

A Two-stage Game-theoretic Method for Residential PV Panels Planning Considering Energy Sharing Mechanism

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Abstract—This paper proposes a novel two-stage game-theoretic residential photovoltaic (PV) panels planning framework for distribution grids with potential PV prosumers. One innovative contribution is that a residential PV panels location-allocation model is integrated with the energy sharing mechanism to increase economic benefits to PV prosumers and meanwhile facilitate the reasonable installation of residential PV panels. The optimization of residential PV panels planning decisions is formulated as a two-stage model. In the first stage, we develop a Stackelberg game based stochastic bi-level energy sharing model to determine the optimal sizing of PV panels with uncertain PV energy output, load demand, and electricity price. Instead of directly solving the proposed bi-level energy sharing problem by using commercial solvers, we develop an efficient descend search algorithm-based solution method which can significantly improve the computation efficiency. In the second stage, we propose a stochastic programming based residential PV panels deployment model for all PV prosumers. This model is formulated as an optimal power flow (OPF) problem to minimize active power loss. Finally, simulations on an IEEE 33-node and 123-node test systems demonstrate the effectiveness of the proposed method.

Index Terms—Residential photovoltaic panels planning, energy sharing mechanism, Stackelberg game, descend search algorithm, optimal power flow problem

NOMENCLATURE

Indices and sets

j Index of nodes/lines/agents

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k/i	Index of prosumer agents/consumer agents
t	Index of time periods
w	Index of scenarios
$\mathcal{N}/\mathcal{L}/\mathcal{T}/\mathcal{W}$	Set of nodes/lines/time periods/scenarios
\mathcal{C}_j	Set of child nodes of node j
$\mathcal{N}^{PV}/\mathcal{N}^C$	Set of prosumer agents/consumer agents
Variables	
P_{jtw}/Q_{jtw}	Active/reactive power flow on distribution line j at time t in scenario w
v_{jtw}	Squared voltage magnitude on node j at t, w
l_{jtw}	Squared line current magnitude on line j at t, w
p_{jtw}/q_{jtw}	Active/reactive power injection on node j at j, t, w
p_{itw}^{PVc}	PV electricity sold to the consumer agent i at t, w
p_{ktw}^{PVg}	PV electricity sold by the prosumer agent k at t, w
$p_{ktw}^{Curtail}$	PV energy curtailment on prosumer agent node k at t, w
P_{jtw}^G	Electricity purchased from the utility grid by agent j at t, w
c_{tw}^U	Uniform price applied to all consumer agents at t, w
u_k^{PV}	Number of PV panels purchased by prosumer agent k
ϕ^{PV}	Total installation number of PV panels
λ_{itw}^L	Dual variable of supply-demand balance (6g)
$\mu_{itw}^{PVc}/\mu_{itw}^G$	Dual variable of inequality constraint (6h)/(6i)
U_{itw}^C	The utility function of consumer agent i at t, w
Rev^C	Revenue of the coalition formed by all prosumer agents
Parameters	
c_{tw}^G/c_{tw}^{MCP}	Electricity price /wholesale market clearing price at t, w
$c_{itw}^{PV}/c_{o\&m}^{PV}$	Investment cost/operation & maintenance cost of each PV panel
c^E	Greenhouse gas emission cost
r^C	The proportion of the incomes of sharing PV electricity to consumer agents via the energy sharing platform
w_i^E	Individual weight factor for greenhouse gas emissions reduction of consumer agent i
c_{itw}^{WTP}	Willingness-to-pay of consumer agent i at t, w
p_w	Probability of uncertainty scenario w
$\gamma_{itw}^{PV}/\gamma_{itw}^L$	PV energy output factor/load factor at t, w
P_{jtw}^L/Q_{jtw}^L	Active/reactive load on node j at t, w
E_{size}^{PV}	Size of each PV panel
E^{PVmax}	PV hosting capacity of the distribution grid
u_k^{PVmax}	Maximum allowable installation number of PV panels on

	node prosumer agent node k
v^{min}/v^{max}	Squared lower/upper bound of voltage magnitude
I_j^{max}	Squared upper bound of current magnitude on distribution line j
η^{PV}	Efficiency of PV panels output
r_j/x_j	Resistance/reactance of distribution line j

I. INTRODUCTION

RESIDENTIAL photovoltaic (PV) panels based distributed renewable generation becomes a promising alternative power generation technology to reduce greenhouse gas emissions and promote a low-carbon lifestyle. Energy policies introduced by many countries encourage end-users to install self-consumed PV systems on their rooftops [1]. As a consequence, PV prosumers have emerged in large numbers [2], which are end-use consumers who can also act as energy providers if they install residential PV panels. In practice, PV prosumers can behave as sellers or buyers according to their net power profiles as well as electricity price when they participate in the energy sharing process. However, the proliferation of PV generators leads to the oversupply of PV energy in some areas, thus, PV curtailment may be needed when all consumers are fully supplied with local PV energy and meanwhile the wholesale market clearing prices are negative. For PV prosumers, they try to maximize the revenue from PV energy sharing to offset the investment, operation and maintenance cost of PV panels. As for the normal consumers without renewable sources, they are interested in reducing their electricity bills and some of them also concern the potential valuation of environmental benefits. Therefore, the energy sharing between PV prosumers and their nearby consumers becomes an effective approach to improve the local PV generation consumption and even reduce negative impacts caused by PV energy integration into the upstream electrical network. In this regard, from the perspective of the residential prosumers, this paper aims to optimally plan residential PV panels for maximizing the revenue of PV prosumers in a distribution grid via energy sharing.

The idea of energy sharing has drawn attention recently. Ref. [3] proposes a two-stage peer-to-peer energy sharing model for an energy building cluster. In Ref. [4], a novel energy sharing strategy is presented for demand-side management in a neighborhood area network (NAN), which is comprised of energy providers, users, and an electricity retailer. Ref. [5] develops a risk aversion energy sharing model based on a devised local energy market for addressing the issues caused by uncertain renewable sources from the perspective of community prosumers. In Ref. [6], a hybrid energy sharing framework of multiple distribution grids is studied with consideration of combined heat and power and demand response. Ref. [7] proposes a building welfare maximization model as well as a game-theoretical energy sharing model to investigate the effect of sharing PV electricity in an apartment building.

For optimal residential PV panels planning, Ref. [8] proposes a probabilistic dynamic allocation model to optimally

determine the capacity of on-site PV generation for a residential energy hub, aiming to minimize the PV energy curtailment as well as consumer cost. In Ref. [9], a data-driven based method is proposed for the detection, verification, and estimation of residential PV system installations. Authors of Ref. [10] endeavor to find the optimal installation capacity of PV generation by splitting the budget between PV generation and energy storage for maximizing the revenue from participating in the electricity market. Ref. [11] determines the energy storage size and PV panels number for a commercial building considering heating ventilation and air conditioning systems. Ref. [12] proposes an optimal two-stage placement method for the heterogeneous distributed generators, including PV generators, in a grid-tied multi-energy microgrid with the consideration of the uncertainties from the renewable energy resources. In Ref. [13], based on a performance indicator and different parameters of the PV system, an economic study is addressed for residential PV panels installation in Santiago, Chile. Ref. [14] presents an expansion planning model of PV and battery systems for the smart house, which considers investment cost, selling price and purchasing price. However, these works only focus on energy sharing or residential PV panels planning. Up to now, the research on integrating energy sharing mechanisms with residential PV panels planning is still at a very early stage.

This paper develops a novel two-stage game-theoretic framework for residential PV panels planning. In the first stage, Stackelberg game theory is used to model the stochastic bi-level energy sharing problem, which is solved by a proposed descend search algorithm. In the second stage, we develop a stochastic programming based optimal power flow (OPF) model to optimally allocate residential PV panels for all PV prosumers with minimum expected active power loss. The main contributions of this paper are threefold,

1) Different from most works related to residential PV panels planning, we innovatively integrate a residential PV panels planning model with the energy sharing mechanism. To our best acknowledge, this has not been studied before. In this way, we can improve economic benefits to PV prosumers and meanwhile facilitate the installation of residential PV panels, which has practical significance.

