Fusion Estimation for A Class of Multi-Rate Power Systems with Randomly Occurring SCADA Measurement Delays ⋆

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Abstract

In this paper, the fusion estimation problem is investigated for a class of multi-rate power systems with randomly occurring delays in supervisory control and data acquisition (SCADA) measurements. The power system is measured by the SCADA and the phasor measurement unit (PMU), and the state updating period of the power system is allowed to be different from the sampling periods of the SCADA and the PMU. The phenomenon of the randomly occurring SCADA measurement delays is characterized by a set of Bernoulli distributed random variables. To facilitate the state estimator design, a new approach is developed to transform the multi-rate power system into single-rate one. First, two local state estimators are designed, respectively, based on the SCADA and the PMU measurements such that upper bounds of the local estimation error covariances are guaranteed at each sampling instant, and such upper bounds are subsequently minimized by appropriately designing the gains of both local state estimators. Then, the asynchronous estimates from the local state estimators are fused by recurring to the covariance intersection fusion scheme. Finally, a simulation experiment is carried out on the IEEE 14-bus system to illustrate the effectiveness of the proposed fusion estimation scheme.

Key words: Power systems; multi-rate systems; covariance intersection fusion; randomly occurring measurement delays.

1 Introduction

As a key module of the energy management systems (EMS), efficient yet accurate state estimation (SE) plays a vital role in the operation and control of the power systems [7–9, 12, 41]. Traditional SE algorithms based on the classical static weighted-least-square (WLS) method has been widely applied in practice, where the measurements generated by the supervisory control and data acquisition (SCADA) unit are used to estimate the state of the power system. In practice, however, the traditional SCADA measurements might exhibit the features of low sampling rate and low accuracy, which makes it difficult to reveal the dynamic behavior of the power systems correctly, thereby impairing the real-time capability and reliability of the state estimation [6, 22, 34, 41].

In order to overcome the weakness of the SCADA, the phasor measurement unit (PMU) has been playing an increasingly important role in the power systems. Compared with the SCADA, the PMU has much higher sampling rate which facilitates accurate yet timely measurements [22]. In this sense, the PMUs are particularly suitable for dynamic state estimation (DSE) of power systems. Unfortunately, due to the high cost of implementing PMU and the limited bandwidth of the communication network, it is impractical to deploy the PMU on every node of the power system in the foreseeable future. As such, an effective way is to simultaneously utilize the measurements from both the SCADA and the PMU in order to achieve an adequate tradeoff between the estimation accuracy and the implementation cost. Consequently, it is of vital importance to develop a SE scheme to use both SCADA and PMU measurements to estimate the state of the power systems [20, 21].

Recently, the SE problem for power systems with mixed SCADA and PMU measurements has attracted increasing research attention, and there are mainly two kinds of estimation schemes in the literature [21, 22]. One estimation scheme is to use a single state estimator where the measurements from the PMUs are mixed with those from the SCADA, see [5, 20, 27]. For instance, a single state estimator for power systems has been designed in [20] by augmenting the measurements from the SCADA and the PMUs. Unfortunately, most of the existing results based on such kind of scheme cannot exert the predominance of the PMU since the SE is only performed at the coarse SCADA time-scale. The second kind of SE scheme is the hybrid one where the estimate from the SCADA-based state estimator is ameliorated by a second estimator which utilizes measurement from PMU only, see [10, 17, 29]. For example, a novel hybrid SE scheme has been proposed in [10], where the SCADA-based estimate and the PMU-based estimate have been obtained independently and then fused. In practical application, however, the advantages of the PMU would be lost if the hybrid estimator is simply worked at the coarse SCADA time-scale while the communication and computation burden would be increased if the hybrid estimation is performed at the PMU sampling rate. It should be pointed out that despite the wide deployment of the

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static WLS method in power systems, the drawbacks such as poor robustness in the presence of non-ideal measurements may degrade the performance of the WLS method severely [18, 30]. Moreover, the WLS method can only forecast the current system states while cannot provide the estimates for the next time step [18]. As such, there is a practical need to develop new approaches for DSE and online monitoring of the power systems with the mixed SCADA and PMU measurements.

So far, Kalman-filter-based DSE approaches have been widely applied in the power systems, see [21, 22, 41]. To estimate the state of the power systems in the presence of the nonlinear measurement, the extend Kalman filtering algorithm has been adopted in [13, 15, 20]. On the other hand, some filtering algorithms have been developed to reduce the computation burden of calculating the Jacobian matrix and better handle the nonlinearities in the power systems, see [29, 40]. For example, in [29], a hybrid DSE method has been proposed, where the SCADA and the PMU measurements have been processed in parallel based on the cubature Kalman filtering algorithm, and then the estimates have been fused. In [40], a robust unscented Kalman filtering algorithm has been designed for DSE for power systems, and the proposed filtering algorithm performs well in the presence of non-Gaussian process and measurement noises.

In a power system with both the PMUs and the SCADA, the measurements are generated with different rates due to the fact that the PMU has a sampling rate of 10–120 samples per second while the SCADA samples every 0.5–2 seconds [16]. Such kind of *multi-rate* systems with *asynchronous* measurements have received considerable research attention in the past decade [16, 28, 33]. For example, in [16], based on the fusion scheme in [33], the fusion estimation problem for the power systems with multi-rate measurements has been studied, while the asynchronicities among the SCADA and PMU have been ignored. Nevertheless, when it comes to the power systems, the fusion estimation problem for *multi-rate multi-sensor* systems with *asynchronous* measurements has not received adequate research attention yet despite its promising engineering significance. Therefore, it is of great necessity to investigate the fusion estimation problem for power systems with both SCADA and PMUs, and this motivates our current research.

In most existing results concerning the state estimation problems, there have been an implicit assumption that the measurement outputs are associated with the current state of the system [1, 2, 14, 38]. This assumption is quite restrictive in practice since the measurement outputs are often subject to unavoidable delays due to the time skewness and long-distance communication [21, 37]. Compared with the synchronized PMU measurements utilizing global position systems (GPS), the phenomenon of time skewness is often encountered in asynchronous SCADA measurements. On the other hand, traditional communication medias (e.g. power line and telephone line), which may suffer from the randomly occurring network-induced communication delays due to limited bandwidth, are still extensively used in the SCADA systems. Since the delayed measurements could largely affect the system stability [21, 22], it is of vital significance to design state estimators that are capable of mitigating the side-effects caused by time delays. Up to now, state estimation problems with measurement delays have received much attention, see e.g. [30, 37]. Despite the progress made so far, little research attention has been paid on the state estimation problem for power systems with randomly occurring SCADA measurement delays.

Based on the above mentioned issues, the main motivations of this paper are to: *1) establish a novel SE framework for the mixed SCADA and PMU measurements which not only takes the advantages of the high sampling rate of the PMU but also efficiently utilizes the SCADA measurements; 2) develop a mechanism to transform the multi-rate system into a single-rate one for the convenience of estimator design; 3) design an appropriate estimator which can better tackle the randomly occurring SCADA measurement delays; and 4) propose a fusion scheme which has a satisfactory performance.* As such, in this paper, we aim to solve the fusion estimation problem for multi-rate power systems with randomly occurring SCADA measurement delays. The main contributions of the paper can be highlighted as follows: *1) the fusion estimation problem is, for the first time, investigated for dynamic state estimation of power systems with multi-rate measurements and randomly occurring SCADA measurement delays; 2) an augmentation method is applied to transform the multi-rate system into a single-rate one, and two local state estimators are then designed, respectively, based on the SCADA and the PMU measurements such that the upper bounds of the local estimation error covariances are minimized at each sampling instant; and 3) a fusion estimation scheme is proposed by recurring to the covariance intersection (CI) fusion method to fuse the asynchronous local estimates.* Finally, intensive numerical simulation is carried out on the IEEE 14-bus system in order to illustrate the effectiveness of the proposed fusion estimation scheme for power systems.

Notation The notations used here are fairly standard. $\text{diag}\{\cdots\}$ and $\text{diag}_m\{*\}$ represent the block-diagonal matrix and $\text{diag}\{\underbrace{*,\cdots,*}_{m}\}$, respectively. $\text{col}_m\{*\}$ stands for $\text{col}\{\underbrace{*,\cdots,*}_{m}\}$. $\circ$ is the Hadamard product which is defined as $[A \circ B]_{ij} = A_{ij} \times B_{ij}$.

