To appear in the *International Journal of General Systems*
Vol. 00, No. 00, Month 20XX, 1–15

H_∞ state estimation for discrete-time memristive recurrent neural networks with stochastic time-delays

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(*Received 00 Month 20XX; accepted 00 Month 20XX*)

This paper deals with the robust H_∞ state estimation problem for a class of memristive recurrent neural networks with stochastic time-delays. The stochastic time-delays under consideration are governed by a Bernoulli distributed stochastic sequence. The purpose of the addressed problem is to design the robust state estimator such that the dynamics of the estimation error is exponentially stable in the mean square, and the prescribed H_∞ performance constraint is met. By utilizing the difference inclusion theory and choosing a proper Lyapunov-Krasovskii functional, the existence condition of the desired estimator is derived. Based on it, the explicit expression of the estimator gain is given in terms of the solution to a linear matrix inequality (LMI). Finally, a numerical example is employed to demonstrate the effectiveness and applicability of the proposed estimation approach.

Keywords: discrete-time; H_∞ state estimation; memristive neural networks; stochastic time-delays

1. Introduction

The past few decades have witnessed constant research interests on various aspects of recurrent neural networks (RNNs) due primarily to their wide applications in many fields such as image processing (Chen et al. 2006), pattern recognition (Nasrabadi and Li 1991), and dynamic optimization (Xia and Wang 2004), etc. In neural networks, owning to the limit of communication capacity, time delays often occur in the signal transmission among neurons. It is well-known that time delays are the main source causing system oscillations and instability. Therefore, the time-delayed neural networks have received increasing research attention, and a great deal of results have been available in the literature, see e.g. Lian and Wang (2015) and the references therein. The time-delays under consideration include constant/time-varying delays, distributed delays and mixed delays. For the RNNs with such types of delays, a lot of results have been obtained on the dynamical behavior analysis of the solution such as global asymptotic stability (Cao, Yuan, and Li 2006), global exponential stability (Arik 2004; Liang and Cao 2003; Liu, Wang, and Liu 2006; Zeng and Wang 2006), robust stability (Zhang, Xu, and Zeng 2009), and delay-dependent

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multistability (Huang and Cao 2010). Recently, a new kind of time-delay, called stochastic time delays, has received increasing research attention. For example, in Zhang, Wang, Li, and Fei (2013), the stochastic time delays have been taken into account in the neural networks and a delay-derivative-dependent stability criterion has been derived.

State estimation problem has been an ongoing research issue that attracts increasing attention from researchers in the area of complex networks (Ding et al. 2012; Shen et al. 2013), especially in the area of neural networks. This is because, in the large-scale neural networks, it is not easy to directly access the neuron states and only partial information about the neuron states is available in terms of the network outputs. Therefore, it is of practice significance to estimate the neurons through the available network outputs. In the past decade, there are a number of papers have attempted to develop state estimation scheme for the neural networks with various types of delays. For example, in Huang, Feng, and Cao (2010), the state estimation problem has been investigated for static neural networks with time-varying delay and, in Kan, Shu, and Li (2014), a robust state estimator has been designed for the discrete-time neural networks where the mixed time-delays have been taken into consideration. Recently, in Bao and Cao (2011), a delay-distribution-dependent state estimation problem has been studied for the discrete-time stochastic neural networks with random delays.

As is well known, in the theoretical modeling of traditional neural circuits, the system parameters are determined by the electric components such as capacitance and resistance. Recently, memristor has received increasing research attention due to its advantages over resistance such as small size, low energy consumption and storage capacity. As the rapid development of the memristor, the memristive recurrent neural networks have stirred a great deal of research interests, and considerable research efforts have been made on the dynamical behavior analysis issues of memristive recurrent neural networks such as stability issues (Zhang, Shen, Yin, and Sun 2013) and synchronization problems (Wu and Zeng 2013). It should be pointed out that, in the existing literature, almost all the memristive recurrent neural networks concerned are continuous-time. Actually, the discrete-time neural networks could be more suitable to model digitally transmitted signals in a dynamical way. Besides, the H_∞ performance index of state estimator could be to ensure that the energy gain from the noise inputs to the estimation error is less than a certain level. However, to the best of the authors' knowledge, the state estimation problem for discrete-time memristive recurrent neural networks (DMRNNs) with stochastic time-delays has not been adequately addressed in the literature yet, not to mention that the H_∞ performance index is imposed simultaneously. It is, therefore, the purpose of this paper to shorten such a gap.

In this paper, the robust H_∞ state estimation problem is investigated for a class of memristive recurrent neural networks with stochastic time-delays. The main contributions of this paper can be summarized as follows: 1) we make the first attempt to address the robust H_∞ state estimation problem for *discrete-time* memristive recurrent neural networks; 2) stochastic time-delays are taken into account in the framework of DMRNNs for the first time; and 3) a robust H_∞ state estimator is designed for *discrete-time* memristive recurrent neural networks, which guarantees that the estimation error system is exponentially mean-square stable and the H_∞ performance is met. Finally, a numerical simulation example is employed to verify the effectiveness of the proposed state estimation scheme.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. $A \otimes B$ denotes the Kronecker product of matrices A and B. I denotes the identity matrix of compatible dimension. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). A^T represents the transpose of A, $Sym\{A\}$ denotes the symmetric matrix $A + A^T$. diag$\{\cdots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $||x||$ describes the Euclidean norm of a vector x. $l_2([0,\infty);\mathbb{R}^m)$ is the space of square-summable m-dimensional vector functions over $[0,\infty)$. co$\{u,v\}$ denotes the closure of the convex hull generated by real numbers u and v.

