On a Class of Observer-Based Fault Diagnosis Schemes under Closed-loop Control: Performance Evaluation and Improvement

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Abstract: This paper deals with a fundamental issue of evaluating the performance of widely used fault detection and diagnosis (FDD) schemes within a closed-loop framework. The focus is to examine how certain implemented controller would impact on the FDD performance and how such performance can be further improved. For this purpose, we consider the FDD problem for a class of linear discrete-time systems (with and without unknown disturbances) under typical proportional-integral (PI) control using observer-based methods. It is revealed that some existing observer-based FDD approaches are no longer applicable in the closed-loop situation due to the feedback control. Furthermore, by appropriately modifying the structure and designing the parameters of the observers, it is proven that the dynamics of closed-loop residuals can be made identical with those of the residuals obtained with known control inputs at each time step. A numerical example is provided to show the effectiveness of the proposed algorithm.

1. Introduction

In the past decades, the fault detection and diagnosis (FDD) problems have been attracting considerable research attention because of the increasing complexity and safety demands in modern industrial systems [1, 2, 3, 4]. To improve the reliability of practical systems, the FDD problems have been extensively studied in a variety of situations such as photovoltaic panels [5], diesel engines [6], induction motors [7], underground cables [8], steam turbines [9], high-speed railways [10], etc. The FDD methods enable us to detect, isolate and estimate possible faults such that severe performance deteriorations and disasters can be prevented as early as possible [11, 12]. So far, a great number of results have been reported on the topic, see e.g. [13, 14, 15, 16] and the references therein.

Among the existing FDD methods, observer-based methods have been investigated for a lot of practical systems [17, 18, 19]. In particular, the classical Luenberger observer has been widely utilized in linear systems under certain completely observable condition owing to its simple structure and stable performance since it was proposed in 1971 [20]. Some modified Luenberger observers have been developed so as to cope with nonlinearities as well [21, 22]. A kind of robust observer has been established in [23] to realize fault detection and isolation by specializing the structures of some significant parameters. When the addressed systems are subject to unknown disturbances, unknown input observer (UIO) methods have been presented to guarantee that the residuals are completely decoupled from disturbances [24]. The UIO techniques have been applied to various systems including uncertain systems [25], distributed systems [26] and singular systems [27].

In practical engineering, many real-world systems have been subjected to closed-loop control under the demands for stability and robustness. With given control laws, the closed-loop controllers can adjust the control inputs on-line according to the system outputs. It should be pointed out that, in most reported FDD literature using observer-based methods, it has been implicitly assumed that the control input is exactly known at every time step. However, such an assumption is not always realistic. For example, the closed-loop control input in practical systems may be unavailable to the FDD unit owing to many reasons such as large scales and distributed structures of the systems. It is often the case that only the prescribed control law, rather than the actual control input, can be used to solve the FDD problems. In such a situation, it would be interesting to examine how the feedback control would influence the performances of the FDD approaches, and this gives rise to the fundamental issue of evaluating the performance of existing schemes within a closed-loop framework. Furthermore, in the case that the FDD performance is indeed degraded by the feedback control, it is of more engineering significance to modify the existing schemes such that they are suitable to be used in closed-loop systems. So far, despite its clear engineering insight, such a closed-loop FDD performance evaluation problem has been largely overlooked due probably to the difficulties in mathematical formulation. It is, therefore, the main aim of this paper to shorten such a gap by initializing the study on the FDD problem under closed-loop control.

In this paper, the FDD problems are investigated for a class of closed-loop systems under PI control that is fairly popular in engineering practice. Some frequently used observer-based FD-D approaches are examined within the closed-loop framework. To be more specific, the UIO-based approach is studied for a class of systems with unknown disturbances, and then both the Luenberger-observer-based method and the robust-observer-based FDD method are discussed in the disturbance-free case. It is shown that some methods are no longer applicable in the closed-loop case when only the control laws are available. Subsequently, the observers are re-designed such that the dynamics of the closed-loop residuals can be the same as those of the residuals achieved with control inputs. A simulation example is presented to show the effectiveness of the proposed method in the closed-loop situation. The main novelty of the paper is twofold: 1) it is illustrated that the performance of the closed-loop residuals may be deteriorated by the effects of the feed-back control; and 2) the structures of the observers can be adjusted and the parameters can be re-calculated in the closed-loop case such that the modified FDD methods are applicable in the closed-loop cases.

The rest of paper is organized as follows. In Section 2, the UIO-based FDD method is discussed for a class of linear closed-loop systems with unknown external disturbances. In Section 3, the Luenberger-observer-based method and robust-observer-based FDD method are studied for a class of linear systems under PI control. An illustrative example is presented in Section 4 and the paper is concluded in Section 5.

Notations. The notation used in the paper is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "T" denotes the transpose. I is the identity matrix with compatible dimension. $\mathrm{diag}_l\{\Diamond\}$ is employed to represent a block-diagonal matrix whose entries are all $\Diamond$.

