Thermal and Mechanical Stresses in Thick Spheres with an Extended FGM Model

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Abstract

In this study, an exact solution for the one-dimensional steady-state thermal and mechanical stresses in a hollow thick sphere made of functionally graded material with a new graded model is presented. The temperature distribution is assumed to be a function of radius. The material properties are graded along the radial direction according to exponential functions of radial direction. The advantage of this model, compared to the other models (linear or power law models), is in satisfaction of the material boundary conditions for several types of material variation profiles. Employing the model, the energy and Navier equations are solved using the generalized Bessel function and the Lagrange method. In addition, a method for solving the governing equations in spherical coordinates is presented. The analytical solution of the heat conduction equation and the Navier equation lead to the temperature profile, radial displacement, radial stress, and hoop stress as a function of radial direction.

Key words: *exponential FGM model; symmetric thermal stresses; hollow sphere*

1 Introduction

Functionally graded materials (FGM) possess properties that vary gradually and continuously with location within the material. The analytical solution for the stresses in spheres and cylinders made of functionally graded materials are given by Lutz and Zimmerman [1,2]. They considered thick spheres and cylinders under radial thermal loads, where radially graded materials with linear composition of the constituent materials were considered. Obata and Noda studied the one-dimensional steady thermal stresses in a functionally graded circular hollow cylinder and a hollow sphere using the perturbation method [3]. By introducing the theory of laminated composites, Ootao et al. treated the theoretical analysis of a three-dimensional thermal stress problem for a nonhomogeneous hollow circular cylinder due to a moving heat source in the axial direction in a transient state [4]. Tanigawa et al. solved the thermal stresses for a semi-infinite body with the assumption that nonhomogeneous material properties are power functions of the thickness direction z [5]. Jabbari, et al. derived analytical solutions for one-dimensional and two-dimensional steady state thermoelastic problems of the functionally graded circular hollow cylinder, where the material properties are expressed as power functions of radius [6,7]. Eslami, et al. obtained an exact solution for the one-dimensional steady-state thermal and mechanical stresses in a hollow thick sphere made of functionally graded material [8]. They assumed the temperature distribution to be a function of radius, with general thermal and mechanical boundary conditions. Ootao and Tanigawa presented the three-dimensional transient thermal stresses of a nonhomogeneous hollow sphere with respect to a rotating heat source [9].

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Functionally graded materials of linear or power law functions for analytical solutions are used in foregoing papers. However, other FGM models satisfying material boundary conditions for more than one profile, are solved numerically.

The dynamic thermoelastic response of functionally graded cylinder and plates are studied by Reddy and Chin [10]. Guven and Baykara presented a functionally graded isotropic hollow sphere with spherically symmetry subjected to internal pressure [11]. They considered a multimaterial sphere composed of great number of concentric homogeneous spheres of the different elasticity moduli. Chen et al. [12] studied the deformation of spherically isotropic hollow sphere of a functionally graded material with laminate approximation theory.

The problem with the power law distribution of the FGM is the restriction imposed by the existence of two constants. The proposed constitutive relations for the power law distribution of the functionally graded materials contain two material constants, see Reddy [10] and Tanigawa [5] models. When either of these models are used to model a FGM structure, such as thick cylinder or sphere, the given boundary conditions at the inside and outside surfaces fix the FGM profile across the thickness. On the other hand, if the FGM constitutive model contains three independent constants, two boundary conditions at the inside and outside surfaces, leaves one more constants to be chosen to comply with the FGM profile.

In the present work, a thick hollow sphere of FGM under one-dimensional steady-state temperature distribution with thermal and mechanical boundary conditions is considered. The thermal and mechanical properties of the sphere are assumed to be expressed by exponential functions of the radial direction with three arbitrary constants. Applying the exponential FGM model, the energy equation leads to the generalized Bessel function [13]. The homogeneous part of the Navier equation also leads to the generalized Bessel function. The Lagrange method is used to solve the particular part of the Navier equation.

2 Derivations

Consider a thick hollow sphere of inside radius a and outside radius b made of functionally graded material. The sphere's material is graded through the radial r-direction. Thus, the material properties are functions of r. Let u be the displacement component along the radial direction. The strain-displacement relations are

$$\epsilon_{rr} = u', \qquad \epsilon_{\theta\theta} = \epsilon_{\phi\phi} = \frac{u}{r} \tag{1}$$

where $(')$ denotes differentiation with respect to r. The Hooke's relations are

$$\begin{aligned} \sigma_{rr} &= \lambda e + 2\mu\epsilon_{rr} - (3\lambda + 2\mu)\alpha T(r) \\ \sigma_{\theta\theta} = \sigma_{\phi\phi} &= \lambda e + 2\mu\epsilon_{\theta\theta} - (3\lambda + 2\mu)\alpha T(r) \end{aligned} \tag{2}$$

where σ_{ii} and ϵ_{ii} $(i = r, \theta, \phi)$ are the stress and strain tensors, $T(r)$ is the temperature distribution determined from the heat conduction equation, α is the coefficient of thermal expansion, e is the strain dilatation, and λ and μ are the Lamé constants related to the modulus of elasticity E and Poisson's ratio ν as

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \qquad \mu = \frac{E}{2(1+\nu)} \tag{3}$$

The equilibrium equation in the radial direction, disregarding the body force and the inertia term, is

$$\sigma'_{rr} + \frac{2}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0 \tag{4}$$