2) Instead of directly solving the proposed bi-level energy sharing problem by using commercial solvers with Mathematical Program with Equilibrium Constraints (MPEC), we develop an efficient descend search algorithm that can significantly enhance computation efficiency. Moreover, in our proposed solution method, the feasibility of voltage constraint and current constraint is checked to ensure the reliability and security of the distribution grid operation during the energy sharing process. Thus, the proposed solution method can address both economic and operating security concerns.

3) To address the conflict of interests between PV prosumers and normal consumers, the leader-followers Stackelberg game theory is used to model a bi-level PV energy sharing problem. Besides, the uncertainties of PV energy output, load demand, as well as electricity price are simultaneously considered in this energy sharing model.

The rest of this paper is organized as follows. In Section II, we introduce the system model, including the branch flow model, consumer agents and the coalition of PV prosumer agents. Section III describes a bi-level energy sharing problem and an efficient search algorithm-based solution method. Section IV presents the optimal PV panels allocation model with the goal of active power loss minimization. Numerical results are given in Section V. Finally, we conclude this paper in Section VI.

II. SYSTEM MODELLING

A. Network Branch Flow Model

In this paper, the branch flow model (BFM) [15] is used to describe the complex power flow equations of the radial electrical network. Consider a radial distribution grid $\mathcal{M} := (\mathcal{N}, \mathcal{L})$, where $\mathcal{N} := \{0, 1, \dots, \mathcal{N}\}$ denotes the node set and $\mathcal{L} := \{0, 1, \dots, \mathcal{L}\}$ denotes the directed line set. Except for the substation node, each node j has a unique ancestor node m and a set of child nodes \mathcal{C}_j . We assume the direction of the line connect node j and its ancestor node m is from m to j . Note that the node n is in the set of the child nodes \mathcal{C}_j of the node j . Therefore, the BFM is given as follows,

$$P_{mj} - r_{mj}l_{mj} + p_j = \sum_{n \in \mathcal{C}_j} P_{jn} \quad j \in \mathcal{N} \quad (1a)$$

$$Q_{mj} - x_{mj}l_{mj} + q_j = \sum_{n \in \mathcal{C}_j} Q_{jn} \quad j \in \mathcal{N} \quad (1b)$$

$$v_j - v_m = 2(r_{mj}P_{mj} + x_{mj}Q_{mj}) - (r_{mj}^2 + x_{mj}^2)l_{mj} \quad j \in \mathcal{N} \quad (1c)$$

$$l_{mj} = \frac{P_{mj}^2 + Q_{mj}^2}{v_j} \quad j \in \mathcal{N} \quad (1d)$$

where the active and reactive power balances at each node are described by (1a) and (1b), respectively; P_{mj} and Q_{mj} represent the active and reactive power flows of line mj (from the ancestor node m to the node j), respectively; p_j and q_j denote the active and reactive power injection (+) or extraction (-) at the node j , respectively; r_{mj} and x_{mj} represent the resistance and reactance of line mj , respectively; l_{mj} is the squared line current magnitude on distribution line mj . The voltage drop/rise on each line can be described by (1c). (1d) denotes the relationship between the power flow, squared bus voltage magnitude and squared line current magnitude, which is expressed by nonconvex. To convexify (1d), it is relaxed into an inequality as (2a) and then reformulated into a second-order cone constraint as (2b) by the second-order cone programming (SOCP) relaxation method [16].

$$l_{mj} \geq \frac{P_{mj}^2 + Q_{mj}^2}{v_j} \quad j \in \mathcal{N} \quad (2a)$$

$$\|(2P_{mj}, 2Q_{mj}, v_j - l_{mj})\|_2 \leq v_j + l_{mj} \quad j \in \mathcal{N} \quad (2b)$$

It has been proved in Ref. [17] that the relaxation is exact as long as the network is radial and the objective function of the OPF problem is strictly increasing in l_{mj} . In our model, the equality in (2b) also holds since we take the network loss into

account and it is a strictly increasing function of l_{mj} .

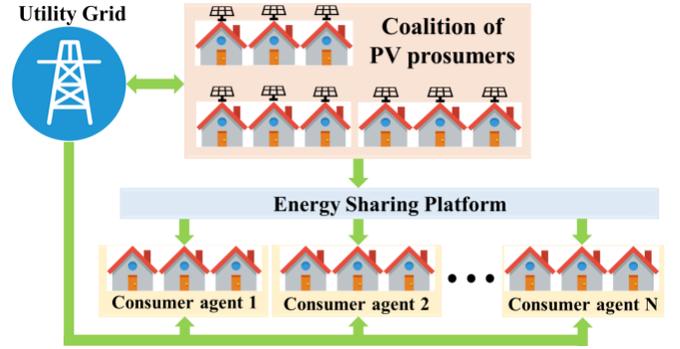


Fig. 1. The proposed energy sharing framework between the coalition and consumer agents

B. Energy Sharing Agent Model

This paper proposes a two-stage residential PV panels planning model, where the first stage is to determine the optimal sizing of PV panels by developing a Stackelberg game-based stochastic bi-level energy sharing model. The proposed energy sharing model is performed during the decision-making process of finding the total PV panel installation capacity. Besides, in the practical operation of the distribution system, our proposed bi-level energy sharing model can be used for trading energy among the coalition, consumers and the utility grid.

For better energy sharing coordination, we introduce two kinds of agents, i.e., consumer agent and prosumer agent. Specifically, the consumer agent denotes the aggregation of customers on the same node and the prosumer agent represents the aggregation of PV prosumers (owners of PV panels) on the same node. On behalf of its local consumers/prosumers, each agent is allowed to participate in the energy sharing process.

1) *Revenue for the coalition of prosumer agents*: In our proposed model, all prosumer agents form a coalition to maximize their common benefits by operating their residential PV panels. This assumption is based on the fact that most individual PV prosumers can only provide a small amount of PV energy, posing some challenges to energy management in power grids. Besides, due to the barriers to renewable energy integration [18-21], especially market entry [22], it is efficient and easy to manage the integrated PV energy. Note that the demands of the coalition are satisfied by the self-generated PV energy firstly, then the surplus PV energy is used for energy sharing with consumers. Therefore, before the coalition participates in the energy sharing process, PV energy trading among the prosumers in the coalition has been finished.

The coalition can be a seller or a buyer according to its loads and PV generations. When the coalition becomes a seller, there is a competitive relationship between the coalition and the distribution system operator (DSO) because they both try to sell electricity to the local consumers. However, when the netload of the coalition is positive, there is a dependency relationship between the coalition and the DSO since the coalition needs to purchase electricity from the utility grid via the DSO. The coalition sells its surplus PV electricity differently to the consumer agents and the utility grid to maximize its revenue.

During the energy sharing with local consumer agents, the coalition needs to pay the bill to the energy sharing platform which acts as an intermediary agent to facilitate energy sharing. Therefore, the revenue of the coalition is defined as follows,

$$\begin{aligned} Rev^C = & \sum_{i \in N^C} \sum_{t \in T} (1-r^C) c_t^U P_{it}^{PVc} + \sum_{k \in N^{PV}} \sum_{t \in T} c_t^{MCP} P_{kt}^{PVg} \\ & - \sum_{k \in N^{PV}} \sum_{t \in T} c_t^G [\gamma_t^L P_k^L - \eta^{PV} \gamma_t^{PV} \phi^{PV} E_{size}^{PV}]^+ \\ & - (\alpha c_{inv}^{PV} + c_{o\&m}^{PV}) \phi^{PV} \end{aligned} \quad (3)$$

where the first term represents the income of selling PV electricity P_{it}^{PVc} to the consumer agents with the uniform price c_t^U . Note that the energy sharing platform charges the coalition intermediary fee in terms of the proportion r^C of this income. The second term is the gain of selling PV electricity P_{kt}^{PVg} to the utility grid with the wholesale market clearing price c_t^{MCP} . The third term denotes that the coalition needs to buy electricity from the utility grid when its demand cannot be satisfied by its PV generation. Note that $[\cdot]^+$ represents the projection operator onto the non-negative orthant, i.e., $[x]^+ = \max(x, 0)$. The last term denotes the investment, operation and maintenance cost of residential PV panels. Note the daily capital recovery factor $\alpha = \frac{r^{PV}(1+r^{PV})^y}{365((1+r^{PV})^y - 1)}$ is to transform the investment cost c_{inv}^{PV} from the planning horizon into the daily horizon, where r^{PV} is the interest rate and y is the planning horizon.