2. UIO method

Consider the following class of linear discrete-time systems with unknown disturbances:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + Ed_k + f_k + w_k, \\ y_k = Cx_k + v_k, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state; $u_k \in \mathbb{R}^p$ is the control input; $y_k \in \mathbb{R}^m$ is measurement output; $d_k \in \mathbb{R}^q$ is the unknown external disturbance; $f_k \in \mathbb{R}^n$ is the additive fault; $w_k \in \mathbb{R}^n$ and $v_k \in \mathbb{R}^m$ are the process noise and measurement noise, respectively; and A, B, C and E are known matrices with appropriate dimensions.

For system (1), the UIO of the following structure is adopted as a discrete version of that in [24] when u_k is attainable at each time step:

$$\begin{cases} z_{k+1} = Fz_k + TBu_k + Ky_k, \\ \hat{x}_{k+1} = z_{k+1} + Hy_{k+1}, \end{cases} \tag{2}$$

where $z_k \in \mathbb{R}^n$ is the observer state and $\hat{x}_k \in \mathbb{R}^n$ is the state estimate. The parameters F, T, K and H are provided as follows:

$$K = K_1 + K_2, \tag{3}$$
$$E = HCE, \tag{4}$$
$$T = I - HC, \tag{5}$$
$$F = A - HCA - K_1C, \tag{6}$$
$$K_2 = FH, \tag{7}$$

and the system matrix F is Schur stable (i.e., all the eigenvalues of F lie within the open unit disk). With the given UIO, the dynamics of estimation error $e_k = x_k - \hat{x}_k$ is governed by:

$$e_{k+1} = Fe_k - Hv_{k+1} - K_1v_k + (I - HC)w_k + (I - HC)f_k. \tag{8}$$

The residual signal is taken as

$$r_k = y_k - C\hat{x}_k = Ce_k + v_k, \tag{9}$$

which is fully decoupled from the disturbance d_k. Necessary and sufficient conditions for (2) to be a UIO for the system defined by (1) are

1. rank(CE)=rank(E);
2. The pair $(C, A - HCA)$ is detectable.

Consider the discrete PI controller

$$u_k = Q_py_k + Q_i\sum_{j=1}^{l} y_{k-j}, \tag{10}$$

where l, Q_p and Q_i are predefined parameters that can ensure the stability of the closed-loop system. When only the control law is available to the observer (rather than the control input), substituting the control law into the original system (1) yields

$$\left\{\begin{array}{rcl} x_{k+1} & = & (A+BQ_pC)x_k + BQ_iC\sum_{j=1}^{l} x_{k-j} + Ed_k \\ & & +f_k + w_k + BQ_pv_k + BQ_i\sum_{j=1}^{l} v_{k-j}, \\ y_k & = & Cx_k + v_k. \end{array}\right. \tag{11}$$

To apply the UIO method, define the following augmented state and measurement noise:

$$\bar{x}_k = [x_k^T, x_{k-1}^T, \ldots, x_{k-l}^T]^T, \quad \bar{v}_k = [v_k^T, v_{k-1}^T, \ldots, v_{k-l}^T]^T.$$

Then, the closed-loop system (11) can be rewritten in the following form:

$$\left\{\begin{array}{rcl} \bar{x}_{k+1} & = & \bar{A}\bar{x}_k + \bar{E}d_k + \bar{D}f_k + \bar{D}w_k + \bar{F}_1\bar{v}_k, \\ y_k & = & \bar{C}\bar{x}_k + \bar{F}_2\bar{v}_k, \end{array}\right. \tag{12}$$

where

$$\bar{A} = \begin{bmatrix} A+BQ_pC & BQ_iC & \cdots & BQ_iC \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix}, \quad \bar{F}_1 = \begin{bmatrix} BQ_p & BQ_i & \cdots & BQ_i \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix},$$

$$\bar{C} = [C, 0, \cdots, 0], \quad \bar{E} = [E^T, 0, \cdots, 0]^T, \quad \bar{D} = [I, 0, \cdots, 0]^T, \quad \bar{F}_2 = [I, 0, \cdots, 0].$$

For system (12), a closed-loop UIO of the following structure is adopted:

$$\left\{\begin{array}{rcl} \bar{z}_{k+1} & = & \bar{F}\bar{z}_k + \bar{K}y_k, \\ \hat{\bar{x}}_{k+1} & = & \bar{z}_{k+1} + \bar{H}y_{k+1}, \end{array}\right. \tag{13}$$

where $\bar{z}_k \in \mathbb{R}^{nl}$ is the observer state and $\hat{\bar{x}}_k \in \mathbb{R}^{nl}$ is the state estimate.

To guarantee that the closed-loop residual is still decoupled from the disturbance, we need to choose $\bar{H}$ such that

$$\bar{E} = \bar{H}\bar{C}\bar{E}. \tag{14}$$

Considering the structure of $\bar{E}$, the matrix $\bar{H}$ should be determined as

$$\bar{H} = [H^T, 0, \cdots, 0]^T \tag{15}$$

and, with $\bar{H}$ given in (15), we have

$$\begin{aligned} \bar{F} =& \bar{A} - \bar{H}\bar{C}\bar{A} - \bar{K}_1\bar{C} \\ =& \begin{bmatrix} (I-HC)(A+BQ_pC) & (I-HC)BQ_iC & \cdots & (I-HC)BQ_iC \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix} - \bar{K}_1\bar{C}. \end{aligned}$$

