LONG MEMORY IN UK REAL GDP, 1851-2013: AN ARFIMA-FIGARCH ANALYSIS

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Abstract

This paper analyses the long-memory properties of both the conditional mean and variance of UK real GDP over the period 1851-2013 by estimating a multivariate ARFIMA-FIGARCH model (with the unemployment rate and inflation as explanatory variables). The results suggest that this series is non-stationary and non-mean-reverting, the null hypotheses of I(0), I(1) and I(2) being rejected in favour of fractional integration - shocks appear to have permanent effects, and therefore policy actions are required to restore equilibrium. The estimate of the long-memory parameter (1.37) is similar to that reported by Candelon and Gil-Alana (2004), implying that aggregate output is not an I(1) process. The presence of long memory in output volatility *(d = 0.80)* is also confirmed.

Keywords: ARFIMA-(FI)GARCH, Dual long memory, Volatility, Fractional impulse-response, Unemployment, Inflation.

JEL classification: B23, C14, C32, C53, C54

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1. Introduction

The persistence and long-memory properties of aggregate output have been extensively analysed in the literature. A number of recent studies suggest that real GDP exhibits long-range dependence and should be modelled as a fractionally integrated process – see, for instance, Hosking (1981, 1984), Granger and Joyeux (1980), Beran (1992, 1994), Baillie (1996), Robinson (1995a, 1995b), Caporale and Gil-Alana. (2009a, 2008b), Gil-Alana (2001, 2003, 2004), Škare and Stjepanović. (2013). Haubrich and Lo (2001) argue instead that macroeconomic variables behave as a hybrid between random walk and white noise processes. The work of Diebold and Rudebusch (1989), Sowell (1992), Gil-Alana and Robinson (1997) among others suggests that fractionally integrated specifications are more appropriate than I(0) or I(1) ones for real GDP. Although long-memory models have become increasingly popular for aggregate output, only a few studies have explored the possibility of long-range dependence in UK real GDP. An exception is the contribution by Candelon and Gil-Alana (2004), who conclude that the log of the UK real GDP series is non-stationary and non-mean-reverting with a long-memory parameter $d = 1.38$ and is best modelled as an ARFIMA (1,1.38,2). They also point out that the estimation of ARFIMA models for real GDP can lead to a better understanding of business cycles.

The present study extends their analysis by estimating both univariate and multivariate models for UK real GDP over the period 1851-2013 and using non-parametric, semi-parametric and parametric methods. The empirical results for the parametric univariate ARFIMA model are inconclusive: they would appear to support the view that real GDP is non-stationary and mean-reverting (exhibiting long memory) but are invalidated by the fact that ARCH effects present in the series are neglected as shown by the Jarque-Bera statistics. A multivariate parametric ARFIMA model is then estimated. Drawing on the theory developed by Phillips (1962), the unemployment rate and prices are included as explanatory variables. Following Gil-Alana (2003), a logistic transformation is used for the unemployment series. This specification provides evidence of long memory in the levels. We then examine the possibility of dual long memory in real GDP: the selected ARFIMA (0, ζ ,0) - FIGARCH (2,d,2) model indeed suggests that both output and its volatility exhibit long memory.

The layout of the paper is as follows. Section 2 discusses the data and the univariate analysis. Section 3 presents the multivariate results. Section 4 offers some concluding remarks.

2. Data and Univariate Analysis

We use annual data on UK real GDP (GDP), the unemployment rate (UNE), and inflation (CPL) over the period 1851-2013. Real GDP is in 2008 millions of pounds, UNE is in percentage terms, and CPL is the consumer price as defined in Hills et al. (2010). The dataset is taken from Hills (2010) and Williamson (2014). The empirical analysis is based on the fractional integration modelling approach used by Hosking (1981, 1984), Granger and Joyeux (1980), Beran (1992, 1994), Baillie (1996), Robinson (1995a, 1995b), Caporale and Gil Alana (2009a, 2008b), Gil-Alana (2001, 2003, 2004), Doornik and Marius (2004), and Škare and Stjepanović (2013). Fractionally integrated I(d) models are a particular case of long memory processes satisfying the condition $0< d \leq 1/2$, and therefore ideally suited to modelling persistence in macroeconomic series.

