(k) \tag{19}$$

where the forward subsystem (S_1) is a known linear time-invariant system with operator $\mathbf{G}$ mapping $v(k)$ to $\vartheta(k)$, and the feedback subsystem (S_2) is an unknown linear time-varying mapping with operator $\mathbf{\Delta}$ which has a block-diagonal structure. A sufficient condition of the input-output stability of the interconnected system formed by (S_1) and (S_2) can be given as a direct result of the SSG theorem.

Before proceeding further, we first present the SSG theorem that will be used in the later derivations.

Lemma 1: (SSG Theorem [16]) Consider the closed-loop system in (19). Assume that the forward subsystem (S_1) is internally stable, the closed-loop system formed by (S_1) and (S_2) is input-output stable for all $\mathbf{G}$ and $\mathbf{\Delta}$ if the following inequality holds:

$$\Upsilon(\mathbf{G}_T) \times \Upsilon(\mathbf{\Delta}_T) < 1 \tag{20}$$

where T is a nonsingular linear matrix, and

$$\Upsilon(\mathbf{G}_T) = \left\| T \circ \mathbf{G} \circ T^{-1} \right\|_\infty, \ \Upsilon(\mathbf{\Delta}_T) = \left\| T \circ \mathbf{\Delta} \circ T^{-1} \right\|_\infty .$$

To cope with the time-varying delay of the filtering error system (16) by the SSG theorem, we need to adopt time-invariant delays as an approximation of the time-varying term and pull the "delay uncertainty" out into the feedback subsystem $\mathbf{\Delta}$. In this paper, we would employ the two-term approximation method adopted in [16]. By defining $d_{12} \triangleq d_2 - d_1$, the term $\mathcal{E}(k-d(k))$ can be expressed as

$$\mathcal{E}(k-d(k)) = \tfrac{1}{2}\left[\mathcal{E}\left(k-d_1\right) + \mathcal{E}\left(k-d_2\right)\right] + \tfrac{d_{12}}{2} v(k) \tag{21}$$

where $\frac{1}{2}\left[\mathcal{E}\left(k-d_1\right) + \mathcal{E}\left(k-d_2\right)\right]$ is the approximation of $\mathcal{E}(k-d(k))$, and $\frac{d_{12}}{2} v(k)$ is the approximation error.

By defining $\vartheta(k) \triangleq \mathcal{E}\left(k+1\right) - \mathcal{E}(k)$, we can obtain

$$\begin{aligned} v(k) &\triangleq \tfrac{2}{d_{12}} \left\{ \mathcal{E}(k-d(k)) - \tfrac{1}{2}\left[\mathcal{E}\left(k-d_1\right) + \mathcal{E}\left(k-d_2\right)\right] \right\} \\ &= \tfrac{1}{d_{12}} \left\{ 2\mathcal{E}(k-d(k)) - \left[\mathcal{E}\left(k-d_1\right) + \mathcal{E}\left(k-d_2\right)\right] \right\} \\ &= \tfrac{1}{d_{12}} \left[\textstyle\sum_{u=k-d_2}^{k-d(k)-1} \vartheta(u) - \sum_{u=k-d(k)}^{k-d_1-1} \vartheta(u) \right] \\ &= \tfrac{1}{d_{12}} \left[\textstyle\sum_{u=k-d_2}^{k-d_1-1} \kappa(u)\vartheta(u) \right] \end{aligned} \tag{22}$$

where

$$\kappa(u) \triangleq \begin{cases} 1, & \text{when } u \le k-d(k)-1 \\ -1, & \text{when } u \ge k-d(k)-1. \end{cases}$$

From (16) and (21), the fuzzy filtering error system (16) can be transformed into the following form:

$$\begin{aligned} \mathcal{E}(k+1) = &\Big[\mathcal{A}(k) - \textstyle\sum_{q=1}^{n} \alpha_q(k)\mathcal{H}_{\mathbf{I}q}(k) - \sum_{q=1}^{n} \beta_q(k) \\ &\times \left(1-\alpha_q(k)\right)\mathcal{H}_{\mathbf{II}q}(k)\Big]\mathcal{E}(k) + \tfrac{d_{12}}{2}\mathcal{A}_d(k)v(k) \\ &+ \tfrac{1}{2}\mathcal{A}_d(k)[\mathcal{E}(k-d_1) + \mathcal{E}(k-d_2)] + \mathfrak{E}(k)w(k) \\ \bar{z}(k) = &\ \mathcal{C}(k)\mathcal{E}(k). \end{aligned} \tag{23}$$

Now, by (23), the interconnection frame of the closed-loop system is presented as

$$\begin{cases} (S_1): \begin{bmatrix} \mathcal{E}(k+1) \\ \vartheta(k) \\ \bar{z}(k) \end{bmatrix} = \begin{bmatrix} \Gamma_1(k) & \mathfrak{E}(k) \\ \Gamma_2(k) & \mathfrak{E}(k) \\ \Gamma_3(k) & \mathbf{0}_{n_z \times n_w} \end{bmatrix} \begin{bmatrix} \varsigma(k) \\ w(k) \end{bmatrix} \\ (S_2): v(k) = \mathbf{\Delta}\vartheta(k) \end{cases} \tag{24}$$

where

$$\begin{aligned} \Gamma_1(k) \triangleq &\Big[\mathcal{A}(k) - \textstyle\sum_{q=1}^{n} \left[\alpha_q(k)\mathcal{H}_{\mathbf{I}q}(k) + \beta_q(k)(1-\alpha_q(k)) \right. \\ &\left. \times \mathcal{H}_{\mathbf{II}q}(k)\right] \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k)\Big] \\ \Gamma_2(k) \triangleq &\Big[\mathcal{A}(k) - \textstyle\sum_{q=1}^{n} \left[\alpha_q(k)\mathcal{H}_{\mathbf{I}q}(k) + \beta_q(k)(1-\alpha_q(k)) \right. \\ &\left. \times \mathcal{H}_{\mathbf{II}q}(k)\right] - I_{2nn_x} \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k)\Big] \\ \Gamma_3(k) \triangleq &\begin{bmatrix} \mathcal{C}(k) & \mathbf{0}_{nn_z \times 6nn_x} \end{bmatrix} \\ \varsigma(k) \triangleq &\begin{bmatrix} \mathcal{E}^T(k) & \mathcal{E}^T(k-d_1) & \mathcal{E}^T(k-d_2) & v^T(k) \end{bmatrix}^T . \end{aligned}$$

Furthermore, the following lemma shows that the scaled gain of the mapping $\mathbf{\Delta}$ in (24) has an upper bound.

Lemma 2: [16] Consider the closed-loop system (24). For any given nonsingular matrix $T \in \mathbb{R}^{2nn_x \times 2nn_x}$, the feedback mapping $\mathbf{\Delta} : \vartheta(k) \to v(k)$ of (22) satisfies $\left\| T \circ \mathbf{\Delta} \circ T^{-1} \right\|_\infty \le 1$.

Remark 3: In view of Lemma 2, we can observe that the l_2-induced norm of (S_2) in (24) is bounded by one. Then, according to Lemma 1, if we can guarantee $\left\| T \circ \mathbf{G} \circ T^{-1} \right\|_\infty < 1$, the whole interconnection system (24) is input-output stable.

B. H_∞ Performance and Stochastic Stability Analysis

Based on Lemmas 1 and 2, the following theorem presents a criterion to ensure that the DFFES (24) is stochastically stable with a prescribed average H_∞ performance.

Theorem 1: Consider the discrete-time DFFES in (24) and suppose that the filter parameters K_{pqj}, H_{pqj}, L_{pj} ($p = 1, 2, \ldots, n$, $q \in \mathcal{N}_p$, $j \in \mathbb{S}$) are given. For integers d_1, d_2 with $d_2 \ge d_1 \ge 1$ and constants $\gamma > 0$, $\bar{\alpha}_q, \bar{\beta}_q > 0$, $\eta_{q1}, \eta_{q2}, \delta_{q1}, \delta_{q2} \ge 0$ ($q \in \mathcal{N}_p$), the DFFES (24) is stochastically stable with an average H_∞ performance γ for any time-varying delay satisfying $d_1 \le d(k) \le d_2$, if $\forall q \in \mathcal{N}_p$, there exist positive definite matrices $P_i, Q_{1i}, Q_{2i}, R_1, R_2, R_3$, $i \in \mathbb{S}$, satisfying

$$\begin{aligned} &\mathcal{G}_{oltii} < 0, (o, l, t, i \in \mathbb{S}) \\ &\mathcal{G}_{oltij} + \mathcal{G}_{oltji} < 0, (o, l, t, i, j \in \mathbb{S}, i < j) \end{aligned} \tag{25}$$

where

$$\begin{aligned} \mathcal{G}_{oltij} &\triangleq \begin{bmatrix} \Lambda_{lti} & \Omega_{oij} \\ * & \Xi_o \end{bmatrix} \\ \Lambda_{lti} &\triangleq \begin{bmatrix} \Lambda_{\mathbf{I}i} & \begin{bmatrix} R_1 & R_2 & \mathbf{0}_{2nn_x} & \mathbf{0}_{2nn_x \times n_w} \end{bmatrix} \\ * & \operatorname{diag}\left\{\Lambda_{\mathbf{II}l}, \Lambda_{\mathbf{III}t}, -R_3, -n\gamma^2 I_{n_w}\right\} \end{bmatrix} \\ \Omega_{oij} &\triangleq \begin{bmatrix} \Psi^T_{\mathbf{III}ij} & d_1\mathcal{W}_{\mathbf{II}ij}\mathcal{R}_1 & d_2\mathcal{W}_{\mathbf{II}ij}\mathcal{R}_2 & \mathcal{W}_{\mathbf{II}ij}\mathcal{R}_3 & \mathcal{W}_{\mathbf{I}ij}\mathcal{P}_o \end{bmatrix} \\ \Xi_o &\triangleq \operatorname{diag}\left\{-I_{nn_z}, -\mathcal{R}_1, -\mathcal{R}_2, -\mathcal{R}_3, -\mathcal{P}_o\right\} \\ \Lambda_{\mathbf{I}i} &\triangleq -P_i - R_1 - R_2 + Q_{1i} + Q_{2i}, \ \Lambda_{\mathbf{II}l} \triangleq -R_1 - Q_{1l} \\ \Lambda_{\mathbf{III}t} &\triangleq -R_2 - Q_{2t}, \ \mathcal{R}_1 \triangleq I_{2n+1} \otimes R_1, \ \mathcal{R}_2 \triangleq I_{2n+1} \otimes R_2 \\ \mathcal{R}_3 &\triangleq I_{2n+1} \otimes R_3, \ \mathcal{P}_o \triangleq I_{2n+1} \otimes P_o \\ \mathcal{W}_{\mathbf{I}ij} &\triangleq \left[\mu_{1,1}\Psi^T_{\mathbf{I}_1 1,ij} \quad \mu_{1,2}\Psi^T_{\mathbf{I}_1 2,ij} \quad \cdots \quad \mu_{1,n}\Psi^T_{\mathbf{I}_1 n,ij} \right. \\ &\quad \left. \mu_{2,1}\Psi^T_{\mathbf{I}_2 1,ij} \quad \mu_{2,2}\Psi^T_{\mathbf{I}_2 2,ij} \quad \cdots \quad \mu_{2,n}\Psi^T_{\mathbf{I}_2 n,ij} \quad \mu_3\Psi^T_{\mathbf{I}_3 ij}\right] \\ \mathcal{W}_{\mathbf{II}ij} &\triangleq \left[\mu_{1,1}\Psi^T_{\mathbf{II}_1 1,ij} \quad \mu_{1,2}\Psi^T_{\mathbf{II}_1 2,ij} \quad \cdots \quad \mu_{1,n}\Psi^T_{\mathbf{II}_1 n,ij} \right. \\ &\quad \left. \mu_{2,1}\Psi^T_{\mathbf{II}_2 1,ij} \quad \mu_{2,2}\Psi^T_{\mathbf{II}_2 2,ij} \quad \cdots \quad \mu_{2,n}\Psi^T_{\mathbf{II}_2 n,ij} \quad \mu_3\Psi^T_{\mathbf{II}_3 ij}\right] \end{aligned}$$