2 Problem Formulation

2.1 Power System Model

In this paper, it is assumed that the power system operates in the quasi-steady state. Let us consider a power system containing N buses described by [13, 20–22]:

$$x_{(k+1)h} = Ax_{kh} + Bu + w_{kh} \tag{1}$$

where $x_k \in \mathbb{R}^{2N}$ is the state vector evolving at a basic period h (h is omitted for brevity in the sequel), $x_k = [V_{1,k}\ \ V_{2,k}\ \cdots\ V_{N,k}\ \ \theta_{1,k}\ \ \theta_{2,k}\ \cdots\ \theta_{N,k}]^T$ with $V_{l,k}$ and $\theta_{l,k}$ representing the voltage magnitude and the voltage phase angle of the bus l ($l = 1, 2, \ldots, N$), respectively. $u \in \mathbb{R}^{2N}$ is the expected steady state, $A \in \mathbb{R}^{2N\times 2N}$ is the transition matrix, $B \triangleq I - A$ is associated with the trend behavior of the state trajectory, and w_k is a zero mean Gaussian white noise with covariance $W_k > 0$. The initial value x_0 of the state is a random variable with mean η_0 and covariance $\Sigma_{0|0}$.

2.2 SCADA Measurement Model with Randomly Occurring Delays

The ideal measurement output $\vec{z}_{s,n_1k} \in \mathbb{R}^{m_1}$ collected by the SCADA units with a sampling period n_1h (n_1 is a positive integer) can be modeled as $\vec{z}_{s,n_1k} = [P^T_{n_1k} \quad Q^T_{n_1k} \quad P^T_{f,n_1k} \quad Q^T_{f,n_1k}]^T$ where $P_{n_1k} \triangleq [P_{1,n_1k} \quad P_{2,n_1k} \quad \cdots \quad P_{n_p,n_1k}]^T$ and $Q_{n_1k} \triangleq [Q_{1,n_1k} \;\; Q_{2,n_1k} \;\cdots\; Q_{n_p,n_1k}]^T$ denote the active and reactive bus power injection measurements, respectively. $P_{f,n_1k} \triangleq [P_{f1,n_1k} \quad P_{f2,n_1k} \quad \cdots \quad P_{fn_s,n_1k}]^T$ and $Q_{f,n_1k} \triangleq [Q_{f1,n_1k} \;\; Q_{f2,n_1k} \;\cdots\; Q_{fn_s,n_1k}]^T$ are the active and reactive power flow measurements, respectively. n_p and n_s represent the number of SCADA units placed at the selected buses and transmission lines, respectively.

Let us consider the typical π-model of a branch i–j, the explicit element for each measurement mentioned above can be expressed as follows (n_1k is omitted for brevity):

$$\begin{aligned}
P_i &= V_i \sum_{j\in\mathcal{N}_i} V_j\big(G_{ij}\cos(\theta_i-\theta_j)+B_{ij}\sin(\theta_i-\theta_j)\big),\\
Q_i &= V_i \sum_{j\in\mathcal{N}_i} V_j\big(G_{ij}\sin(\theta_i-\theta_j)-B_{ij}\cos(\theta_{i,k}-\theta_j)\big),\\
P_{f,i} &= V_i^2(g_{si}+g_{ij}) - V_iV_j\big(g_{ij}\cos(\theta_i-\theta_j)\\
&\quad + b_{ij}\sin(\theta_i-\theta_j)\big),\\
Q_{f,i} &= -V_i^2(b_{si}+b_{ij}) - V_iV_j\big(g_{ij}\sin(\theta_i-\theta_j)\\
&\quad - b_{ij}\cos(\theta_i-\theta_j)\big)
\end{aligned} \tag{2}$$

where $G_{ij}+jB_{ij}$ is the (i,j)-th element of the complex bus admittance matrix, $g_{ij}+jb_{ij}$ is the admittance of the series branch connecting buses i and j, $g_{si}+jb_{si}$ is the admittance of the shunt branch connected to bus i, and $\mathcal{N}_i$ is the set of bus numbers which are directly connected to bus i.

Taking the measurement noise and the randomly occurring measurement delays into consideration, we obtain the following measurement model for SCADA units:

$$z_{s,n_1k} = \Lambda_{n_1k}\vec{z}_{s,n_1k} + (I-\Lambda_{n_1k})\vec{z}_{s,n_1(k-1)} \tag{3}$$

where

$$\vec{z}_{s,n_1k} \triangleq h_s(x_{n_1k}) + v_{s,n_1k}. \tag{4}$$

Here, $h_s(\cdot)$ is determined by (2). z_{s,n_1k} is the actual measurement output. v_{s,n_1k} is the measurement noise on the SCADA units, which is a Gaussian white noise with zero mean and covariance $R_{s,n_1k} > 0$. Λ_{n_1k} is defined as $\Lambda_{n_1k} \triangleq \mathrm{diag}\{\lambda_{1,n_1k} \;\; \lambda_{2,n_1k} \;\cdots\; \lambda_{m_1,n_1k}\}$ with λ_{i,n_1k} ($i=1,2,\ldots,m_1$) being mutually independent random variables. λ_{i,n_1k} is also uncorrelated with w_k, ν_{s,n_1k} and x_0. Furthermore, the random variable λ_{i,n_1k} satisfies Bernoulli distribution and takes values on 0 or 1 with

$$\mathrm{Prob}\{\lambda_{i,n_1k}=1\}=\mu_i, \quad \mathrm{Prob}\{\lambda_{i,n_1k}=0\}=1-\mu_i$$

where $\mu_i \in [0,1]$ is a known scalar.

2.3 PMU Measurement Model

The measurement $z_{p,n_2k} \in \mathbb{R}^{m_2}$ of the PMUs with a sampling period n_2h (n_2 is a positive integer) can be modeled as $z_{p,n_2k} = [z^T_{p,1,n_2k} \;\; z^T_{p,2,n_2k} \;\cdots\; z^T_{p,m_2,n_2k}]^T$. Suppose that the l-th PMU ($l=1,2,\ldots,m_2$) is installed at the bus i, and the bus i is directly connected to bus j. Then, z_{p,l,n_2k} is described as $z_{p,l,n_2k} = [I_{r,ij,n_2k} \;\; I_{i,ij,n_2k}]^T$ where I_{r,ij,n_2k} and I_{i,ij,n_2k} are the real and imaginary current between the bus i and j, respectively. To be more specific, I_{r,ij,n_2k} and I_{i,ij,n_2k} can be described as follows (n_2k is omitted for brevity):

$$\begin{aligned}
I_{r,ij} &= V_i\big((g_{si}+g_{ij})\cos\theta_i - (b_{si}+b_{ij})\sin\theta_i\big)\\
&\quad - V_j\big(g_{ij}\cos\theta_j - b_{ij}\sin\theta_j\big),\\
I_{i,ij} &= V_i\big((b_{si}+b_{ij})\cos\theta_i + (g_{si}+g_{ij})\sin\theta_i\big)\\
&\quad - V_j\big(g_{ij}\sin\theta_j + b_{ij}\cos\theta_j\big).
\end{aligned} \tag{5}$$

Taking the measurement noise into consideration, we obtain the following measurement model for PMUs:

$$z_{p,n_2k} = h_p(x_{n_2k}) + v_{p,n_2k} \tag{6}$$

where $h_p(\cdot)$ is determined by (5), v_{p,n_2k} is the measurement noise on the PMU that is also a Gaussian white noise with zero mean and covariance $R_{p,n_2k} > 0$.

