Active Fault Tolerant Control for an Internet-based Networked Three-Tank System

Xiao He, Zidong Wang, Liguo Qin and D. H. Zhou

Abstract

This paper is concerned with the active fault tolerant control (FTC) problem for an Internet-based networked three-tank system (INTTS) serving as a benchmark system for evaluating networked FTC algorithms. The INTTS has two parts located at Tsinghua University in China and at the University of South Wales in the UK, respectively, which are connected via Internet. With the INTTS as an experimental platform, the active FTC problem is investigated for a class of nonlinear networked systems subject to partial actuator failures. A binary switching random sequence with a known distribution is employed to describe the packet dropout phenomena induced by network cables with limited-capacity. Once a specific actuator failure is detected and confirmed by a fault diagnosis unit, the control law is then reconfigured based on the information of the detected fault. Both the stability and the H_∞ disturbance attenuation performance are guaranteed for the closed-loop system by using the remaining reliable actuators. Extensive experiments are carried out on the active fault tolerant control problem of the INTTS with partial actuator failures and the effectiveness of the proposed scheme is illustrated.

Keywords

Active fault tolerant control, Internet-based networked three-tank system, actuator failure, controller reconfiguration, random packet dropout.

I. Introduction

Manufactured by the Amira Automation Company in Germany, the Three-Tank System (TTS) DTS 200 has been used as a benchmark system to evaluate the algorithms for the fault detection and diagnosis (FDD) problems and the fault tolerant control (FTC) problems in the past decades. So far, the TTS DTS 200 has been widely adopted in developing FDD and/or FTC techniques due mainly to the facts that its nonlinear mathematical model can be precisely established and the leakage/sensor/actuator faults can be easily realized in the equipment manually. The profile of DTS 200

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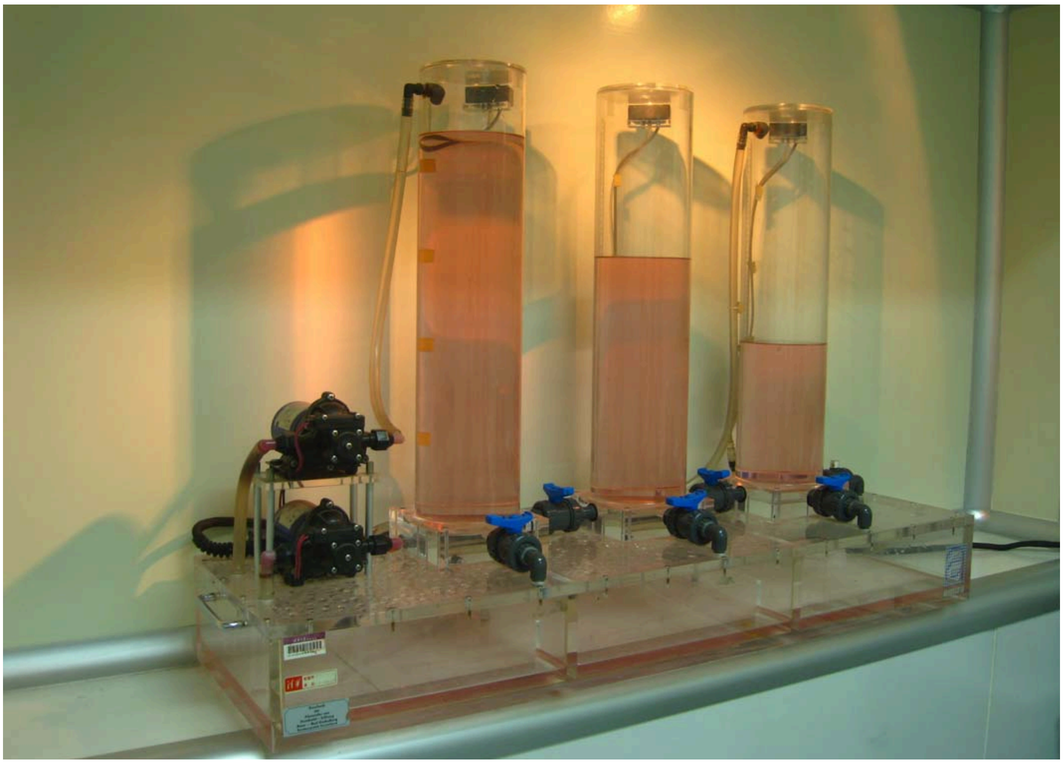

Fig. 1. A benchmark system: DTS 200

is shown in Fig. 1 and the detailed description of this system can be found in [1, 2]. Based on this benchmark system, much work has been done on FDD and/or FTC problems, see e.g. [3–9].

The rapid development of network technologies has recently stimulated wide applications of the so-called networked systems (NSs). The NSs are defined as the control/sensing systems using communication network cable to exchange data (instead of using ideal point-to-point wires). This kind of systems has many advantages as compared with the traditional point-to-point systems and many successful applications have been reported in the literature during the past ten years [10, 11]. To better verify the networked filtering/FDD/FTC algorithms, we have modified the TTS into the so-called Internet-based networked three-tank system (INTTS) [2]. The INTTS has been built up by using the NetCon platform [12] that connects the TTS at Tsinghua University in China and the controller/fault diagnosis unit at the University of South Wales in the UK through Internet, see Fig. 2. Based on the INTTS, the problems of networked state estimation, fault detection and isolation as well as fault tolerant control have been investigated [13, 14]. Note that the main difficulties in dealing with these networked analysis/design problems stem from the incomplete signal phenomena in a networked environment, which include communication delays, packet dropouts, signal quantizations as well as multiple packet transmissions. These phenomena hinder the direct usage of traditional FDD/FTC algorithms on networked systems [16, 17].

An FTC system is a closed-loop control system capable of maintaining its stability and desired