2. Problem Formulation

Consider the following class of DMRNNs consisting of n neurons:

$$\begin{cases} x(k+1) = \breve{D}(x(k))x(k) + \breve{A}(x(k))f(x(k)) + \delta(k)\breve{B}(x(k))g(x(k-\tau_1)) \\ \qquad\qquad + (1-\delta(k))\breve{B}(x(k))g(x(k-\tau_2)) + L\varsigma(k), \\ x(k) = \Psi(k), \quad k \in \Gamma \end{cases} \tag{1}$$

where $x(k) = \begin{bmatrix} x_1(k)\ x_2(k) \cdots x_n(k) \end{bmatrix}^T$ is the neural state vector; $\breve{D}(x(k)) =$ diag$\{d_1(x_1(k)), d_2(x_2(k)), \cdots, d_n(x_n(k))\}$ is the self-feedback matrix; $\breve{A}(x(k)) = \big(a_{ij}(x_i(k))\big)_{n\times n}$ and $\breve{B}(x(k)) = \big(b_{ij}(x_i(k))\big)_{n\times n}$ are the connection and the delayed connection weight matrices, respectively; $\varsigma(k) = \begin{bmatrix} \varsigma_1(k)\ \varsigma_2(k) \cdots \varsigma_n(k) \end{bmatrix}^T$ is the external disturbance input vector belonging to $l_2([0,\infty);\mathbb{R}^n)$, $L = \text{diag}\{l_1, l_2, \cdots, l_n\}$ is the disturbance intensity; $f(x(k)) = \begin{bmatrix} f_1(x_1(k))\ f_2(x_2(k)) \cdots f_n(x_n(k)) \end{bmatrix}^T$ and $g(x(k)) = \begin{bmatrix} g_1(x_1(k))\ g_2(x_2(k)) \cdots g_n(x_n(k)) \end{bmatrix}^T$ are the nonlinear functions standing for the neuron activation functions; τ_1 and τ_2 are two positive scalars denoting the transmission delays, and $\delta(k)$ is a stochastic variable accounting for the randomly occurring transmission delay. $\Psi(k) = \begin{bmatrix} \psi_1(k)\ \psi_2(k) \cdots \psi_n(k) \end{bmatrix}^T$ describes the initial condition, and $\psi_i(k)$ define on $\Gamma := \{-\max\{\tau_1,\tau_2\}, -\max\{\tau_1,\tau_2\}+1, \cdots, 0\}$.

For the neuron activation functions $f(x(k))$ and $g(x(k))$ are assumed to be continuous and satisfy the following conditions (Liu, Wang, and Liu 2006):

$$\begin{cases} [f(x) - f(y) - \rho_f^1(x-y)]^T[f(x) - f(y) - \rho_f^2(x-y)] \leq 0, \\ [g(x) - g(y) - \rho_g^1(x-y)]^T[g(x) - g(y) - \rho_g^2(x-y)] \leq 0 \end{cases} \tag{2}$$

for all $x, y \in \mathbb{R}^n$ $(x \neq y)$, where ρ_f^1, ρ_f^2, ρ_g^1 and ρ_g^2 are real matrices.

The stochastic variable $\delta(k)$ is a Bernoulli-distributed white sequence taking values on 0 or 1 with the following probabilities

$$\text{Prob}\{\delta(k) = 1\} = \bar{\delta}, \quad \text{Prob}\{\delta(k) = 0\} = 1 - \bar{\delta} \tag{3}$$

where $\bar{\delta} \in [0,1]$ is a known constant.

Along the similar lines in Chua (1971); Wu and Zeng (2013), $d_i(x(k))$, $a_{ij}(x(k))$ and $b_{ij}(x(k))$ are state-dependent functions with the form

$$d_i(x_i(\cdot)) = \begin{cases} \hat{d}_i, & |x_i(\cdot)| > \kappa_i, \\ \check{d}_i, & |x_i(\cdot)| \leq \kappa_i, \end{cases} \quad s_{ij}(x_i(\cdot)) = \begin{cases} \hat{s}_{ij}, & |x_i(\cdot)| > \kappa_i, \\ \check{s}_{ij}, & |x_i(\cdot)| \leq \kappa_i \end{cases}$$

where s stands a or b, the switching jumps $\kappa_i > 0$, $|\hat{d}_i| < 1$, $|\check{d}_i| < 1$, $\hat{s}_{ij}$ and $\check{s}_{ij}$ are constants. Then, denoting $d_i^- = \min\{\hat{d}_i, \check{d}_i\}$, $d_i^+ = \max\{\hat{d}_i, \check{d}_i\}$, $a_{ij}^- = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, $a_{ij}^+ = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $b_{ij}^- = \min\{\hat{b}_{ij}, \check{b}_{ij}\}$, $b_{ij}^+ = \max\{\hat{b}_{ij}, \check{b}_{ij}\}$, $D^- = \mathrm{diag}\{d_1^-, d_2^-, \cdots, d_n^-\}$, $D^+ = \mathrm{diag}\{d_1^+, d_2^+, \cdots, d_n^+\}$, $A^- = (a_{ij}^-)_{n\times n}$, $A^+ = (a_{ij}^+)_{n\times n}$, $B^- = (b_{ij}^-)_{n\times n}$, $B^+ = (b_{ij}^+)_{n\times n}$, one has $\breve{D}(x(k)) \in \big[D^-, D^+\big]$, $\breve{A}(x(k)) \in \big[A^-, A^+\big]$ and $\breve{B}(x(k)) \in \big[B^-, B^+\big]$.

Define

$$\bar{D} := \frac{D^+ + D^-}{2} = \mathrm{diag}\Big\{\frac{d_1^+ + d_1^-}{2}, \frac{d_2^+ + d_2^-}{2}, \cdots, \frac{d_n^+ + d_n^-}{2}\Big\},$$
$$\bar{A} := \frac{A^+ + A^-}{2} = \Big(\frac{a_{ij}^+ + a_{ij}^-}{2}\Big)_{n\times n}, \quad \bar{B} := \frac{B^+ + B^-}{2} = \Big(\frac{b_{ij}^+ + b_{ij}^-}{2}\Big)_{n\times n}.$$

The matrices $\breve{D}(x(k))$, $\breve{A}(x(k))$ and $\breve{B}(x(k))$ can be written as

$$\breve{D}(x(k)) = \bar{D} + \Delta D(k), \quad \breve{A}(x(k)) = \bar{A} + \Delta A(k), \quad \breve{B}(x(k)) = \bar{B} + \Delta B(k) \tag{4}$$

where $\Delta D(k) = \sum_{i=1}^n e_i s_i(k) e_i^T$, $\Delta A(k) = \sum_{i,j=1}^n e_i t_{ij}(k) e_j^T$ and $\Delta B(k) = \sum_{i,j=1}^n e_i p_{ij}(k) e_j^T$, $e_k \in \mathbb{R}^n$ is the column vector with the kth element being 1 and others being 0, $s_i(k)$, $t_{ij}(k)$ and $p_{ij}(k)$ are unkonwn scalars satisfying $|s_i(k)| \le \tilde{d}_i$, $|t_{ij}(k)| \le \tilde{a}_{ij}$ and $|p_{ij}(k)| \le \tilde{b}_{ij}$ with $\tilde{d}_j = \frac{d_j^+ - d_j^-}{2}$, $\tilde{a}_{ij} = \frac{a_{ij}^+ - a_{ij}^-}{2}$ and $\tilde{b}_{ij} = \frac{b_{ij}^+ - b_{ij}^-}{2}$. $\Delta D(k), \Delta A(k)$ and $\Delta B(k)$ are the parameter matrices of the following structures

