Household Tax Evasion*

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Abstract

Household members share public goods and make intra-household transfers. We show how these features of the household interact with the tax evasion decision, and identify the dimensions in which household evasion differs from individual evasion. In the model we present two members of a household choose how much to contribute to a household public good and how much self-employment income to evade. We are interested in how different evasion possibilities interact with the contribution decisions to the household public good and the role of income transfers within the household. We show the household evasion decision differs from the individual decision because it affects the outcome of the household contribution game. When household members are taxed as individuals neutrality applies when choices are not constrained. If the evasion level of one household member is constrained then an income transfer can generate a Pareto improvement. When the household members are jointly taxed there is a couple constraint on strategies and corner solutions can emerge.

1 Introduction

All the available evidence confirms that the shadow economy is significant in size. Elgin and Oztunali (2012) analyze data from 161 countries and estimate the mean size of the shadow economy to be 20 percent of GDP in OECD-EU countries, 43 percent in Sub-Saharan Africa, and 28 percent for the World. A similar level of magnitude is estimated by Medina and Schneider (2018), with an average size of the shadow economy for their set of 158 countries of 32 percent

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over the period 1991 to 2015. For individual countries, their estimates range from 62 percent for Bolivia down to Switzerland with 7 percent. The size of the shadow economy reveals how significant tax evasion is as an economic phenomenon and this is reflected in the very extensive literature that investigates the motivation behind the decision of an individual to evade tax. The initial model of expected utility maximization (Allingham and Sandmo, 1971) has been extended to include psychological costs (Gordon, 1989), the social setting (Myles and Naylor, 1996, Gamannossi degl'Innocenti and Rablen, 2020), and behavioral preferences (Hashimzade *et al.*, 2013, Piolatto and Rablen, 2017). Despite the large body of research, there has been no previous analysis of the evasion decision within a household setting. This is especially surprising given that the household has non-compliance options - especially the transfer of income between household members - that are not available to an individual. The household setting also raises interesting analytical issues due to the involvement of strategic interaction in the provision of household public goods.

The empirical literature on the determinants of tax evasion should necessarily be treated with caution due to the fact that evasion is an illegal activity with resulting data limitations.[1] Even so, the literature has consistently reported the significance of household structure. Both Clotfelter (1983) and Feinstein (1991) demonstrated that married taxpayers evade more than unmarried. This is only partial evidence since our argument is built on the difference in evasion opportunities between individuals living alone and those who have formed households - and therefore not marriage *per se*. To appreciate the more recent evidence, observe that in many tax systems the employed pay a withholding tax so have little or no opportunity to evade. To study evasion the focus is placed on the self-employed who are responsible for an annual income declaration and, hence, have the option of mis-reporting. Johansson (2015) showed that households with one self-employed member failed to declare 16.5 percent of income whereas households with two self-employed members failed to declare 42 percent of income.[2] Cabral *et al.* (2019) report the same finding concerning the effect of the number of self-employed in the household with a similar order of magnitude. This evidence clearly indicates that there is a household effect at work and one that merits investigation. Although there has been extensive experimental research on evasion, we are not aware of any work that explores household effects on tax evasion since all of the experimental evidence is based on experiments that have involved individual decision making.

There are three significant reasons why the study of the household is important for understanding tax evasion. First, the household has a greater range of non-compliance options than the individual. An option for the household, but not for the individual, is the transfer of income between members to optimize the allocation of evasion. Income can be transferred directly but, for the purposes of tax evasion, it is transfers via the creation of nominal partnerships or

[1] The level of evasion is usually taken to be the level of undeclared income revealed in audit, but audits are well known to be be imperfect, see Feinstein (1991).

[2] This is not just a consequence of having two self-employed who can misreport since the amount of evasion more than doubles.

employment that are important. Suppose that one member of the household is in receipt of income that is not subject to third-party reporting. Typically, such income arises from the operation of a private business, with the business owner being responsible for declaring the income to the revenue service. This creates the opportunity for under-reporting. An individual can do no more than choose the optimal level of evasion. A member of a household has a second option: they can make the other household member a nominal partner in the business, share the business income, and then both under-report. The second option allows the household to achieve a better balance of risk across the two members but also impacts on the household contribution game. If the two members of the household face different marginal tax rates then the benefits of income transfers are more obvious.[3] The member facing the higher rate can make the other a partner in the business, which lowers the average tax rate and permits evasion, or offer them nominal employment which only lowers the average tax rate. If the employment is truly nominal, then it can also be viewed as an act of non-compliance involving statement of false information.

Second, by their nature, households are defined by the shared provision and consumption of household public goods. The tax evasion decision of each member of the household interacts with the process for providing these public goods. An act of evasion that is not detected by the revenue service permits a greater contribution to the household public good. Conversely, detected evasion reduces what can be provided. The evasion decision therefore has implications that span the household so from the household perspective it becomes more than just a pair of separate individual decisions.

Thirdly, personal tax systems differ in whether they impose separate taxation or joint taxation of households and the structure of taxation will affect the evasion decisions of the household members. Separate taxation involves each household member being treated as an individual so that income transfers within the household will directly affect individual tax liabilities. Joint taxation can involve full or limited income splitting. For example, in France and in Germany the incomes of spouses are added, the sum is divided by two, and the tax function for a single individual is applied to each half. Thus, the total tax bill (after taking children into account in France) is twice the tax bill for the average income of the spouses. In the United States spouses may choose to file jointly or separately: according to IRS guidance, a joint return may attract higher tax reliefs and a lower total tax; depending on the level of earnings and on the difference in spouses' earnings, so married couples may face a "bonus" or a "penalty" when filing jointly. The two tax treatments of the household imply different outcomes when the tax system is progressive. We assume a linear tax system for simplicity, and focus instead on the consequence of the allocation of liability when evasion is detected.

A central division in the household decision literature is between cooperative and non-cooperative models of the household. We focus upon the non-

[3]Shortage of space prevents a treatment of this case in the present paper. The issue will be explored in a later paper.

cooperative approach in this paper and plan to consider the cooperative model in subsequent work. The analysis of the paper embeds the tax evasion decision within a model of household public good allocation (Apps and Rees, 2009). Since income transfers are central to the evasion strategy, the analysis also draws on neutrality results from the literature on the private provision of public goods (Bergstrom *et al.*, 1986, Itaya *et al.* 1997, 2002, and Warr 1983). The main message of the analysis is that the neutrality results have strong implications for the evasion strategy and that household evasion behavior is markedly differently to individual behavior.

Section 2 describes the model we use and discusses alternative assumptions concerning the choice process. Section 3 analyses the evasion decision by a non-cooperative household when there is separate taxation. Section 4 considers the implications of joint taxation. Several extensions of the basic model are reviewed in section 5. Section 6 provides a discussion of the policy implications of the analysis and concludes the paper. Proofs are given in the appendix.

2 Household Decisions

The intention of the model is to capture the household evasion decision, and how this interacts with the provision of a household public good. We adopt the standard assumption that a household consists of two members who make individual decisions while benefiting jointly from a public good. Unlike much of the literature we make no attempt to assign roles or labels to the two household members. We also assume that the household members have the same preferences. A potential explanation for the experimental finding of differences in male and female compliance behavior is variation of risk aversion across genders. If males are, on average, less risk averse than females, then the difference in risk aversion is an additional explanation for why the behavior of a household may differ from that of two individuals acting independently. We choose not to make any distinctions between the two members of our household for two reasons. First, to focus attention on the role of the basic opportunities that distinguish a household from two individuals. Second, to reflect the fact that historical gender roles are increasingly irrelevant within households. Since we assume no such differences our results identify only the consequences of strategic interaction within the household without conflating these with the effects of heterogeneity of household members.

A household consists of two members, labeled by $j = 1, 2$. The gross income of member j is given by Y^j where

$$Y^j = Y_e^j + Y_s^j, \tag{1}$$

with

$$Y_e^j \geq 0, \quad Y_s^j \geq 0. \tag{2}$$

Y_e^j is employment income and is known by the revenue service. It is not possible to be non-compliant with respect to this income, either because it is subject to third-party reporting and so any false report will be audited with certainty, or because it is subject to a withholding tax. Y_s^j denotes self-employment income. This income is unobserved by the revenue service and the taxpayer is responsible for filing a report. It is therefore possible to make a false declaration of self-employment income to the revenue service.