To make the dynamics of the closed-loop residuals as similar as possible with that of residuals obtained with control inputs u_k, one needs to choose

$$\bar{K}_1 = [(K_1 + BQ_p - HCBQ_p)^T, 0, \cdots, 0]^T \tag{16}$$

and it then follows that

$$\bar{F} = \begin{bmatrix} F & (I-HC)BQ_iC & \cdots & (I-HC)BQ_iC \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix}. \tag{17}$$

Similar to (3) and (7), $\bar{K}_2$ and $\bar{K}$ are given as below to eliminate the terms directly related to y_k in the estimation error dynamics:

$$\bar{K} = \bar{K}_1 + \bar{K}_2, \tag{18}$$

$$\bar{K}_2 = \bar{F}\bar{H}. \tag{19}$$

Considering the structures of $\bar{F}$, $\bar{K}$ and $\bar{H}$, the augmented estimation error $\bar{e}_k = \bar{x}_k - \hat{\bar{x}}_k$ can be partitioned as $\bar{e}_k = \left[e_k^T, \ldots, e_{k-l}^T\right]^T$. Based on the parameters obtained in (15)-(19), we have

$$\begin{aligned} e_{k+1} =& Fe_k + (I-HC)BQ_iC\sum_{j=1}^{l} e_{k-j} - K_1v_k + (I-HC)BQ_i\sum_{j=1}^{l} v_{k-j} \\ & - Hv_{k+1} + (I-HC)w_k + (I-HC)f_k. \end{aligned} \tag{20}$$

From (20), it is straightforward to see that the estimation error dynamics of the closed-loop system may be *unstable* even if F is Schur stable owing to the effects of the delayed errors $e_{k-1}, \ldots, e_{k-l}$. *Such influences result from the integral part of the feedback control.* Considering the sparse nature of $\bar{C}$ in the closed-loop situation, the eigenvalues of $\bar{F}$ cannot be assigned effectively no matter how $\bar{K}_1$ is selected. To tackle such a problem, the structure of the closed-loop UIO should be appropriately changed.

Introducing the augmented measurement $\bar{y}_k = [y_k^T, y_{k-1}^T, \ldots, y_{k-l}^T]^T$, we have

$$\bar{y}_k = \tilde{C}\bar{x}_k + \bar{v}_k, \tag{21}$$

where $\tilde{C} = \mathrm{diag}_l\{C\}$.

Now, consider a closed-loop UIO in the following form for system (12):

$$\begin{cases} \bar{z}_{k+1} = \bar{F}\bar{z}_k + \bar{K}\bar{y}_k, \\ \hat{\bar{x}}_{k+1} = \bar{z}_{k+1} + \bar{H}y_{k+1}, \end{cases} \tag{22}$$

where $\bar{z}_k \in \mathbb{R}^{nl}$ and $\hat{\bar{x}}_k \in \mathbb{R}^{nl}$ are the same as defined above. The modified UIO will be developed in the next theorem.

Theorem 1. Consider the UIO (22) for the closed-loop system (1) with the PI controller (10). For the initial error $\bar{e}_0 = [e_0^T, 0, \ldots, 0]^T$, if $\bar{H}$ is given by (15), and $\bar{K}$ and $\bar{F}$ are calculated as follows:

$$\bar{K} = \bar{K}_1 + \bar{K}_2, \tag{23}$$

$$\bar{F} = \bar{A} - \bar{H}\bar{C}\bar{A} - \bar{K}_1\tilde{C}, \tag{24}$$

$$\bar{K}_1 = \begin{bmatrix} K_1 + (I-HC)BQ_p & (I-HC)BQ_i & \cdots & (I-HC)BQ_i \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}, \tag{25}$$

$$\bar{K}_2 = \bar{F}\bar{H}\bar{F}_2, \tag{26}$$

then the dynamics of the closed-loop residual $r_k = y_k - \bar{C}\hat{\bar{x}}_k$ will be identical with that in (8)-(9).

Proof. Based on (12) and (22), we have

$$\begin{aligned}\bar{e}_{k+1} &= \bar{x}_{k+1} - (z_{k+1} + \bar{H}y_{k+1}) \\ &= (I - \bar{H}\bar{C})\bar{x}_{k+1} - \bar{H}\bar{F}_2\bar{v}_k - \left[\bar{F}z_k + (\bar{K}_1 + \bar{K}_2)\bar{y}_k\right].\end{aligned} \tag{27}$$

Considering the definition of $\bar{y}_k$, we obtain that

$$\bar{e}_{k+1} = (I - \bar{H}\bar{C})\bar{x}_{k+1} - \bar{H}\bar{F}_2\bar{v}_{k+1} - \bar{F}(\bar{x}_k - \bar{e}_k - \bar{H}y_k) - \bar{K}_1(\tilde{C}\bar{x}_k + \bar{v}_k) - \bar{K}_2\bar{y}_k. \tag{28}$$