To obtain the equilibrium equation in terms of the displacement component for the FGM sphere, the functional relationship of the material properties must be known. The sphere's material is assumed to be described with an exponential function of the radial direction as

$$E(r) = \frac{\beta_1}{r^{\gamma_1}} e^{\frac{\lambda_1}{\theta_1} r^{\theta_1}}, \qquad \alpha(r) = \frac{\beta_2}{r^{\gamma_2}} e^{\frac{\lambda_2}{\theta_2} r^{\theta_2}} \tag{5}$$

where β_i, λ_i, θ_i, and γ_i $(i = 1, 2)$ are unknowns to be found. The constants β_i and λ_i $(i = 1, 2)$ are used to satisfy the inner and outer material boundary conditions, respectively, as

$$\lambda_i = \frac{\theta_i}{b^{\theta_i} - a^{\theta_i}} ln \left[\frac{E(b)}{E(a)} \left(\frac{b}{a} \right)^{\gamma_i} \right]$$
$$\beta_i = E(a) a^{\gamma_i} e^{-\frac{\lambda_i}{\theta_i} a^{\theta_i}}, \qquad i = 1, 2. \tag{6}$$

Since θ_i $(i = 1, 2)$ is further restricted, the choice of γ_i $(i = 1, 2)$ produces different material profile variations between the inside and outside surfaces. We may further assume that the Poisson's ratio is constant.

Using Eqs. (1) through (5), the Navier equation in term of the displacement becomes

$$r^2 u''(r) + (\lambda_1 r^{\theta_1} - \gamma_1 + 2) r u'(r) + \left[\frac{2\nu}{1-\nu} \lambda_1 r^{\theta_1} - \frac{2\nu}{1-\nu} \gamma_1 - 2 \right] u(r) =$$
$$\beta_2 e^{\frac{\lambda_2}{\theta_2} r^{\theta_2} - \gamma_2 ln r} \frac{1+\nu}{1-\nu} r \left[(\lambda_1 r^{\theta_1} + \lambda_2 r^{\theta_2} - (\gamma_1 + \gamma_2)) T(r) + r T'(r) \right]. \tag{7}$$

3 Heat Conduction Problem

The heat conduction equation in the steady-state condition for the radial temperature distribution and the thermal boundary conditions for a FGM hollow sphere are given, respectively, as

$$\frac{1}{r^2} (r^2 k(r) T'(r))' = 0 \tag{8}$$

$$C_{11} T'(a) + C_{12} T(a) = f_1$$
$$C_{21} T'(b) + C_{22} T(b) = f_2 \tag{9}$$

where $k = k(r)$ is the thermal conduction coefficient, C_{ij} are the constant thermal parameters related to the conduction and convection coefficients, and f_1 and f_2 are known constants on the inside and outside radii, respectively. It is assumed that the thermal conduction coefficient $k(r)$ is an exponential function of r as

$$k(r) = \frac{\beta_3}{r^{\gamma_3}} e^{\frac{\lambda_3}{\theta_3} r^{\theta_3}} \tag{10}$$

where β_3 and λ_3 are obtained from Eq. (6). Substituting $i = 3$, θ_3 is further restricted and γ_3 is free to be chosen to provide different material conduction profiles along the radius. Using Eq. (10), the energy equation becomes

$$r^2 T''(r) + r \left[(2 - \gamma_3) + \lambda_3 r^{\theta_3} \right] T' = 0 \tag{11}$$

For $\theta_3 = \gamma_3 - 1$ (which produce a restriction for θ_3), Eq. (11) becomes the generalized Bessel equation [13], which has a general solution as

$$T(r) = r^{\alpha_3} e^{-\zeta_3 r^{\theta_3}} \left[A I_{\frac{1}{2}}(\zeta_3 r^{\theta_3}) + B I_{-\frac{1}{2}}(\zeta_3 r^{\theta_3}) \right] \quad (12)$$

where

$$\begin{aligned} \alpha_3 &= \frac{\theta_3}{2} \\ \zeta_3 &= \frac{\lambda_3}{2\theta_3} \end{aligned} \quad (13)$$

The constants A and B are unknowns to be found using the thermal boundary conditions (9), and $I_{\pm\frac{1}{2}}$ is the modified Bessel function of the first kind and the $\pm\frac{1}{2}$th order.

4 Solution of the Navier Equation

To solve the Navier equation, the boundary conditions at the inner and outer surfaces of the sphere are required. The general kinematic or forced boundary conditions at the inside and outside surfaces may be assumed of the form

$$\begin{aligned} u(a) &= \chi_1 \\ u(b) &= \chi_2 \\ \sigma_{rr}(a) &= \chi_3 \\ \sigma_{rr}(b) &= \chi_4 \end{aligned} \quad (14)$$

where the constants χ_1 to χ_4 are known on the inner and outer radii, respectively. Equations (14) let stress or displacement boundary conditions (e.g. traction-free or fixed-surface) to be applied. For thermal stresses, any mathematical or applied selection of Eqs. (14) is applicable. But in Mechanical stresses, displacement and stress boundary conditions can not be selected both in inner or outer surfaces, due to boundary value problem.

