

1 **Modeling alkali metal emissions in large-eddy simulation of a preheated**
2 **pulverized-coal turbulent jet flame using tabulated chemistry**

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14 **Abstract**

15 The numerical modeling of alkali metal reacting dynamics in turbulent pulverized-coal
16 combustion is discussed using tabulated sodium chemistry in Large Eddy Simulation (LES). A
17 lookup table is constructed from a detailed sodium chemistry mechanism including 5 sodium
18 species, i.e., Na, NaO, NaO₂, NaOH and Na₂O₂H₂, and 24 elementary reactions. This sodium
19 chemistry table contains four coordinates, i.e., the equivalence ratio, the mass fraction of the
20 sodium element, the gas-phase temperature, and a progress variable. The table is first
21 validated against the detailed sodium chemistry mechanism by zero-dimensional simulations.
22 Then, LES of a turbulent pulverized-coal jet flame is performed and major coal-flame
23 parameters compared against experiments. The chemical percolation devolatilization (CPD)

24 model and the partially stirred reactor (PaSR) model are employed to predict coal pyrolysis
25 and gas-phase combustion, respectively. The response of the 5 sodium species in the
26 pulverized-coal jet flame is subsequently examined. Finally, a systematic global sensitivity
27 analysis of the sodium lookup table is performed and the accuracy of the proposed tabulated
28 sodium chemistry approach has been calibrated.

29

30 *Keywords:* Large-eddy simulation; Tabulated chemistry; Pulverized-coal combustion; Sodium;
31 Alkali metal

32

33 **1. Introduction**

34 Coal is used to produce approximately 40% of the worldwide electricity. In China the
35 figure is more than 70% [1]. In the near and medium future, it can be expected that the
36 utilization of coal will continue due to its broad availability and the overall flexibility of coal
37 combustion systems [2]. In practice, the presence of sodium in coal leads to rapid ash
38 deposition on heat exchangers of the boilers [3]. This high deposition rate of ash causes
39 unscheduled shutdown of the boilers in order to clean the combustion systems to secure their
40 efficiency [4]. Sodium species also form complex chloride and sulfur compounds, which can
41 foul and corrode heat transfer surfaces within coal-fired boilers [5]. These sodium-induced
42 issues severely limit the utilization of coal with a relatively high concentration of sodium,
43 such as Zhundong coal. The newly explored 390-billion-ton Zhundong coalmine in China is
44 able to be used for more than a century under the current consumption rate of coal [6], if the

45 sodium-induced fouling and corrosion issues can be overcome.

46 Such issue is not limited to coal. In the quest for cleaner fuels, biomass provides a
47 renewable energy source which is foreseen as a candidate to complement fossil fuels [7]. The
48 elemental composition of biomass, such as straw, is usually rich in potassium [8]. Since both
49 potassium (K) and sodium (Na) belong to alkali metals, burning biomass also causes massive
50 ash deposition, promoting corrosion in the boilers [9]. Therefore, strong motivations exist to
51 better understand the fundamental mechanisms driving the formation and the destruction of
52 alkali metal species during coal and biomass combustion. A better understanding of these
53 mechanisms is important and necessary for the development of technologies ready for
54 reducing, or capturing, these harmful species produced by coal and biomass combustion.

55 To understand alkali release mechanisms and reacting dynamics, online measurements
56 using advanced laser diagnostics have been attempted recently [10, 11], together with offline
57 measuring techniques [12], e.g. sampling measurements that can be used to analyse the bulk
58 composition of fly ash and ash deposits. For instance, van Eyk et al. [10] developed a
59 quantitative Planar Laser-induced Fluorescence (PLIF) technique to measure the
60 concentration of atomic Na and its release process in the plume of a burning coal particle. It is,
61 however, difficult to obtain the information on the release of atomic Na during the pyrolysis
62 stage, because of the strong scattering due to soot particles. Furthermore, other sodium
63 compounds such as NaOH and NaCl can hardly be measured simultaneously using PLIF. To
64 overcome these limitations, Laser-induced Breakdown Spectroscopy (LIBS) was used to
65 detect the alkali metal element (Na/K) in all relating species and measure the total released

66 amount. Using LIBS, the time-resolved release processes of sodium and potassium can be
67 obtained during all coal combustion stages, including the sooting pyrolysis stage [11].

68 On the numerical side, the final forms of sodium species in post-combustion gases can
69 now be modeled with well-established reaction mechanisms, as the one discussed by Glarborg
70 and Marshall [13]. A model for sodium release during the combustion of a single Loy Yang
71 brown coal char particle has been proposed recently [14]. However, to the best of our
72 knowledge, the dynamic release and reaction characteristics of sodium during turbulent
73 pulverized-coal combustion (PCC) have not been studied, despite their key role in providing
74 important physical insights into alkali metal emissions of industrial coal-fired furnaces.
75 Large-eddy simulation of pulverized-coal combustion (PCC-LES) has been reported in the
76 literature as a valuable tool [15-18]. Recently, PCC-LES research has mainly focused on
77 laboratory-scale pulverized-coal jet flames [19-23]. Tabulated flamelets have been used for
78 modeling the flame scales unresolved by the mesh [2, 24] and studies on NO_x predictions with
79 LES [25], which suggests that the minor chemical species, which are similar to sodium
80 species in PCC and have much longer chemical timescales than those of the major chemical
81 species, can be addressed within this modeling framework.

82 To predict sodium species in PCC-LES, two major points need to be addressed. The first
83 one is predicting the release of sodium species from pulverized-coal particles. The amount of
84 the sodium volatiles released from coal particles and their exact chemical composition need to
85 be determined to provide initial conditions for the subsequent reactions of sodium species in
86 the gas phase. In the most recent model for the release of sodium during the combustion of a

87 single Loy Yang brown coal char particle [14], the information on both the total releasable
88 amount and the exact chemical composition of the released sodium species is still out of reach.
89 In addition, the pyrolysis stage is not included in the modeling.

90 The second issue concerns the chemical kinetics of sodium species in the gas phase. As
91 noted above, a detailed reaction mechanism for sodium and potassium has been proposed by
92 Glarborg and Marshall [13]. This chemical scheme includes 105 elementary reactions, and it
93 would be computationally expensive to directly employ it in LES, instead a tabulated
94 chemistry approach is retained [26].

95 Detailed-chemistry tabulations based on canonical combustion problems, such as
96 homogeneous reactors or one-dimensional premixed or non-premixed flames, thus featuring
97 strong similarities with flamelet modeling, have received extensive attention over the past
98 fifteen years [27-31]. Based on the combination of the premixed flamelet and progress
99 variable concepts, two equivalent approaches, i.e., the flame prolongation of intrinsic
100 low-dimensional manifolds (FPI) [32] and flamelet-generated manifolds (FGM) [33] have
101 been successfully developed and employed in numerical simulations of gas [34] and
102 multiphase [35, 36] combustion [37]. In both FPI and FGM, the species distributions observed
103 through a reference flame are mapped over a progress variable, so that the knowledge of this
104 single variable is sufficient to retrieve the local flow composition. Such methodology was in
105 fact first employed in Reynolds-averaged Navier-Stokes (RANS) modeling of turbulent
106 premixed flames [38], well before chemistry tabulation was discussed *per se* in the literature.
107 In addition to modeling hydrocarbon combustion, tabulated chemistry methods have also been

108 applied to the prediction of minor species such as NO_x [39]. Because of the separation of time
109 scales between the oxidation of the hydrocarbon and the production of thermal NO_x , novel
110 approaches based on tabulated chemistry have been developed to predict NO_x [25, 34, 40].
111 For instance, NOMANI (Nitrogen Oxide emission model with one-dimensional MANifold)
112 developed by Pecquery et al. [25] uses two separate progress variables, one for hydrocarbon
113 combustion and another for NO_x reactions.

114 In the present work, the second key issue of sodium predictions in PCC is addressed
115 using detailed chemistry tabulation, namely, the introduction of the chemical kinetics of
116 gaseous sodium species in a turbulent reacting flow. Since currently detailed information
117 about the sodium species released from a burning coal particle is still unavailable,
118 assumptions are formulated to provide these initial conditions. Therefore, the model that is
119 developed will need in a close future full validation against experimental measurements.

120 The paper is organized as follows: details on the tabulation method for sodium chemistry
121 are presented in Section 2, including the development and the validation of the chemical table.
122 The LES framework is given in Section 3. The subsequent Sections 4 and 5 are devoted to the
123 analysis of the behavior of sodium chemical species in the numerical simulation of a turbulent
124 pulverized-coal jet flame ignited by a pilot of preheated gases. In the pulverized-coal jet flame
125 at the laboratory scale studied in the present LES, char burnout stays limited, it could
126 therefore be neglected [22]. A first set of LES without sodium chemistry includes char
127 combustion. In a second set, char combustion is neglected and sodium reactions are added.

128

129 **2. Tabulation of detailed sodium chemistry**

130 ***2.1. Problem formulation***

131 Because atomic sodium is predicted to be the favoured species in a flame environment
132 [41], it is the alkali released from coal in the simulations. Moreover, because the sodium
133 generated inside the porous structure of a coal particle will be transported outward by the
134 volatile yielded during the pyrolysis stage, the sodium release rate is assumed to be
135 proportional to the pyrolysis rate, or the volatile release rate (the source terms in the transport
136 equations for volatile species mass fractions).

137 The detailed reaction mechanism of alkali metal species by Glarborg and Marshall [13]
138 include 105 elementary reactions over the atomic elements Na, K, C, H, O, S and Cl. In the
139 present study, only the subset with the atomic elements Na, C, H and O is considered, which
140 includes 5 sodium species, i.e., Na, NaO, NaO₂, NaOH and Na₂O₂H₂, and 24 elementary
141 reactions.

142 Apart from the reactions of sodium species, hydrocarbon volatile combustion has been
143 modeled by a partially stirred reactor concept [42, 43]. Since the magnitude of sodium species
144 is very small, i.e., at the ppm level, and the reactions of sodium species are slower than
145 volatile combustion, one-way coupling is considered in the interaction between the sodium
146 reactions and volatile combustion, i.e., the effects of hydrocarbon volatile combustion on the
147 reactions of sodium species are considered, but the effects of sodium reactions on
148 hydrocarbon combustion are ignored.

149

150 **2.2. Table coordinates: initial conditions**

151 The initial conditions of the chemical trajectories of sodium-related species are defined from
152 three parameters: (i) the equivalence ratio ϕ of the mixture; (ii) the mass fraction of the
153 sodium element Y_{sodium} ; and (iii) the gas-phase temperature T . The first two allow for
154 describing the mixing between three streams: (a) the primary pulverized-coal-particle-laden
155 air jet, (b) the coflow and (c) the volatile stream originated from coal particles. The gas-phase
156 temperature accounts for the effects of temperature on sodium reactions, including
157 temperature variations due to gas-phase combustion; heat loss due to radiation; and the heat
158 exchange between the gas and particle phases.

159 The range of variation, or the upper and lower bounds, of Y_{sodium} and T as table
160 coordinates can be obtained from LES results of the pulverized-coal flame under investigation.
161 The scatter plot in Fig. 1 shows the distribution of the mass fraction of the sodium element
162 (Y_{sodium}) versus the equivalence ratio ϕ from an instantaneous LES result of a coflow-heated
163 pulverized-coal jet flame studied in this work. Please note that Y_{sodium} is physically equivalent
164 to the mass fraction of a volatile species because the release rates of volatile gas and sodium
165 vapor are assumed to be proportional to each other (see Section 2.1). Therefore in Fig. 1, a
166 higher Y_{sodium} also indicates that more volatile species exists at the local grid cell at each
167 equivalence ratio. The detailed flame parameters will be given later.

168 The two parameters ϕ and Y_{sodium} can therefore be used to quantify the mixing among the
169 three feeding streams in the current flame configuration. Based on the compositions of atomic
170 C (carbon), H (hydrogen) and O (oxygen) in each of the three feeding streams, the theoretical

171 upper and lower bounds of Y_{sodium} can be computed and are also given in Fig. 1. The upper
172 bound indicates a mixture of volatile and the primary air jet, and the lower bound indicates a
173 mixture of volatile and the hot co-flow. Since the primary air jet flow carries pulverized-coal
174 particles that generate volatile, a pure mixture between volatile and the co-flow cannot form.
175 Since volatile is generated from a pulverized-coal particle after it is heated by the co-flow, a
176 pure mixture between volatile and the primary air jet cannot form either. Therefore, both the
177 theoretical upper and lower bounds are not reached by the LES results. In the present study,
178 the two theoretical bounds have been used as the upper and lower limits of Y_{sodium} to build the
179 chemical lookup table for sodium chemistry at each equivalence ratio.

180 Figure 2 shows the distribution of the gas-phase temperature (T) versus equivalence ratio.
181 Due to the effects of the high-temperature coflow, the density of the scatter data points
182 becomes higher in the region where the equivalence ratio is between 0 and 0.5 (Fig. 2). Also,
183 because the gas-phase temperature is not only governed by combustion, but it is also affected
184 by the high-temperature coflow, radiation and the heat exchange between the gas and particle
185 phases, fluctuations of temperature exist for a fixed value of the equivalence ratio. The line
186 plots in Fig. 2 indicate the upper and lower bounds employed in the chemical lookup table.
187 The conditional mean and fluctuation of temperature for a given equivalence ratio lies within
188 these bounds. Almost the whole span of the gas-phase temperature is covered by the table,
189 except for a few data samples, which have been intentionally avoided in the lookup table.
190 These points are very few compared to the other points and statistically they do not contribute
191 much to the conditional mean nor to the conditional fluctuations. Moreover, the accuracy of

192 the interpolation used to retrieve data from the table decreases with the range spanned and
193 including these few points would, in the end, add more uncertainties in the modeling loop.
194 The lower bound is chosen to keep the temperature range for all the equivalence ratios less
195 than 700 K.

196

197 **2.3. Table coordinates: progress variable**

198 After defining the three parameters characterizing the initial conditions of the trajectories
199 used to build the lookup table, the time evolution along these trajectories is remapped into a
200 progress variable space. The progress variable Y_c , representing the progress of chemical
201 reactions, is defined as a linear combination of the mass fractions of the chemical species:

$$202 \quad Y_c(t, Z) = \sum_{i=1}^n \alpha_i Y_i(t, Z), \quad (1)$$

203 where Y_i is the mass fraction of the i th species, α_i is the corresponding coefficient, t is time, Z
204 denotes the initial conditions, i.e., ϕ , Y_{sodium} and T . In the present study, 5 sodium species are
205 considered. The progress variable is therefore expressed as:

$$206 \quad Y_c(t, Z) = \alpha_1 Y_{Na}(t, Z) + \alpha_2 Y_{NaO}(t, Z) + \alpha_3 Y_{NaO_2}(t, Z) + \alpha_4 Y_{NaOH}(t, Z) + \alpha_5 Y_{Na_2O_2H_2}(t, Z). \quad (2)$$

207 For the tabulation to be effective [26], the progress variable Y_c should monotonically
208 evolve with the time t along all the chemical trajectories, so that the mass fractions of all
209 chemical species can be expressed as single-valued functions of Y_c . Then, $Y_c(t, Z)$ can be
210 uniquely inverted to $t(Y_c, Z)$, and $Y_i(t, Z) = Y_i(t(Y_c), Z)$ can then be expressed as $Y_i(Y_c, Z)$,
211 which is the evolution of chemical reactions in the progress variable space. In Niu et al. [26] it
212 was also discussed how the gradient of the species concentrations in the progress variable

213 space $\partial Y_i(Y_c, Z)/\partial Y_c$ should not be too large. Otherwise, a small deviation in the prediction of
 214 Y_c would lead to large errors in Y_i when reading species mass fractions in the table.