According to (12) and (24), (28) can be written as

$$\begin{aligned}\bar{e}_{k+1} = &\bar{F}\bar{e}_k - \bar{K}_1\bar{v}_k - \bar{H}\bar{F}_2\bar{v}_{k+1} + (I - \bar{H}\bar{C})\bar{E}d_k + (I - \bar{H}\bar{C})\bar{D}f_k + (I - \bar{H}\bar{C})\bar{D}w_k \\ &+ (I - \bar{H}\bar{C})\bar{F}_1\bar{v}_k + \bar{F}\bar{H}y_k - \bar{K}_2\bar{y}_k.\end{aligned} \tag{29}$$

From (15), (26) and the fact $y_k = \bar{F}_2\bar{y}_k$, it follows that

$$\bar{e}_{k+1} = \bar{F}\bar{e}_k - \bar{K}_1\bar{v}_k - \bar{H}\bar{F}_2\bar{v}_{k+1} + (I - \bar{H}\bar{C})\bar{D}f_k + (I - \bar{H}\bar{C})\bar{D}w_k + (I - \bar{H}\bar{C})\bar{F}_1\bar{v}_k. \tag{30}$$

Now, we are in a position to investigate $\bar{F}$. With (24) and (25), we have

$$\bar{F} = \bar{A} - \bar{H}\bar{C}\bar{A} - \bar{K}_1\tilde{C} \tag{31}$$

$$= \begin{bmatrix} (I-HC)(A+BQ_pC) & (I-HC)BQ_iC & \cdots & (I-HC)BQ_iC \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix} \tag{32}$$

$$- \begin{bmatrix} K_1C + (I-HC)BQ_pC & (I-HC)BQ_iC & \cdots & (I-HC)BQ_iC \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \tag{33}$$

and it follows that

$$\bar{F} = \begin{bmatrix} F & 0 & \cdots & 0 \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix}. \tag{34}$$

With the proposed UIO parameters, the augmented estimation error $\bar{e}_k$ can still be partitioned as $\bar{e}_k = \left[e_k^T, \ldots, e_{k-l}^T\right]^T$. Taking $\bar{F}$ in (34) into account, we have

$$e_{k+1} = Fe_k - Hv_{k+1} - K_1v_k + (I - HC)w_k + (I - HC)f_k, \tag{35}$$

and

$$r_k = Ce_k + v_k, \tag{36}$$

which means that the dynamics of the closed-loop estimation error and the residual are exactly the same as those in (8) and (9) if the initial error $\bar{e}_0 = [e_0^T, 0, \ldots, 0]^T$. This concludes the proof. □

Remark 1. A modified UIO has been presented in Theorem 1 for a class of linear discrete-time system under PI control. The UIO has been developed with the control law, rather than the control input u_k. By doing so, the real-time signal transmission from the controller to the FDD unit can be avoided, and the relationship between the feedback control and the FDD performance can be established. At each time step, some historical measurements $y_{k-1}, y_{k-2}, \dots, y_{k-l}$ are utilized to deal with the effects of the feedback control. It is noted that when, only proportion control $u_k = Q_p y_k$ is employed, the UIO in (2) can be applied in the closed-loop case simply by setting that $\bar{K}_1 = K_1 + BQ_p - HCBQ_p$ and the dynamics of the closed-loop estimation error will be identical with that in (8) when the initial values are the same. In other words, the structure of the UIO is adjusted to specifically cope with the influences from the integral control. In the next section, a linear system without unknown disturbances will be considered, and both the Luenberger-observer-based method and the robust-observer-based method will be modified in the closed-loop case under PI control.

3. Luenberger-observer-based and robust-observer-based method

In this section, the Luenberger-observer-based and robust-observer-based method will be investigated for a class of linear systems under PI control.

Consider the following discrete-time system:

$$\left\{\begin{array}{rcl} x_{k+1} &=& Ax_k + Bu_k + f_k + w_k, \\ y_k &=& Cx_k + v_k, \end{array}\right. \tag{37}$$

where the variables x_k, u_k, y_k, f_k, w_k and v_k and the same as defined in Section 2. The conventional Luenberger observer which directly uses the control input is of the following structure [20]:

$$\hat{x}_{k+1} = A\hat{x}_k + Bu_k + L(y_k - C\hat{x}_k), \tag{38}$$

where L is determined such that $A - LC$ is stable. The dynamics of the estimation error $e_k = x_k - \hat{x}_k$ and residual $r_k = y_k - C\hat{x}_k$ can be obtained, respectively, as follows:

$$e_{k+1} = (A - LC)e_k + w_k + f_k - Lv_k, \tag{39}$$

$$r_k = Ce_k + v_k. \tag{40}$$

Considering the same PI control (10), the system (37) can be written as

$$\left\{\begin{array}{rcl} \bar{x}_{k+1} &=& \bar{A}\bar{x}_k + \bar{D}f_k + \bar{D}w_k + \bar{F}_1\bar{v}_k, \\ y_k &=& \bar{C}\bar{x}_k + \bar{F}_2\bar{v}_k, \end{array}\right. \tag{41}$$

where $\bar{x}_k$, $\bar{v}_k$, $\bar{A}$, $\bar{D}$, $\bar{F}_1$, $\bar{C}$ and $\bar{F}_2$ have been defined previously. For system (41), the following Luenberger observer is selected:

$$\hat{\bar{x}}_{k+1} = \bar{A}\hat{\bar{x}}_k + \bar{L}(y_k - \bar{C}\hat{\bar{x}}_k). \tag{42}$$

Following the similar procedure in the previous section, if we choose

$$\bar{L} = [(L + BQ_p)^T, 0, \cdots, 0]^T, \tag{43}$$

then

$$e_{k+1} =(A-LC)e_k + BQ_iC\sum_{j=1}^{l} e_{k-j} - Lv_k + BQ_i\sum_{j=1}^{l} v_{k-j} + w_k + f_k, \tag{44}$$

from which we can conclude that the effects of the delayed errors may lead to unsatisfactory dynamics of the estimation error in the closed-loop system. To deal with this problem, we would resort to, once again, the augmented measurement output $\bar{y}_k$.