2.4 State Estimator

Let us convert the multi-rate system into a single-rate one. By applying the relation (1) recursively, for $i=1,2$ and a nonnegative integer l, we have

$$\begin{aligned}
x_{n_ik+l+1} &= Ax_{n_ik+l} + Bu + w_{n_ik+l}\\
&= A^{(l+1)}x_{n_ik} + \sum_{i=0}^{l} A^i(Bu + w_{n_ik+l-i}).
\end{aligned} \tag{7}$$

Denoting

$$\begin{aligned}
\tilde{A}_i &\triangleq A^{n_i}, \; \bar{A}_i \triangleq \Big[A^{n_i-1} \;\; A^{n_i-2} \;\cdots\; I\Big],\\
\bar{B} &\triangleq \mathrm{col}_{n_i}\{B\}, \; \bar{w}_{n_ik} \triangleq \Big[w^T_{n_ik} \;\; w^T_{n_ik+1} \;\cdots\; w^T_{n_i(k+1)-1}\Big]^T,
\end{aligned}$$

we have (for $i=1,2$) that

$$x_{n_i(k+1)} = \tilde{A}_i x_{n_ik} + \bar{A}_i\bar{B}u + \bar{A}_i\bar{w}_{n_ik}. \tag{8}$$

Then, the local state estimators using the measurements from SCADA units (denoted by LSES) and PMUs (denoted by LSEP) are of the following form:

$$\text{LSES}\begin{cases}
\hat{x}_{n_1k|n_1(k-1)} = \tilde{A}_1\hat{x}_{n_1(k-1)|n_1(k-1)} + \bar{A}_1\bar{B}u,\\
\hat{x}_{n_1k|n_1k} = \hat{x}_{n_1k|n_1(k-1)} + K_{n_1k}\big(z_{s,n_1k}\\
\qquad - \bar{\Lambda}h_s(\hat{x}_{n_1k|n_1(k-1)})\\
\qquad - (I-\bar{\Lambda})h_s(\hat{x}_{n_1(k-1)|n_1(k-2)})\big)
\end{cases} \tag{9}$$

and

$$\text{LSEP}\begin{cases}
\hat{x}_{n_2k|n_2(k-1)} = \tilde{A}_2\hat{x}_{n_2(k-1)|n_2(k-1)} + \bar{A}_2\bar{B}u,\\
\hat{x}_{n_2k|n_2k} = \hat{x}_{n_2k|n_2(k-1)} + G_{n_2k}\big(z_{p,n_2k}\\
\qquad - h_p(\hat{x}_{n_2k|n_2(k-1)})\big)
\end{cases} \tag{10}$$

where $\hat{x}_{n_ik|n_i(k-1)}$ and $\hat{x}_{n_ik|n_ik}$ are the one-step prediction and the estimate of x_{n_ik} ($i=1,2$), respectively. K_{n_1k} and G_{n_2k} are the local state estimator gain matrices to be designed, and $\bar{\Lambda} \triangleq \mathbb{E}\{\Lambda_{n_1k}\} = \mathrm{diag}\{\mu_1, \mu_2, \ldots, \mu_{m_1}\}$.

The purpose of this paper is to:

(1) design the asynchronous local state estimators in the form of (9) and (10) such that the upper bounds of the local estimation error covariances are minimized at the corresponding sampling instant by choosing appropriate local state estimator gain matrices K_{n_1k} and G_{n_2k}; and
(2) develop a fusion scheme to fuse the asynchronous local estimates generated by the local state estimators (9) and (10).

3 Main Results

The flowchart of the proposed estimation scheme is summarized in Fig. 1.

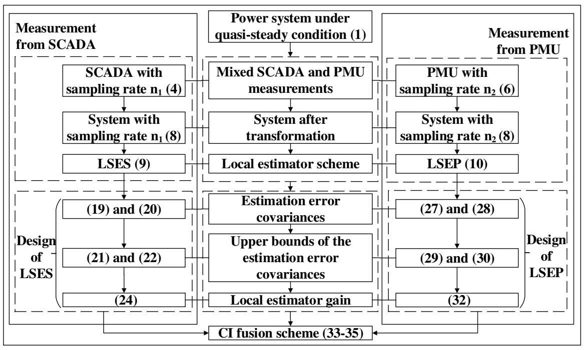

Fig. 1. Flowchart of the proposed fusion estimation scheme.

Lemma 1 *[32] Given matrices A, H, F, and M with compatible dimensions such that $FF^T \le I$. Let U be a symmetric positive-definite matrix and a be an arbitrary positive constant such that $a^{-1}I - MUM^T > 0$. Then, the following matrix inequality holds:*

$$(A + HFM)U(A + HFM)^T \le A(U^{-1} - aM^TM)^{-1}A^T + a^{-1}HH^T.$$

Lemma 2 *For matrices X, Y and a positive scalar λ, the following inequality holds:*

$$XY^T + YX^T \le \lambda XX^T + \lambda^{-1}YY^T.$$

Lemma 3 *[19] Let $A = [a_{ij}]_{n\times n}$ be a real-valued matrix and $B = \mathrm{diag}\{b_1, b_2, \ldots, b_n\}$ be a diagonal stochastic matrix. Then*

$$\mathbb{E}\{BAB^T\} = \begin{bmatrix} \mathbb{E}\{b_1^2\} & \mathbb{E}\{b_1b_2\} & \cdots & \mathbb{E}\{b_1b_p\} \\ \mathbb{E}\{b_1b_2\} & \mathbb{E}\{b_2^2\} & \cdots & \mathbb{E}\{b_2b_p\} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbb{E}\{b_pb_1\} & \mathbb{E}\{b_pb_2\} & \cdots & \mathbb{E}\{b_p^2\} \end{bmatrix} \circ A$$

where $\circ$ is the Hadamard product.

Denote the one-step prediction error $\tilde{x}_{n_ik|n_i(k-1)} \triangleq x_{n_ik} - \hat{x}_{n_ik|n_i(k-1)}$ and the estimation error $\tilde{x}_{n_ik|n_ik} \triangleq x_{n_ik} - \hat{x}_{n_ik|n_ik}$, respectively $(i = 1, 2)$. By using the Taylor series expansion for $h_s(x_{n_1k})$ and $h_p(x_{n_2k})$ around $\hat{x}_{n_1k|n_1(k-1)}$ and $\hat{x}_{n_2k|n_2(k-1)}$, respectively, we have

$$\begin{aligned} h_s(x_{n_1k}) =& h_s(\hat{x}_{n_1k|n_1(k-1)}) + H_{s,n_1k}\tilde{x}_{n_1k|n_1(k-1)} \\ &+ o_s(|\tilde{x}_{n_1k|n_1(k-1)}|), \end{aligned} \tag{11}$$

$$\begin{aligned} h_p(x_{n_2k}) =& h_p(\hat{x}_{n_2k|n_2(k-1)}) + H_{p,n_2k}\tilde{x}_{n_2k|n_2(k-1)} \\ &+ o_p(|\tilde{x}_{n_2k|n_2(k-1)}|) \end{aligned} \tag{12}$$

where

$$H_{s,n_1k} \triangleq (\partial h_s(x_{n_1k})/\partial x_{n_1k})|_{x_{n_1k}=\hat{x}_{n_1k|n_1(k-1)}},$$
$$H_{p,n_2k} \triangleq (\partial h_p(x_{n_2k})/\partial x_{n_2k})|_{x_{n2k}=\hat{x}_{n_2k|n_2(k-1)}}.$$

Following [3] and [23], the high-order terms of the Taylor series expansion $o_s(|\tilde{x}_{n_1k|n_1(k-1)}|)$ and $o_p(|\tilde{x}_{n_2k|n_2(k-1)}|)$ can be transformed into:

$$o_s(|\tilde{x}_{n_1k|n_1(k-1)}|) = C_{s,n_1k}\aleph_{s,n_1k}L_{s,n_1k}\tilde{x}_{n_1k|n_1(k-1)}, \tag{13}$$

$$o_p(|\tilde{x}_{n_2k|n_2(k-1)}|) = C_{p,n_2k}\aleph_{p,n_2k}L_{p,n_2k}\tilde{x}_{n_2k|n_2(k-1)} \tag{14}$$

where C_{s,n_1k} and C_{p,n_2k} are the problem-dependent scaling matrices, L_{s,n_1k} and L_{p,n_2k} are the tuning matrices providing extra freedom degrees, $\aleph_{s,n_1k}$ and $\aleph_{p,n_2k}$ are the unknown time-varying matrices accounting for the linearization errors satisfying

$$\aleph_{s,n_1k}\aleph_{s,n_1k}^T \leqslant I, \tag{15}$$

$$\aleph_{p,n_2k}\aleph_{p,n_2k}^T \leqslant I. \tag{16}$$

3.1 Design of the LSES

From (8), (9), (11) and (13), the one-step prediction error and the estimation error of the LSES are obtained as