with

$$\Psi_{\mathbf{I}_1 q,ij} \triangleq \begin{bmatrix} \mathcal{A}_{ij} - n\mathcal{H}_{\mathbf{I}q,ij} & \frac{1}{2}\mathcal{A}_{di} & \frac{1}{2}\mathcal{A}_{di} & \frac{d_{12}}{2}\mathcal{A}_{di} & \mathfrak{E}_{ij} \end{bmatrix}$$
$$\Psi_{\mathbf{I}_2 q,ij} \triangleq \begin{bmatrix} \mathcal{A}_{ij} - n\mathcal{H}_{\mathbf{II}q,ij} & \frac{1}{2}\mathcal{A}_{di} & \frac{1}{2}\mathcal{A}_{di} & \frac{d_{12}}{2}\mathcal{A}_{di} & \mathfrak{E}_{ij} \end{bmatrix}$$
$$\Psi_{\mathbf{I}_3 ij} \triangleq \begin{bmatrix} \mathcal{A}_{ij} & \frac{1}{2}\mathcal{A}_{di} & \frac{1}{2}\mathcal{A}_{di} & \frac{d_{12}}{2}\mathcal{A}_{di} & \mathfrak{E}_{ij} \end{bmatrix}$$
$$\Psi_{\mathbf{II}_1 q,ij} \triangleq \begin{bmatrix} \mathcal{A}_{ij} - n\mathcal{H}_{\mathbf{I}q,ij} - I_{2nn_x} & \frac{1}{2}\mathcal{A}_{di} & \frac{1}{2}\mathcal{A}_{di} & \frac{d_{12}}{2}\mathcal{A}_{di} & \mathfrak{E}_{ij} \end{bmatrix}$$
$$\Psi_{\mathbf{II}_2 q,ij} \triangleq \begin{bmatrix} \mathcal{A}_{ij} - n\mathcal{H}_{\mathbf{II}q,ij} - I_{2nn_x} & \frac{1}{2}\mathcal{A}_{di} & \frac{1}{2}\mathcal{A}_{di} & \frac{d_{12}}{2}\mathcal{A}_{di} & \mathfrak{E}_{ij} \end{bmatrix}$$
$$\Psi_{\mathbf{II}_3 ij} \triangleq \begin{bmatrix} \mathcal{A}_{ij} - I_{2nn_x} & \frac{1}{2}\mathcal{A}_{di} & \frac{1}{2}\mathcal{A}_{di} & \frac{d_{12}}{2}\mathcal{A}_{di} & \mathfrak{E}_{ij} \end{bmatrix}$$
$$\Psi_{\mathbf{III}ij} \triangleq \begin{bmatrix} \mathcal{C}_{ij} & \mathbf{0}_{nn_z \times (6nn_x + n_w)} \end{bmatrix}$$

and

$$\mathcal{A}_{ij} \triangleq \begin{bmatrix} \bar{A}_i & \mathbf{0}_{nn_x} \\ \bar{A}_i - \bar{K}_j & \bar{K}_j \end{bmatrix}, \ \mathcal{A}_{di} \triangleq \begin{bmatrix} \bar{A}_{di} & \mathbf{0}_{nn_x} \\ \bar{A}_{di} & \mathbf{0}_{nn_x} \end{bmatrix}$$
$$\mathfrak{E}_{ij} \triangleq \begin{bmatrix} \bar{E}_i \\ \bar{E}_i - \bar{H}_j \bar{F}_i \end{bmatrix}, \ \mathcal{H}_{\mathbf{I}q,ij} \triangleq \begin{bmatrix} \mathbf{0}_{nn_x} & \mathbf{0}_{nn_x} \\ \bar{H}_j \mathcal{I}_q \bar{B}_i & \mathbf{0}_{nn_x} \end{bmatrix}$$
$$\mathcal{C}_{ij} \triangleq \begin{bmatrix} \bar{C}_i^T - \bar{L}_j^T \\ \bar{L}_j^T \end{bmatrix}^T, \ \mathcal{H}_{\mathbf{II}q,ij} \triangleq \begin{bmatrix} \mathbf{0}_{nn_x} & \mathbf{0}_{nn_x} \\ \bar{H}_j \mathcal{I}_q \bar{D}_i & \mathbf{0}_{nn_x} \end{bmatrix}$$
$$\mu_{1q} \triangleq \sqrt{\frac{\bar{\alpha}_q + \eta_{q1}}{n}}, \ \mu_{2q} \triangleq \sqrt{\frac{(\bar{\beta}_q + \delta_{q1})(1 - \bar{\alpha}_q + \eta_{q2})}{n}}$$
$$\mu_3 \triangleq \sqrt{\frac{\sum_{q=1}^{n}[1 - \bar{\alpha}_q + \eta_{q2} - (\bar{\beta}_q - \delta_{q2})(1 - \bar{\alpha}_q - \eta_{q1})]}{n}}.$$

Proof: See Appendix. ■

C. Distributed H_∞ Filter Design

In this subsection, sufficient conditions on the existence of the desired distributed H_∞ fuzzy filters in the form of (9) will be provided ensuring that the closed-loop system in (24) is stochastically stable with a guaranteed average H_∞ performance.

Theorem 2: Consider the discrete-time DFFES in (24) with uncertain packet dropout rates. Given integers d_1, d_2 with $d_2 \geq d_1 \geq 1$ and constants $\gamma, \bar{\alpha}_q, \bar{\beta}_q > 0$, $\eta_{q1}, \eta_{q2}, \delta_{q1}, \delta_{q2} \geq 0$ ($q \in \mathcal{N}_p$), the DFFES is stochastically stable with an average H_∞ performance γ, if $\forall p = 1, 2, \ldots, n$, $q \in \mathcal{N}_p$, there exist positive definite matrices $P_i, Q_{1i}, Q_{2i}, \tilde{R}_1, \tilde{R}_2, \tilde{R}_3$, $i \in \mathbb{S}$, matrices $\bar{T}_{1pqj}, \bar{T}_{2pqj}, S_{pj}, X_p, Z_p$, $j \in \mathbb{S}$, and some positive scalars $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$, satisfying

$$\tilde{\mathcal{G}}_{oltii} < 0, (o, l, t, i \in \mathbb{S})$$
$$\tilde{\mathcal{G}}_{oltij} + \tilde{\mathcal{G}}_{oltji} < 0, (o, l, t, i, j \in \mathbb{S}, i < j) \tag{26}$$

where

$$\tilde{\mathcal{G}}_{oltij} \triangleq \begin{bmatrix} \tilde{\Lambda}_{lti} & \tilde{\Omega}_{ij} \\ * & \tilde{\Xi}_o \end{bmatrix}$$
$$\tilde{\Lambda}_{lti} \triangleq \begin{bmatrix} \tilde{\Lambda}_{\mathbf{I}i} & \begin{bmatrix} \varepsilon_1^{-1}\tilde{R}_1 & \varepsilon_2^{-1}\tilde{R}_2 & \mathbf{0}_{2nn_x} & \mathbf{0}_{2nn_x \times n_w} \end{bmatrix} \\ * & \mathrm{diag}\left\{ \tilde{\Lambda}_{\mathbf{II}l}, \tilde{\Lambda}_{\mathbf{III}t}, -\tilde{R}_3, -n\gamma^2 I_{n_w} \right\} \end{bmatrix}$$
$$\tilde{\Omega}_{ij} \triangleq \begin{bmatrix} \tilde{\Psi}_{\mathbf{III}ij}^T & d_1 \tilde{\mathcal{W}}_{\mathbf{II}ij} & \varepsilon_4 d_2 \tilde{\mathcal{W}}_{\mathbf{II}ij} & \tilde{\mathcal{W}}_{\mathbf{II}ij} & \tilde{\mathcal{W}}_{\mathbf{I}ij} \end{bmatrix}$$
$$\tilde{\Xi}_o \triangleq \mathrm{diag}\left\{ I_{nn_z} - sym(\mathcal{Z}), \tilde{\mathcal{R}}_1, \tilde{\mathcal{R}}_2, \tilde{\mathcal{R}}_3, \tilde{\mathcal{P}}_o \right\}$$
$$\tilde{\Lambda}_{\mathbf{I}i} \triangleq -P_i - \varepsilon_1^{-2}\tilde{R}_1 - \varepsilon_2^{-2}\tilde{R}_2 + Q_{1i} + Q_{2i}$$
$$\tilde{\Lambda}_{\mathbf{II}l} \triangleq -\tilde{R}_1 - \varepsilon_1^2 Q_{1l}, \ \tilde{\Lambda}_{\mathbf{III}t} \triangleq -\tilde{R}_2 - \varepsilon_2^2 Q_{2t}$$
$$\tilde{\mathcal{R}}_1 \triangleq I_{2n+1} \otimes (\tilde{R}_1 - sym(\varepsilon_1 \mathcal{X}))$$
$$\tilde{\mathcal{R}}_2 \triangleq I_{2n+1} \otimes (\tilde{R}_2 - sym(\varepsilon_2 \varepsilon_4 \mathcal{X}))$$
$$\tilde{\mathcal{R}}_3 \triangleq I_{2n+1} \otimes (\tilde{R}_3 - sym(\varepsilon_3 \mathcal{X}))$$
$$\tilde{P}_o \triangleq I_{2n+1} \otimes (P_o - sym(\mathcal{X}))$$
$$\tilde{\mathcal{W}}_{\mathbf{I}ij} \triangleq [\, \mu_{1,1}\tilde{\Psi}_{\mathbf{I}_1 1,ij}^T \ \mu_{1,2}\tilde{\Psi}_{\mathbf{I}_1 2,ij}^T \ \cdots \ \mu_{1,n}\tilde{\Psi}_{\mathbf{I}_1 n,ij}^T$$
$$\mu_{2,1}\tilde{\Psi}_{\mathbf{I}_2 1,ij}^T \ \mu_{2,2}\tilde{\Psi}_{\mathbf{I}_2 2,ij}^T \ \cdots \ \mu_{2,n}\tilde{\Psi}_{\mathbf{I}_2 n,ij}^T \ \mu_3 \tilde{\Psi}_{\mathbf{I}_3 ij}^T \,]$$
$$\tilde{\mathcal{W}}_{\mathbf{II}ij} \triangleq [\, \mu_{1,1}\tilde{\Psi}_{\mathbf{II}_1 1,ij}^T \ \mu_{1,2}\tilde{\Psi}_{\mathbf{II}_1 2,ij}^T \ \cdots \ \mu_{1,n}\tilde{\Psi}_{\mathbf{II}_1 n,ij}^T$$
$$\mu_{2,1}\tilde{\Psi}_{\mathbf{II}_2 1,ij}^T \ \mu_{2,2}\tilde{\Psi}_{\mathbf{II}_2 2,ij}^T \ \cdots \ \mu_{2,n}\tilde{\Psi}_{\mathbf{II}_2 n,ij}^T \ \mu_3 \tilde{\Psi}_{\mathbf{II}_3 ij}^T \,]$$

