Unified approaches based effective capacity analysis over composite $\alpha - \eta - \mu$-gamma fading channels

H. Al-Hmood and H. S. Al-Raweshidy

This letter analyses the effective capacity of communications systems using unified models. In order to obtain a simple closed-form mathematically tractable expression, two different unified approximate models are used. The normalised effective capacity over fading channels is defined as the number of the parameters that provide better practical results than the traditional distributions such as Nakagami-m and Nakagami-$\kappa$ fading channels. The MoG distribution: It can be observed that (9) includes a Meijer’s $G$-function of two variables which can be evaluated by using the MATHEMATICA program that is implemented in [15].

The PDF of $\alpha - \eta - \mu$-gamma fading can be calculated via averaging the PDF of $\alpha - \eta - \mu$-gamma fading channels. This letter analyses the effective capacity of communications systems over composite fading channels such as Nakagami-$\kappa$ and Weibull/gamma [2, 6]. In [7], the $\kappa - \mu$ shadowing fading channel is a more generalised model which is utilised to model fading channels. However, no works have been dedicated to analyse the effective capacity over wireless fading channels [2]. To represent the line-of-sight (LoS), non-LoS (NLoS), and non-linearity communication scenarios of wireless fading channels, the $\kappa - \mu$ and Nakagami-$\kappa$ distributions are utilised to model the fading channel.

The impact of shadowing fading is also considered in the analysis of the effective capacity of communication systems over composite fading channels such as Nakagami-$\kappa$ and Weibull/gamma [2, 6]. In [7], the $\kappa - \mu$ shadowing fading channel is a more generalised model which is utilised to model fading channels. Furthermore, the unified framework in [6] is based on the moment generating function (MGF) of the instantaneous signal-to-noise ratio (SNR) that cannot be obtained in exact closed-form expression.

Motivated by the above approach and the parametric model of the effective capacity, this letter provides two different frameworks by using mixture gamma (MG) [8] and mixture of Gaussian (MoG) distributions [9]. These distributions have been widely utilised in the analysis of digital communication systems [10, 11]. This is because they provide simple closed-form analytic expression of the performance metrics. To this effect, the effective capacity over composite $\alpha - \eta - \mu$-gamma fading channel is more generalised than the aforementioned channels is analysed using MG and MoG distributions. The main difference between the MG and MoG distributions is the number of the parameters that is required to achieve a minimum mean square (MSE) between the probability density function (PDF) of the exact and approximate models.

**System model:** The normalised effective capacity over fading channels is expressed by [7, eq. (1)]

$$ R = -\frac{1}{A} \log_2 \left( E \left( (1 + \gamma)^{-A} \right) \right) $$

where $E \{ \}$ stands for the expectation and $A \triangleq \theta T B / \log_2$ with $\theta$, $T$, and $B$ denote the delay exponent, block duration, and bandwidth of the system, respectively.

**The MG distribution:** Using a MG distribution, the PDF of the instantaneous SNR can be written as [8, eq. (1)]

$$ f_\gamma(\gamma) = \sum_{i=1}^{S} \phi_i \gamma^{\mu_i - 1} e^{-\gamma/\mu_i} $$

where $S$ is the number of Gamma distributions which is obtained via calculating the minimum MSE between (2) and exact PDF and $\phi_i$, $\mu_i$, and $\xi_i$ correspond to the parameters of $i$ Gamma component.

**The MoG distribution:** The PDF of the instantaneous SNR, $\gamma$, can be expressed using a MoG as [9, eq. (24)]

$$ f_\gamma(\gamma) = \sum_{i=1}^{N} \frac{\rho_i}{\sqrt{2\pi \psi_i}} e^{-\left( \frac{\gamma - \mu_i}{2\psi_i} \right)^2} $$

where $N$ is the number of Gaussian components that provides minimum MSE and $\rho_i$, $\mu_i$, and $\psi_i$ are the weight, mean, and variance of the $i$th component, respectively. Moreover, $\sum_{i=1}^{N} \rho_i = 1$ with $\rho_i > 0$.

