Recursive Secure Filtering over Gilbert-Elliott Channels in Sensor Networks: The Distributed Case

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***Abstract*—This paper is concerned with the recursive secure filtering problem for a class of discrete-time systems subject to unreliable communication due to the security vulnerability of sensor networks. The unreliable communication, caused probably by denial-of-service cyber-attacks, is described by the well-known Gilbert-Elliott model. The addressed nonlinearities are applicable for some of the most investigated stochastic nonlinear models, including the well-known state-dependent multiplicative noises as special cases. The aim of this paper is to design a novel distributed filter that uses the information not only from the individual node itself but also from its neighboring nodes according to the given topology. In order to improve the security of designed filter, a χ^2 detector is utilized to detect abnormal innovations. By means of the failure and recovery rates of the Gilbert-Elliott channels, sufficient conditions are established to ensure the existence of an upper bound on the estimation error covariance, and then the desired filter parameters are designed by minimizing the trace of such an upper bound. The asymptotic boundedness of the estimation error covariance is subsequently investigated. Finally, a simulation example on the target tracking problem is employed to verify the effectiveness and the security of the proposed filtering scheme.**

***Index Terms*—Secure filtering; sensor networks; distributed filtering; Gilbert-Elliott channels; χ^2 test.**

I. Introduction

A typical sensor network (SN) consists of a large number of smart sensors spatially deployed in some predetermined areas of interest. Powered by batteries, sensor nodes are usually linked together according to a given topology to carry out the tasks of information collection and processing in a collaborative way [5], [10], [11], [18], [24]. Benefiting from the configuration convenience and deployment flexibility [12], [32], SNs have been regarded as a kind of fashionable information sensing/processing platform with promising application prospect, thereby attracting ever-increasing research attention from various engineering areas [25], [28], [31]. In SNs, it is quite common that collected information needs to be exchanged among neighboring nodes and/or transmitted to a fusion center for target state estimation. Due to the resource limitation on both energy usage and communication bandwidth, the *centralized* data processing at the fusion center could be infeasible or inefficient especially for large scale SNs. As such, in parallel with the quiet evolution of centralized fusion techniques, the distributed filtering algorithms have received considerable research interest in the past few years. It should be pointed out that the main idea of the distributed filtering algorithms is to decentralize the function of fusion center to each intelligent sensor [14].

For distributed filtering algorithms over SNs, there are two critical issues deserving intensive investigations: 1) how to fuse the measurements or predicted states from the individual node itself and its neighboring nodes in order to improve the filtering performance [7], and 2) how to design numerically appealing filtering algorithms with desired *scalability* towards *real-time* implementation. According to the way of utilizing the obtained information, existing recursive distributed filtering strategies can be generally divided into four categories. The first category consists of the Kalman-consensus filtering algorithms with a typical two-stage form [1], in which an L-step calculation needs to be implemented at each instant to achieve the consensus performance. The second category is composed of the diffusion Kalman filtering algorithms [2] which actually replace the consensus strategies in the first category by diffusion strategies after the measurement update. The third category comprises the weighted innovation-sum-based algorithms [8] whose innovation terms involve the weighted sum of errors of the measurements from its neighboring nodes and the predicted measurement from the individual node itself. The last category is made of the so-called prediction-error-based algorithms [23] which replace the innovation in the third category by the weighted sum of errors of predicted states from its neighboring nodes and the individual node itself. Obviously, for all the four categories, the scalability requirement becomes particularly important with the introduced weighted sum.

In practical engineering, it is not uncommon that the communication-based measurements suffer from the security vulnerability of SNs and therefore become unreliable. On the one hand, adversaries can jam the shared network channel to prevent designed filters from communicating with their neighboring filters within a distributed architecture. This kind of phenomenon can be regarded as denial-of-service attacks or successive packet dropouts from different engineering points of view [34]. In this case, the channel states are temporally correlated and cannot be exactly described via the conventional Bernoulli distributed white sequence. Fortunately, a time-

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homogeneous binary Markov process can be employed to describe the transformation characteristics of the channel states [29], for which the corresponding model is referred to as the Gilbert-Elliott channel [14], [25]. On the other hand, if not protected by hardware/software strategies, smart sensor nodes could be vulnerable to malware damages and the stored data on sensor nodes might be corrupted, which leads to significant deviation from its real measurements [8]. In this scenario, the filtering performance would be unavoidably deteriorated if the corrupted data are utilized to estimate the system states [20]. Accordingly, it is of great importance to propose a suitable filter structure capable of cyber defense in order to facilitate the target monitoring in a secure manner.

When cyber security is a major concern, much progress has been made on the general filtering problems so far, see e.g. [21], [26], [27] for resilient filtering problems, [33] for the attack scheduling problems, and [19], [22] for the attack detection problems. To be more specific, when the addressed linear system is $2s$-sparse observable, two state reconstruction algorithms have been proposed in [26] via a batch of sensor measurements subject to sparse malicious attacks. Some optimal schemes of attack scheduling with energy constraints have been designed in [33] to decide whether to jam the channel. Up to now, to the best of the authors' knowledge, the filtering problem with a defense strategy over sensor networks has not been adequately addressed yet, which still remains as a challenging research topic. Obviously, the main challenges stem from the rather stringent security requirements, which are identified as follows: 1) the designed defense strategies should be realizable from the engineering point of view; 2) it is essentially difficult to estimate the inspection probability of corrupted data for distributed filters with designed defense strategies; and 3) it is nontrivial to design the desired filter gains due to the complicated calculation of the error covariances. Note that the single-sensor-based centralized filtering schemes without defense strategies have been thoroughly examined by constructing a sequence of stopping times, see e.g. [29]. Unfortunately, such an approach is no longer applicable to the distributed secure filter design problems to be addressed in this paper because the estimation error covariance cannot be accurately calculated at two adjacent moments of stopping times.

Summarizing the above discussions, in this paper, we focus our attention on the distributed secure filtering problem with a defense strategy over the Gilbert-Elliott channels. The main contributions are highlighted as follows: *1) a novel distributed filter is designed by embedding a χ^2 detector to identify unreliable measurements due to malicious attacks or outliers; 2) the gains of the designed distributed filters are dependent on the solution to a Riccati-like difference equation, and the computational complexity is therefore unrelated to the scale of underlying SNs; and 3) rigorous analysis is carried out on the failure and recovery rates of channels in order to ensure the boundedness of designed secure filtering algorithm.*

The rest of this paper is organized as follows. Section II briefly introduces the problem under consideration. In Section III, the evolutions of both the one-step prediction error covariance and the estimation error covariance are derived. Then, the filter gains are designed to ensure the existence of an locally optimized upper bound on the estimation error covariance at each sampling instant. For the designed filters, a boundedness condition on the estimation error covariance is proposed in Section IV. An illustrative example is provided in Section V to show the effectiveness of the proposed method. Finally, the paper is concluded in Section VI.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. $\mathbb{N}_m$ means the positive integer set $\{1,2,\cdots,m\}$. I denotes the identity matrix of compatible dimension. $[A]_i$ means a row vector whose elements come from the ith row of matrix A. $\mathcal{N}(0,1)$ denotes the Gaussian distribution with mean 0 and variance 1, and χ^2_m stands for the chi-square (i.e. χ^2) distribution with m freedom degrees.

II. Problem Formulation and Preliminaries

The triple $\mathscr{G}=(\mathcal{V},\mathcal{E},\mathcal{H})$ in this paper is employed to describe the underlying SNs. For this triple, $\mathcal{V}=\mathbb{N}_m$ and $\mathcal{E}\in\mathbb{N}_m\times\mathbb{N}_m$ stand for, respectively, the sets of nodes and edges, and $\mathcal{H}=[h_{ij}]_{m\times m}$ with nonnegative adjacency element h_{ij} represents the weighted adjacency matrix. An edge of $\mathscr{G}$ is usually denoted by the ordered pair (i,j), and the adjacency elements associated with the edges is positive, i.e., $h_{ij}>0 \Longleftrightarrow (i,j)\in\mathcal{E}$, which means that sensor i can receive information from sensor j. The set of neighbors of node i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}:(i,j)\in\mathscr{E}\}$, and $\lfloor\mathcal{N}_i\rfloor$ is the number of neighbors of node i. In addition, the Laplacian matrix of this graph is defined as $\mathcal{G}=\mathcal{H}-\text{diag}\{\sum_{j=1}^m h_{1j},\sum_{j=2}^m h_{2j},\cdots,\sum_{j=1}^m h_{mj}\}$. For the purpose of convenience, we further assume that the topology graph of SNs is a strongly connected directed graph and the row sum of weighted adjacency matrix $\mathcal{H}$ is equal to one. Under this assumption, we always have $\sum_{j=1}^m h_{ij}^2\le 1$.