$$\Delta D(k) = \mathcal{H} F_1(k) E_1, \quad \Delta A(k) = \mathcal{H} F_2(k) E_2, \quad \Delta B(k) = \mathcal{H} F_3(k) E_3 \tag{5}$$

where $\mathcal{H} = \big[H_1\ H_2 \cdots H_n\big]$ and $E_i = \big[E_{i1}^T\ E_{i2}^T \cdots E_{in}^T\big]^T$ $(i = 1, 2, 3)$ are known real constant matrices with $H_i = \underbrace{\big[e_i\ e_i \cdots e_i\big]}_{n}$, $E_{1j} = \Big[e_1^T\ e_2^T \cdots \tilde{d}_j e_j^T \cdots e_n^T\Big]$, $E_{2j} = \big[\tilde{a}_{j1} e_1^T\ \tilde{a}_{j2} e_2^T \cdots \tilde{a}_{jn} e_n^T\big]$ and $E_{3j} = \big[\tilde{b}_{j1} e_1^T\ \tilde{b}_{j1} e_2^T \cdots \tilde{b}_{j1} e_n^T\big]$. $F_i(k)$ $(i = 1, 2, 3)$ are unknown time-varying matrices which are given by

$$F_i(k) = \mathrm{diag}\{F_{i1}(k), \cdots, F_{in}(k)\}, \ F_{1j}(k) = \mathrm{diag}\{\underbrace{0, \cdots, 0}_{j-1}, s_j(k)\tilde{d}_j^{-1}, \underbrace{0, \cdots, 0}_{n-j}\},$$
$$F_{2j}(k) = \mathrm{diag}\{t_{j1}(k)\tilde{a}_{j1}^{-1}, \cdots, t_{jn}(k)\tilde{a}_{jn}^{-1}\}, \ F_{3j}(k) = \mathrm{diag}\{p_{j1}(k)\tilde{b}_{j1}^{-1}, \cdots, p_{jn}(k)\tilde{b}_{jn}^{-1}\}.$$

It is not difficult to verify that the matrices $F_i(k)$ $(i = 1, 2, 3)$ satisfy $F_i^T(k) F_i(k) \le I$.

Suppose that the measurement output and the output to be estimated of the neural network (1) are given as follows:

$$y(k) = Cx(k) + N\xi(k), \tag{6}$$
$$z(k) = Mx(k) \tag{7}$$

where $y(k) \in \mathbb{R}^m$ is the measurement output, $z(k) \in \mathbb{R}^r$ is the output to be estimated, $\xi(k) \in \mathbb{R}^l$ is the disturbance input belonging to $l_2([0, \infty); \mathbb{R}^l)$.

In order to estimate the neuron state $x(k)$, we employ the following state estimator

$$\begin{cases} \hat{x}(k+1) = \bar{D}\hat{x}(k) + \bar{A}f(\hat{x}(k)) + \bar{\delta}\bar{B}g(\hat{x}(k-\tau_1)) \\ \qquad\qquad + (1-\bar{\delta})\bar{B}g(\hat{x}(k-\tau_2)) + K(y(k) - C\hat{x}(k)), \\ \hat{z}(k) = M\hat{x}(k), \\ \hat{x}(k) = 0, \quad k \in \Gamma \end{cases} \tag{8}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the estimate of the neuron state $x(k)$, $\hat{z}(k) \in \mathbb{R}^r$ is the estimate of the output $z(k)$, and $K \in \mathbb{R}^{n\times m}$ is the estimator gain to be determined.

From (1), (6), (7) and (8), the dynamics of the estimation error can be obtained as follows:

$$\begin{cases} e(k+1) = (\bar{D} - KC)e(k) + \Delta D(k)x(k) + \bar{A}\tilde{f}(k) + \Delta A(k)f(x(k)) \\ \qquad\qquad + \bar{\delta}\bar{B}\tilde{g}^1(k) + (\delta(k) - \bar{\delta})\breve{B}g(x(k-\tau_1)) + \bar{\delta}\Delta B(k)g(x(k-\tau_1)) \\ \qquad\qquad + (\bar{\delta} - \delta(k))\breve{B}g(x(k-\tau_2)) + (1-\bar{\delta})\bar{B}\tilde{g}^2(k) + (1-\bar{\delta})\Delta B(k) \\ \qquad\qquad \times g(x(k-\tau_2)) + L\varsigma(k) - KN\xi(k), \\ e(k) = \Psi(k), \quad k \in \Gamma \end{cases} \tag{9}$$

where $e(k) = x(k) - \hat{x}(k)$, $\tilde{f}(k) = f(x(k)) - f(\hat{x}(k))$ and $\tilde{g}^r(k) = g(x(k-\tau_r)) - g(\hat{x}(k-\tau_r))$ $(r = 1, 2)$.

Then, setting $\eta(k) = \left[x^T(k)\ e^T(k)\right]^T$ and letting the output estimation error be $\tilde{z}(k) = z(k) - \hat{z}(k)$, we have the following augmented system

$$\begin{cases} \eta(k+1) = \tilde{W}_1\eta(k) + \tilde{W}_2\vec{f}(k) + \bar{\delta}\tilde{W}_3\vec{g}^1(k) + (\delta(k) - \bar{\delta})\tilde{W}_4\vec{g}^1(k) \\ \qquad\qquad + (\bar{\delta} - \delta(k))\tilde{W}_4\vec{g}^2(k) + (1-\bar{\delta})\tilde{W}_3\vec{g}^2(k) + W_5\zeta(k), \\ \tilde{z}(k) = \vec{M}\eta(k), \\ \eta(k) = \left[x^T(k)\ \psi^T(k)\right]^T, \quad k \in \Gamma \end{cases} \tag{10}$$

where

$$\vec{f}(k) = \left[f^T(x(k))\ \tilde{f}^T(k)\right]^T, \quad \vec{g}^r(k) = \left[g^T(x(k-\tau_r))\ (\tilde{g}^r(k))^T\right]^T \ (r = 1, 2),$$

$$\vec{M} = \left[0\ M\right], \quad \zeta(k) = \left[\varsigma^T(k)\ \xi^T(k)\right]^T, \quad \tilde{W}_1 = W_1 + \Delta\mathcal{D}(k), \quad \tilde{W}_2 = W_2 + \Delta\mathcal{A}(k),$$