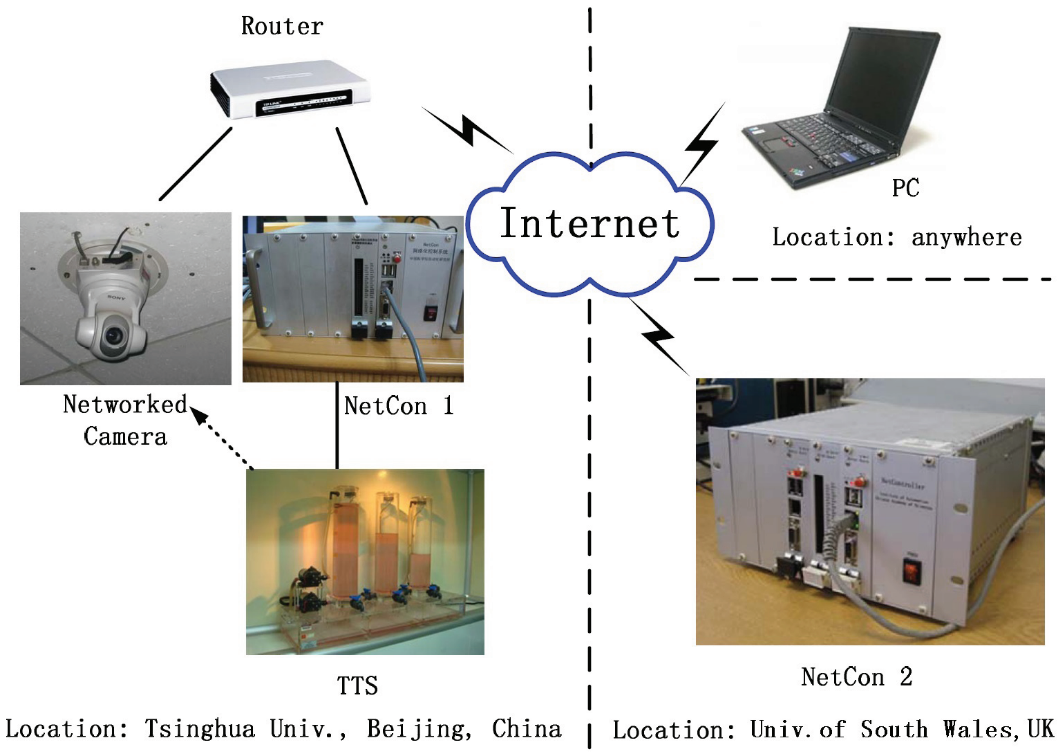

Fig. 2. Setup of INTTS

performance even in the presence of faults [18]. The analytical redundancy-based FTCS can generally be classified into two categories: *the passive FTCS* where controllers are designed fixedly to be robust against a specific class of faults as well as *the active FTCS* where the system component failures are reacted actively and the controller is reconfigured to accommodate the faults, see [19–21]. FTC results for traditional systems can be found in [22, 23]. However, compared with the fruitful FDD results for networked systems, the FTC problems in networked environments have gained relatively less research attention and the corresponding results have been scattered [24, 25].

It should be noticed that most existing FTC results for NSs have been concerned with the passive FTC strategies. For a passive FTC strategy, the same controller is used before and after the fault occurrence, and this may give rise to the inability for the controlled system to achieve its best performance in the healthy case. Apparently, it would be less conservative yet more challenging to research into the active FTC problem for NCSs than its passive counterpart. Unfortunately, to date, there have been very few results reported for the FTC problems of NCSs especially in the practical applications. Furthermore, as is well known, the plant and signal are inevitably simulant for numerical simulations and, therefore, it is of more significance to consider a physical system in order to better verify and implement the designed FTC algorithms and methods in practice.

In [15], an active fault tolerant control problem has been investigated for an INTTS with partial actuator faults, where only the linear case has been considered. In our present paper, a more realistic nonlinear system model of INTTS is employed, based on which the active fault tolerant control problem is thoroughly investigated. The main contributions of this paper are highlighted as follows: 1) an active fault tolerant control problem is studied for a nonlinear networked system with partial actuator failures, where the addressed kind of nonlinearity is more general than the usually investigated Lipschitz-type nonlinearity; 2) a new fault isolation technique is proposed which is proven to be more efficient than the residual contribution degree approach proposed in [2]; 3) experiments over a practical INTTS are conducted and the proposed techniques are verified via a real networked system.

Notation. The notations used throughout this paper are fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. $P > 0$ (respectively, $P < 0$) means that the matrix P is real, symmetric and positive definite (respectively, negative definite). The subscript "T" denotes the matrix transpose. $\Pr\{\cdot\}$ represents the occurrence probability of the event $\cdot$, and $\mathbb{E}\{y\}$ is the mathematical expectation of a stochastic variable y. $l_2[0,\infty)$ is the space of all square-summable vector functions over $[0,\infty)$, and $\|x\|$ is the standard l_2 norm of x, i.e., $\|x\| = (x^T x)^{1/2}$, and $\mathbb{Z}^+$ stands for the set of nonnegative integers. $x^{(i)}$ stands for the ith component of vector x. In symmetric block matrices, we use "$*$" to represent a term that is induced by symmetry, Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. System Description and Problem Formulation

The controlled system considered in this paper is an INTTS that has been built as a benchmark system for the evaluation of networked fault diagnosis and FTC algorithms. It connects the equipment at Tsinghua University (Beijing, China) with that at the University of South Wales (Wales, UK) through Internet. The whole INTTS system is composed of a TTS DTS200 [9], two NetCon equipment used for the implementation of control and fault diagnosis algorithms as well as the Internet interface, a networked camera used for real time on line monitoring, a PC used for downloading the algorithms and operational observation, as well as the Internet used for data transmission. The hardware structure is shown in Fig. 2, and the detailed description of INTTS can be found in [2].

In this paper, we are interested in investigating the active fault tolerant control problem for the INTTS with partial actuator faults, see Fig. 3 for the block diagram. The random packet dropout phenomena in both the forward and backward data transmission processes are taken into account. Once an actuator fault is detected and isolated by the fault diagnosis unit comprising a fault detection filter and a fault isolation strategy, the controller is reconfigured so as to guarantee system performance by using the remaining reliable actuators.

After model simplification and discretization [2], the mathematical model of the TTS can be de-

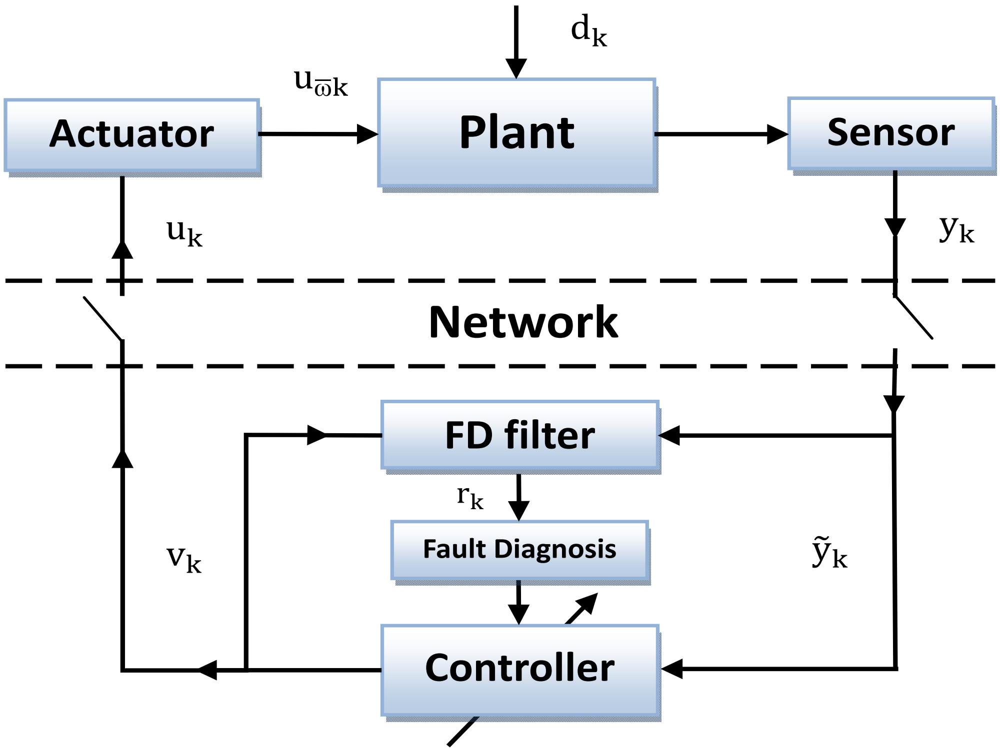

Fig. 3. Block diagram of the active fault tolerant control scheme

scribed by the following nonlinear discrete-time system:

$$\begin{cases} x_{k+1} = Ax_k + Hg(x_k) + Bu_k + Dd_k, \\ z_k = Ex_k, \\ y_k = Cx_k, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state vector (water levels in the three tanks); $u_k \in \mathbb{R}^{n_u}$ is the control input (water inflow of the three pumps); $d_k \in \mathbb{R}^{n_d}$ is the disturbance belonging to $l_2[0, \ \infty)$; $y_k \in \mathbb{R}^{n_y}$ is the measurement output (water heights in three tanks) and $z_k \in \mathbb{R}^{n_z}$ is the controlled output (the weighted average of the offsets of the three water levels from the desired value). $g(x_k)$ is a vector-valued nonlinear function satisfying the sector-bounded condition [27, 28]

$$[g(x_k) - T_1 x_k]^T [g(x_k) - T_2 x_k] \leq 0, \quad \forall x_k \in \mathbb{R}^{n_x}, \tag{2}$$

where T_1 and T_2 are known real constant matrices and $T = T_1 - T_2$ is symmetric positive definite matrices. Let $x_0 \in \mathbb{R}^{n_x}$represent the known initial condition. All the system matrices are known, real, and constant with appropriate dimensions.