The standard long-memory approach is to model aggregate output around a deterministic trend or random walk with a stationary component. By contrast, our hypothesis is that long-range dependence in UK real GDP is driven by the dynamics of unemployment and inflation, in the spirit of Phillips (1962). As a first step, we examine the sample autocorrelation function (ACF - not displayed here). The autocorrelations decay slowly and point to long-range dependence. The unemployment and inflation series clearly follow a long-memory process

$$\lim_{n\to\infty}\sum_{i=-n}^{n}|\rho_i| = \infty \tag{1}$$

whilst at first glance output follows a short-memory process:

$$\lim_{n\to\infty}\sum_{i=-n}^{n}|\rho_i| = k \tag{2}$$

As noted by Perron (2006), stationary short-memory processes with level shifts tend to generate spurious long memory. This could the case with UK real GDP, the ACF rapidly decaying to zero but then exhibiting spikes around lag 20. The cross-correlation between aggregate output growth and unemployment is persistent and negative over time. In fact, a ten-year cycle is apparent, the initial impact of output growth on unemployment being strong and then slowly decaying to zero. The cross-correlogram between output growth and prices is positive at lags 1 – 3 and turns negative at lag

20. Switches from positive to negative cross-correlations occur throughout the sample period. Unemployment and output growth also display time-varying cross-correlations. Changes in unemployment affect aggregate output positively in the short run and quickly converge to zero, then the effect becomes negative and more switches occur over the sample. Unemployment is more affected by demand side disturbances while inflation by supply side shocks. However, UK aggregate output is driven by more than one type of shocks, suggesting a combination of Okun's and Phillip's Law. Its cycles tend to be irregular, which is another indication of long memory. Another noticeable feature is the presence of level shifts. Haubrich and Lo (2001) point to the possibility that production shocks follow a fractionally integrated process generating long-run dependence in output. In a similar spirit, we explain the persistence in UK real output in terms of the dynamics of the unemployment and inflation series: as noticed by Diebold and Rudebusch (1989), the presence of a large permanent component in aggregate output conflicts with traditional economic theories.

Throughout 1850 - 1940 unemployment did not follow any particular equilibrium path, with high volatility in both inflation and unemployment. In the period 1940-1970 unemployment was closer to the equilibrium rate and inflation more stable. Oil shocks in the 70's and financial shocks at the beginning of 2005 both moved unemployment away from equilibrium (see Gil-Alana et al., 2003). Prices in the UK appear to be characterised by time-varying volatility over the subperiod 1850 - 1936. For the period 1851-2013 the estimated Okun's (1962) coefficient is -0.509490, implying that an increase in the UK real output growth of 2% was followed by a fall in the unemployment rate by 0.51 percentage points.

Diebold and Rudebusch (1989) also find that inflation and unemployment shocks have persistent effects on output growth. Its decomposition into permanent and transitory component requires an accurate estimation of the order of (fractional) integration (the knife edge distinction problem between I(0) and I(1) series), as pointed out by Michelacci and Zaffaroni (2000), Silverberg and Verspagen (2000), Mayoral (2006), and Caporale and Gil-Alana (2009a).

We estimate a dynamic model for UK aggregate output:

$$\Phi(L)Y_t = \beta_1^{'}\Theta(L)x_t + \varepsilon_t \qquad (3)$$

with x_t being a set of variables affecting UK real GDP, specifically unemployment and prices, ε_t a white noise process, Y_t UK real GDP, and Φ and Θ respectively autoregressive and moving average matrix polynomials with all roots lying outside the unit circle. All three series exhibit non-normality, with negative skewness in real GDP and positive one in unemployment and inflation (Jarque and Bera, 1987). It is well known that standard ADF test have very low power (Diebold and Rudebusch, 1991, Banerjee et al., 1993, Hassler and Wolters, 1994). Lee and Schmidt (1996) suggest using KPSS unit root test for identifying fractionally integrated processes. Therefore, we carry out both. Rejections on the basis of both ADF and KPSS for all series indicates that real GDP, unemployment and inflation series cannot be described as either I(0) or I(1) processes, therefore we apply a variety of fractional integration methods to test the I(d) hypothesis. First we use the nonparametric method of Robinson and Lobato (1998), Lo (1991)'s modified R/S and Giraitis et al. (2003) V/S test. The results are summarised in Table 1.