with

$$\tilde{\Psi}_{\mathbf{I}_1 q,ij} \triangleq \big[\mathcal{X}\mathcal{A}_{0i} + \mathbf{I}\mathcal{T}_j \mathfrak{I}_1 - n\mathbf{I}\mathcal{T}_j \mathcal{B}_{qi} \quad \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \quad \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j \mathcal{F}_i \big]$$
$$\tilde{\Psi}_{\mathbf{I}_2 q,ij} \triangleq \big[\mathcal{X}\mathcal{A}_{0i} + \mathbf{I}\mathcal{T}_j \mathfrak{I}_1 - n\mathbf{I}\mathcal{T}_j \mathcal{D}_{qi} \quad \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \quad \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j \mathcal{F}_i \big]$$
$$\tilde{\Psi}_{\mathbf{I}_3 ij} \triangleq \big[\mathcal{X}\mathcal{A}_{0i} + \mathbf{I}\mathcal{T}_j \mathfrak{I}_1 \quad \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \quad \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j \mathcal{F}_i \big]$$
$$\tilde{\Psi}_{\mathbf{II}_1 q,ij} \triangleq \big[\mathcal{X}\mathcal{A}_{0i} + \mathbf{I}\mathcal{T}_j \mathfrak{I}_1 - n\mathbf{I}\mathcal{T}_j \mathcal{B}_{qi} - \mathcal{X} \quad \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \quad \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j \mathcal{F}_i \big]$$
$$\tilde{\Psi}_{\mathbf{II}_2 q,ij} \triangleq \big[\mathcal{X}\mathcal{A}_{0i} + \mathbf{I}\mathcal{T}_j \mathfrak{I}_1 - n\mathbf{I}\mathcal{T}_j \mathcal{D}_{qi} - \mathcal{X} \quad \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \quad \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j \mathcal{F}_i \big]$$
$$\tilde{\Psi}_{\mathbf{II}_3 ij} \triangleq \big[\mathcal{X}\mathcal{A}_{0i} + \mathbf{I}\mathcal{T}_j \mathfrak{I}_1 - \mathcal{X} \quad \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \quad \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \quad \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j \mathcal{F}_i \big]$$
$$\tilde{\Psi}_{\mathbf{III}ij} \triangleq \big[\mathcal{Z}\mathcal{C}_{0i} + \mathcal{S}_j \mathfrak{I}_2 \quad \mathbf{0}_{nn_z \times (6nn_x + n_w)} \big]$$

and

$$\mathcal{A}_{0i} \triangleq \begin{bmatrix} \bar{A}_i & \mathbf{0}_{nn_x} \\ \bar{A}_i & \mathbf{0}_{nn_x} \end{bmatrix}, \ \mathcal{B}_{qi} \triangleq \begin{bmatrix} \mathbf{0}_{nn_x} & \mathbf{0}_{nn_x} \\ \mathcal{I}_q \bar{B}_i & \mathbf{0}_{nn_y \times nn_x} \end{bmatrix}, \ \mathbf{I} \triangleq \begin{bmatrix} \mathbf{0}_{nn_x} \\ I_{nn_x} \end{bmatrix}$$
$$\mathcal{C}_{0i} \triangleq \begin{bmatrix} \bar{C}_i^T \\ \mathbf{0}_{nn_x \times nn_z} \end{bmatrix}^T, \ \mathcal{D}_{qi} \triangleq \begin{bmatrix} \mathbf{0}_{nn_x} & \mathbf{0}_{nn_x} \\ \mathcal{I}_q \bar{D}_i & \mathbf{0}_{nn_y \times nn_x} \end{bmatrix}, \ \mathfrak{E}_{0i} \triangleq \begin{bmatrix} \bar{E}_i \\ \bar{E}_i \end{bmatrix}$$
$$\mathcal{F}_i \triangleq \begin{bmatrix} \mathbf{0}_{nn_x \times n_w} \\ -\bar{F}_i \end{bmatrix}, \ \mathfrak{I}_1 \triangleq \begin{bmatrix} -I_{nn_x} & I_{nn_x} \\ \mathbf{0}_{nn_y \times nn_x} & \mathbf{0}_{nn_y \times nn_x} \end{bmatrix}$$
$$\mathfrak{I}_2 \triangleq \begin{bmatrix} -I_{nn_x} & I_{nn_x} \end{bmatrix}, \ \mathcal{X} \triangleq \mathrm{diag}\{\bar{X}, \bar{X}\}, \ \bar{X} \triangleq \mathrm{diag}_n\{X_p\}_p$$
$$\mathcal{S}_j \triangleq \mathrm{diag}_n\{S_{pj}\}_p, \ \mathcal{Z} \triangleq \mathrm{diag}_n\{Z_p\}_p$$
$$\mathcal{T}_j \triangleq \begin{bmatrix} \bar{T}_{1j} & \bar{T}_{2j} \end{bmatrix}, \ \bar{T}_{1j} \triangleq \begin{bmatrix} \bar{T}_{1pqj} \end{bmatrix}_{n \times n}^{p \times q}, \ \bar{T}_{2j} \triangleq \begin{bmatrix} \bar{T}_{2pqj} \end{bmatrix}_{n \times n}^{p \times q}.$$

Furthermore, if the inequalities in (26) are feasible, the matrices $\bar{K}_j$, $\bar{H}_j$, and $\bar{L}_j$ are given as

$$\bar{K}_j = \bar{X}^{-1}\bar{T}_{1j}, \ \bar{H}_j = \bar{X}^{-1}\bar{T}_{2j}, \ \bar{L}_j = \mathcal{Z}^{-1}\mathcal{S}_j. \tag{27}$$

Then, combining with (13) and (14), the desired filter parameters K_{pqj}, H_{pqj}, and L_{pj} can be obtained.

Proof: See Appendix. ■

Remark 4: Based on Theorem 2, the average H_∞ performance can be optimized for the underlying distributed filtering problem via two channels. A noteworthy fact is that when we set the matrix $D_{pi} = \mathbf{0}_{n_y \times n_x}$ in (4), the conditions in Theorem 2 reduce to those corresponding to the single channel case. As expected, the obtained optimized γ in the presence of one more redundant channel, denoted as $\gamma_{\min}$, should be less than the one in single channel, which will be verified in next section.

IV. Illustrative example

Let us consider the following nonlinear system with time-varying delays [20] :

$$\begin{cases} x_1(k+1) = -\left[\mathcal{R}x_1(k) + (1-\mathcal{R})x_1(k-d(k))\right]^2 \\ \qquad\qquad + 0.3x_2(k) + w(k) \\ x_2(k+1) = \mathcal{R}x_1(k) + (1-\mathcal{R})x_1(k-d(k)) \end{cases}$$

where the constant $\mathcal{R} \in [0,1]$ is the retarded coefficient.

In the example, it is assumed that the SN is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{L})$ with the set of nodes $\mathcal{V} = \{1,2,3\}$, the set of edges $\mathcal{E} = \{(1,1),(1,3),(2,2),(2,3),(3,1),(3,3)\}$, and the adjacency matrix $\mathcal{L}$ is given as

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

For each p $(p = 1,2,3)$, the pth sensor node is described in the form of (7).

Let $\boldsymbol{\theta}(k) = \mathcal{R}x_1(k) + (1-\mathcal{R})x_1(k-d(k))$. It is assumed that $\boldsymbol{\theta}(k) \in (-\mathcal{M}, \mathcal{M})$ where $\mathcal{M} > 0$. We set the fuzzy basis functions as

$$h_1(\boldsymbol{\theta}(k)) = \frac{1}{2}\left(1 - \frac{\boldsymbol{\theta}(k)}{\mathcal{M}}\right), \; h_2(\boldsymbol{\theta}(k)) = \frac{1}{2}\left(1 + \frac{\boldsymbol{\theta}(k)}{\mathcal{M}}\right) \quad (28)$$

where $h_1((\boldsymbol{\theta}(k)), h_2(\boldsymbol{\theta}(k)) \in [0,1]$, and $h_1(\boldsymbol{\theta}(k)) + h_2(\boldsymbol{\theta}(k)) = 1$.

From (28), it can be seen that $h_1(\boldsymbol{\theta}(k)) = 0, h_2(\boldsymbol{\theta}(k)) = 1$ when $\boldsymbol{\theta}(k)$ is about $\mathcal{M}$, and $h_1(\boldsymbol{\theta}(k)) = 1, h_2(\boldsymbol{\theta}(k)) = 0$ when $\boldsymbol{\theta}(k)$ is about $-\mathcal{M}$. Then, the underlying fuzzy model is as follows.