$$\tilde{W}_3 = W_3 + \Delta\mathcal{B}(k), \quad \tilde{W}_4 = W_4 + \Delta\mathcal{B}(k), \quad W_2 = \mathrm{diag}\{\bar{A}, \bar{A}\}, \quad W_3 = \mathrm{diag}\{\bar{B}, \bar{B}\},$$

$$W_1 = \begin{bmatrix} \bar{D} & 0 \\ 0 & \bar{D} - KC \end{bmatrix}, \quad W_4 = \begin{bmatrix} \breve{B} & 0 \\ \breve{B} & 0 \end{bmatrix}, \quad W_5 = \begin{bmatrix} L & 0 \\ L & -KN \end{bmatrix},$$

$$\Delta\mathcal{D}(k) = \begin{bmatrix} \Delta D(k) & 0 \\ \Delta D(k) & 0 \end{bmatrix}, \quad \Delta\mathcal{A}(k) = \begin{bmatrix} \Delta A(k) & 0 \\ \Delta A(k) & 0 \end{bmatrix}, \quad \Delta\mathcal{B}(k) = \begin{bmatrix} \Delta B(k) & 0 \\ \Delta B(k) & 0 \end{bmatrix}.$$

Definition 1. *The augmented system (10) with $\zeta(k) = 0$ is said to be exponentially mean-square stable if there exist constants $\epsilon > 0$ and $0 < \lambda < 1$ such that*

$$\mathbb{E}\{\|\eta(k)\|^2\} \leq \epsilon\lambda^k \max_{i\in\Gamma} \mathbb{E}\{\|\eta(i)\|^2\}, \quad \forall k \in \mathbb{N}.$$

The aim of this paper is to design an H_∞ state estimator for DMRNNs with stochastic time-delays given by (1). More specifically, we are interested in looking for the gain matrix K such that the following two requirements are met simultaneously:

1) The augmented system (10) with $\zeta(k) = 0$ is exponentially mean-square stable;

2) Under zero initial conditions, for a given disturbance attention level $\gamma > 0$ and all nonzero $\zeta(k)$, the output estimation error $\tilde{z}(k)$ satisfies

$$\sum\nolimits_{k=0}^{\infty} \mathbb{E}\{\|\tilde{z}(k)\|^2\} \leq \gamma^2 \sum\nolimits_{k=0}^{\infty} \|\zeta(k)\|^2. \tag{11}$$

3. MAIN RESULTS

In this section, the stability and the H_∞ performance are analyzed for the augmented system (10). A sufficient condition is established to guarantee that the augmented system (10) is exponentially mean-square stable and the H_∞ performance is achieved. Then, the explicit expression of the desired estimator gain is given in terms of the solution to certain matrix inequality.

Theorem 1. *Let the estimator parameter K be given. The augmented system (10) with $\zeta(k) = 0$ is exponentially mean-square stable if there exist positive definite matrices $P = \mathrm{diag}\{P_1, P_2\}$, $Q_i = \mathrm{diag}\{Q_{i1}, Q_{i2}\}$ $(i = 1, 2)$ and positive scalars λ_j $(j = 1, 2, 3)$ satisfying the following inequality:*

$$\tilde{\Phi} = \begin{bmatrix} \tilde{\Theta}_{11} & 0 & 0 & \tilde{\Theta}_{14} & \Theta_{15} & \Theta_{16} \\ * & \tilde{\Theta}_{22} & 0 & 0 & \tilde{\Theta}_{25} & 0 \\ * & * & \tilde{\Theta}_{33} & 0 & 0 & \tilde{\Theta}_{36} \\ * & * & * & \tilde{\Theta}_{44} & \Theta_{45} & \Theta_{46} \\ * & * & * & * & \tilde{\Theta}_{55} & \Theta_{56} \\ * & * & * & * & * & \tilde{\Theta}_{66} \end{bmatrix} < 0, \tag{12}$$

where

$$\begin{aligned}
&\phi_1^f = I \otimes \mathrm{Sym}\{\tfrac{1}{2}\rho_1^{fT}\rho_2^f\}, \quad \phi_2^f = I \otimes (\rho_1^f + \rho_2^f)/2, \\
&\phi_1^g = I \otimes \mathrm{Sym}\{\tfrac{1}{2}\rho_1^{gT}\rho_2^g\}, \quad \phi_2^g = I \otimes (\rho_1^g + \rho_2^g)/2 \ (r = 1, 2), \\
&\tilde{\Theta}_{11} = \tilde{W}_1^T P \tilde{W}_1 - P + Q_1 + Q_2 - \lambda_1 \phi_1^{\vec{f}}, \quad \tilde{\Theta}_{14} = \tilde{W}_1^T P \tilde{W}_2 + \lambda_1 \phi_2^{\vec{f}T}, \\
&\Theta_{15} = \bar{\delta} \tilde{W}_1^T P \tilde{W}_3, \quad \Theta_{16} = (1 - \bar{\delta}) \tilde{W}_1^T P \tilde{W}_3, \quad \tilde{\Theta}_{22} = -Q_1 - \lambda_2 \phi_1^{\vec{g}^1}, \\
&\tilde{\Theta}_{25} = \lambda_2 \phi_2^{\vec{g}^{1T}}, \quad \tilde{\Theta}_{33} = -Q_2 - \lambda_3 \phi_1^{\vec{g}^2}, \quad \tilde{\Theta}_{36} = \lambda_3 \phi_2^{\vec{g}^{2T}}, \\
&\tilde{\Theta}_{44} = \tilde{W}_2^T P \tilde{W}_2 - \lambda_1 I, \quad \Theta_{45} = \bar{\delta} \tilde{W}_2^T P \tilde{W}_3, \quad \tilde{\Theta}_{55} = \bar{\delta}^2 \tilde{W}_3^T P \tilde{W}_3 - \lambda_2 I, \\
&\Theta_{46} = (1 - \bar{\delta}) \tilde{W}_2^T P \tilde{W}_3, \quad \Theta_{56} = \bar{\delta}(1 - \bar{\delta}) \tilde{W}_3^T P \tilde{W}_3, \quad \tilde{\Theta}_{66} = (1 - \bar{\delta})^2 \tilde{W}_3^T P \tilde{W}_3 - \lambda_3 I.
\end{aligned}$$

Proof: Construct the following Lyapunov-Krasovskii functional for system (10)

$$V(\eta(k)) = V_1(\eta(k)) + V_2(\eta(k)) + V_3(\eta(k)) \tag{13}$$

where $V_1(\eta(k)) = \eta(k)^T P \eta(k)$, $V_2(\eta(k)) = \sum_{i=k-\tau_1}^{k-1} \eta(i)^T Q_1 \eta(i)$ and $V_3(\eta(k)) = \sum_{i=k-\tau_2}^{k-1} \eta(i)^T Q_2 \eta(i)$.