Table 1 Non-parametric test statistics for long memory

Series	Robinson -Lobato (d = 0)	Lo's Modified R/S (d = 0)	Giraitis et al. V/S (d = 0)	Robinson's estimated (d)
Real GDP (level)	0.537	2.017**	0.365**	0.484
Real GDP (first difference)	2.179**	1.992**	0.313**	0.309
Log Real GDP (level)	0.586	2.130**	0.385**	0.483
Log Real GDP (first difference)	-0.977	1.039	0.074	0.068
Unemployment rate	-0.571	1.283	0.0772	0.392
Unemployment rate (first difference)	-1.510	0.850	0.0275	-0.414
Inflation	0.428	1.960**	0.358**	0.481
Inflation (first difference)	0.635	2.012**	0.363**	0.451

Source: Authors' calculations
Notes: denotes significance at ** 5% level and rejection of the null of short memory

For real GDP the null of I(0) stationarity is rejected on the basis of Lo's (1991) modified R/S and Giraitis et al. (2003) V/S test. The Robinson and Lobato (1998) test does not lead to a rejection of the null of short memory. No evidence of long memory is found for real GDP growth. Overall, the non-parametric test results provide evidence against the unit root hypothesis and in favour of long memory (fractional integration) in UK real GDP, with the estimated order of integration *(d)* ranging from 0.068 to 0.484. For the unemployment rate series, there is evidence of short-memory behaviour, in line with Gil-Alana (2001), Gil-Alana et al. (2003), Caporale and Gil-Alana (2008b). The same holds for inflation.

We also estimate the fractional differencing parameter (d) using the semi-parametric procedures of Geweke and Porter-Hudak (1983) as modified in Phillips (2007), Moulines and Soulier (1999) and Robinson (1995a, Robinson 1995b). Table 2 show the corresponding results, which vary considerably depending on the method used and the series considered.

Table 2 Semi-parametric test statistics for long memory – estimated (d) in columns

Series	Phillips (2007)	Moulines/ Soulier (1999)	Robinson's (1995a)	Robinson's (1995b) Whittle ML
Real GDP (level)	0.996*	1.016*	0.844*	0.709*
Real GDP (first difference)	0.166*	0.679*	0.507*	0.988*
Log Real GDP (level)	1.135*	1.003*	0.832*	0.785*
Log Real GDP (first difference)	0.449*	-0.008	0.263*	0.599*
Unemployment rate (level)	-0.493*	0.494*	0.792*	-2.693*
Unemployment rate (first difference)	-1.855*	-0.319	-0.001*	-1.835*
Inflation (level)	1.079*	0.987*	0.826*	0.452*
Inflation (first difference)	2.368*	0.771*	0.912*	0.678*

Source: Authors' calculations
Notes: * indicates statistical significance at the 1% level

The test results indicate the presence of long memory ($d > 0$) and in some cases no mean reversion ($d > 1$). The overall conclusion for UK real GDP is that it exhibits long memory with d between $0.4 < d < 1$. i.e. it is non-stationary but mean-reverting.

For the unemployment rate series the estimated d values are between –2.6 and 0.8. The Moulines/Souliner and Robinson (1995a) test results provide evidence of fractional integration ($0.494 \leq d \leq 0.794$). Phillips (2007) and Robinson's (1995b) Whittle ML estimates of d are in the interval $-2.693 \leq d \leq -0.493$. Overall, the test results for the differenced unemployment rate series indicate anti-persistence in unemployment with d between $-1.855 \leq d \leq -0.001$, in line with Gil-Alana (2001), Gil-Alana et al. (2003), and Caporale and Gil-Alana (2008b).