Plant rule 1: IF $\boldsymbol{\theta}(k)$ is about $-\mathcal{M}$, THEN

$$\begin{cases} x(k+1) = A_1x(k) + A_{d1}x(k-d(k)) + E_1w(k) \\ y_p(k) = \alpha_p(k)B_{p1}x(k) + (1-\alpha_p(k))\beta_p(k)D_{p1}x(k) \\ \qquad\quad + F_{p1}w(k) \\ z(k) = C_1x(k) \end{cases}$$

Plant rule 2: IF $\boldsymbol{\theta}(k)$ is about $\mathcal{M}$, THEN

$$\begin{cases} x(k+1) = A_2x(k) + A_{d2}x(k-d(k)) + E_2w(k) \\ y_p(k) = \alpha_p(k)B_{p2}x(k) + (1-\alpha_p(k))\beta_p(k)D_{p2}x(k) \\ \qquad\quad + F_{p2}w(k) \\ z(k) = C_2x(k) \end{cases}$$

where

$$A_1 = \begin{bmatrix} \mathcal{R}\mathcal{M} & 0.3 \\ \mathcal{R} & 0 \end{bmatrix}, \; A_{d1} = \begin{bmatrix} (1-\mathcal{R})\mathcal{M} & 0.3 \\ 1-\mathcal{R} & 0 \end{bmatrix}, \; E_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\mathcal{R}\mathcal{M} & 0.3 \\ \mathcal{R} & 0 \end{bmatrix}, \; A_{d2} = \begin{bmatrix} -(1-\mathcal{R})\mathcal{M} & 0.3 \\ 1-\mathcal{R} & 0 \end{bmatrix}$$

$$B_{1i} = \begin{bmatrix} 0.2 & 0.18 \end{bmatrix}, \; D_{1i} = \begin{bmatrix} 0.08 & 0.12 \end{bmatrix}, \; F_{1i} = 0.2$$

$$B_{2i} = \begin{bmatrix} 0.16 & 0.2 \end{bmatrix}, \; D_{2i} = \begin{bmatrix} 0.12 & 0.14 \end{bmatrix}, \; F_{2i} = 0.17$$

$$B_{3i} = \begin{bmatrix} 0.14 & 0.16 \end{bmatrix}, \; D_{3i} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \; F_{3i} = 0.15$$

$$C_i = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; \forall i = 1,2.$$

We suppose that the time-varying state delay satisfies $1 \leq d(k) \leq 3$. The values of the parameters $\mathcal{R}$ and $\mathcal{M}$ are given by $\mathcal{R} = 0.8$ and $\mathcal{M} = 0.1$, respectively. The nominal expectations of packet arrivals of the communication channels are taken as $\bar{\alpha}_1 = 0.9$, $\bar{\alpha}_2 = 0.8$, $\bar{\alpha}_3 = 0.85$, $\bar{\beta}_1 = 0.65$, $\bar{\beta}_2 = 0.6$, and $\bar{\beta}_3 = 0.7$, respectively. Without loss of generality, we suppose $\eta_{p2} = 2\eta_{p1}$ and $\delta_{p2} = 2\delta_{p1}$ $(p = 1,2,3)$. A comparison between minimum average H_∞ performance $\gamma_{\min}$ obtained based on different cases is given in Table I with the choice of $\varepsilon_1 = 7$, $\varepsilon_2 = 25$, $\varepsilon_3 = 8$ and $\varepsilon_4 = 0.08$. The parameters of the desired distributed filters for case I can be calculated with the optimized performance index as follows:

TABLE I
Minimum Values of Average H_∞ Performance $\gamma_{\min}$ for Different Cases.

Cases	$\gamma_{\min}$
I) Two channels ($\eta_{p2} = 0.02, \delta_{p2} = 0.03$)	3.6594
II) Two channels ($\eta_{p2} = \delta_{p2} = 0$)	2.8002
III) Single channel ($\eta_{p2} = 0.02, D_p = [0 \;\; 0]$)	3.7799
IV) Single channel ($\eta_{p2} = 0, D_p = [0 \;\; 0]$)	2.8576

$$K_{111} = \begin{bmatrix} 0.3523 & 0.2876 \\ 0.4173 & -0.1474 \end{bmatrix}, \; K_{131} = \begin{bmatrix} -0.1032 & -0.1007 \\ -0.1155 & -0.0329 \end{bmatrix}$$

$$K_{221} = \begin{bmatrix} 0.2701 & 0.3155 \\ 0.5257 & -0.2232 \end{bmatrix}, \; K_{231} = \begin{bmatrix} 0.0315 & -0.0337 \\ 0.0394 & 0.0326 \end{bmatrix}$$

$$K_{311} = \begin{bmatrix} 0.0272 & 0.0456 \\ -0.0851 & -0.1745 \end{bmatrix}, \; K_{331} = \begin{bmatrix} 0.2159 & 0.2497 \\ 0.6110 & -0.0282 \end{bmatrix}$$

$$K_{112} = \begin{bmatrix} -0.1029 & 0.2531 \\ 1.3240 & 0.2909 \end{bmatrix}, \; K_{132} = \begin{bmatrix} -0.0331 & -0.0369 \\ -0.0181 & 0.0724 \end{bmatrix}$$

$$K_{222} = \begin{bmatrix} -0.0624 & 0.2830 \\ 0.9946 & 0.1175 \end{bmatrix}, \; K_{232} = \begin{bmatrix} -0.1172 & -0.0784 \\ 0.2074 & 0.1567 \end{bmatrix}$$

$$K_{312} = \begin{bmatrix} -0.1356 & -0.0566 \\ 0.1417 & 0.1431 \end{bmatrix}, \; K_{332} = \begin{bmatrix} -0.0563 & 0.2608 \\ 1.2944 & 0.2560 \end{bmatrix}$$

$$H_{111} = \begin{bmatrix} 0.8502 \\ 1.1495 \end{bmatrix}, \; H_{131} = \begin{bmatrix} 0.9040 \\ -0.0345 \end{bmatrix}, \; H_{221} = \begin{bmatrix} 0.9173 \\ 0.8051 \end{bmatrix}$$

$$H_{231} = \begin{bmatrix} 0.5748 \\ 0.2013 \end{bmatrix}, \; H_{311} = \begin{bmatrix} 0.3877 \\ 0.6801 \end{bmatrix}, \; H_{331} = \begin{bmatrix} 0.8079 \\ -0.1022 \end{bmatrix}$$

$$H_{112} = \begin{bmatrix} 1.4821 \\ -2.4257 \end{bmatrix}, \; H_{132} = \begin{bmatrix} 0.6354 \\ -0.7688 \end{bmatrix}, \; H_{222} = \begin{bmatrix} 0.9696 \\ -1.6934 \end{bmatrix}$$

$$H_{232} = \begin{bmatrix} 0.5413 \\ -0.8294 \end{bmatrix}, \; H_{312} = \begin{bmatrix} 0.5862 \\ -0.3957 \end{bmatrix}, \; H_{332} = \begin{bmatrix} 1.2317 \\ -2.1529 \end{bmatrix}$$

$$L_{11} = \begin{bmatrix} 0.7724 & -0.0448 \end{bmatrix}, \; L_{21} = \begin{bmatrix} 0.6927 & 0.0173 \end{bmatrix}$$

$$L_{31} = \begin{bmatrix} 0.7100 & -0.0208 \end{bmatrix}, \; L_{12} = \begin{bmatrix} 0.7526 & -0.1087 \end{bmatrix}$$

$$L_{22} = \begin{bmatrix} 0.8860 & -0.0496 \end{bmatrix}, \; L_{32} = \begin{bmatrix} 0.6971 & -0.1549 \end{bmatrix}$$

From Table I, we can observe that $\gamma_{\min}$ increases as the uncertainties of the packet dropout rates occur, and the average H_∞ performance of the system becomes better if two channels are adopted at each node.

In order to test the performance of the designed filters, we assume zero-initial conditions and $\hat{x}_p(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ $(p = 1,2,3)$ with external disturbance $w(k) = e^{-0.2k}\cos(k)$. The output $z(k)$ and its estimates from the filter p $(p = 1,2,3)$ for case I are shown in Fig. 2, which shows the effectiveness of the designed distributed filters for the system against the time-varying delays and the uncertain packet dropout rates. The resulting disturbance attenuation ratios $\frac{1}{\sqrt{3}}\sqrt{\sum_{i=0}^{k} \bar{z}(i)^T\bar{z}(i)} / \sqrt{\sum_{i=0}^{k} w(i)^T w(i)}$ can be computed

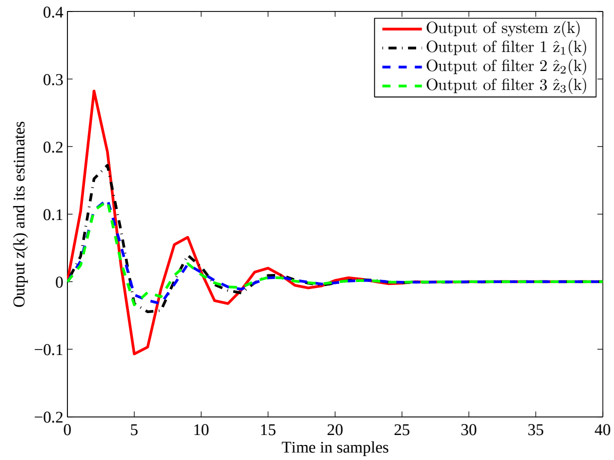

Fig. 2. Output $z(k)$ and its estimates for sensor networks with redundant channels subject to uncertain packet dropout rates.

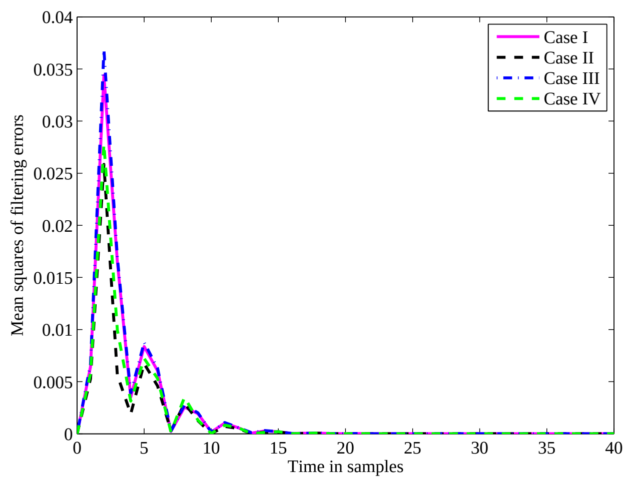

Fig. 3. Mean squares of filtering errors of three sensor nodes of different cases.

as $\gamma = 0.6456$ for case I, $\gamma' = 0.5907$ for case II, $\tilde{\gamma} = 0.6679$ for case III and $\tilde{\gamma}' = 0.6013$ for case IV, which are all less than the corresponding $\gamma_{\min}$. It can be seen that $\gamma > \gamma'$, which further shows the necessity of considering the uncertainties of packet dropout rates in the design phase. It can also be seen that $\tilde{\gamma} > \gamma$ and $\tilde{\gamma}' > \gamma'$, which also illustrates the performance benefiting from redundant channels. The mean squares of filtering errors of three sensor nodes $\frac{1}{3}\sum_{p=1}^{3}\tilde{z}_p^2(k)$ for different cases are shown in Fig. 3.