The utility of household member j is derived from consumption of a private good in quantity x^j and a household public good in total quantity G. The public good is the sum of contributions from the two household members, so

$$G = g^1 + g^2. \tag{3}$$

Preferences over $\{x^j, G\}$ are represented by the utility function

$$U^j = U\left(x^j, G\right). \tag{4}$$

Three assumptions are placed on the structure of the utility function. Assumption 1 is standard strict concavity but allowing for potential separability in utility between private and public good, and implies that each household member is strictly risk averse. The second assumption is an Inada condition that is used to rule out the possibility of zero or negative consumption. Without this assumption, it is possible that a household member may choose a level of evasion that forces negative consumption in the state in which they are caught evading. The role of this assumption becomes clearer following the discussion of commitment below. Assumption 3 is required to determine the responses of the household members to changes in income levels. The first part is decreasing absolute risk aversion which is typically assumed in the analysis of tax evasion to ensure that the level of evasion increases with income (see Yitzhaki, 1974). The second part is an equivalent restriction for the cross-derivative between public and private good. The assumption is satisfied, for example, if there is separability between public and private good.[4] It is stronger than the standard normality assumption used in the literature on the private provision of public goods (Bergstrom *et al.* 1986, Faias *et al.* 2020) but is required to maintain predictable comparative statics in the presence of the risk introduced by tax evasion.[5]

Assumption 1: $U_{xx} < 0, U_{GG} < 0, U_{xG} \geq 0, U_{xx}U_{GG} - U_{xG}U_{xG} > 0$.

Assumption 2: $\forall G, \lim_{x^j \to 0} U_x = +\infty$.

Assumption 3: For any pair of income levels $\bar{Y}$and $\hat{Y}$ with $\bar{Y} > \hat{Y}$, and any level of public good provision $G > 0$:

$$\left(-\frac{U_{xx}\left(\bar{Y}, G\right)}{U_x\left(\bar{Y}, G\right)}\right) - \left(-\frac{U_{xx}\left(\hat{Y}, G\right)}{U_x\left(\hat{Y}, G\right)}\right) < 0,$$

[4]Section 4.2 uses the utility function $U = \ln(x) + \ln(G)$.

[5]Normality requires $0 < \frac{U_{xx} - U_{xG}}{U_{xx} - 2U_{xG} + U_{GG}} < 1$, which is ensured by assumption 1. Expressed as a differential, the second part of assumption 3 requires $\frac{\partial}{\partial Y}\left(-\frac{U_{xG}}{U_x}\right) < 0$ or $U_{xG}U_{xx} - U_xU_{xxG} < 0$. This is not implied by assumption 1.

$$\left(-\frac{U_{xG}\left(\bar{Y},G\right)}{U_x\left(\bar{Y},G\right)}\right)-\left(-\frac{U_{xG}\left(\hat{Y},G\right)}{U_x\left(\hat{Y},G\right)}\right)\leq 0.$$

The possibility of audit and punishment implies the net income (after payment of tax and a fine if audited) of a household member who evades is a random variable at the time the evasion decision is made. In the standard model of individual evasion behavior this randomness has led the focus to be placed on the details of decision-making with risk or uncertainty. The household setting with contribution to a public good creates an additional implication of the randomness. How this further aspect is resolved is a key component in the description of the non-cooperative game between household members. To see what is involved, consider a household member who chooses to evade and is subsequently audited and fined. The question that has to be answered is how planned consumption and contribution to the public good are adjusted to take account of the lower-than-expected net income. This matters directly for the solution of the non-cooperative game because the two household members are linked via provision of the public good.

The question can be answered in three ways that differ in the assumption made about commitment. The first possibility is to assume that a commitment is made to the contribution to the public good so that all randomness is captured in the level of private consumption. This assumption is the most convenient for solving the model and the closest analog to the analysis of individual evasion. Restricting randomness to private consumption ensures the level of public good is deterministic which simplifies the analysis of strategic interaction. The second possibility is to assume the level of private consumption is committed so that contribution to the public good becomes random. In this case the strategic interaction can be modeled as a contribution game in which the "type" of each household member is unknown when decisions are made. The two types in the game will be distinguished by the level of public good provision. The third alternative is to assume that neither private consumption nor public good are committed. This situation could be modeled using a three-stage game in which an evasion decision is made at the first stage, "nature" selects whether or not to audit at the second stage, and consumption and contribution are determined at the third stage.

The approach we take is to focus on the case in which the public good contribution is committed and private consumption is random. This case is the most analytically tractable so makes the central results clearer, and it provides the closest parallel with the individual evasion model. It also has some justification in terms of household organization and dynamics, since failure to deliver promised contributions is likely to lead to household dissolution.[6] With commitment, some choices of public good and evasion will imply negative consumption

[6] The model is static so all events happen simultaneously. If a time element was added it would seem natural to assume auditing to occur after public good had been made thus supporting the commitment assumption. As noted by a referee, this argument is reinforced if a time endowment is divided between paid work and home production of a public good.

of the private good if caught. The imposition of the Inada condition ensures that this outcome is avoided.

3 Individual Taxation

This section analyzes the compliance decision when the household members are taxed as individuals and have individual responsibility for correct payment of taxes. We first characterize the equilibrium in the absence of intra-household transfers and then analyze the incentive for transfers.

The income of household member j, after tax at rate t and a possible fine at rate f on evaded tax, is a random variable, $\tilde{Y}^j$, where

$$\tilde{Y}^j = \begin{cases} Y^{j,c} = (1-t)\,Y^j - tfe^j, \text{ w/probability } p, \\ Y^{j,n} = (1-t)\,Y^j + te^j, \text{ w/probability } 1-p. \end{cases} \tag{5}$$

It is assumed that a commitment is made by j to provide a level of public good g^j. Consequently, the level of private consumption is a random variable, $\tilde{x}^j$, determined as the residual,

$$\tilde{x}^j = \begin{cases} x^{j,c} = Y^{j,c} - g^j, \text{ w/probability } p, \\ x^{j,n} = Y^{j,n} - g^j, \text{ w/probability } 1-p. \end{cases} \tag{6}$$

Under these assumptions the level of expected utility of member j is

$$\mathcal{E}U^j\left(\tilde{x}^j, G\right) = pU\left(Y^{j,c} - g^j, G\right) + (1-p)\,U\left(Y^{j,n} - g^j, G\right). \tag{7}$$

The decision problem is to choose the levels of evasion and public good provision taking as given the public good contribution of the other member

$$\max_{\{e^j, g^j\}} \mathcal{E}U^j\left(\tilde{x}^j, G\right) \quad \text{given } g^i,\ i \neq j, \tag{8}$$

subject to the constraints

$$\begin{aligned} g^j &\geq 0, \\ e^j &\geq 0, \\ Y_s^j &\geq e^j. \end{aligned} \tag{9}$$

The first result determines the necessary and sufficient condition for a member of the household to choose to evade if they have a strictly positive amount of self-employment income.

Lemma 1 *If $Y_s^j > 0$ then the optimal choice $\hat{e}^j > 0$ if $p < \frac{1}{1+f}$.*

The result given in lemma 1 is identical to the condition for non-compliance to take place in the standard individual evasion model.[7] Two observations are

Auditing will only occur after the period of work (and home production) is completed so time in home production is necessarily committed.

[7] It would also be modified in the same way as the standard analysis (see Hashimzade *et al.*, 2013, for details) if behavioral preferences involving probability transformations were introduced. Further potential extensions are discussed in section 5.

worth making about this condition. First, it does not depend on the level of public good provision. So, no matter what are the levels of income and public good provision, the condition for evasion to occur is unchanged. Second, the same condition applies to both household members so the incentive to begin evading is not dependent on the distribution of income within the household.

The next result explores the effect of an intra-household transfer when both household members have income from self-employment, both are choosing to evade, and the upper limit on evasion is not binding. This is the key neutrality result that provides an explanation for the later results. It applies only when the income difference between household members is not so great as to constrain one of the members either in public good provision or in evasion. Denote the income levels in the absence of any transfer by $\left\{\hat{Y}_e^j, \hat{Y}_s^j\right\}$ and the associated optimal choices by $\left\{\hat{e}^j, \hat{g}^j\right\}$. Similarly, the income levels and optimal choices if a transfer takes place are $\left\{\bar{Y}_e^j, \bar{Y}_s^j\right\}$ and $\left\{\bar{e}^j, \bar{g}^j\right\}$ respectively. Without loss of generality, we choose $\bar{Y}_e^1 + \bar{Y}_s^1 = \hat{Y}_e^1 + \hat{Y}_s^1 - \Delta y$ and $\bar{Y}_e^2 + \bar{Y}_s^2 = \hat{Y}_e^2 + \hat{Y}_s^2 + \Delta y$. Using this notation we can state the central neutrality result.

Theorem 1 *If* $\hat{Y}_s^j > \hat{e}^j > 0$*, and* $\hat{g}^j > 0$, $j = 1, 2$, *then:*
i) $\hat{Y}^{1,c} - \hat{g}^1 = \hat{Y}^{2,c} - \hat{g}^2$, $\hat{Y}^{1,n} - \hat{g}^1 = \hat{Y}^{2,n} - \hat{g}^2$*, and* $\hat{e}^1 = \hat{e}^2$;
ii) $\hat{g}^1 - \hat{g}^2 = (\hat{Y}_e^1 + \hat{Y}_s^1) - (\hat{Y}_e^2 + \hat{Y}_s^2)$;
iii) $\bar{g}^1 = \hat{g}^1 - \Delta y$, $\bar{g}^2 = \hat{g}^2 + \Delta y$*, and* $\bar{e}^1 = \hat{e}^1$, $\bar{e}^2 = \hat{e}^2$.