215 In the present case, a progress variable $Y_{c,a}$ is first defined as the total mass fraction of the
 216 sodium element present in the products of the sodium reactions. The mass of the sodium
 217 element is conserved during the sodium reactions and $Y_{c,a}$ monotonically increases along with
 218 the consumption of the reactant. Because the mass fraction of $\text{Na}_2\text{O}_2\text{H}_2$ is several orders of
 219 magnitude smaller than that of the other sodium species, $Y_{c,b}$ is added to $Y_{c,a}$ to reduce the
 220 gradient of the mass fraction of $\text{Na}_2\text{O}_2\text{H}_2$ in the progress variable space, thereby improving
 221 the accuracy of the chemistry table on the predictions of this minor sodium species.

222 The progress variable Y_c then reads:

$$223 \quad Y_c = Y_{c,a} + Y_{c,b}, \quad (3)$$

224 with,

$$225 \quad Y_{c,a} = (M_{\text{Na}}/M_{\text{NaO}}) Y_{\text{NaO}} + (M_{\text{Na}}/M_{\text{NaO}_2}) Y_{\text{NaO}_2}$$

$$226 \quad + (M_{\text{Na}}/M_{\text{NaOH}}) Y_{\text{NaOH}} + (2M_{\text{Na}}/M_{\text{Na}_2\text{O}_2\text{H}_2}) Y_{\text{Na}_2\text{O}_2\text{H}_2}, \quad (4)$$

$$227 \quad Y_{c,b} = (2M_{\text{Na}}/M_{\text{Na}_2\text{O}_2\text{H}_2}) Y_{\text{Na}_2\text{O}_2\text{H}_2} \times (10^4 - 1), \quad (5)$$

228 where M denotes the molecular weight (see Table 1). The amplification factor $(10^4 - 1)$ was
 229 determined via a trial-and-error approach to achieve the best performance of the chemistry
 230 table on tracing $\text{Na}_2\text{O}_2\text{H}_2$ and other sodium species, as shown in Section 2.5 the verification
 231 study.

232

233 **2.4. Build-up of the chemical lookup table**

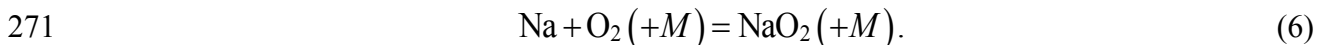
234 The procedure of building up the sodium chemistry table is as follows. For each initial
235 condition $Z(\phi, Y_{sodium}, T)$, a chemical equilibrium state of hydrocarbon combustion is
236 computed with the open-source software CANTERA [44] together with GRI-3.0 [45],
237 excluding the sodium reactions. Then, a zero-dimensional simulation of sodium reactions is
238 performed using CANTERA in combination with GRI-3.0 and the detailed sodium reaction
239 mechanism [13]. The zero-dimensional simulation is run for 2.0 seconds in total, which is
240 much longer than the residence time of a fluid particle of the jet flow in the LES. It is also
241 sufficiently long for the sodium reactions reaching the chemical equilibrium state except for a
242 limited number of cases under some low-temperature conditions. The obtained chemical
243 trajectory is then remapped into the progress variable space. Specifically, for each value of the
244 progress variable Y_c , the corresponding concentration Y_i of the i -th sodium species and also
245 the source term ω_{Y_c} in the transport equation for the progress variable are obtained and stored
246 into the chemistry table. ω_{Y_c} is calculated based on a linear combination of the source of Y_i ,
247 using the same coefficients given in Table 1. The progress variable Y_c is normalized by the
248 final maximum value before being stored in the table, and the normalized progress variable
249 $C(t, Z) = Y_c(t, Z) / Y_{c,max}(2.0, Z)$ monotonically evolves from zero to unity for any single
250 chemical trajectory of the sodium reactions. The same procedure will be looped for all the
251 initial conditions for the obtained sodium chemistry table to include all the chemical
252 trajectories.

253 The equivalence ratio ϕ in the range of 0.0 to 1.85 is discretized over 100 points on a

254 non-uniform grid. The grid is refined around $\phi = 1.0$, where the species composition changes
 255 rapidly. For each equivalence ratio, the lower and upper limits of the mass fraction of the
 256 sodium element Y_{sodium} and gas-phase temperature T can be obtained from Figs. 1 and 2. Both
 257 of them are discretized on a uniform grid, with 30 points for Y_{sodium} and 50 for T . Finally, for
 258 the normalized progress variable C , a non-uniform grid with 100 points is used, with grid
 259 refinement at smaller C values, i.e., the initial stage of the sodium reactions, because
 260 conditions exist with strong variations in this zone. In summary, the sodium chemical table
 261 features $100 \times 30 \times 50 \times 100$ data points for $\phi \times Y_{sodium} \times T \times C$. On each of these data points
 262 the mass fractions of the 5 sodium species and the source term ω_{Yc} in the transport equation
 263 for the progress variable are stored. The size of this complete table is about 700 MB.

264 Figures 3-6 show the evolution of sodium reactions in the progress variable space under
 265 representative initial conditions defined by ϕ , Y_{sodium} and T . Three equivalence ratios ($\phi =$
 266 0.508, 1.0 and 1.508) are selected to illustrate fuel-lean, stoichiometric and fuel-rich flame
 267 conditions, with three typical temperatures at each equivalence ratio, to illustrate the effects of
 268 the gas-phase temperature on the sodium reactions (see Figs. 3-5).

269 Under fuel-lean condition ($\phi = 0.508$), NaO_2 and NaOH are the main sodium products
 270 (Fig. 3). The oxidation path from Na to NaO_2 occurs through a very fast 3-body process [46]:



272 The NaO_2 reacts with the radicals H and OH in the products of hydrocarbon combustion,
 273 producing NaOH [46]. Figure 3 shows that the oxidation of Na towards NaO_2 is not favored
 274 at high temperature. When the temperature increases from 1005 K to 1396 K, the peak

275 concentration of NaO_2 decreased by almost one order of magnitude. NaOH mass fraction
276 stays very small at the lower temperatures (1005 K in Fig. 3); but it becomes an important
277 product at higher temperatures (1208 K and 1396 K in Fig. 3). At the end equilibrium point of
278 the chemical trajectories, almost all the sodium is transformed into NaOH at these higher
279 temperatures.

280 The source term ω_{Yc} of the progress variable that is solved with the flow is shown in Fig.
281 3f. Large values of ω_{Yc} at the early stage of the chemical trajectory are caused by the
282 abovementioned fast oxidation of Na towards NaO_2 . It should be noted that the source term
283 has a relatively small but non-zero value towards the ‘long tail’, i.e., $C = 0.1-1.0$, which is
284 illustrated in the enlarged view. As mentioned above, denser grids are used to capture the
285 rapid variation of species trajectories in their early stage.

286 Figure 4 shows the evolution of sodium properties under the stoichiometric condition (ϕ
287 = 1.0) for three temperatures (1721 K, 1878 K and 2022 K). Compared to the fuel-lean
288 condition (Fig. 3), the most significant difference is that NaO_2 is no longer one of the major
289 sodium products and the stiff variations at low values of the progress variable almost
290 disappear. Indeed, because of the stoichiometric condition of the products contained in the
291 mixture, the O_2 concentration is extremely low and the influence of the sodium oxidation
292 reaction (6) becomes minor. At all the three temperatures, Na is gradually transformed into
293 NaOH via reactions with hydrocarbon combustion products. As the temperature rises, the
294 production of NaOH decreases, which indicates that the sodium reactions are again not
295 favored at high temperature. It can also be noted that part of Na remains untransformed at the

296 end of the trajectories.

297 The evolution of the mass fractions of sodium species under fuel-rich condition ($\phi =$
298 1.508) is shown in Fig. 5, also at three different temperatures (1522 K, 1629 K and 1727 K).
299 Na is gradually transformed into NaOH, and the conversion rate of Na here is much lower.
300 Besides, the values of the progress variable source term are also reduced compared to the
301 stoichiometric condition (Fig. 4), indicating an expected overall slower Na chemistry under
302 fuel-rich environment. Figure 6 shows the species response for various levels of Y_{sodium} under
303 the stoichiometric condition. The initial sodium concentration impacts on the levels of species
304 mass fractions, but does not profoundly modify the overall shape of the response versus
305 progress variable.

306

307 ***2.5. Verification of the chemistry table against detailed mechanism***

308 Before coupling the lookup-table of sodium chemistry with unsteady flow simulations,
309 verification tests are performed in which it is attempted to recover the time evolution of the
310 chemistry from solving the progress variable only (i.e. $dY_c/dt = \omega_{Y_c}$), species mass fractions
311 being simply read from the table. Homogeneous cases at various equivalence ratios and initial
312 conditions, in terms of sodium concentrations and temperature, are considered. Results are
313 compared against those directly obtained from the detailed chemical system [13] using
314 CANTERA and GRI-3.0. The source of the progress variable is read from the table and Y_c is
315 advanced in time with a first-order scheme. To test the multi-linear interpolations, the
316 equivalence ratios, the initial temperatures and sodium mass fractions are chosen not to be

317 exactly on the grid points of the lookup table.

318 Figures 7-9 show the comparison of the sodium chemistry predicted by the lookup table
319 against those directly obtained from the detailed mechanism. For all the 9 initial conditions
320 chosen, the predictions of the chemistry table closely follow those of the detailed chemistry
321 [13]. Minor deviations can be found in the predictions of the mass fractions of NaO and
322 Na₂O₂H₂ under the fuel-lean condition (Fig. 7), due to interpolation errors. However, NaO
323 and Na₂O₂H₂ are not major sodium species during the reactions and therefore these minor
324 errors are well acceptable. From Figs. 7-9, it can also be found that the reaction time of 2.0 s
325 is sufficient for most of the cases to reach the chemical equilibrium of the sodium reactions,
326 except for the case at the low temperature, $T = 1000$ K, and under the fuel-lean condition.
327 Because a duration of 2.0 seconds is much longer than the flow residence time in the jet
328 simulated thereafter, it is not necessary to include the full chemical trajectory until the
329 chemical equilibrium state for this particular case, thereby allowing us to focus the grid points
330 of the lookup table in the first stage of the chemical evolution. This verification has also been
331 performed under various other initial conditions (not shown for the sake of brevity),
332 confirming the reliability of the lookup table.

333

334 **3. Large-eddy simulation**

335 ***3.1. Gas phase modeling***

336 The filtered three-dimensional Navier-Stokes (NS) equations in the low-Mach-number
337 form for mass, momentum, species and temperature are solved for the gas phase [43, 47]. The

338 transport by unresolved subgrid scale (SGS) velocity fluctuations is modeled with the
 339 Germano dynamic model [48]. The balance equations for the mass fractions of N₂, O₂, H₂O,
 340 CO₂, CH₄, CO, C₂H₂, H₂, tar (9 species for coal/hydrocarbon combustion) and Y_{sodium} , Y_c (2
 341 scalars for sodium chemistry) are solved:

$$342 \quad \frac{\partial \bar{\rho} Y_n}{\partial t} + \frac{\partial \bar{\rho} u_j Y_n}{\partial x_j} = \frac{\partial}{\partial x_j} \left(\bar{\rho} \bar{D}_{diff} \frac{\partial Y_n}{\partial x_j} - q_{sgs,Y,n,j} \right) + \bar{\omega}_{Y,n} + \bar{S}_{Y,p,n}, \quad (7)$$

343 where $\bar{\rho}$ is the density of the gas mixture (kg/m³), Y_n is the mass fraction of species
 344 (kg/kg), n means the n th chemical species, u_j is gas velocity (m/s), x_j is the coordinate (m),
 345 \bar{D}_{diff} is the molecular mass-diffusion coefficient (m²/s), $q_{sgs,Y,n,j}$ is the SGS term (kg/m² s), $\bar{\omega}_{Y,n}$
 346 is the source term due to chemical reactions (kg/m³ s), $\bar{S}_{Y,p,n}$ is the source term due to the
 347 discrete phase (devolatilization and surface reaction) (kg/m³ s). It should be mentioned that
 348 for Y_{sodium} , $\bar{\omega}_{Y,n}$ is zero, because it is actually the conserved mass fraction of the sodium
 349 element originating from coal particles during sodium reactions. While for the progress
 350 variable Y_c , $\bar{S}_{Y,p,n}$ is zero and $\bar{\omega}_{Y,n}$ is obtained from the sodium-chemistry table (see
 351 Section 3.5).

352 Pulverized-coal particles are modeled as point sources and two-way interactions between
 353 the gas phase and particles are considered. The source terms due to coal pyrolysis, volatile
 354 combustion and char combustion in the species mass fraction equations are defined in
 355 Sections 3.3 and 3.4. In the temperature equation, the radiation, heat exchange between the
 356 gas phase and coal particles, and heat release of coal combustion are considered. In the
 357 present study, the Lewis number (Le = 1.0) and Prandtl number (Pr = 0.7) are assumed to be
 358 constant.

359

360 **3.2. Particle phase modeling**

361 The momentum equation of a coal particle in the Lagrangian framework is cast in

$$362 \quad \frac{du_{p,j}}{dt} = \frac{f}{\tau_p}(u_j - u_{p,j}) + W_{sgs,j}, \quad (8)$$

363 where $u_{p,j}$ is the velocity of the particle (m/s) and u_j is gas velocity (m/s). The dynamic
364 response time of the particle (s) is $\tau_p = \rho_p d_p^2 / 18\bar{\mu}$, where ρ_p is the particle density (kg/m³),
365 and d_p is the particle diameter (m). f is the drag coefficient. Following Jones et al. [49], a
366 stochastic Markov model is used to incorporate the effects of the unresolved SGS turbulence
367 ($W_{sgs,j}$) into the particle acceleration.

368 The particle temperature is obtained by solving

$$369 \quad \frac{dT_p}{dt} = \frac{(Q_{conv} + Q_{rad} + Q_{dev} + Q_{char})}{m_p C_{P,p}}, \quad (9)$$

370 where T_p is the temperature of the particle (K), m_p the particle mass (kg). $C_{P,p}$ is the specific
371 heat capacity (J/kg K) of the solid phase and determined by
372 $C_{P,p} = 836.0 + 1.53 \times (T_p - 273.0) - 5.4 \times 10^{-4} (T_p - 273.0)^2$ [47]. The heat transfer due to
373 convection, radiation, pyrolysis (devolatilization) and char combustion is
374 $Q_{conv} = Nu C_{P,g} m_p (T - T_p) / 3 Pr \tau_p$, $Q_{rad} = \varepsilon_p \pi d_p^2 \sigma (T_R^4 - T_p^4)$, $Q_{dev} = -\Delta h_{dev} dm_{vol} / dt$, and
375 $Q_{char} = -\Delta h_{char} dm_{char} / dt$, respectively, where $C_{P,g}$ is the specific heat capacity of the gas
376 phase (J/kg K), T the gas temperature (K). Nu is the Nusselt number, calculated by the
377 Ranz-Marshall correlations [50]. The radiation temperature (K) is estimated by
378 $T_R = (G/4\sigma)^{1/4}$, where G is the incident radiation (W/m²), which is determined here by the

379 Discrete Ordinates Method (DOM) [21, 23, 51]. σ is the Stefan-Boltzmann constant. The
380 particle emissivity ε_p is set to 0.9 [52]. The gas absorption coefficient is determined by the
381 weighted-sum-of-the-gray-gases model (WSGGM) [53]. The effects of the subgrid-scale
382 gas-phase temperature and the Stefan flow on pulverized-coal-particle heating have been
383 found to be negligible for the cases considered in the present study and therefore have not
384 been taken into account in Q_{conv} . Finally, the rate of change of the mass of each coal particle
385 (dm_p/dt) is equal to the sum of the coal pyrolysis rate (dm_{vol}/dt) and char combustion rate
386 (dm_{char}/dt).