V. Conclusion

In this paper, the distributed H_∞ filtering problem has been investigated for a class of discrete-time DFFESs with time-varying delays and redundant channels subject to uncertain norm-bounded packet dropout rates. Sufficient conditions for the existence of the desired distributed filters, in a basic two-channel case, are established by employing the scaled small gain theorem to ensure that the closed-loop system is stochastically stable and achieves a prescribed average H_∞ performance index. The necessity of allowing for the uncertain packet dropout rates was shown, and it was also demonstrated that the achieved average H_∞ performance of the distributed filtering can be improved by including redundant channels. One future work will be to consider real Henon mapping systems with more complicated dynamics rather than the simplified one in the section of Illustrative Example.

Appendix

Proof of Theorem 1: We choose the following fuzzy-basis-dependent Lyapunov–Krasovskii functional

$$V(\mathcal{E}(k),k) \triangleq \sum_{m=1}^{3} V_m(\mathcal{E}(k),k) \tag{29}$$

where

$$\begin{cases} V_1(\mathcal{E}(k),k) \triangleq \mathcal{E}^T(k)\bar{P}(k)\mathcal{E}(k) \\ V_2(\mathcal{E}(k),k) \triangleq \sum_{l=1}^{2}\sum_{u=k-d_l}^{k-1}\mathcal{E}^T(u)\bar{Q}_l(u)\mathcal{E}(u) \\ V_3(\mathcal{E}(k),k) \triangleq \sum_{l=1}^{2}\sum_{u=-d_l}^{-1}\sum_{v=k+u}^{k-1} d_l\vartheta^T(v)R_l\vartheta(v) \end{cases} \tag{30}$$

with

$$\bar{P}(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))P_i, \;\; \bar{Q}_1(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))Q_{1i}$$
$$\bar{Q}_2(k) \triangleq \sum_{i=1}^{s} h_i(\boldsymbol{\theta}(k))Q_{2i}.$$

Moreover, we define

$$J \triangleq E\left\{\sum_{k=0}^{\infty}[\vartheta^T(k)R_3\vartheta(k) - \nu^T(k)R_3\nu(k)]\right\} \tag{31}$$

and

$$J_{cl} \triangleq E\left\{\sum_{k=0}^{\infty}[\bar{z}^T(k)\bar{z}(k) - n\gamma^2 w^T(k)w(k)]\right\} \tag{32}$$

where $R_3 = T^T T$.

Then, taking the forward difference of $V(\mathcal{E}(k),k)$, we have

$$\begin{aligned} \Delta V_1 &\triangleq E\left\{V_1(\mathcal{E}(k+1),k+1)\,|\mathcal{E}(k),k\right\} - V_1(\mathcal{E}(k),k) \\ &= \mathcal{E}^T(k+1)\bar{P}(k+1)\mathcal{E}(k+1) - \mathcal{E}^T(k)\bar{P}(k)\mathcal{E}(k) \\ &= E\{\xi^T(k)\Psi_1^T(k)\bar{P}(k+1)\Psi_1(k)\xi(k)\} - \mathcal{E}^T(k)\bar{P}(k)\mathcal{E}(k) \\ &\le E\left\{\frac{1}{n}\sum_{q=1}^{n}\left\{\left(\alpha_q(k)\beta_q(k) - \alpha_q^2(k)\beta_q(k)\right)n^2\mathcal{E}^T(k)\right.\right. \\ &\quad \times[\mathcal{H}_{\mathbf{I}q}^T(k)\bar{P}(k+1)\mathcal{H}_{\mathbf{I}q}(k) + \mathcal{H}_{\mathbf{II}q}^T(k)\bar{P}(k+1)\mathcal{H}_{\mathbf{II}q}(k)] \\ &\quad \times\mathcal{E}(k) + \xi^T(k)\left[\Psi_{1q}^T(k)\bar{P}(k+1)\Psi_{1q}(k) + \Psi_{2q}^T(k)\right. \\ &\quad \left.\left.\left.\times\bar{P}(k+1)\Psi_{2q}(k) - \Psi_{\mathbf{I}_3}^T(k)\bar{P}(k+1)\Psi_{\mathbf{I}_3}(k)\right]\xi(k)\right\}\right\} \\ &\quad - \mathcal{E}^T(k)\bar{P}(k)\mathcal{E}(k) \\ &\le \sum_{q=1}^{n}\left[\mu_{1,q}^2\xi^T(k)\Psi_{\mathbf{I}_1q}^T(k)\bar{P}(k+1)\Psi_{\mathbf{I}_1q}(k)\xi(k)\right. \\ &\quad \left.+\mu_{2,q}^2\xi^T(k)\Psi_{\mathbf{I}_2q}^T(k)\bar{P}(k+1)\Psi_{\mathbf{I}_2q}(k)\xi(k)\right] + \mu_3^2\xi^T(k) \\ &\quad \times\Psi_{\mathbf{I}_3}^T(k)\bar{P}(k+1)\Psi_{\mathbf{I}_3}(k)\xi(k) - \mathcal{E}^T(k)\bar{P}(k)\mathcal{E}(k) \end{aligned} \tag{33}$$

$$\begin{aligned} \Delta V_2 &\triangleq E\left\{V_2(\mathcal{E}(k+1),k+1)\,|\mathcal{E}(k),k\right\} - V_2(\mathcal{E}(k),k) \\ &= \sum_{l=1}^{2}\left[\mathcal{E}^T(k)\bar{Q}_l(k)\mathcal{E}(k)\right. \\ &\quad \left.-\mathcal{E}^T(k-d_l)\bar{Q}_l(k-d_l)\mathcal{E}(k-d_l)\right] \end{aligned} \tag{34}$$

$$\Delta V_3 \triangleq E\left\{V_3\left(\mathcal{E}(k+1),k+1\right)|\mathcal{E}(k),k\right\} - V_3\left(\mathcal{E}(k),k\right) = E\left\{\sum_{l=1}^{2}\left[\sum_{u=-d_l}^{-1} d_l\vartheta^T(k)R_l\vartheta(k) - d_l\sum_{u=-d_l}^{-1}\vartheta^T(k+u)R_l\vartheta(k+u)\right]\right\} \quad (35)$$

where

$$\Psi_1(k) \triangleq \left[\mathcal{A}(k) - \sum_{q=1}^{n}\alpha_q(k)\mathcal{H}_{\mathbf{I}q}(k) - \sum_{q=1}^{n}\beta_q(k) \times \left(1-\alpha_q(k)\right)\mathcal{H}_{\mathbf{II}q}(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\Psi_{1q}(k) \triangleq \left[\mathcal{A}(k) - \alpha_q(k)n\mathcal{H}_{\mathbf{I}q}(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\Psi_{2q}(k) \triangleq \left[\mathcal{A}(k) - \left(\beta_q(k) - \alpha_q(k)\beta_q(k)\right)n\mathcal{H}_{\mathbf{II}q}(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\Psi_{\mathbf{I}_1q}(k) \triangleq \left[\mathcal{A}(k) - n\mathcal{H}_{\mathbf{I}q}(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\Psi_{\mathbf{I}_2q}(k) \triangleq \left[\mathcal{A}(k) - n\mathcal{H}_{\mathbf{II}q}(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\Psi_{\mathbf{I}_3}(k) \triangleq \left[\mathcal{A}(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\xi(k) \triangleq \left[\mathcal{E}^T(k) \ \ \mathcal{E}^T(k-d_1) \ \ \mathcal{E}^T(k-d_2) \ \ v^T(k) \ \ w^T(k)\right]^T.$$

According to the Jensen inequality, $\forall l = 1,2$, one has

$$-d_l\sum_{u=k-d_l}^{k-1}\vartheta^T(u)R_l\vartheta(u) \le -\left[\mathcal{E}(k)-\mathcal{E}(k-d_l)\right]^T R_l\left[\mathcal{E}(k)-\mathcal{E}(k-d_l)\right] \quad (36)$$

From (35) and (36), we have

$$\begin{aligned}\Delta V_3 &= \sum_{l=1}^{2}\Big\{E\{d_l^2\vartheta^T(k)R_l\vartheta(k)\} - \left[\mathcal{E}(k)-\mathcal{E}(k-d_l)\right]^T R_l\left[\mathcal{E}(k)-\mathcal{E}(k-d_l)\right]\Big\}\\ &\le \sum_{l=1}^{2}\Big\{d_l^2\xi^T(k)\left\{\mu_1^2\Psi_{\mathbf{II}_1q}^T(k)R_l\Psi_{\mathbf{II}_1q}(k) + \mu_2^2 \times\Psi_{\mathbf{II}_2q}^T(k)R_l\Psi_{\mathbf{II}_2q}(k) + \mu_3^2\Psi_{\mathbf{II}_3}^T(k)R_l\Psi_{\mathbf{II}_3}(k)\right\}\xi(k)\\ &\quad - \left[\mathcal{E}(k)-\mathcal{E}(k-d_l)\right]^T R_l\left[\mathcal{E}(k)-\mathcal{E}(k-d_l)\right]\Big\}\end{aligned} \quad (37)$$

where

$$\Psi_{\mathbf{II}_1q}(k) \triangleq \left[\mathcal{A}(k) - n\mathcal{H}_{\mathbf{I}q}(k) - I_{2nn_x} \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\Psi_{\mathbf{II}_2q}(k) \triangleq \left[\mathcal{A}(k) - n\mathcal{H}_{\mathbf{II}q}(k) - I_{2nn_x} \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right]$$
$$\Psi_{\mathbf{II}_3}(k) \triangleq \left[\mathcal{A}(k) - I_{2nn_x} \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{1}{2}\mathcal{A}_d(k) \quad \tfrac{d_{12}}{2}\mathcal{A}_d(k) \quad \mathfrak{E}(k)\right].$$

Similarly, we can obtain

$$E\left\{\vartheta^T(k)R_3\vartheta(k)|\mathcal{E}(k),k\right\} \le \xi^T(k)\left\{\mu_1^2\Psi_{\mathbf{II}_1q}^T(k)R_3\Psi_{\mathbf{II}_1q}(k) + \mu_2^2\Psi_{\mathbf{II}_2q}^T(k)R_3\Psi_{\mathbf{II}_2q}(k) + \mu_3^2\Psi_{\mathbf{II}_3}^T(k)R_3\Psi_{\mathbf{II}_3}(k)\right\}\xi(k) \quad (38)$$

and

$$E\left\{\bar{z}^T(k)\bar{z}(k)|\xi(k),k\right\} = \xi^T(k)\Psi_{\mathbf{III}}^T(k)\Psi_{\mathbf{III}}(k)\xi(k) \quad (39)$$

where $\Psi_{\mathbf{III}}(k) \triangleq \left[\mathcal{C}(k) \quad \mathbf{0}_{nn_z\times(6nn_x+n_w)}\right]$.