In the case of $\zeta(k)=0$, we calculate the difference of $V_1(k)$ along (10) as follows:

$$\begin{aligned}
&\mathbb{E}\{\Delta V_1(\eta(k))\}\\
=&\mathbb{E}\{[\eta^T(k)\tilde{W}_1^T P\tilde{W}_1\eta(k)+\vec{f}^T(k)\tilde{W}_2^T P\tilde{W}_2\vec{f}(k)+\bar{\delta}^2\vec{g}^{1T}(k)\tilde{W}_3^T P\tilde{W}_3\vec{g}^1(k)\\
&+(1-\bar{\delta})^2\vec{g}^{2T}(k)\tilde{W}_3^T P\tilde{W}_3\vec{g}^2(k)+2\eta^T(k)\tilde{W}_1^T P\tilde{W}_2\vec{f}(k)+2\bar{\delta}\eta^T(k)\tilde{W}_1^T P\tilde{W}_3\\
&\times\vec{g}^1(k)+2(1-\bar{\delta})\eta^T(k)\tilde{W}_1^T P\tilde{W}_3\vec{g}^2(k)+2\bar{\delta}\vec{f}^T(k)\tilde{W}_2^T P\tilde{W}_3\vec{g}^1(k)+2(1-\bar{\delta})\\
&\times\vec{f}^T(k)\tilde{W}_2^T P\tilde{W}_3\vec{g}^2(k)+2\bar{\delta}(1-\bar{\delta})\vec{g}^{1T}(k)\tilde{W}_3^T P\tilde{W}_3\vec{g}^2(k)]-\eta^T(k)P\eta(k)\}.
\end{aligned}\tag{14}$$

Similarly, we obtain

$$\mathbb{E}\{\Delta V_2(\eta(k))\}=\mathbb{E}\{\eta^T(k)Q_1\eta(k)-\eta^T(k-\tau_1)Q_1\eta(k-\tau_1)\},\tag{15}$$

$$\mathbb{E}\{\Delta V_3(\eta(k))\}=\mathbb{E}\{\eta^T(k)Q_2\eta(k)-\eta^T(k-\tau_2)Q_2\eta(k-\tau_2)\}.\tag{16}$$

By setting $\vartheta(k)=\left[\eta^T(k)\ \eta^T(k-\tau_1)\ \eta^T(k-\tau_2)\ \vec{f}^T(k)\ \vec{g}^{1T}(k)\ \vec{g}^{2T}(k)\right]^T$, the combination of (14)–(16) results in

$$\mathbb{E}\{V(\eta(k+1))-V(\eta(k))\}=\sum_{i=1}^{3}\mathbb{E}\{\Delta V_i(\eta(k))\}=\mathbb{E}\{\vartheta^T(k)\Phi\vartheta(k)\}\tag{17}$$

where

$$\Phi=\begin{bmatrix}\Theta_{11} & \Pi_{12}\\ * & \Pi_{22}\end{bmatrix},\quad \Pi_{22}=\begin{bmatrix}\Theta_{22} & 0 & 0 & 0 & 0\\ * & \Theta_{33} & 0 & 0 & 0\\ * & * & \Theta_{44} & \Theta_{45} & \Theta_{46}\\ * & * & * & \Theta_{55} & \Theta_{56}\\ * & * & * & * & \Theta_{66}\end{bmatrix},\quad \Pi_{12}=\begin{bmatrix}0 & 0 & \Theta_{14} & \Theta_{15} & \Theta_{16}\end{bmatrix},$$

$$\Theta_{11}=\tilde{W}_1^T P\tilde{W}_1-P+Q_1+Q_2,\quad \Theta_{14}=\tilde{W}_1^T P\tilde{W}_2,\quad \Theta_{22}=-Q_1,$$

$$\Theta_{33}=-Q_2,\quad \Theta_{44}=\tilde{W}_2^T P\tilde{W}_2,\quad \Theta_{55}=\bar{\delta}^2\tilde{W}_3^T P\tilde{W}_3,\quad \Theta_{66}=(1-\bar{\delta})^2\tilde{W}_3^T P\tilde{W}_3$$

and $\Theta_{15},\Theta_{16},\Theta_{45},\Theta_{46},\Theta_{56}$ are defined in Theorem 1.

Taking (2) into consideration, one has

$$\begin{aligned}
&\mathbb{E}\{V(\eta(k+1))-V(\eta(k))\}\\
\le&\mathbb{E}\Big\{\sum_{i=1}^{3}\mathbb{E}\{\Delta V_i(\eta(k))\}-\lambda_1[\vec{f}(k)-(I\otimes\rho_1^f)\eta(k)]^T[\vec{f}(k)-(I\otimes\rho_2^f)\eta(k)]\\
&-\sum_{r=1}^{2}\lambda_{r+1}[\vec{g}^r(k)-(I\otimes\rho_1^g)\eta(k-\tau_r)]^T[\vec{g}^r(k)-(I\otimes\rho_2^g)\eta(k-\tau_r)]\\
\le&\mathbb{E}\{\vartheta^T(k)\tilde{\Phi}\vartheta(k)\}
\end{aligned}\tag{18}$$

where $\tilde{\Phi}$ is defined by (12). Since $\tilde{\Phi}<0$, we have $\mathbb{E}\{V(\eta(k+1))-V(\eta(k))\}\le -\epsilon\mathbb{E}\{\|\eta(k)\|\}$ where $\epsilon=-\lambda_{\max}(\Psi)$. Then, by following the similar analysis in Liang, Wang, and Liu (2009), the exponential mean-square stability of the augmented system (10) with $\zeta(k)=0$ can be shown and hence this proof is complete.

In the following theorem, the H_∞ performance analysis is conducted and a sufficient condition is derived to guarantee that the output estimation error $\tilde{z}(k)$ satisfies the H_∞ performance constraint (11).