Equation (7) is an ordinary differential equation, having the general and particular solutions. The homogeneous part of the Navier equation for the radial displacement u is

$$r^2 u''(r) + (\lambda_1 r^{\theta_1} - \gamma_1 + 2) r u'(r) + \left[\frac{2\nu}{1-\nu} \lambda_1 r^{\theta_1} - \frac{2\nu}{1-\nu} \gamma_1 - 2 \right] u(r) = 0 \quad (15)$$

Substituting $\theta_i = \gamma_i - 1 + \frac{4\nu}{1-\nu}$; $i = 1, 2$ (which produce a restriction for θ_1 and θ_2) in Eq. (15), leads to the generalized Bessel differential equation [13], which has a general solution as

$$u_g(r) = r^{\alpha_1} e^{-\zeta_1 r^{\theta_1}} \left[C I_p(\zeta_1 r^{\theta_1}) + D I_{-p}(\zeta_1 r^{\theta_1}) \right] \quad (16)$$

where

$$\begin{aligned} p &= \frac{1}{\theta_1} \sqrt{ (\frac{\gamma_1 - 1}{2})^2 + \frac{2\nu\gamma_1}{1-\nu} + 2 } \\ \alpha_1 &= \frac{1}{2}\theta_1 - \frac{2\nu}{1-\nu} \\ \zeta_i &= \frac{\lambda_i}{2\theta_i} \qquad (i = 1, 2) \end{aligned} \quad (17)$$

The constants C and D are unknowns to be found, and I_p is the modified Bessel function of the first kind and the pth order. Since p is in general a noninteger, I_p and I_{-p} are independent functions. In case where p is an integer, the second kind of modified Bessel function K_p should be used, instead of I_{-p}.

The particular solution $u_p(r)$ is obtained using the method of variation of parameters. Equation (7) may be written in the form of variation of parameters formula as

$$u_p''(r) + f(r)u_p'(r) + g(r)u_p(r) = h(r) \tag{18}$$

where

$$\begin{aligned}
f(r) &= \frac{1}{r}\left(\lambda_1 r^{\theta_1} - \theta_1 + \frac{1+3\nu}{1-\nu}\right) \\
g(r) &= \frac{1}{r^2}\left[\frac{2\nu}{1-\nu}\left(\lambda_1 r^{\theta_1} - \theta_1 - \frac{1-5\nu}{1-\nu}\right) - 2\right] \\
h(r) &= \beta_2 \frac{1+\nu}{1-\nu} r^{\alpha_3 - 1} e^{\left(\frac{\lambda_2}{\theta_2} r^{\theta_2} - \gamma_2 ln r - \zeta_3 r^{\theta_3}\right)} \times \\
&\{\left[\left(\lambda_1 r^{\theta_1} - \gamma_1\right) + \left(\lambda_2 r^{\theta_2} - \gamma_2\right) - \frac{1}{2}\left(\lambda_3 r^{\theta_3} - \gamma_3 + 1\right)\right]\left[A I_{\frac{1}{2}}(\zeta_3 r^{\theta_3}) + B I_{-\frac{1}{2}}(\zeta_3 r^{\theta_3})\right] + \\
&r\left[A I'_{\frac{1}{2}}(\zeta_3 r^{\theta_3}) + B I'_{-\frac{1}{2}}(\zeta_3 r^{\theta_3})\right]\}
\end{aligned} \tag{19}$$

Let $u_{g_1}(r)$ and $u_{g_2}(r)$ be two independent linear general solutions of Eq. (18). Using Eq. (16), gives

$$\begin{aligned}
u_{g_1}(r) &= r^{\alpha_1} e^{-\zeta_1 r^{\theta_1}} I_p(\zeta_1 r^{\theta_1}) \\
u_{g_2}(r) &= r^{\alpha_1} e^{-\zeta_1 r^{\theta_1}} I_{-p}(\zeta_1 r^{\theta_1})
\end{aligned} \tag{20}$$

Therefore, the particular solution $u_p(r)$, according to the method of variation of parameters becomes

$$u_p(r) = -u_{g_1}(r)\int \frac{u_{g_2}(r)}{w(r)} h(r)\, dr + u_{g_2}(r)\int \frac{u_{g_1}(r)}{w(r)} h(r)\, dr \tag{21}$$

where $w(r)$ is the Wronskian determinant of $u_{g_1}(r)$ and $u_{g_2}(r)$ as

$$w(r) = u_{g_1}(r)u'_{g_2}(r) - u_{g_2}(r)u'_{g_1}(r) \tag{22}$$

$$\begin{aligned}
u'_{g_1}(r) &= \left[(\alpha_1 - \zeta_1\theta_1 r^{\theta_1}) I_p(\zeta_1 r^{\theta_1}) + r I'_p(\zeta_1 r^{\theta_1})\right] r^{\alpha_1 - 1} e^{-\zeta_1 r^{\theta_1}} \\
u'_{g_2}(r) &= \left[(\alpha_1 - \zeta_1\theta_1 r^{\theta_1}) I_{-p}(\zeta_1 r^{\theta_1}) + r I'_{-p}(\zeta_1 r^{\theta_1})\right] r^{\alpha_1 - 1} e^{-\zeta_1 r^{\theta_1}}
\end{aligned} \tag{23}$$

where $I'_{\pm p}(\zeta_1 r^{\theta_1})$ is the derivative of $I_{\pm p}(\zeta_1 r^{\theta_1})$ with respect to r as

$$I'_{\pm p}(\zeta_1 r^{\theta_1}) = \zeta_1\theta_1 r^{\theta_1 - 1} I_{\pm p+1}(\zeta_1 r^{\theta_1}) \pm \frac{p\theta_1}{r} I_{\pm p}(\zeta_1 r^{\theta_1}) \tag{24}$$