Then, by performing a congruence transformation $\text{diag}\{I_{2nn_x}, I_{2nn_x}, I_{2nn_x}, I_{2nn_x}, I_{n_w}, -\Xi_o^{-1}\}$ to the inequalities in (25), we obtain the following inequalities:

$$\check{\mathcal{G}}_{oltii} < 0, (o,l,t,i \in \mathbb{S})$$
$$\check{\mathcal{G}}_{oltij} + \check{\mathcal{G}}_{oltji} < 0, (o,l,t,i,j \in \mathbb{S}, i<j) \quad (40)$$

where

$$\check{\mathcal{G}}_{oltij} \triangleq \begin{bmatrix}\Lambda_{lti} & \check{\Omega}_{ij}\\ * & \Xi_o^{-1}\end{bmatrix}$$
$$\check{\Omega}_{ij} \triangleq \left[\Psi_{\mathbf{III}ij}^T \quad d_1\mathcal{W}_{\mathbf{II}ij} \quad d_2\mathcal{W}_{\mathbf{II}ij} \quad \mathcal{W}_{\mathbf{II}ij} \quad \mathcal{W}_{\mathbf{I}ij}\right].$$

Moreover, we define

$$\mathcal{G}(k) \triangleq \sum_{o=1}^{s}\sum_{l=1}^{s}\sum_{t=1}^{s}h_o(\boldsymbol{\theta}(k+1))h_l(\boldsymbol{\theta}(k-d_1))h_t(\boldsymbol{\theta}(k-d_2)) \times\left\{\sum_{i=1}^{s}h_i(\boldsymbol{\theta}(k))\check{\mathcal{G}}_{oltii} + \sum_{i=1}^{s-1}\sum_{j=i+1}^{s}h_i(\boldsymbol{\theta}(k)) \times h_j(\boldsymbol{\theta}(k))\left(\check{\mathcal{G}}_{oltij}+\check{\mathcal{G}}_{oltji}\right)\right\}. \quad (41)$$

Then, combining (15), (17), (25), (40) and (41), and the definitions of $\bar{P}(k), \bar{Q}_1(k), \bar{Q}_2(k), R_1, R_2, R_3$, we have

$$\mathcal{G}(k) < 0 \quad (42)$$

where

$$\mathcal{G}(k) \triangleq \begin{bmatrix}\Lambda(k) & \Omega(k)\\ * & \Xi^{-1}(k)\end{bmatrix}$$
$$\Lambda(k) \triangleq \begin{bmatrix}\Lambda_{\mathbf{I}}(k) & \left[R_1 \quad R_2 \quad \mathbf{0}_{2nn_x} \quad \mathbf{0}_{2nn_x\times n_w}\right]\\ * & \text{diag}\left\{\Lambda_{\mathbf{II}}(k), \Lambda_{\mathbf{III}}(k), -R_3, -n\gamma^2 I_{n_w}\right\}\end{bmatrix}$$
$$\Lambda_{\mathbf{I}}(k) \triangleq -\bar{P}(k) - R_1 - R_2 + \bar{Q}_1(k) + \bar{Q}_2(k)$$
$$\Lambda_{\mathbf{II}}(k) \triangleq -R_1 - \bar{Q}_1(k-d_1), \ \Lambda_{\mathbf{III}}(k) \triangleq -R_2 - \bar{Q}_2(k-d_2)$$
$$\Omega(k) \triangleq \left[\Psi_{\mathbf{III}}^T(k) \quad d_1\mathcal{W}_{\mathbf{II}}(k) \quad d_2\mathcal{W}_{\mathbf{II}}(k) \quad \mathcal{W}_{\mathbf{II}}(k) \quad \mathcal{W}_{\mathbf{I}}(k)\right]$$
$$\mathcal{W}_{\mathbf{I}}(k) \triangleq \left[\mu_{1,1}\Psi_{\mathbf{I}_11}^T(k) \quad \mu_{1,2}\Psi_{\mathbf{I}_12}^T(k) \quad \cdots \quad \mu_{1,n}\Psi_{\mathbf{I}_1n}^T(k) \quad \mu_{2,1}\Psi_{\mathbf{I}_21}^T(k) \quad \mu_{2,2}\Psi_{\mathbf{I}_22}^T(k) \quad \cdots \quad \mu_{2,n}\Psi_{\mathbf{I}_2n}^T(k) \quad \mu_3\Psi_{\mathbf{I}_3}^T(k)\right]$$
$$\mathcal{W}_{\mathbf{II}}(k) \triangleq \left[\mu_{1,1}\Psi_{\mathbf{II}_11}^T(k) \quad \mu_{1,2}\Psi_{\mathbf{II}_12}^T(k) \quad \cdots \quad \mu_{1,n}\Psi_{\mathbf{II}_1n}^T(k) \quad \mu_{2,1}\Psi_{\mathbf{II}_21}^T(k) \quad \mu_{2,2}\Psi_{\mathbf{II}_22}^T(k) \quad \cdots \quad \mu_{2,n}\Psi_{\mathbf{II}_2n}^T(k) \quad \mu_3\Psi_{\mathbf{II}_3}^T(k)\right]$$
$$\Xi^{-1}(k) \triangleq diag\left\{-I_{nn_z}, -\mathcal{R}_1^{-1}, -\mathcal{R}_2^{-1}, -\mathcal{R}_3^{-1}, -\boldsymbol{P}(k+1)\right\}$$

with $\boldsymbol{P}(k+1) \triangleq I_{2n+1}\otimes\left[\sum_{o=1}^{s}h_o(\boldsymbol{\theta}(k+1))P_o^{-1}\right]$, $o \in \mathbb{S}$.

Combining (33), (34) and (37), and considering the zero inputs, i.e., $\left[v^T(k) \quad w^T(k)\right] = \mathbf{0}_{1\times(2nn_x+n_w)}$, we have

$$\Delta V \triangleq \Delta V_1 + \Delta V_2 + \Delta V_3 = \zeta^T(k)\Sigma(k)\zeta(k)$$

where

$$\begin{aligned}\Sigma(k) \triangleq\ & \Lambda'(k) + \sum_{q=1}^{n}\Big[\mu_{1,q}^2\phi_{\mathbf{I}_1q}^T(k)\bar{P}(k+1)\phi_{\mathbf{I}_1q}(k) + \mu_{2,q}^2\phi_{\mathbf{I}_2q}^T(k)\bar{P}(k+1)\phi_{\mathbf{I}_2q}(k)\Big] + \mu_3^2\phi_{\mathbf{I}_3}^T(k)\\ &\times\bar{P}(k+1)\phi_{\mathbf{I}_3}(k) + \sum_{l=1}^{2}\Big\{d_l^2\sum_{q=1}^{n}\Big[\mu_{1,q}^2 \times\phi_{\mathbf{II}_1q}^T(k)R_l\phi_{\mathbf{II}_1q}(k) + \mu_{2,q}^2\phi_{\mathbf{II}_2q}^T(k)R_l\phi_{\mathbf{II}_2q}(k)\Big]\\ &+ d_l^2\mu_3^2\phi_{\mathbf{II}_3}^T(k)R_l\phi_{\mathbf{II}_3}(k)\Big\}\end{aligned}$$

$$
\begin{aligned}
\Lambda'(k) &\triangleq \begin{bmatrix} \Lambda_{\mathbf{I}}(k) & \begin{bmatrix} R_1 & R_2 \end{bmatrix} \\ * & \mathrm{diag}\left\{\Lambda_{\mathbf{II}}(k), \Lambda_{\mathbf{III}}(k)\right\} \end{bmatrix} \\
\phi_{\mathbf{I}_1 q}(k) &\triangleq \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{I}q}(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\phi_{\mathbf{I}_2 q}(k) &\triangleq \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{II}q}(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\phi_{\mathbf{I}_3}(k) &\triangleq \begin{bmatrix} \mathcal{A}(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\phi_{\mathbf{II}_1 q}(k) &\triangleq \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{I}q}(k) - I_{2nn_x} & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\phi_{\mathbf{II}_2 q}(k) &\triangleq \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{II}q}(k) - I_{2nn_x} & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\phi_{\mathbf{II}_3}(k) &\triangleq \begin{bmatrix} \mathcal{A}(k) - I_{2nn_x} & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\zeta(k) &\triangleq \begin{bmatrix} \mathcal{E}^T(k) & \mathcal{E}^T(k-d_1) & \mathcal{E}^T(k-d_2) \end{bmatrix}^T.
\end{aligned}
$$

By performing a congruence transformation to (42) and by Schur complement, we have $\Sigma(k) < 0$, which implies $\Delta V < 0$. Thus, the forward subsystem (S_1) in (24) is internally stable.

Considering the index J defined in (31), together with (29), (33), (34), (37) and (38) in the case of $w(k) = \mathbf{0}_{n_w \times 1}$, and under zero initial condition, i.e., $V(\mathcal{E}(0), 0) = 0$, we have

$$
\begin{aligned}
J &\le J + V(\mathcal{E}(\infty), \infty) - V(\mathcal{E}(0), 0) \\
&= E\left\{\sum_{k=0}^{\infty}\left[\vartheta^T(k)R_3\vartheta(k) - v^T(k)R_3 v(k) + \Delta V\right]\right\} \\
&= \sum_{k=0}^{\infty}\varsigma^T(k)\Phi(k)\varsigma(k) \qquad (43)
\end{aligned}
$$

where

$$
\begin{aligned}
\Phi(k) \triangleq\ & \Lambda''(k) + \sum_{q=1}^{n}\Big[\mu_{1,q}^2\psi_{\mathbf{I}_1 q}^T(k)\bar{P}(k+1)\psi_{\mathbf{I}_1 q}(k) \\
&+\mu_{2,q}^2\psi_{\mathbf{I}_2 q}^T(k)\bar{P}(k+1)\psi_{\mathbf{I}_2 q}(k)\Big] + \mu_3^2\psi_{\mathbf{I}_3}^T(k) \\
&\times \bar{P}(k+1)\psi_{\mathbf{I}_3}(k) + \sum_{l=1}^{2}\Big\{d_l^2\sum_{q=1}^{n}[\mu_{1,q}^2 \\
&\times\psi_{\mathbf{II}_1 q}^T(k)R_l\psi_{\mathbf{II}_1 q}(k) + \mu_{2,q}^2\psi_{\mathbf{II}_2 q}^T(k)R_l\psi_{\mathbf{II}_2 q}(k)] \\
&+d_l^2\mu_3^2\psi_{\mathbf{II}_3}^T(k)R_l\psi_{\mathbf{II}_3}(k)\Big\} + \mu_3^2\psi_{\mathbf{II}_3}^T(k)R_3\psi_{\mathbf{II}_3}(k) \\
&+ \sum_{q=1}^{n}[\mu_{1,q}^2\psi_{\mathbf{II}_1 q}^T(k)R_3\psi_{\mathbf{II}_1 q}(k) \\
&+ \mu_{2,q}^2\psi_{\mathbf{II}_2 q}^T(k)R_3\psi_{\mathbf{II}_2 q}(k)] \\
\Lambda''(k) \triangleq\ & \begin{bmatrix} \Lambda_{\mathbf{I}}(k) & \begin{bmatrix} R_1 & R_2 & \mathbf{0}_{2nn_x} \end{bmatrix} \\ * & \mathrm{diag}\left\{\Lambda_{\mathbf{II}}(k), \Lambda_{\mathbf{III}}(k), -R_3\right\} \end{bmatrix} \\
\psi_{\mathbf{I}_1 q}(k) \triangleq\ & \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{I}q}(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{d_{12}}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\psi_{\mathbf{I}_2 q}(k) \triangleq\ & \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{II}q}(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{d_{12}}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\psi_{\mathbf{I}_3}(k) \triangleq\ & \begin{bmatrix} \mathcal{A}(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{d_{12}}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\psi_{\mathbf{II}_1 q}(k) \triangleq\ & \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{I}q}(k) - I_{2nn_x} & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{d_{12}}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\psi_{\mathbf{II}_2 q}(k) \triangleq\ & \begin{bmatrix} \mathcal{A}(k) - n\mathcal{H}_{\mathbf{II}q}(k) - I_{2nn_x} & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{d_{12}}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\psi_{\mathbf{II}_3}(k) \triangleq\ & \begin{bmatrix} \mathcal{A}(k) - I_{2nn_x} & \frac{1}{2}\mathcal{A}_d(k) & \frac{1}{2}\mathcal{A}_d(k) & \frac{d_{12}}{2}\mathcal{A}_d(k) \end{bmatrix} \\
\varsigma(k) \triangleq\ & \begin{bmatrix} \mathcal{E}^T(k) & \mathcal{E}^T(k-d_1) & \mathcal{E}^T(k-d_2) & v^T(k) \end{bmatrix}^T.
\end{aligned}
$$