2) *Consumer agent utility function:* The utility function of each consumer agent i is composed of three terms as follows,

$$U_i^C = - \sum_{t \in T} c_t^U P_{it}^{PVc} - \sum_{t \in T} c_t^G P_{it}^G - \sum_{t \in T} w_i^E c^E P_{it}^G \quad (4)$$

where the first term and second term represent the cost of electricity purchased from the coalition and the utility grid, respectively. Generally, the conventional consumer utility function only includes the cost of purchasing electricity. However, it is not rational enough since some consumers have environmental awareness so they are interested in reducing greenhouse gas emissions. Therefore, the third term in (4) is for emissions reduction with the individual weight factor w_i^E . The load demand P_{it}^L of each consumer agent i can be satisfied by buying electricity from the coalition (P_{it}^{PVc}) and/or the utility grid (P_{it}^G), i.e., $P_{it}^L = P_{it}^{PVc} + P_{it}^G$. Thus, by replacing P_{it}^G in (4) by $P_{it}^L - P_{it}^{PVc}$, the willingness-to-pay (WTP) c_{it}^{WTP} for purchasing local PV electricity of consumer agent i can be characterized as follows,

$$c_{it}^{WTP} = c_t^G + w_i^E c^E \quad i \in N^C, t \in T \quad (5)$$

Note that the individual weight factor w_i^E is introduced to express the preference for emissions reduction of consumer agent i : 1) $w_i^E > 0$ means that the consumer agent i has strong environmental awareness so his WTP c_{it}^{WTP} is higher than the real-time electricity price c_t^G ; 2) $w_i^E = 0$ means that the WTP of the consumer agent i equals to the real-time electricity price; 3) $w_i^E < 0$ means that the environmental awareness of consumer agent i is weak so his WTP is lower than the real-time electricity price.

III. BI-LEVEL ENERGY SHARING MODEL FOR DETERMINING OPTIMAL PV PANELS INSTALLATION CAPACITY

A. Uncertainty Characterization

In this paper, we consider three sources of uncertainties in sizing the PV panels installation capacity, i.e., PV energy output, load demand, and electricity price. The stochastic programming-based approach is employed to deal with these three uncertainty sources, which are represented by a set of representative scenarios. We assume that local historical public data of the electricity prices, loads and PV generations can be collected by the coalition and a well-established backward-reduction algorithm [23] can be used to select the representatives with probabilities, which can distinguish the importance of each scenario. Each representative scenario $w \in \mathcal{W}$ consists of three vectors, given as,

$$w = \{\gamma_{nw}^{PV}, \gamma_{nw}^L, c_{nw}^G\} \quad t \in T, w \in \mathcal{W}$$

Besides, p_w is defined as the occurrence probability of scenario w , and the sum of probabilities of all considered representative scenarios equals to one, i.e., $\sum_{w \in \mathcal{W}} p_w = 1$.

B. Stackelberg Game Model

In a leader-follower structure, the Stackelberg game is suitable for studying the decision-making processes of followers in response to the decision taken by the leaders [24]. In this paper, we model the coalition as the leader which determines the uniform price of PV electricity and participating consumer agents act as followers to purchase electricity from different sources, e.g., utility grid and coalition, according to the time-varying price. Correspondingly, the Stackelberg game is defined as follows,

$G = \{(Coalition \cup N^C), \{c_{nw}^U\}, \{P_{itw}^{PVc}\}, \{P_{itw}^G\}, \{Rev^C\}, \{U_{itw}^C\}\}$ where $(Coalition \cup N^C)$ denote the player sets, the coalition acts as the leader and the consumer agents in the set N^C take the roles of followers in response to the strategy of the coalition; $\{c_{itw}^U\}$ is the strategy set of the coalition; $\{P_{itw}^{PVc}\}$ and $\{P_{itw}^G\}$ are the strategy sets of consumer agents; $\{Rev^C\}$ and $\{U_{itw}^C\}$ are the revenue (3) of the coalition and the utility function (4) of the consumer agent, respectively.

C. Bi-level Energy Sharing Model

Stackelberg game theory is adopted to model the energy sharing negotiation [25]. Here, the coalition is the leader who sets the uniform prices for PV electricity and consumer agents are the followers who react to decisions taken by the leader. Thus, the stochastic programming-based bi-level energy sharing model is formulated as follows,

$$\begin{aligned} \max Rev^C := & \sum_{w \in \mathcal{W}} p_w [\sum_{i \in N^C} \sum_{t \in T} (1-r^C) c_{itw}^U P_{itw}^{PVc} + \sum_{k \in N^{PV}} \sum_{t \in T} c_{ktw}^{MCP} P_{ktw}^{PVg} \\ & - \sum_{k \in N^{PV}} \sum_{t \in T} c_{ktw}^G [\gamma_{ktw}^L P_k^L - \eta^{PV} \gamma_{ktw}^{PV} \phi^{PV} E_{size}^{PV}]^+] - (\alpha c_{inv}^{PV} + c_{o\&m}^{PV}) \phi^{PV} \end{aligned} \quad (6a)$$

over $\{c_{nw}^U, \phi^{PV}, P_{itw}^{PVc}, P_{itw}^G\}_{i \in N^C, t \in T, w \in \mathcal{W}}$

$$\text{s.t.} \quad (6b)$$

$$\sum_{i \in \mathbb{N}^C} P_{itw}^{PVc} + \sum_{k \in \mathbb{N}^{PV}} P_{ktw}^{PVg} + \sum_{k \in \mathbb{N}^{PV}} P_{ktw}^{Curtail}$$

$$= [\eta^{PV} \gamma_{tw}^{PV} \phi^{PV} E_{size}^{PV} - \sum_{k \in \mathbb{N}^{PV}} \gamma_{tw}^L P_k^L]^+ \quad t \in \mathbb{T}, w \in \mathbb{W}$$

$$\phi^{PV} E_{size}^{PV} \leq E^{PVmax}, \phi^{PV} \in \mathbb{Z}^+ \quad (6c)$$

$$P_{itw}^{PVc}, P_{ktw}^{PVg}, P_{ktw}^{Curtail} \geq 0$$

$$i \in \mathbb{N}^C, k \in \mathbb{N}^{PV}, t \in \mathbb{T}, w \in \mathbb{W} \quad (6d)$$

$$\max U_{itw}^C := -c_{itw}^U P_{itw}^{PVc} - c_{itw}^{WTP} P_{itw}^G \quad (6e)$$

$$\text{over } \{P_{itw}^{PVc}, P_{itw}^G\}_{i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W}}$$

$$\text{s.t. } P_{itw}^{PVc} + P_{itw}^G = \gamma_{itw}^L P_i^L : \lambda_{itw}^L \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (6f)$$

$$P_{itw}^{PVc} \geq 0 : \mu_{itw}^{PVc} \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (6g)$$

$$P_{itw}^G \geq 0 : \mu_{itw}^G \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (6h)$$

where the upper-level problem (6a)-(6h) is to maximize revenue over considered time periods in all scenarios. (6b) describes the dispatch of surplus PV electricity, which can be either fed into consumer agents (P_{itw}^{PVc}), sold to the utility grid (P_{ktw}^{PVg}), or curtailed ($P_{ktw}^{Curtail}$). Note that the right side of (6b) can be defined as the surplus PV electricity that can be supplied to the energy sharing process. It should be noted that PV energy curtailment becomes an optimal option in some time periods where all consumer agents are fully provided with local PV generation and the wholesale market clearing prices are negative [26]. For security reasons, (6c) is added to ensure that total PV panels installation capacity cannot exceed the PV hosting capacity of the grid. (6d) guarantees that the upper-level variables are non-negative.