$$\tilde{x}_{n_1k|n_1(k-1)} = \tilde{A}\tilde{x}_{n_1(k-1)|n_1(k-1)} + \bar{A}_1\bar{w}_{n_1(k-1)} \tag{17}$$

and

$$\begin{aligned} &\tilde{x}_{n_1k|n_1k} \\ =&(I - K_{n_1k}\Lambda_{n_1k}\Phi_{s,n_1k})\tilde{x}_{n_1k|n_1(k-1)} \\ &- K_{n_1k}(\Lambda_{n_1k} - \bar{\Lambda})h_s(\hat{x}_{n_1k|n_1(k-1)}) - K_{n_1k}\Lambda_{n_1k}v_{s,n_1k} \\ &- K_{n_1k}(I - \Lambda_{n_1k})\Phi_{s,n_1(k-1)}\tilde{x}_{n_1(k-1)|n_1(k-2)} \\ &+ K_{n_1k}(\Lambda_{n_1k} - \bar{\Lambda})h_s(\hat{x}_{n_1(k-1)|n_1(k-2)}) \\ &- K_{n_1k}(I - \Lambda_{n_1k})v_{s,n_1(k-1)} \end{aligned} \tag{18}$$

where $\Phi_{s,n_1k} \triangleq H_{s,n_1k} + C_{s,n_1k}\aleph_{s,n_1k}L_{s,n_1k}$.

The one-step prediction error covariance $P_{s,n_1k|n_1(k-1)} \triangleq \mathbb{E}\{\tilde{x}_{n_1k|n_1(k-1)}\tilde{x}_{n_1k|n_1(k-1)}^T\}$ is given by

$$\begin{aligned} &P_{s,n_1k|n_1(k-1)} \\ =&\tilde{A}_1P_{s,n_1(k-1)|n_1(k-1)}\tilde{A}_1^T + \bar{A}_1\bar{W}_{n_1(k-1)}\bar{A}_1^T \end{aligned} \tag{19}$$

where $\bar{W}_{n_1(k-1)} \triangleq \mathbb{E}\{\bar{w}_{n_1(k-1)}\bar{w}_{n_1(k-1)}^T\}$ can be written as $\bar{W}_{n_1(k-1)} = \mathrm{diag}\{W_{n_1(k-1)}, W_{n_1(k-1)+1}, \cdots, W_{n_1k-1}\}$.

Similarly, the estimation error covariance $P_{s,n_1k|n_1(k-1)} \triangleq \mathbb{E}\{\tilde{x}_{n_1k|n_1(k-1)}\tilde{x}_{n_1k|n_1(k-1)}^T\}$ can be written as

$$P_{s,n_1k|n_1k}$$

$$
\begin{aligned}
=&\Theta_{1,n_1k}P_{s,n_1k|n_1(k-1)}\Theta_{1,n_1k}^T \\
&+K_{n_1k}\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1k|n_1(k-1)}\}K_{n_1k}^T \\
&+K_{n_1k}\bar{\Lambda}R_{s,n_1k}\bar{\Lambda}K_{n_1k}^T \\
&+\Theta_{2,n_1k}P_{s,n_1(k-1)|n_1(k-2)}\Theta_{2,n_1k}^T \\
&+K_{n_1k}\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1(k-1)|n_1(k-2)}\}K_{n_1k}^T \\
&+K_{n_1k}(I-\bar{\Lambda})R_{s,n_1(k-1)}(I-\bar{\Lambda})K_{n_1k}^T \\
&-\mathscr{L}_{1,n_1k}-\mathscr{L}_{1,n_1k}^T-\mathscr{L}_{2,n_1k}-\mathscr{L}_{2,n_1k}^T \\
&-\mathscr{L}_{3,n_1k}-\mathscr{L}_{3,n_1k}^T+\mathscr{L}_{4,n_1k}+\mathscr{L}_{4,n_1k}^T \\
&+\mathscr{L}_{5,n_1k}+\mathscr{L}_{5,n_1k}^T-\mathscr{L}_{6,n_1k}-\mathscr{L}_{6,n_1k}^T \qquad (20)
\end{aligned}
$$

where

$$
\begin{aligned}
&\Theta_{1,n_1k}\triangleq I-K_{n_1k}\bar{\Lambda}\Phi_{s,n_1k}, \\
&\Theta_{2,n_1k}\triangleq K_{n_1k}(I-\bar{\Lambda})\Phi_{s,n_1(k-1)}, \\
&\Theta_{3,n_1k}\triangleq K_{n_1k}(\Lambda_{n_1k}-\bar{\Lambda}), \\
&\Psi_{n_1k|n_1(k-1)}\triangleq h_s(\hat{x}_{n_1k|n_1(k-1)})h_s^T(\hat{x}_{n_1k|n_1(k-1)}), \\
&\tilde{\Lambda}\triangleq \mathrm{diag}\{\mu_1-\mu_1^2,\mu_2-\mu_2^2,\ldots,\mu_{m_1}-\mu_{m_1}^2\}, \\
&\mathscr{L}_{1,n_1k}\triangleq\Theta_{1,n_1k}\mathbb{E}\{\tilde{x}_{n_1k|n_1(k-1)}h_s^T(\hat{x}_{n_1k|n_1(k-1)})\}\Theta_{3,n_1k}^T, \\
&\mathscr{L}_{2,n_1k}\triangleq\Theta_{2,n_1k}\mathbb{E}\{\tilde{x}_{n_1(k-1)|n_1(k-2)} \\
&\qquad\times h_s^T(\hat{x}_{n_1(k-1)|n_1(k-2)})\}\Theta_{3,n_1k}^T, \\
&\mathscr{L}_{3,n_1k}\triangleq\Theta_{1,n_1k}\mathbb{E}\{\tilde{x}_{n_1k|n_1(k-1)}\tilde{x}_{n_1(k-1)|n_1(k-2)}^T\}\Theta_{2,n_1k}^T, \\
&\mathscr{L}_{4,n_1k}\triangleq\Theta_{1,n_1k}\mathbb{E}\{\tilde{x}_{n_1k|n_1(k-1)} \\
&\qquad\times h_s^T(\hat{x}_{n_1(k-1)|n_1(k-2)})\}\Theta_{3,n_1k}^T, \\
&\mathscr{L}_{5,n_1k}\triangleq K_{n_1k}\tilde{\Lambda}\circ\mathbb{E}\{h_s(\hat{x}_{n_1k|n_1(k-1)}) \\
&\qquad\times\tilde{x}_{n_1(k-1)|n_1(k-2)}^T\}\Theta_{2,n_1k}^T, \\
&\mathscr{L}_{6,n_1k}\triangleq\Theta_{3,n_1k}\mathbb{E}\{h_s(\hat{x}_{n_1k|n_1(k-1)}) \\
&\qquad\times h_s^T(\hat{x}_{n_1(k-1)|n_1(k-2)})\}\Theta_{3,n_1k}^T.
\end{aligned}
$$

Theorem 1 *For positive scalars γ_{1,n_1k} and ε_{i,n_1k} ($i=1,\ldots,6$) under the initial condition $\Sigma_{s,0|0}=P_{s,0|0}>0$, assume that the following two difference equations*

$$
\begin{aligned}
\Sigma_{s,n_1k|n_1(k-1)}=&\tilde{A}_1\Sigma_{s,n_1(k-1)|n_1(k-1)}\tilde{A}_1^T \\
&+\bar{A}_1\bar{W}_{n_1(k-1)}\bar{A}_1^T \qquad (21)
\end{aligned}
$$

and

$$
\begin{aligned}
\Sigma_{s,n_1k|n_1k}=&\kappa_{1,n_1k}\bar{\Theta}_{1,n_1k}\Omega_{s,n_1k}^{-1}\bar{\Theta}_{1,n_1k}^T \\
&+K_{n_1k}\Upsilon_{n_1k}K_{n_1k}^T \qquad (22)
\end{aligned}
$$