Consider a closed-loop Luenberger observer in the following form:

$$\hat{\bar{x}}_{k+1} = \bar{A}\hat{\bar{x}}_k + \bar{L}(\bar{y}_k - \tilde{C}\hat{\bar{x}}_k), \tag{45}$$

where $\bar{L}$ is to be given in the next theorem.

Theorem 2. Consider the Luenberger observer (45) for the closed-loop system (37) with the PI controller (10). If the initial error $\bar{e}_0 = [e_0^T, 0, \dots, 0]^T$ and $\bar{L}$ is derived as:

$$\bar{L} = \begin{bmatrix} L + BQ_p & BQ_i & \cdots & BQ_i \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}, \tag{46}$$

then the dynamics of the closed-loop residual $r_k = y_k - \bar{C}\hat{\bar{x}}_k$ will be identical with that in (39)-(40).

Proof. Noticing the fact that

$$\begin{aligned} &\bar{A} - \bar{L}\tilde{C} \\ =&\begin{bmatrix} A + BQ_pC & BQ_iC & \cdots & BQ_iC \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix} - \begin{bmatrix} LC + BQ_pC & BQ_iC & \cdots & BQ_iC \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix} \\ =&\begin{bmatrix} A - LC & 0 & \cdots & 0 \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix}, \end{aligned}$$

Theorem 2 can be easily obtained and the proof is omitted here for conciseness. □

Now, let us discuss the robust observer presented in [23]. For system (37), the following observer is taken into account in the discrete situation:

$$\begin{cases} z_{k+1} = Fz_k + TBu_k + Gy_k, \\ r_{k+1} = Kz_{k+1} + Py_{k+1} \end{cases} \tag{47}$$

where

$$TA - FT = GC, \tag{48}$$
$$KT + PC = 0, \tag{49}$$
$$F \text{ is stable.} \tag{50}$$

For such an observer, r_k is the residual and the estimation error is defined as $e_k = z_k - Tx_k$. Based on (37) and (47), we have

$$e_{k+1} = Fe_k - Tw_k - Tf_k + Gv_k, \tag{51}$$

and

$$r_k = Ke_k + Pv_k. \tag{52}$$

For system (41), the following robust observer is selected:

$$\begin{cases} \bar{z}_{k+1} &= \bar{F}\bar{z}_k + \bar{G}y_k, \\ r_{k+1} &= \bar{K}\bar{z}_{k+1} + \bar{P}y_{k+1}, \end{cases} \tag{53}$$

With hope to make dynamics of the closed-loop error as similar as possible with that of the error obtained with control inputs, we choose the following parameters:

$$\bar{T} = \mathrm{diag}_l\{T\}, \tag{54}$$

$$\bar{F} = \begin{bmatrix} F & 0 & \cdots & 0 \\ I & & & \\ & \ddots & & \\ & & I & 0 \end{bmatrix}, \tag{55}$$

$$\bar{K} = [K, 0, \cdots, 0], \tag{56}$$

$$\bar{P} = P, \tag{57}$$

$$\bar{G} = [(G + TBQ_p)^T, 0, \cdots, 0]^T, \tag{58}$$

and then we have

$$e_{k+1} = Fe_k - TBQ_iC\sum_{j=1}^{l} e_{k-j} + Gv_k - TBQ_i\sum_{j=1}^{l} v_{k-j} - Tw_k - Tf_k. \tag{59}$$

Again, we need the augmented measurement to eliminate the influences of the integral control. For this purpose, we consider a closed-loop robust observer in the following form for system (37):

$$\begin{cases} \bar{z}_{k+1} &= \bar{F}\bar{z}_k + \bar{G}\bar{y}_k, \\ r_{k+1} &= \bar{K}\bar{z}_{k+1} + \bar{P}y_{k+1}. \end{cases} \tag{60}$$

In virtue of Theorem 1, the following results are easily accessible.

Theorem 3. Consider the robust observer (60) for the closed-loop system (37) with the PI controller (10). For the initial error $\bar{e}_0 = [e_0^T, 0, \ldots, 0]^T$, if $\bar{T}$, $\bar{F}$, $\bar{K}$ and $\bar{P}$ are given by (54)-(57) and $\bar{G}$ is adopted as:

$$\bar{G} = \begin{bmatrix} G + TBQ_p & TBQ_i & \cdots & TBQ_i \\ 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}, \tag{61}$$

then the dynamics of the closed-loop residual r_k will be identical with that in (51)-(52).