We consider the following discrete-time system:

$$x_{k+1}=Ax_k+Bw_k \tag{1}$$

with measurements

$$y_{i,k}=C_ix_k+D\nu_{i,k},\quad i\in\mathbb{N}_m \tag{2}$$

where $x_k\in\mathbb{R}^{n_x}$ is the state of target plant that cannot be observed directly, $y_{i,k}\in\mathbb{R}^{n_y}$ is the measurement output from sensor i, and $\{w_k\}_{k\ge0}$ and $\{\nu_{i,k}\}_{k\ge0}$ are independent and identically distributed (i.i.d) sequences obeying the Gaussian distribution $\mathcal{N}(0,I)$. All stochastic variables and the initial state x_0 are mutually independent. A, B, C_i ($i\in\mathbb{N}_m$), and D are known matrices with compatible dimensions.

Assumption 1: The pairs $(A,\sqrt{BB^T})$ and $((I-\kappa\mathcal{H})\otimes A,C)$ are, respectively, stabilizable and observable, where $C=\text{diag}\{C_1,C_2,\cdots,C_m\}$.

In this paper, the signal is transmitted over shared Gilbert-Elliott channels, under which the packet loss process is modeled by a time-homogeneous two-state Markov chain with the state space $\{0,1\}$. To be precise, this Markov sequence is denoted as $\{\delta_k\}_{k\ge0}$ and its transition probability matrix is as follows

$$\mathbf{P}=\begin{bmatrix}1-q & q\\ p & 1-p\end{bmatrix}$$

where $p = \mathbb{P}\{\delta_{k+1} = 0|\delta_k = 1\}$ and $q = \mathbb{P}\{\delta_{k+1} = 1|\delta_k = 0\}$ are called the failure rate and the recovery rate, respectively. For presentation convenience, we denote the received information $\hat{x}^{r+}_{i,k}$ by neighbors of filter i as

$$\hat{x}^{r+}_{i,k} = \begin{cases} \hat{x}^{+}_{i,k}, & \delta_k = 1; \\ 0, & \delta_k = 0, \end{cases} \tag{3}$$

where the second case in (3) means that $\hat{x}^{r+}_{i,k}$ is set as zero when no information from filter i is received. Introduce the following indicator function:

$$\delta^{r}_{i,k} = \begin{cases} 1, & \sum_{j\in\mathcal{N}_i} \|\hat{x}^{r+}_{j,k}\| > 0; \\ 0, & \sum_{j\in\mathcal{N}_i} \|\hat{x}^{r+}_{j,k}\| = 0, \end{cases} \tag{4}$$

which can be utilized to detect the channel state δ_k. In light of the probability theory, one has $\mathbb{P}\{\sum_{j\in\mathcal{N}_i} \|\hat{x}^{r+}_{j,k}\| = 0|\delta_k = 1\} = 0$ (or $\mathbb{P}\{\delta^{r}_{i,k} = \delta_k\} = 1$ a.s.) because x_k and $y_{i,k}$ are driven by Gaussian white noises with continuous probability density functions. Therefore, variables $\delta^{r}_{i,k}$ and δ_k have the same statistical characteristics. Furthermore, one has

$$\begin{aligned} &\mathbb{P}\{\delta^{r}_{i,k+1} = 0|\delta^{r}_{i,k} = 1\} \\ =& \mathbb{P}\{\delta^{r}_{i,k+1} = 0, \delta_{k+1} = 0|\delta^{r}_{i,k} = 1\} \\ &+ \mathbb{P}\{\delta^{r}_{i,k+1} = 0, \delta_{k+1} = 1|\delta^{r}_{i,k} = 1\} \\ =& \mathbb{P}\{\delta_{k+1} = 0|\delta^{r}_{i,k} = 1, \delta_k = 1\} \\ &+ \mathbb{P}\{\delta_{k+1} = 0|\delta^{r}_{i,k} = 1, \delta_k = 0\} \\ =& \mathbb{P}\{\delta_{k+1} = 0|\delta_k = 1\} \end{aligned}$$

and

$$\mathbb{P}\{\delta^{r}_{i,k+1} = 1|\delta^{r}_{i,k} = 0\} = \mathbb{P}\{\delta_{k+1} = 1|\delta_k = 0\}.$$

Denote $\hat{x}^{-}_{i,k}$ and $\hat{x}^{+}_{i,k}$ as the one-step prediction and estimate of the target state x_k at instant k, respectively. Furthermore, define the corresponding prediction and estimation error covariance as follows:

$$\begin{aligned} P^{-}_{i,k} &= \mathbb{E}\{(x_k - \hat{x}^{-}_{i,k})(x_k - \hat{x}^{-}_{i,k})^T\}, \\ P^{+}_{i,k} &= \mathbb{E}\{(x_k - \hat{x}^{+}_{i,k})(x_k - \hat{x}^{+}_{i,k})^T\}. \end{aligned}$$

In what follows, according to the famous Kalman filtering theory, the innovation, denoted as $\eta_{i,k} = y_{i,k} - C_i\hat{x}^{-}_{i,k}$, obeys the Gaussian distribution with variance $P_{\eta,i,k} = C_iP^{-}_{i,k}C_i^T + DD^T$. As such, the square of the Mahalanobis distance of the above innovation is χ^2 distributed, that is

$$M_{i,k} = \eta^T_{i,k}P^{-1}_{\eta,i,k}\eta_{i,k} \sim \chi^2_{n^y} \tag{5}$$

with the freedom degree n^y. In light of the hypothesis test, for a given level σ, one has

$$\mathbb{P}(M_{i,k} < \chi^2_{n^y,\sigma}) = 1 - \sigma$$

where $\chi^2_{n^y,\sigma}$ is usually called as the σ-quantile [3].

Due to the vulnerability of communication networks, the adversary may overhear and modify the information in the transmitted data packets in order to yield a larger estimation error, which will produce some negative impacts on the operation of systems. In this paper, we only consider the case that attackers do not have knowledge of full network topology and system parameters. In other words, they cannot carry out stealth attack. In this paper, the attack model is model by

$$y_{i,k} = C_ix_k + D\nu_{i,k} + n_{i,k}, \quad i \in \mathbb{N}_m$$

where $n_{i,k}$ is any unknown data injected by attackers. For this kind of scenario, (5) is adopted to detect abnormal measurements or sensor attacks. In order to describe this inspection mechanism in filters, we introduce the following indicator function:

$$\vartheta_{i,k} = \begin{cases} 1, & M_{i,k} < \chi^2_{n^y,\sigma}, \\ 0, & \text{otherwise}. \end{cases} \tag{6}$$

By means of (6), we construct the following distributed secure filter on sensor i:

$$\begin{cases} \hat{\xi}^{+}_{i,k} = \sum_{j\in\mathcal{N}_i} h_{ij}\big(\hat{x}^{r+}_{j,k} - \delta^{r}_{i,k}\hat{x}^{+}_{i,k}\big), \\ \hat{x}^{-}_{i,k+1} = A\hat{x}^{+}_{i,k} + \kappa A\hat{\xi}^{+}_{i,k}, \\ \hat{x}^{+}_{i,k+1} = \hat{x}^{-}_{i,k+1} + \vartheta_{i,k+1}K_{i,k+1}\eta_{i,k+1} \end{cases} \tag{7}$$

where $\kappa \in (0,\ 1)$ is a predetermined coupling strength and $K_{i,k+1}$ is the filter gain to be determined. This kind of filters is also named as distributed secure filters due to the utilization of secure detectors of abnormal measurements or sensor attacks.