The complete solution for $u(r)$ is the sum of the general and particular solutions and is

$$u(r) = u_g(r) + u_p(r) \tag{25}$$

Therefore, using Eqs. (16) and (21) yields

$$u(r) = \left[C - \int Q_2(r)\,dr\right] u_{g_1}(r) + \left[D + \int Q_1(r)\,dr\right] u_{g_2}(r) \tag{26}$$

where

$$\begin{aligned} Q_1(r) &= \int \frac{u_{g_1}(r)}{w(r)} h(r) \\ Q_2(r) &= \int \frac{u_{g_2}(r)}{w(r)} h(r) \end{aligned} \tag{27}$$

Substituting Eq. (26), with the help of Eq. (24), into Eqs. (1) and (2), the strains and stresses are obtained as

$$\begin{aligned} \epsilon_{rr}(r) = r^{\alpha_1 - 1} e^{-\zeta_1 r^{\theta_1}} \{ [C - &\int Q_2(r)\,dr] \left[(\alpha_1 - \zeta_1 \theta_1 r^{\theta_1} + p\theta_1) I_p(\zeta_1 r^{\theta_1}) + \zeta_1 \theta_1 r^{\theta_1} I_{p+1}(\zeta_1 r^{\theta_1}) \right] + \\ &[D + \int Q_1(r)\,dr] \left[(\alpha_1 - \zeta_1 \theta_1 r^{\theta_1} - p\theta_1) I_{-p}(\zeta_1 r^{\theta_1}) + \zeta_1 \theta_1 r^{\theta_1} I_{-p+1}(\zeta_1 r^{\theta_1}) \right] - \\ &Q_2(r) r I_p(\zeta_1 r^{\theta_1}) + Q_1(r) r I_{-p}(\zeta_1 r^{\theta_1}) \} \end{aligned} \tag{28}$$

$$\epsilon_{\theta\theta}(r) = r^{\alpha_1 - 1} e^{-\zeta_1 r^{\theta_1}} \left\{ \left[C - \int Q_2(r)\,dr\right] I_p(\zeta_1 r^{\theta_1}) + \left[D + \int Q_1(r)\,dr\right] I_{-p}(\zeta_1 r^{\theta_1}) \right\} \tag{29}$$

$$\begin{aligned} &\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \end{bmatrix} = \langle \{ [C - \int Q_2(r)\,dr] \left[(\alpha_1 - \zeta_1 \theta_1 r^{\theta_1} + p\theta_1) I_p(\zeta_1 r^{\theta_1}) + \zeta_1 \theta_1 r^{\theta_1} I_{p+1}(\zeta_1 r^{\theta_1}) \right] + \\ &[D + \int Q_1(r)\,dr] \left[(\alpha_1 - \zeta_1 \theta_1 r^{\theta_1} - p\theta_1) I_{-p}(\zeta_1 r^{\theta_1}) + \zeta_1 \theta_1 r^{\theta_1} I_{-p+1}(\zeta_1 r^{\theta_1}) \right] - \\ &Q_2(r) r I_p(\zeta_1 r^{\theta_1}) + Q_1(r) r I_{-p}(\zeta_1 r^{\theta_1}) \} \begin{bmatrix} 1-\nu \\ \nu \end{bmatrix} + \{ [C - \int Q_2(r)\,dr] I_p(\zeta_1 r^{\theta_1}) + \\ &[D + \int Q_1(r)\,dr] I_{-p}(\zeta_1 r^{\theta_1}) \} \begin{bmatrix} 2\nu \\ 1 \end{bmatrix} \rangle \frac{\beta_1}{(1+\nu)(1-2\nu)} r^{-\alpha_1 - 2} e^{\zeta_1 r^{\theta_1}} \\ &- \frac{\beta_1 \beta_2}{1 - 2\nu} \frac{e^{\left(2\zeta_1 r^{\theta_1} + 2\zeta_2 r^{\theta_2} - \zeta_3 r^{\theta_3}\right)}}{r^{(2\alpha_1 + 2\alpha_2 - \alpha_3 + 2)}} \left[A I_{\frac{1}{2}}(\zeta_3 r^{\theta_3}) + B I_{-\frac{1}{2}}(\zeta_3 r^{\theta_3}) \right] \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned} \tag{30}$$

Equations (28) through (30) contain two unknowns C and D. Therefore, two boundary conditions are required to evaluate the two unknowns. These boundary conditions may be arbitrary (physically meaningful) selected from the mechanical boundary conditions given in Eqs. (14). Thus, C and D are obtained solving a system of two algebraic equations, selected arbitrary from Eqs. (14).

5 Results

The analytical solution obtained in the previous section may be checked for a number of examples. As first example, consider a thick hollow sphere of inner radius $a = 40cm$ and outer radius $b = 50cm$. Poisson's ratio is assumed to be constant and is taken to be 0.3, and the modulus of elasticity, thermal coefficient of expansion and thermal conduction