For the inflation series there is evidence of long memory and persistence. The estimated values of *d* are in the interval $0.678 \leq d \leq 2.368$, in line with Franses and Marius (1997).

The long-memory results could be biased as a result of overlooking structural breaks. We test for their possible presence using the same procedure as in Caporale and Gil-Alana (2008a, 2009b) and the Bai and Perron (2003) test. Even when accounting for breaks in this way, the long-memory results (not displayed here) are essentially the same.

As in Gil-Alana (2004), the adopted ARFIMA(p,d,q) specification for UK real GDP is the following:

$$\begin{aligned} &Y_t = \alpha + \beta t + x_t,\ t = 1,2... \\ &(1-L)^d x_t = u_t,\ t = 1,2,...,\end{aligned} \quad (4)$$

Table 3 summarises the estimated ARFIMA(p,d,q) models following the procedure of Sowell (1992) with AR and MA polynomials

Table 3 Maximum likelihood estimation of ARFIMA(p,d,q) models for real GDP growth rate

		LM test		AR parameters			MA parameters			Criterions	
ARMA	d	LM d=0	LM d=1	$\phi 1$	$\phi 2$	$\phi 3$	$\theta 1$	$\theta 2$	$\theta 3$	AIC	SIC
(0,0)’	0.36***	68.1	35.6	-	-	-	-	-	-	398.9	402.0
(1,0)	0.25***	4.23	19.9	0.20	-	-	-	-	-	395.1	399.8
(0,1)	0.27***	27.9	0.77’	-	-	-	-0.19	-	-	398.3	402.9
(1,1)	0.25***	2.60’	2.91’	0.10	-	-	-0.10	-	-	396.0	402.2
(2,0)	0.88***	--	0.00’	0.44	-0.02	-	-	-	-	391.5	399.2
(0,2)	1.09***	16.5	1.45’	-	-	-	0.68	0.29	-	397.5	403.6
(2,1)	0.88***	--	0.00’	0.44	-0.02	-	0.96	-	-	391.5	399.2
(1,2)’	0.68***	0.29’	3.61’	0.43	-	-	0.73	0.19	-	392.9	400.6
(2,2)	0.27***	1.67’	--	0.49	-0.57	-	0.29	-0.52	-	395.0	404.2
(3,0)	0.30***	2.57’	8.40	0.15	-0.03	-0.11	-	-	-	394.9	399.5
(0,3)	0.91***	5.54	4.59	-	-	-	0.51	0.22	0.19	397.1	404.8
(3,1)	--	--	--	--	--	--	--	--	--	--	--
(3,2)	--	--	--	--	--	--	--	--	--	--	--
(1,3)	--	--	--	--	--	--	--	--	--	--	--
(2,3)	0.24**	0.07’	--	0.28	-0.69	-	-0.96	-0.10	-0.07	395.1	405.9

(3,3)	--	--	--	--	--	--	--	--	--	--	--

Source: Authors' calculations
Notes: ** statistically significant *(d)* at 5 and *** 1 percent level, ' – non rejection values for the null of *(d=0, d=1)* at 95% significance level. Bolded – best model specification using AIC – Akaike (1974) and SIC – Schwartz (1978) information criteria

The order of integration for real GDP ranges between 0.24 and 1.09 depending on the specification and the order of the AR and MA components. In fact, these do not contribute significantly to the model fit (they have a low p value), suggesting that the models presented in Table 3 are over-parameterised. On the whole, the estimated ARFIMA models for UK real GDP series provide evidence of long memory. Next, to take into account the possibility of dependence between aggregate output and other series, we estimate long-memory ARFIMA – GARCH, ARFIMA - FIGARCH models for UK real GDP with unemployment rate and inflation as the explanatory variables, on the basis of the theory of Phillips (1962).