Remark 1: In comparison with some existing schemes, the constructed distributed filter (7) exhibits distinct novelty in that the innovation inspection is introduced to remove the abnormal data or outliers that might result from false data-injection attacks or abnormal interferences of sensors. Specifically, in case the innovation is abnormal, we have that $\vartheta_{i,k} = 0$ and therefore the negative impact from the data abnormality is minimized. Furthermore, the model of Gilbert-Elliott channels are capable of describing the phenomenon of denial of service (DoS) attacks implemented by jamming the shared network medium, where the real-time state of the communication channel, δ_k, is commonly detectable under the designed scheme.

Remark 2: In practical engineering, the inspection threshold $\chi^2_{n^y,\sigma}$ can be predetermined according to a given detection probability σ and the degree of freedom n^y. Furthermore, such a threshold (essentially a σ-quantile) can be found from the χ^2 distribution table. It follows from (6) that the implementation of attack detection mainly depends on the square of the Mahalanobis distance $M_{i,k}$ and therefore the detection real-time is high benefiting from its low calculation burden. Other detection approaches include Bayesian detection approaches with binary hypothesis, weighted least square approaches, artificial-intelligence-based detection strategies, and so forth.

Remark 3: In comparison with the fusion-aware consensus mechanism in [17], the role of the added consensus term in the employed filter (7) in this paper is just to reduce the disagreement potential, and the corresponding filtering error dynamic is, essentially, a large-scale system, see a similar structure in Assumption 3 in [31]. When omitting the behavior of the attack inspection (6) as well as the packet loss, the boundedness of error dynamic is definitely dependent on Assumption 1, which is similar to the collective observability in [17]. As such, there is no doubt that the main challenges for the addressed filtering issues are how to realize the distributed design of filter gains and how to disclose the impact on the boundedness from the failure and recovery rates of channels.

Denote the known information as $\mathcal{I}_{i,k}=\{\delta^r_{i,k},\vartheta_{i,k+1}\}$ on filter i. The aim of this paper is highlighted as twofold:

R1) Design a Kalman-type distributed secure filter with the form (7) and the known information $\mathcal{I}_{i,k}$ such that an upper bound of estimation error covariance is guaranteed in the presence of abnormal measurements or sensor attacks, that is, there exists a sequence of positive-definite matrices $\Pi^+_{i,k|k}$ satisfying

$$P^+_{i,k}\leq \Pi^+_{i,k},\quad \forall k>0 \tag{8}$$

where $M_{i,k}$ in the innovation inspection (6) is taken as $M_{i,k}=\eta^T_{i,k}\Gamma^{-1}_{\eta,i,k}\eta_{i,k}$ with $\Gamma_{\eta,i,k}$ being a positive-definite matrix to be designed. Furthermore, the sequence of upper bounds $\Pi^+_{i,k}$ is minimized via the designed filter parameters $K_{i,k}$;

R2) For the designed filter parameters $K_{i,k}$, find a condition on the failure rate p and the recovery rate q, under which the sequence $\Pi^+_{i,k}$ is asymptotically bounded as time tends to infinity.

III. Distributed Filter Design

In this section, we first deal with the unbiasedness, and then discuss the lower/upper bounds on both prediction and estimation error covariance of the proposed distributed secure filter. Furthermore, by optimizing the upper bound of the trace of the estimation error covariance, we aim to develop a new design scheme for the desired filter gain in terms of the solution to a Riccati-like difference equation.

Before proceeding further, we introduce the following mathematical operation. Specially, for two positive-definite matrices X and Y, we define the operations:

$$\min\{X,Y\}=\begin{cases} X, & X\leq Y\\ Y, & Y<X\\ \lambda^{XY}_{\min}I, & \text{otherwise}\end{cases}$$
$$\max\{X,Y\}=\begin{cases} Y, & X\leq Y\\ X, & Y<X\\ \lambda^{XY}_{\max}I, & \text{otherwise}\end{cases}$$

where $\lambda^{XY}_{\max}$ and $\lambda^{XY}_{\min}$ stand for, respectively, the maximum eigenvalue and minimum eigenvalue in all eigenvalues of X and Y.

In what follows, for the discrete-time system (1), if $\hat{x}^+_0=\mathbb{E}\{x_0\}$ for any $i\in\mathbb{N}_m$, one has

$$\begin{aligned}&\mathbb{E}\{x_{k+1}-\hat{x}^-_{i,k+1}\}\\ &=\mathbb{E}\{A(x_k-\hat{x}^+_{i,k})+Bw_k-\kappa A\hat{\xi}^+_{i,k}\}\\ &=-\kappa A\sum_{j\in\mathcal{N}_i}h_{ij}\mathbb{E}\{\delta^r_{i,k}\}\mathbb{E}\big\{\big(\hat{x}^+_{j,k}-x_k\big)+\big(x_k-\hat{x}^+_{i,k}\big)\big\}\\ &\quad-\kappa A\sum_{j\in\mathcal{N}_i}h_{ij}\mathbb{E}\{\delta_k-\delta^r_{i,k}\}\hat{x}^+_{j,k}\\ &=0,\end{aligned}\tag{9}$$

and

$$\begin{aligned}&\mathbb{E}\{x_{k+1}-\hat{x}^+_{i,k+1}\}\\ &=\mathbb{E}\{x_{k+1}-\hat{x}^-_{i,k+1}\}\\ &\quad-\mathbb{E}\{\vartheta_{i,k+1}K_{i,k+1}(C_i(x_{k+1}-\hat{x}^-_{i,k+1})+D\nu_{i,k})\}\\ &=0,\end{aligned}\tag{10}$$

which implies that the proposed filter is unbiased, that is, $\mathbb{E}\{x_k-\hat{x}^+_{i,k}\}=0$.

Lemma 1: For the distributed secure filter (7) with the known information $\mathcal{I}_{i,k}$, the covariance $P^-_{i,k+1}$ of one-step prediction errors and the covariance $P^+_{i,k+1}$ of estimation errors satisfy

$$P^-_{i,k+1}\leq(1+\kappa\delta^r_{i,k})AP^+_{i,k}A^T+\mathcal{Q}_{i,k},\tag{11}$$
$$\begin{aligned}P^+_{i,k+1}&=P^-_{i,k+1}-\vartheta_{i,k+1}K_{i,k+1}C_iP^-_{i,k+1}\\ &\quad-\vartheta_{i,k+1}P^-_{i,k+1}C_i^TK^T_{i,k+1}\\ &\quad+\vartheta_{i,k+1}K_{i,k+1}P_{\eta,i,k+1}K^T_{i,k+1}\end{aligned}\tag{12}$$

where

$$\begin{aligned}\Xi^+_{i,k}&=\sum_{j\in\mathcal{N}_i\cup\{i\}}2P^+_{j,k},\\ \mathcal{Q}_{i,k}&=BB^T+\delta^r_{i,k}\kappa(1+\kappa)A\Xi^+_{i,k}A^T.\end{aligned}$$

Proof: Define $e^+_{i,k}=x_k-\hat{x}^+_{i,k}$ and its derivative vector

$$e^+_k=[\begin{array}{cccc}e^{+T}_{1,k} & e^{+T}_{2,k} & \cdots & e^{+T}_{m,k}\end{array}]^T.$$

Then, the conditional expectation of $\hat{\xi}^+_{i,k}(\hat{\xi}^+_{i,k})^T$ is calculated as

$$\begin{aligned}&\mathbb{E}\big\{\hat{\xi}^+_{i,k}(\hat{\xi}^+_{i,k})^T\big|\delta^r_{i,k}\big\}\\ &=\delta^r_{i,k}\mathbb{E}\big\{([\mathcal{G}]_i\otimes I)e^+_ke^{+T}_k([\mathcal{G}]^T_i\otimes I)\big\}\\ &\quad+\mathbb{E}\{(\delta_k-\delta^r_{i,k})^2\big|\delta^r_{i,k}\}\Big(\sum_{s\in\mathcal{N}_i}h_{ij}\hat{x}^+_{j,k}\Big)\Big(\sum_{s\in\mathcal{N}_i}h_{ij}\hat{x}^+_{j,k}\Big)^T\\ &=\delta^r_{i,k}\mathbb{E}\Big\{P^+_{i,k}-e^+_{i,k}\Big(\sum_{j\in\mathcal{N}_i}h_{ij}e^+_{j,k}\Big)^T-\sum_{j\in\mathcal{N}_i}h_{ij}e^+_{j,k}(e^+_{i,k})^T\\ &\quad+\sum_{j\in\mathcal{N}_i}\sum_{s\in\mathcal{N}_i}h_{ij}h_{is}e^+_{j,k}(e^+_{s,k})^T\Big\}\\ &\leq\delta^r_{i,k}\Big(P^+_{i,k}+\sum_{j\in\mathcal{N}_i}h^2_{ij}P^+_{i,k}\\ &\quad+\sum_{j\in\mathcal{N}_i}P^+_{j,k}+\sum_{j\in\mathcal{N}_i}\sum_{s\in\mathcal{N}_i}h^2_{is}P^+_{j,k}\Big)\\ &\leq 2\delta^r_{i,k}\textstyle\sum_{j\in\mathcal{N}_i\cup\{i\}}P^+_{j,k}.\end{aligned}\tag{13}$$