where

$$
\begin{aligned}
&\Upsilon_{n_1k} \\
\triangleq&\kappa_{1,n_1k}\gamma_{1,n_1k}^{-1}\bar{\Lambda}C_{s,n_1k}C_{s,n_1k}^T\bar{\Lambda} \\
&+\kappa_{2,n_1k}(\tilde{\Lambda}\circ\mathrm{tr}(\Psi_{n_1k|n_1(k-1)})I) \\
&+\kappa_{3,n_1k}(I-\bar{\Lambda})H_{s,n_1(k-1)}\Omega_{s,n_1(k-1)}^{-1}H_{s,n_1(k-1)}^T(I-\bar{\Lambda}) \\
&+\kappa_{3,n_1k}\gamma_{1,n_1(k-1)}^{-1}(I-\bar{\Lambda})C_{s,n_1(k-1)}C_{s,n_1(k-1)}^T(I-\bar{\Lambda}) \\
&+\kappa_{4,n_1k}(\tilde{\Lambda}\circ\mathrm{tr}(\Psi_{n_1(k-1)|n_1(k-2)})I) \\
&+\bar{\Lambda}R_{s,n_1k}\bar{\Lambda}+(I-\bar{\Lambda})R_{s,n_1(k-1)}(I-\bar{\Lambda})
\end{aligned}
$$

with

$$
\begin{aligned}
\bar{\Theta}_{1,n_1k}=&I-K_{n_1k}\bar{\Lambda}H_{s,n_1k}, \\
\Omega_{s,n_1k}=&\Sigma_{s,n_1k|n_1(k-1)}^{-1}-\gamma_{1,n_1k}L_{s,n_1k}^TL_{s,n_1k}, \\
\kappa_{1,n_1k}=&1+\varepsilon_{1,n_1k}+\varepsilon_{3,n_1k}+\varepsilon_{4,n_1k}, \\
\kappa_{2,n_1k}=&1+\varepsilon_{1,n_1k}^{-1}+\varepsilon_{5,n_1k}+\varepsilon_{6,n_1k}, \\
\kappa_{3,n_1k}=&1+\varepsilon_{2,n_1k}+\varepsilon_{3,n_1k}^{-1}+\varepsilon_{5,n_1k}^{-1}, \\
\kappa_{4,n_1k}=&1+\varepsilon_{2,n_1k}^{-1}+\varepsilon_{4,n_1k}^{-1}+\varepsilon_{6,n_1k}^{-1}
\end{aligned}
$$

have positive-definite solutions $\Sigma_{s,n_1k|n_1(k-1)}$ and $\Sigma_{s,n_1k|n_1k}$ such that the following constraint

$$
\gamma_{1,n_1k}^{-1}I-L_{s,n_1k}\Sigma_{s,n_1k|n_1(k-1)}L_{s,n_1k}^T>0 \qquad (23)
$$

is satisfied. Then, the matrix $\Sigma_{s,n_1k|n_1k}$, which is an upper bound of $P_{s,n_1k|n_1k}$, can be minimized with

$$
\begin{aligned}
K_{n_1k}=&\kappa_{1,n_1k}\Omega_{s,n_1k}^{-1}H_{s,n_1k}^T\bar{\Lambda}(\kappa_{1,n_1k}\bar{\Lambda}H_{s,n_1k} \\
&\times\Omega_{s,n_1k}^{-1}H_{s,n_1k}^T\bar{\Lambda}+\Upsilon_{n_1k})^{-1}. \qquad (24)
\end{aligned}
$$

3.2 Design of The LSEP

From (8), (10), (12) and (14), the one-step prediction error and the estimation error of the LSEP are derived as

$$
\tilde{x}_{n_2k|n_2(k-1)}=\tilde{A}\tilde{x}_{n_2(k-1)|n_2(k-1)}+\bar{A}_2\bar{w}_{n_2(k-1)} \qquad (25)
$$

and

$$
\tilde{x}_{n_2k|n_2k}=\Pi_{p,n_2k}\tilde{x}_{n_2k|n_2(k-1)}-G_{n_2k}v_{p,n_2k} \qquad (26)
$$

where

$$
\begin{aligned}
\Pi_{p,n_2k}\triangleq&I-G_{n_2k}\Phi_{p,n_2k}, \\
\Phi_{p,n_2k}\triangleq&H_{p,n_2k}+C_{p,n_2k}\aleph_{p,n_2k}L_{p,n_2k}.
\end{aligned}
$$

The one-step prediction error covariance $P_{p,n_2k|n_2(k-1)}\triangleq\mathbb{E}\{\tilde{x}_{n_2k|n_2(k-1)}\tilde{x}_{n_2k|n_2(k-1)}^T\}$ is given by

$$
\begin{aligned}
&P_{p,n_2k|n_2(k-1)} \\
=&\tilde{A}_2P_{p,n_2(k-1)|n_2(k-1)}\tilde{A}_2^T+\bar{A}_2\bar{W}_{n_2(k-1)}\bar{A}_2^T \qquad (27)
\end{aligned}
$$

where $\bar{W}_{n_2(k-1)}\triangleq\mathbb{E}\{\bar{w}_{n_2(k-1)}\bar{w}_{n_2(k-1)}^T\}$ can be written as $\bar{W}_{n_2(k-1)}=\mathrm{diag}\{W_{n_2(k-1)},W_{n_2(k-1)+1},\cdots,W_{n_2k-1}\}$.

Similarly, the estimation error covariance $P_{p,n_2k|n_2k}\triangleq\mathbb{E}\{\tilde{x}_{n_2k|n_2k}\tilde{x}_{n_2k|n_2k}^T\}$ can be written as

$$
\begin{aligned}
&P_{p,n_2k|n_2k} \\
=&\Pi_{p,n_2k}P_{p,n_2k|n_2(k-1)}\Pi_{p,n_2k}^T+G_{n_2k}R_{p,n_2k}G_{n_2k}^T. \qquad (28)
\end{aligned}
$$

Theorem 2 *For positive scalar γ_{2,n_2k} with initial condition $\Sigma_{p,0|0}=P_{p,0|0}>0$, if the following two difference equations*

$$
\begin{aligned}
&\Sigma_{p,n_2k|n_2(k-1)} \\
=&\tilde{A}_2\Sigma_{p,n_2(k-1)|n_2(k-1)}\tilde{A}_2^T+\bar{A}_2\bar{W}_{n_2(k-1)}\bar{A}_1^T \qquad (29)
\end{aligned}
$$

and

$$
\Sigma_{p,n_2k|n_2k}
$$

$$=\bar{\Pi}_{p,n_2k}\Omega^{-1}_{p,n_2k}\bar{\Pi}^T_{p,n_2k}+\gamma^{-1}_{2,n_2k}G_{n_2k}C_{p,n_2k}C^T_{p,n_2k}G^T_{n_2k} + G_{n_2k}R_{p,n_2k}G^T_{n_2k} \tag{30}$$

with

$$\bar{\Pi}_{p,n_2k} =I-G_{n_2k}H_{p,n_2k},$$
$$\Omega_{p,n_2k} =\Sigma^{-1}_{p,n_2k|n_2(k-1)}-\gamma_{2,n_2k}L^T_{p,n_2k}L_{p,n_2k}$$

have positive definite solutions $\Sigma_{p,n_2k|n_2(k-1)}$ *and* $\Sigma_{p,n_2k|n_2k}$ *such that the following constraint*

$$\gamma^{-1}_{2,n_2k}I-L_{p,n_2k}\Sigma_{p,n_2k|n_2(k-1)}L^T_{p,n_2k}>0 \tag{31}$$

holds, then the matrix $\Sigma_{p,n_2k|n_2k}$*, which is an upper bound of* $P_{p,n_2k|n_2k}$*, can be minimized with the estimator gain*

$$G_{n_2k} =\Omega^{-1}_{p,n_2k}H^T_{p,n_2k}\big[H_{p,n_2k}\Omega^{-1}_{p,n_2k}H^T_{p,n_2k} + \gamma^{-1}_{2,n_2k}C_{p,n_2k}C^T_{p,n_2k}+R_{p,n_2k}\big]^{-1}. \tag{32}$$

Remark 1 *Due to the existence of the nonlinearities and the randomly occurring SCADA measurement delays, it is impossible to obtain the exact values of* $P_{s,n_1k|n_1k}$ *and* $P_{p,n_2k|n_2k}$*. An alternative way is to look for their locally minimal upper bounds by appropriately designing* K_{n_1k} *and* G_{n_2k}*. Moreover, the conservativeness can be reduced effectively by adjusting the scalars* $\gamma_{1,n1(k+1)}$*,* $\varepsilon_{i,n1(k+1)}$ $(i=1,\ldots,6)$ *and* $\gamma_{2,n2(k+1)}$*.*