3. Multivariate Analysis

The estimated multivariate ARFIMA(p,d,q) specification is the following:

$$\begin{aligned} Y_t &= \mu + \sum_{k=0}^{4} \psi_{1,k} UNE_{t-k} + \sum_{k=0}^{4} \psi_{2,k} CPL_{t-k} + v_t \\ (1-L)^d v_t &= w_t,\ t = 1,1,.... \end{aligned} \tag{4}$$

where Y_t = UK real GDP (log of real GDP, log differenced real GDP, first difference real GDP and real GDP in levels), UNE = unemployment rate and CPL = consumer price level as defined in Hills et al. (2010), with w_t assumed to be white noise or AR(1), AR(2). The autoregressive $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$ and moving average matrix polynomials $\Theta(L) = 1 - \theta_1 L - \ldots - \theta_q L^q$ have all roots lying outside the unit circle. We focus on the linkages between output, unemployment and prices as in the model of Phillips (1962), and therefore no other variables are included in the model.

Standard diagnostic tests indicated the existence of GARCH effects which could result in wrong inference for *(d)* and spurious forecasting. It is not surprising to find that these are important for UK real GDP. The possible negative impact of high uncertainty and volatility on aggregate output (through lower investment and expected returns driving demand down) has been highlighted by Keynes (1936), Bernanke (1983), Pindyck (1991), Ramey and Ramey (1991) among others. However, the impact could also be positive (see Solow (1956), Mirman (1971), Blackburn (1999) and

(Black 1987): when facing macroeconomic uncertainty, investors might seek safety by increasing aggregate savings leading to a higher future equilibrium growth rate. In Friedman (1968), Phelps (1968) and Lucas (1972) the effect is neutral.

Given the evidence above, we added a GARCH component to the ARFIMA(p,d,q) model of Granger and Joyeux (1980), Hosking (1984) and Baillie et al. (1996) as follows:

$$\begin{aligned} &\phi(L)(1-L)^{d}(Y_t-\mu-b'x_{1t}-\delta\sigma_t)=\theta(L)\varepsilon_t \\ &\varepsilon_t\big|\Omega_{t-1}\sim D(0,\sigma_t^2) \qquad . \qquad (5) \\ &\beta(L)\sigma_t^2=+\alpha(L)\varepsilon_t^2+\gamma'x_{2t} \end{aligned}$$

with Y_t = UK real GDP (log of real GDP, log differenced real GDP, first difference real GDP and real GDP in levels).

Table 4. Estimation Results for ARFIMA-GARCH models

	RGDP (1,d,1) (1,1)		LRGDP (0,d,0)(1,0)		DRGDP (0,d,0)(1,0)		LDRGDP (1,d,0)(1,1)	
	Normal	Stud.-t	Normal	Stud.-t	Normal	Stud.-t	Normal	Stud.-t
$\hat{\mu}$	108.2*	83.59*	4.287*	4.268*	20.29	4.997	0.027	-0.01*
$\hat{\psi}_1$	-10.82*	-7.63*	-0.125*	-0.104*	0.048	0.499*	-0.009	0.036*
$\hat{\psi}_2$	-29.22*	-8.43	-0.006*	0.004*	-18.18	-3.294	0.000	0.000
$\hat{d}$	1.246*	2.112*	1.370*	1.431*	0.305*	0.281*	-0.520	-0.30*
$\hat{\theta}_1$	1.003*	0.167	-	-	-	-	0.633	0.498*
$\hat{\theta}_2$	-	-	-	-	-	-	-	-
$\hat{\Phi}_1$	0.912*	0.887*	-	-	-	-	-0.224	-
$\hat{\Phi}_2$	-	-	-	-	-	-	-	-
$\hat{\omega}$	3.097*	1.691	0.021	0.022	5.160*	6.569*	0.018	0.020
$\hat{\alpha}$	1.230*	1.075*	0.361*	0.370*	1.280*	2.355	0.804*	0.33*
$\hat{\beta}$	0.258	0.884*	-	-	-	-	0.642	-0.22*
$\hat{\upsilon}$	-	1.951	-	2.207	-	1.609	-	2.273
LM(A)	(0.416)	(0.945)	(0.194)	(0.243)	(0.562)	(0.062)	(0.849)	(0.37)