Parts (i) and (ii) of Theorem 1 are an extension of Itaya *et al.* (1997) and show that the difference in income levels is identical to the difference in contributions to the public good and the private consumption levels are the same for both household members. The extension here is that the evasion levels are also identical. Part (iii) of the theorem captures the effect of a transfer and is an extension of the standard neutrality result of Warr (1983) and Bergstrom *et al.* (1986) that a transfer of income is met by an offsetting change in contribution to the public good (see also Faias *et al.*, 2020). It should be noted that the theorem requires the transfer to be sufficiently small so that the level of income from self-employment in the presence of a transfer is sufficiently high to allow the level of evasion in the absence of the transfer to be a feasible choice. Other than this restriction, it does not matter whether the transfer is made from employment income or self-employment income. The effect of the transfer on evasion is more surprising: the levels of evasion by the two household members are identical with and without the transfer. Consequently, when the conditions of the theorem apply the process of public good provision within the household makes the level of evasion independent of individual incomes. This outcome arises because provision to the public good equalizes the incomes of the two household members net of contribution, so they are in an equal position after taking contribution into account and make the same "gamble" on tax evasion.[8] This ensures that the risk from evasion is shared equally between the two members of the household so that the private contribution game induces perfect risk sharing in the household.

[8]Or, following the terminology of Cowell and Gordon (1988), the same investment in the risky evasion asset.

The first corollary of Theorem 1 describes the effect on choices of an increase in self-employment income for one of the household members. The proof of this result uses standard comparative statics analysis.

Corollary 1 *Let the choices* $\{\hat{e}^j, \hat{g}^j\}$ *satisfy the conditions of Theorem 1 given incomes* $\left\{\hat{Y}_e^j, \hat{Y}_s^j\right\}$*. For incomes* $\left\{\breve{Y}_e^j, \breve{Y}_s^j\right\}$ *where* $\breve{Y}_s^1 > \hat{Y}_s^1$*,* $\breve{Y}_s^2 = \hat{Y}_s^2$*,* $\breve{Y}_e^1 = \hat{Y}_e^1$*, and* $\breve{Y}_e^2 = \hat{Y}_e^2$*, the resulting choices* $\{\breve{e}^j, \breve{g}^j\}$ *are such that* $\breve{e}^j > \hat{e}^j$*,* $\breve{g}^j > \hat{g}^j$ *for* $j = 1, 2$*, and* $\breve{e}^1 = \breve{e}^2$*.*

This corollary shows that an increase in self-employment income for *either* member of the household increases the level of evasion by *both* members of the household. The intuition for this follows from recalling that the discussion of theorem 1 observed that contribution to the public good equalizes the net-of-contribution incomes of the two household members. Consequently, an increase in the self-employment income of one household member is effectively an increase in the net-of-contribution income for both. The amount gambled on the risky evasion asset is then increased by both household members because of decreasing absolute risk aversion. It should again be emphasized that this requires the evasion constraint to be non-binding for the household member with the lower level of self-employment income. The fact that a household member engages in greater evasion despite no change in own-income level has significant implications for audit strategy. Many revenue services select audit targets by using predictive analytics. The result shows that the predictions will be improved by the inclusion of household factors in the modeling. Using only individual variables will not pick up the intra-household effects identified in the corollary.

The next corollary determines the effect of an increase in the employment income of one member of the household.

Corollary 2 *Let choices* $\{\hat{e}^j, \hat{g}^j\}$ *satisfy the conditions of theorem 1 given incomes* $\left\{\hat{Y}_e^j, \hat{Y}_s^j\right\}$*. For incomes* $\left\{\breve{Y}_e^j, \breve{Y}_s^j\right\}$ *where* $\breve{Y}_e^1 > \hat{Y}_e^1$*,* $\breve{Y}_e^2 = \hat{Y}_e^2$*,* $\breve{Y}_s^1 = \hat{Y}_s^1$ *and* $\breve{Y}_s^2 = \hat{Y}_s^2$*, the resulting choices* $\{\breve{e}^j, \breve{g}^j\}$ *are such that* $\breve{e}^j > \hat{e}^j$*,* $\breve{g}^j > \hat{g}^j$ *for* $j = 1, 2$*, and* $\breve{e}^1 = \breve{e}^2$*.*

The content of the corollary is that an increase in employment income for one household member will increase the level of evasion for both. The intuition mirrors that for corollary 1: neither household member faces a binding constraint on evasion (some self-employment income is declared) so the increase in net-of-contribution income due to the additional employment income can still be met with an increase in the evasion gamble. The important policy observation is that it is not just self-employment income that matters for evasion. The receipt of self-employment income makes it possible to evade, but it is total household income that determines the extent of evasion. This is because household public good provision effectively results in income pooling when both household members are at an interior optimum so the source of an income increase does not matter for behavior.

Theorem 1 holds when neither household member is at a corner solution in the choice of evasion level or public good contribution, so each must have sufficient self-employment income and total income. When neutrality applies an intra-household transfer that leaves both members at an interior solution does not change consumption levels, public good provision, or the levels of evasion. In contrast, an intra-household transfer will have an impact when one of the household members would otherwise be constrained. We now consider constrained choices. To make the discussion relevant for tax evasion we assume that mechanisms exist through which a transfer be made in employment income or in self-employment income. For example, the transfer can take the form of employment income if one household member provides nominal employment for the other, and it can be self-employment income if the other household member is engaged as a nominal business partner. In both cases the transfer is engineered to appear as a justifiable income flow. The form of transfer matters for subsequent changes in behavior. A transfer of self-employment income can relax both the public good contribution constraint and the evasion constraint, whereas a transfer of employment income can relax only the public good constraint.

In the private provision of public good model Itaya *et al.* (1997) show that with two or more potential contributors a transfer can raise social welfare when a potential contributor to the public good is at a corner solution. Cornes and Sandler (2000) show that a transfer can be a Pareto improvement if there are three or more potential contributors. Theorem 2 below shows that if one household member is constrained with respect to the evasion choice because of an insufficiency of self-employment income then there are circumstances in which an intra-household transfer can lead to a Pareto-improvement. The intuition for the result is that the constrained household member responds to the increase in self-employment income by increasing evasion and contributing more to the household public good. Since the member receiving the transfer is initially constrained and evasion is a gamble with a positive expected payoff, the marginal relaxation of the constraint due to additional income can induce an increase in contribution to the public good that is sufficiently large to offset the reduction in contribution and private consumption of the member providing the transfer. This result is the first to show how a transfer with two potential contributors can create a Pareto improvement. If the sufficient condition of the theorem is satisfied then one household member will willingly make a transfer of income to the other - and will enjoy a utility increase from so doing. It should be observed that this applies even though both face the same marginal tax rate - it is the asymmetry between the constrained and the unconstrained that permits the theorem to hold.

Theorem 2 *If* $\bar{y}_s^1 > \hat{e}^1$, $\bar{y}_s^2 = \hat{e}^2 < \hat{e}^1$, $\hat{g}^1 > 0$ *and* $\hat{g}^2 \geq 0$ *then a transfer* s *from 1 to 2 such that* $\hat{y}_s^1 = \bar{y}_s^1 - s > \hat{e}^1$, $\hat{y}_s^2 = \bar{y}_s^2 + s > \hat{e}^2$ *and* $\hat{g}^1 > s$ *is a Pareto improvement when* $\frac{\partial g^1}{\partial s} > -(1-t)$ *and* $\frac{\partial g^2}{\partial s} > (1-t)$.

The results of this section have shown that with independent taxation the fundamental neutrality result in the private provision of public goods carries over to the model with tax evasion provided both household members are unconstrained at the Nash equilibrium. Furthermore, the two members choose the same level of tax evasion even with differences in income levels. More surprisingly, an increase in either employment income or self-employment income of one member will increase the evasion of both members. When income is transferred to a household member who is constrained in the evasion choice it is possible that a Pareto improvement arises. These results demonstrate that the tax evasion behavior of the household is significantly different to that of the individual.

4 Joint Taxation

Joint taxation of households occurs when the tax liability depends on the incomes of both households members. There are a variety of systems in operation for joint taxation. In Germany, for example, the incomes of spouses are summed and each spouse pays tax, calculated as for a single taxpayer, on half of the total income. This is called *full income splitting.* As a result, under a progressive tax system the lower-earning spouse may end up facing a higher marginal tax rate, and the higher-earning spouse a lower marginal tax rate, than they would if taxed separately. The total tax burden, however, is never higher than the sum of the tax burdens of two individuals with the same earnings, and can be substantially lower (see Bach *et al*, 2013). There are other countries in which income splitting is limited, with only certain parts of income being summed. In the United States spouses can choose whether to file jointly or separately, and the total tax burden for a married couple filing jointly may exceed (a "marriage penalty") or fall short (a "marriage bonus") of the sum of tax burdens under separate filing. To measure the consequences of these rules, Bick and Fuchs-Schündeln (2017) use the difference between the average tax rate of a married woman (since in their sample women are typically the secondary earners) and the average tax rate of a single-earner household as the measure of the "degree of jointness".