Proof. The proof of Theorem 3 is similar to that of Theorem 1 and is therefore omitted. □

Remark 2. So far, several observer-based FDD methods (including UIO-based method, Luenberger-observer-based method and robust-observer-based method) have been examined in the closed-loop situations. It is revealed that the effects of the integral control may degrade the performances of the observers when only control laws are available. Fortunately, by appropriately changing the structure of the observers, the dynamics of the closed-loop residuals can be made exactly the same as that of residuals obtained with known control inputs. The scalar l and the matrices Q_p and Q_i quantify the effects of the controller on the dimensions and parameterizations of the observers, respectively. According to Theorems 1-3, the parameters of the closed-loop observers can be constructed with the open-loop observer parameters and the original system parameters after several matrix additions and matrix multiplications. The computation burden of the presented method would not get significantly increased even with higher parameter dimensions, which makes the method easy to implement in practice. It is also worth mentioning that the results obtained in the paper can be easily extended to time-varying systems by using Theorems 1-3 at each time step. For uncertain systems in which the directions of the uncertainties are known, the UIO-based FDD method can be utilized after formulating the uncertainties as parts of unknown inputs. When the directions of the uncertainties are unavailable, the H_∞ observer can be used to constrain the attenuation level from the uncertainties to the residuals. The structure of the observer should be modified, and the parameters should be re-designed accordingly. For the systems under the network-based environments, the network-induced phenomena such as time delays and packet dropouts [28, 29] will lead to that the influences of the possible faults on the residuals are stochastic and the dynamics of residuals is directly dependent on the stochastic variables and the original system states. In such a case, it makes more sense to study the FDD problem and adjust the observer parameters in the robust or mean-square sense, and the residual evaluation problem becomes more challenging. In the next section, an illustrative example will be provided to show the effectiveness of the proposed closed-loop algorithm in the presence of the PI controller.

4. A Simulation Example

Inspired by the three-tank system proposed in [1], a system (1) with the following parameters is considered:

$$A = \begin{bmatrix} 0.9908 & 0 & 0.0091 \\ 0 & 0.9856 & 0.0072 \\ 0.0091 & 0.0072 & 0.9836 \end{bmatrix}, B = \begin{bmatrix} 64.6627 & 0.0007 & 0.2978 \\ 0.0007 & 64.4908 & 0.2358 \\ 0.2978 & 0.2358 & 64.4217 \end{bmatrix}, E = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

The closed-loop control is designed as $u_k = Q_p y_k + Q_i \sum_{i=1}^{2} y_{k-i}$ where

$$Q_p = \begin{bmatrix} -0.0029 & 0.0087 \\ 0.0001 & -0.0057 \\ -0.0372 & -1.8862 \end{bmatrix}, Q_i = \begin{bmatrix} 0 & 0.0001 \\ -0.004 & 0 \\ 0.001 & 0.001 \end{bmatrix}.$$

The disturbances d_k, w_k and v_k are mutually independent Gaussian distributed sequences whose variances are $1 \times 10^{-10} I$, $1 \times 10^{-12} I$ and $1 \times 10^{-12} I$, respectively. The observer parameters in

(3)-(7) are determined as

$$H = \begin{bmatrix} 0 & 2 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, K_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.3 \\ 0 & 0 \end{bmatrix}, K_2 = \begin{bmatrix} 0 & -0.7896 \\ 0 & -0.3 \\ 0 & 0.0254 \end{bmatrix},$$

$$F = \begin{bmatrix} 0.5908 & -1.9712 & -0.0053 \\ 0 & -0.3 & 0 \\ 0.0091 & 0.0072 & 0.9836 \end{bmatrix}.$$

In such a case, F is stable and its eigenvalues are 0.5909, 0.9835 and -0.3.

The additive fault f_k is in the following form:

$$f_k = \begin{cases} [0,0,0]^T, & \text{if} \quad k \leq 20, \\ [-8 \times 10^{-6}, 0, 0]^T, & \text{otherwise}. \end{cases}$$

In the fault-free case, the average Euclidean norm of the residual obtained with the proposed closed-loop observer is 1.9983×10^{-4} in 50 Monte-Carlo simulations. With the traditional open-loop observer, the average Euclidean norm of the residual is 4.4595×10^{-4}. Fig. 1 plots the norm of residual achieved with the conventional UIO method (13) in the fault-free case. It can be seen that: 1) the closed-loop UIO can achieve better estimation performance than the open-loop one due to the consideration of the feedback control; 2) the residual obtained with the open-loop observer cannot converge even when there is no fault in the system. As a result, the conventional UIO is no longer applicable in the closed-loop case. Fig. 2 depicts the norm of residual obtained with the closed-loop UIO method (22) in the faulty situation and the threshold obtained from 100 Monte-Carlo experiments. We can conclude that the closed-loop UIO method proposed in (22) could help us to detect the fault effectively thanks to our efforts to deal with the closed-loop system dynamics.