LM(ARCH)	(0.923)	(0.931)	(0.973)	(0.977)	(1)	(1)	(0.304)	(0.243)
LM(FF)	(0.111)	(0.814)	(0.189)	(0.517)	(0.474)	(0.192)	(0.282)	(0.266)
NORM(JB)	(0.06)	(0.000)	(0.042)	(0.013)	(0.000)	(0.000)	(0.000)	(0.000)

Source: Authors' calculations

Notes: * statistically significant at the 5 percent level. The best model specification is chosen using AIC – Akaike (1974) and SIC – Schwartz (1978) information criteria. LM(A) = Lagrange multiplier autocorrelation test, LM(ARCH) = Lagrange multiplier neglected ARCH
test, LM(FF) = Lagrange multiplier functional form test, NORM(JB) = normality Jarque – Bera test.

Table 4 shows the estimated parameters along with a set of diagnostic tests (normality, heteroscedasticity, functional form, (ARCH) effects, Ljung and Box, serial correlation). AIC and BIC as well as the LR tests imply that the best model is that for the log of UK real GDP (LRGDP) with a Gaussian distribution. The estimated order of integration for this series based on the ARFIMA (0,d,0) - GARCH(1,0) specification is 1.37, implying that the unit root null cannot be rejected. This result is in line with the findings of Candelon and Gil-Alana (2004), whose estimate was 1.38. For the transformed series there is evidence of long memory, with the exception of the first differenced one, which exhibits short memory with the test statistics not rejecting the I(0) hypothesis. For the real GDP series with a Gaussian distribution one can reject the I(0) but not the I(1) null, both are rejected with a Student's t-distribution. The best model for the first differences implies a rejection of the null of I(0) offering evidence of long memory with a fractional integration parameter *(d)* between 0.28 – 0.35. There is also evidence of significant GARCH effects.

The residuals of the models presented in Table 4 are characterised by both kurtosis and skewness as indicated by the Jarque-Bera (JB) test results. To capture long memory in both the conditional mean and variance we next estimate an ARFIMA-FIGARCH model adding a FIGARCH component to the ARFIMA model of Granger and Joyeux (1980), Hosking (1984) and Baillie et al. (1996), Andersen et al. (2003). The adopted specification is the following:

$$\begin{aligned} &\Psi(L)(1-L)^{\zeta}(y_t-\mu)=\Theta(L)\varepsilon_t, \\ &\varepsilon_t = z_t\sigma_t,\ z_t \sim N(0,1), \\ &\varphi(L)(1-L)^{d}\varepsilon_t^2=\omega+\left[1-\beta(L)\right]v_t \end{aligned} \quad . \qquad (6)$$

The estimation results are presented in Table 5. Both long-memory parameters (the conditional mean ζ and variance *d*) are significantly different from zero and indicate persistence both in real GDP and its volatility. Table 5 shows the estimated ARFIMA-FIGARCH models with a Gaussian and Student's t-distribution respectively. On the whole, the UK real GDP series appears to be characterised by long memory, non-stationarity and non-mean-reverting behaviour, as also found by Haubrich and Lo (2001) and Blanchard and Quah (1993).