A second significant aspect of the tax system is the assignment of responsibility for undeclared income. In principle, it is possible to have individual liability so that the responsibility falls upon the household member whose income has been discovered to be falsely reported. However, in practice, joint responsibility is more common so that either spouse can be made liable for paying the unpaid tax and the penalty. According to the IRS guidance[9] "*both spouses on a married filing jointly return are generally held responsible for all the tax due even if one spouse earned all the income or claimed improper deductions or credits.*" However, in certain circumstances only one spouse can be held responsible for underpayment. In particular, this is the case the other spouse can prove that he or she was unaware of the fraud.

The analysis we now present concentrates on the case of full income splitting. In principle, limited income splitting could be investigated using the same

[9] www.irs.gov/taxtopics/tc205

methods. We briefly consider the case of individual responsibility for undeclared income but place a greater focus upon joint responsibility. Joint responsibility raises additional analytical questions so merits a more detailed treatment.

4.1 Individual responsibility

The assumptions in this section are that the household is taxed jointly under a system of full income splitting but there is individual responsibility if undeclared income is discovered. Consequently, each household member pays tax on half of the total household income but is penalized only on his or her own undeclared income.

Under these assumptions the level of expected utility of household member j is

$$\mathcal{E}U^j\left(\tilde{x}^j, G\right) = pU\left(Y^{j,c} - g^j, G\right) + (1-p)\,U\left(Y^{j,n} - g^j, G\right), \tag{10}$$

where full income splitting and individual responsibility imply the income levels in the two states are

$$Y^{j,c} = Y^j - \frac{t}{2}\left(Y^j + Y^i\right) - tfe^j + \frac{t}{2}\left(e^i - e^j\right), \tag{11}$$

$$Y^{j,n} = Y^j - \frac{t}{2}\left(Y^j + Y^i\right) + \frac{t}{2}\left(e^j + ei\right). \tag{12}$$

It can be seen from (11) and (12) that there is now strategic interaction through both the public good and the level of evasion. Both of these have a positive externality on the other player.

The additional dimension of strategic interaction changes the equilibrium of the game but does not alter the conclusion that there will be neutrality in the level of public good with respect to an income transfer when the two household members are unconstrained. Furthermore, the public good provision implies, via the necessary conditions for the choice of strategies, that the two household members evade the same amount when unconstrained. Denote the income levels in the absence of a transfer by $\left\{\hat{Y}_e^j, \hat{Y}_s^j\right\}$ and the associated optimal choices by $\left\{\hat{e}^j, \hat{g}^j\right\}$. Similarly, the income levels and optimal choices if a transfer takes place are denoted by $\left\{\bar{Y}_e^j, \bar{Y}_s^j\right\}$ and $\left\{\bar{e}^j, \bar{g}^j\right\}$ respectively. Without loss of generality, we choose $\bar{Y}_e^1 + \bar{Y}_s^1 = \hat{Y}_e^1 + \hat{Y}_s^1 - \Delta y$ and $\bar{Y}_e^2 + \bar{Y}_s^2 = \hat{Y}_e^2 + \hat{Y}_s^2 + \Delta y$. Using this notation we can state a neutrality result identical to theorem 1.

Theorem 3 *If* $\hat{Y}_s^j > \hat{e}^j > 0$, *and* $\hat{g}^j > 0$, $j = 1, 2$, *then:*
i) $\hat{Y}^{1,c} - \hat{g}^1 = \hat{Y}^{2,c} - \hat{g}^2$, $\hat{Y}^{1,n} - \hat{g}^1 = \hat{Y}^{2,n} - \hat{g}^2$, *and* $\hat{e}^1 = \hat{e}^2$;
ii) $\hat{g}^1 - \hat{g}^2 = (\hat{Y}_e^1 + \hat{Y}_s^1) - (\hat{Y}_e^2 + \hat{Y}_s^2)$;
iii) $\bar{g}^1 = \hat{g}^1 - \Delta y$, $\bar{g}^2 = \hat{g}^2 + \Delta y$, *and* $\bar{e}^1 = \hat{e}^1$, $\bar{e}^2 = \hat{e}^2$.

The neutrality result implies that the results of the previous section concerning the cases with constraints can also be proved using the slight modifications of the previous arguments. The additional feature is that there is a direct positive externality of the evasion choice. This makes it more likely that a Pareto improvement can be established.

4.2 Joint responsibility

Joint responsibility makes both parties liable for the payment of the fine on undeclared income. Consequently, a pair of equilibrium strategies for the household contribution game must ensure that the two members make sufficient fine payments to meet the revenue service demand. This places an additional constraint upon the strategy choices and requires the use of an extended definition of equilibrium for the game.

It is assumed that each member chooses a strategy $\{e^i, g^i, z^i\}$, where z^i is the amount contributed to the payment of fine when caught evading. The income level when not caught, an event which occurs with probability $1-p$, is

$$Y^{i,n} = (1-t)Y^i + te^i, \tag{13}$$

and the income when caught, which occurs with probability p, is

$$Y^{i,c} = (1-t)Y^i + te^i - z^i. \tag{14}$$

Four points need to be noted. First, the probability with joint responsibility is not directly comparable to that with individual responsibility because it is the probability of one or both members of the household being detected. Second, these income levels differ from those in the individual responsibility case because of the operation of joint responsibility. To illustrate the second point, consider the extreme case of household member j paying the entire fine after evasion by member i. Then the evader, i, will gain from evasion whatever the state, so the benefit, te^i, appears in both income levels. Correspondingly, since the fine is paid by the other household member, the punishment arising from detection for i, z^i, will be zero. Third, the income levels in (13) and (14) are based on the standard assumption that, when an audit takes place, all evaded income is detected. Finally, joint responsibility introduces risk pooling within the household since both members make a contribution to any fines that are accrued. The extent to which the risk is pooled (i.e. the relative values of $z^1$and z^2) is determined endogenously in the equilibrium of the game.

To match the definitions of net income, the pair of strategies $\{e^i, g^i, z^i\}$, $i = 1, 2$, must satisfy the constraint

$$z^1 + z^2 - t(1+f)\left(e^1 + e^2\right) \geq 0, \tag{15}$$

which ensures that aggregate household payments match the total of fines that need to be paid. Equation (15) is termed a *couple constraint* and the non-cooperative household plays a *couple constrained game*. The Nash equilibria of the game can be characterized using the theorem of Rosen (1965) which is stated in the appendix.

Using the Rosen theorem, we can place the evasion model in the notation of the couple-constrained game as follows. The objective functions are

$$\varphi^i = pU^i\left(Y^{i,c} - g^i - z^i, g^1 + g^2\right) + (1-p)\,U^i\left(Y^{i,n} - g^i, g^1 + g^2\right) \;\; i = 1, 2,$$

and the constraints are

$$\begin{aligned}
h^1 &= (1-t)Y^1 + te^1 - g^1 - z^1 \geq 0,\ h^2 = (1-t)Y^2 + te^2 - g^2 - z^2 \geq 0, \\
h^3 &= e^1 \geq 0,\ h^4 = e^2 \geq 0, \\
h^5 &= g^1 \geq 0,\ h^6 = g^2 \geq 0, \\
h^7 &= z^1 \geq 0,\ h^8 = z^2 \geq 0, \\
h^9 &= Y_s^1 - e^1 \geq 0,\ h^{10} = Y_s^2 - e^2 \geq 0, \\
h^{11} &= z^1 + z^2 - t(1+f)e^1 - t(1+f)e^2 \geq 0.
\end{aligned}$$

The Lagrange multiplier corresponding to constraint i is denoted λ_i.

Before investigating the general solution it is informative to consider the case of logarithmic utility

$$U^j(x,G) = \ln(x) + \ln(G),\ \theta > 0. \tag{16}$$

Assume that equilibrium of the game has $e^i > 0, g^i > 0, z^i > 0$, $i = 1, 2$, so that all the constraints other than h^{11} are slack and multipliers λ_1 to λ_{10} are 0. In this case, the necessary conditions for the choice of g^1 and g^2 (see (41) and (44) in the appendix) imply

$$\frac{p}{(1-t)Y^1 - g^1 + te^1 - z^1} + \frac{1-p}{(1-t)Y^1 - g^1 + te^1} = \frac{1}{g^1+g^2}, \tag{17}$$

$$\frac{p}{(1-t)Y^2 - g^2 + te^2 - z^2} + \frac{1-p}{(1-t)Y^2 - g^2 + te^2} = \frac{1}{g^1+g^2}. \tag{18}$$

Substituting (17) and (18) into the necessary conditions for the choice of $e^1$and e^2 (see (40) and (43) in the appendix) gives

$$\lambda_{11} = \frac{r_1}{1+f}\frac{1}{g^1+g^2}, \tag{19}$$

$$\lambda_{11} = \frac{r_2}{1+f}\frac{1}{g^1+g^2}. \tag{20}$$

In the definition of the couple-constrained equilibrium (and the Rosen theorem characterizing the equilibrium) r_1 and r_2 are pre-determined constants that are not derived as part of the equilibrium. The interpretation is that they determine the allocation of effort to meet the couple constraint across the two players. Observe that the two solutions (19) and (20) for λ_{11} are inconsistent whenever $r_1 \neq r_2$. This implies that it is only possible to have an interior solution for all the choice variables of both household members if the constants r_1 and r_2 are equal. In any other case, one or more of the choice variables must be zero for one of the household members.