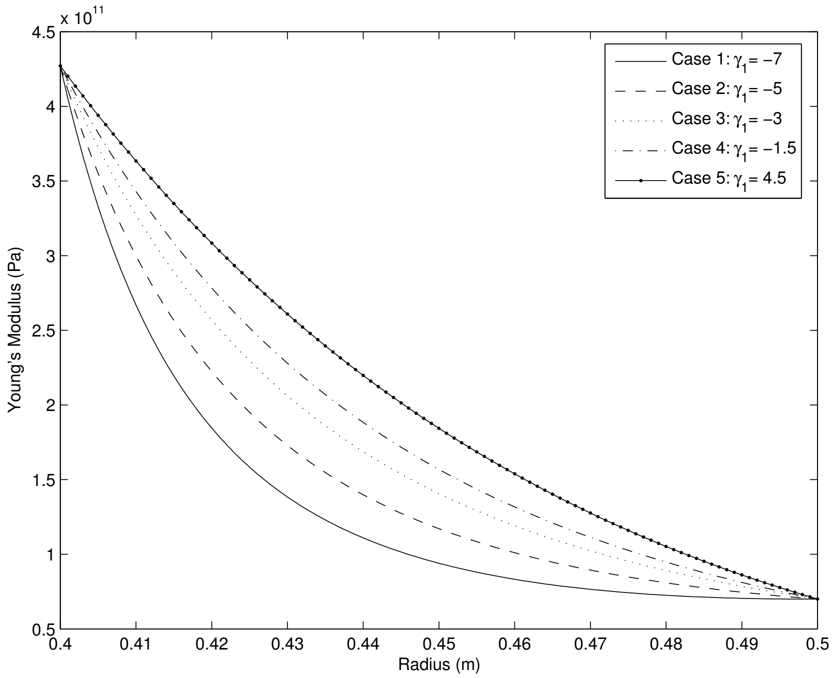

Figure 1: Young's Modulus along the radius of a functionally graded sphere.

at the inner radius are $E(a) = 427$ GPa, $\alpha(a) = 3.4 \times 10^{-6}/K$ and $K(a) = 65W/mK$, respectively, and those of the outer radius are $E(b) = 70$ GPa, $\alpha(b) = 23.4 \times 10^{-6}/K$ and $K(b) = 233W/mK$, respectively.

Various FGM variation profiles across the thickness are produced by choosing arbitrary magnitudes for γ_1, γ_2, and γ_3. These constants (γ_1, γ_2, γ_3) provide various material property profiles for the modulus of elasticity, thermal expansion coefficient, and the conductivity coefficient.

The boundary conditions for temperature are taken as $T(a) = 50K$ and $T(b) = 0K$. The hollow sphere may be assumed to be traction-free at inner and outer surfaces. The variation of modulus of elasticity, coefficient of thermal expanssion and heat conduction coefficient along the radial direction are plotted in Figs. (1) through (3). These figures are plotted for different values of γ's. The curve associated with the largest negative value of γ's represent more rich ceramic, and that of largest positive value of γ's represent more metal rich FGM. For different FGM profiles, given in Figs. (1) to (3), temperature profile, radial displacement, radial stress, and hoop stress along the radial direction are plotted in Figs. (4) through (7). Figure (4) shows the temperature distribution across the wall thickness. This figure shows that the temperatures are higher in ceramic rich FGMs. Figure (5) shows the resulting thermoelastic radial displacement due to the given temperature variations. As seen, the radial displacements are lower in ceramic rich FGM spheres. Figure (6) represents the radial stress along the radial direction satisfying the traction free boundary conditions. The radial stresses are zero at the inner and outer surfaces, due to the assumed boundary conditions. The circumferential stress versus radial direction is shown in Fig. (7). It is observed that the circumferential stress variations along the radial direction is lower in metal rich FGMs. As seen, the mechanical hoop stress distribution is compressive at the inside surface (ceramic) and tensile at the outside surface (metal).

Now, consider a thick sphere under mechanical stresses. The inside pressure is assumed to be 100 MPa, and outside pressure zero. The resulting radial displacements due to the

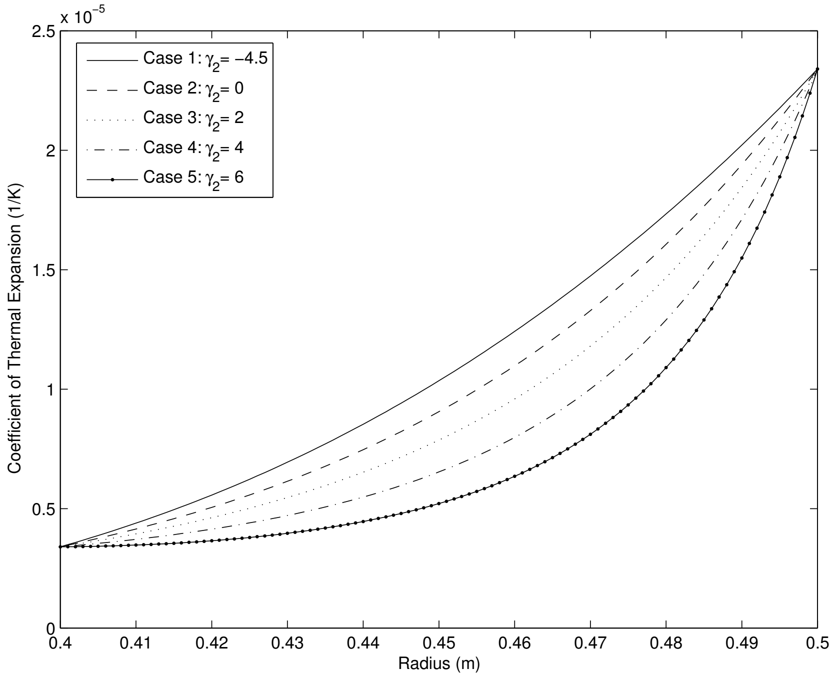

Figure 2: Coefficient of thermal expansion along the radius of a functionally graded sphere.