Table 5: Estimation results for ARFIMA-FIGARCH

	RGDP		LRGDP		DRGDP		LDRGDP	
	(0, ζ ,1) (2,d,1)		(0, ζ ,0)(2,d,2)		(0, ζ ,0)(2,d,1)		(0, ζ ,0)(1,d,1)	
	Normal	Stud.-t	Normal	Stud.-t	Normal	Stud.-t	Normal	Stud.-t
$\hat{\mu}$	76.55*	32.42*	4.414*	4.361*	2.405	3.645	0.015*	0.023
$\hat{\psi}_1$	-25.44*	-10.53*	-0.183*	-0.140*	-0.155	-2.136	0.008*	-0.002
$\hat{\psi}_2$	-11.38*	-8.22*	-0.013*	-0.012*	0.237*	0.374*	-0.000	-0.000
$\hat{\psi}_3$	0.714	-0.037	0.004	0.001	-4.280	-3.600*	-0.020	-0.02*
$\hat{\zeta}$	1.769*	1.541*	1.598*	1.535*	0.271*	0.278*	0.184*	0.09
$\hat{\theta}_1$	-	-	-	-	-	-	-	-
$\hat{\theta}_2$	-	-	-	-	-	-	-	-
$\hat{\Phi}_1$	0.236	0.282*	-	-	-	-	-	-
$\hat{\Phi}_2$	-	-	-	-	-	-	-	-
$\hat{\omega}$	3.113*	6.371*	0.000	0.000	1.808*	1.892*	0.015*	0.013*
φ_1	1.367*	0.281*	0.218*	-0.150	1.557*	1.583*	-0.63*	-0.67*
φ_2	-0.276*	-0.025*	0.840*	0.197	-0.541*	-0.559*	-	-
β_1	0.610*	0.978*	0.268*	-0.486	0.947*	0.944*	0.677*	0.75*
β_2	-	-	0.706*	-0.492	-	-	-	-
$\hat{d}$	0.011	1.032*	0.800*	0.678*	-0.320	-0.188	0.193*	0.311*
$\hat{\upsilon}$	-	2.785*	-	2.303	-	1.957*	-	2.058*
LM(A)	(0.179)	(0.943)	(0.543)	(0.217)	(0.076)	(0.042)	(0.247)	(0.606)
LM(ARCH)	(0.567)	(0.588)	(0.273)	(0.558)	(0.131)	(0.245)	(0.583)	(0.352)
LM(FF)	(0.446)	(0.502)	(0.473)	(0.571)	(0.436)	(0.885)	(0.826)	(0.213)

NORM(JB)	(0.146)	(0.078)	(0.442)	(0.001)	(0.000)	(0.000)	(0.000)	(0.000)
Q(20)	15.36	11.53	21.34	19.20	20.75	18.16	23.43	21.34
$Q_s(20)$	7.57	5.78	13.99	15.29	8.92	7.92	11.58	12.19
R^2	0.99	0.99	0.99	0.99	0.29	0.28	0.03	0.03

Source: Authors' calculations

Notes: * statistically significant at 5 percent level. Best model specification using AIC – Akaike (1974) and SIC – Schwartz (1978) information criteria. LM(A) = Lagrange multiplier autocorrelation test, LM(ARCH) = Lagrange multiplier neglected ARCH test, LM(FF) = Lagrange multiplier functional form test, NORM(JB) = normality Jarque – Bera test.

For real output both the I(0) and I(1) hypotheses are rejected, and the long-memory estimates for the conditional mean range between 0.18 and 1.769, being higher than in Candelon and Gil-Alana (2004). The same holds for the (log) real GDP series, whilst for the first differenced series the I(0) null is rejected in favour of long memory (fractional integration). The presence of long memory for both real GDP and the log series as well as their first differences is supported by the ARFIMA-FIGARCH estimates. Table 5 shows that the long-memory coefficient ζ is statistically significant for all these four series; for real and log real GDP $\zeta > 1$, whilst for the differenced series $0 < \zeta < 0.5$. Long memory in the conditional variance is also confirmed for the log real GDP series (with Student's t distribution) and its first differences. For the real GDP series, the unit root null cannot be rejected at the 5% statistical significance level for the conditional variance, whilst for the differenced series the null of short memory $d = 0$ cannot be rejected.

The preferred specification for UK real GDP is an ARFIMA $(0, \zeta, 1)$ - FIGARCH (2,d,1) and for its log an ARFIMA $(0, \zeta, 0)$ - FIGARCH (2,d,2). Model adequacy is confirmed by the diagnostic tests. Figure 1 shows the predicted values and the residuals from the selected specification for the log of the real GDP series: it can be seen that the model fits the data quite well.