The lower-level problem (6e)-(6h) aims to maximize the utility function of each consumer agent in each time period and scenario. (6f) balances the supply and demand for each consumer agent node. In (6g) and (6h), the decision variables of this lower-level problem are limited to positive values. Since the lower-level problem is linear and continuous, it can be substituted by Karush–Kuhn–Tucker (KKT) conditions [27]. To facilitate the understanding of readers, the proposed bi-level energy sharing model (6) can be written as follows,

$$\min_{\{x, y, \lambda, \mu\}} f_1(x, y, \lambda, \mu) \quad (7a)$$

$$\text{s.t. } h_1(x, y, \lambda, \mu) = 0 \quad (7b)$$

$$g_2(x, y, \lambda, \mu) \geq 0 \quad (7c)$$

$$\min_{\{y, \lambda, \mu\}} f_2(x, y) \quad (7d)$$

$$\text{s.t. } h_2(x, y) = 0 : \lambda \quad (7e)$$

$$g_2(x, y) \geq 0 : \mu \quad (7f)$$

The KKT conditions of the lower-level problem (7d)-(7f) can be implemented in the upper-level problem (7a)-(7c), given as follows,

$$\min_{\{x, y, \lambda, \mu\}} f_1(x, y, \lambda, \mu) \quad (8a)$$

$$\text{s.t. } h_1(x, y, \lambda, \mu) = 0 \quad (8b)$$

$$g_2(x, y, \lambda, \mu) \geq 0 \quad (8c)$$

$$\nabla_y f_2(x, y) + \lambda \nabla_y h_2(x, y) + \mu \nabla_y g_2(x, y) = 0 \quad (8d)$$

$$h_2(x, y) = 0 \quad (8e)$$

$$g_2(x, y) \geq 0 \perp \mu \geq 0 \quad (8f)$$

The Lagrangian is introduced as follows,

$$L = -c_{itw}^U P_{itw}^{PVc} - c_{itw}^{WTP} P_{itw}^G - \lambda_{itw}^L (P_{itw}^{PVc} + P_{itw}^G - P_{itw}^L) - \mu_{itw}^{PVc} P_{itw}^{PVc} - \mu_{itw}^G P_{itw}^G \quad (9)$$

Therefore, the lower-level problem can be replaced by KKT conditions, described as follows,

$$\frac{\partial L}{\partial \lambda_{itw}^L} = P_{itw}^{PVc} + P_{itw}^G - P_{itw}^L = 0 \quad (10a)$$

$$\frac{\partial L}{\partial P_{itw}^{PVc}} = c_{itw}^{WTP} - \lambda_{itw} - \mu_{itw}^{PVc} = 0 \quad (10b)$$

$$\frac{\partial L}{\partial P_{itw}^G} = c_{itw}^{WTP} - \lambda_{itw} - \mu_{itw}^G = 0 \quad (10c)$$

$$P_{itw}^{PVc} \geq 0 \perp \mu_{itw}^{PVc} \geq 0 \quad (10d)$$

$$P_{itw}^G \geq 0 \perp \mu_{itw}^G \geq 0 \quad (10e)$$

Then the stochastic bi-level energy sharing model (6) can be formulated as mathematical programming with equilibrium constraints (MPEC) model [28], as follows,

$$\max Rev^C := \sum_{w \in \mathbb{W}} p_w [\sum_{i \in \mathbb{N}^C} \sum_{t \in \mathbb{T}} (1-r^C) c_{itw}^U P_{itw}^{PVc} + \sum_{k \in \mathbb{N}^{PV}} \sum_{t \in \mathbb{T}} c_{ktw}^{MCP} P_{ktw}^{PVg} - \sum_{k \in \mathbb{N}^{PV}} \sum_{t \in \mathbb{T}} c_{ktw}^G [\gamma_{ktw}^L P_k^L - \eta^{PV} \gamma_{ktw}^{PV} \phi^{PV} E_{size}^{PV}]^+] - (\alpha c_{itw}^{PV} + c_{o\&m}^{PV}) \phi^{PV} \quad (11a)$$

$$\text{over } \{c_{itw}^U, P_{itw}^{PVc}, P_{itw}^G, \phi^{PV}, \lambda_{itw}^L, \mu_{itw}^{PVc}, \mu_{itw}^G\}_{i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W}}$$

$$\text{s.t. (6b)-(6d), (10a)-(10e)} \quad (11b)$$

D. Linearization of Bi-level Energy Sharing Model

The MPEC model (11) contains two nonlinearities, 1) the nonlinear term $c_{itw}^U P_{itw}^{PVc}$ in the objective function (11a); 2) the complementarity constraints (10d) and (10e). This may make the problem unsolvable. Therefore, the original model (11) needs to be linearized. Firstly, the strong duality condition can be used to obtain a linear expression for $c_{itw}^U P_{itw}^{PVc}$. As stated in the strong duality theorem [29], if a problem is convex, the objective functions of the primal and dual problems have the same value at the optimum. Therefore, the primary objective function (6e) of the lower-level problem is equal to its dual objective function, as follows,

$$c_{itw}^U P_{itw}^{PVc} + c_{itw}^{WTP} P_{itw}^G = \lambda_{itw}^L \gamma_{itw}^L P_i^L \quad (12)$$

Then we can obtain the following expression for the nonlinear term $c_{itw}^U P_{itw}^{PVc}$,

$$c_{itw}^U P_{itw}^{PVc} = \lambda_{itw}^L \gamma_{itw}^L P_i^L - c_{itw}^{WTP} P_{itw}^G \quad (13)$$

Secondly, the complementarity constraints (10d) and (10e) can be linearized using linear expressions proposed in Ref. [30], so the following constraints should be added,

$$P_{itw}^{PVc}, P_{itw}^G \geq 0 \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (14a)$$

$$\mu_{itw}^{PVc}, \mu_{itw}^G \geq 0 \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (14b)$$

$$P_{itw}^{PVc} \leq (1 - u_{itw}^{PVc}) M \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (14c)$$

$$P_{itw}^G \leq (1 - u_{itw}^G) M \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (14d)$$

$$\mu_{itw}^{PVc} \leq u_{itw}^{PVc} M \quad i \in \mathbb{N}^C, t \in \mathbb{T}, w \in \mathbb{W} \quad (14e)$$

$$\mu_{i,w}^G \leq u_{i,w}^G M \quad i \in N^C, t \in T, w \in W \quad (14f)$$

$$u_{i,w}^{Pvc}, u_{i,w}^G \in \{0,1\} \quad (14g)$$

Finally, the linearized MPEC model can be written as follows,

$$\begin{aligned} \max \quad & \sum_{w \in W} p_w \left[\sum_{i \in N^C} \sum_{t \in T} (1-r^C)(\lambda_{i,w}^L \gamma_{i,w}^L P_i^L - c_{i,w}^{WTP} P_{i,w}^G) \right. \\ & + \sum_{k \in N^{PV}} \sum_{t \in T} c_{i,w}^{MCP} P_{k,t}^{PVg} - \sum_{k \in N^{PV}} \sum_{t \in T} c_{i,w}^G [\gamma_{i,w}^L P_k^L - \eta^{PV} \gamma_{i,w}^{PV} \phi^{PV} E_{size}^{PV}]^+ \\ & \left. - (\alpha c_{i,w}^{PV} + c_{o\&m}^{PV}) \phi^{PV} \right] \end{aligned} \quad (15a)$$

$$\begin{aligned} \text{over } \quad & \{c_{i,w}^U, P_{i,w}^{Pvc}, P_{i,w}^G, \phi^{PV}, \lambda_{i,w}^{PVc}, \mu_{i,w}^{Pvc}, \mu_{i,w}^G\}_{i \in N^C, t \in T, w \in W} \\ \text{s.t. } \quad & (6b)-(6h), (10a)-(10c), (14a)-(14g) \end{aligned} \quad (15b)$$

E. Descend Search based Solution Algorithm

Although the linearized MPEC model (15) can be directly solved by cutting-edge solvers, such as CLPEX [31] and Gurobi [32], its computation burden may be quite heavy due to a very large number of scenarios and mixed-integer variables. In this regard, this paper develops an efficient solution algorithm for solving the linearized MPEC model.