387

388 ***3.3. Coal pyrolysis and combustion models***

389 The CPD model, proposed by Fletcher et al. [54], is directly incorporated into the LES
390 framework to model the pyrolysis process of each coal particle [47]. As one of the current
391 state-of-the-art formulation for coal pyrolysis, CPD describes the formation of volatile
392 (including light gases and heavy tar) based on the unique chemical structure of different coals.
393 Its performance on predicting the pyrolysis rate and volatile yield composition has been
394 validated over a wide range of heating rates, temperatures and coal ranks [54-56]. In our
395 previous work [52, 57], CPD was used in the pyrolysis of a single coal particle under various
396 operating conditions. Good agreement against experimental measurements was achieved on
397 key pyrolysis parameters, such as the time history of the particle residual mass and the
398 temperature at the center of the particle. We also conducted further validations of the coupling
399 of CPD with turbulent flow transport in LES and detail on its implementation may be found in

400 [43, 47]. It is briefly repeated here for completeness. The CPD model was originally
401 developed to predict the volatile yields with time. When incorporated into LES framework,
402 the variables that represent the particle pyrolysis status in the CPD model are recorded and
403 updated at every time step for each particle. When the new particle temperature is known at
404 the end of the time step, the CPD model can predict the volatile yields of the particle at the
405 new time according to the instantaneous heating rate of the particle. By subtracting the
406 volatile yields at the previous time, the volatile release of the particle in the present time step,
407 and the mass and species source terms due to the pyrolysis of the particle can be calculated.
408 The same procedure will be used for all the particles in the computational domain.

409 After the volatile matter is completely released, heterogeneous combustion of the residual
410 char occurs. The char oxidation reaction is assumed to be $C(s) + 0.5 O_2 \rightarrow CO$, using the
411 kinetic/diffusion surface reaction model by Baum & Street [58]. Char oxidation is enabled in
412 the LES of the pulverized-coal jet flame for validation (Section 4). However, in Section 5,
413 where the sodium chemistry is investigated, the char reaction model is disabled due to very
414 limited char burnout in the turbulent jet.

415

416 ***3.4. Gas-phase combustion model***

417 Because the objective of this work is the modeling of sodium chemistry, a simple
418 description has been chosen for the turbulent burning rate controlling the heat-release in the
419 pulverized-coal turbulent flame. The LES-PaSR model [42, 43] is used for the combustion of
420 the volatiles, with a two-step reaction mechanism for the oxidation of methane [59], while

421 other species react according to a single-step chemistry [59, 60]. In this approach, the effects
 422 of the unresolved SGS fluctuations of species and temperature on the non-linear burning rates
 423 are accounted for by a decomposition of the turbulent flow into fine reactive/diffusive layers,
 424 where most of the chemical reactions take place, and their surroundings. The volume fraction
 425 of the fine structures is estimated as $\kappa = \tau_c / (\tau_c + \tau_m)$, where τ_c and τ_m are a chemical time scale
 426 and a subgrid mixing time scale, respectively. The filtered reaction rate is then expressed as
 427 $\bar{\omega}_{Y,n} \approx \kappa \omega(\bar{\rho}, Y_n, T)$.

428 Notice however that this description of the flame within the subgrid does not explicitly
 429 account for any specific flame topologies. For example, if the gas phase equivalence ratio at
 430 the LES cell center is lean and at the same time the gas at the coal surface is rich then an
 431 envelope flame could exist, but it would not be resolved and would not be accounted for in
 432 the convection heat transfer model in an explicit manner in the modeling.

433

434 **3.5. Modeling sodium reactions**

435 Two variables need to be transported as two coordinates of the chemical lookup table of
 436 sodium, i.e., Y_{sodium} and Y_c . The Eulerian production source of Y_{sodium} is obtained through the
 437 simplified sodium release model of Section 2.1. The other two coordinates of the chemical
 438 lookup table are ϕ and T . The gas-phase temperature T is directly transported in the LES
 439 solver, and the equivalence ratio can be computed as

$$440 \quad \phi = \frac{2X_C + X_H/2}{X_O}, \quad (10)$$

441 where X_C , X_H , and X_O denote the mole fractions of atomic carbon, hydrogen and oxygen,

442 respectively. The source of the progress variable Y_c , i.e., ω_{Y_c} , is read from the table to then
 443 advance in time the Eulerian set of the equations. As usually done with lookup tables, Y_c is
 444 normalized to define a progress variable between zero and unity. As shown above, the sodium
 445 chemistry is much slower than any of the characteristic flow times. Therefore, mesh nodes
 446 values are directly used without accounting for unresolved SGS fluctuations.

447 The chemical lookup table, which is based on trajectories obtained for a fixed value of
 448 the temperature, cannot account for the variations of the sodium compositions at a chemical
 449 equilibrium (the progress variable equals unity) due to the evolution of the local temperature.
 450 To track these additional variations of the progress of sodium reactions, a term has been added
 451 to ω_{Y_c} to consider the relaxation towards the new equilibrium conditions. Following Galpin et
 452 al. [61], the additional source term can be written as:

$$453 \quad \omega_{Y_c}^{\text{RTE}} = \frac{Y_c^{\text{Eq}}(\bar{\phi}, Y_{\text{sodium}}, T) - Y_c}{\gamma \delta t} \quad (11)$$

454 where δt is the time step used in the LES solver, γ is a relaxation coefficient. Then the source
 455 term of the progress variable reads:

$$456 \quad \bar{\omega}_{Y_c} = \omega_{Y_c}^{\text{TAB}}(\bar{\phi}, Y_{\text{sodium}}, T) + \beta \omega_{Y_c}^{\text{RTE}} \quad (12)$$

457 with $\beta = 0$ as long as the sodium reactions do not reach chemical equilibrium, i.e., $Y_c < Y_c^{\text{Eq}}$,
 458 otherwise $\beta = 1$. $\omega_{Y_c}^{\text{TAB}}$ is the source term obtained from the lookup table. The value of γ is
 459 set to be 1.0 in the present study.

460

461 **3.6. Numerical schemes**

462 The time advancement uses a second-order Crank-Nicolson scheme. All terms in the
463 momentum equations, and the scalar diffusion terms in the species and temperature equations,
464 are discretized with a second-order central-difference scheme. To secure scalar boundedness,
465 a third-order weighted essentially non-oscillatory (WENO) scheme [62] is used for the
466 advection terms in the species and temperature equations [63].

467

468 **4. LES of a pulverized-coal jet flame: validation**

469 The pulverized-coal turbulent jet flame experimentally and numerically studied in [64]
470 and [19, 20] is simulated. The laboratory-scale coflow-heated pulverized-coal jet flame was
471 measured at the Hitachi Power & Industrial Systems R& D Laboratory [64]. A mixture of
472 pulverized-coal particles and air are injected through a central nozzle, the inner diameter of
473 which is 7 mm. The Reynolds number of the primary air jet is about 4500. To stabilize the
474 pulverized-coal jet flame, hot coflow gas produced by catalytic combustion of propane is used
475 to ignite coal particles. A high-speed camera was employed to capture the ignition phenomena
476 of the pulverized-coal jet and the lift-off height was then determined from the images. Gas
477 temperature was measured by a Pt/Pt-Rh thermocouple with a diameter of 100 μm and
478 corrected for heat loss of radiation. Coal burnout was measured by collecting and analyzing
479 burning particles using a stainless-steel water cooling sampling probe with an inner diameter
480 of 10 mm.

481

482 **4.1. Computational setup**

483 The computational domain spans 500 mm in length and 300 mm in width. The mesh
484 contains 1.52 million cells, whose characteristic size varies between 0.3 mm (e.g., at the edge
485 of the nozzle and in the main reaction zones) and 5.2 mm (e.g., at the downstream exit of the
486 domain). After reaching a resolution of 300 μm at the edge of the nozzle, the statistical flow
487 properties collected from LES have been found weakly sensitive to grid resolution.

488 The operating conditions are summarized in Table 2. The primary inlet is located at the
489 center of the $x = 0$ face (x is the streamwise direction), with a nozzle diameter (D) of 7 mm.
490 Cold air (300 K) and coal particles are injected through this nozzle. The coal particles are
491 ignited by the coflowing preheated gas (1510 K). To provide the relevant inflow boundary
492 conditions for the LES, a separate, pre-processed pipe-flow LES with periodic streamwise
493 boundary conditions has been carried out to generate an inflow database [65]. Convective
494 boundary conditions are applied at the streamwise exit of the domain, while zero-gradient
495 boundary conditions are applied at the side of the domain. A particle size distribution given
496 by Yamamoto et al. [19] is prescribed at the inlet. The location of a given particle injected at
497 the nozzle is determined according to a random uniform distribution over the primary inlet
498 face, while its velocity equals the local gas velocity. The coal analysis data [19] is listed in
499 Table 3. The initial density of coal particles is 1400 kg/m^3 [20]. Three inlet stoichiometric
500 ratios (SRs) are considered (0.14, 0.22, 0.36) by varying the coal mass flow rate, where the
501 SR is defined as: $\text{SR} = (\text{the inlet oxygen mass}) / (\text{oxygen mass that is required for completely}$
502 $\text{burning the coal at the inlet})$.

503

504 **4.2. Results and discussion**

505 *4.2.1. Instantaneous gas temperature*

506 The instantaneous gas temperature fields of the pulverized-coal jet flame at three different
507 SRs are shown in Fig. 10. Similar to the findings in [19], three combustion regions can be
508 observed in the ignition process. The first one is governed by heat-transfer (turbulent mixing
509 and radiation) between the flow and the particles (pre-heating region). There only some
510 isolated flame structures (IFSs) are observed. Some particles with relatively small diameters
511 are ignited at first on the jet periphery, i.e. in the mixing layer with the hot-coflow, but the
512 heat release is not strong enough to ignite adjacent coal particles, which results in the reported
513 isolated burning flow areas. The second is a growing flame (GF) region, where more and
514 more particles ignite, followed by a rapid spreading of the flame. The final one is a
515 continuous flame (CF) region, where the pulverized-coal jet flame achieves a stable
516 combustion state. As the inlet air/fuel stoichiometric ratio increases, the ignition position of
517 the pulverized-coal jet flame moves further downstream, and the flame tends to be weaker.

518

519 *4.2.2. Quantitative comparisons on the flame characteristics*

520 Following Yamamoto et al. [19], the pulverized-coal jet is considered to be ignited when
521 the gas temperature reaches 1560 K, which is 50 K higher than the preheated co-flow gas
522 temperature. The lift-off height is then defined as the distance from the primary nozzle exit to
523 the location where the growing flame region and the iso-surface of the ignition temperature (T

524 = 1560 K) appear. As shown in Fig. 11, the lift-off heights predicted by the present CPD-LES
525 at three different SRs are in good agreement with the experimental data [64] and the LES
526 results of Yamamoto et al. (LES-Yamamoto) [19].

527 Figure 12 compares the mean gas temperature along the jet centerline against the
528 experimental measurements and previous LES by Yamamoto et al. for SR = 0.14. Above the
529 streamwise location $x = 100$ mm, the collected statistics agree well with the experimental data
530 and previous LES. For $x < 100$ mm, which is the preheating region, the temperature increase
531 on the centerline is slower in the actual CPD-LES than in the previously reported LES, with
532 an underestimation of the temperature compared to the measurements at $x = 100$ mm. The
533 deviation may be due to the inflow boundary conditions of the LES.

534 Figure 13 shows the comparison of the coal burnout along the centerline at SR = 0.22.
535 Both the CPD-LES and LES by Yamamoto et al. achieve good agreement with the
536 experimental data at $x = 50, 150$ and 400 mm. However, the two LES methods give different
537 predictions in the region of $150 < x < 400$ mm. Compared with the experimental data, the
538 present CPD-LES slightly over-predicts the coal burnout at $x = 300$ mm, for which the LES
539 by Yamamoto et al. under-predicts. It is found that at $x = 400$ mm the coal burnout by char
540 combustion is only 4.2% (daf basis), and this value is 1.9% in the LES results of Yamamoto
541 et al. (not shown in the figure). Since the influence of char combustion is weak compared to
542 pyrolysis in this pulverized-coal jet flame [19], the difference between the two LES results is
543 mainly due to the prediction of pyrolysis. The CPD model used in the present simulations
544 predicts faster pyrolysis mechanisms compared to the modified SFOM pyrolysis model [19,

545 66] used in the LES by Yamamoto et al. [19]. Despite these departures, which are
546 unavoidable considering the complexity of the physical phenomena that need to be addressed
547 in coal combustion, overall these simulations capture the global flame structure, which allows
548 for proceeding with the introduction of the modeling of sodium chemistry in the turbulent
549 flame.

550

551 **5. Sodium reacting dynamics in a pulverized-coal jet flame**

552 ***5.1. Computational setup***

553 The computational setup is similar to that of Section 4.1 except for some minor
554 adjustments. Properties of Loy Yang brown coal are used, for which sodium data are available
555 [67]. Its coal analysis data are listed in Table 4. The mass fraction of the sodium element in
556 the coal is 0.067%. The percentage of sodium that is releasable during the coal pyrolysis stage
557 is set to 19.1%, according to [67]. Then the proportional factor of sodium release to volatile
558 release can be determined as $0.067\% \times 19.1\% \times m_p / m_{vol}$, where m_p is the mass of a
559 pulverized-coal particle and m_{vol} is the mass of volatile yields. The char reaction model is
560 turned off because it was found to weakly contribute in this turbulent flame. The inlet
561 stoichiometric ratio $SR = 0.22$ is considered.

562

563 ***5.2. Results and discussion***

564 A snapshot of the turbulent pulverized-coal jet flame is shown in Fig. 14. Only 0.1% of
565 the total number of particles is plotted. Each particle is colored by its temperature and its size

566 in this plot is proportional to its diameter. The gas-phase temperature field is also shown. As
567 in the simulations reported above, after their injection with the cold primary air jet, the coal
568 particles are gradually heated up by the high-temperature coflow. Then, pyrolysis takes place
569 and the discrete solid phase releases volatiles. These gaseous volatile fuels are subsequently
570 ignited to burn around the pulverized-coal particles, which will further enhance the pyrolysis
571 reaction and helps achieve stable combustion of the pulverized-coal jet.

572 Sodium is released along with volatile species during the coal pyrolysis process. The
573 mass fraction of NaOH, one of the major sodium species, is shown on the left side of Fig. 14,
574 with the isoline of $\phi = 1$ superimposed. The equivalence ratio ϕ is calculated based on the
575 molar fractions of atomic carbon, hydrogen and oxygen (see Eq.(10)). It is therefore
576 conserved during gas-phase combustion, but not conserved during turbulent mixing. Since the
577 equivalence ratio is zero for the primary air jet and small ($= 0.52$) for the coflow, large values
578 of ϕ are mainly due to the volatile stream originating from coal particles (Fig. 15). The isoline
579 of $\phi = 1$ denotes the stoichiometric condition for reactions between the volatile fuels and the
580 oxidizer, which also indicates the region of the highest temperature (Figs. 14 and 2). From Fig.
581 14, it can be seen that NaOH is formed in both fuel-lean (outside the isoline) and fuel-rich
582 (inside the isoline) regions. The highest NaOH concentration is observed in the regions close
583 to the iso-surface defined by $\phi = 1$, which is in accordance with the results shown in Figs. 7-9.

584 Figure 16 shows the instantaneous fields of the mass fractions of the other 4 sodium
585 species, i.e., Na, NaO, NaO₂ and Na₂O₂H₂, still together with the isoline of $\phi = 1$. It is seen
586 that Na, the atomic sodium, features a higher concentration in fuel-rich regions (Fig. 16a).

587 This is explained by the fact that Na is the assumed outgassing species released along with the
588 volatiles from coal particles, accordingly Na is favored in a fuel-rich environment, as also
589 reported above in Figs. 7-9. NaO is found to be produced under the stoichiometric condition
590 ($\phi = 1$), but the magnitude of its mass fraction is two orders of magnitude lower than those of
591 Na and NaOH (Fig. 16b). Both NaO₂ (Fig. 16c) and Na₂O₂H₂ (Fig. 16d) are then rather
592 generated under fuel-lean conditions. However, the concentration of NaO₂ reaches a
593 considerably higher value near the ignition region of the pulverized-coal jet flame – a flow
594 zone where sodium vapor is already generated from coal particles, but the temperature is still
595 low. Finally, the concentration of Na₂O₂H₂ is found to be very low in the whole domain.