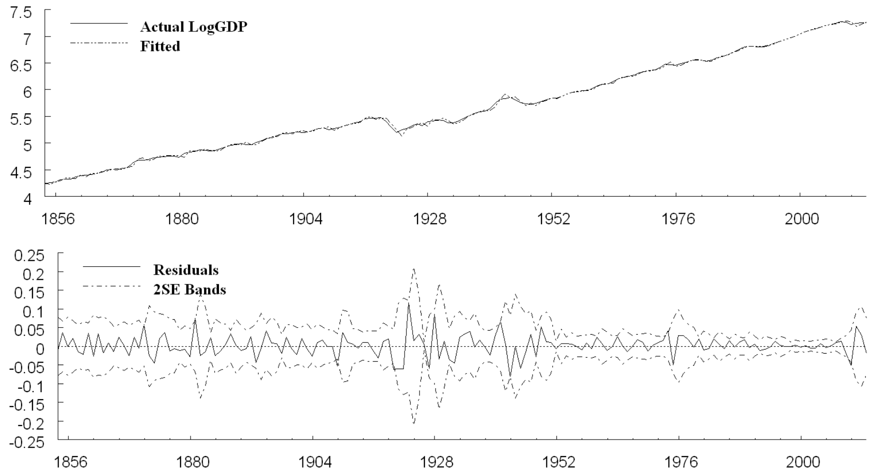

Fig. 1 Predicted values and residuals from the best specification for the log of the UK real GDP series ARFIMA (0, ζ ,0) - FIGARCH (2,d,2) model

The results in Table 5 indicate the presence of long memory in UK output volatility, with the FIGARCH estimates of d ranging from 0.19 to 1.03. This parameter is statistically significant at the 5% level and implies a rejection of the null $d = 0$ (GARCH model) as well as $d = 1$ (IGARCH model), except for the real GDP series in levels with a Student's t-distribution ($d = 1.03$). Figure 2 shows the impulse responses of real GDP to to unemployment and price shocks. The impact of this type of shocks appears to be permanent, whilst there are no significant effects of price shocks. Output volatility is also highly persistent, as indicated by the FIGARCH long-memory parameters.

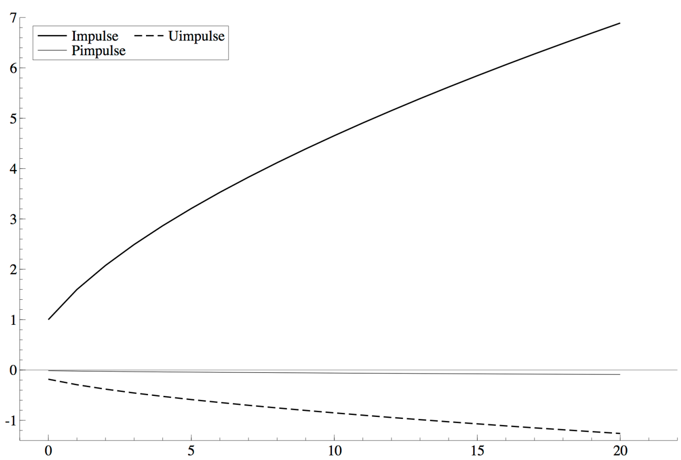

Fig. 2 Impulse responses to unemployment and price shocks for UK real GDP from the ARFIMA (0, ζ ,0) - FIGARCH (2,d,2) model

5. Conclusions

This study investigates the long-memory properties of UK real GDP over the period 1851-2013. It extends the work of Candelon and Gil-Alana (2004) by providing evidence of dual memory in both the conditional mean and variance through an ARFIMA-FIGARCH specification that takes into account GARCH effects in the output series. The results confirm that UK real GDP is non-stationary and non-mean-reverting, and exhibits long memory, with shocks having permanent effects, our estimate of *d* is in fact very close to that reported in Candelon and Gil-Alana (2004). The results hold whether a normal or Student's t-distribution is assumed. The choice of the unemployment rate and prices as the explanatory variables in the multivariate model is motivated by the economic theory developed by Phillips (1962). Our results reveal that 20% of the variance in UK real GDP over the period 1851-2013 can be explained by unemployment shocks, while price shocks do not play any role. Future work could consider an extended multivariate model with additional explanatory variables.

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