As the leader in the Stackelberg game, the coalition sets the uniform price for its PV electricity sold to the consumer agent. According to the definition stated in Ref. [25], the bi-level energy sharing model (6) reaches the Stackelberg Equilibrium (SE) when all players acquire the optimal solutions. Therefore, our proposed game-theoretic framework reaches a SE as long as the coalition finds the optimal uniform price and meanwhile all consumer agents choose their electricity consumption. In this paper, it is assumed the coalition knows the load curves of all the consumer agents in the paper. This assumption is reasonable in practice since the coalition can acquire the load information of each consumer agent by the long-term observation via the nonintrusive load monitoring [33]. Besides, the consumer agent is on behalf of all consumers connected to the same node, so the exact load demand of each consumer cannot be obtained by the coalition. In this regard, the privacy of individual consumers can be protected. Therefore, the optimal uniform price set by the coalition is equal to the WTP offered by the consumer agents. Thus, the coalition prefers to sell its surplus PV electricity to the consumer agent with the highest WTP, followed by the consumer agent with the second-highest WTP, etc. Thus, the surplus PV electricity will be dispatched in the descending order by the WTP of consumer agents. Besides, because the wholesale market clearing price is always lower than the WTP, so the coalition prefers to sell its PV electricity firstly to the consumer agents and the second to the utility grid. Note that for the extreme situation where the uniform price equals the electricity price, we assume that the local consumers prefer to purchase the electricity from the collation firstly due to their environmental awareness. This can facilitate the utilization of local PV energy and reduce the negative impacts on the utility grid caused by PV energy integration.

In this regard, we develop a descend search algorithm to calculate the uniform price, as shown in Algorithm 2. The idea

Algorithm 1: Solution algorithm for solving the proposed bi-level energy sharing problem

1. Initialize the revenue Rev^C and residential PV panels installation number ϕ^{PV} . Initialize the iteration index $\sigma = 0$ and set the step size ϕ_{Step}^{PV} .
2. **Repeat:**
3. $\sigma = \sigma + 1$;
4. $\phi^{PV(\sigma)} = \phi^{PV(\sigma-1)} + \phi_{Step}^{PV}$;
5. **For the coalition do**
6. | Execute **Algorithm 2**;
6. **End**
7. Receive $Rev^{C(\sigma)}$ from executing **Algorithm 2**;
8. **If** $Rev^{C(\sigma)} > Rev^{C(\sigma-1)}$ **then**
9. | $Rev_{opt}^C = Rev^{C(\sigma)}$;
10. | $\phi_{opt}^{PV} = \phi^{PV(\sigma)}$;
11. | **Else** $Rev_{opt}^C = Rev^{C(\sigma-1)}$;
12. | $\phi_{opt}^{PV} = \phi^{PV(\sigma-1)}$;
13. **End**
14. **Until** $\phi^{PV} E_{per}^{PV} \geq E^{PVmax}$;
15. **Return** $Rev_{opt}^C, \phi_{opt}^{PV}$;
16. **End procedure**

Algorithm 2: Descend search algorithm for seeking the optimal internal uniform price

1. Receive $\phi^{PV(\sigma)}$ from **Algorithm 1**;
2. **For all** $w \in W$ **do**
3. | **For all** $t \in T$ **do**
4. | | **For all** $i \in N^C$ **do** ▷ Sorted descending by WTP $c_{i,w}^{wtp}$
5. | | | $N^{*(\sigma)} \leftarrow N^{*(\sigma)} \cup \{i\}$;
6. | | | **If** $c_{i,w}^{wtp} > c_{i,w}^{MCP}$ **then continue**
7. | | | **Else** go back to step 3
8. | | | **End**
9. | | | $c_{i,w}^{U(\sigma)} \leftarrow c_{i,w}^{wtp}$;
10. | | | $P_{i,w}^{PVg(\sigma)} \leftarrow (\eta^{PV} \gamma_{i,w}^{PV} \phi^{PV(\sigma)} E_{per}^{PV} - \sum_{k \in N^{PV}} \gamma_{i,w}^L P_k^L) - \sum_{i \in N^{*(\sigma)}} \gamma_{i,w}^L P_i^L$;
11. | | | **If** $P_{i,w}^{PVg(\sigma)} \geq 0$ **then**
12. | | | | $Rev_{i,w}^{C(\sigma)} \leftarrow \sum_{i \in N^{*(\sigma)}} (1-r^C) c_{i,w}^{U(\sigma)} \gamma_{i,w}^L P_i^L + [c_{i,w}^{MCP}]^+ P_{i,w}^{PVg(\sigma)} - (\alpha c_{i,w}^{PV} + c_{o\&m}^{PV}) \phi^{PV(\sigma)}$;
13. | | | | **Else**
14. | | | | **If** $(\eta^{PV} \gamma_{i,w}^{PV} \phi^{PV(\sigma)} E_{per}^{PV} - \sum_{k \in N^{PV}} \gamma_{i,w}^L P_k^L) \geq 0$
15. | | | | | $Rev_{i,w}^{C(\sigma)} \leftarrow (1-r^C) c_{i,w}^{U(\sigma)} (\eta^{PV} \gamma_{i,w}^{PV} \phi^{PV(\sigma)} E_{per}^{PV} - \sum_{k \in N^{PV}} \gamma_{i,w}^L P_k^L) - (\alpha c_{i,w}^{PV} + c_{o\&m}^{PV}) \phi^{PV(\sigma)}$;
16. | | | | | **Else** $Rev_{i,w}^{C(\sigma)} \leftarrow c_{i,w}^G (\sum_{k \in N^{PV}} \gamma_{i,w}^L P_k^L - \eta^{PV} \gamma_{i,w}^{PV} \phi^{PV(\sigma)} E_{per}^{PV}) - (\alpha c_{i,w}^{PV} + c_{o\&m}^{PV}) \phi^{PV(\sigma)}$;
17. | | | | **End**
18. | | | **End**
19. | **End**
20. | Check power flow by using BFM (1)
21. | **If** $l_{i,w} \leq l_i^{max}$ && $v^{min} \leq v_{i,w} \leq v^{max}$ **then continue**
22. | **Else** go back to step 4
23. | **End**
24. | **End**
25. **End**
26. $b \leftarrow \text{argmax}(Rev_{i,w}^{C(\sigma)})$; ▷ Find the optimal result
27. $c_{i,w}^{U(\sigma)} \leftarrow c_{i,w}^{wtp}$;
28. $Rev^{C(\sigma)} \leftarrow \sum_{w \in W} p_w \sum_{t \in T} Rev_{i,w}^{C(\sigma)}$;
29. **Return** $Rev_{i,w}^{C(\sigma)}, c_{i,w}^{U(\sigma)}$;
30. **End procedure**

of this search algorithm is to iteratively update the revenue from selling surplus PV electricity to consumer agents, whereby the consumer agents are ranked in descending order by their WTP. Finally, the uniform price that brings about the highest revenue to the coalition will be returned. Therefore, the optimal uniform price can be found by using Algorithm 2