596 The mean and RMS mass fractions of the 5 sodium species along the jet centerline are
597 shown in Fig. 17, with the fuel-lean and fuel-rich regions also illustrated. For the mean mass
598 fractions, it can be found that Na features a single-peak distribution along the centerline and
599 has the highest concentration among the 5 sodium species in the fuel-rich region; while the
600 profile of Y_{NaOH} is a double-peak distribution with the peak concentrations achieved in the
601 stoichiometric regions. The concentration of NaO₂ can be considerably high in the fuel-lean
602 regions but very minor in the fuel-rich region. Finally, the concentrations of NaO and
603 Na₂O₂H₂ stay very low all along the centerline, similar to their instantaneous mass fractions
604 illustrated in Fig. 16. For the RMS mass fractions, the fluctuations of the 5 sodium species
605 can achieve the same order of magnitude as the corresponding mean values, which indicates
606 the distribution of the sodium species is considerably affected by the turbulence. The highest
607 fluctuation of Na is found near the upstream stoichiometric region, where the pulverized-coal

608 jet is ignited. The turbulence of the jet flow leads to the instability of the ignition process and
609 then results in a high fluctuation of sodium release.

610

611 ***5.3. Sensitivity analysis of the tabulated sodium chemistry***

612 To investigate the uncertainties of the sodium species results predicted by the tabulated
613 sodium chemistry approach, a systematic global sensitivity analysis [68] has been performed.
614 Considering the computational cost of performing such a global sensitivity analysis study
615 directly on LES would be very high, here a systematic quantification of the uncertainties
616 generated by the sodium lookup table and the global sensitivity of sodium species results to
617 the variation of the four table input parameters (ϕ , Y_{sodium} , T and C) have been assessed.

618 Three representative conditions (see Table 5), which illustrate fuel-lean, stoichiometric
619 and fuel-rich flame conditions, are selected to perform the uncertainty quantification and
620 global sensitivity analysis. The mean values of Y_{sodium} and T are typical values at each
621 condition, while the mean values of C are set as 0.5 to represent the midpoint of the sodium
622 reactions. The uncertainty range of ϕ , Y_{sodium} , T and C is determined considering the available
623 values stored in the chemistry table, the empirical magnitude of the uncertainty and also the
624 discrepancy between the LES and experimental measurements in Section 4 the validation
625 study. All the input uncertain parameters are described by a semicircle Beta distribution
626 ($Beta(3/2, 3/2)$) to avoid unrealistic values that could easily appear with a Normal distribution
627 [69]. 10,000 random sampling values are generated from the Beta distribution for each
628 uncertain parameter and used as the chemistry table inputs. The response sodium species

629 results from the chemistry table are then collected and analyzed. Figure 18 illustrates the
630 mean mass fractions of the 5 sodium species along with the standard deviations under the
631 fuel-lean, stoichiometric and fuel-rich conditions. For the two major sodium species NaOH
632 and Na predicted in the pulverized-coal jet flame, the uncertainty of their mass fractions is
633 found to be small. Considering the other sodium species, the uncertainty of Y_{NaO_2} is also small
634 under the fuel-lean condition. NaO_2 is known from Fig. 17 to mainly form in fuel-lean regions
635 in the flame; while the uncertainties of Y_{NaO} and $Y_{\text{Na}_2\text{O}_2\text{H}_2}$ can be large.

636 An optimized Monte Carlo method proposed by Sobol' et al. [70] has been employed to
637 compute the first-order main-effect sensitivity indices [71], which quantify the contribution of
638 each uncertain parameter (ϕ , Y_{sodium} , T and C) to the uncertainty of the 5 sodium species.
639 Figures 19-21 shows the obtained main-effect sensitivity indices under the fuel-lean,
640 stoichiometric and fuel-rich conditions. It can be observed that the predicted mass fractions of
641 NaOH and Na are strongly sensitive to Y_{sodium} under the fuel-lean and fuel-rich conditions,
642 respectively. Since Y_{sodium} is determined from the sodium release model during the LES, it
643 means the accuracy of the predictions of the two major sodium species NaOH and Na can
644 benefit from a more accurate sodium release model. A similar phenomenon is observed for
645 NaO_2 in the fuel-lean condition. For the minor sodium species NaO and $\text{Na}_2\text{O}_2\text{H}_2$, the
646 prediction of the tabulated sodium chemistry approach is found to be highly sensitive to the
647 temperature in the fuel-lean and fuel-rich conditions. Finally, all the 5 sodium species are
648 found to have a significant sensitivity to ϕ under the stoichiometric condition. This result can
649 be anticipated since a small uncertainty of ϕ can result in a large variation of the gas

650 compositions near the stoichiometric condition, which in turn leads to a significant impact on
651 the sodium reactions.

652

653 **6. Conclusions**

654 A tabulated chemistry method for predicting sodium species in LES of turbulent
655 pulverized-coal combustion has been discussed. One-way coupling is used for the interaction
656 between the sodium reactions and volatile combustion, i.e., the former having no influence on
657 the latter, which is expected because of the large difference in both their respective
658 characteristic time scales and their relative contribution to the total mass. In a first
659 approximation, the sodium release rate from a pulverized-coal particle is also assumed to be
660 proportional to the pyrolysis rate. The chemical lookup table is built from a series of
661 zero-dimensional simulations, which are representative of the time evolution of sodium
662 species mass fractions in a detailed chemistry mechanism [13], which includes 5 sodium
663 species, i.e., Na, NaO, NaO₂, NaOH and Na₂O₂H₂, reacting over 24 elementary reactions.
664 Three parameters, i.e., the equivalence ratio, the mass fraction of the sodium element and the
665 gas-phase temperature, define the initial conditions of these chemical trajectories. The three
666 parameters, along with the progress variable that represents the progress of sodium reactions,
667 are then the four coordinates of the sodium chemical lookup table. A preliminary verification
668 study of the tabulation strategy has been reported, in which the species mass fractions
669 retrieved from solving the time evolution of the single progress variable are compared against
670 the fully detailed chemistry solutions.

671 The sodium chemistry table is then coupled to LES in the configuration of a laboratory
672 pulverized-coal turbulent jet flame for which experimental results of major coal-flame
673 parameters exist. Statistics are collected and results compared against measurements and
674 previous simulations of the literature. Good agreements between the experimental
675 measurements and the LES results have been achieved on gas temperature and lift-off height,
676 while the LES tends to slightly over-predict the coal burnout. In a second part, the combustion
677 of the Loy Yang brown coal is simulated, for which sodium data are available. The results
678 show that Na and NaOH are the two major sodium species in the pulverized-coal turbulent jet
679 flame. Na, the atomic sodium, has a high concentration in fuel-rich regions; while the highest
680 NaOH concentration is found in regions close to the stoichiometric condition. The
681 fluctuations of the 5 sodium species can achieve the same order of magnitude as the
682 corresponding mean values at the jet centerline, indicating the distribution of the sodium
683 species is considerably affected by the turbulence of the jet flow. The accuracy of the
684 proposed tabulated sodium chemistry approach has been calibrated via a systematic global
685 sensitivity analysis of the sodium lookup table. The uncertainties of the two major sodium
686 species Na and NaOH are found to be small. The predicted mass fractions of Na and NaOH
687 have a strong sensitivity to the sodium release, which means the accuracy of the predictions of
688 the two major sodium species can benefit from a more accurate sodium release model.

689 In the perspective of this modeling work, developing sodium species measurements in
690 pulverized-coal laboratory jet-flames appears as a mandatory step, in order to perform further
691 validation against experimental data and gain more confidence in the proposed modeling.

692 Finally, it should be pointed out that the proposed formalism can be readily extended to the
693 modeling of the emission of potassium in turbulent multiphase biomass combustion.

694

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703

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Table 1. Coefficients of the progress variable for sodium reactions.

Species	α_i
Na	0.0
NaO	$M_{Na}/M_{NaO} = 23.0/39.0$
NaO ₂	$M_{Na}/M_{NaO_2} = 23.0/55.0$
NaOH	$M_{Na}/M_{NaOH} = 23.0/40.0$
Na ₂ O ₂ H ₂	$2M_{Na}/M_{Na_2O_2H_2} \times 10^4 = 46.0/80.0 \times 10^4$

Table 2. Operating conditions.

	Primary jet	Preheated gas
Average velocity (m/s)	10	4.8
Temperature (K)	300	1510
Mass fraction		
N ₂	0.768	0.760
O ₂	0.232	0.101
CO ₂	0.0	0.093
H ₂ O	0.0	0.045

Table 3. Coal analysis [19].

<i>Ultimate analysis (dry-ash-free basis, wt%)</i>	
C	83.1
H	4.6
O	9.9
N	1.9
S	0.5
<i>Proximate analysis (dry basis, wt%)</i>	
Volatile matter	31.1
Fixed carbon	54.0
Ash	14.9

Table 4. Analysis of Loy Yang brown coal [67].

<i>Dry basis, wt%</i>	
C	67.8
H	5.20
N	0.57
S	0.24
Cl	0.06
Ash	0.80
O	25.3
<i>In ash, wt%</i>	
SiO ₂	12.9
Al ₂ O ₃	31.4
Fe ₂ O ₃	6.70
TiO ₂	0.70
K ₂ O	0.77
MgO	12.2
Na ₂ O	11.3
CaO	5.60
SO ₃	16.9

Table 5. Uncertainty in input parameters of the sodium chemistry lookup table.

Uncertain parameter	Mean values			Uncertainty range	
	Fuel-lean	Stoichiometric	Fuel-rich	Limits	σ
ϕ	0.5	1.0	1.5	± 0.1	0.05
$Y_{sodium} (\times 10^{-5})$	0.8	2.35	4.0	± 0.5	0.25
T (K)	1200	1850	1600	± 100	50
C	0.5	0.5	0.5	± 0.1	0.05

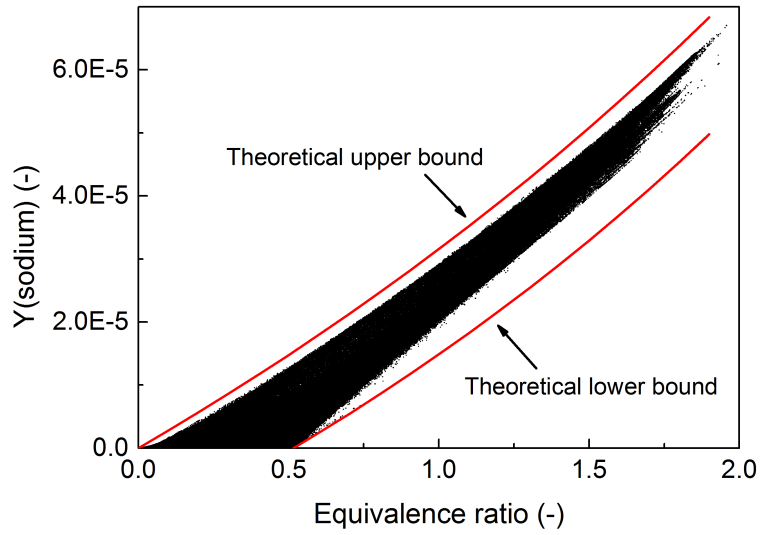


Figure 1. Sodium element mass fraction Y_{sodium} vs equivalence ratio ϕ . Scatter plot: LES results at one time instant. Lines: upper and lower bounds, which are used as the upper and lower limits of Y_{sodium} and ϕ in the chemical lookup table.

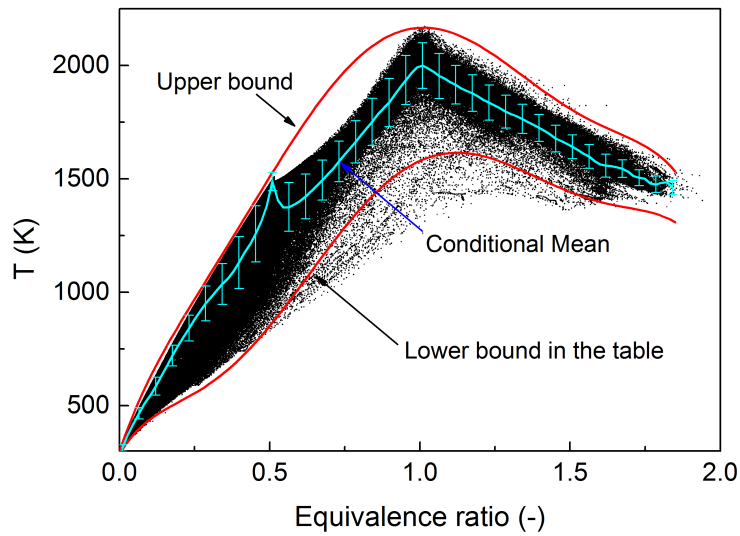


Figure 2. Gas-phase temperature vs equivalence ratio. Scatter plot: LES results at one time instant. Lines: upper and lower bounds used in the chemical lookup table, and the conditional mean and fluctuation of LES results.

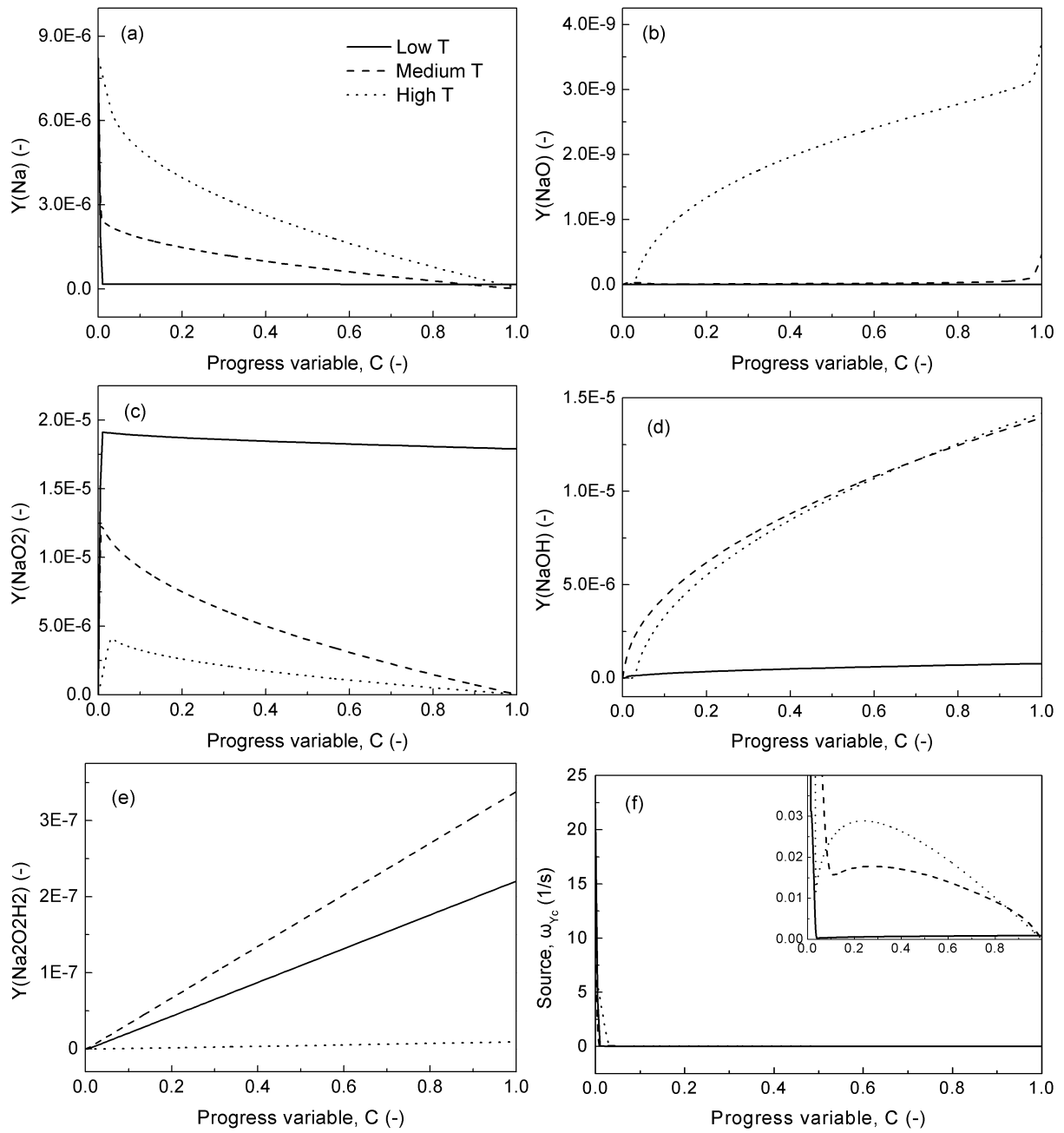


Figure 3. Species mass fractions and progress variable source vs progress variable at $\phi = 0.508$ and $Y_{sodium} = 0.82E-5$. Solid line: $T = 1005$ K; Dashed line: 1208 K; Dotted line: 1396 K.