Remark 1: Since the three-tank system is essentially nonlinear, the term $g(x_k)$ is preserved in the modeling process in order to characterize the nonlinearities resulting from the unmodeled dynamics, linearization errors, external disturbances, etc. The nonlinearity descriptions in (2), called sector-bounded nonlinearities, are quite general that include the usual Lipschitz conditions as a special case.

Assumption 1: The measurement matrix of system (1), C, is a matrix with full column rank.

Due to limited bandwidth of the communication channel, data packets carrying measurement signals or control signals could be lost during the transmission process and such phenomena can be modeled by

$$\left\{ \begin{array}{rcl} \tilde{y}_k & = & \lambda_{1k} y_k, \\ u_k & = & \lambda_{2k} v_k, \end{array} \right. \tag{3}$$

where $\tilde{y}_k \in \mathbb{R}^{n_y}$ is the measurement received at the controller node and $v_k \in \mathbb{R}^{n_u}$ is the control signal calculated in the same place before being transmitted to the plant. λ_{1k} and λ_{2k} are two Bernoulli distributed stochastic variables that reflect the random data dropouts occurred in the sensor-to-controller channel and controller-to-actuator channel, respectively.

Assumption 2: The mathematical expectations

$$\mathbb{E}\{\lambda_{1k}\} = \beta_1 \; and \; \mathbb{E}\{\lambda_{2k}\} = \beta_2,$$

as well as the correlation coefficient ρ between λ_{1k} and λ_{2k} are all known scalars that can be obtained *a priori* through statistical tests, see [29].

In the present work, we focus on the situation where the data transmission process may suffer from random packet dropout and partial actuator faults. We are interested in the analysis and design problem of an active fault-tolerant controller, which includes a fault diagnosis unit followed by a controller reconfiguration strategy. The fault diagnosis, unit is used to detect and locate the faulty actuator. Once an actuator fault is diagnosed, the control gain is then reconfigured to a proper one corresponding to the faulty actuator. The plant is stabilized with certain performance achieved via manipulating the remaining reliable actuators before the faulty actuator is repaired or replaced.

Consider a fault detection filter of the following form:

$$\left\{ \begin{array}{rcl} \tilde{x}_{k+1} & = & G\tilde{x}_k + N\tilde{y}_k + Mv_k, \\ r_k & = & L\tilde{x}_k. \end{array} \right. \tag{4}$$

where $\tilde{x}_k$ is the filter state and r_k is the residual signal that indicates whether there is a fault. G, N, M and L are the filter parameters to be designed.

The controller is chosen as follows:

$$v_k = K_{FTC}\tilde{y}_k, \tag{5}$$

where

$$K_{FTC} = \left\{ \begin{array}{ll} K_h, & \text{Fault free} \\ K_f, & \text{Actuator faults} \end{array} \right. \tag{6}$$

The control gains, in both the faulty case and the fault free case, will be designed within an H_∞ framework. Furthermore, the fault tolerant controller (with respect to partial actuator faults) can be realized with the following strategy: when there is no fault detected, a normal H_∞ control gain K_h is utilized and, once some specific actuator faults are confirmed, the control gain is switched to K_f that is designed according to the faulty actuators.

Assumption 3: Although fault may occur with one or more actuators, the whole system can still be stabilized by the remaining reliable actuators.

Assumption 4: here is at most one fault that can occur at a certain time instant.

We are now in a position to state the main problems to be addressed in this paper. The purpose of this paper is to deal with the following three interrelated problems.

Problem 1: Design an H_∞ controller (6) for system (1) with gain K_f in the fault-free situation.

Problem 2: Design a fault detection filter (4) for system (1) and a fault isolation strategy to diagnose the actuator fault.

Problem 3: Design an H_∞ fault tolerant control law (6) for system (1) with gains K_f corresponding to the controller with a specific failure actuator. Once an actuator fault is determined, the control gain is switched to K_f.

Remark 2: Note that Problem 1 is actually a special case of Problem 3 if no fault occurs, and this indicates that Problem 1 can be solved by solving Problem 3 with a constraint that there is no actuator fault.

III. Main Results

In this section, the main results of this paper are obtained that include the design of a fault diagnosis unit and a controller reconfiguration strategy.

Let $\Omega \subseteq \{1, 2, \dots, n_u\}$ denote the set of actuators that are susceptible to fail and $\bar{\Omega} = \{1, 2, \dots, n_u\} \backslash \Omega$ represent the set of reliable actuators. Based on this, the matrix B can be decomposed as

$$B = B_{\bar{\omega}} + B_\omega, \tag{7}$$

where $B_{\bar{\omega}}$ and B_ω are determined from B by setting the columns corresponding to $\bar{\Omega}$ and Ω as zeros, respectively. The outputs of the faulty actuators $u_{\omega k}$ are considered to be arbitrary signals belonging to $l_2[0, \infty)$, which can be regarded as external disturbances of the system. The output of the reliable actuators $u_{\bar{\omega} k}$ is used for the control purpose.

A. The Fault Diagnosis Unit

A.1 The Residual Generation Filter Design

By introducing an auxiliary term, system (1) can be rewritten as

$$\left\{\begin{array}{rcl} x_{k+1} &=& Ax_k + Hg(x_k) + Bu_k + Dd_k + B_\omega f_k, \\ z_k &=& Ex_k, \\ y_k &=& Cx_k, \end{array}\right. \tag{8}$$

where $f_k \in \mathbb{R}^{n_u}$ is the fault signal and B_ω is the same as defined in (7).

Remark 3: Note that $B_\omega f_k$ is employed to characterize the actuator fault. $f_k = 0$ represents the normal case with no fault; $f_k = -u_k$ corresponds to the case that there is one or more actuator failure faults; and $f_k = \alpha u_k$ describes partial actuator gain faults with a gain $1+\alpha$.

Define $\eta_k = \begin{bmatrix} x_k^T & \tilde{x}_k^T \end{bmatrix}^T$, $\zeta_k = \begin{bmatrix} f_k^T & d_k^T \end{bmatrix}^T$ and $\tilde{r}_k = r_k - f_k$. According to (3)-(5) and (8), we have the following augmented system:

$$\begin{cases} \eta_{k+1} &= \mathfrak{A}\eta_k + (\lambda_k - \beta)\mathfrak{X}\eta_k + \mathfrak{H}g(Z\eta_k) + \mathfrak{B}\zeta_k, \\ \tilde{r}_k &= \mathfrak{C}\eta_k + \mathfrak{D}\zeta_k. \end{cases} \tag{9}$$

where $\lambda_k := \lambda_{1k}\lambda_{2k}$, $\beta = \mathbb{E}\{\lambda_k\}$ and

$$\begin{aligned} \mathfrak{A} &= \begin{bmatrix} A + \beta B K_h C & 0 \\ \beta(N + M K_h C) & G \end{bmatrix}, \\ \mathfrak{X} &= \begin{bmatrix} B K_h C & 0 \\ (N + M K_h C) & 0 \end{bmatrix}, \quad \mathfrak{H} = \begin{bmatrix} H \\ 0 \end{bmatrix}, \\ \mathfrak{B} &= \begin{bmatrix} B_\omega & D \\ 0 & 0 \end{bmatrix}, \quad \mathfrak{C} = \begin{bmatrix} 0 & L \end{bmatrix}, \\ \mathfrak{D} &= \begin{bmatrix} -I & 0 \end{bmatrix}, \quad Z = \begin{bmatrix} I & 0 \end{bmatrix}. \end{aligned} \tag{10}$$

From the above manipulations, the original fault detection filter design problem can be converted into an H_∞ filter design problem. Moreover, Problem 2 can be reformulated as follows: design a fault detection filter of the form (4) such that, for all possible packet dropouts, the augmented system (9) is asymptotically mean-square stable [30] and the H_∞ disturbance attenuation level

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|\tilde{r}_k\|^2\right\} \leq \gamma_1^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|\zeta_k\|^2\right\} \tag{11}$$

is achieved for a prescribed scalar γ_1.