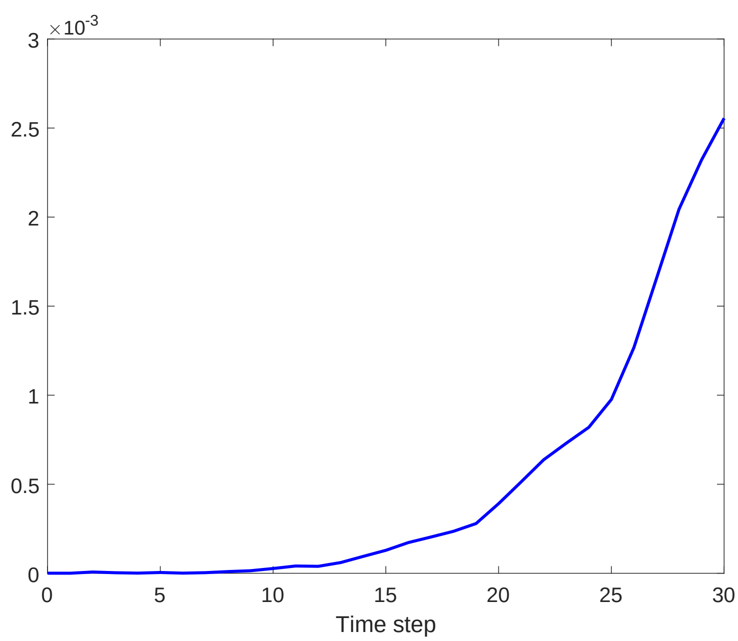

Fig. 1. *Residual obtained with (13) in the fault-free case*

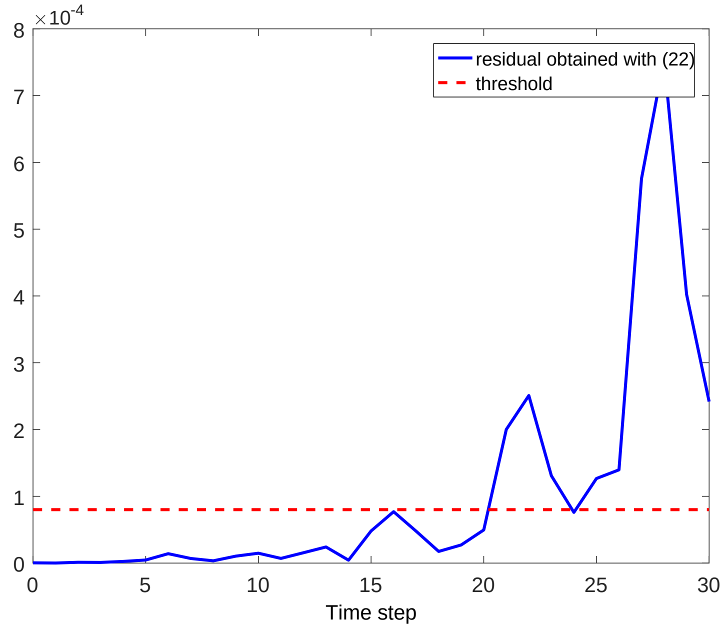

Fig. 2. *Residual obtained with (22) and the threshold*

5. Conclusion

In this paper, the FDD problems under closed-loop control have been investigated with a class of observer-based methods. A discrete-time linear system with/without unknown inputs under PI control has been considered. Several observer-based FDD methods have been taken into account including UIO method, Luenberger observer method and robust observer method. It has been shown that the approaches may be no longer applicable in closed-loop situations when only control laws are available due to the integral feedback control. Then the structures of the observers have been modified and the parameters have been adjusted such that the dynamics of closed-loop residuals are identical with that of the residuals obtained with known control inputs. A numerical example has been presented to show the effectiveness of the proposed algorithm. Future research topics would include the extension of our results to systems with more complex controls (e.g. sliding-mode control and adaptive control), and more complicated practical systems such as uncertain systems, nonlinear systems and networked systems.

6. References

[1] He, X., Wang, Z., Liu, Y., *et al.*: 'Least-squares fault detection and diagnosis for networked sensing systems using a direct state estimation approach', IEEE Transactions on Industrial Informatics, 2013, **9**, (3), pp. 1670–1679

[2] Peng, K., Zhang, K., You, B., *et al.*: 'A quality-based nonlinear fault diagnosis framework focusing on industrial multimode batch processes', IEEE Transactions on Industrial Electronics, 2016, **63**, (4), pp. 2615–2624

[3] Zhang, Y., Wang, Z., Zou, L., *et al.*: 'Fault detection filter design for networked multi-rate

systems with fading measurements and randomly occurring faults', IET Control Theory & Applications, 2016, **10**, (5), pp. 573–581

[4] Zhang, S., Wang, Z., Ding, D., *et al.*: 'On design of robust fault detection filter in finite-frequency domain with regional pole assignment', IEEE Transactions on Circuits and Systems II-Express Briefs, 2015, **62**, (4), pp. 382–386

[5] Wang, W., Liu, A.C.-F., Chung, H.S.-H., *et al.*: 'Fault diagnosis of photovoltaic panels using dynamic current-voltage characteristics', IEEE Transactions on Power Electronics, 2016, **31**, (2), pp. 1588–1599

[6] Wu, W., Lin, T.R., Tan, A.C.C.: 'Normalization and source separation of acoustic emission signals for condition monitoring and fault detection of multi-cylinder diesel engines', Mechanical Systems and Signal Processing, 2015, **64–65**, pp. 479–497