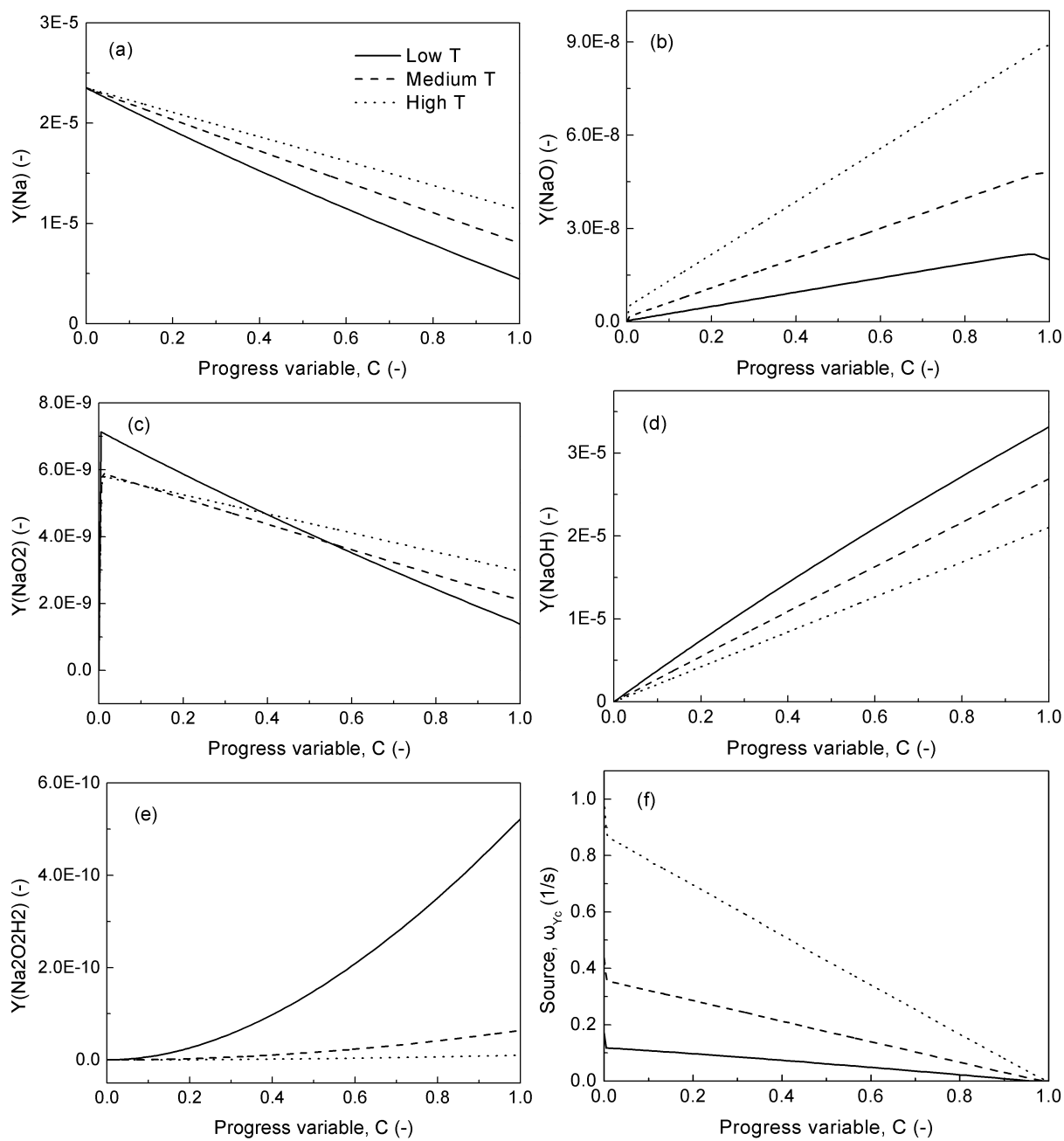


Figure 4. Species mass fractions and progress variable source vs progress variable at $\phi = 1.0$ and $Y_{sodium} = 2.35E-5$. Solid line: $T = 1721$ K. Dashed line: 1878 K. Dotted line: 2022 K.

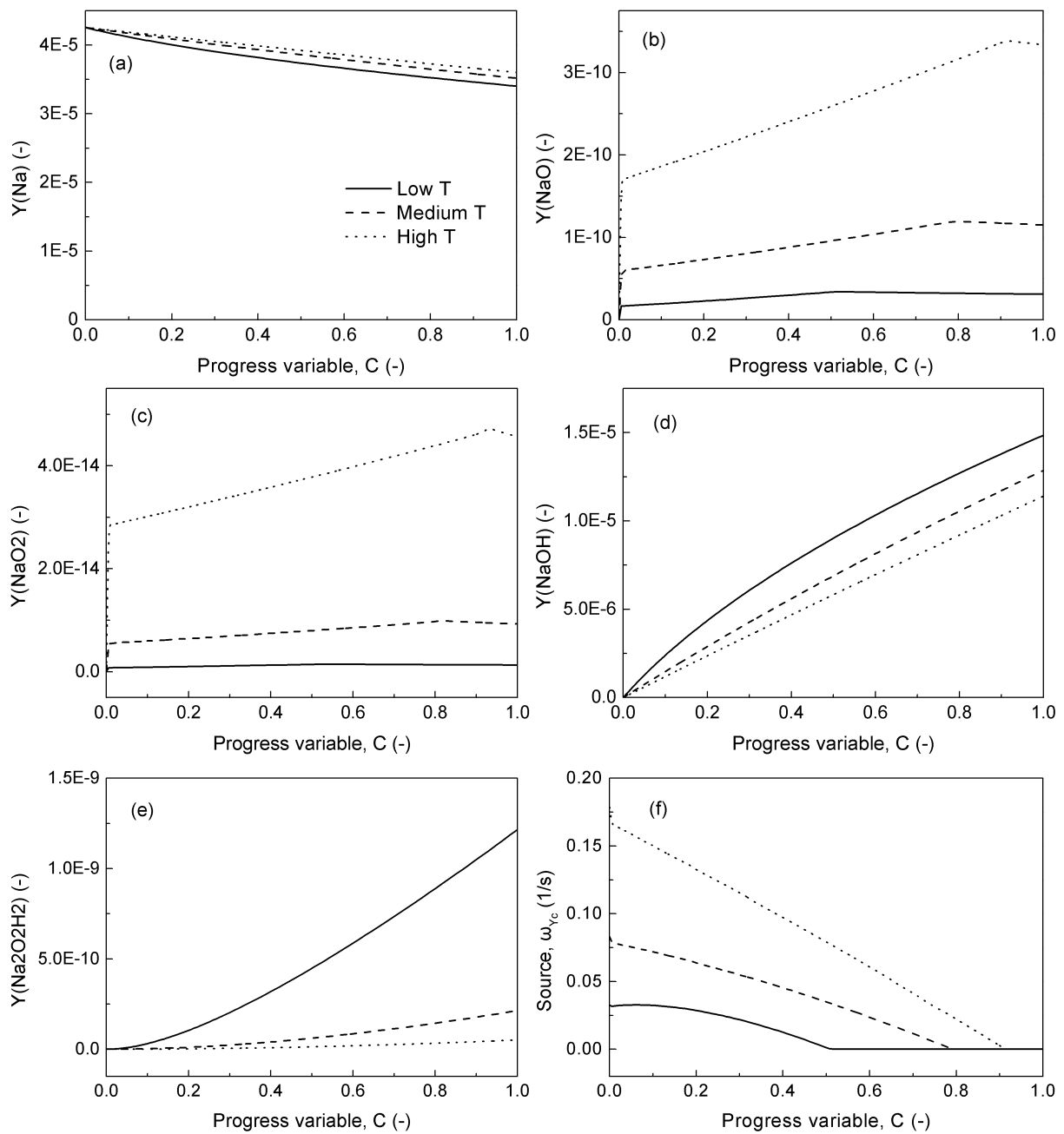


Figure 5. Species mass fractions and progress variable source vs progress variable at $\phi = 1.508$ and $Y_{\text{sodium}} = 4.26\text{E-}5$. Solid line $T = 1522$ K. Dashed line: 1629 K. Dotted line: 1727 K.

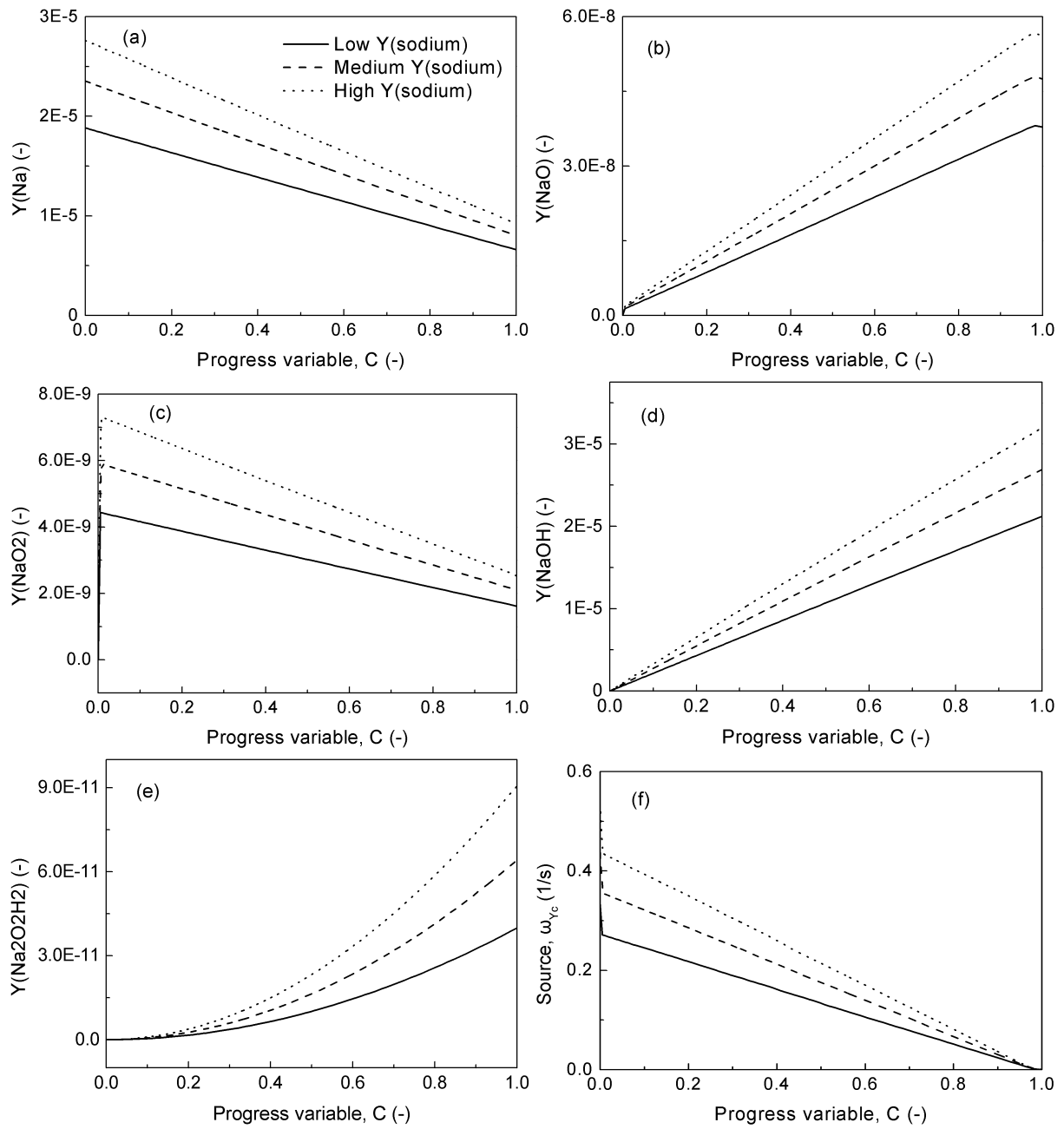
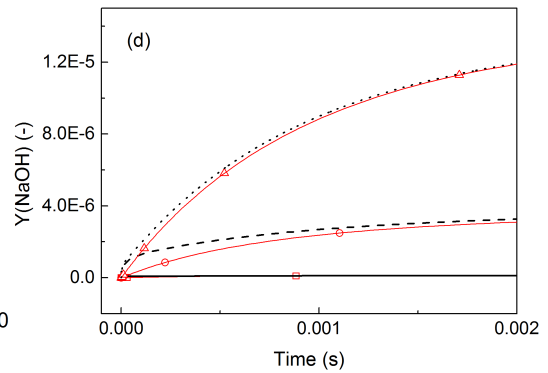
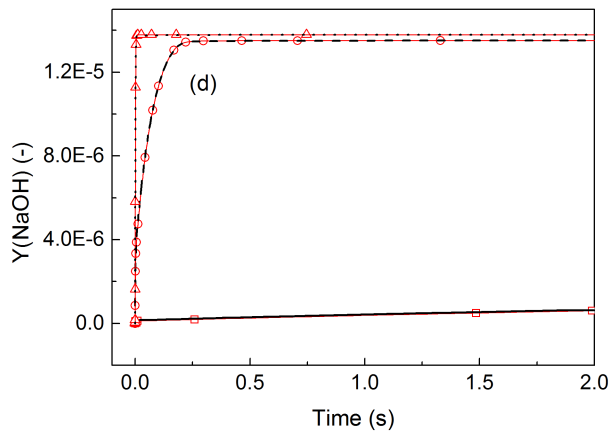
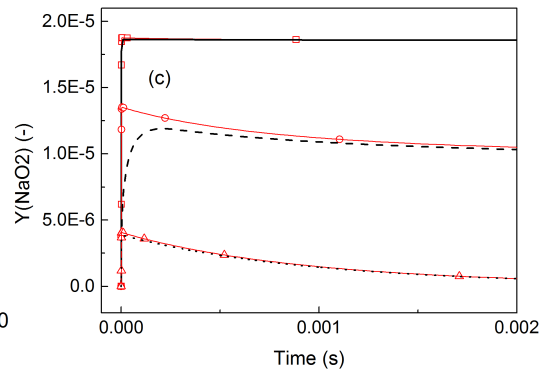
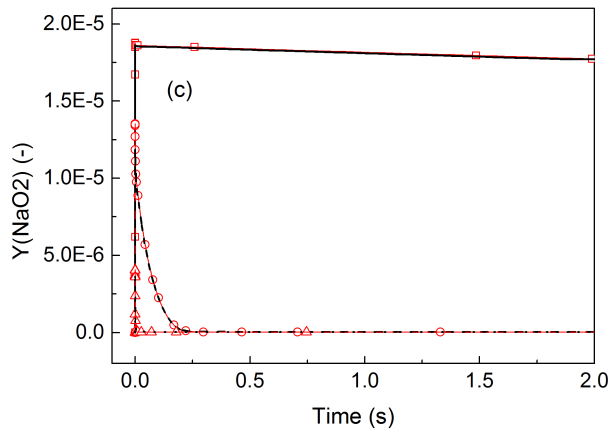
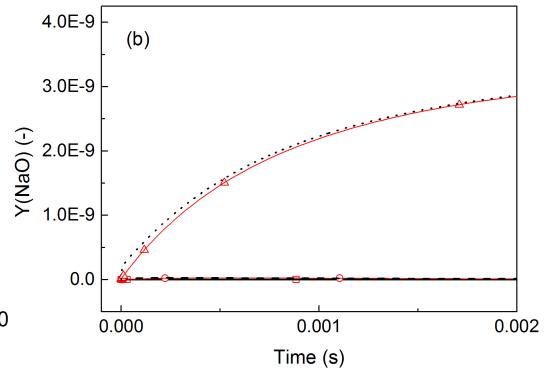
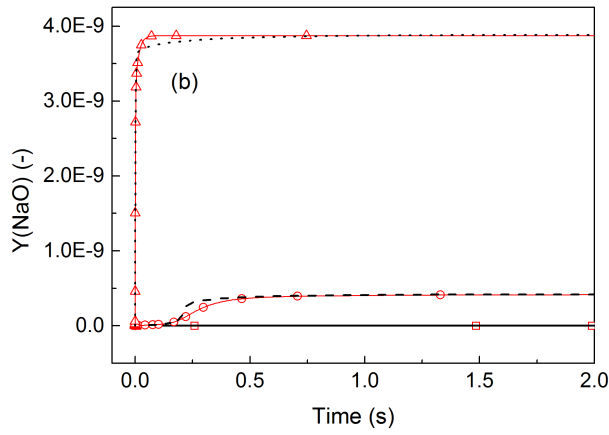
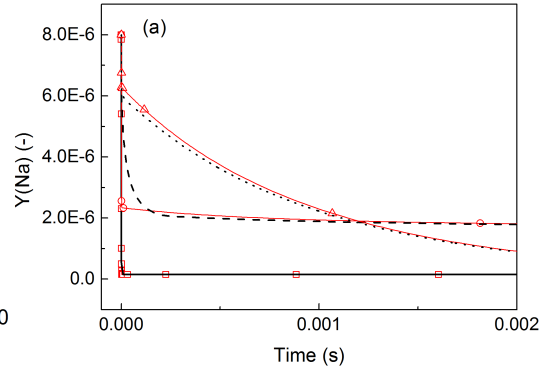
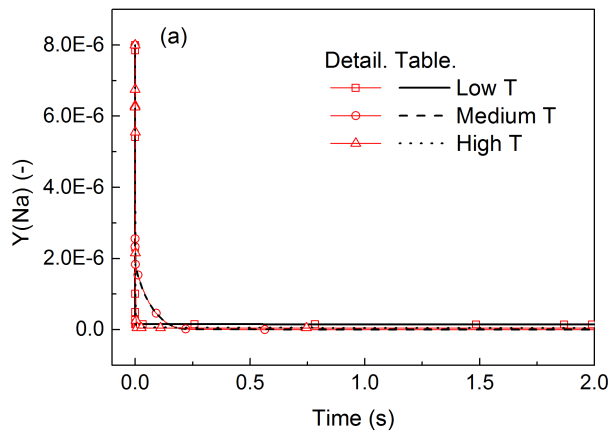