The following theorem is provided to solve Problem 2.

Theorem 1: Consider the INTTS (1), (3) and (5) with a given control gain K_h. For possible partial actuator faults, a desired fault detection filter exists if there exist matrices $0 < V^T = V \in \mathbb{R}^{n_x \times n_x}$, $0 < F^T = F \in \mathbb{R}^{n_x \times n_x}$, $\bar{G} \in \mathbb{R}^{n_x \times n_x}$, $\bar{N} \in \mathbb{R}^{n_x \times n_y}$, $\bar{M} \in \mathbb{R}^{n_x \times n_u}$, $\bar{L} \in \mathbb{R}^{n_u \times n_x}$, as well as a scalar δ such that the following linear matrix inequality (LMI) holds

$$\begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 & \bar{L} & 0 & -I & 0 \\ * & -V & -F & 0 & 0 & \Phi_{26} & \Phi_{27} & 0 & 0 & 0 \\ * & * & -F & 0 & 0 & \Phi_{36} & \Phi_{37} & 0 & 0 & 0 \\ * & * & * & -V & -F & \Phi_{46} & \Phi_{47} & VH & VB_\omega & VD \\ * & * & * & * & -F & \Phi_{56} & \Phi_{57} & FH & FB_\omega & FD \\ * & * & * & * & * & -V-\delta\mathcal{T}_1 & -F-\delta\mathcal{T}_1 & -\delta\mathcal{T}_2 & 0 & 0 \\ * & * & * & * & * & * & -F-\delta\mathcal{T}_1 & -\delta\mathcal{T}_2 & 0 & 0 \\ * & * & * & * & * & * & * & -\delta I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\gamma_1^2 I & 0 \\ * & * & * & * & * & * & * & * & * & -\gamma_1^2 I \end{bmatrix} < 0, \tag{12}$$

where $\Phi_{26} = \mu VBK_hC + \mu\bar{N}C + \mu\bar{M}K_hC$, $\Phi_{27} = \mu VBK_hC + \mu\bar{N}C + \mu\bar{M}K_hC$, $\Phi_{36} = \mu FBK_hC$, $\Phi_{37} = \mu FBK_hC$, $\Phi_{46} = VA + \beta VBK_hC + \beta\bar{N}C + \beta\bar{M}K_hC$, $\Phi_{47} = VA + \beta VBK_hC + \beta\bar{N}C + \beta\bar{M}K_hC + \bar{G}$, $\Phi_{56} = FA + \beta FBK_hC$, $\Phi_{57} = FA + \beta FBK_hC$, $\mu = \sqrt{\beta(1-\beta)}$, $\mathcal{T}_1 = (T_1^T T_2 + T_2^T T_1)/2$, $\mathcal{T}_2 = -(T_1^T + T_2^T)/2$, and $\beta = \mathbb{E}\{\lambda_{1k}\lambda_{2k}\} = \rho\sqrt{\beta_1\beta_2(1-\beta_1)(1-\beta_2)} + \beta_1\beta_2$. Furthermore, if (12) is feasible, the gain matrices of the desired fault detection filter are given by

$$\begin{aligned} G &= (F-V)^{-1}\bar{G}, \quad N = (F-V)^{-1}\bar{N}, \\ M &= (F-V)^{-1}\bar{M}, \quad L = \bar{L}. \end{aligned} \tag{13}$$

Proof: The proof can be conducted along the similar line of Theorem 1 in [31] by using the techniques developed in [27, 28] to handle the nonlinearity, and therefore the proof is omitted for brevity. ■

A.2 Fault Detection Strategy

Having generated the residual by the fault detection filter, we consider a residual evaluation function:

$$J_k = \left\{ \sum_{s=0}^{k} r_s^T r_s \right\}^{1/2}. \tag{14}$$

It can be inferred that the output of the residual evaluation function J_k is a scalar at a certain time instant k. The occurrence of the faults can be alarmed by comparing $J(k)$ with a prescribed threshold J_{th} according to the following rule:

$$\begin{aligned} \Delta J_{k,\mathcal{L}} > J_{th}^{\mathcal{L}} &\implies \text{fault detected}, \\ \Delta J_{k,\mathcal{L}} \leq J_{th}^{\mathcal{L}} &\implies \text{no faults}, \end{aligned} \tag{15}$$

where

$$\Delta J_{k,\mathcal{L}} := J_k - J_{k-\mathcal{L}}, \quad J_{th}^{\mathcal{L}} = \sup_{k\in\mathbb{Z}+,\ d_k\in l_2,\ f_k=0} \mathbb{E}\{J_k - J_{k-\mathcal{L}}\}. \tag{16}$$

and $\mathcal{L}$ is the length of a time window considered for evaluating the residual for a fault alarm.

A.3 Fault Isolation Strategy

Consider a new vector $\tilde{J}_k$ whose ith component is given by

$$\tilde{J}_k^{(i)} := \left\{ \sum_{s=0}^{k} (r_s^{(i)})^2 \right\}^{1/2} \tag{17}$$

which is called the fault isolation residual evaluation signal. Once an actuator fault is detected, a Weighted Residual Component Comparison (WRCC) approach is employed to locate the fault. The main idea of the WRCC approach is explained as follows.

Let $OA(\beta_1, \beta_2, \ldots, \beta_n)$ denote an ordered array with $\beta_1 < \beta_2 < \ldots < \beta_n$ for a set $\{\beta_1, \beta_2, \ldots, \beta_n\}$. $SE_\beta^{(n)}$ is defined as the set of all possible ordered arrays. It is obvious that $OA(\beta_1, \beta_2, \ldots, \beta_n) \in SE_\beta^{(n)}$. Note that, for an n_u-dimensional fault input, $\tilde{J}_k^{(i)}$ can be generated by the fault detection filter designed in Subsection III-A.1.

At the time instant k_d when the fault is detected, there is a set $\{\tilde{J}_{k_d}^{(1)}, \tilde{J}_{k_d}^{(2)}, \ldots, \tilde{J}_{k_d}^{(n_u)}\}$, where the n_u components of the residual evaluation signal $\tilde{J}_{k_d}$ are its elements. For an arbitrary fault vector f_k, if one can find a set of scalars $\{\alpha_1, \alpha_2, \ldots, \alpha_{n_u}\}$ such that, for any $1 < i < n_u$, $\bar{J}_{k_d}^{(i)} = \alpha_i \tilde{J}_{k_d}^{(i)}$, then there exists a subset $SB_{\bar{J}}^{(n_u)} \subset SE_{\bar{J}}^{(n_u)}$ that has one-to-one mapping with the fault set $\{f_{k_d}^{(1)}, f_{k_d}^{(2)}, \ldots, f_{k_d}^{(n_u)}\}$. Once the scalars are determined, we can obtain an ordered array at the time instant k_d and, if it belongs to $SB_{\bar{J}}^{(n_u)}$, we can isolate the fault based on the one-to-one mapping relationship.

