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Contact Modelling in Isogeometric Analysis: Application to Sheet Metal Forming Processes

Rui P.R. Cardoso[1], **O.B. Adetoro**[2] **and D. Adan**[2]

[1]Department of Mechanical, Aerospace and Civil Engineering, Brunel University London, UB8 3PH Uxbridge, London, UK

[2]Department of Engineering Design and Mathematics, University of the West of England, BS16 1QY Bristol, UK

E-mail: `Rui.Cardoso@brunel.ac.uk`

Abstract. Isogeometric Analysis (IGA) has been growing in popularity in the past few years essentially due to the extra flexibility it introduces with the use of higher degrees in the basis functions leading to higher convergence rates. IGA also offers the capability of easily reproducing discontinuous displacement and/or strain fields by just manipulating the multiplicity of the knot parametric coordinates. Another advantage of IGA is that it uses the Non-Uniform Rational B-Splines (NURBS) basis functions, that are very common in CAD solid modelling, and consequently it makes easier the transition from CAD models to numerical analysis. In this work it is explored the contact analysis in IGA for both implicit and explicit time integration schemes. Special focus will be given on contact search and contact detection techniques under NURBS patches for both the rigid tools and the deformed sheet blank.

1. Introduction

The constant drive for change in computational methods has created an increased demand for new computational tools as are the cases of meshless methods and, more recently, Isogeometric Analysis (IGA) based on Non-Uniform Rational B-Splines (NURBS). IGA is a powerful numerical tool for engineers and scientists as far as it allows accurate predictions of structural and fluid mechanics directly on the CAD geometries and at low computational costs. Besides many other advantages of IGA, numerical analysis directly on NURBS objects avoids the time consuming step of mesh generation, providing in this way a more competitive approach for numerical simulations in general. NURBS are widely used for geometrical modelling in many CAD software packages and, in many cases, are the standard for the interchange of many graphical formats (e.g., IGES, STEP, ACIS, STL, etc.). IGA was introduced successfully in the work of Hughes et al. [1] and then it evolved and as a result many other reference publications appeared over the term of the past few years. The isogeometric concept uses the same approximation basis functions for the geometry and for the displacement field at the NURBS's control points. The main difference from the finite element method is that the basis functions in IGA are always positive and they are not interpolatory, which causes some difficulties to handle essential boundary conditions.

In this work the contact analysis is implemented under the IGA framework and NURBS geometric approximation functions are going to be defined for contact analysis for both the rigid tools and the deformable blank sheet. The contact weak forms are developed with Lagrange multipliers for the implicit time integration scheme and with the penalty formulation for the explicit dynamic method. Special focus is given on the contact search and contact detection under IGA with NURBS objects.

2. Non-Uniform Rational B-Splines (NURBS) and IsoGeometric Analysis (IGA)

Detailed overview of B-Spline curves, B-Splines surfaces and NURBS can be found in Piegl and Tiller [2] and Cottrell et al. [3].

2.1. Knot vectors

An open knot vector is a set of non-negative parametric coordinates which are repeated $p+1$ times at the beginning and at the end of the vector (p is the order of the polynomial basis functions). For 1D:

$$\Xi = \left\{ \ \xi_1, \ldots, \xi_{p+1}, \ldots, \xi_{m+1}, \ldots, \xi_{m+p+1}, \ \right\} \tag{1}$$

where m is the number of control points or basis functions.

2.2. Control points and basis functions

For a specific local parametric coordinate ξ from an open knot vector and for a degree p of the polynomial, the basis functions are obtained recursively from the following formulae [3]:

$$N_I^p = \frac{\xi - \xi_I}{\xi_{I+p} - \xi_I} N_I^{p-1}(\xi) + \frac{\xi_{I+p+1} - \xi}{\xi_{I+p+1} - \xi_I} N_{I+1}^{p-1}(\xi), \tag{2}$$

where I is the index for the basis functions.