By performing a congruence transformation to (42) and by Schur complement, we obtain $\Phi(k) < 0$, which implies

$$
E\{\Delta V + \vartheta^T(k)R_3\vartheta(k) - v^T(k)R_3 v(k)\} < 0. \qquad (44)
$$

From (43), it is easy to see that $J < 0$, which implies $\left\|T \circ \mathbf{G} \circ T^{-1}\right\|_{\infty} < 1$, such that the whole interconnection system in (24) is input-output stable under $w(k) = \mathbf{0}_{n_w \times 1}$.

Based on (44), we choose the following Lyapunov–Krasovskii functional which can demonstrate the stochastic stability of the system (24) with $w(k) = \mathbf{0}_{n_w \times 1}$:

$$
V_{cl}(\mathcal{E}(k), k) \triangleq V(\mathcal{E}(k), k) + \frac{1}{d_{12}}\sum_{u=-d_2}^{-d_1-1}\sum_{v=k+u}^{k-1}\vartheta^T(v)R_3\vartheta(v). \qquad (45)
$$

By virtue of Jensen inequality, taking the forward difference of $V_{cl}(\mathcal{E}(k), k)$ along the trajectory of the closed-loop system consisting of (S_1) and (S_2), we have

$$
\begin{aligned}
&\Delta V_{cl} \\
&= E\{\Delta V + \vartheta^T(k)R_3\vartheta(k) - \tfrac{1}{d_{12}}\textstyle\sum_{u=k-d_2}^{k-d_1-1}\vartheta^T(u)R_3\vartheta(u)\} \\
&\le E\Big\{\Delta V + \vartheta^T(k)R_3\vartheta(k) - d_{12}^{-2}\Big(\textstyle\sum_{u=k-d_2}^{k-d_1-1}\vartheta(u)\Big)^T \\
&\quad \times R_3\Big(\textstyle\sum_{u=k-d_2}^{k-d_1-1}\vartheta(u)\Big)\Big\} \\
&= E\Big\{\Delta V + \vartheta^T(k)R_3\vartheta(k) - \Big(d_{12}^{-1}\textstyle\sum_{u=k-d_2}^{k-d_1-1}\kappa(u)\vartheta(u)\Big)^T \\
&\quad \times R_3\Big(d_{12}^{-1}\textstyle\sum_{u=k-d_2}^{k-d_1-1}\kappa(u)\vartheta(u)\Big)\Big\} \\
&= E\{\Delta V + \vartheta^T(k)R_3\vartheta(k) - v^T(k)R_3 v(k)\} < 0.
\end{aligned}
$$

Then, it follows that $\Phi(k) < 0$, thus we can always find $\sigma < 0$ such that

$$
\begin{aligned}
\Delta V_{cl} &\triangleq E\left\{V_{cl}\left(\mathcal{E}\left(k+1\right), k+1\right)|\mathcal{E}(k), k\right\} - V_{cl}(\mathcal{E}(k), k) \\
&\le \lambda_{\max}(\Phi(k))\varsigma^T(k)\varsigma(k) \\
&\le \sigma\mathcal{E}^T(k)\mathcal{E}(k). \qquad (46)
\end{aligned}
$$

where $\sigma = \lambda_{\max}(\Phi(k))$, then $\sigma < 0$. From (46), one has

$$
E\left\{V_{cl}\left(\mathcal{E}\left(k+1\right), k+1\right)\right\} - V_{cl}(\mathcal{E}(k), k) \le \sigma|\mathcal{E}(k)|^2. \qquad (47)
$$

Take mathematical expectation on both sides of (47). For any $h \ge 1$, summing up the inequality in (47) on both sides from $k = 0, ..., h$, we have

$$
E\{V_{cl}(\mathcal{E}(h+1), h+1)\} - V_{cl}(\mathcal{E}(0), 0) \le \sigma E\left\{\sum_{k=0}^{h}|\mathcal{E}(k)|^2\right\}
$$

which implies

$$
\begin{aligned}
&E\left\{\sum_{k=0}^{h}|\mathcal{E}(k)|^2\right\} \\
&\le \sigma^{-1}\left\{E\left\{V_{cl}\left(\mathcal{E}(h+1), h+1\right)\right\} - E\left\{V_{cl}(\mathcal{E}(0), 0)\right\}\right\} \\
&\le -\sigma^{-1}E\left\{V_{cl}(\mathcal{E}(0), 0)\right\}.
\end{aligned}
$$

When $h = \infty$, we arrive at

$$
E\left\{\sum_{k=0}^{\infty}|\mathcal{E}(k)|^2 \mid \mathcal{E}(0), 0\right\} \le -\sigma^{-1}\left\{V_{cl}(\mathcal{E}(0), 0)\right\} < \infty
$$

which implies that the closed-loop system (24) is stochastically stable.

Consider the index J_{cl} defined in (32) for any nonzero $w(k) \in l_2[0, \infty)$. Together with (29), (33), (34), (37)–(39) and (45), and according to the fact that $V_{cl}(\mathcal{E}(\infty), \infty) \ge 0$, under zero initial condition, i.e., $V_{cl}(\mathcal{E}(0), 0) = 0$, we have

$$
\begin{aligned}
J_{cl} &\le J_{cl} + V_{cl}(\mathcal{E}(\infty), \infty) - V_{cl}(\mathcal{E}(0), 0) \\
&= E\left\{\sum_{k=0}^{\infty}\left[\bar{z}^T(k)\bar{z}(k) - n\gamma^2 w^T(k)w(k) + \Delta V_{cl}\right]\right\} \\
&= \sum_{k=0}^{\infty}\xi^T(k)\digamma(k)\xi(k)
\end{aligned}
$$

where

$$\begin{aligned}\digamma(k) \triangleq\ & \Lambda(k) + \textstyle\sum_{q=1}^{n} \left[\mu_{1,q}^2 \Psi_{\mathbf{I}_1 q}^T(k)\bar{P}(k+1)\Psi_{\mathbf{I}_1 q}(k)\right.\\ & \left.+\mu_{2,q}^2 \Psi_{\mathbf{I}_2 q}^T(k)\bar{P}(k+1)\Psi_{\mathbf{I}_2 q}(k)\right] + \mu_3^2 \Psi_{\mathbf{I}_3}^T(k)\\ & \times \bar{P}(k+1)\Psi_{\mathbf{I}_3}(k) + \textstyle\sum_{l=1}^{2}\left\{d_l^2 \sum_{q=1}^{n}\left[\mu_{1,q}^2\Psi_{\mathbf{II}_1 q}^T(k)\right.\right.\\ & \left.\times R_l \Psi_{\mathbf{II}_1 q}(k) + \mu_{2,q}^2 \Psi_{\mathbf{II}_2 q}^T(k) R_l \Psi_{\mathbf{II}_2 q}(k)\right]\\ & \left.+d_l^2\mu_3^2\Psi_{\mathbf{II}_3}^T(k)R_l\Psi_{\mathbf{II}_3}(k)\right\} + \Psi_{\mathbf{III}}^T(k)\Psi_{\mathbf{III}}(k)\\ & + \textstyle\sum_{q=1}^{n}\left[\mu_{1,q}^2\Psi_{\mathbf{II}_1 q}^T(k)R_3\Psi_{\mathbf{II}_1 q}(k) + \mu_{2,q}^2\Psi_{\mathbf{II}_2 q}^T(k)\right.\\ & \left.\times R_3\Psi_{\mathbf{II}_2 q}(k)\right] + \mu_3^2\Psi_{\mathbf{II}_3}^T(k)R_3\Psi_{\mathbf{II}_3}(k). \qquad (48)\end{aligned}$$

By Schur complement, it follows from (48) that (42) is a sufficient condition ensuring $\digamma(k) < 0$, which implies $J_{cl} < 0$. Then, we have $\|\bar{z}(k)\|_{E_2}^2 \leq n\gamma^2 \|w(k)\|_2^2$, which implies that the DFFES (24) is stochastically stable with an average H_∞ performance γ. ■

Proof of Theorem 2: Firstly, we define the matrix $\mathcal{K}_j \triangleq \begin{bmatrix}\bar{K}_j & \bar{H}_j\end{bmatrix}$. From (27) and the definition of $\mathcal{K}_j$ and $\mathcal{T}_j$, we have

$$\mathbf{I}\mathcal{T}_j = \mathcal{X}\mathbf{I}\mathcal{K}_j,\ \mathcal{S}_j = \mathcal{Z}\bar{L}_j. \qquad (49)$$

For each $o \in \mathbb{S}$, $R_1, R_2, R_3, P_o > 0$, we have the following inequalities:

$$\begin{aligned}-\mathcal{X}P_o^{-1}\mathcal{X}^T \leq P_o - sym(\mathcal{X}),\ & -\mathcal{Z}\mathcal{Z}^T \leq I_{nn_z} - sym(\mathcal{Z})\\ -\varepsilon_1^2\mathcal{X}\tilde{R}_1^{-1}\mathcal{X}^T &\leq \tilde{R}_1 - sym(\varepsilon_1\mathcal{X})\\ -\varepsilon_2^2\varepsilon_4^2\mathcal{X}\tilde{R}_2^{-1}\mathcal{X}^T &\leq \tilde{R}_2 - sym(\varepsilon_2\varepsilon_4\mathcal{X})\\ -\varepsilon_3^2\mathcal{X}\tilde{R}_3^{-1}\mathcal{X}^T &\leq \tilde{R}_3 - sym(\varepsilon_3\mathcal{X}). \qquad (50)\end{aligned}$$