[7] Goktas, T., Arkan, M., Ozguven, O.F.: 'Detection of rotor fault in three-phase induction motor in case of low-frequency load oscillation', Electrical Engineering, 2015, **97**, (4), pp. 337–345

[8] Ghanbari, T.: 'Kalman filter based incipient fault detection method for underground cables', IET Generation, Transmission & Distribution, 2015, **9**, (14), pp. 1988–1997

[9] Chen, J., Jiang, D., Liu, C.: 'Fault analysis and optimal balancing of bowing of steam turbine rotor under long-term service', Journal of Engineering for Gas Turbines and Power, 2015, **137**, (11), pp. 112503-1–112503-10

[10] Liu, P., Yang, L., Gao, Z., *et al.*: 'Fault tree analysis combined with quantitative analysis for high-speed railway accidents', Safety Science, 2015, **79**, pp. 344–357

[11] Hwang, I., Kim, S., Kim, Y., *et al.*: 'A survey of fault detection, isolation, reconfiguration methods', IEEE Transactions on Control Systems Technology, 2010, **18**, (3), pp. 636–653

[12] Li, S., Wu, R., Gao, Z., *et al.*: 'Fault service restoration in distribution network based on breadth-first search and improved ant colony algorithm', Journal of Shandong University of Science and Technology, 2016, **35**, (2), pp. 105–112

[13] Li, Z., Wang, Z., Ding, D., *et al.*: 'H_∞ fault estimation with randomly occurring uncertainties, quantization effects and successive packet dropouts: the finite-horizon case', International Journal of Robust and Nonlinear Control, 2015, **25**, (15), pp. 2671–2686

[14] Liu, Y., Wang, Z., He, X., *et al.*: 'Filtering and fault detection for nonlinear systems with polynomial approximation', Automatica, 2015, **54**, pp. 348–359

[15] Dong, H., Wang, Z., Ding, S.X., *et al.*: 'Finite-horizon estimation of randomly occurring faults for a class of nonlinear time-varying systems', IEEE Transactions on Industrial Informatics, 2014, **50**, (12), pp. 3182–3189

[16] Wang, D., Shen, B., Wang, Z., *et al.*: 'Reliable H_∞ filtering for stochastic spatial-temporal systems with sensor saturations and failures', IET Control Theory & Applications, 2015, **9**, (17), pp. 2590–2597

[17] Ríos, H., Davila, J., Fridman, L., *et al.*: 'Fault detection and isolation for nonlinear systems via high-order-sliding-mode multiple-observer', International Journal of Robust and Nonlinear Control, 2015, **25**, (16), pp. 2871–2893

[18] Li, X., Zhu, F.: 'Simultaneous time-varying actuator and sensor fault reconstruction based on PI observer for LPV systems', International Journal of Adaptive Control and Signal Processing, 2015, **29**, (9), pp. 1086–1098

[19] Hassanabadia, A.H., Shafieea, M., Puig, V.: 'UIO design for singular delayed LPV systems with application to actuator fault detection and isolation', International Journal of Systems Science, 2016, **47**, (1), pp. 107–121

[20] Luenberger, D.G.: 'An introduction to observers', IEEE Transactions on Automatic Control, 1971, **16**, (6), pp. 596–602

[21] Zeitz, M.: 'The extended Luenberger observer for nonlinear-systems', Systems & Control Letters, 1987, **9**, (2), pp. 149–156

[22] Ciccarella, G., Dallamora, M., Germani, A.: 'A Luenberger-like observer for nonlinear-systems', International Journal of Control, 1993, **57**, (3), pp. 537–556

[23] Ge, W., Fang, C.-Z.: 'Detection of faulty components via robust observation', International Journal of Control, 1988, **47**, (2), pp. 581–599

[24] Chen, J., Patton, R.J., Zhang, H.Y.: 'Design of unknown input observers and robust fault-detection filters', International Journal of Control, 1996, **63**, (1), pp. 85–105

[25] Bagherpour, E.A., Hairiyazdi, M.R., Mahjoob, M.J.: 'Residual generation with unknown input observer for linear systems in the presence of unmatched uncertainties', Journal of Mechanical Science and Technology, 2014, **28**, (12), pp. 5159–5167

[26] Shames, I., Teixeira, A.M.H., Sandberg, H., *et al.*: 'Distributed fault detection for interconnected second-order systems', Automatica, 2011, **47**, (12), pp. 2757–2764

[27] Marx, B., Koenig, D., Ragot, J.: 'Design of observers for Takagi-Sugeno descriptor systems with unknown inputs and application to fault diagnosis', IET Control Theory & Applications, 2007, **1**, (5), pp. 1487–1495

[28] Qiu, J., Gao, H., Ding, S.X.: 'Recent advances on fuzzy-model-based nonlinear networked control systems: a survey', IEEE Transactions on Industrial Electronics, 2016, **63**, (2), pp. 1207–1217

[29] Wang, T., Gao, H., Qiu, J.: 'A combined adaptive neural network and nonlinear model predictive control for multirate networked industrial process control', IEEE Transactions on Neural Networks & Learning Systems, 2016, **27**, (2), pp. 416–425