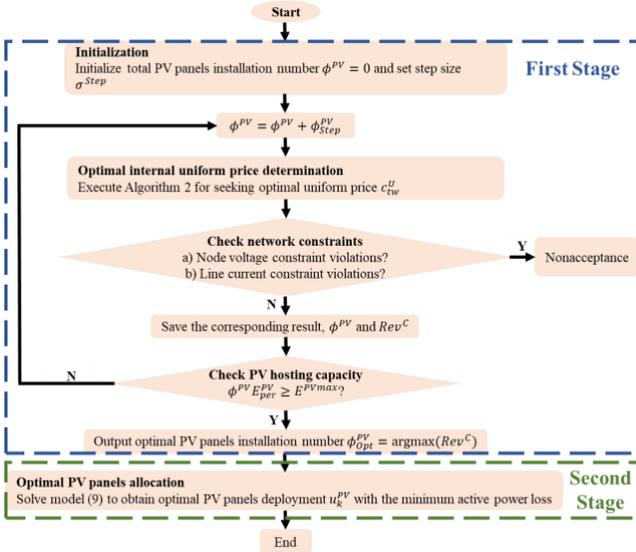


Fig. 2. Flowchart of our proposed two-stage game-theoretic residential PV panels planning framework

instead of directly solving the problem (8). Moreover, to ensure the reliability and security of the distribution grid during the energy sharing process, the feasibility voltage constraint and current constraint are checked by using the BFM. Our proposed descend search algorithm-based solution method is given as Algorithm 1. Besides, Fig. 2. is plotted to depict the flowchart of our proposed two-stage game-theoretic residential PV panels planning framework.

IV. STOCHASTIC OPTIMAL PV PANELS ALLOCATION IN THE COALITION OF PROSUMER AGENTS

After the first stage, the optimal installation capacity of residential PV panels for the coalition can be obtained. Then the second stage of our proposed two-stage planning framework concerns the optimal residential PV panels allocation among the prosumer agents of the coalition. At this stage, the power loss becomes the most important variable since it changes in terms of the allocation result of PV panels. The primary goal of the second-stage model is to minimize active power loss so that economic loss can be reduced. The stochastic optimal residential PV panels allocation model is as follows,

$$\min \sum_{w \in W} p_w \sum_{t \in T} \sum_{j \in N} r_{mj} l_{mjtw} \quad (16a)$$

over

$$\{P_{mktw}, Q_{mktw}, P_{mitw}, Q_{mitw}, v_{jtw}, l_{mjtw}, u_k^{PV}\}_{k \in N^{PV}, i \in N^C, t \in T, w \in W, j \in N}$$

s.t.

$$P_{mktw} - r_{mk} l_{mktw} + \eta^{PV} \gamma_{tw}^{PV} u_k^{PV} E_{size}^{PV} - \gamma_{tw}^L P_k^L = \sum_{n \in C_k} P_{kntw}$$

$$k \in N^{PV}, t \in T, w \in W \quad (16b)$$

$$P_{mitw} - r_{mi} l_{mitw} - \gamma_{tw}^L P_i^L = \sum_{n \in C_i} P_{intw} \quad i \in N^C, t \in T, w \in W \quad (16c)$$

$$Q_{mktw} - x_{mk} l_{mktw} - \gamma_{tw}^L Q_k^L = \sum_{n \in C_k} Q_{kntw} \quad k \in N^{PV}, t \in T, w \in W \quad (16d)$$

$$Q_{mitw} - x_{mi} l_{mitw} - \gamma_{tw}^L Q_i^L = \sum_{n \in C_i} Q_{intw} \quad i \in N^C, t \in T, w \in W \quad (16e)$$

$$v_{jtw} - v_{mtw} = 2(r_{mj} P_{mjtw} + x_{mj} Q_{mjtw}) - (r_{mj}^2 + x_{mj}^2) l_{mjtw} \quad j \in N, t \in T, w \in W \quad (16f)$$

$$\|(2P_{mjtw}, 2Q_{mjtw}, v_{jtw} - l_{mjtw})\|_2 \leq v_{jtw} + l_{mjtw} \quad j \in N, t \in T, w \in W \quad (16g)$$

$$v^{min} \leq v_{jtw} \leq v^{max} \quad j \in N, t \in T, w \in W \quad (16h)$$

$$0 \leq l_{mjtw} \leq l_{mj}^{max} \quad j \in N, t \in T, w \in W \quad (16i)$$

$$u_k^{PV} \leq u_k^{PVmax} \quad k \in N^{PV} \quad (16j)$$

$$\sum_{k \in N^{PV}} u_k^{PV} = \phi^{PV}, u_k^{PV} \in Z^+ \quad (16k)$$

where the objective function (16a) is to minimize the expected active power loss of all considered uncertainty scenarios. (16b) and (16c) describe the active power balance on the node with the prosumer agent and consumer agent, respectively. (16d) and (16e) describe the reactive power balance on each node. (16f) and (16f) are derived from (1c) and (2), respectively. (16h) is the voltage constraint and (16i) is the current constraint. (16j) guarantees that the residential PV panels installation number at each prosumer agent node cannot exceed its maximum allowable installation number, which is determined by the installation conditions. (16k) ensures that the total installation number of residential PV panels equals the optimal value obtained from the first stage.

V. NUMERICAL RESULTS

A. Implementation on IEEE 33-node Distribution System

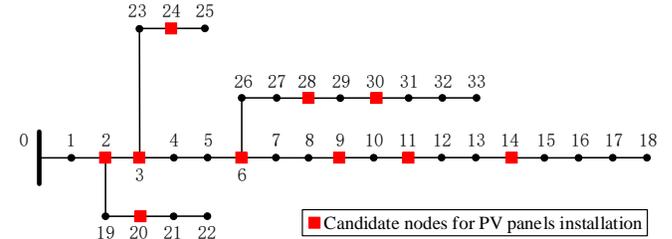


Fig. 3. IEEE 33-node distribution grid with candidate nodes for PV panels installation

In this section, we employ the modified IEEE 33-node distribution grid to test our proposed residential PV panels planning model. Detailed information on this test system can be referred to [34]. Fig. 4 depicts 125 (5*5*5) uncertainty scenarios, consisting of five load scenarios, five PV output scenarios and five electricity price scenarios. For the sake of simplicity, the electricity price \$c_{tw}^G\$ is assumed to be equal for all agents in the test system and it consists of the market clearing price \$c_{tw}^{MCP}\$, generation markup (0.013 \$/kWh) and delivery charge (0.036 \$/kWh) [35], i.e., \$c_{tw}^G = c_{tw}^{MCP} + 0.013 + 0.036\$ \$/kWh. (Source: Power Smart Pricing). Due to the characteristic of residential PV panels output, the length of the entire time period \$\mathcal{T}\$ is thirteen hours (6:00-18:00) and the duration of each time period is one hour. We randomly assume

the following weight factors to describe different consumer agent preferences on WTP: $w_i^E = [6.3, 8.2, -7.5, 8.3, 2.6, -8, -4.4, 0.9, 9.2, 9.3, -6.8, 9.4, 9.1, -0.3, 6, -7.2, -1.6, 8.3, 5.8, 9.2, 3.1, -9.3, 7]$ for consumer agents 1, 4, 5, 7, 8, 10, 12, 13, 15, 16, 17, 18, 19, 21, 22, 23, 25, 26, 27, 29, 31, 32 and 33, respectively. Note that these weight factors have significant effects on optimal residential PV panels planning results and in practice, they can be adjustable depending on the preferences of consumer agents on a case by case basis. Other parameters including greenhouse gas emission price, per residential PV panel size, and residential PV panel investment, maintenance and operation cost are summarized in Table I. Table II lists the optimal residential PV panels allocation result of our proposed two-stage game-theoretic model. All case studies are implemented by using MATLAB on a computer with an Intel Core i7 of 2.4 GHz and 12GB memory.