Figure 6. Species mass fractions and progress variable source vs progress variable at $\phi = 1.0$ and $T = 1878$ K. Solid line: $Y_{\text{sodium}} = 1.88\text{E-}5$. Dashed line: $2.35\text{E-}5$. Dotted line: $2.76\text{E-}5$.



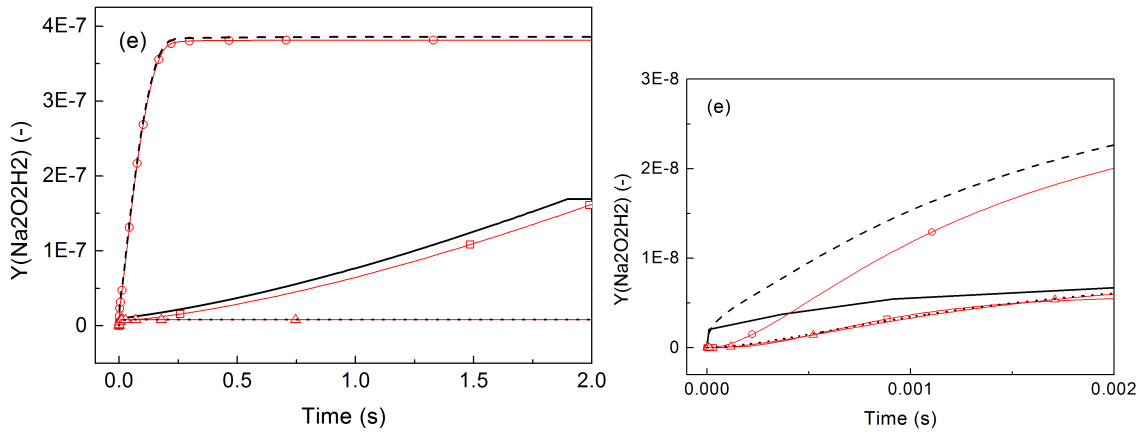
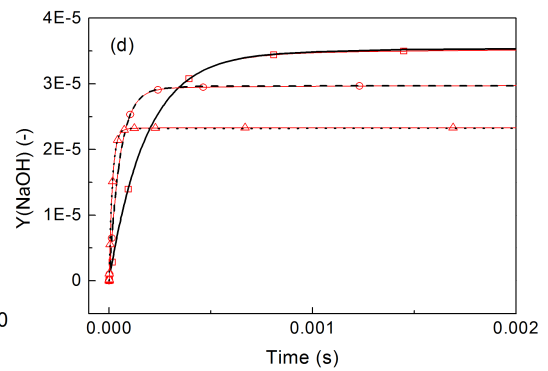
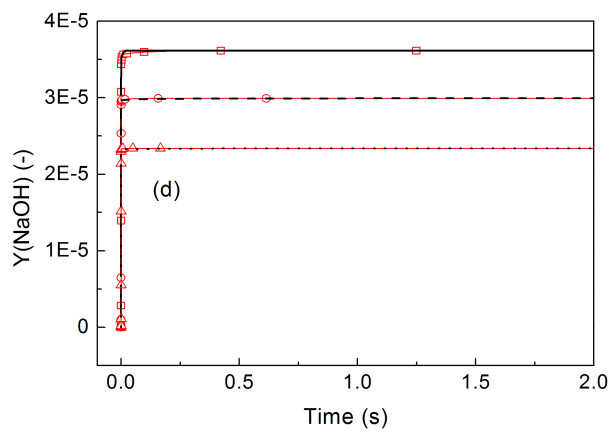
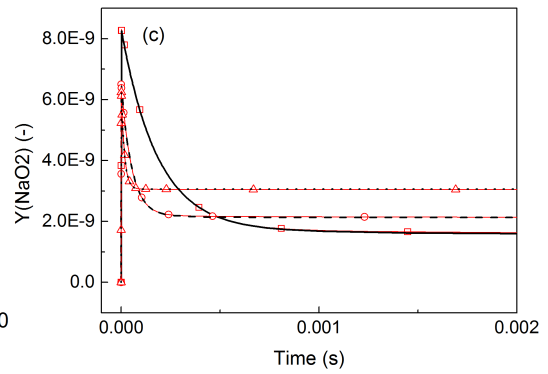
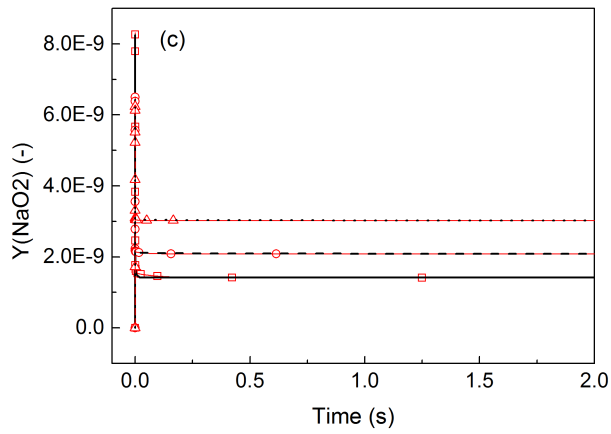
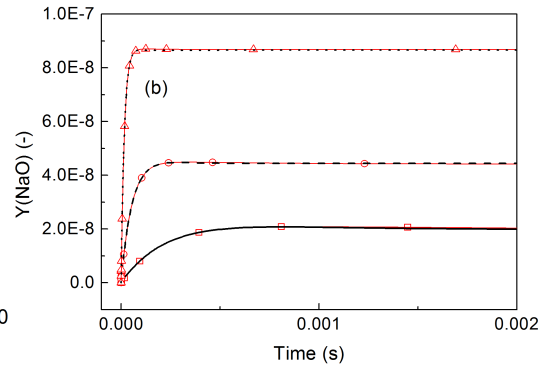
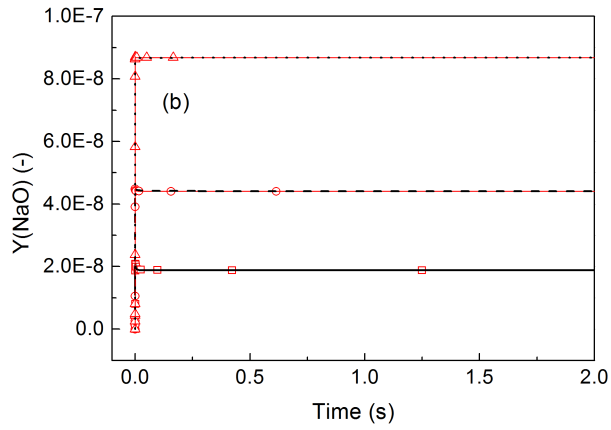
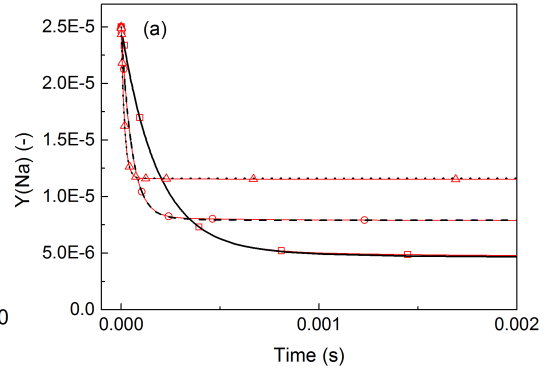
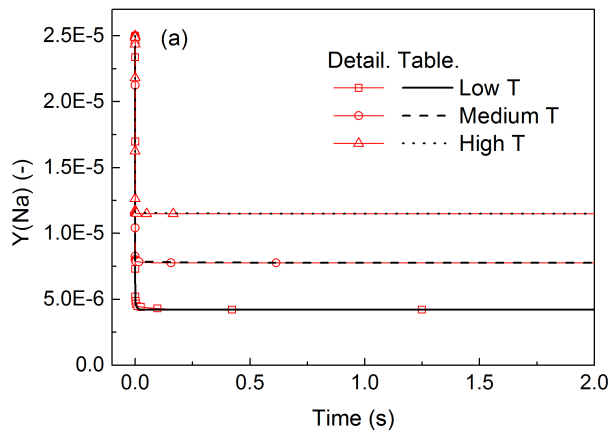


Figure 7. Species mass fractions versus time. Symbols: solving detailed mechanism [13]. Lines: solving progress variable. Square and solid-line: $T = 1000$ K; Circle and dashed-line: 1200 K; Triangle and dotted-line: 1400 K. $\phi = 0.493$ and $Y_{sodium} = 0.8E-5$. The first-stage stiff evolutions are zoomed in and shown on the right side.



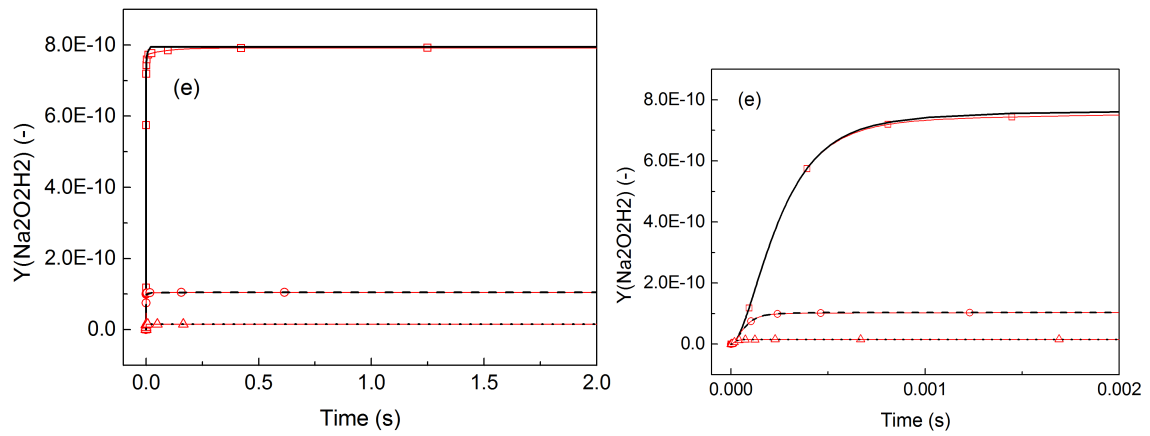
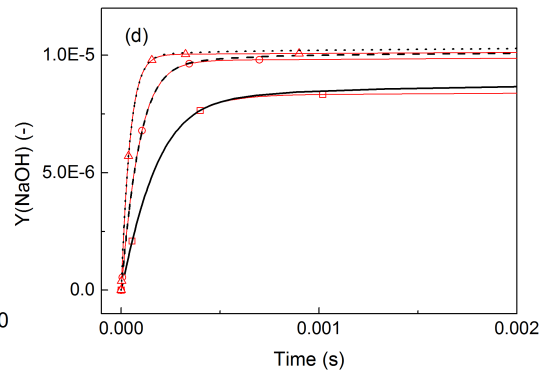
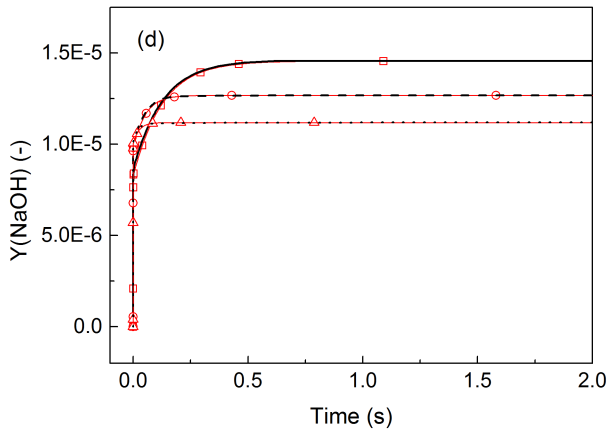
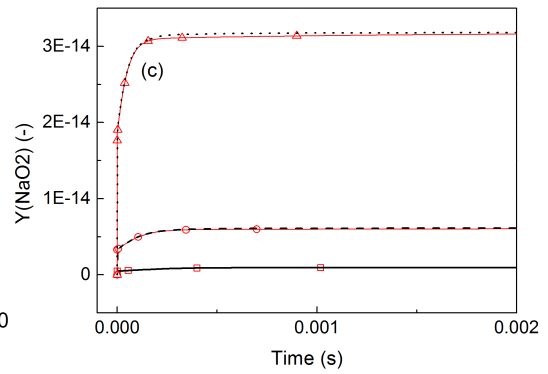
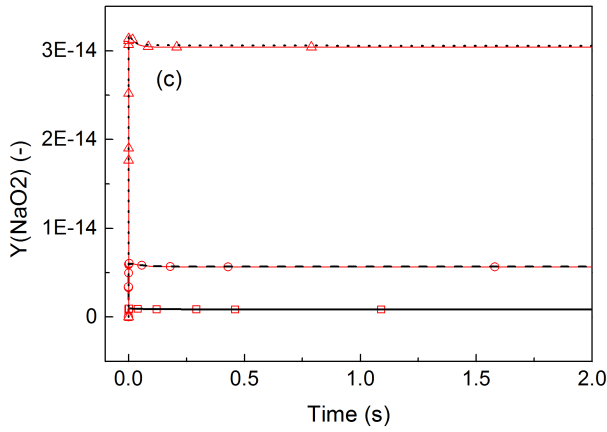
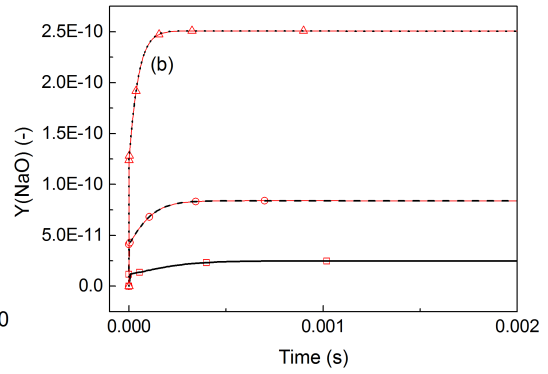
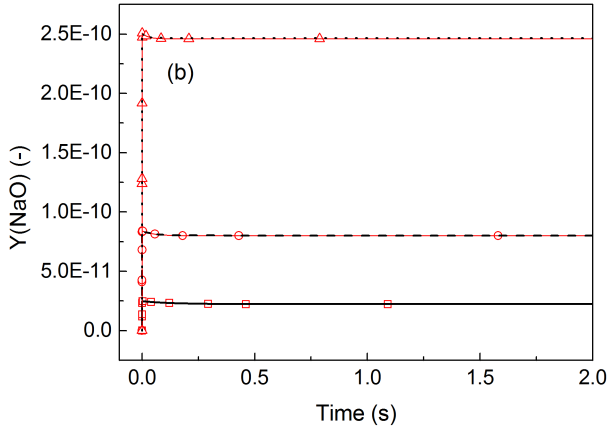
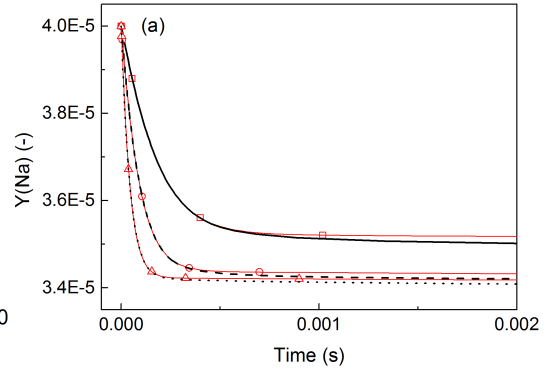
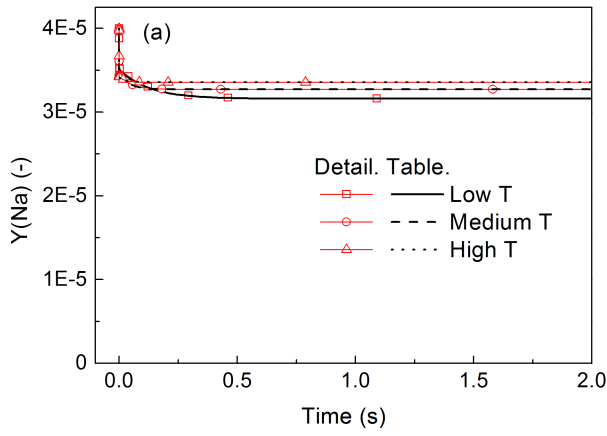