Furthermore, we denote

$$R_1 \triangleq \varepsilon_1^{-2}\tilde{R}_1,\ R_2 \triangleq \varepsilon_2^{-2}\tilde{R}_2,\ R_3 \triangleq \varepsilon_3^{-2}\tilde{R}_3. \qquad (51)$$

Then, combining (26), (49), (50) and (51), we can obtain

$$\begin{aligned}&\acute{\mathcal{G}}_{oltii} < 0, (o,l,t,i \in \mathbb{S})\\ &\acute{\mathcal{G}}_{oltij} + \acute{\mathcal{G}}_{oltij} < 0, (o,l,t,i,j \in \mathbb{S}, i<j) \qquad (52)\end{aligned}$$

where

$$\begin{aligned}\acute{\mathcal{G}}_{oltij} &\triangleq \begin{bmatrix}\acute{\Lambda}_{lti} & \acute{\Omega}_{ij}\\ * & \acute{\Xi}_o\end{bmatrix}\\ \acute{\Lambda}_{lti} &\triangleq \begin{bmatrix}\Lambda_{\mathbf{I}i} & \begin{bmatrix}\varepsilon_1 R_1 & \varepsilon_2 R_2 & \mathbf{0}_{2nn_x} & \mathbf{0}_{2nn_x\times n_w}\end{bmatrix}\\ * & \mathrm{diag}\left\{\acute{\Lambda}_{\mathbf{II}l}, \acute{\Lambda}_{\mathbf{III}t}, -\varepsilon_3^2 R_3, -n\gamma^2 I_{n_w}\right\}\end{bmatrix}\end{aligned}$$

$$\begin{aligned}\acute{\Omega}_{ij} &\triangleq [\ \acute{\Psi}_{\mathbf{III}ij}^T \ \ d_1\acute{\mathcal{W}}_{\mathbf{II}ij} \ \ \varepsilon_4 d_2\acute{\mathcal{W}}_{\mathbf{II}ij} \ \ \acute{\mathcal{W}}_{\mathbf{II}ij} \ \ \acute{\mathcal{W}}_{\mathbf{I}ij}\ ]\\ \acute{\Xi}_o &\triangleq \mathrm{diag}\{-\mathcal{Z}\mathcal{Z}^T, -\acute{\mathcal{R}}_1, -\acute{\mathcal{R}}_2, -\acute{\mathcal{R}}_3, -\acute{\mathcal{P}}_o\}\\ \acute{\Lambda}_{\mathbf{II}l} &\triangleq -\varepsilon_1^2 R_1 - \varepsilon_1^2 Q_{1l},\ \acute{\Lambda}_{\mathbf{III}t} \triangleq -\varepsilon_2^2 R_2 - \varepsilon_2^2 Q_{2t}\\ \acute{\mathcal{R}}_1 &\triangleq I_{2n+1}\otimes\mathcal{X}R_1^{-1}\mathcal{X}^T,\ \acute{\mathcal{R}}_2 \triangleq I_{2n+1}\otimes\varepsilon_4^2\mathcal{X}R_2^{-1}\mathcal{X}^T\\ \acute{\mathcal{R}}_3 &\triangleq I_{2n+1}\otimes\mathcal{X}R_3^{-1}\mathcal{X}^T,\ \acute{\mathcal{P}}_o \triangleq I_{2n+1}\otimes\mathcal{X}P_o^{-1}\mathcal{X}^T\\ \tilde{\mathcal{W}}_{\mathbf{I}ij} &\triangleq [\,\mu_{1,1}\acute{\Psi}_{\mathbf{I}_1 1,ij}^T\ \ \mu_{1,2}\acute{\Psi}_{\mathbf{I}_1 2,ij}^T\ \cdots\ \mu_{1,n}\acute{\Psi}_{\mathbf{I}_1 n,ij}^T\\ &\quad \mu_{2,1}\acute{\Psi}_{\mathbf{I}_2 1,ij}^T\ \ \mu_{2,2}\acute{\Psi}_{\mathbf{I}_2 2,ij}^T\ \cdots\ \mu_{2,n}\acute{\Psi}_{\mathbf{I}_2 n,ij}^T\ \ \mu_3\acute{\Psi}_{\mathbf{I}_3 ij}^T]\\ \tilde{\mathcal{W}}_{\mathbf{II}ij} &\triangleq [\,\mu_{1,1}\acute{\Psi}_{\mathbf{II}_1 1,ij}^T\ \ \mu_{1,2}\acute{\Psi}_{\mathbf{II}_1 2,ij}^T\ \cdots\ \mu_{1,n}\acute{\Psi}_{\mathbf{II}_1 n,ij}^T\\ &\quad \mu_{2,1}\acute{\Psi}_{\mathbf{II}_2 1,ij}^T\ \ \mu_{2,2}\acute{\Psi}_{\mathbf{II}_2 2,ij}^T\ \cdots\ \mu_{2,n}\acute{\Psi}_{\mathbf{II}_2 n,ij}^T\ \ \mu_3\acute{\Psi}_{\mathbf{II}_3 ij}^T]\end{aligned}$$

with

$$\begin{aligned}\acute{\Psi}_{\mathbf{I}_1 q,ij} &\triangleq \left[\mathcal{X}(\mathcal{A}_{0i} + \mathbf{I}\mathcal{K}_j\mathfrak{I}_1 - n\mathbf{I}\mathcal{K}_j\mathcal{B}_{qi}) \ \ \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di}\right.\\ &\qquad \left.\tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \ \ \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \ \ \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j\mathcal{F}_i\right]\\ \acute{\Psi}_{\mathbf{I}_2 q,ij} &\triangleq \left[\mathcal{X}(\mathcal{A}_{0i} + \mathbf{I}\mathcal{K}_j\mathfrak{I}_1 - n\mathbf{I}\mathcal{K}_j\mathcal{D}_{qi}) \ \ \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di}\right.\\ &\qquad \left.\tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \ \ \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \ \ \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j\mathcal{F}_i\right]\\ \acute{\Psi}_{\mathbf{I}_3 ij} &\triangleq \left[\mathcal{X}(\mathcal{A}_{0i} + \mathbf{I}\mathcal{K}_j\mathfrak{I}_1) \ \ \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di} \ \ \tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di}\right.\\ &\qquad \left.\tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \ \ \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j\mathcal{F}_i\right]\\ \acute{\Psi}_{\mathbf{II}_1 q,ij} &\triangleq \left[\mathcal{X}\left(\mathcal{A}_{0i} + \mathbf{I}\mathcal{K}_j\mathfrak{I}_1 - n\mathbf{I}\mathcal{K}_j\mathcal{B}_{qi} - I_{2nn_x}\right) \ \ \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di}\right.\\ &\qquad \left.\tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \ \ \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \ \ \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j\mathcal{F}_i\right]\\ \acute{\Psi}_{\mathbf{II}_2 q,ij} &\triangleq \left[\mathcal{X}\left(\mathcal{A}_{0i} + \mathbf{I}\mathcal{K}_j\mathfrak{I}_1 - n\mathbf{I}\mathcal{K}_j\mathcal{D}_{qi} - I_{2nn_x}\right) \ \ \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di}\right.\\ &\qquad \left.\tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \ \ \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \ \ \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j\mathcal{F}_i\right]\\ \acute{\Psi}_{\mathbf{II}_3 ij} &\triangleq \left[\mathcal{X}\left(\mathcal{A}_{0i} + \mathbf{I}\mathcal{K}_j\mathfrak{I}_1 - I_{2nn_x}\right) \ \ \tfrac{\varepsilon_1}{2}\mathcal{X}\mathcal{A}_{di}\right.\\ &\qquad \left.\tfrac{\varepsilon_2}{2}\mathcal{X}\mathcal{A}_{di} \ \ \tfrac{\varepsilon_3 d_{12}}{2}\mathcal{X}\mathcal{A}_{di} \ \ \mathcal{X}\mathfrak{E}_{0i} + \mathbf{I}\mathcal{T}_j\mathcal{F}_i\right]\\ \acute{\Psi}_{\mathbf{III}ij} &\triangleq \left[\mathcal{Z}\left(\mathcal{C}_{0i} + \bar{L}_j\mathfrak{I}_2\right) \quad \mathbf{0}_{nn_z\times(6nn_x+n_w)}\right].\end{aligned}$$

According to the definitions of $\mathcal{A}_{ij}, \mathfrak{E}_{ij}, \mathcal{C}_{ij}, \mathcal{H}_{\mathbf{I}q,ij}, \mathcal{H}_{\mathbf{II}q,ij}$, $\mathcal{A}_{0i}, \mathfrak{E}_{0i}, \mathcal{C}_{0i}, \mathbf{I}, \mathcal{K}_j, \bar{L}_j, \mathfrak{I}_1, \mathfrak{I}_2, \mathcal{F}_i, \mathcal{B}_i$ and $\mathcal{D}_i$, we rewrite the matrices $\mathcal{A}_{ij}, \mathfrak{E}_{ij}, \mathcal{C}_{ij}, \mathcal{H}_{\mathbf{I}q,ij}$, $\mathcal{H}_{\mathbf{II}q,ij}$ in Theorem 1 in the following form:

$$\begin{aligned}&\mathcal{A}_{ij} = \mathcal{A}_{0i} + \mathbf{I}\mathcal{K}_j\mathfrak{I}_1,\ \mathcal{C}_{ij} = \mathcal{C}_{0i} + \bar{L}_j\mathfrak{I}_2,\ \mathcal{H}_{\mathbf{I}q,ij} = \mathbf{I}\mathcal{K}_j\mathcal{B}_i\\ &\mathfrak{E}_{ij} = \mathfrak{E}_{0i} + \mathbf{I}\mathcal{K}_j\mathcal{F}_i,\ \mathcal{H}_{\mathbf{II}q,ij} = \mathbf{I}\mathcal{K}_j\mathcal{D}_i. \qquad (53)\end{aligned}$$

Pre- and post-multiply the inequalities in (52) with $\mathrm{diag}\{I_{2nn_x}, \varepsilon_1^{-1}I_{2nn_x}, \varepsilon_2^{-1}I_{2nn_x}, \varepsilon_3^{-1}I_{2nn_x}, I_{n_w}, \mathcal{Z}^{-1}, I_{2n+1}\otimes X^{-1}, I_{2n+1}\otimes\varepsilon_4^{-1}X^{-1}, I_{2n+1}\otimes X^{-1}\}$ and its transpose, respectively. Then, bearing (53) in mind, we can obtain (40). The rest of the proof follows directly from Theorem 1. ■

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