Given this observation, consider the case of $r_1 = r_2$. The necessary conditions cannot be solved for the individual choices but can be solved for the aggregate outcome at the household level. Letting $Y = Y^1 + Y^2$, the solution is given by

$$G = \frac{(1-t)}{3}Y, \tag{21}$$

and

$$E=\frac{2\left(1-t\right)\left[1-p\left(1+f\right)\right]}{3ft}Y, \tag{22}$$

where $E=e^{1}+e^{2}$. It can be seen that this solution necessarily satisfies neutrality because it is aggregate income that is the determinant of aggregate choices. A transfer will have no effect on this equilibrium provided it does not cause any of the constraints to bind. When $r_1>r_2$ the solution generalizes to

$$G=\frac{r_1}{2r_1+r_2}(1-t)Y, \tag{23}$$

$$E=\frac{r_1+r_2}{2r_1+r_2}\frac{\left(1-t\right)\left[1-p\left(1+f\right)\right]}{ft}Y, \tag{24}$$

so neutrality remains satisfied. The extent of risk pooling in equilibrium can be observed from the fine payments

$$z^{i}=\frac{r_i}{2r_1+r_2}\frac{\left(1-t\right)\left[1-p(1+f)\right]\left(1+f\right)}{f}Y,\ \ i=1,2.$$

Both contribute to the fine but in proportion to the weights that are placed on satisfying the couple constraint.

The potential number of solutions for the couple-constrained equilibrium[10] restricts the general results that can be demonstrated once the assumption of a specific utility function is dropped. Two results can be given. The first demonstrates a neutrality theorem for the joint responsibility case provided that both household members contribute to the public good. What the result does not do is characterize when both will contribute. As the log utility example has illustrated this will require special conditions to apply. The second result demonstrates that the necessary condition required for an interior equilibrium with log utility function ($r^1=r^2$) holds for the couple-constrained equilibrium generally when utility is separable in private and public good. This supports the inference from the log utility case that an interior solution, and hence neutrality, rarely arises in the couple-constrained case.

To demonstrate the first result, denote the income levels in the absence of a transfer by $\left\{\hat{Y}_e^j,\hat{Y}_s^j\right\}$ and the associated optimal choices by $\left\{\hat{e}^j,\hat{g}^j,\hat{z}^j\right\}$. Similarly, the income levels and optimal choices if there is a transfer are $\left\{\bar{Y}_e^j,\bar{Y}_s^j\right\}$ and $\left\{\bar{e}^j,\bar{g}^j,\bar{z}^j\right\}$ respectively. Without loss of generality, we choose $\bar{Y}_e^1+\bar{Y}_s^1=\hat{Y}_e^1+\hat{Y}_s^1-\Delta y$ and $\bar{Y}_e^2+\bar{Y}_s^2=\hat{Y}_e^2+\hat{Y}_s^2+\Delta y$. It should be noted that the theorem requires only that public good contributions are positive. Both payment toward the fine and evasion can be constrained at 0.

Theorem 4 *The equilibrium is neutral with respect to the transfer when $\hat{g}^1>0$ and $\hat{g}^2>0$. In particular, $\bar{g}^1=\hat{g}^1-\Delta y$ and $\bar{g}^2=\hat{g}^2+\Delta y$, $\bar{e}^j=\hat{e}^j$, $\bar{z}^j=\hat{z}^j$, $j=1,2$ is the equilibrium for incomes $\left\{\bar{Y}_e^j,\bar{Y}_s^j\right\}$.*

[10]Potentially, any combination of the 11 constraint may be binding so there 2^{11}combinations that have to be checked for each scenario.

The second result shows the condition required for both household members to contribute to the public good when utility is separable between consumption and public good. It should be noticed that the result does not need to assume that both household members make a positive contribution toward meeting the punishment when evasion is discovered.

Theorem 5 *If* $U_G^{1,k} = U_G^{2,k} = \varphi(G)$ *for* $k = c, n$ *and all* x^1, x^2, *and* G, *the equilibrium is an interior solution* $\left(x^j > 0,\ Y_s^j > e^j > 0,\ g^j > 0,\ j = 1, 2\right)$ *for strategies if, and only if,* $r^1 = r^2$.

Joint responsibility for tax evasion makes both household members liable for any fines and leads to results that are significantly different to those obtained with individual responsibility. The couple constraint binds the strategies of the two household members and introduces linearity into the analysis that generates corner solutions in situations where the individual model would have interior solutions. The use of Rosen's concept of the couple-constrained equilibrium has introduced additional variables, r^1 and r^2 as part of the equilibrium definition. In the present context r^i can be interpreted as the measuring the extent to which member i is excused from contributing toward the punishment (recall that $\frac{\lambda_{11}}{r^i}$ is the shadow price of the punishment constraint for i). As a positive theory of household behavior the introduction of the r^i leaves something to be desired since a key element of the solution becomes external to the analysis.

5 Extensions

The results above have been derived in the most basic setting compatible with the aims of the analysis. The results have demonstrated when neutrality applies and the effect of income increases and income transfers. In this section we explore the implications of applying some extensions of the individual tax evasion model to the household setting.

5.1 Unregistered income

The main analysis has assumed that the household can potentially have income from employment - which is reported to the revenue service by the employer - and income from self-employment - which has to be reported by the taxpayer. What we did not do was to probe deeper into the nature of self-employment. The form of self-employment in the model could be interpreted equally as either a registered firm operating in the formal sector or an unregistered firm operating in an informal sector. What it does not describe is the situation in which the household makes a choice between these two options. We now sketch how our model can be extended to include the choice of sector in which to operate.

The key characteristic of registration in the context of our model is that the revenue service is aware of the existence of the firm and, consequently, expects to receive a tax return from the firm. In contrast, an unregistered firm is unknown and no return is expected. The revenue service will only obtain tax information from an unregistered firm through a process of discovery. This suggests extending the model by assuming that the probability of being audited is higher for a registered firm than for an unregistered firm. Correspondingly, the fine for an unregistered firm if discovered can be assumed to be higher to capture additional punishment for operating as unregistered. Since a firm cannot be both registered and unregistered, the household faces a discrete choice between types.

Formally, let p^r be the probability of detection for a registered firm and p^u the probability for a firm that is unregistered, where $p^r > p^u$. Similarly, the corresponding rates of fine are denoted $f^r$and f^u, with $f^u > f^r$. Considering the case of independent taxation, we can then re-state (5) as

$$\tilde{Y}^{ji} = \begin{cases} Y^{j,c_i} = (1-t)\, Y^j - tf^i e^j, \text{ w/probability } p^i, \\ Y^{j,n_i} = (1-t)\, Y^j + te^j, \text{ w/probability } 1-p^i,\ i = r, u. \end{cases} \tag{25}$$

For each $i = r, u$ the optimization can be extended to $\max_{\{e^{ji}, g^{ji}\}} \mathcal{E}U^{ji}\left(\tilde{x}^{ji}, G\right)$, and the Nash equilibrium determined. Denote the value of utility at the Nash equilibrium with firm type i by V^{ji}. Household member j then contrasts V^{jr} to V^{ju}, and supports r or u according to which is the larger. If both household members prefer r to u, or vice versa, then the choice is clear. In contrast, if there is a division of opinion then a tie-breaking rule, such as a coin toss or a decisive household member, can be adopted to determine the outcome. This process determines the equilibrium choices for the household and the nature of self-employment.

Conditional on the choice to operate as registered or unregistered, the results already established continue to apply. Hence, whether the firm is registered or unregistered, there will be neutrality to income transfers when both household members are at an interior solution and additional income will increase the level of evasion of both household members. What can be changed is the incentive of the household to establish a firm in the unregistered sector compared to the choice of two individuals. Forming a household raises the total income of both members (because of the economy in public good provision). The impact of this can be analyzed by viewing the choice of registered or unregistered as the choice between two lotteries with different probabilities and different payoffs. Forming a household will make operation of an unregistered firm more likely if the increase in real income leads to a preference for a lottery with a lower probability of detection but a higher fine when detected.

5.2 Evasion costs

A major theme of the tax evasion literature has been to make the models consistent with the empirical observation that many taxpayers choose to declare honestly. The model in this paper shares the common feature of many others that there is a single sufficient condition (see lemma 1) that determines if evasion will take place. This sufficient condition is the same for all households, so when it is satisfied all changes take place at the *intensive* margin. Only if evaders and non-evaders exist side-by-side, and with potential movement between the two groups, can there be any changes at the *extensive* margin. The literature has introduced several different ways in which extensions can be made to change this, and our household model can be extended to incorporate all of these.