Remark 4: In the practical fault diagnosis over the INTTS, it can be observed that the faults occurring in the same position lead to the same residual form no matter how much the amplitude of the faults is. In view of this, we propose the aforementioned WRCC approach to isolate the possible fault. Compared with the fault isolation method based on the residual contribution degree (RCD) proposed in [2], this approach can utilize more information from the residual evaluation signal as opposed to simply comparing the magnitudes of its components. Furthermore, for the subset $SE_{\bar{J}}^{(n_u)}$, there are $n_u!$ elements in total and it is generally easy to find a subset $SB_{\bar{J}}^{(n_u)}$ with n_u elements which has a one-to-one mapping with the fault set.

B. Fault Tolerant Controller Design

Let us now design a fault tolerant control strategy. Denoting $w_k = \begin{bmatrix} u_{\omega k}^T & d_k^T \end{bmatrix}^T$, we can rewrite the closed-loop dynamics as

$$\left\{ \begin{array}{rcl} x_{k+1} &=& \mathcal{A}x_k + (\lambda_k - \beta)\mathcal{X}x_k + Hg(x_k) + \mathcal{B}w_k, \\ z_k &=& \mathcal{C}x_k, \end{array} \right. \tag{18}$$

where $\mathcal{A} = A + \beta B_{\bar{\omega}}KC$, $\mathcal{X} = B_{\bar{\omega}}KC$, $\mathcal{B} = [B_\omega \;\; D]$, $\mathcal{C} = E$.

As pointed out in Remark 2, Problem 1 can be regarded as a special case of Problem 3 with a constraint $\Omega = \varnothing$. This indicates that, in both the faulty case and fault-free case, the control gain (6) of a fault tolerant controller (5) can be determined by solving Problem 3. The main procedure is to find a fault tolerant control gain K_f such that, for a specific actuator fault, the whole system (18) is asymptotically mean-square stable [30] and the H_∞ disturbance attenuation level from w_k to z_k

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|z_k\|^2\right\} \leq \gamma_2^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|w_k\|^2\right\} \tag{19}$$

is made as small as possible.

In this subsection, a performance analysis result is firstly provided for a known controller, based on which a fault tolerant controller can be further designed.

Theorem 2: Consider the INTTS (1) and (3) with a given control law (5) and a specific actuator faults Ω. For a given scalar γ_2, the closed-loop system (18) is asymptotically mean-square stable and a prescribed H_∞ attenuation level is achieved if there are a positive definite matrix $P = P^T > 0$ and a scalar δ such that

$$\begin{bmatrix} -P & 0 & \mu P\mathcal{X} & 0 & 0 & 0 \\ * & -P & P\mathcal{A} & PH & P\mathcal{B} & 0 \\ * & * & -P - \delta\mathcal{T}_1 P & -\delta\mathcal{T}_2 & 0 & \mathcal{C}^T \\ * & * & * & -\delta I & 0 & 0 \\ * & * & * & * & -\gamma_2^2 I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{20}$$

holds where $\mathcal{T}_1$, $\mathcal{T}_2$, and μ are the same as defined in Theorem 1.

Proof: Let $P > 0$ and consider a Lyapunov function of the following form:

$$V_k = x_k^T P x_k. \tag{21}$$

In the case of $w_k = 0$, we take the following relationship into consideration

$$\begin{bmatrix} x_k \\ g(x_k) \end{bmatrix}^T \begin{bmatrix} \mathcal{T}_1 & \mathcal{T}_2 \\ \mathcal{T}_2^T & I \end{bmatrix} \begin{bmatrix} x_k \\ g(x_k) \end{bmatrix} \leq 0.$$

For a positive scalar δ, we have

$$\begin{aligned} \Delta V_k &= \mathbb{E}\{V_{k+1}\} - V_k = \mathbb{E}\left\{x_{k+1}^T P x_{k+1}\right\} - x_k^T P x_k \\ &\leq \begin{bmatrix} x_k \\ g(x_k) \end{bmatrix}^T \times \\ &\quad \left\{ \begin{bmatrix} \mathcal{A}^T P \mathcal{A} + \mu^2 \mathcal{X}^T P \mathcal{X} - P & \mathcal{A}^T P H \\ * & H^T P H \end{bmatrix} \right. \\ &\quad \left. - \begin{bmatrix} \delta \mathcal{T}_1 & \delta \mathcal{T}_2 \\ * & \delta I \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & I \end{bmatrix} \right\} \times \begin{bmatrix} x_k \\ g(x_k) \end{bmatrix} \\ &:= \begin{bmatrix} x_k \\ g(x_k) \end{bmatrix}^T \Psi \begin{bmatrix} x_k \\ g(x_k) \end{bmatrix}. \end{aligned} \tag{22}$$

From Schur Complement Lemma, $\Psi < 0$ is equivalent to

$$\begin{bmatrix} -P & 0 & \sqrt{(1-\beta)\beta} P \mathcal{X} & 0 \\ * & -P & P\mathcal{A} & PH \\ * & * & -P - \delta \mathcal{T}_1 & -\delta \mathcal{T}_2 \\ * & * & * & -\delta I \end{bmatrix} < 0. \tag{23}$$

It follows from (20) that (23) holds, i.e., $\Delta V_k < 0$. Therefore, it can be confirmed that the closed-loop system (18) is asymptotically mean-square stable [30].

Let us now consider the H_∞ disturbance rejection attenuation performance. In the zero initial state situation, for any $w_k \neq 0$, we can obtain $\Delta V_k + \mathbb{E}\left\{z_k^T z_k\right\} - \gamma_2^2 \mathbb{E}\left\{w_k^T w_k\right\} < 0$. Summing up this inequality from 0 to ∞ with respect to k yields

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{|z_k|^2\right\} < \gamma_2^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{|w_k|^2\right\} - \mathbb{E}\left\{V_\infty\right\} + \mathbb{E}\left\{V_0\right\},$$

which implies that (19) is true and the proof is now complete. ■

Theorem 2 provides a fault tolerant controller analysis result for a given controller. The design procedure of the fault tolerant controller is outlined in the following theorem.

Theorem 3: Consider the INTTS (1) with network-induced packet dropouts (3). For a given scalar γ_2, in case that the actuators in Ω fail, there exists a control law (5) such that the closed-loop system (18) is asymptotically mean-square stable and the prescribed H_∞ disturbance attenuation level (19) is fulfilled if there exist matrices $X = X^T > 0$, $\bar{K}$ and a scalar δ satisfying

$$\begin{bmatrix} -X & 0 & \mu B_{\bar{\omega}}\bar{K} & 0 & 0 & 0 & 0 \\ * & -X & AX + \beta B_{\bar{\omega}}\bar{K} & H & B_\omega & D & 0 \\ * & * & -X - \delta X\mathcal{T}_1 & -\delta X\mathcal{T}_2 & 0 & 0 & XE^T \\ * & * & * & -\delta I & 0 & 0 & 0 \\ * & * & * & * & -\gamma_2^2 I & 0 & 0 \\ * & * & * & * & 0 & -\gamma_2^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{24}$$

where μ is defined in Theorem 1. Moreover, if (24) holds, the desired control law can be obtained from the following equation:

$$K = \bar{K}X^{-1}C^{\dagger}. \tag{25}$$

where $C^{\dagger}$ is the left inverse matrix of C.