**MG distribution based analysis:** It can be noted that (1) can be expressed as

$$ R = -\frac{1}{A} \log_2 \left( \int_{0}^{\infty} (1 + \gamma)^{-A} f_\gamma(\gamma) d\gamma \right) $$

Substituting (2) in (4), this yields

$$ R = -\frac{1}{A} \log_2 \left( \sum_{i=1}^{S} \phi_i \Gamma(\theta_i, 1) \left( \frac{1}{\theta_i} + 1 - A; \xi_i \right) \right) $$

Employing [2, eq. (9)] to compute the integration in (4), the following unified closed-form is obtained

$$ R = -\frac{1}{A} \log_2 \left( \sum_{i=1}^{S} \phi_i \Gamma(\theta_i, 1) \left( \frac{1}{\theta_i} + 1 - A; \xi_i \right) \right) $$

where $\Gamma(\cdot, \cdot)$ is the incomplete Gamma function and $U(\cdot; \cdot; \cdot)$ is the Tricomi hypergeometric function defined in [12, eq. (39)].

**MoG distribution based analysis:** When (3) is inserted in (4), the integral cannot be solved in exact closed-form. Accordingly, we express the functions of the integral in terms of Meijer G-function by using [13, eq. (10)], [13, eq. (11)], and [14, eq. (01.03.26.0115.01)]

$$ (1 + \gamma)^{-A} = \frac{1}{\Gamma(A)} \sum_{n=0}^{\infty} \frac{\Gamma(1 - n - A)}{\Gamma(n + 1)} \gamma^n $$

$$ e^{-\gamma/\mu} = \sum_{n=0}^{\infty} \frac{\mu^n}{n!} \gamma^n $$

$$ e^{\gamma/\mu} = \sum_{n=0}^{\infty} \frac{\mu^n}{n!} \gamma^n $$

$$ \int_{0}^{\infty} \frac{\gamma^n}{\sqrt{\mu}} e^{-\gamma/\mu} d\gamma = \frac{\mu^n}{n!} \sqrt{\pi} \mu^{n/2} \left( \frac{\mu^{n/2}}{n/2} \right) $$

$$ \int_{0}^{\infty} \frac{\gamma^n}{\sqrt{\mu}} e^{\gamma/\mu} d\gamma = \frac{\mu^n}{n!} \sqrt{\pi} \mu^{n/2} \left( \frac{\mu^{n/2}}{n/2} \right) $$

$$ \int_{0}^{\infty} \frac{\gamma^n}{\sqrt{\mu}} d\gamma = \frac{\mu^n}{n!} \sqrt{\pi} \mu^{n/2} \left( \frac{\mu^{n/2}}{n/2} \right) $$

$$ \text{With the aid of} [15, eq. (9)], \text{the integral in (8) can be computed in exact closed-form as follows} $$

$$ R = -\frac{1}{A} \log_2 \left( \sum_{i=1}^{M} \frac{\rho_i}{\sqrt{2\pi \psi_i}} \int_{0}^{\infty} e^{-\left( \frac{\gamma - \mu_i}{2\psi_i} \right)^2} \left( \frac{1}{\theta_i} + 1 - A; \xi_i \right) \right)$$

It can be observed that (9) includes a Meijer’s $G$-function of two variables which can be evaluated by using the MATHEMATICA program that is implemented in [15].
Effective Capacity (bits/s/Hz)

\[ f_s(\gamma) = \frac{\sqrt{\pi \alpha h^\alpha \mu^{\alpha + \frac{1}{2}}} \gamma^{\frac{\alpha}{2} (\mu + \frac{1}{2}) - 1}}{\Gamma(\mu) \Gamma(\mu) \Gamma(H - \mu - \frac{1}{2})} \times \int_0^\infty x^{H - \mu - \frac{1}{2} - 1} e^{-\frac{2\mu x^2}{\gamma^2}} I_{\mu - \frac{1}{2}} \left( \frac{2\mu x^2}{\gamma^2} \right) dx \]  