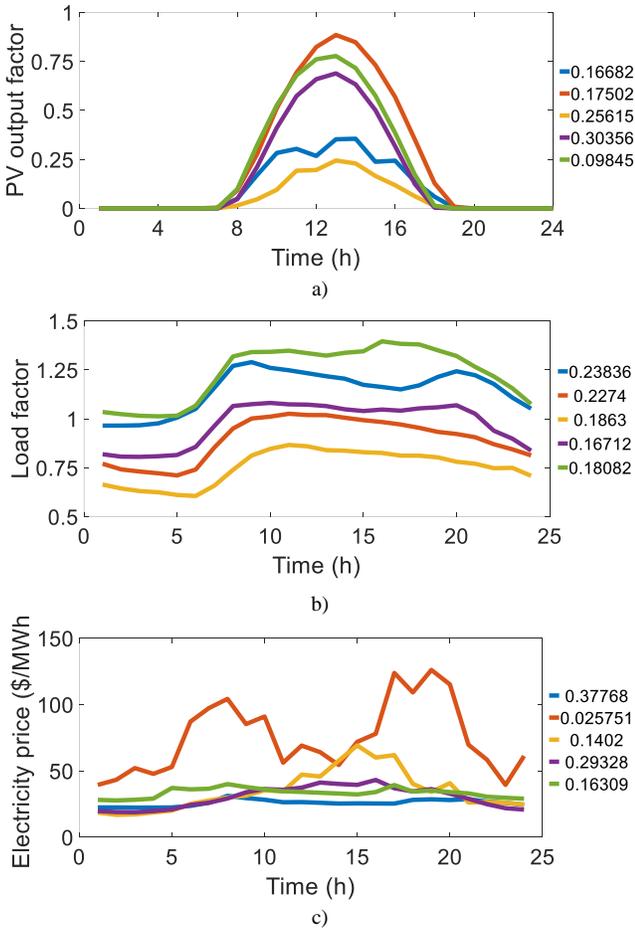


Fig. 4. Uncertainty scenarios of a) load factor γ_{tw}^L , b) PV output factor γ_{tw}^{PV} , and c) electricity price c_{tw}^E

TABLE I
RELATED PARAMETERS IN THE 33-NODE CASE

Parameter name	Parameter value
Greenhouse gas emission price	0.0125 \$/kWh [36]
Per residential PV panel size	200 kVA [37]
Residential PV panel planning cost	3.05 \$/W [37]
Residential PV panel output efficiency	95% [37]
Planning horizon	20 years
Depreciation rate of PV panels	5%

TABLE II
OPTIMAL RESIDENTIAL PV PANELS PLANNING RESULT IN 33-NODE SYSTEM

Candidate locations (nodes)	PV panels capacity (kVA)
2	148.4
3	129
6	259
9	269
11	63.2
14	165.6
20	151.8
24	218.6
28	240
30	255.4

1) From the Perspective of the Coalition

TABLE III
RESIDENTIAL PV PANELS PLANNING SIZE AND COST AND DAILY REVENUE WITH AND WITHOUT ENERGY SHARING IN 33-NODE TEST SYSTEM

Energy sharing mechanism	No	Yes
Residential PV panels planning size	6.4*10 ² kVA	1.9*10 ³ kVA
Residential PV panels planning cost	1.45*10 ⁶ \$	4.3*10 ⁶ \$
Daily revenue	32.45 \$	110.27 \$

In this subsection, we demonstrate the results from the perspective of the coalition. Table III lists comparisons on the PV panels planning size and cost as well as the daily revenue with and without energy sharing. We can see from this table that the PV panels installation capacity with energy sharing is more than that without consideration of energy sharing, so its corresponding PV panels planning cost is higher. This is because local consumer agents with good environmental awareness would like to purchase PV electricity from the coalition, so more PV panels will be invested. Besides, it also can be observed that daily revenue improves significantly after energy sharing between the coalition and consumer agents, which can bring approximately 77.8 \$ revenue improvement each day.

Fig. 5 illustrates the hourly revenue of the coalition in representative scenario 6 and scenario 12. As shown in this figure, hourly revenue is negative in the morning and becomes positive from noon. The main reason is that the residential PV panels only can provide a small amount of PV electricity in the morning, but the PV production is sufficient at midday. Fig. 6 depicts the dispatch of PV electricity provided by the coalition in representative scenario 4. It can be seen that the coalition sells more PV electricity to the consumer agents in each time period. The coalition allocates the PV electricity in a way to maximize its revenue, so it sells PV electricity differently to the utility grid and the consumer agents in each time period. According to the WTP equation (5), we can know that the wholesale market clearing price c_{tw}^{MCP} is always lower than the WTP c_{tw}^{WTP} , so the coalition prefers to sell its PV electricity firstly to the consumer agents and then to the utility grid. Fig. 7 shows the hourly optimal uniform price in representative scenario 15. It can be observed from this figure that the uniform price is high in time periods of non-peak PV generation (6:00-10:00 and 16:00-18:00) and relatively low in time periods of peak PV generation (10:00-16:00). This is because the PV electricity can be consumed by local consumers during

the daytime, the coalition sets low uniform prices so its PV electricity can be dispatched to more consumer agents. While in the morning or early night, the coalition would like to choose a high uniform price to maximize its revenue with low PV production.

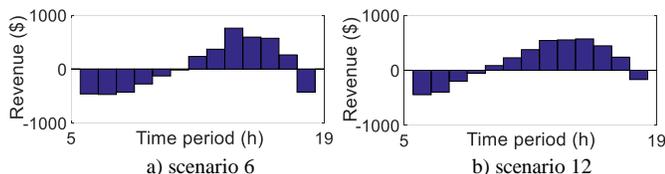


Fig. 5. Hourly revenue of the coalition in representative a) scenario 6 and b) scenario 12

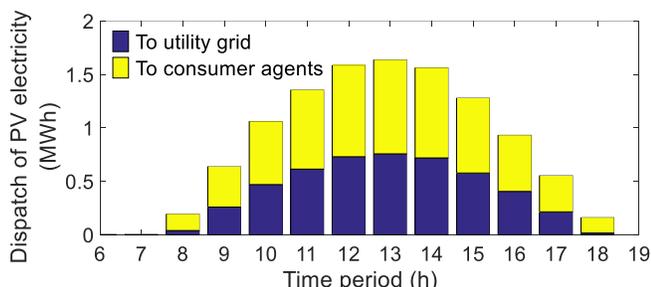


Fig. 6. PV generation sold to the consumer agents and sold to the utility grid in representative scenario 4

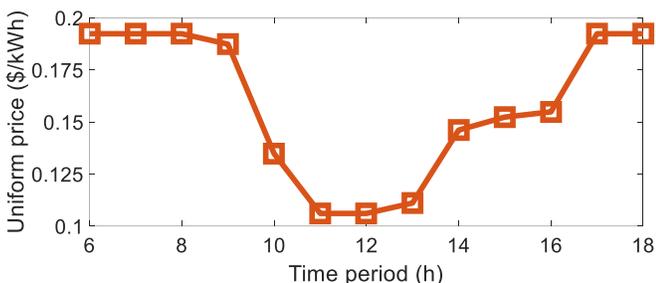


Fig. 7. Optimal uniform price of each time period in representative scenario 15

TABLE IV
COMPUTATION PERFORMANCE ON SOLVING PROPOSED BI-LEVEL ENERGY SHARING PROBLEM BY DIFFERENT METHODS CONSIDERING DIFFERENT NUMBER OF OPERATION SCENARIOS