Proof: From the definitions of $\mathcal{A}$, $\mathcal{X}$, $\mathcal{B}$ and $\mathcal{C}$, (20) can be rewritten as

$$\begin{bmatrix} -P & 0 & \mu P B_{\bar{\omega}} K C & 0 & 0 & 0 & 0 \\ * & -P & PA + P\beta B_{\bar{\omega}} K C & PH & PB_{\omega} & PD & 0 \\ * & * & -P - \delta \mathcal{T}_1 P & -\delta \mathcal{T}_2 & 0 & 0 & E^T \\ * & * & * & -\delta I & 0 & 0 & 0 \\ * & * & * & * & -\gamma_2^2 I & 0 & 0 \\ * & * & * & * & * & -\gamma_2^2 I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0. \tag{26}$$

Defining $X = P^{-1}$ and performing the congruence transformation to (26) by diag $\{X,\ X,\ X,\ I,\ I,\ I\}$, we obtain (24) by introducing a new matrix variable $\bar{K} = KCX$. This is equivalent to (20). According to Theorem 2, the closed-loop system (18) is asymptotically mean-square stable and also satisfies (19). If (23) is solvable, from the full column property of C and X, we can obtain the control law from (25) and this concludes the proof. ■

Remark 5: Although the condition in Theorem 3 guaranteeing the existence of a satisfactory fault tolerant controller is a nonlinear matrix inequality in terms of the product of δ and X, we could first fix the variable δ and solve the optimization problem by using the interior-point methods, then decrease δ and repeat the optimization procedure until we achieve a minimum disturbance attenuation level γ.

Algorithm 1: The whole active fault tolerant control algorithm can be implemented for INTTS according to the following steps:

Step 1: System Modeling.

Setup the mathematical model of the INTTS, determine system parameters and choose proper parameters of the sector-bounded nonlinearity.

Step 2: Residual Generation Filter design.

Design a residual generation filter in the form of (4) using Theorem 1.

Step 3: H_∞ Controller Design.

Calculate the parameters of the controllers in the form of (5) by using Theorem 3. Both the controller for fault free case and the faulty case are designed.

Step 4: Determine Threshold J_{th}. Based on the output of the residual evaluation signal, determine a proper time length $\mathcal{L}$ in (16) and a threshold for the sake of detect the fault, using the logic (15). Equation (16) is a theoretical formula of a threshold, however, in practice, we use Monte Carlo experiments to get a best threshold.

Step 5: Alarm the Fault. By comparing the output J_k of the residual evaluation function and the predefined threshold J_{th}, one can alarm the fault and get the fault detected time k_d, which is defined as the first time J_k exceeds J_{th}.

Step 6: Isolate the Fault. By using the WRCC approach, one can isolate the fault.

Step 7: Reconfigure the Control Law. Use the fault information provided by the fault diagnosis unit, reconfigure the controller to the H_∞ control law a prior designed without the failure actuator.

IV. EXPERIMENTAL RESULTS

To illustrate the effectiveness of the proposed methodology, in this section, we conducted two experiments on the INTTS. Parameters of TTS can be found in [1] and the packet dropout property of the Internet can be obtained from experiments, see [2] for details. Different from [2], in our experiments, data packets over communication network with delays larger than one step are discarded and only missing effects are considered.

In a real TTS, there are two pumps supplying water for Tank 1 and Tank 2. Three sensors are equipped for measuring the water levels in Tank 1, Tank 2, and Tank 3, respectively. Consider that the nominal water inflow is $u_1 = 3 \times 10^{-5}(m^3/s)$, $u_2 = 2 \times 10^{-5}(m^3/s)$, then the equilibrium point of TTS is $h_1 = 0.318(m)$, $h_2 = 0.151(m)$, $h_3 = 0.231(m)$. The states of the system are water heights of Tanks $1 \sim 3$; the inputs are inflows of pumps $1 \sim 2$; the measurements are the same as the system states since $C = I$; and the controlled outputs are the deviations of the real water levels from their setpoints. Choosing a sampling time $Ts = 1(s)$, the simplified mathematical model (1) is obtained after the discretization treatment. This model is in incremental form and its parameters are given as follows:

$$A = \begin{bmatrix} 0.9889 & 0.0001 & 0.0110 \\ 0.0001 & 0.9774 & 0.0119 \\ 0.0110 & 0.0119 & 0.9770 \end{bmatrix}, \quad B = D = \begin{bmatrix} 64.5993 & 0.0015 \\ 0.0015 & 64.2236 \\ 0.3604 & 0.3909 \end{bmatrix},$$

$$C = I_3, \quad H = I_3, \quad E = [0.3\ 0.6\ 0.1]\,.$$

Based on some preliminary experiments, we have the nonlinearity term $g(x_k)$ bounded by (2) with the following parameters:

$$T_1 = 1 \times 10^{-5} \times I_3 \quad and \quad T_1 = 1.5 \times 10^{-5} \times I_3,$$

from which we can get

$$\mathcal{T}_1 = 1.5 \times 10^{-10} \times I_3 \quad and \quad \mathcal{T}_2 = -1.25 \times 10^{-5} \times I_3.$$

For the packet dropout phenomenon, we get $\beta_1 = \beta_2 = 0.9$ and $\rho = 1$ from some tests, and therefore we have $\beta = 0.9$. For a fault-free system ($\Omega = \varnothing$), we set $\delta = 1$ and use Theorem 3 to obtain an sub-optimal H_∞ disturbance attenuation level $\gamma_h = 65.6611$ with the following control gain

$$K_h = \begin{bmatrix} -0.0160 & -0.0012 & -0.0387 \\ -0.0005 & -0.0163 & 0.0163 \end{bmatrix}.$$

Assume that at most one pump may fail during the simulation. Again, using Theorem 3, we can obtain the following fault tolerant control gains with respect to the failures of Pump 1 and Pump 2:

$$K_{f1} = \begin{bmatrix} 0 & 0 & 0 \\ -0.0086 & -0.0169 & -0.0032 \end{bmatrix},$$
$$K_{f2} = \begin{bmatrix} -0.0170 & -0.0336 & -0.0062 \\ 0 & 0 & 0 \end{bmatrix}.$$

Also, the corresponding H_∞ disturbance attenuation levels are calculated as $\gamma_{f1} = 72.0335$ and $\gamma_{f2} = 88.1046$. It can be seen that both of them are larger than γ_h.

As for the fault detection filter, Theorem 1 is utilized to obtain the following gain matrices with a sub-optimal H_∞ attenuation level $\gamma_{filter} = 1.0014$:

$$\begin{aligned} G &= \begin{bmatrix} -0.1261 & -0.0977 & -0.9216 \\ -0.1390 & -0.1271 & 1.7613 \\ -0.0068 & -0.0069 & 0.8752 \end{bmatrix}, \\ N &= 10^{11} \times \begin{bmatrix} -1.8548 & -0.3760 & -4.2223 \\ -0.2729 & 1.2104 & -2.7377 \\ -0.0158 & 0.0061 & -0.0459 \end{bmatrix}, \\ M &= 10^{13} \times \begin{bmatrix} -1.1547 & -0.1477 \\ -0.3836 & -0.7682 \\ -0.0100 & 0.0044 \end{bmatrix}, \\ L &= 10^{-3} \times \begin{bmatrix} 0.0013 & -0.0002 & 0.3009 \\ 0.0002 & 0.0007 & 0.3156 \end{bmatrix}. \end{aligned}$$

In this section, we consider two cases of the actuator faults, one is the failure fault of Pump 1 and the other one is the failure of Pump 2.