In what follows, noting that $\delta^r_{i,k}$ takes a value in $\{0,1\}$, we calculate that

$$\begin{aligned}&\mathbb{E}\big\{\big(x_k-\hat{x}^+_{i,k}\big)(\hat{\xi}^+_{i,k})^T\big|\mathcal{I}_{i,k}\big\}\\ &=\mathbb{E}\big\{\delta^r_{i,k}\big(x_k-\hat{x}^+_{i,k}\big)(\hat{\xi}^+_{i,k})^T\big|\mathcal{I}_{i,k}\big\}\\ &\leq\frac{1}{2}(\delta^r_{i,k}P^+_{i,k}+\mathbb{E}\big\{\hat{\xi}^+_{i,k}(\hat{\xi}^+_{i,k})^T\big|\mathcal{I}_{i,k}\big\})\\ &\leq\frac{\delta^r_{i,k}}{2}(P^+_{i,k}+\Xi^+_{i,k}).\end{aligned}\tag{14}$$

Then, it follows from (14) that

$$\begin{aligned}P^-_{i,k+1}&=\mathbb{E}\{(x_{k+1}-\hat{x}^-_{i,k+1})(x_{k+1}-\hat{x}^-_{i,k+1})^T\}\\ &=\mathbb{E}\big\{\big(A(x_k-\hat{x}^+_{i,k})+Bw_k-\kappa A\hat{\xi}^+_{i,k}\big)\\ &\quad\times\big(A(x_k-\hat{x}^+_{i,k})+Bw_k-\kappa A\hat{\xi}^+_{i,k}\big)^T|\mathcal{I}_{i,k}\big\}\end{aligned}$$

$$
\begin{aligned}
&\leq (1+\kappa\delta^r_{i,k})AP^+_{i,k}A^T + BB^T \\
&\quad + \kappa(1+\kappa)A\mathbb{E}\{\hat{\xi}^+_{i,k}(\hat{\xi}^+_{i,k})^T|\mathcal{I}_{i,k}\}A^T, \qquad (15)
\end{aligned}
$$

which yields the relationship (11). Furthermore, according to the unbiasedness we have that

$$
\begin{aligned}
&P^+_{i,k+1} \\
&= \mathbb{E}\{(x_{k+1}-\hat{x}^+_{i,k+1})(x_{k+1}-\hat{x}^+_{i,k+1})^T\} \\
&= \mathbb{E}\big\{\big((I-\vartheta_{i,k+1}K_{i,k+1}C_i)(x_{k+1}-\hat{x}^-_{i,k+1}) \\
&\quad - \vartheta_{i,k+1}K_{i,k+1}D\nu_{i,k}\big)\big((I-\vartheta_{i,k+1}K_{i,k+1}C_i) \\
&\quad \times (x_{k+1}-\hat{x}^-_{i,k+1}) - \vartheta_{i,k+1}K_{i,k+1}D\nu_{i,k}\big)^T|\mathcal{I}_{i,k}\big\} \\
&= (I-\vartheta_{i,k+1}K_{i,k+1}C_i)P^-_{i,k+1}(I-\vartheta_{i,k+1}K_{i,k+1}C_i)^T \\
&\quad + \vartheta_{i,k+1}K_{i,k+1}DD^TK^T_{i,k+1}, \qquad (16)
\end{aligned}
$$

which results in (12). The proof is now complete. ■

Performing some necessary operations to (11) and (12), we have the following recursive equations on the lower and upper bounds of covariance matrices $P^+_{i,k}$.

Theorem 1: Let $\Gamma^-_{i,k} \leq P^-_{i,k}$, $\Gamma^+_{i,k} \leq P^+_{i,k}$ and $P^+_{i,k} \leq \Pi^+_{i,k}$. For the distributed secure filter (7) with the known information $\mathcal{I}_{i,k}$, a set of lower bounds $(\Gamma^-_{i,k+1}, \Gamma^+_{i,k+1})$ of the covariance matrix pair $(P^-_{i,k+1}, P^+_{i,k+1})$ is calculated by

$$
\Gamma^-_{i,k+1} = \begin{cases} \tilde{\mathcal{Q}}_{1i,k} + BB^T, & \tilde{\mathcal{Q}}_{1i,k} \geq 0 \\ BB^T, & \text{otherwise} \end{cases} \qquad (17)
$$

$$
\Gamma^+_{i,k+1} = \min\{\Gamma^-_{i,k+1}, \tilde{\mathcal{S}}_{1i,k+1}\} \qquad (18)
$$

where

$$
\begin{aligned}
\tilde{\Xi}^+_{i,k} &= \sum_{j\in\mathcal{N}_i\cup\{i\}} 2\Pi^+_{j,k}, \\
\tilde{\mathcal{S}}_{1i,k} &= K_{i,k}DD^TK^T_{i,k} \qquad (19) \\
&\quad + (I-K_{i,k}C_i)\Gamma^-_{i,k}(I-K_{i,k}C_i)^T, \\
\tilde{\mathcal{Q}}_{1i,k} &= (1-\kappa\delta^r_{i,k})A\Gamma^+_{i,k}A^T + \delta^r_{i,k}\kappa(\kappa-1)A\tilde{\Xi}^+_{i,k}A^T
\end{aligned}
$$

Proof: First, it is straightforward to see that

$$
\begin{aligned}
&\mathbb{E}\big\{\big(x_k-\hat{x}^+_{i,k}\big)(\hat{\xi}^+_{i,k})^T|\mathcal{I}_{i,k}\big\} \\
&\geq -\frac{\delta^r_{i,k}P^+_{i,k} + \mathbb{E}\{\hat{\xi}^+_{i,k}(\hat{\xi}^+_{i,k})^T\big|\delta^r_{i,k}\}}{2}. \qquad (20)
\end{aligned}
$$

Along the same line in deriving (15), one has

$$
\begin{aligned}
P^-_{i,k+1} &\geq (1-\kappa\delta^r_{i,k})AP^+_{i,k}A^T + BB^T \\
&\quad + \kappa(\kappa-1)A\mathbb{E}\{\hat{\xi}^+_{i,k}(\hat{\xi}^+_{i,k})^T\big|\delta^r_{i,k}\}A^T \\
&\geq (1-\kappa\delta^r_{i,k})AP^+_{i,k}A^T \\
&\quad + BB^T + \delta^r_{i,k}\kappa(\kappa-1)A\tilde{\Xi}^+_{i,k}A^T.
\end{aligned}
$$

Next, let us denote

$$
\begin{aligned}
\bar{P}^-_{i,k+1} &= (1-\kappa\delta^r_{i,k})AP^+_{i,k}A^T \\
&\quad + BB^T + \delta^r_{i,k}\kappa(\kappa-1)A\tilde{\Xi}^+_{i,k}A^T.
\end{aligned}
$$

If $\tilde{\mathcal{Q}}_{1i,k} \geq 0$, subtracting (17) from the above equation leads to

$$
\begin{aligned}
&(1-\kappa\delta^r_{i,k})A(P^+_{i,k}-\Gamma^+_{i,k})A^T \\
&= \bar{P}^-_{i,k+1} - \Gamma^-_{i,k+1} \\
&\leq P^-_{i,k+1} - \Gamma^-_{i,k+1}. \qquad (21)
\end{aligned}
$$

Furthermore, considering that $\tilde{\mathcal{Q}}_{1i,k}$ could be indefinite, one further has

$$
\begin{aligned}
P^-_{i,k+1} &= \mathbb{E}\big\{\big(A(x_k-\hat{x}^+_{i,k}) - \kappa A\hat{\xi}^+_{i,k}\big) \\
&\quad \times \big(A(x_k-\hat{x}^+_{i,k}) - \kappa A\hat{\xi}^+_{i,k}\big)^T|\mathcal{I}_{i,k}\big\} + BB^T \\
&\geq \Gamma^-_{i,k+1}. \qquad (22)
\end{aligned}
$$

Taking (21) and (22) into consideration, we have that $\Gamma^-_{i,k+1} \leq P^-_{i,k+1}$ when $\Gamma^+_{i,k} \leq P^+_{i,k}$.