Figure 8. Species mass fractions versus time. Symbol: solving detailed mechanism [13]. Lines: solving progress variable. Square and solid-line: $T = 1700$ K; Circle and dashed-line: 1850 K; Triangle and dotted-line: 2000 K. $\phi = 1.0$ and $Y_{\text{sodium}} = 2.5\text{E-}5$. The first-stage stiff evolutions are zoomed in and shown on the right side.



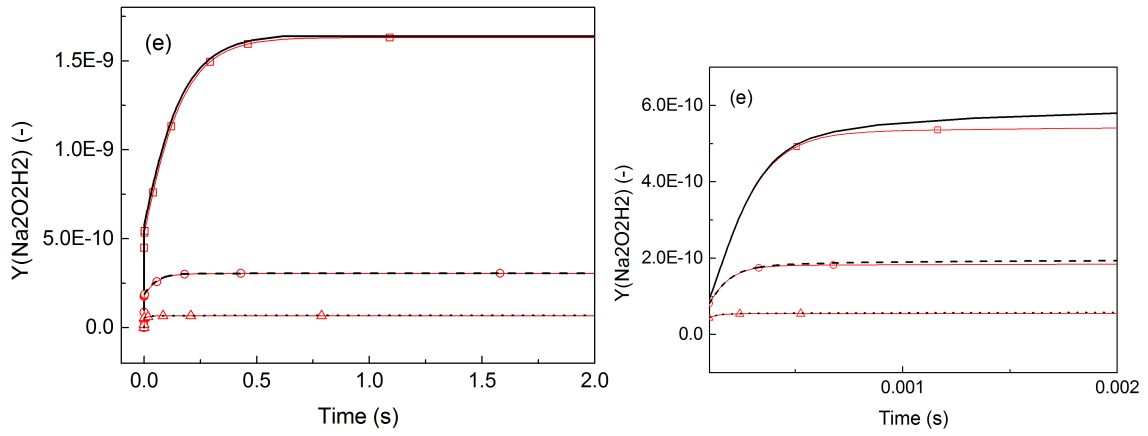


Figure 9. Species mass fractions versus time. Symbol: solving detailed mechanism [13]. Lines: solving progress variable. Square and solid-line: $T = 1500$ K; Circle and dashed-line: 1600 K; Triangle and dotted-line: 1700 K. $\phi = 1.494$ and $Y_{sodium} = 4.0E-5$. The first-stage stiff evolutions are zoomed in and shown on the right side.

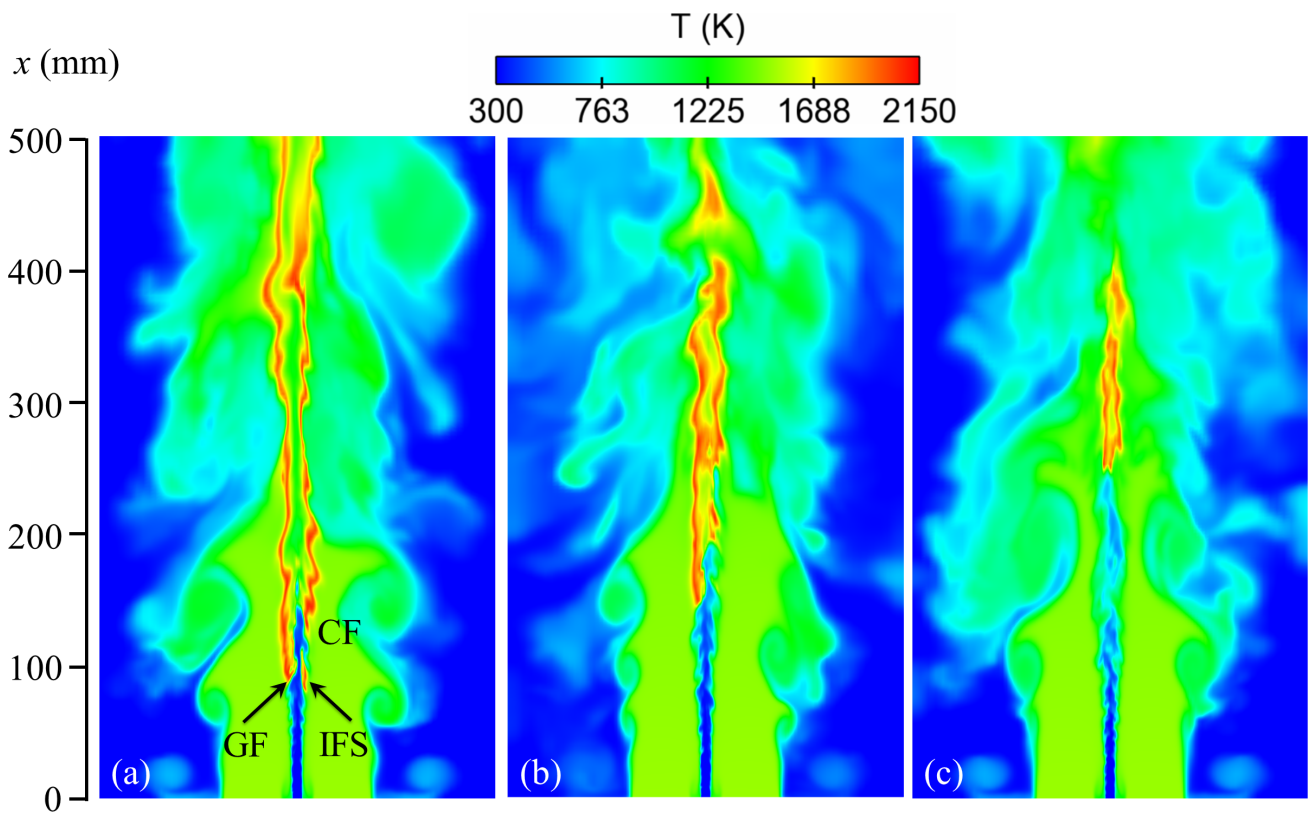


Figure 10. 2D snapshot of instantaneous gas temperature fields for SR = 0.14 (a), 0.22 (b) and 0.36 (c).

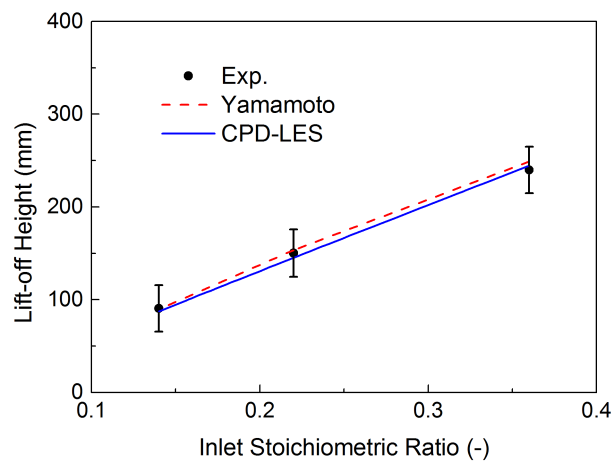


Figure 11. Comparison of the lift-off height at the three SRs.

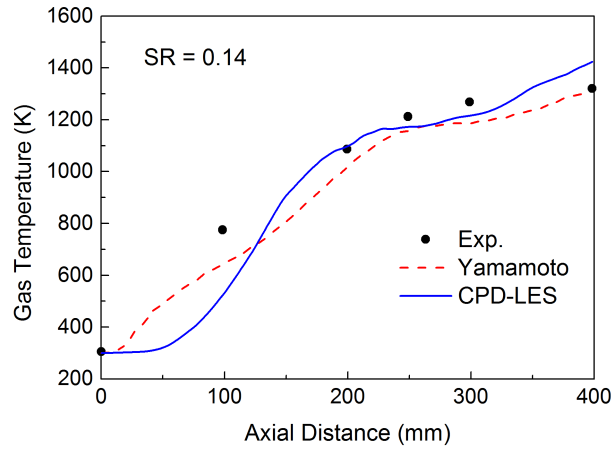


Figure 12. Averaged gas temperature along the jet centerline.

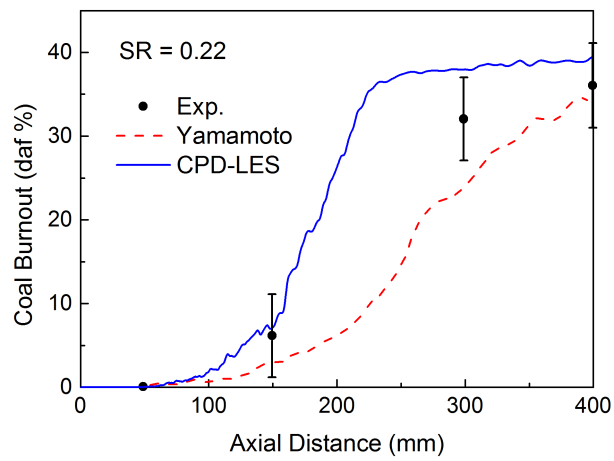


Figure 13. Coal burnout (%) along the jet centerline.

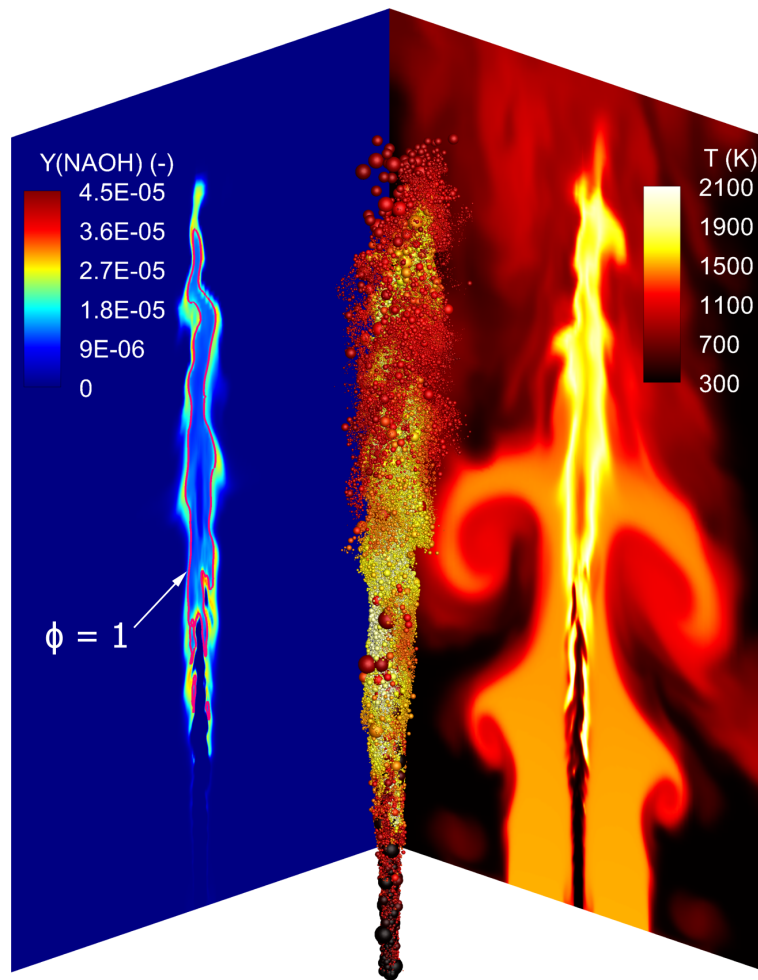


Figure 14. Instantaneous pulverized-coal particle distribution. Mass fraction of NaOH (left) and gas-phase temperature (right), 2D snapshot through the jet centerline. The pulverized-coal particles are colored by the particle temperature. The size of coal particles is also illustrated. The diameter of the largest particle is 180 μm .

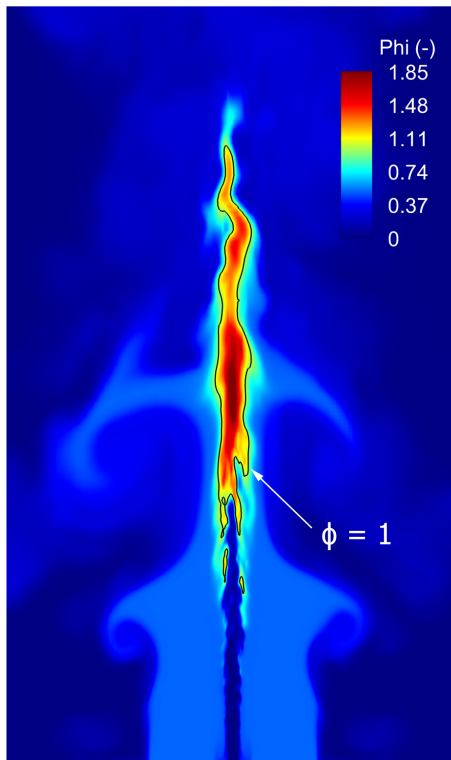


Figure 15. 2D snapshot of the equivalence ratio (ϕ).