Theorem 2. *Let the estimator parameter K be given. The augmented system (10) with $\zeta(k) = 0$ is exponentially mean-square stable and the output estimation error $\tilde{z}(k)$ satisfies the H_∞ performance constraint (11) under the zero initial condition for all nonzero $\zeta(k)$ if there exist positive definite matrices $P = \mathrm{diag}\{P_1, P_2\}$, $Q_i = \mathrm{diag}\{Q_{i1}, Q_{i2}\}$ $(i = 1, 2)$ and positive scalars λ_j $(j = 1, 2, 3)$ satisfying the following inequality:*

$$\hat{\Phi} = \begin{bmatrix} \hat{\Theta}_{11} & 0 & 0 & \tilde{\Theta}_{14} & \Theta_{15} & \Theta_{16} & \Theta_{17} \\ * & \tilde{\Theta}_{22} & 0 & 0 & \tilde{\Theta}_{25} & 0 & 0 \\ * & * & \tilde{\Theta}_{33} & 0 & 0 & \tilde{\Theta}_{36} & 0 \\ * & * & * & \tilde{\Theta}_{44} & \Theta_{45} & \Theta_{46} & \Theta_{47} \\ * & * & * & * & \tilde{\Theta}_{55} & \Theta_{56} & \Theta_{57} \\ * & * & * & * & * & \tilde{\Theta}_{66} & \Theta_{67} \\ * & * & * & * & * & * & \Theta_{77} \end{bmatrix} < 0, \tag{19}$$

where $\hat{\Theta}_{11} = \tilde{W}_1^T P \tilde{W}_1 - P + Q_1 + Q_2 + \vec{M}^T \vec{M} - \lambda_1 \phi_1^f$, $\Theta_{77} = -\gamma^2 I + W_5^T P W_5$, $\Theta_{17} = \tilde{W}_1^T P W_5$, $\Theta_{47} = \tilde{W}_2^T P W_5$, $\Theta_{57} = \bar{\delta} \tilde{W}_3^T P W_5$, $\Theta_{67} = (1 - \bar{\delta}) \tilde{W}_3^T P W_5$ and other parameters are defined in Theorem 1.

Proof: The proof of the exponential mean-square stability for (10) in the case of $\zeta(k) = 0$ follows immediately from Theorem 1 since that the inequality (12) is implied by (19). For the H_∞ performance analysis, we choose the same Lyapunov-Krasovskii functional and calculate the difference of $V(\eta(k))$ along (10) as follows:

$$\mathbb{E}\{V(\eta(k+1)) - V(\eta(k)) + \|\tilde{z}(k)\|^2 - \gamma^2 \|\zeta(k)\|^2\} = \mathbb{E}\{\varpi^T(k) \bar{\Phi} \varpi(k)\}$$

where

$$\varpi(k) = \left[\eta^T(k) \ \eta^T(k - \tau_1) \ \eta^T(k - \tau_2) \ \vec{f}^T(k) \ \vec{g}^{1T}(k) \ \vec{g}^{2T}(k) \ \zeta^T(k)\right]^T$$

and

$$\bar{\Phi} = \begin{bmatrix} \bar{\Theta}_{11} & \Pi_{12} & \Theta_{17} \\ * & \Pi_{22} & \Pi_{23} \\ * & * & \Theta_{77} \end{bmatrix}$$

with $\bar{\Theta}_{11} = \tilde{W}_1^T P \tilde{W}_1 - P + Q_1 + Q_2 + \vec{M}^T \vec{M}$, $\Pi_{23} = \left[0 \ 0 \ \Theta_{47}^T \ \Theta_{57}^T \ \Theta_{67}^T\right]^T$ and other parameters defined in Theorems 1 and 2.

Using (2) and (19), we have

$$\mathbb{E}\{V(\eta(k+1)) - V(\eta(k)) + \|\tilde{z}(k)\|^2 - \gamma^2 \|\zeta(k)\|^2\} < 0. \tag{20}$$

Under the zero initial condition, summing up (20) from 0 to ∞ with respect to k, and considering $\mathbb{E}\{V(\eta(\infty)\} \geq 0$, we obtain (11), which accomplishes the proof of Theorem 2.

According to the H_∞ performance analysis conducted in Theorem 2, a design method of the H_∞ state estimator for (1) is provided in Theorem 3.

Theorem 3. *Consider the system (1) and let the disturbance attenuation level $\gamma > 0$ be given. The augmented system (10) is exponentially stable in mean square and the H_∞ performance constraint (11) is met for all nonzero $\zeta(k)$ under the* *zero initial condition if there exist positive definite matrices* $P = \mathrm{diag}\{P_1, P_2\}$, $Q_i = \mathrm{diag}\{Q_{i1}, Q_{i2}\}$ $(i = 1, 2)$, *matrix* X *and positive scalars* ε, λ_j $(j = 1, 2, 3)$ *satisfying the following inequality:*

$$\begin{bmatrix} \check{\Phi} & \check{H} & \varepsilon \check{E}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{21}$$

where

$$\check{\Phi} = \begin{bmatrix} \check{\Pi}_{11} & \check{\Pi}_{12} \\ * & \check{\Theta}_{88} \end{bmatrix}, \ \check{\Pi}_{11} = \begin{bmatrix} \check{\Theta}_{11} & 0 & 0 & \check{\Theta}_{14} & 0 & 0 & 0 \\ * & \tilde{\Theta}_{22} & 0 & 0 & \tilde{\Theta}_{25} & 0 & 0 \\ * & * & \tilde{\Theta}_{33} & 0 & 0 & \tilde{\Theta}_{36} & 0 \\ * & * & * & \check{\Theta}_{44} & 0 & 0 & \Theta_{47} \\ * & * & * & * & \check{\Theta}_{55} & 0 & \Theta_{57} \\ * & * & * & * & * & \check{\Theta}_{66} & \Theta_{67} \\ * & * & * & * & * & * & \check{\Theta}_{77} \end{bmatrix}, \ \check{\Pi}_{12} = \begin{bmatrix} \check{\Theta}_{18} \\ 0 \\ 0 \\ \check{\Theta}_{48} \\ \check{\Theta}_{58} \\ \check{\Theta}_{68} \\ \check{\Theta}_{78} \end{bmatrix},$$

$$\check{H} = \begin{bmatrix} 0\,0\,0\,0\,0\,0\,0\,\mathcal{H}^T \check{P}^T \\ 0\,0\,0\,0\,0\,0\,0\,\mathcal{H}^T \check{P}^T \\ 0\,0\,0\,0\,0\,0\,0\,\mathcal{H}^T \check{P}^T \end{bmatrix}^T, \ \check{P} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, \ \check{E} = \begin{bmatrix} S_{18} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{48} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{58} & S_{68} & 0 & 0 \end{bmatrix}, \tag{22}$$