On the other hand, when $\vartheta_{i,k+1}=0$, one can see from (12) and (18) that

$$
\begin{aligned}
P^+_{i,k+1} &= P^-_{i,k+1} \geq \Gamma^-_{i,k+1} \\
&\geq \min\{\Gamma^-_{i,k+1}, \tilde{\mathcal{S}}_{1i,k}\} = \Gamma^+_{i,k+1}. \qquad (23)
\end{aligned}
$$

In addition, when $\vartheta_{i,k+1}=1$, we derive that

$$
\begin{aligned}
&P^+_{i,k+1} - \Gamma^+_{i,k+1} \\
&= P^-_{i,k+1} - K_{i,k+1}C_iP^-_{i,k+1} - P^-_{i,k+1}C_i^TK^T_{i,k+1} \\
&\quad + K_{i,k+1}P_{\eta,i,k+1}K^T_{i,k+1} - \min\{\Gamma^-_{i,k+1}, \tilde{\mathcal{S}}_{1i,k}\} \\
&\geq (I-K_{i,k+1}C_i)\Upsilon^-_{\Gamma,i,k+1}(I-K_{i,k+1}C_i)^T \\
&\geq 0 \qquad (24)
\end{aligned}
$$

where $\Upsilon^-_{\Gamma,i,k+1} = P^-_{i,k+1} - \Gamma^-_{i,k+1} \geq 0$.

In light of (23) and (24), we conclude that $\Gamma^+_{i,k+1} \leq P^+_{i,k+1}$ when $\Gamma^-_{i,k+1} \leq P^-_{i,k+1}$. Finally, together with (21) and (22), one confirms that the iterative conditions (17)-(18) are true, and this completes the proof. ■

Considering the practical implementation of the innovation inspection (6), the improved inspection on innovation (2) by resorting to this lower bound is

$$
\tilde{\vartheta}_{i,k} = \begin{cases} 1, & \eta^T_{i,k}\Gamma^{-1}_{\eta,i,k}\eta_{i,k} < \chi^2_{n^y,\sigma}, \\ 0, & \text{otherwise}, \end{cases} \qquad (25)
$$

where $\Gamma_{\eta,i,k} = C_i\Gamma^-_{i,k}C_i^T + DD^T$. It should be pointed out that the resultant modified scheme is $\mathbb{P}\{\tilde{\vartheta}_{i,k}=1\} \leq \mathbb{P}\{\vartheta_{i,k}=1\}$ and therefore inevitably increases the conservatism of attack detection.

Theorem 2: Let $\Pi^-_{i,k} \geq P^-_{i,k}$ and $\Pi^+_{i,k} \geq P^+_{i,k}$. For the distributed secure filter (7) with the known information $\mathcal{I}_{i,k}$, a set of feasible upper bounds $(\Pi^-_{i,k+1}, \Pi^+_{i,k+1})$ of the covariance matrix pair $(P^-_{i,k+1}, P^+_{i,k+1})$ is computed by

$$
\Pi^-_{i,k+1} = (1+\kappa\delta^r_{i,k})A\Pi^+_{i,k}A^T + \tilde{\mathcal{Q}}_{2i,k}, \qquad (26)
$$

$$
\begin{aligned}
\Pi^+_{i,k+1} &= \tilde{\vartheta}_{i,k+1}\tilde{\mathcal{S}}_{2i,k+1} \\
&\quad + (1-\tilde{\vartheta}_{i,k+1})\max\{\Pi^-_{i,k+1}, \tilde{\mathcal{S}}_{2i,k+1}\} \qquad (27)
\end{aligned}
$$

where

$$
\begin{aligned}
\tilde{\mathcal{S}}_{2i,k+1} &= K_{i,k+1}DD^TK^T_{i,k+1} \\
&\quad + (I-K_{i,k+1}C_i)\Pi^-_{i,k+1}(I-K_{i,k+1}C_i)^T, \\
\tilde{\mathcal{Q}}_{2i,k} &= BB^T + \delta^r_{i,k}\kappa(1+\kappa)A\tilde{\Xi}^+_{i,k}A^T.
\end{aligned}
$$

Proof: First, for analysis convenience, let us denote

$$
\tilde{P}^-_{i,k+1} = (1+\kappa\delta^r_{i,k})AP^+_{i,k}A^T + \tilde{\mathcal{Q}}_{2i,k}.
$$

Obviously, one has $P^-_{i,k+1} \leq \tilde{P}^-_{i,k+1}$. Then, subtracting the above equation from (26) leads to

$$\begin{aligned}(1+\kappa\delta^r_{i,k})&A(\Pi^+_{i,k}-P^+_{i,k})A^T\\ &=\Pi^-_{i,k+1}-\tilde{P}^-_{i,k+1}\\ &\leq \Pi^-_{i,k+1}-P^-_{i,k+1}. \end{aligned} \tag{28}$$

Therefore, we obtain $P^-_{i,k+1} \leq \Pi^-_{i,k+1}$ when $P^+_{i,k} \leq \Pi^+_{i,k}$.

In what follows, we will address the relationship between $\Pi^+_{i,k+1}$ and $P^+_{i,k+1}$. In doing so, we first obtain

$$\eta^T_{i,k+1}P^{-1}_{\eta,i,k+1}\eta_{i,k+1} \leq \eta^T_{i,k+1}\Gamma^{-1}_{\eta,i,k+1}\eta_{i,k+1}.$$

Then, noting the definition (6) and (25), one has the following three cases:

- *Case 1)*: $\vartheta_{i,k+1}=0$ and $\tilde{\vartheta}_{i,k+1}=0$;
- *Case 2)*: $\vartheta_{i,k+1}=1$ and $\tilde{\vartheta}_{i,k+1}=0$;
- *Case 3)*: $\vartheta_{i,k+1}=1$ and $\tilde{\vartheta}_{i,k+1}=1$.

For the purpose of simplicity, we define

$$\begin{aligned}&\Upsilon^-_{\Pi,i,k+1}\\ &=(I-K_{i,k+1}C_i)(\Pi^-_{i,k+1}-P^-_{i,k+1})(I-K_{i,k+1}C_i)^T.\end{aligned}$$

Now, for *Case 1)* and *Case 2)*, we obtain from (12) and (27) that

$$\begin{aligned}&\Pi^+_{i,k+1}-P^+_{i,k+1}\\ =\ &\max\{\Pi^-_{i,k+1},\tilde{\mathcal{S}}_{2i,k+1}\}-(I-\vartheta_{i,k+1}K_{i,k+1}C_i)\\ &\times P^-_{i,k+1}(I-\vartheta_{i,k+1}K_{i,k+1}C_i)^T\\ &+\vartheta_{i,k+1}K_{i,k+1}DD^TK^T_{i,k+1}\\ \geq\ &\begin{cases}\Pi^-_{i,k+1}-P^-_{i,k+1}, & \vartheta_{i,k+1}=0,\\ \Upsilon^-_{\Pi,i,k+1}, & \vartheta_{i,k+1}=1.\end{cases}\end{aligned} \tag{29}$$

Obviously, one has $\Pi^+_{i,k+1}-P^+_{i,k+1} \geq 0$ when $P^-_{i,k+1} \leq \Pi^-_{i,k+1}$. Then, for *Case 3)*, one has

$$\Pi^+_{i,k+1}-P^+_{i,k+1} \geq \Upsilon^-_{\Pi,i,k+1} \geq 0. \tag{30}$$

Obviously, we conclude that the pair $(\Pi^-_{i,k+1},\Pi^+_{i,k+1})$ is a set of feasible upper bounds, and the proof is complete. ■