Remark 2 *Up to now, under the quasi-static model framework mentioned in* (1)*, two local estimators have been designed. However, when it comes to the complex situations such as the fault estimation or the decentralized control, the Markov process is a useful tool [4, 31, 39] and the latest results have been reported in [35, 36].*

3.3 Fusion Scheme

Denote the least common multiple of n_1 and n_2 as L. We aim to fuse the local estimates at time instants $k = m_kL$ ($m_k = 0, 1, 2, \ldots, n$). By recurring to the CI fusion scheme in [11], the fusion scheme is given as follows:

$$\hat{x}^0_{k|k} =\Sigma^0_{k|k}\big(\omega_1\Sigma^{-1}_{r,m_kL|m_kL}\hat{x}_{m_kL|m_kL,n_1} + \omega_2\Sigma^{-1}_{p,m_kL|m_kL}\hat{x}_{m_kL|m_kL,n_2}\big),$$
$$\Sigma^0_{k|k} =\big(\omega_1\Sigma^{-1}_{r,m_kL|m_kL}+\omega_2\Sigma^{-1}_{p,m_kL|m_kL}\big)^{-1} \tag{33}$$

where $\hat{x}^0_{k|k}$ and $\Sigma^0_{k|k}$ are the fused estimate and covariance, respectively, and $\omega_i \geq 0$ ($i = 1, 2$) are the weights.

The objective is to minimize the performance index J, i.e.:

$$\min J = \min\{\mathrm{tr}\{\Sigma^0_{k|k}\}\},$$
$$\text{s.t.} \sum_{i=1}^{2}\omega_i = 1,\ \omega_i \geq 0. \tag{34}$$

Theorem 3 *For the multi-rate system* (8) *with local estimators* (9) *and* (10)*, the CI fusion scheme* (33)*–*(34) *is consistent, that is*

$$\bar{\Sigma}_{k|k} = \mathbb{E}\{(x_{k|k}-\hat{x}^0_{k|k})(x_{k|k}-\hat{x}^0_{k|k})^T\} \leq \Sigma^0_{k|k}. \tag{35}$$

Remark 3 *For the fusion estimation problem, a significant number of schemes have been proposed, see e.g. [11, 24–26, 28]. It should be pointed out that most of the fusion approaches are obtained under the single-rate environment, and it is difficult to expand these results to the multi-rate multi-sensor systems. Moreover, in many scenarios, the cross-covariances cannot be solved due to the complexity of the algorithms or the unknown terms [11]. In order to cope with such situations, some schemes such as the CI fusion and the sequential fusion have been proposed, see [11, 28]. As such, in this paper, we use the CI fusion scheme to derive the fused estimate.*

Remark 4 *In this paper, we take a close look at the fusion estimation problem for a class of multi-rate power systems with randomly occurring SCADA measurement delays characterized by a set of Bernoulli distributed random variables, where the measurements from both the SCADA and the PMUs are utilized. The overall system is a multi-rate one since the state updating period of the power system is allowed to be different from that of the SCADA and the PMUs. A new approach is first developed to transform the multi-rate power system into a single-rate one. In Theorems 1 and 2, two local state estimators are designed, respectively, based on the SCADA and the PMU measurements such that upper bounds of the local estimation error covariances are ensured at each sampling instant, and such upper bounds are subsequently minimized by appropriately designing the gains of both local state estimators. Then, in Theorem 3, the asynchronous estimates from the local state estimators are fused by recurring to the CI fusion scheme. Numerical simulation is finally carried out on the IEEE 14-bus system to validate the usefulness of the proposed fusion estimation scheme.*

Remark 5 *In this paper, a systematic investigation is initiated on the fusion estimation problem for multi-rate power systems with unconventional measurements from both the SCADA and the PMUs. The main novelties of this paper are outlined as follows: 1) the research problem addressed is new that represents the first of few attempts to deal with the dynamic state estimation of power systems with multi-rate measurements and randomly occurring SCADA measurement delays; 2) the two local estimation algorithms based on the SCADA and the PMU measurements are new that ensure the existence and the minimization of the upper bounds of the local estimation error covariances at each sampling instant; and 3) the fusion estimation scheme is new that applies the CI fusion method to fuse the asynchronous local estimates.*

4 Simulation Results

In this section, the proposed fusion estimation scheme is tested in the IEEE 14-bus system with the aid of Matpower package [42]. The IEEE 14-bus system is modeled as (1) with parameters $A = \mathrm{diag}_{28}\{0.9\}$, $B = \mathrm{diag}_{28}\{0.1\}$ and $W_k = \mathrm{diag}_{28}\{0.01^2\}$. The expected steady state u is given in Table 1. The initial value x_0 is a random variable with mean value u and covariance $\Sigma_{0|0} = \mathrm{diag}_{28}\{0.01^2\}$. The measurement configuration is the same as the one used in [20]. The covariances of the measurement noises are $R_{s,n_1k} = \mathrm{diag}_{42}\{0.02\}$ and $R_{p,n_2k} = \mathrm{diag}_{22}\{0.02^2\}$. Other parameters are taken as $\varepsilon_{1,n_1k} = 1$, $\varepsilon_{2,n_1k} = 4$, $\varepsilon_{3,n_1k} = 50$, $\varepsilon_{4,n_1k} = 10$, $\varepsilon_{5,n_1k} = 1$, $\varepsilon_{6,n_1k} = 2$, $L_{r,n_1k} = 0.001I_{28}$, $\gamma_{1,n_1k} = 100$,

$\gamma_{2,n_2k} = 5$, $C_{r,n_1k} = 0.01I_{42}$, $L_{p,n_2k} = 0.001I_{28}$ and $C_{p,n_2k} = 0.01I_{22}$. The mean square error (MSE) is applied to evaluate the estimation accuracy, i.e., $\text{MSE}_i(k) = \frac{1}{T}\sum_{t=1}^{T}(x_i(k) - \hat{x}_i(k))^2$, where T is the number of samples. For the sake of saving space, only the voltage of bus 2 (i.e., x_k^2) is taken for illustration.

Table 1
Expected States of IEEE 14-bus System

Bus	1	2	3	4	5	6	7
V (p.u.)	1.060	1.045	1.010	1.018	1.020	1.070	1.062
θ (°)	0.000	-4.983	-12.725	-10.313	-8.744	-14.211	-13.360
Bus	8	9	10	11	12	13	14
V (p.u.)	1.090	1.056	1.051	1.057	1.055	1.050	1.036
θ (°)	-13.360	-14.939	-15.097	-14.791	-15.076	-15.156	-16.034

Case 1 *Conventional Method Versus the Proposed Method*

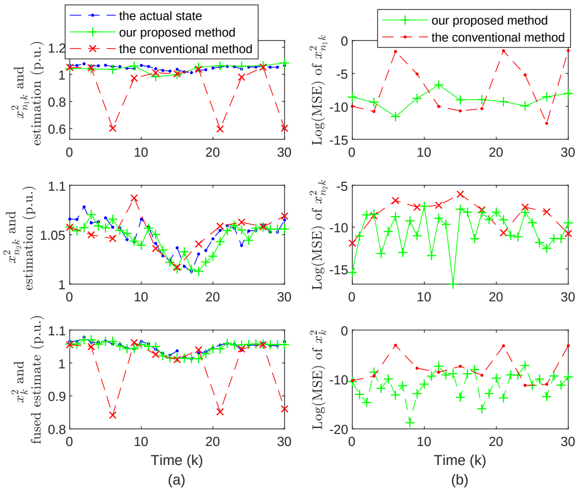

Fig. 2. Test results under case 1. (a) x_k^2 and its estimate. (b) Log(MSE) of $\hat{x}_k^2$.