$$\check{\Theta}_{11} = -P + Q_1 + Q_2 + \vec{M}^T \vec{M} - \lambda_1 \phi_1^f, \ \check{\Theta}_{14} = \lambda_1 \phi_2^{fT}, \ \check{\Theta}_{18} = W_1^T P^T,$$
$$\check{\Theta}_{44} = -\lambda_1 I, \ \check{\Theta}_{48} = W_2^T P^T, \ \check{\Theta}_{55} = -\lambda_2 I, \ \check{\Theta}_{58} = \bar{\delta} W_3^T P^T, \ \check{\Theta}_{66} = -\lambda_3 I,$$
$$\check{\Theta}_{68} = (1 - \bar{\delta}) W_3^T P^T, \ \check{\Theta}_{77} = -\gamma^2 I, \ \check{\Theta}_{78} = W_5^T P^T, \ \check{\Theta}_{88} = -P,$$
$$S_{18} = \begin{bmatrix} E_1 & 0 \end{bmatrix}, \ S_{48} = \begin{bmatrix} E_2 & 0 \end{bmatrix}, \ S_{58} = \begin{bmatrix} \bar{\delta} E_3 & 0 \end{bmatrix}, \ S_{68} = \begin{bmatrix} (1 - \bar{\delta}) E_3 & 0 \end{bmatrix}$$

and other parameters are defined in Theorems 1 and 2. Moreover, if the above inequality is solvable, the state estimator gain can be determined by $K = P_2^{-1} X$.

Proof: In order to eliminate the uncertainties in (19), we use Schur complement lemma and obtain

$$\Xi = \begin{bmatrix} \check{\Pi}_{11} & \Xi_{12} \\ * & \check{\Theta}_{88} \end{bmatrix} < 0 \tag{23}$$

where $\Xi_{12} = \begin{bmatrix} \Lambda_{18}^T & 0 & 0 & \Lambda_{48}^T & \Lambda_{58}^T & \Lambda_{68}^T & \check{\Theta}_{78}^T \end{bmatrix}^T$,

$$\begin{aligned} \Lambda_{18} &= \check{\Theta}_{18} + S_{18}^T F_1^T(k) \mathcal{H}^T \check{P}^T, \quad & \Lambda_{48} &= \check{\Theta}_{48} + S_{48}^T F_2^T(k) \mathcal{H}^T \check{P}^T, \\ \Lambda_{58} &= \check{\Theta}_{58} + S_{58}^T F_3^T(k) \mathcal{H}^T \check{P}^T, \quad & \Lambda_{68} &= \check{\Theta}_{68} + S_{68}^T F_3^T(k) \mathcal{H}^T \check{P}^T \end{aligned} \tag{24}$$

and other parameters are defined in Theorems 2 and 3.

By considering $X = P_2 K$, it follows from (24) that

$$\check{\Phi} + \check{H} \check{F}(k) \check{E} + (\check{H} \check{F}(k) \check{E})^T < 0 \tag{25}$$

where $\check{F}(k) = \mathrm{diag}\{F_1(k), F_2(k), F_3(k)\}$ and other parameters have been defined in (22). According to S-procedure Lemma, it can be easily shown that inequality (25) is implied by (21). The rest of the proof follows Theorem 2 immediately.

Remark 1. In Theorem 3, the H_∞ state estimator is designed for discrete-time memristive recurrent neural networks with stochastic time-delays in terms of the solution to LMI (21). Note that, for a standard LMI system, the algorithm has a polynomial-time complexity. Fortunately, research on LMI optimization is a very active area in the applied mathematics, optimization and the operations research community, and substantial speed-ups can be expected in the future.

To this end, the H_∞ state estimation problem addressed in Section 2 has been solved in terms of a solution to the LMI (21). In the next section, the effectiveness of the developed state estimation approach will be verified by an illustrative example.

4. An Illustrative Example

In this section, a numerical simulation example is given to show the effectiveness of the derived results. The system parameters of the DMRNNs are set as follows

$$d_1(x_1(\cdot)) = \begin{cases} 0.4, & |x_1(\cdot)| > 1, \\ 0.6, & |x_1(\cdot)| \le 1, \end{cases} \qquad d_2(x_2(\cdot)) = \begin{cases} 0.6, & |x_2(\cdot)| > 1, \\ 0.4, & |x_2(\cdot)| \le 1, \end{cases}$$

$$a_{11}(x_1(\cdot)) = \begin{cases} 0.5, & |x_1(\cdot)| > 1, \\ 0.2, & |x_1(\cdot)| \le 1, \end{cases} \qquad a_{12}(x_1(\cdot)) = \begin{cases} 0.2, & |x_1(\cdot)| > 1, \\ -0.3, & |x_1(\cdot)| \le 1, \end{cases}$$

$$a_{21}(x_2(\cdot)) = \begin{cases} 0.3, & |x_2(\cdot)| > 1, \\ 0.15, & |x_2(\cdot)| \le 1, \end{cases} \qquad a_{22}(x_2(\cdot)) = \begin{cases} 0.6, & |x_2(\cdot)| > 1, \\ -0.18, & |x_2(\cdot)| \le 1, \end{cases}$$

$$b_{11}(x_1(\cdot)) = \begin{cases} 0.2, & |x_1(\cdot)| > 1, \\ 0.5, & |x_1(\cdot)| \le 1, \end{cases} \qquad b_{12}(x_1(\cdot)) = \begin{cases} 0.3, & |x_1(\cdot)| > 1, \\ 0.2, & |x_1(\cdot)| \le 1, \end{cases}$$

$$b_{21}(x_2(\cdot)) = \begin{cases} 0.2, & |x_2(\cdot)| > 1, \\ -0.1, & |x_2(\cdot)| \le 1, \end{cases} \qquad b_{22}(x_2(\cdot)) = \begin{cases} -0.3, & |x_2(\cdot)| > 1, \\ 0.1, & |x_2(\cdot)| \le 1, \end{cases}$$

$$\Delta D(k) = \begin{bmatrix} 0.1\sin(0.6k) & 0 \\ 0 & 0.1\sin(0.6k) \end{bmatrix}, \quad \Delta A(k) = \begin{bmatrix} 0.15\sin(0.8k) & 0.3\sin(0.8k) \\ 0.07\sin(0.8k) & 0.39\sin(0.8k) \end{bmatrix},$$

$$\Delta B(k) = \begin{bmatrix} 0.15\cos(0.5k) & 0.05\cos(0.5k) \\ 0.15\cos(0.5k) & 0.2\cos(0.5k) \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix},$$

$$N = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad L = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.15 \end{bmatrix}, \quad M = \begin{bmatrix} 0.35 & 0.3 \end{bmatrix}.$$

The activation functions $f(x(k))$ and $g(x(k))$ are chosen as

$$f(x(k)) = \begin{bmatrix} \tanh(-0.3x_1(k)) \\ \tanh(0.5x_2(k)) \end{bmatrix}, \quad g(x(k)) = \begin{bmatrix} \tanh(0.10x_1(k)) \\ 0.02x_2(k) - 0.06\tanh(x_2(k)) \end{bmatrix},$$

which satisfy the constraint (2) with

$$\rho_f^1 = \begin{bmatrix} -0.3 & 0 \\ 0 & 0 \end{bmatrix}, \quad \rho_f^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \rho_g^1 = \begin{bmatrix} 0 & 0 \\ 0 & -0.04 \end{bmatrix}, \quad \rho_g^2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.02 \end{bmatrix}.$$

In the example, the probability is taken as $\bar{\delta} = 0.86$, the stochastic time delays are set as $\tau_1 = 1$, $\tau_2 = 3$, and the disturbance attenuation level is chosen as $\gamma = 0.95$.