Next, one has

$$\frac{\partial \mathrm{Tr}(\tilde{\mathcal{S}}_{2i,k+1})}{\partial K_{i,k+1}} = -2\Pi^-_{i,k+1}C_i^T+2K_{i,k+1}\Pi_{\eta,i,k+1} \tag{31}$$

where $\Pi_{\eta,i,k+1}=C_i\Pi^-_{i,k+1}C_i^T+DD^T$. Clearly, when selecting

$$K_{i,k+1}=\Pi^-_{i,k+1}C_i^T\Pi^{-1}_{\eta,i,k+1}, \tag{32}$$

the trace of matrix $\tilde{\mathcal{S}}_{2i,k+1}$ is minimized, and the corresponding matrix is

$$\begin{aligned}\mathcal{S}^*_{i,k+1}&=\min_{K_{i,k+1}}\{\tilde{\mathcal{S}}_{2i,k+1}\}\\ &=\Pi^-_{i,k+1}-K_{i,k+1}C_i\Pi^-_{i,k+1}.\end{aligned} \tag{33}$$

Under this selection, $\Pi^-_{i,k+1} \geq \mathcal{S}^*_{i,k+1}$ is always satisfied.

Recalling the objective **R1**), the updated equation in (7) will be replaced by

$$\hat{x}^+_{i,k+1}=\hat{x}^-_{i,k+1}+\tilde{\vartheta}_{i,k+1}K_{i,k+1}\eta_{i,k+1} \tag{34}$$

and, therefore, we can obtain following theorem readily.

Theorem 3: For the discrete-time system (1) with the measurement (2) inspected by (25), let the gain matrix $K_{i,k+1}$ of the distributed secure filter (7) be calculated by (32) and the estimate of state x_{k+1} be updated by (34). Then, the objective **R1**) under the known information $\mathcal{I}_{i,k}=\{\delta^r_{i,k},\tilde{\vartheta}_{i,k+1}\}$ is achieved with the locally optimized upper bounds of error covariance given by

$$\begin{cases}\Pi^-_{i,k+1}=(1+\kappa\delta^r_{i,k})A\Pi^+_{i,k}A^T+\mathcal{Q}_{2i,k},\\ \Pi^+_{i,k+1}=\Pi^-_{i,k+1}-\tilde{\vartheta}_{i,k+1}K_{i,k+1}C_i\Pi^-_{i,k+1}.\end{cases} \tag{35}$$

Remark 4: The above theorem combing with (4) and (25) consists of the critical core of the distributed recursive filtering scheme. For each step of the proposed scheme, the implementation on node i includes 2 times of the matrix inversion operation (i.e. $\Gamma^{-1}_{\eta,i,k}$ and $\Pi^{-1}_{\eta,i,k+1}$) and $52+3\lfloor\mathcal{N}_i\rfloor$ times of the matrix multiplication operation. With the help of the dimensions of $x_k \in \mathbb{R}^{n_x}$, it is not difficult to calculate the overall computational complexity on node i as $O(n_x^3)$, which is independence of the scale of whole sensor networks. Due to the sparseness of sensor networks, such an algorithm with low computation burden is really suitable for online application.

IV. Performance Analysis

In this section, we will analyze the boundedness of estimation error covariance of the proposed distributed secure filters in the mean-square sense. To this end, the following assumption is necessary.

Assumption 2: There exist positive real numbers $\bar{\alpha}$, $\underline{c}$, $\bar{c}$, $\lfloor\bar{\mathcal{N}}\rfloor$ and ν such that the following conditions on the bounds of the system and measurement matrices, the number of neighbors as well as the probability of the innovation inspection are fulfilled:

$$\begin{gathered}\|A\| \leq \bar{\alpha},\quad \underline{c} \leq \|C_i\| \leq \bar{c},\\ 1+\lfloor\mathcal{N}_i\rfloor \leq \lfloor\bar{\mathcal{N}}\rfloor,\quad \left\|BB^T\right\| \leq \bar{\tau},\\ \mathbb{P}\{\tilde{\vartheta}_{i,k+1}=0,\vartheta_{i,k+1}=1\} \leq \nu.\end{gathered}$$

Considering $\tilde{\mathcal{Q}}_{2i,k}$ in (26), one has

$$\mathbb{E}\big\{\mathcal{Q}_{i,k}\big|\delta_k\big\} \leq BB^T+\delta^r_{i,k}\mathbb{E}\{\tilde{\Pi}^+_{\mathcal{N}_i,k}\} \tag{36}$$

where

$$\tilde{\Pi}^+_{\mathcal{N}_i,k}=2\kappa(1+\kappa)\sum\nolimits_{j\in\mathcal{N}_i\cup\{i\}}A\Pi^+_{j,k}A^T.$$

In light of the above inequality, one has the conditional expectation on $\Pi^-_{i,k+1}$:

$$\mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=0\} \leq A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T \tag{37}$$

and

$$\begin{aligned}&\mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=1\}\\ &\leq (1+\kappa)A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T+\mathbb{E}\{\tilde{\Pi}^+_{\mathcal{N}_i,k}\}.\end{aligned} \tag{38}$$

Furthermore, taking the transition probability (describing Gilbert-Elliott channels) into consideration, one further has that

$$\mathbb{E}\{\Pi^-_{i,k+1}\}$$

$$
\begin{aligned}
&= \mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=0\}\mathbb{P}\{\delta^r_{i,k}=0\}\\
&\quad + \mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=1\}\mathbb{P}\{\delta^r_{i,k}=1\}\\
&= \mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=0\}\mathbb{P}\{\delta^r_{i,k}=0|\delta^r_{i,k-1}=0\}\mathbb{P}\{\delta^r_{i,k-1}=0\}\\
&\quad + \mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=0\}\mathbb{P}\{\delta^r_{i,k}=0|\delta^r_{i,k-1}=1\}\mathbb{P}\{\delta^r_{i,k-1}=1\}\\
&\quad + \mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=1\}\mathbb{P}\{\delta^r_{i,k}=1|\delta^r_{i,k-1}=0\}\mathbb{P}\{\delta^r_{i,k-1}=0\}\\
&\quad + \mathbb{E}\{\Pi^-_{i,k+1}|\delta^r_{i,k}=1\}\mathbb{P}\{\delta^r_{i,k}=1|\delta^r_{i,k-1}=1\}\mathbb{P}\{\delta^r_{i,k-1}=1\}\\
&\le (1-q)(A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T)\mathbb{P}\{\delta^r_{i,k-1}=0\}\\
&\quad + p(A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T)\mathbb{P}\{\delta^r_{i,k-1}=1\}\\
&\quad + q((1+\kappa)A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T\\
&\quad + \mathbb{E}\{\tilde{\Pi}^+_{\mathcal{N}_i,k}\})\mathbb{P}\{\delta^r_{i,k-1}=0\}\\
&\quad + (1-p)\big((1+\kappa)A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T\\
&\quad + \mathbb{E}\{\tilde{\Pi}^+_{\mathcal{N}_i,k}\}\big)\mathbb{P}\{\delta^r_{i,k-1}=1\}.
\end{aligned} \tag{39}
$$

Similarly, it is straightforward to see that

$$
\begin{aligned}
&\mathbb{E}\{\Pi^+_{i,k+1}\}\\
&= \mathbb{E}\{\Pi^+_{i,k+1}|\tilde{\vartheta}_{i,k+1}=1\}\mathbb{P}\{\tilde{\vartheta}_{i,k+1}=1\}\\
&\quad + \mathbb{E}\{\Pi^+_{i,k+1}|\tilde{\vartheta}_{i,k+1}=0\}\mathbb{P}\{\tilde{\vartheta}_{i,k+1}=0\}\\
&\le \mathbb{E}\{\Pi^+_{i,k+1}|\tilde{\vartheta}_{i,k+1}=1\}\mathbb{P}\{\tilde{\vartheta}_{i,k+1}=1\}\\
&\quad + \mathbb{E}\{\Pi^+_{i,k+1}|\tilde{\vartheta}_{i,k+1}=0\}\mathbb{P}\{\tilde{\vartheta}_{i,k+1}=0\}\\
&\le (1-\sigma)\mathbb{E}\{\Pi^+_{i,k+1}|\tilde{\vartheta}_{i,k+1}=1\}\\
&\quad + (\sigma+\nu)\mathbb{E}\{\Pi^+_{i,k+1}|\tilde{\vartheta}_{i,k+1}=0\}.
\end{aligned} \tag{40}
$$

Denote the known information set as $\mathbb{I}_{i,k} = \{\delta^r_{i,k},\ \tilde{\vartheta}_{i,k+1}|i\in\mathcal{N}_i\}$. According to the above preparation, we are ready to present the following result.