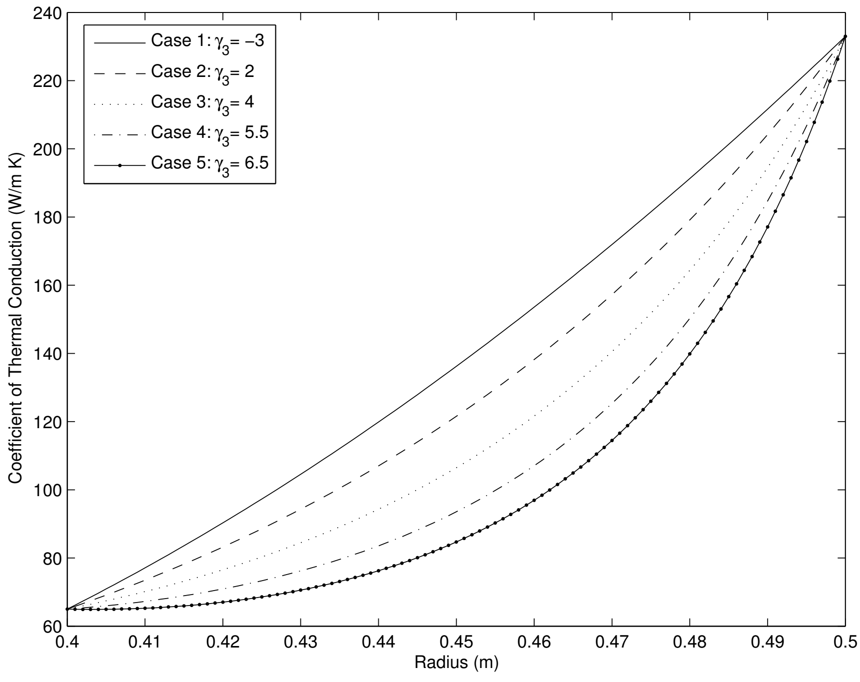

Figure 3: Coefficient of thermal conduction along the radius of a functionally graded sphere.

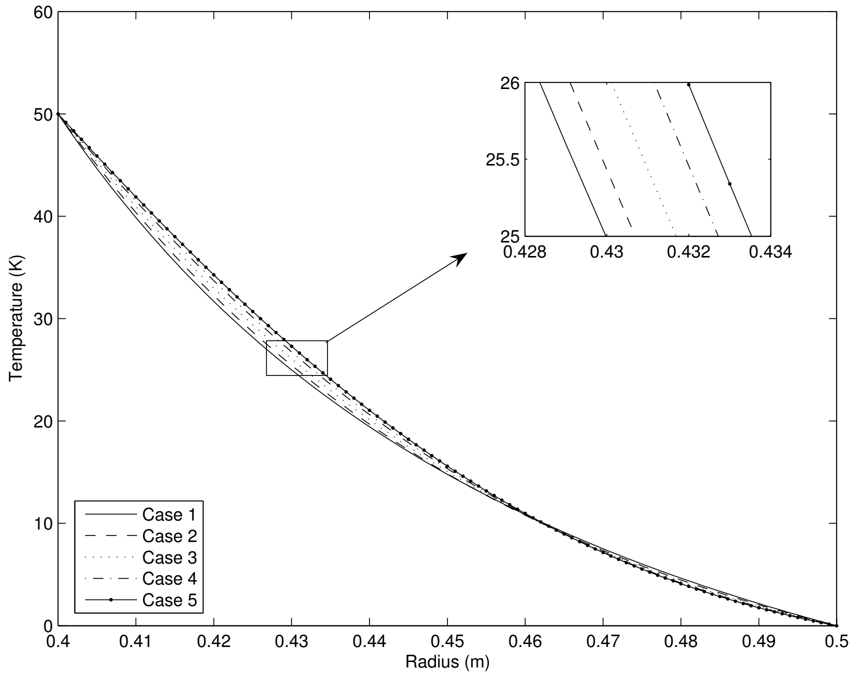

Figure 4: Temperature distribution.

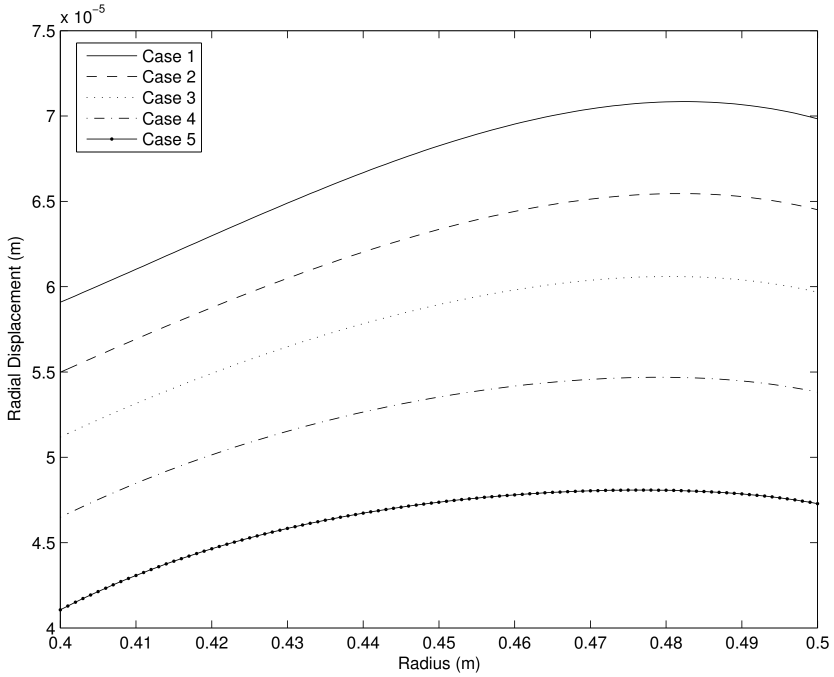

Figure 5: Thermal radial displacement.