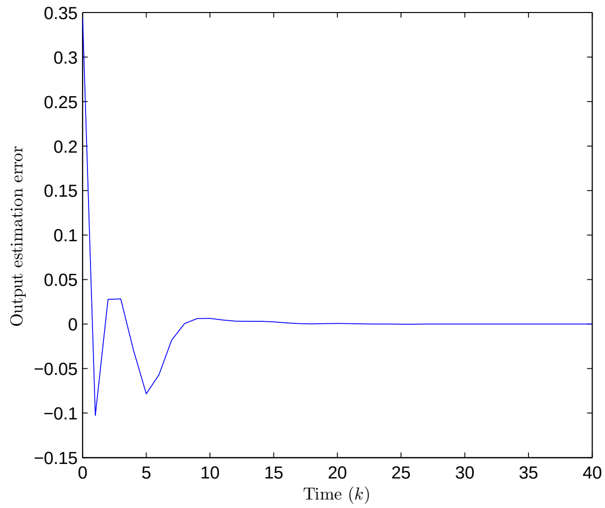

Figure 1. Output estimation error $\tilde{z}(k)$.

By solving the LMI (21) in Theorem 3 with the help of Matlab toolbox, we can obtain P_2 and X as follows:

$$P_2 = \begin{bmatrix} 0.1846 & 0.0533 \\ 0.0533 & 0.1436 \end{bmatrix}, \quad X = \begin{bmatrix} -0.4065 & 0.4750 \\ 0.3992 & -0.0277 \end{bmatrix}.$$

Then, according to $K = P_2^{-1}X$, the desired estimator parameter is designed as

$$K = \begin{bmatrix} -3.3658 & 2.9443 \\ 4.0281 & -1.2856 \end{bmatrix}.$$

In the simulation, the external disturbance inputs are assumed to be $\varsigma_1(k) = \varsigma_2(k) = \xi_1(k) = \xi_2(k) = 3\exp(-0.3k)\cos(0.2k)$. Simulation results are shown in Figs. 1-3. The output estimation error $\tilde{z}(k)$ is presented in Fig. 1. Figs. 2 and 3 plot the state and those estimates for node 1 and node 2, respectively. Simulation results show that the estimator designed works well.

5. Conclusions

In this paper, the robust H_∞ state estimation problem has been studied for a class of DMRNNs with stochastic time delays. The stochastic time delays under consideration are assumed to switch randomly between two values according to the Bernoulli distribution. In order to estimate the neuron states, an estimator has been constructed. By employing the difference inclusion theory and the stochastic analysis technique, a sufficient condition has been obtained to ensure the exponential mean-square stability of the output estimation error dynamics and the prescribed H_∞ performance requirement is met. Based on the derived sufficient condition, the explicit expression of the desired estimator gain has been given. Finally, a numerical example has been provided to show the usefulness and effectiveness of the proposed estimator design method.

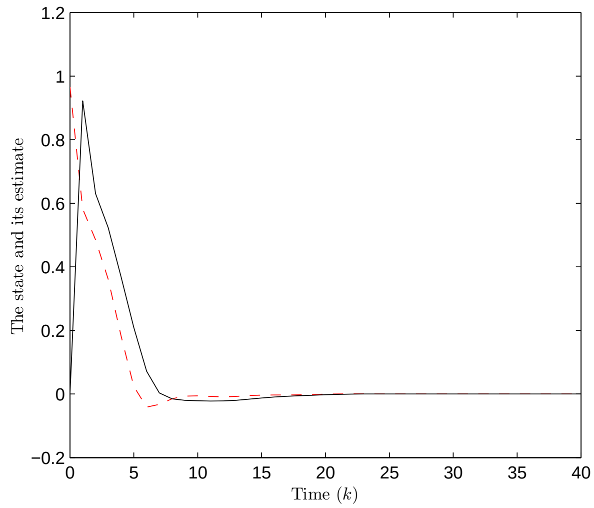

Figure 2. The state and its estimate of node 1.

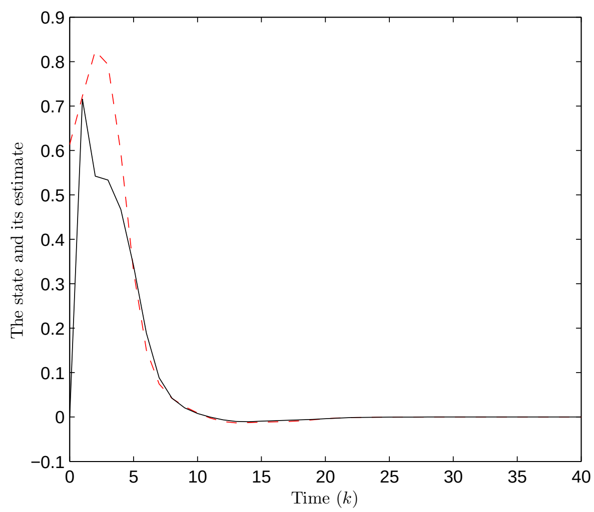

Figure 3. The state and its estimate of node 2.

Funding

This work was supported in part by the National Natural Science Foundation of China [grant number 61329301], [grant number 61134009] and [grant number 61473076], the Shu Guang project of Shanghai Municipal Education Commission and Shanghai Education Development Foundation [grant number 13SG34], the Fundamental Research Funds for the Central Universities, the DHU Distinguished Young Professor Program, and the Alexandervon Humboldt Foundation of Germany.

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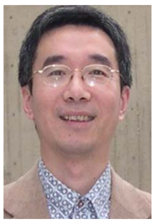

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