An extensive margin has been motivated by the psychological costs of evasion (Gordon, 1989), the existence of a social norm (Kim, 2003, Myles and Naylor, 1996, Traxler, 2010), and by administrative costs (Alm, 1988, Lin and Yang, 2001). The common feature of all these approaches is that there is an additional cost when choosing to evade over and above the fine if evasion is discovered. Denote this cost by $C(e^1, e^2, z^j)$, where z^j is a vector of parameters that can include individual characteristics of j (such as the personal psychological cost or evaluation of the social custom) and social characteristics (such as the number of other evaders). In the household setting, the cost includes the level of evasion of both household members since, for example, one member can bear a psychological cost when the other member evades. The level of expected utility if evasion takes place (again, illustrated for independent taxation) is then

$$\begin{aligned}\mathcal{E}U^j\left(\tilde{x}^j, G\right) &= pU\left(Y^{j,c} - g^j, G\right) + (1-p)\,U\left(Y^{j,n} - g^j, G\right) \\ &\quad - C(e^j, e^i, z^j),\ i, j = 1, 2, i \neq j. \end{aligned} \tag{26}$$

If j chooses not to evade, the level of utility is

$$U^j = U\left(Y^j - g^j, G\right) - C(0, e^i, z^j)\, i, j = 1, 2, i \neq j. \tag{27}$$

The decision making within the non-cooperative household can be represented as a two-stage game. At the first stage, the two household members simultaneously choose either "Evade" or "Not Evade". If Evade is chosen, then the subsequent payoff is determined by (26) and if Not Evade then the payoff is determined by (27). Each household member who chooses Evade then makes a second move where the level of evasion is chosen. The game can be solved by backward induction to determine the equilibrium. This may involve none, one, or two household members evading.

The introduction of the additional costs into the household decision problem can affect the incentive for the household to make transfers. To see this, consider the case in which the two household members have identical characteristics. If the costs of evasion are convex in evasion levels, then there is an additional motivation for transfers of income within the household in order to reach the best division of costs between the members. Alternatively, if the characteristics differ (say one member bears a higher psychological cost of own evasion), then there is motivation for transferring income to the lower-cost person to engage in more evasion and provide more of the public good. In both cases, the consideration of additional costs highlights the role of transfers within the household and the fact that the household setting will make a difference. In addition, costs can also have an impact on the extensive margin in these sense that two taxpayers who did not evade when single may choose two evade after forming a household (and *vice versa*). The direction of this effect is dependent on how individual costs aggregate into household costs.

5.3 Labor supply

In common with much of the literature on individual tax evasion the analysis of previous sections has been based on the assumption that incomes, from employment and self-employment, were fixed. There are several contributions in which this assumption has been relaxed by introducing a labor supply decision into the model. As shown by Pencavel (1979) variable labor supply implies the comparative statics effects of parameter changes cannot be unambiguously signed in contrast to the situation with fixed income. Andersen (1977) assumes separability of consumption and labor, and demonstrates that an increase in the tax rate, holding utility constant, reduces labor supply but increases the level of evaded income. A further extension has been to consider the allocation of labor supply between a registered market and an unregistered market. Isachen and Strom (1980) assume separability to show an increase in the tax rate reduces the proportion of time spent in the official labor market and increases tax evasion. Cowell (1985) reviews the results that are obtained under different sets of assumptions. Goerke (2005) uses this framework to demonstrate that a switch from payroll to income tax may not be neutral with evasion.

The discussion in section 5.1 of the choice whether to operate the firm in a registered market or an unregistered market can be applied directly to model a discrete choice of labor supply to one or the other. With labor supply that can be varied between markets, our model of employment and self-employment is very similar to the analysis of labor supply to regulated/unregulated markets. This can be seen by assuming that there is an available labor time which we normalize at 1 unit. This unit of time is divided between labor time in employment, ℓ_e^j, labor time self-employment, ℓ_s^j, and leisure, ℓ^j, with the constraint

$$\ell^j + \ell_e^j + \ell_s^j = 1. \tag{28}$$

Then, defining employment and self-employment wages w_e and w_s respectively, income from employment is $Y_e^j = w_e \ell_e^j$ and income from self-employment is $Y_s^j = w_s \ell_s^j$,so

$$Y^j = w_e \ell_e^j + w_s \ell_s^j. \tag{29}$$

Given this, $\widetilde{Y}^j$ remains as defined in (5). The level of expected utility of member j extended to include leisure is

$$\mathcal{E}U^j \left(\tilde{x}^j, G\right) = pU\left(Y^{j,c} - g^j, G, \ell^j\right) + (1-p)\, U\left(Y^{j,n} - g^j, G, \ell^j\right). \tag{30}$$

The optimization in (8) can then be amended to include the time allocation $\left\{\ell^j, \ell_e^j, \ell_s^j\right\}$ as an additional choice variable and the constraint set revised to include (28) and $\ell^j \geq 0, \ell_e^j \geq 0, \ell_s^j \geq 0$.

This extended model will still satisfy neutrality of transfers if both household members are at an interior solution. Holding the time allocation fixed, the neutrality argument applies directly as before. But then there is no reason to for labor supply to change in response to a transfer, so the neutrality extends. Notice that this neutrality result does not depend on either ℓ_e^j or ℓ_s^j being strictly positive - it holds if all labor time is in employment or all labor time is in self-employment. The only requirement is that the level of evasion is not constrained by the income from self-employment. The analysis of the impact of income transfers when evasion is constrained for one household member has to take into account the impact of labor supply adjustments. As noted in the literature cited above, the competing income and substitution effects prevent a clear extension of the earlier results without restricting the structure of utility.

The addition of variable labor supply to the model makes it interesting to reconsider what is implied by the household using nominal employment as a device to transfer income. In such a case, a distinction would need to be made between the actual time that a household member spends in self-employment, ℓ_s^j, and the nominal time, ℓ_n^j. The actual time is bound by the constraint (28) but the nominal time is not, so if this device is used $\ell_n^j > \ell_s^j$ and $\ell^j + \ell_e^j + \ell_n^j > 1$ for the household member receiving a transfer and the converse apply for the member making the transfer. A choice of transfer can then be modeled by making ℓ_n^j the variable that determines the size of the transfer, taking into account that the actual income from self-employment must equal the nominal income

$$w_s \left[\ell_s^1 + \ell_s^2\right] = w_s \left[\ell_n^1 + \ell_n^2\right].$$

5.4 Samuelson Rule Provision

The role of private provision in determining the results we have described can be emphasized by considering an alternative model of public good provision within the household. The analysis of the cooperative household merits a fuller treatment but the intermediate case of Samuelson rule provision with non-cooperative evasion is worth brief investigation.

Assume that the level of public good is chosen by the household in an efficient manner (meaning that provision satisfies the Samuelson rule) and that there is a known cost-share arrangement in place. After the level of public good is determined the household members choose their personal level of evasion in a non-cooperative way. Denote the cost-share of household member j by s^j. For household member j the level of evasion maximizes

$$\mathcal{E}U^j\left(\tilde{x}^j, G\right) = pU\left(Y^{j,c} - s^j G, G\right) + (1-p)\,U\left(Y^{j,n} - s^j G, G\right). \qquad (31)$$

Express the chosen level of evasion as $e^j = e^j(G)$. A level of provision that satisfies the Samuelson rule for public good provision can be derived by choosing G to maximize the utility of household member 1 subject to member 2 reaching a pre-assigned utility level:

$$max_{\{G\}} pU\left((1-t)\,Y^1 - tfe^1(G) - s^1 G, G\right)$$

$$+\,(1-p)\,U\left((1-t)\,Y^1 + te^1(G) - s^1 G, G\right)$$

subject to

$$pU\left((1-t)\,Y^2 - tfe^2(G) - s^2 G, G\right)$$

$$+ (1-p)\, U\left((1-t)\, Y^2 + te^2(G) - s^2G, G\right) \geq \bar{U}^2$$

Two points can be noted. First, the envelope condition applies to the constrained optimization so that the impact of public good provision on evasion does not enter the Samuelson rule. Second, the choice of evasion levels reduces to two independent choice problems so that the strategic interaction is eliminated.

Consequently, although the quantity of public good provision will affect the extent of evasion, this model reduces effectively to two independent optimizations for evasion choice. Hence, the outcome will have the properties of the standard individual evasion model. This demonstrates the significance of non-cooperative private provision for generating the neutrality results described in previous sections.

6 Conclusions

The extensive literature on tax evasion has modeled the decision problem of an individual taxpayer. This overlooks the fact that many taxpayers make decisions within a household setting and that tax systems (in some countries) use household income as the tax base. This paper makes a first step to addressing this omission by setting the tax evasion decision within the context of a household that shares a public good. This adds two additional elements to the analysis: the public good links the evasion decisions of the two household members and joint liability links the responsibility for detected evasion. These elements have significant impacts upon household behavior.