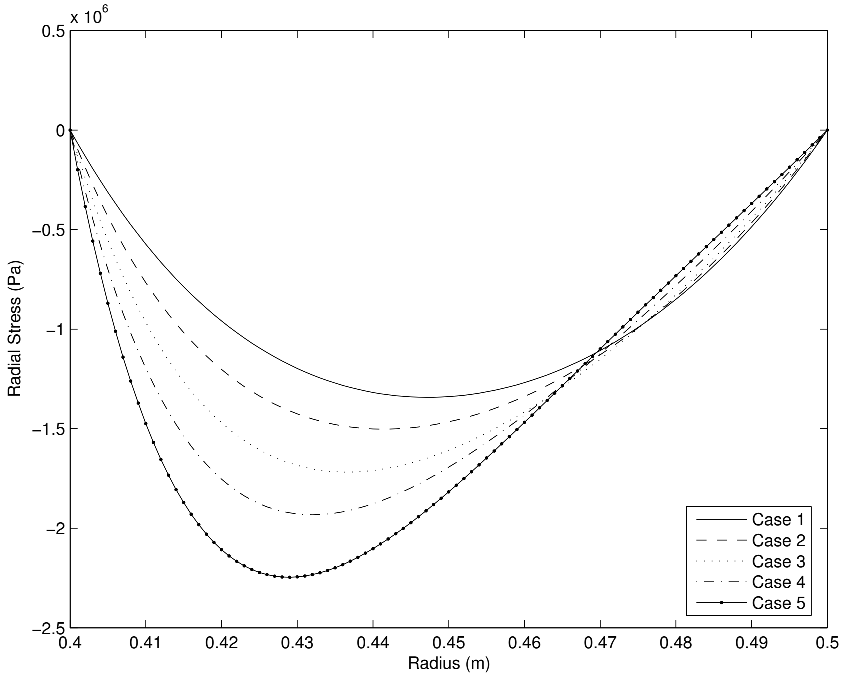

Figure 6: Radial thermal stress.

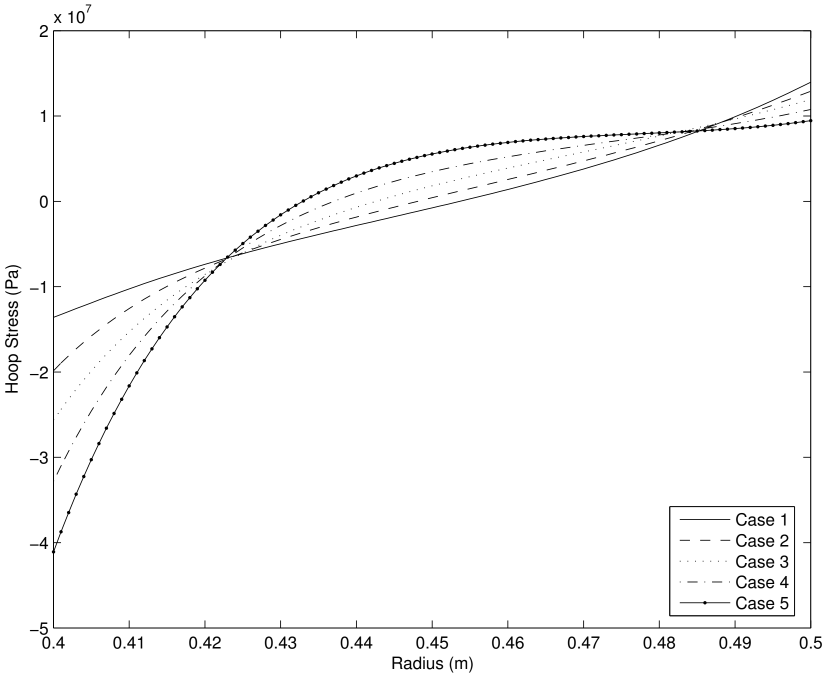

Figure 7: Hoop thermal stress.

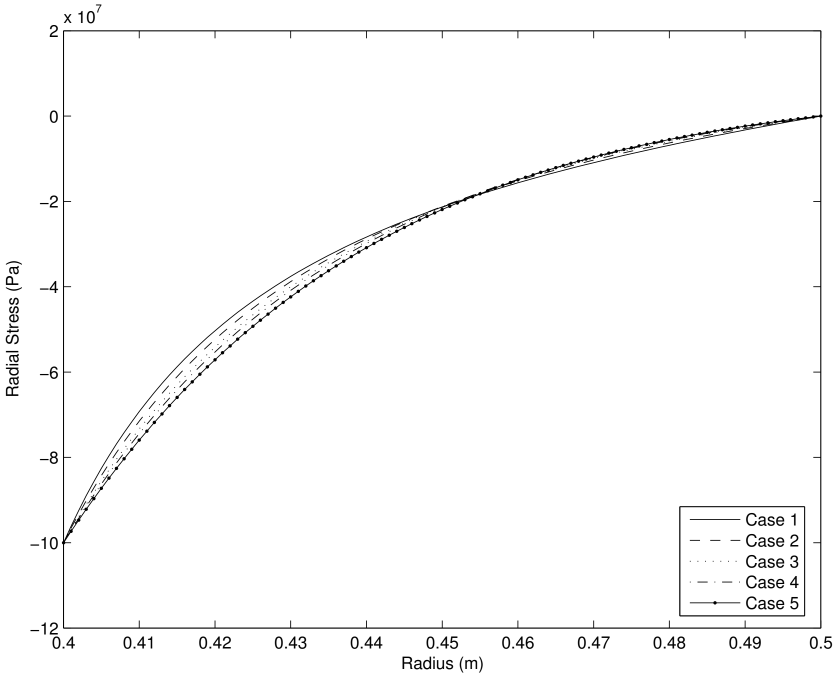

Figure 8: Mechanical radial displacement.