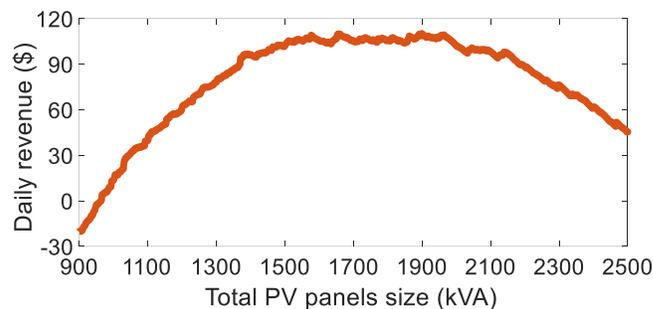
No. of operation scenarios	Computation time (sec)	
	MILP method	Proposed method
1 (1*1*1)	2.91	4.38
8 (2*2*2)	120.57	20.05
27 (3*3*3)	3961.12	67.66
64 (4*4*4)	-	160.38
125 (5*5*5)	-	313.25

To investigate the computation performance on our proposed solution method (see Algorithm 1 and Algorithm 2) for solving energy sharing problem (6), the conventional mixed-integer linear programming (MILP) method for solving our formulated MPEC problem (11) is used as the benchmark. Table IV lists the computation time of solving the proposed bi-level energy sharing problem by these two methods considering the different number of operation scenarios. As shown in this table, both two solution methods can be used to solve the proposed problem with low considered scenario numbers (e.g. 1, 8, 27) while our proposed method shows a clear advantage over the first

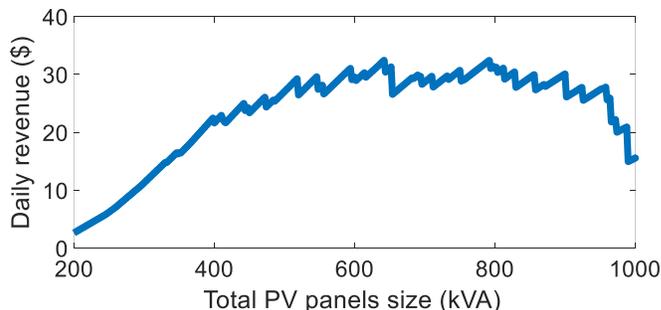
method. With high scenarios numbers (e.g. 64, 125), a huge computation burden may be involved so the commercial solver cannot handle the original problem. By contrast, our solution algorithm can solve the same problem with acceptable computation time. Note that the solution obtained by these two methods satisfies all conditions in each considered scenario, so the solution becomes more robust with the consideration of more operation scenarios [38]. In this regard, the robustness and reliability of the solution obtained by our proposed solution method can be more guaranteed since this method can take a large number of uncertainty scenarios into account.

2) The Tradeoff between Total Residential PV Panels Size and the Daily Coalition Revenue

In this subsection, a sensitivity analysis of the total residential PV panels size with the daily coalition revenue is performed. Fig. 8 depicts the tradeoff curve between the total residential PV panel size and the daily coalition revenue with and without energy sharing, respectively. With energy sharing, fast-growing daily revenue can be observed with increasing residential PV panels size until the investment capacity reaches about 1,900 kVA, then the daily revenue declines after more residential PV panels installation. While without energy sharing, the daily coalition revenue shows a rapid increase at first as increasing residential PV panels installation capacity, then it becomes fluctuant and finally performs a gradual decrease. This is because larger residential PV panels investment leads to high planning costs, which cannot be covered by the economic benefit of PV energy sharing.



a) with energy sharing mechanism



b) without energy sharing mechanism

Fig. 8. Tradeoff curve between the total PV panel size and the daily coalition revenue a) with and b) without energy sharing mechanism in the 33-node system.

B. Implementation on IEEE 123-node Distribution System

The IEEE 123-node distribution grid [39] is adopted as the large-scale test system to verify the proposed planning model

and solution algorithm. In this case, uncertainty scenarios including load factor, PV energy output factor, and electricity price are the same as those in the 33-node case. Other parameters including greenhouse gas emission price, per residential PV panel size, and residential PV panel investment, maintenance and operation cost are also the same as those in the 33-node case.

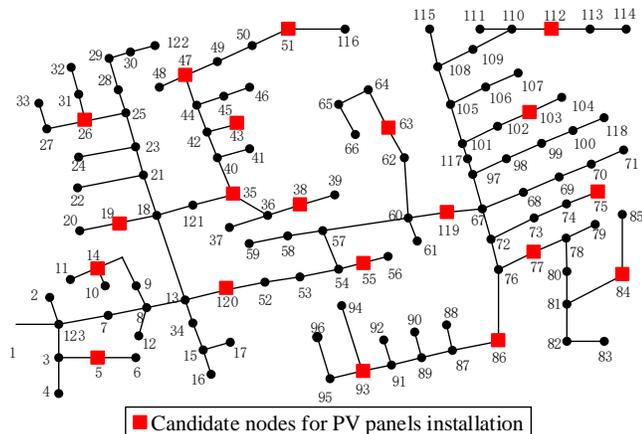


Fig. 9. IEEE 123-node distribution grid with candidate nodes for PV panels installation

TABLE V
OPTIMAL RESIDENTIAL PV PANELS PLANNING RESULT IN 123-NODE SYSTEM

Locations (nodes)	PV panels capacity (kVA)	Locations (nodes)	PV panels capacity (kVA)
5	298.67	63	279.10
14	97.37	75	64.01
19	663.21	77	386.75
26	394.31	84	241.06
35	87.65	86	634.42
38	222.52	93	444.33
43	67.30	103	305.76
47	96.56	112	70.35
51	139.54	119	15.68
55	1300.40	120	191.07

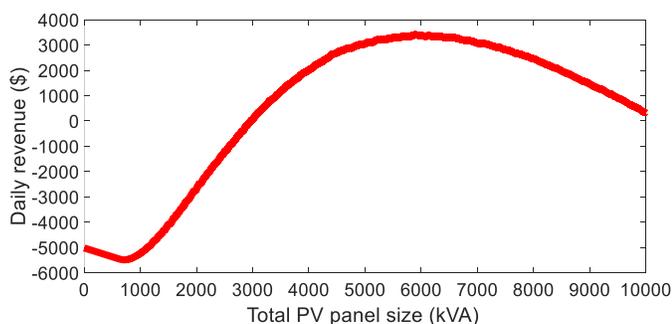


Fig. 10. Tradeoff curve between the total PV panel size and the daily coalition revenue with energy sharing in the 123-node system.

Fig. 9 depicts this modified test system by selecting twenty candidate nodes for PV panels installation, i.e., 5, 14, 19, 26, 35, 38, 43, 47, 51, 55, 63, 75, 77, 84, 86, 93, 103, 112, 119 and 120. Besides, random weight factors in the range of [-10,10] are assumed to define the consumer agent preferences on WTP. Residential PV panels planning result is listed in Table V. Fig. 10 is plotted to demonstrate the tradeoff between the total residential PV panel size and the daily coalition revenue with

the energy sharing mechanism. As shown in this figure, the daily revenue rises rapidly with the increase of the PV panel size. After the PV panel size reaches approximately 6,000 kVA, the daily revenue decreases with the increasing PV panel size.

VI. CONCLUSION

In this paper, we develop a novel two-stage game-theoretic framework of residential PV panels planning. In the first stage, Stackelberg game theory is used to model the stochastic bi-level energy sharing problem, which is solved by our proposed descend search algorithm-based solution method, so the optimal installation capacity of residential PV panels can be obtained. In the second stage, we develop a stochastic programming-based optimal OPF model to optimally allocate residential PV panels for all PV prosumers with minimum expected active power loss. Finally, the IEEE 33-node and 123-node test systems are used to demonstrate the effectiveness of our proposed framework. As shown in numerical results, with the consideration of energy sharing mechanism during the PV panels planning stage, not only economic benefits to PV prosumers can be improved but also the residential PV panels installation is facilitated.

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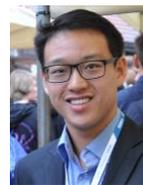
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