The Non-Uniform Rational B-Splines (NURBS) are rational polynomials obtained from a weighted linear combination of the basis functions with their control points as coefficients. In this way, a NURBS solid is constructed from a three-dimensional knot set $\Xi \times H \times Z$ and a net of control points A_{IJK} and weights W_{IJK}:

$${}_{W}T^{p,q,r}(\xi, \eta, \zeta) = \frac{\sum_{k,I,J=1}^{l,n,m} N_K^r(\zeta) N_I^q(\eta) N_J^p(\xi) W_{IJK} A_{IJK}}{W}. \tag{3}$$

with:

$$W = \sum_{k=1}^{l} \sum_{I=1}^{n} \sum_{J=1}^{m} N_K^r(\zeta) N_I^q(\eta) N_J^p(\xi) W_{IJK} \tag{4}$$

being the NURBS weight function for a solid.

The rational basis functions from equation (3) are commonly used for the approximation of the kinematics of the continuum giving rise to the well-known IsoGeometric Analysis (IGA). Many works were already published on the development of different formulations for IGA making use of the NURBS approximations from equation (3). Cardoso and Cesar de Sa [4] used the enhanced assumed strain method in combination with NURBS basis functions for the increase of the subspace of incompressible deformations and, more recently, Cardoso and Cesar de Sa [5] used the Moving Least Square (MLS) approximation to project the kinematics from a fully-integrated NURBS patch into a reduced integrated space so that different kinds of locking pathologies could be alleviated or avoided. In this work, the formulation of Cardoso and Cesar de Sa [5] for a reduced integrated 3D NURBS patch is going to be utilised.

3. Contact Analysis in IGA

3.1. Contact search/detection in IGA

In Figure 1 (b) it can be seen clearly the non-interpolatory nature of NURBS in such that the control points (represented by the white circles in the figure) are not in the curve. Instead, the dark squares in the figure represent the coordinates of a point in the curve after being approximated by the basis functions from equation (3) from a knot value defined in the parametric space of the knot vectors.

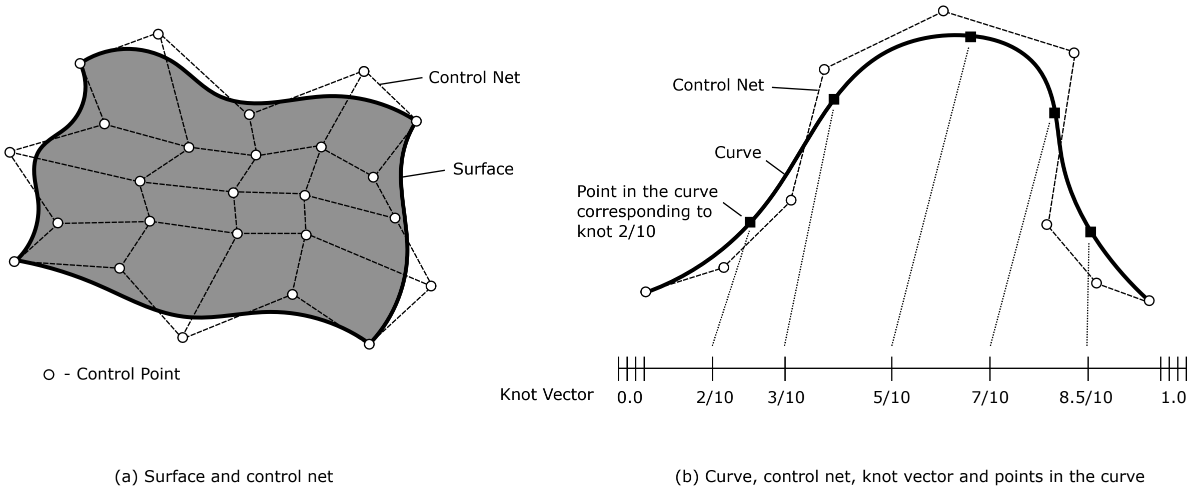

Figure 1. Definitions for control points, control net and knot vector. (a) NURBS surface and its control net. (b) 2D curve with its control points, knot vector in the parametric space and points in the curve after approximation with NURBS basis functions.

For contact search/detection is fundamental that the algorithm goes through the physical coordinates on the curve rather than through the control net. For example, the calculation of the gap in contact needs to be done from the detection of penetration between points in the curves/surfaces rather than the control points on their control nets. Due to the non-interlapolatory character of NURBS, special methods, such as point inversion [2] for the determination of the parametric knot coordinates corresponding to a minimum distance to a contacting point needs to be used.