The case of independent taxation leads to an analysis similar to the standard model of the non-cooperative household. When the equilibrium evasion levels and public good provision levels are unconstrained then an extended form of neutrality to income transfers applies in which neither evasion or public good provision are affected. Each member of the unconstrained household chooses the same level of evasion even if there are differences in employment and self-employment incomes. Furthermore, an increase in either form of income for either household member will raise the evasion level of both. When one of the household members is constrained, there are circumstances in which a transfer of income from the other can raise the utility level of both.

Joint taxation significantly changes the nature of the game played by the household. The joint responsibility to meet the punishment levied after evasion is detected creates a couple constraint that must be satisfied by equilibrium strategy choices. As usual, neutrality applies when both household members contribute to the public good, but the nature of the couple-constrained equilibrium is that corner solutions that remove neutrality emerge in cases for which individual taxation would have interior solutions. When one member of the household is constrained at a zero evasion level their choice will not be affected by small changes in income - a result that does not apply to the individual problem.

The results of this paper have several notable policy implications. Our central theme has been that the household can transfer income between members by creating a nominal partnership to conduct business, or through nominal employment, in a way that enhances evasion opportunities. The results have shown that the transfer of income can increase the expected utility of the household even when there are no differences in marginal tax rates and that the total amount of evasion by the two members of the household increases. Consequently, the revenue service should be aware that a household is likely to evade more than two identical, but separate, individuals. This impacts on the selection of audit targets using predictive analytics. The empirical evidence supports the contention that households with two self-employed members evade proportionately more than households with one self-employed, and our theory explains why. Belonging to a household should be a positive indicator in a predictive model and should raise the level of hidden income an audit is expected to discover. Continuing this theme, any tax audit that captures evasion by one member of a household should be extended to the second since the theory supports a presumption that the second household member will also be evading.

The results also support the argument that the revenue service should look carefully at the economic justification for a business partnership formed by two members of same household. The formation of a business partnership that involves no real economic contribution by one partner is the practical interpretation of the income transfers in the theoretical analysis. In this regard, it is illuminating to draw a parallel with the policy treatment of tax avoidance. The UK revenue service (HM Revenue and Customs) introduced legislation in 2013 to permit the penalization of *abusive tax avoidance* which was defined as activities which had no reasonable economic rationale but could only be justified by pursuit of a reduction in tax liability. The same concept is at work in the present case when the formation of a partnership takes place only for reasons of tax evasion, and a very similar test of "reasonableness" could be applied to determine the economic justification for a partnership.[11] We have analyzed a uniform tax system but these arguments apply even more strongly when the two household members face different marginal rates. The difference in this case is that the employment of the low-rate taxpayer by the high-rate taxpayer in a nominal job can reduce the total tax liability. This provision of nominal employment can also fall under the interpretation of aggressive tax avoidance. Hence, employment of one household member by another should be given the same scrutiny as the formation of partnerships.

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Appendix

Proof of lemma 1 The optimal choice will be $\hat{e}^j = 0$ if the derivative of (7) with respect to e^j is negative when evaluated at $e^j = 0$. Hence, $\hat{e}^j = 0$ if

$$-tfpU_x\left(Y^{j,c} - g^j, G\right) + (1-p)\,tU_x\left(Y^{j,n} - g^j, G\right) < 0$$

Since $Y^{j,c} = Y^{j,n}$ when $\hat{e}^j = 0$, this condition reduces to

$$\frac{pf}{1-p} < 1,$$

which can be re-arranged to give the condition in the statement.

Proof of theorem 1 To prove (i) observe that at incomes $\hat{Y}_e^j$, $\hat{Y}_s^j$ the optimal choices $\hat{e}^j$, $\hat{g}^j$ satisfy

$$-fpU_x\left(\hat{Y}^{j,c}-\hat{g}^j,\hat{g}^1+\hat{g}^2\right)+(1-p)\,U_x\left(\hat{Y}^{j,n}-\hat{g}^j,\hat{g}^1+\hat{g}^2\right)=0, \tag{32}$$

$$-pU_x\left(\hat{Y}^{j,c}-\hat{g}^j,\hat{g}^1+\hat{g}^2\right)-(1-p)\,U_x\left(\hat{Y}^{j,n}-\hat{g}^j,\hat{g}^1+\hat{g}^2\right)$$
$$+pU_G\left(\hat{Y}^{j,c}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)+(1-p)\,U_G\left(\hat{Y}^{j,n}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)=0. \tag{33}$$

For any pair $\left\{\hat{g}^1,\hat{g}^2\right\}$, assumption 1 implies there is a unique solution $\hat{Y}^{j,c}-\hat{g}^j$, $\hat{Y}^{j,n}-\hat{g}^j$ to (32) and (33). This solution is independent of j, proving the first part of (i). Since $\hat{Y}^{1,c}-\hat{g}^1=\hat{Y}^{2,c}-\hat{g}^2$, it follows that $\hat{g}^1=\hat{Y}^{1,c}-\hat{Y}^{2,c}+\hat{g}^2$. Substituting into $\hat{Y}^{1,n}-\hat{g}^1=\hat{Y}^{2,n}-\hat{g}^2$ gives $\hat{e}^1=\hat{e}^2$.

(ii) follows directly from the fact that $\hat{e}^1=\hat{e}^2$.

After the transfer the choices satisfy

$$-fpU_x\left(\bar{Y}^{1,c}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)+(1-p)\,U_x\left(\bar{Y}^{1,n}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)=0, \tag{34}$$

$$-pU_x\left(\bar{Y}^{1,c}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)-(1-p)\,U_x\left(\bar{Y}^{1,n}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)$$
$$+pU_G\left(\bar{Y}^{1,c}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)+(1-p)\,U_G\left(\bar{Y}^{1,n}-\hat{g}^1,\hat{g}^1+\hat{g}^2\right)=0. \tag{35}$$

Setting $\bar{Y}^{1,c}=\hat{Y}^{1,c}-\Delta y$, $\bar{Y}^{1,n}=\hat{Y}^{1,n}-\Delta y$ (which imply $\bar{e}^1=\hat{e}^1$), $\bar{g}^2=\hat{g}^1-\Delta y$, and $\bar{g}^2=\hat{g}^2+\Delta y$, gives

$$\bar{Y}^{1,c}-\bar{g}^1=(\hat{Y}^{1,c}-\Delta y)-\left(\hat{g}^1-\Delta y\right)=\hat{Y}^{1,c}-\hat{g}^1,$$

and

$$\bar{g}^1+\bar{g}^2=(\hat{g}^1-\Delta y)+(\hat{g}^2+\Delta y)=\hat{g}^1+\hat{g}^2.$$

Comparison of (34-35) to (32-33) and the necessary conditions for 2 completes the proof of (iii).

Proof of corollary 1 The proof is a standard application of comparative statics using assumption 1 and 2 to evaluate the responses. The details are provided in the supplementary materials.

Proof of corollary 2 Repeat the argument of corollary 1 with the subscripts e and s interchanged.

Proof of theorem 2 To abbreviate the notation, define

$$U_x^{j,s}\equiv\frac{\partial U\left(x^{j,s},G\right)}{\partial x^{j,s}},U_G^{j,s}\equiv\frac{\partial U\left(x^{j,s},G\right)}{\partial G},\ s=c,n.$$

The effect of the transfer on the expected utility of household member 1 (using the envelope theorem for e^1 and g^1) can then be written

$$\begin{aligned}d\mathcal{E}U^1 \quad &= \quad -pU_x^{1,c}\,(1-t)\,ds+pU_G^{1,c}\frac{\partial g^2}{\partial s}ds \\ &\quad\ -(1-p)\,U_x^{1,n}\,(1-t)\,ds+(1-p)\,U_G^{1,n}\frac{\partial g^2}{\partial s}ds.\end{aligned} \tag{36}$$

The necessary condition for the choice of $g^1$is

$$-pU_x^{1,c} + pU_G^{1,c} - (1-p)\,U_x^{1,n} + (1-p)\,U_G^{1,n} = 0. \tag{37}$$

Substituting from (37) into (36) gives

$$d\mathcal{E}U^1 = \left[pU_x^{1,c} + (1-p)\,U_x^{1,n}\right]\left[\frac{\partial g^2}{\partial s} - (1-t)\right] ds.$$

Hence, the necessary and sufficient condition for ${}^{d\mathcal{E}U^1}/_{ds} > 0$ is

$$\frac{\partial g^2}{\partial s} > (1-t)\,. \tag{38}$$

The corresponding calculation for 2 is different because 2 is initially constrained with e. We have

$$\begin{aligned} d\mathcal{E}U^2 \quad = \quad & pU_x^{2,c}\,(1-t)\,ds + pU_G^{2,c}\frac{\partial g^1}{\partial s}ds \\ & + (1-p)\,U_x^{2,n}\,(1-t)\,ds + (1-p)\,U_G^{2,n}\frac{\partial g^1}{\partial s}ds \\ & - tfpU_x^{2,c}\frac{\partial e^2}{\partial s} + t(1-p)U_x^{2,n}\frac{\partial e^2}{\partial s} \end{aligned}$$

If e is not constrained after transfer then the envelope condition gives

$$-tfpU_x^{2,c} + t(1-p)U_x^{2,n} = 0.$$

If e remains constrained after the transfer then

$$-tfpU_x^{2,c} + t(1-p)U_x^{2,n} > 0.$$

Since $\frac{\partial e^2}{\partial s} > 0$. it follows that

$$-tfpU_x^{2,c}\frac{\partial e^2}{\partial s} + t(1-p)U_x^{2,n}\frac{\partial e^2}{\partial s} \geq 0.$$

For 2 the necessary condition for choice of g^2 is

$$-pU_x^{2,c} + pU_G^{2,c} - (1-p)\,U_x^{2,n} + (1-p)\,U_G^{2,n} = 0.$$

So the sufficient condition for ${}^{d\mathcal{E}U^2}/_{ds} > 0$ is

$$\frac{\partial g^1}{\partial s} > -(1-t) \tag{39}$$

Conditions (38) and (39) are sufficient for the Pareto improvement.