(10)

where \( \alpha, \mu, b, \) and \( \Omega \) stand for the non-linearity severe parameter, the number of multipath clusters, shadowing index, and mean power, respectively. Moreover, where \( I_{\mu}(.) \) is the modified Bessel function of the first kind and 4th order [12]. The parameters \( H \) and \( \beta \) are related to \( \eta \) which represents a relationship between the quadrature and in-phase scattered components, into two formats, format 1 and format 2. In the former, \( H = (\eta - 1 - \eta)/4 \) and \( h = (2 + \eta - 1 - \eta)/4 \) with \( \eta \in (0, \infty) \) denotes the power ratio between the components whereas in the latter \( H = \eta/(1 - \eta^2) \) and \( h = 1/(1 - \eta^2) \) with \( \eta \in (-1, 1) \) refers to the correlation coefficient between the components [16].

Using \( z = \frac{2b\mu x^2}{\gamma^2} \) and following the same steps in [11], (10) can be expressed by a MG distribution with the following parameters

\[ \theta_{b} = \frac{b \sqrt{\Gamma(\mu)}}{\sqrt{\Gamma(2 + \mu)}} \left( \frac{\mu b^2}{\pi \gamma^2} \right)^{\frac{1}{4}}, \quad \theta_{\gamma} = \frac{b \sqrt{\Gamma(\mu)}}{\sqrt{\Gamma(2 + \mu)}} \left( \frac{\mu b^2}{\pi \gamma^2} \right)^{\frac{1}{4}} \]

(11)

Numerical results: Fig. 1 and Fig. 2 show the simulated and numerical effective capacity of \( \alpha - \eta - \mu \text{gamma} \) fading (format 1) against the average SNR, \( \tilde{\gamma} \), using MG distribution and delay exponent, \( \theta \), using MoG approach, respectively. The number of components for both distributions, namely, \( S \) and \( N \), are chosen to achieve MSE \( \leq 10^{-8} \). In Fig. 2, the parameters have been calculated by following the same procedure in [9]. From both figures, it can be observed that the effective capacity becomes better when \( \mu, \eta \) and \( \theta \) increase. This is because higher \( \mu, \eta \) and \( \theta \) mean the number of multipath clusters is large, the received power is high and the shadowing impact is low, respectively.

Conclusion: In this letter, we have used MG and MoG distributions to analyse the effective capacity over \( \alpha - \eta - \mu \text{gamma} \) fading channels. These distributions can be employed to approximate with high accuracy the PDF of a wide range of distributions that are used in modelling the wireless channels. Although the MG distribution leads to simple expression, its not applicable for all fading channels. Therefore, we have utilised the MoG distribution. To this effect, unified simple closed-form mathematically tractable expressions are derived. The results have showed different scenarios that have not been yet investigated in the technical literature such as \( \eta - \mu \) and \( \alpha - \mu \text{gamma} \) fading channels.

Hussien Al-Hmood (Electrical and Electronics Engineering Department, University of Thi-Qar, Thi-Qar, Iraq)

E-mail: hussien.al-hmood@brunel.ac.uk, eng.utq.edu.iq

H. S. Al-Raweshidy (Electronic and Computer Engineering Department, College of Engineering, Design and Physical Sciences, Brunel University London, UK)

References

10. Al-Hmood, H., and Al-Raweshidy, H.S.: ‘Unified modeling of composite \( \kappa - \mu \text{gamma} \), \( \kappa - \mu \text{gamma} \), and \( \alpha - \mu \text{gamma} \) fading channels using a mixture gamma distribution with applications to energy detection’, IEEE Antennas and Wirel. Propag. Lett., 2016, 17, pp. 1019–1022
16. Fraidenraich, G. and Yacoub, M.D.: ‘The \( \alpha - \eta - \mu \) and \( \alpha - \kappa - \mu \text{gamma} \) fading distributions’, Proc. IEEE ISSSSTA, Manaus, Brazil, 2006.