Theorem 4: For the discrete-time system (1) with the measurement (2) inspected by (25), let the gain matrix $K_{i,k+1}$ of the distributed secure filter (7) be calculated by (32) and the estimate of state x_{k+1} be updated by (34). Then, under Assumption 2, the estimation error covariance under the known sequences $\{\mathbb{I}_{i,k}\}$ is mean-square bounded, i.e.,

$$
\lim_{k\to\infty}\sum_{i\in\mathbb{N}_m}\left\|\mathbb{E}\{\Pi^+_{i,k}\}\right\|<\infty \tag{41}
$$

if the failure rate p, the recovery rate q and the coupling strength κ satisfy

$$
\xi_q = \zeta\bar{\alpha}^2\big(1-q+q(1+\kappa)(1+\kappa\lfloor\bar{\mathcal{N}}\rfloor)\big)<1, \tag{42}
$$

$$
\xi_p = \zeta\bar{\alpha}^2\big(p+(1-p)(1+\kappa)(1+\kappa\lfloor\bar{\mathcal{N}}\rfloor)\big)<1 \tag{43}
$$

where

$$
\zeta=\frac{(1+\nu)\underline{c}^2+\bar{c}^2(1-\sigma)}{\underline{c}^2}.
$$

Proof: For the benefits of boundedness analysis, the probability theory is exploited to tackle the randomness from both the communication channels and the innovation inspection. For the simplicity, we denote

$$
\mathbf{P}_{k,0}=\mathbb{P}\{\delta^r_{i,k-1}=0\},\quad \mathbf{P}_{k,1}=\mathbb{P}\{\delta^r_{i,k-1}=1\}.
$$

Obviously, one has $\mathbf{P}_{k,0}+\mathbf{P}_{k,1}=1$ and it is clear from (39) that

$$
\begin{aligned}
&\mathbb{E}\{\Pi^-_{i,k+1}\}\\
&\le \Big((1-q+q(1+\kappa))A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T\\
&\quad + q\tilde{\kappa}\sum\nolimits_{j\in\mathcal{N}_i\cup\{i\}}A\mathbb{E}\{\Pi^+_{j,k}\}A^T\Big)\mathbf{P}_{k,0}\\
&\quad + \Big((p+(1-p)(1+\kappa))A\mathbb{E}\{\Pi^+_{i,k}\}A^T+BB^T\\
&\quad + (1-p)\tilde{\kappa}\sum\nolimits_{j\in\mathcal{N}_i\cup\{i\}}A\mathbb{E}\{\Pi^+_{j,k}\}A^T\Big)\mathbf{P}_{k,1}
\end{aligned} \tag{44}
$$

where $\tilde{\kappa}=2\kappa(1+\kappa)$. By resorting to the property of norm operation, the above inequality is further manipulated as follows:

$$
\begin{aligned}
&\|\mathbb{E}\{\Pi^-_{i,k+1}\}\|\\
&\le \Big((1-q+q(1+\kappa))\bar{\alpha}^2\|\mathbb{E}\{\Pi^+_{i,k}\}\|+\bar{\tau}\\
&\quad + q\tilde{\kappa}\bar{\alpha}^2\sum\nolimits_{j\in\mathcal{N}_i\cup\{i\}}\|\mathbb{E}\{\Pi^+_{j,k}\}\|\Big)\mathbf{P}_{k,0}\\
&\quad + \Big((p+(1-p)(1+\kappa))\bar{\alpha}^2\|\mathbb{E}\{\Pi^+_{i,k}\}\|+\bar{\tau}\\
&\quad + (1-p)\tilde{\kappa}\bar{\alpha}^2\sum\nolimits_{j\in\mathcal{N}_i\cup\{i\}}\|\mathbb{E}\{\Pi^+_{j,k}\}\|\Big)\mathbf{P}_{k,1}.
\end{aligned} \tag{45}
$$

On the other hand, along the similar line used in [16], it follows from (40) that

$$
\begin{aligned}
&\|\mathbb{E}\{\Pi^+_{i,k+1}\}\|\\
&\le \|(\sigma+\nu)\mathbb{E}\{\Pi^-_{i,k+1}\}\|\\
&\quad + \|(1-\sigma)\mathbb{E}\{(I-K_{i,k+1}C_i)\Pi^-_{i,k+1}\}\|\\
&\le \frac{(1+\nu)\underline{c}^2+\bar{c}^2(1-\sigma)}{\underline{c}^2}\|\mathbb{E}\{\Pi^-_{i,k+1}\}\|.
\end{aligned} \tag{46}
$$

Next, taking (45) and (46) into consideration, one has that

$$
\begin{aligned}
&\sum_{i\in\mathbb{N}_m}\left\|\mathbb{E}\{\Pi^+_{i,k+1}\}\right\|\\
&\le \Big((1-q+q(1+\kappa))\zeta\bar{\alpha}^2\sum_{i\in\mathbb{N}_m}\|\mathbb{E}\{\Pi^+_{i,k}\}\|+m\zeta\bar{\tau}\\
&\quad + q\zeta\tilde{\kappa}\bar{\alpha}^2\sum_{i\in\mathbb{N}_m}\sum\nolimits_{j\in\mathcal{N}_i\cup\{i\}}\|\mathbb{E}\{\Pi^+_{j,k}\}\|\Big)\mathbf{P}_{k,0}\\
&\quad + \Big((p+(1-p)(1+\kappa))\zeta\bar{\alpha}^2\sum_{i\in\mathbb{N}_m}\|\mathbb{E}\{\Pi^+_{i,k}\}\|+m\zeta\bar{\tau}\\
&\quad + (1-p)\zeta\tilde{\kappa}\bar{\alpha}^2\sum_{i\in\mathbb{N}_m}\sum\nolimits_{j\in\mathcal{N}_i\cup\{i\}}\|\mathbb{E}\{\Pi^+_{j,k}\}\|\Big)\mathbf{P}_{k,1}\\
&\le \xi_q\mathbf{P}_{k,0}\sum_{i\in\mathbb{N}_m}\|\mathbb{E}\{\Pi^+_{i,k}\}\|\\
&\quad + \xi_p\mathbf{P}_{k,1}\sum_{i\in\mathbb{N}_m}\|\mathbb{E}\{\Pi^+_{i,k}\}\|+m\zeta\bar{\tau}\\
&\le \max\{\xi_q,\xi_p\}\sum_{i\in\mathbb{N}_m}\|\mathbb{E}\{\Pi^+_{i,k}\}\|+m\zeta\bar{\tau}\\
&< \sum_{i\in\mathbb{N}_m}\left\|\mathbb{E}\{\Pi^+_{i,k}\}\right\|+m\zeta\bar{\tau},
\end{aligned} \tag{47}
$$

which means that the sequence $\sum_{i\in\mathbb{N}_m}\|\mathbb{E}\{\Pi^+_{i,k}\}\|$ is convergent. Because of the fact that $\Pi^+_{i,k}\ge P^+_{i,k}$, we conclude that the estimation error covariance is mean-square bounded, which ends the proof. ■

Remark 5: So far, a distributed recursive secure filtering algorithm is developed for a class of discrete-time stochastic nonlinear systems subject to unreliable communication due