Proof of theorem 3 The proof of part (i) repeats that of theorem 1 using the necessary conditions for $j = 1, 2$

$$
\begin{aligned}
0 &= -pU_x\left(\hat{Y}^{j,c} - \hat{g}^j, \hat{G}\right)\left(tf + \frac{t}{2}\right) + (1-p)\,U_x\left(\hat{Y}^{j,n} - \hat{g}^j, \hat{G}\right)\frac{t}{2}, \\
0 &= -pU_x\left(\hat{Y}^{j,c} - \hat{g}^j, \hat{G}\right) + pU_G\left(\hat{Y}^{j,c} - \hat{g}^j, \hat{G}\right) - (1-p)\,U_x\left(\hat{Y}^{j,n} - \hat{g}^j, \hat{G}\right) \\
&\quad + (1-p)\,U_G\left(\hat{Y}^{j,n} - \hat{g}^j, \hat{G}\right),
\end{aligned}
$$

where $\hat{G} = \hat{g}^1 + \hat{g}^2$. To establish that $\hat{e}^1 = \hat{e}^2$ use $\hat{Y}^{1,c} - \hat{g}^1 = \hat{Y}^{2,c} - \hat{g}^2$ and $\hat{Y}^{1,n} - \hat{g}^1 = \hat{Y}^{2,n} - \hat{g}^2$ to write

$$
\begin{aligned}
& \hat{Y}^j - \frac{t}{2}\left(\hat{Y}^j + \hat{Y}^i\right) - tf\hat{e}^j + \frac{t}{2}\left(\hat{e}^i - \hat{e}^j\right) - \hat{g}^j \\
=\; & \hat{Y}^i - \frac{t}{2}\left(\hat{Y}^j + \hat{Y}^i\right) - tf\hat{e}^i + \frac{t}{2}\left(\hat{e}^j - \hat{e}^i\right) - \hat{g}^i, \\
& \hat{Y}^j - \frac{t}{2}\left(\hat{Y}^j + \hat{Y}^i\right) + \frac{t}{2}\left(\hat{e}^j + \hat{e}i\right) - \hat{g}^j \\
=\; & \hat{Y}^i - \frac{t}{2}\left(\hat{Y}^j + \hat{Y}^i\right) + \frac{t}{2}\left(\hat{e}^j + \hat{e}^i\right) - \hat{g}^i.
\end{aligned}
$$

These conditions prove the result by reducing to

$$tf\hat{e}^i + t\hat{e}^i = tf\hat{e}^j + t\hat{e}^j.$$

Part (ii) follows directly, and part (iii) has the same proof as part (iii) of theorem 1.

Rosen Theorem To state the equilibrium characterization for a general couple constrained game, denote the strategy of player i by $\mathbf{x}^i = \left\{x_1^i, ..., x_{n_i}^i\right\}$, the payoff function by $\varphi^i\left(\mathbf{x}^1, \mathbf{x}^2\right)$, and the m constraints on the strategies by $h^j\left(\mathbf{x}^1, \mathbf{x}^2\right) \geq 0$. The Nash equilibria of the coupled-constrained game are characterized in the following theorem.

Theorem 6 *(Rosen, 1965) For positive integers r_1 and r_2 the couple constrained equilibrium is the solution to the Kuhn-Tucker conditions*

$$
\begin{aligned}
r_i \frac{\partial \varphi^i}{\partial x_{k_i}^i} + \sum_{j=1}^{m} \lambda_j \frac{\partial h^j}{\partial x_{k_i}^i} &= 0, \;\; i = 1, 2, \;\; k_i = 1, ..., n_i, \\
\sum_{j=1}^{m} \lambda_j h^j &= 0,
\end{aligned}
$$

with $\lambda_j \geq 0$.

r_1 and r_2 are pre-determined constants that are not derived as part of the equilibrium. More formally, $\rho_{ij} \equiv \frac{\lambda_j}{r_i}$ is player i's shadow price of constraint

j. From this relationship, it can be the that the r_is, can be subject to a normalization without affecting the equilibrium strategies. Doing so generates a *normalized couple-constrained equilibrium.*

Proof of theorem 5

Using theorem 6 the couple-constrained Nash equilibrium satisfies the complementary slackness conditions for the constraints and multipliers and the necessary conditions

$$
\begin{aligned}
r_1 ptU_x^{1,c} + r_1 (1-p) tU_x^{1,n} + \lambda_1 t + \lambda_3 - \lambda_9 - t(1+f)\lambda_{11} &= 0, && (40)\\
r_1 \left[p\left(-U_x^{1,c} + U_G^{1,c}\right) + (1-p)\left(-U_x^{1,n} + U_G^{1,n}\right)\right] - \lambda_1 + \lambda_5 &= 0, && (41)\\
-r_1 pU_x^{1,c} - \lambda_1 + \lambda_7 + \lambda_{11} &= 0, && (42)\\
r_2 ptU_x^{2,c} + r_2 (1-p) tU_x^{2,n} + \lambda_2 t + \lambda_4 - \lambda_{10} - t(1+f)\lambda_{11} &= 0, && (43)\\
r_2 \left[p\left(-U_x^{2,c} + U_G^{12,c}\right) + (1-p)\left(-U_x^{2,n} + U_G^{2,n}\right)\right] - \lambda_2 + \lambda_6 &= 0, && (44)\\
-r_2 pU_x^{2,c} - \lambda_2 + \lambda_8 + \lambda_{11} &= 0. && (45)
\end{aligned}
$$

At an interior equilibrium $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = \lambda_9 = \lambda_{10} = 0$. So the optimal choices $\left\{\hat{e}^j, \hat{g}^j, \hat{z}^j\right\}$ satisfy

$$
\begin{aligned}
r_1 ptU_x^{1,c} + r_1 (1-p) tU_x^{1,n} - t(1+f)\lambda_{11} &= 0, && (46)\\
r_1 \left[p\left(-U_x^{1,c} + U_G^{1,c}\right) + (1-p)\left(-U_x^{1,n} + U_G^{1,n}\right)\right] &= 0, && (47)\\
-r_1 pU_x^{1,c} + \lambda_7 + \lambda_{11} &= 0, && (48)\\
r_2 ptU_x^{2,c} + r_2 (1-p) tU_x^{2,n} - t(1+f)\lambda_{11} &= 0, && (49)\\
r_2 \left[p\left(-U_x^{2,c} + U_G^{2,c}\right) + (1-p)\left(-U_x^{2,n} + U_G^{2,n}\right)\right] &= 0, && (50)\\
-r_2 pU_x^{2,c} + \lambda_8 + \lambda_{11} &= 0. && (51)
\end{aligned}
$$

Combining (46) and (47) gives

$$\lambda_{11} = \frac{r_1 \left[pU_G^{1,c} + (1-p) U_G^{1,n}\right]}{1+f}, \qquad (52)$$

while combining (49) and (50) gives

$$\lambda_{11} = \frac{r_2 \left[pU_G^{2,c} + (1-p) U_G^{2,n}\right]}{1+f}. \qquad (53)$$

Since $U_G^{j,c} = U_G^{j,n} = \varphi(G)$, (52) and (53) imply

$$\frac{r_1 \varphi(G)}{1+f} = \frac{r_2 \varphi(G)}{1+f}.$$

This can only hold if $r^1 = r^2$.

Proof of theorem 4 Observe that the values $\bar{g}^1 = \hat{g}^1 - \Delta y$ and $\bar{g}^2 = \hat{g}^2 + \Delta y$, $\bar{e}^j = \hat{e}^j$, $\bar{z}^j = \hat{z}^j$, $j = 1, 2$ imply $\bar{Y}^{j,c} - \bar{g}^j - \bar{z}^j = \hat{Y}^{j,c} - \hat{g}^j - \hat{z}^j$, $\bar{Y}^{j,n} - \bar{g}^j = \hat{Y}^{j,n} - \hat{g}^j$, and $\bar{g}^1 + \bar{g}^2 = \hat{g}^1 + \hat{g}^2$. Hence, all marginal utilities are unchanged so the strategies $\left\{\bar{e}^j, \bar{g}^j, \bar{z}^j\right\}$, $j = 1, 2$, satisfy the necessary conditions.