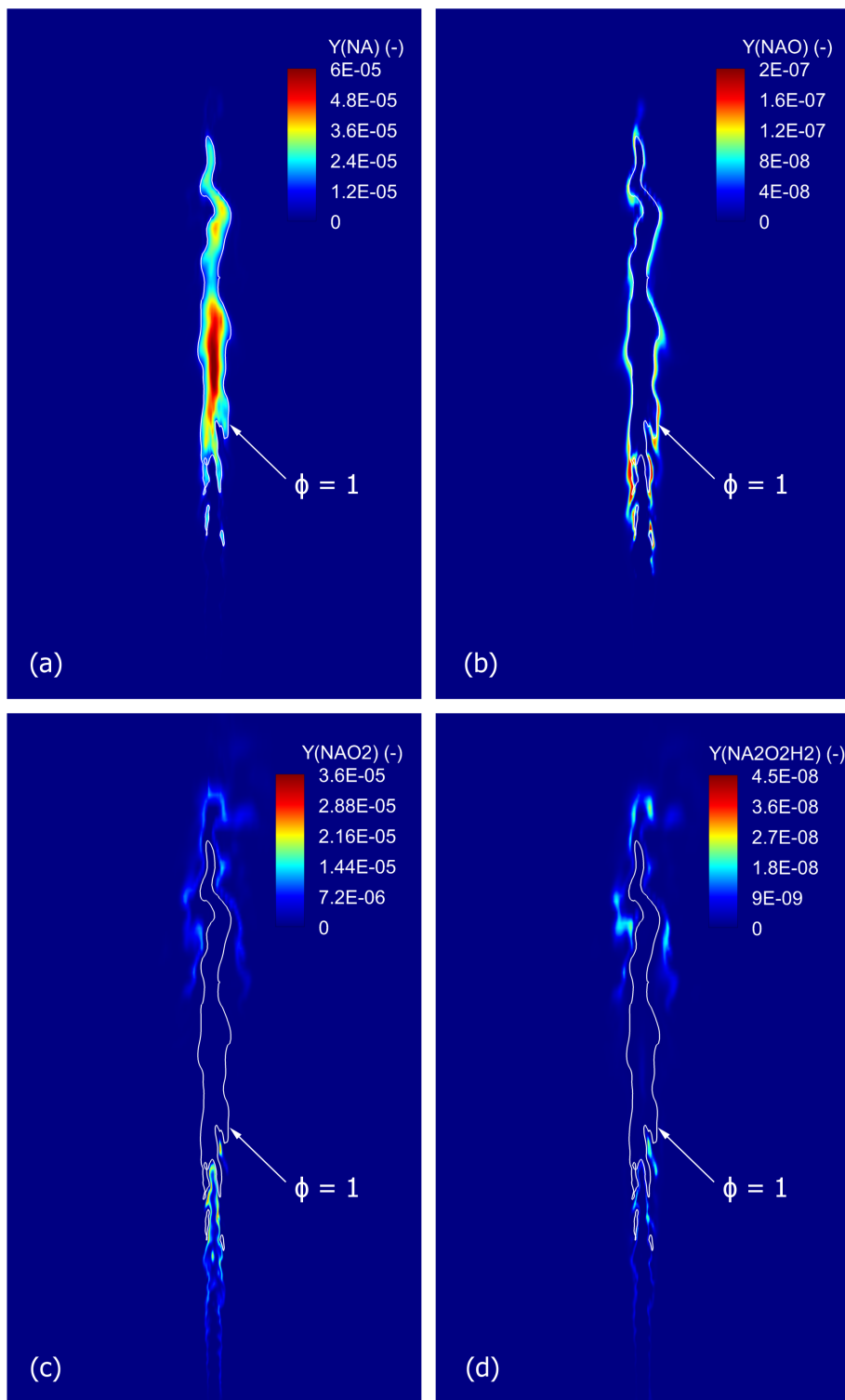


Figure 16. 2D snapshots of the mass fractions of (a) Na, (b) NaO, (c) NaO₂, and (d) Na₂O₂H₂.

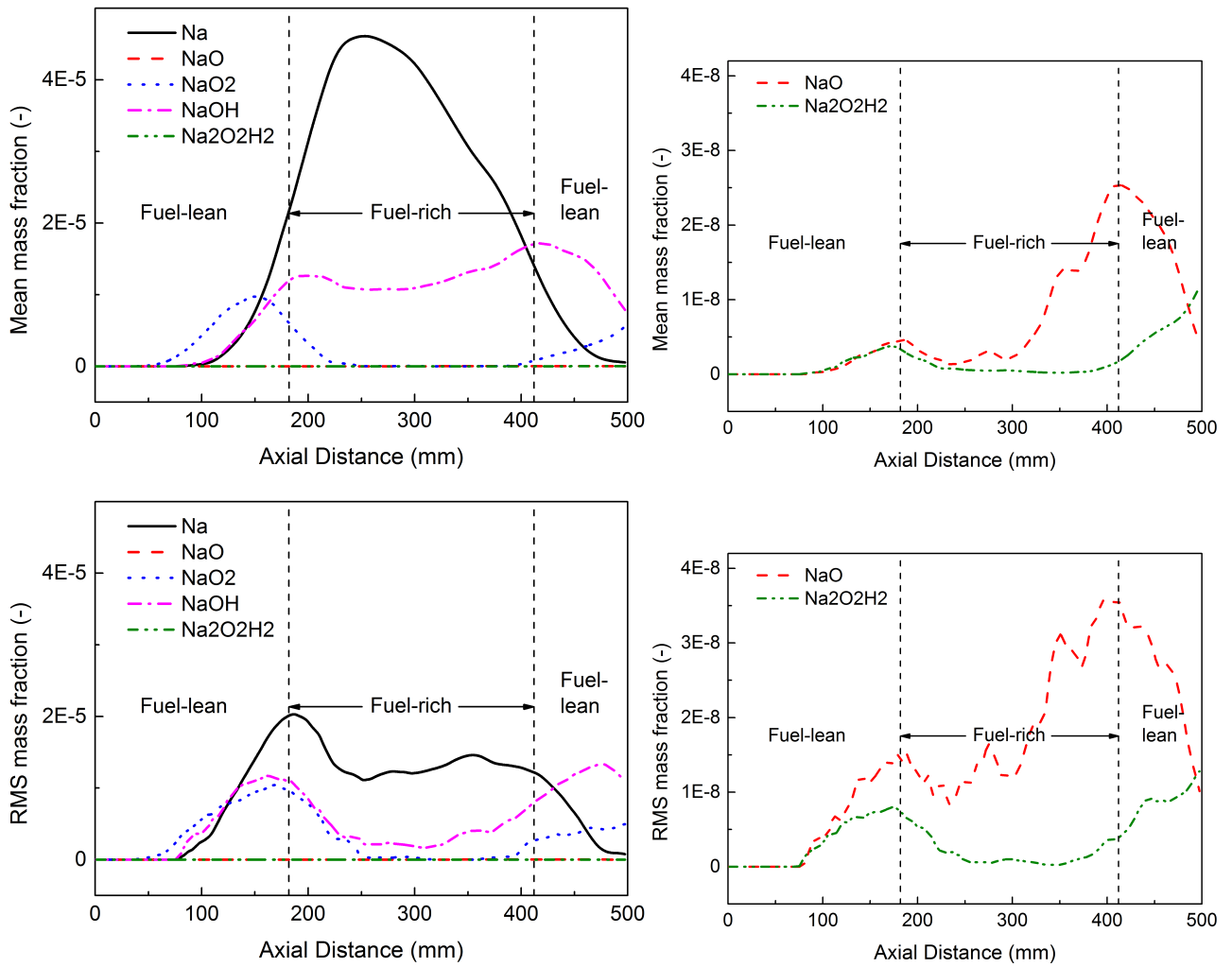


Figure 17. Mean and RMS mass fractions of the 5 sodium species along the jet centerline. The profiles for the two minor species NaO and Na₂O₂H₂ are zoomed in and shown on the right side.

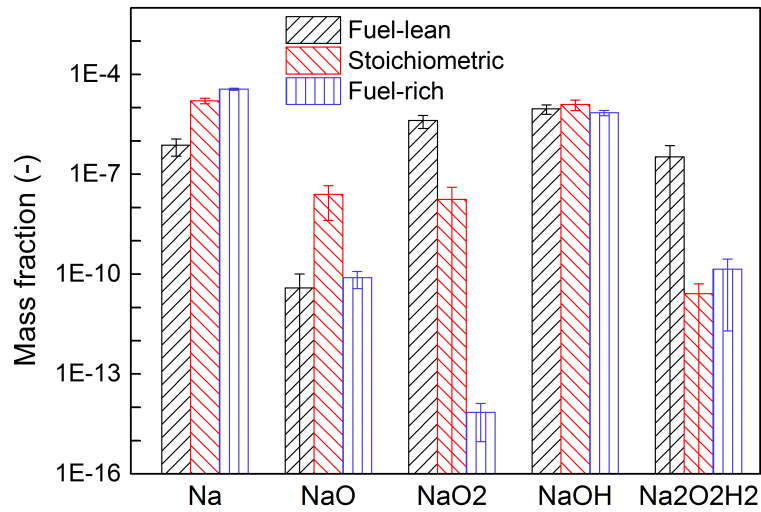


Figure 18. The response mass fractions of the 5 sodium species from the chemistry table under the fuel-lean, stoichiometric and fuel-rich conditions. In some cases, the lower end of the error bar is not shown because the standard deviation is larger than the mean value.

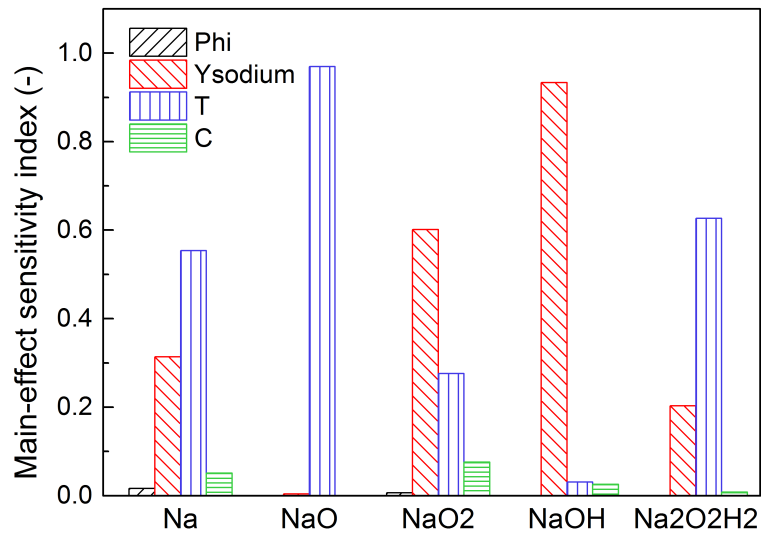


Figure 19. Main-effect sensitivity indices for the 5 sodium species under the fuel-lean condition.

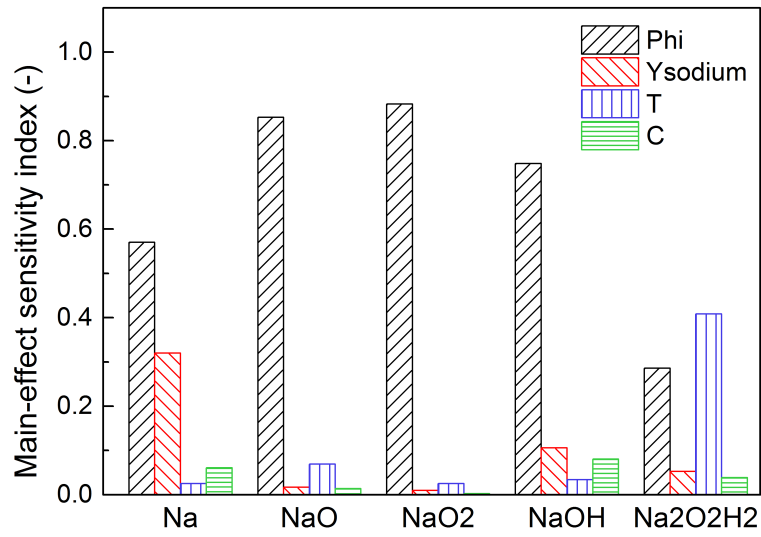


Figure 20. Main-effect sensitivity indices for the 5 sodium species under the stoichiometric condition.

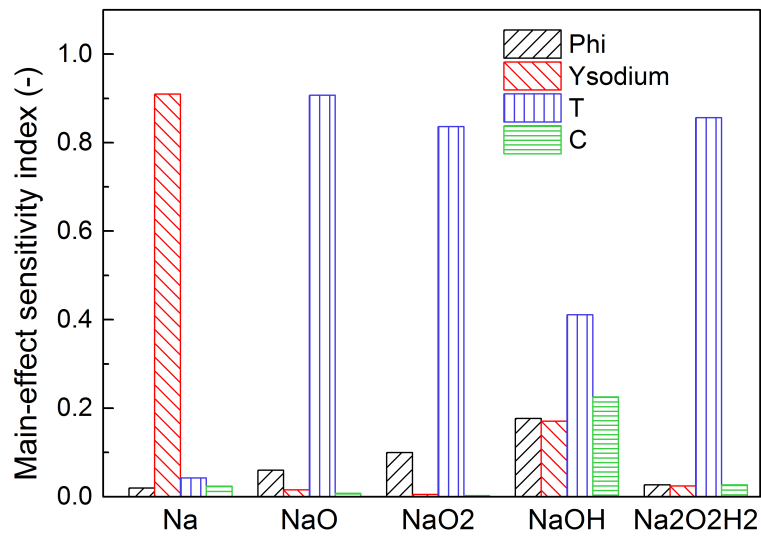


Figure 21. Main-effect sensitivity indices for the 5 sodium species under the fuel-rich condition.

Figure captions

Figure 1. Sodium element mass fraction Y_{sodium} vs equivalence ratio ϕ . Scatter plot: LES results at one time instant. Lines: upper and lower bounds, which are used as the upper and lower limits of Y_{sodium} and ϕ in the chemical lookup table.

Figure 2. Gas-phase temperature vs equivalence ratio. Scatter plot: LES results at one time instant. Lines: upper and lower bounds used in the chemical lookup table, and the conditional mean and fluctuation of LES results.

Figure 3. Species mass fractions and progress variable source vs progress variable at $\phi = 0.508$ and $Y_{sodium} = 0.82E-5$. Solid line: $T = 1005$ K; Dashed line: 1208 K; Dotted line: 1396 K.

Figure 4. Species mass fractions and progress variable source vs progress variable at $\phi = 1.0$ and $Y_{sodium} = 2.35E-5$. Solid line: $T = 1721$ K. Dashed line: 1878 K. Dotted line: 2022 K.

Figure 5. Species mass fractions and progress variable source vs progress variable at $\phi = 1.508$ and $Y_{sodium} = 4.26E-5$. Solid line $T = 1522$ K. Dashed line: 1629 K. Dotted line: 1727 K.

Figure 6. Species mass fractions and progress variable source vs progress variable at $\phi = 1.0$ and $T = 1878$ K. Solid line: $Y_{sodium} = 1.88E-5$. Dashed line: $2.35E-5$. Dotted line: $2.76E-5$.

Figure 7. Species mass fractions versus time. Symbols: solving detailed mechanism [13]. Lines: solving progress variable. Square and solid-line: $T = 1000$ K; Circle and dashed-line: 1200 K; Triangle and dotted-line: 1400 K. $\phi = 0.493$ and $Y_{sodium} = 0.8E-5$. The first-stage stiff evolutions are zoomed in and shown on the right side.

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Figure 20. Main-effect sensitivity indices for the 5 sodium species under the stoichiometric condition.

Figure 21. Main-effect sensitivity indices for the 5 sodium species under the fuel-rich condition.

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