In this case, comparisons between the conventional method and our proposed one are carried out. The probability density function for the delay Λ_{n1k} is $\text{Prob}\{\Lambda_{i,n_1k} = 0\} = 0.3$ (i.e., $\mu_i = 0.3$). The simulation results based on the conventional approach are plotted as red line in Fig. 2, where the randomly occurring SCADA measurement delays are not handled and the algorithm is simply performed at the coarse SCADA time-scale (i.e., $n_1 = n_2 = 3$). Moreover, the simulation results with our proposed method are plotted as green line in Fig. 2, where the randomly occurring SCADA measurement delays are handled and the different sampling rates of SCADA and PMU are also considered (i.e., $n_1 = 3, n_2 = 2$). From Fig. 2 (a), we can find that the local estimates from the LSES and the LSEP are plotted in the first and second subfigures, respectively, and the third subfigure depicts the fused estimate. Similarly, the corresponding log(MSEs) of the local estimates and the fused estimate are plotted in Fig. 2 (b), respectively.

From Fig. 2, we can find: 1) the randomly occurring SCADA measurement delays are better tackled; and 2) the multi-rate measurements are properly handled instead of processing them at a coarse time resolution likes the conventional one.

Case 2 *Different SCADA Measurement Delay Probabilities*

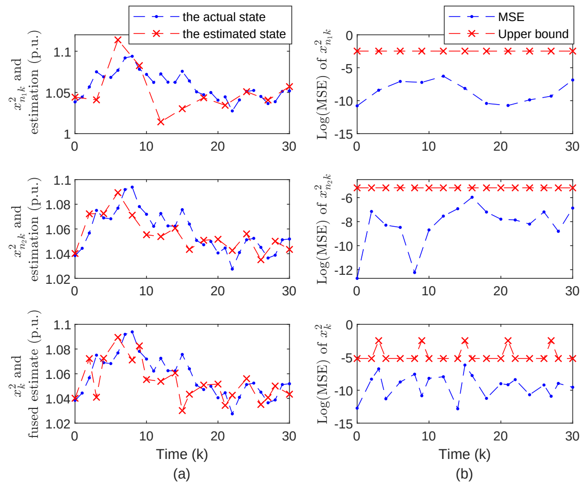

Fig. 3. Test results under fixed delay probability 0.4. (a) x_k^2 and its estimate. (b) Log(MSE) of $\hat{x}_k^2$ and its upper bound.

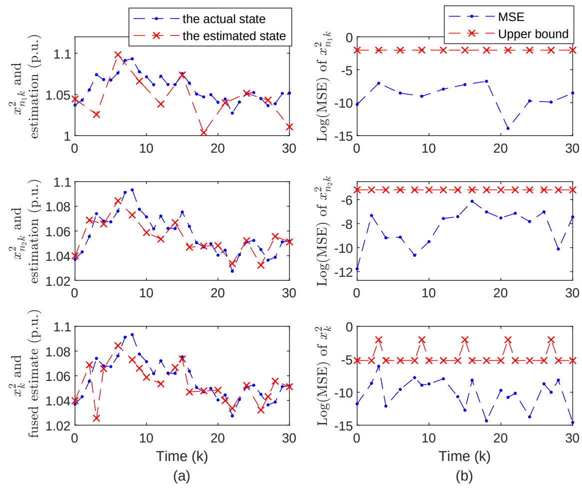

Fig. 4. Test results under random delay probability. (a) x_k^2 and its estimate. (b) Log(MSE) of $\hat{x}_k^2$ and its upper bound.

In this case, different probabilities of the randomly occurring SCADA measurement delays are considered. The sampling rates of the SCADA and PMU are chosen as $n_1 = 3$ and $n_2 = 2$, respectively. For comparison, first, we chose the Λ_{n1k} as $\text{Prob}\{\Lambda_{i,n_1k} = 0\} = 0.4$. Then, a more general case is considered where Λ_{n1k} is chosen as $\text{Prob}\{\Lambda_{i,n_1k} = 0\} = \mu_i$ with μ_i obeys the uniform distribution over the interval $[0.4, 0.8]$ at each iteration. The corresponding results are shown in Figs. 3 and 4, respectively.

From Figs. 3 and 4, we can find that: 1) the performance index is satisfied since the MSE stays below the relevant upper bound; 2) the fused estimate outperforms the local estimates since not only the advantages of the high sampling rate of the PMU are taken but also the SCADA measurements are efficiently used; and 3) the fusion estimation accuracy is still acceptable even with the severe SCADA measurement delays.

5 Conclusion

In this paper, we have investigated the fusion estimation problem for multi-rate power systems with randomly occurring SCADA measurement delays. A set of Bernoulli distributed random variables have been used to characterize the random SCADA measurement delays. In order to maximize the advantages of the high sampling rate of the PMU and utilize the SCADA measurements efficiently, an augmentation method has been proposed to process the asynchronous measurements with different sampling rates. Two local state estimators have been designed based on, respectively, the SCADA and the PMU measurement such that the local estimation error covariances have certain upper bounds. Then, such upper bounds have been minimized at each sampling instant by properly designing the local state estimator gains. Furthermore, the asynchronous local estimates have been fused by utilizing the CI fusion method and the consistency of the fused estimate has been guaranteed. Finally, a simulation based on the IEEE 14-bus system has been provided and the corresponding results have illustrated the effectiveness of the proposed fusion estimation scheme since: 1) the proposed fusion estimation scheme outperforms the conventional one; 2) the impact of the randomly occurring SCADA measurement delays has been better tackled; and 3) the fusion estimate outperforms the local estimates.

A Proof of Theorem 1

First, let us handle the right-hand side of (20) term by term. From Lemma 1, we have

$$\begin{aligned}
&\Theta_{1,n_1k}P_{s,n_1k|n_1(k-1)}\Theta_{1,n_1k}^T\\
\leq&\bar{\Theta}_{1,n_1k}\Gamma_{s,n_1k}^{-1}\bar{\Theta}_{1,n_1k}^T+\gamma_{1,n_1k}^{-1}K_{n_1k}\bar{\Lambda}C_{s,n_1k}C_{s,n_1k}^T\bar{\Lambda}K_{n_1k}^T,\\
&\Theta_{2,n_1k}P_{s,n_1(k-1)|n_1(k-2)}\Theta_{2,n_1k}^T\\
\leq&\bar{\Theta}_{2,n_1k}\Gamma_{s,n_1(k-1)}^{-1}\bar{\Theta}_{2,n_1k}^T\\
&+\gamma_{1,n_1(k-1)}^{-1}\bar{\Theta}_{3,n_1k}C_{s,n_1(k-1)}C_{s,n_1(k-1)}^T\bar{\Theta}_{3,n_1k}^T
\end{aligned}$$

with

$$\begin{aligned}
\Gamma_{s,n_1k}=&P_{s,n_1k|n_1(k-1)}^{-1}-\gamma_{1,n_1k}L_{s,n_1k}^TL_{s,n_1k},\\
\bar{\Theta}_{2,n_1k}=&K_{n_1k}(I-\bar{\Lambda})H_{s,n_1(k-1)},\\
\bar{\Theta}_{3,n_1k}=&K_{n_1k}(I-\bar{\Lambda})
\end{aligned}$$

if

$$\gamma_{1,n_1k}^{-1}I-L_{s,n_1k}\Sigma_{s,n_1k|n_1(k-1)}L_{s,n_1k}^T>0.$$

Moreover, the second and fifth terms on the right-hand side of (20) can be tackled as the following form:

$$\begin{aligned}
&K_{n_1k}(\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1k|n_1(k-1)}\})K_{n_1k}^T\\
\leq&K_{n_1k}(\tilde{\Lambda}\circ\operatorname{tr}(\Psi_{n_1k|n_1(k-1)})I)K_{n_1k}^T.
\end{aligned}$$