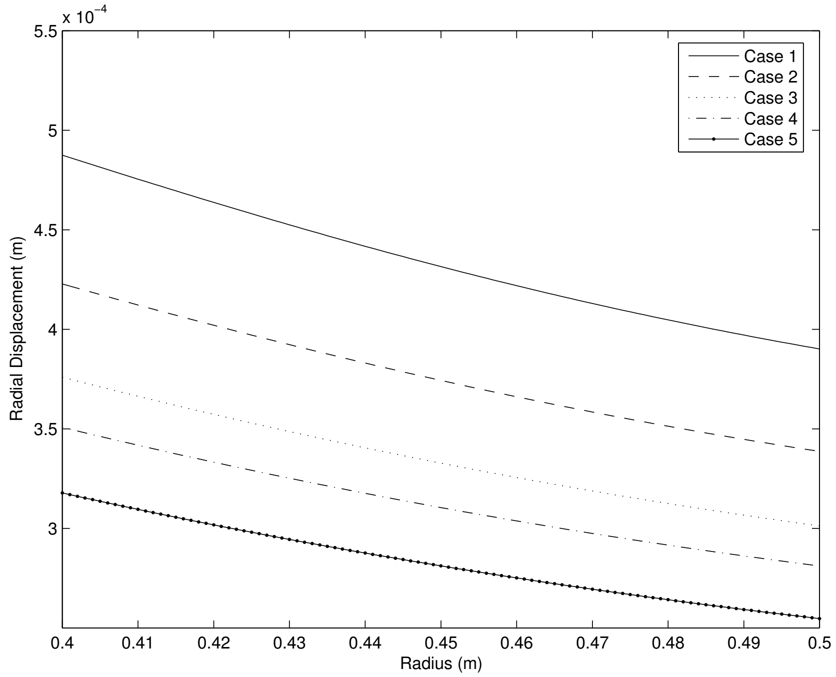

Figure 9: Radial mechanical stress.

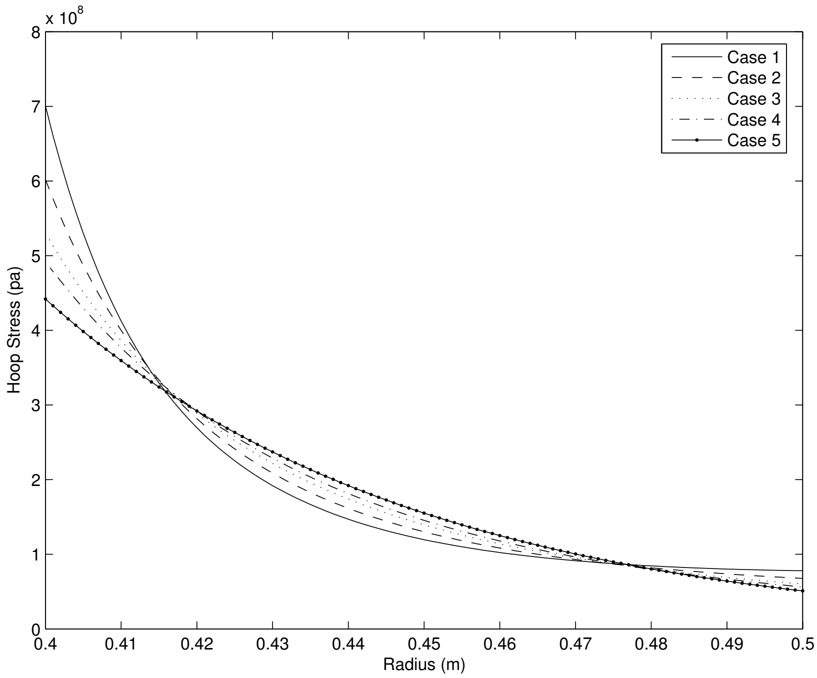

Figure 10: Hoop mechanical stress.

given mechanical boundary condition, for different profiles of functionally graded materials, are plotted in Fig. (8). The radial displacements are higer in metal rich functionally graded spheres. In Fig. (9) the radial stress along the sphere thickness is plotted. It is observed that the radial stress increases as the ceramic density of FGM sphere increases. The mechanical circumferential stress versus the radial direction is shown in Fig. (10). It is seen that the circumferential stress variations decrease in ceramic rich cases.

A comparison between final results show that under the pure thermal boundary condition, maximum radial and hoop stresses happen in the ceramic rich sphere. However, under the pure mechanical boundary condition, maximum hoop stresses happen in metal rich one. It is interesting and acceptable that in both pure thermal and pure mechanical boundary conditions, radial displacement is minimum in ceramic rich sphere.

6 Conclusions

This paper presents an analytical solution to obtain the spherically symmetric thermal and mechanical stresses in a thick hollow sphere made of functionally graded material. The material properties are assumed to be graded along the radial direction according to the exponential functions of radial direction. The constitutive law of FGM is assumed to be of exponential type with three constants, allowing different variation profiles of FGM with fix material boundary conditions. The mechanical and thermal stresses are obtained through the direct method of solution of the Navier equation.

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