Case 1: Failure of Pump 1, Start from $k = 1s$, Fault occurs at $k = 300s$.

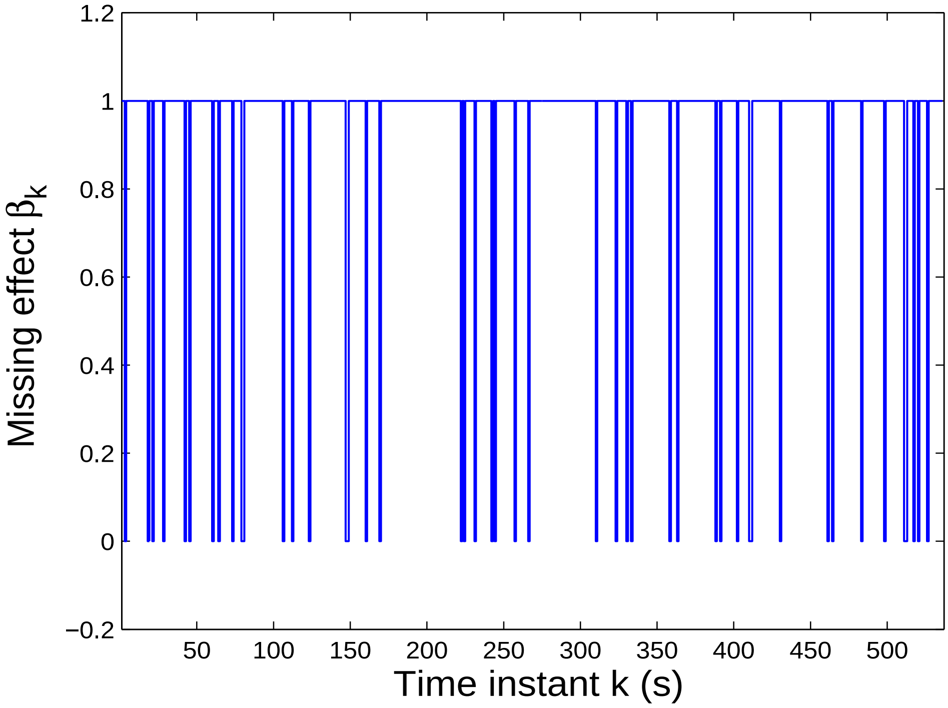

Fig. 4. Packet dropout phenomenon

The experiment results are shown in Figs. 4∼6. Fig. 4 illustrates that whether the packet is missing during the transmission from sensor to actuator via the network cable. $\beta_k = 0$ means that the data is missing and $\beta_k = 1$ corresponds to successful transmission. Fig. 5 shows the fault detection and isolation result. $\Delta J_{k,\mathcal{L}}$ is the incremental real-time residual evaluation function, where we set $\mathcal{L} = 5$. We choose $J_{th}^{\mathcal{L}} = 220$ from Monte Carlo experiments. It can be seen from Fig. 5 that $\Delta J_{322,5} = 225.4296 > J_{th}^{\mathcal{L}} = 220$, which means the failure can be detected at $k = 322s$ by using the logic (15). The detecting time is 22 s later after the fault occurs.

For the fault isolation, we use the WRCC approach proposed in Subsection III-A.1. For the two possible failures that may occur in Pump 1 or Pump 2, we set the scalars as $\alpha_1 = 1$, $\alpha_2 = 0.97$. This can be obtained from some initial experiments. At time instant $k = 321s$, the one-to-one map relationship between the weighted ordered array and the possible fault set is as follows:

$$\begin{cases} OA(\bar{J}_{321}^{(1)}, \bar{J}_{321}^{(2)}) & \Longleftrightarrow & f_k^{(1)}, \\ OA(\bar{J}_{321}^{(2)}, \bar{J}_{321}^{(1)}) & \Longleftrightarrow & f_k^{(2)}. \end{cases}$$

In the present experiment, we have $\bar{J}_{322}^{(1)} = 481.0639$ and $\bar{J}_{322}^{(2)} = 471.6604$. From the above relationship, we can refer that the fault occurs in pump 1. The bottom figure in Fig. 5 provides the fault detection and isolation information. The solid line stands for the true fault with 1 for failure of pump 1 and 2 for pump 2. The dashed line shows the fault detection and isolation result with 0 for no fault, 1 for failure of pump 1 and 2 for failure of pump 2.