The kinematic constraint of zero gap is imposed through the use of Lagrange multipliers in implicit time integration and through the use of a penalty parameter for the explicit time integration scheme. The stationary of the energy in its weak form for the explicit time integration becomes:

$$\delta\pi(\mathbf{u}) = \int_\Omega \left(\delta\boldsymbol{\epsilon}(\mathbf{u}) : \boldsymbol{\sigma}(\mathbf{u}) - \delta\mathbf{u}\cdot\mathbf{b} + \delta\mathbf{u}\cdot m\ddot{\mathbf{u}}\right) d\Omega - \int_{\Gamma_t} \delta\mathbf{u}\cdot\bar{\mathbf{t}}\, d\Gamma_t + \alpha\int_{\Gamma_u} \delta\left(\mathbf{u}-\bar{\mathbf{u}}\right) d\Gamma_u = 0 \quad (5)$$

where $\delta\bar{\mathbf{u}}$ is the vector with the penetration gap, which is obtained from the procedures schematically described in Figure 2. For implicit analysis, the stationary of the energy is written as:

$$\begin{aligned} \int_\Omega \left(\delta\boldsymbol{\epsilon}(\mathbf{u}) : \boldsymbol{\sigma}(\mathbf{u}) - \delta\mathbf{u}\cdot\mathbf{b} + \delta\mathbf{u}\cdot m\ddot{\mathbf{u}}\right) d\Omega - \int_{\Gamma_t} \delta\mathbf{u}\cdot\bar{\mathbf{t}}\, d\Gamma_t + \lambda\int_{\Gamma_u} \delta\left(\mathbf{u}-\bar{\mathbf{u}}\right) d\Gamma_u &= 0 \\ \delta\lambda\int_{\Gamma_u} \left(\mathbf{u}-\bar{\mathbf{u}}\right) d\Gamma_u &= 0 \quad (6) \end{aligned}$$

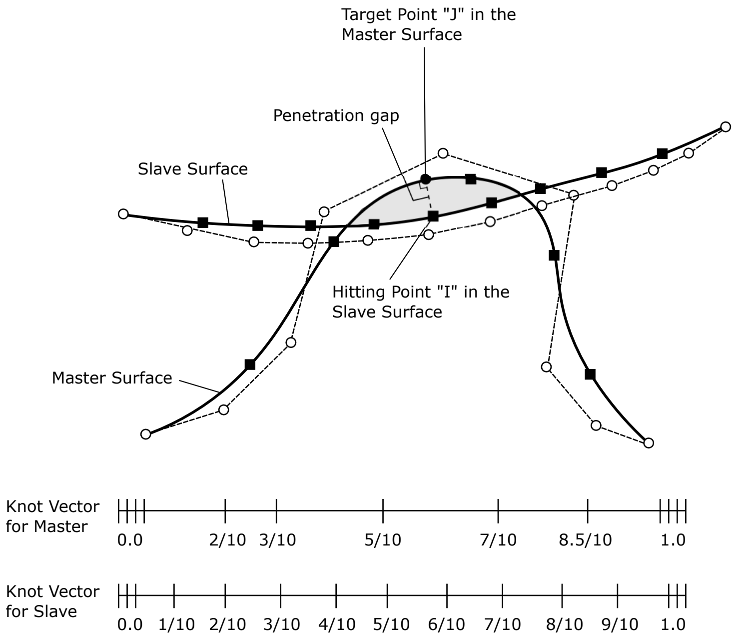

Figure 2. Contact detection between a master surface (rigid dies) and a slave surface (deformable blank sheet).

where λ are the additional Lagrange multipliers. The weak form equations for contact analysis in IGA are in fact the same as for the finite element method, however they differ in the way the kinematic quantities are interpolated/approximated.

4. Conclusions

In this work it is given a brief overview of the main challenges of developing contact formulations for IGA. The major difficulty in modelling contact in IGA is due to the non-interpolatory character of IGA, which introduces some complexities in contact search and on the determination of the penetration gap for the imposition of the kinematic constraints. The weak forms are similar to the weak forms used in the finite element method but however the approximation of the kinematic variables is performed from the use of the NURBS basis functions from equation (3).

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