By using Lemmas 2 and 3, it is obvious that

$$\begin{aligned}
&\mathscr{L}_{1,n_1k}+\mathscr{L}_{1,n_1k}^T\\
\leq&\varepsilon_{1,n_1k}\Theta_{1,n_1k}P_{s,n_1k|n_1(k-1)}\Theta_{1,n_1k}^T\\
&+\varepsilon_{1,n_1k}^{-1}K_{n_1k}(\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1k|n_1(k-1)}\})K_{n_1k}^T,\\
&\mathscr{L}_{2,n_1k}+\mathscr{L}_{2,n_1k}^T\\
\leq&\varepsilon_{2,n_1k}\Theta_{2,n_1k}P_{s,n_1(k-1)|n_1(k-2)}\Theta_{2,n_1k}^T\\
&+\varepsilon_{2,n_1k}^{-1}K_{n_1k}(\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1(k-1)|n_1(k-2)}\})K_{n_1k}^T,\\
&\mathscr{L}_{3,n_1k}+\mathscr{L}_{3,n_1k}^T\\
\leq&\varepsilon_{3,n_1k}\Theta_{1,n_1k}P_{s,n_1k|n_1(k-1)}\Theta_{1,n_1k}^T\\
&+\varepsilon_{3,n_1k}^{-1}\Theta_{2,n_1k}P_{s,n_1(k-1)|n_1(k-2)}\Theta_{2,n_1k}^T,\\
&\mathscr{L}_{4,n_1k}+\mathscr{L}_{4,n_1k}^T\\
\leq&\varepsilon_{4,n_1k}\Theta_{1,n_1k}P_{s,n_1k|n_1(k-1)}\Theta_{1,n_1k}^T\\
&+\varepsilon_{4,n_1k}^{-1}K_{n_1k}(\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1(k-1)|n_1(k-2)}\})K_{n_1k}^T,\\
&\mathscr{L}_{5,n_1k}+\mathscr{L}_{5,n_1k}^T\\
\leq&\varepsilon_{5,n_1k}K_{n_1k}(\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1k|n_1(k-1)}\})K_{n_1k}^T\\
&+\varepsilon_{5,n_1k}^{-1}\Theta_{2,n_1k}P_{s,n_1(k-1)|n_1(k-2)}\Theta_{2,n_1k}^T,\\
&\mathscr{L}_{6,n_1k}+\mathscr{L}_{6,n_1k}^T\\
\leq&\varepsilon_{6,n_1k}K_{n_1k}(\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1k|n_1(k-1)}\})K_{n_1k}^T\\
&+\varepsilon_{6,n_1k}^{-1}K_{n_1k}(\tilde{\Lambda}\circ\mathbb{E}\{\Psi_{n_1(k-1)|n_1(k-2)}\})K_{n_1k}^T.
\end{aligned}$$

Summarizing the above discussions, we have

$$\begin{aligned}
&P_{s,n_1k|n_1k}\\
\leq&\kappa_{1,n_1k}\bar{\Theta}_{1,n_1k}\Gamma_{s,n_1k}^{-1}\bar{\Theta}_{1,n_1k}^T\\
&+\kappa_{1,n_1k}\gamma_{1,n_1k}^{-1}K_{n_1k}\bar{\Lambda}C_{s,n_1k}C_{s,n_1k}^T\bar{\Lambda}K_{n_1k}^T\\
&+\kappa_{2,n_1k}K_{n_1k}(\tilde{\Lambda}\circ\operatorname{tr}(\Psi_{n_1k|n_1(k-1)})I)K_{n_1k}^T\\
&+\kappa_{3,n_1k}\bar{\Theta}_{2,n_1k}\Gamma_{s,n_1(k-1)}^{-1}\bar{\Theta}_{2,n_1k}^T\\
&+\kappa_{3,n_1k}\gamma_{1,n_1(k-1)}^{-1}\bar{\Theta}_{3,n_1k}C_{s,n_1(k-1)}C_{s,n_1(k-1)}^T\bar{\Theta}_{3,n_1k}^T\\
&+\kappa_{4,n_1k}K_{n_1k}(\tilde{\Lambda}\circ\operatorname{tr}(\Psi_{n_1(k-1)|n_1(k-2)})I)K_{n_1k}^T\\
&+K_{n_1k}\bar{\Lambda}R_{s,n_1k}\bar{\Lambda}K_{n_1k}^T+\bar{\Theta}_{3,n_1k}R_{s,n_1(k-1)}\bar{\Theta}_{3,n_1k}^T.
\end{aligned}$$

Based on the mathematical induction method, it is not difficult to verify that $P_{s,n_1k|n_1k}\leq\Sigma_{s,n_1k|n_1k}$.

Next, taking the partial derivatives of $\Sigma_{s,n_1k|n_1k}$ with respect to K_{n_1k} and letting the derivative be zero, we have

$$\begin{aligned}
&\frac{\partial\operatorname{tr}(\Sigma_{s,n_1k|n_1k})}{\partial K_{n_1k}}\\
=&-2\kappa_{1,n_1k}\bar{\Theta}_{1,n_1k}\Omega_{s,n_1k}^{-1}H_{s,n_1k}^T\bar{\Lambda}\\
&+2\kappa_{1,n_1k}\gamma_{1,n_1k}^{-1}K_{n_1k}\bar{\Lambda}C_{s,n_1k}C_{s,n_1k}^T\bar{\Lambda}\\
&+2\kappa_{2,n_1k}K_{n_1k}(\tilde{\Lambda}\circ\operatorname{tr}(\Psi_{n_1k|n_1(k-1)})I)\\
&+2\kappa_{3,n_1k}\bar{\Theta}_{2,n_1k}\Omega_{s,n_1(k-1)}^{-1}H_{s,n_1(k-1)}^T\big(I-\bar{\Lambda}\big)\\
&+2\kappa_{3,n_1k}\gamma_{1,n_1(k-1)}^{-1}\bar{\Theta}_{3,n_1k}C_{s,n_1(k-1)}C_{s,n_1(k-1)}^T(I-\bar{\Lambda})\\
&+2\kappa_{4,n_1k}K_{n_1k}(\tilde{\Lambda}\circ\operatorname{tr}(\Psi_{n_1(k-1)|n_1(k-2)})I)\\
&+2K_{n_1k}\bar{\Lambda}R_{s,n_1k}\bar{\Lambda}+2\bar{\Theta}_{3,n_1k}R_{s,n_1(k-1)}(I-\bar{\Lambda})=0.
\end{aligned}$$

Based on the above equation, the estimator gain K_{n_1k} that minimizes $\Sigma_{s,n_1k|n_1k}$ is obtained as (24). The proof is complete.

B Proof of Theorem 2

Based on Lemma 1, the first term of the right-hand side of (28) satisfies

$$\begin{aligned}&\Pi_{n_2k}P_{p,n_2k|n_2(k-1)}\Pi_{n_2k}^T\\ \leq&\bar{\Pi}_{n_2k}\Gamma_{p,n_2k}^{-1}\bar{\Pi}_{n_2k}^T+\gamma_{2,n_2k}^{-1}G_{n_2k}C_{p,n_2k}C_{p,n_2k}^TG_{n_2k}^T\end{aligned}$$

with

$$\Gamma_{p,n_2k}=P_{p,n_2k|n_2(k-1)}^{-1}-\gamma_{2,n_2k}L_{p,n_2k}^TL_{p,n_2k}$$

if

$$\gamma_{2,n_2k}^{-1}I-L_{p,n_2k}\Sigma_{p,n_2k|n_2(k-1)}L_{p,n_2k}^T>0.$$

Then, we have

$$\begin{aligned}P_{p,n_2k|n_2k}\leq&\bar{\Pi}_{n_2k}\Gamma_{p,n_2k}^{-1}\bar{\Pi}_{n_2k}^T+\gamma_{2,n_2k}^{-1}G_{n_2k}C_{p,n_2k}\\&\times C_{p,n_2k}^TG_{n_2k}^T+G_{n_2k}R_{p,n_2k}G_{n_2k}^T.\end{aligned}$$

Again, based on the mathematical induction approach, we can conclude that $P_{p,n_2k|n_2k}\leq\Sigma_{p,n_2k|n_2k}$.

Next, taking the partial derivatives of $\Sigma_{p,n_2k|n_2k}$ with respect to G_{n_2k} and letting the derivative be zero, we have

$$\begin{aligned}&\frac{\partial\text{tr}(\Sigma_{p,n_2k|n_2k})}{\partial G_{n_2k}}\\=&-2\bar{\Pi}_{n_2k}\Gamma_{n_2k}^{-1}H_{p,n_2k}^T+2\gamma_{2,n_2k}^{-1}G_{n_2k}C_{p,n_2k}C_{p,n_2k}^T\\&+2G_{n_2k}R_{p,n_2k}=0.\end{aligned}$$

Through some algebraic manipulations, the estimator gain G_{n_2k} that minimizes $\Sigma_{p,n_2k|n_2k}$ is obtained as (32). The proof is complete.

C Proof of Theorem 3

The proof of Theorem 3 is readily accessible from [11], and is thus omitted here.

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