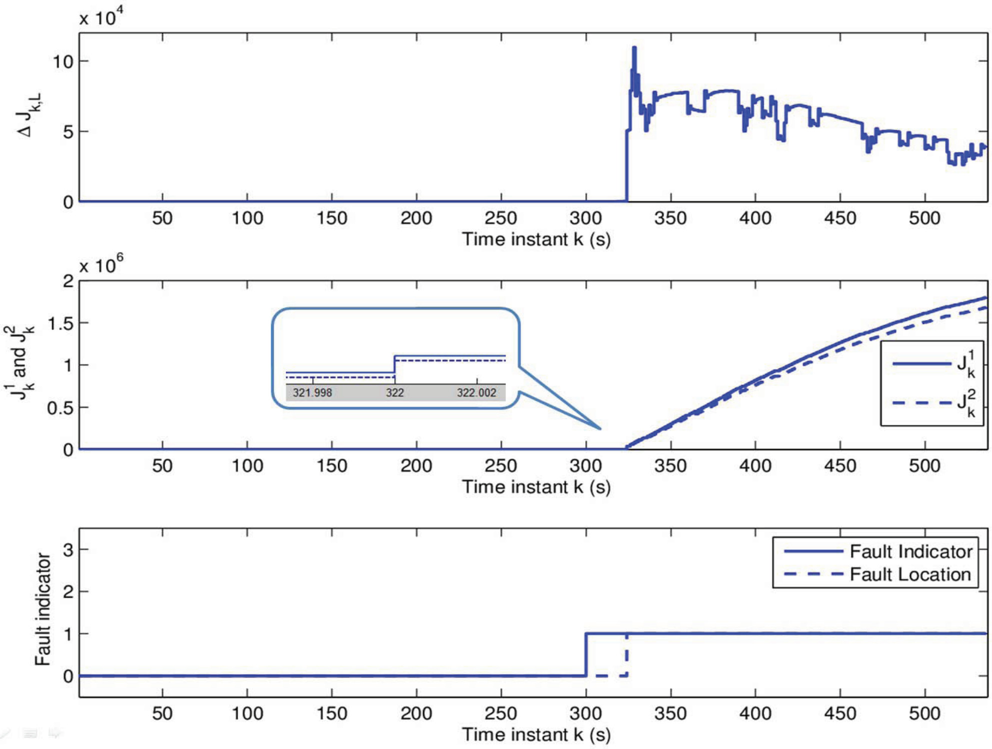

Fig. 5. Fault detection and isolation results in Case 1

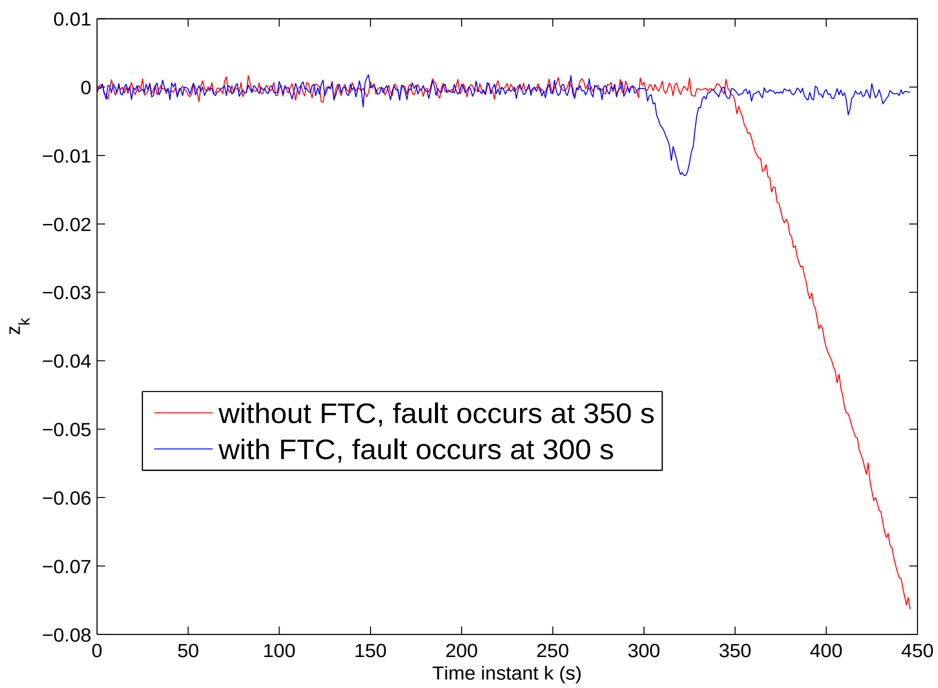

Fig. 6. Controlled outputs under controller K_h and K_{FTC} in Case 1

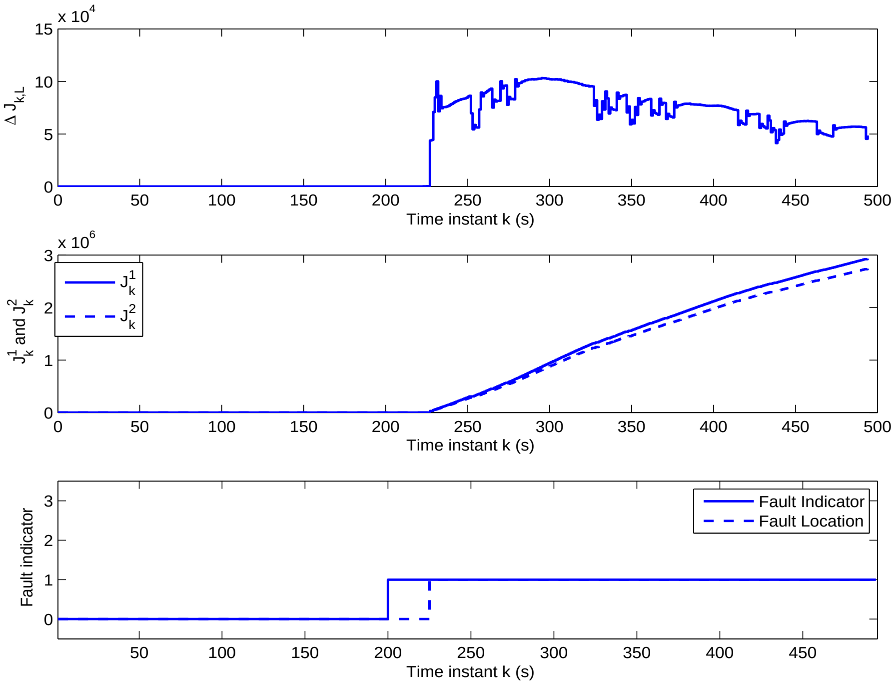

Fig. 7. Fault detection and isolation results in Case 2

We are now ready to make a comparison between a fixed H_∞ controller K_h and a fault tolerant controller K_{FTC} in the faulty case. Fig. 6 demonstrates the evolutions of controlled output z_k under both fixed controller K_h (red line) and fault tolerant controller K_{FTC} (blue line). It can be seen that under the fixed control law K_h, when there is a failure fault with Pump 1, water levels descend gradually with time increase and the deviation is more than $7cm$ after $100s$. However, under the fault tolerant controller (5), although the control performance is worse than that in the fault free case, water levels can be controlled with an accuracy of $\pm 1cm$.

Case 2: Failure of Pump 2, Start from $k = 1s$, Fault occurs at $k = 200s$.

Experiment results are shown in Figs. 7$\sim$8, which are similar with the results in Case 1. Fig. 7 shows the fault detection and isolation result. For the comparison between a fixed H_∞ controller K_h and a fault tolerant controller K_{FTC} in the faulty case, Fig. 8 provides the evolutions of controlled output z_k under both fixed controller K_h (red line) and fault tolerant controller K_{FTC} (blue line). We can observe that under the fixed control law K_h, when there is a failure fault with Pump 2, the fault tolerant control strategy can guarantee the scalability of the whole INTTS while the fixed controller cannot.

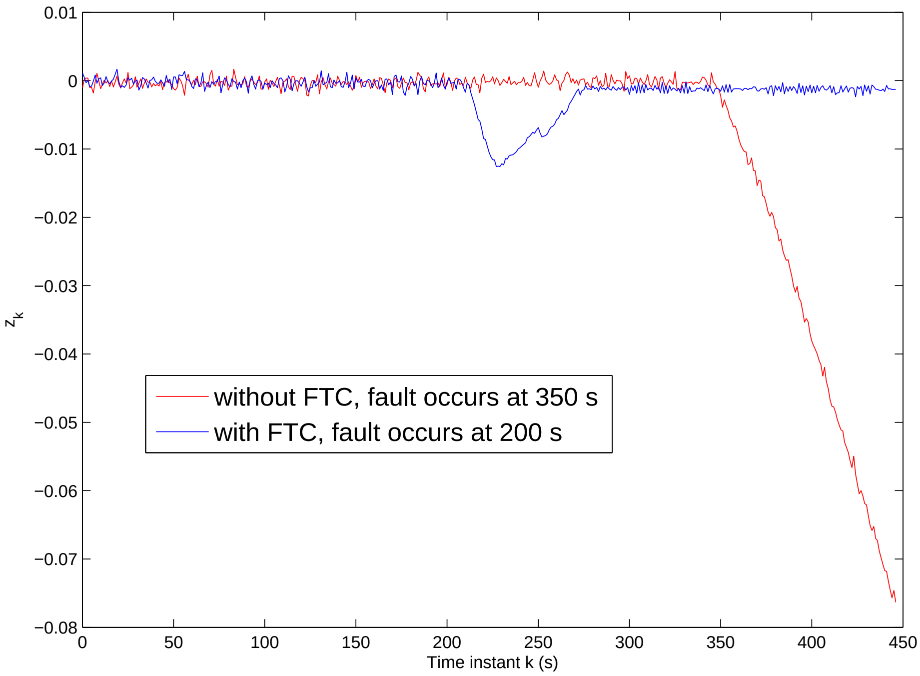

Fig. 8. Controlled outputs under controller K_h and K_{FTC} in Case 2

V. Conclusion

INTTS is a benchmark system that can verify networked FDD/FTC algorithms. Based on the INTTS, this paper has proposed an active fault tolerant control strategy for nonlinear networked systems with possible actuator failures. A Bernoulli distributed stochastic sequence has been utilized to describe the random packet dropout introduced by the limited-capacity of network transmission. A networked fault tolerant control has been designed in an active framework, which includes the design of a fault detection and isolation unit and a controller reconfiguration strategy. Real-time experiments on the real INTTS have been provided and the effectiveness of the proposed technique has been verified by two cases with actuator failure faults.

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