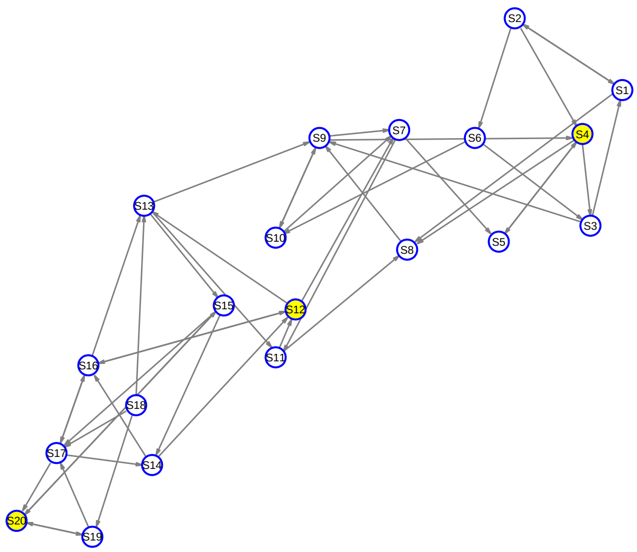

Fig. 1. The communication topology of sensor network

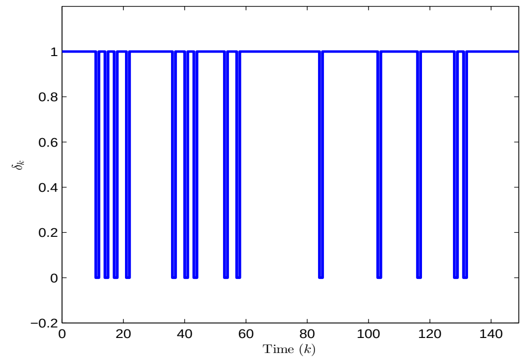

Fig. 2. The state δ_k of Gilbert-Elliott channels

probably to security vulnerability of sensor networks. Sufficient conditions are characterized in terms of the failure and recovery rates of the Gilbert-Elliott channels so as to guarantee the existence of an upper bound on the estimation error covariance, and then the desired filter parameters are designed by minimizing the trace of such an upper bound. The asymptotic boundedness of the estimation error covariance is also investigated. It should be pointed out that, in the main results in Theorems 3 and 4, all the system information has been reflected including the system parameters, the statistics of the nonlinear functions and the noises, the innovation inspection function, the topology information and the failure/recovery rates of the Gilbert-Elliott channels.

Remark 6: Our main results distinguish from some existing ones in that: 1) the proposed distributed secure filter is equipped with a novel χ^2 detector so as to exclude the unreliable measurements due possibly to malicious attacks or outliers; 2) the proposed filter design algorithm exhibits the desired scalability and the computational burden is unrelated to the dimension of the underlying SNs; and 3) quantitative results are established by means of the failure/recovery rates of Gilbert-Elliott channels in order to ensure the asymptotic boundedness of designed secure filtering algorithm.

V. Simulation Examples

In this section, a simulation example is presented to illustrate the effectiveness of the proposed filtering scheme.

The adopted SN consists of 20 nodes, which are randomly deployed in an area and the topology utilized in this paper is shown in Fig. 1. Furthermore, the adjacency element h_{ij} is set as $1/\lfloor \mathcal{N}_i \rfloor$ if node i and node j are connected.

Consider a maneuvering target with state

$$x_k = [\ x_{1,k} \quad x_{2,k} \quad x_{3,k}\ ]^T$$

and system parameters

$$A = \begin{bmatrix} 1 & T & T^2/2 \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad B = 0.2I, \quad D = 0.05$$

$$C_i = \begin{cases} [\ 4.5 \quad -0.8 \quad -0.5\ ], & i < 8, \\ [\ 4.0 \quad -0.8 \quad 1.0\ ], & i \geq 8. \end{cases}$$

Here, the elements $x_{1,k}$, $x_{2,k}$ and $x_{3,k}$ stand for, respectively, the position, the velocity and the acceleration of the target, and the sampling period is selected as $T = 0.05s$. According to the security requirement, the σ-quantile $\chi^2_{n^y,\sigma}$ is 25.

For the addressed distributed filter, the failure rate p, the recovery rate q, and the coupled strength κ are, respectively, selected as 0.1, 0.92 and 0.004. For the given parameters, a sequence of channel states $\{\delta_k\}$ over time-horizon $k \in [0,\ 150]$ are generated via Matlab software and further plotted in Fig. 2. In addition, we randomly produce both the initial state and the initial estimate.

Without loss of generality, we only analyze the filtering results on Sensors 4, 12 and 20 under two cases:

- *Case A)*: all sensors are normal;
- *Case B)*: Sensor 4 and Sensor 12 are subject to attacks.

Specially, we assume that the attack strength is 15, and the attack instants are involved in the intervals $[90,\ 95]$ and $[50,\ 55]$ for Sensor 4 and Sensor 12, respectively. The attacked measurements are shown in Fig. 3 and the curves of normal measurements can be obtained by subtracting 15 from corresponding ones at attack instants. The position of the tracked target and its estimation on Sensors 4, 12 and 20 are drawn in Fig. 4. Combining Fig. 4(a) and Fig. 4(b), we can find that there is no obvious effect on the filtering performance with or without attacks benefiting from the capability of designed detection scheme. When this inspection is removed from our designed distributed filtering scheme, the corresponding filtering curves are plotted in Fig. 4(c). In comparison with Fig. 4(b), we can easily find that the filtering performance is degraded at attack instants. As such, the superiority and reliability of the proposed distributed filtering scheme have been clearly verified.

In what follows, a comparison with traditional filters based on linear matrix inequalities (LMIs) is implemented to further

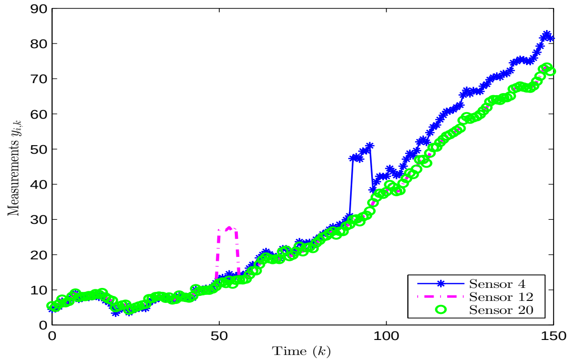

Fig. 3. Measurements subject to attacks

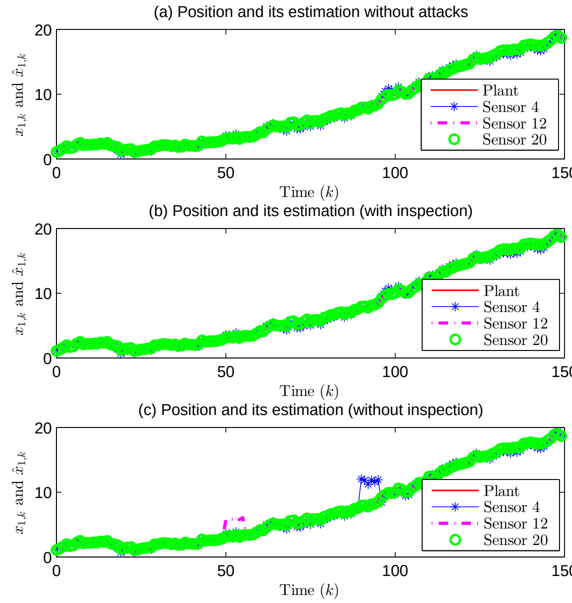

Fig. 4. Position $x_{1,k}$ and its estimation

verify the reliability of the proposed scheme. To this end, a simplified version of results in [9], [13] can be easily obtained over sensor networks. In this simulation, all system parameters, channel states and various noises are the same with ones in the above test. In order to solve the corresponding LMIs, we further select the parameters $\chi_1 = 0.98$ and $\varepsilon_1 = 10$ (corresponding to Corollary 2 in [9]), and then obtain the filter gains $K = [\ 0.4177 \quad 0.5799 \quad 0.4029\ ]^T$ for node $i < 8$ and $K = [\ 0.2883 \quad 0.6150 \quad 0.4457\ ]^T$ for other nodes. The measurements and the implementation results are plotted in Fig. 5 and Fig. 6, respectively. Obviously, these two schemes have similar filtering performance according to the trajectories of the target dynamics when there are no attacks. Furthermore, the reliability of developed scheme is clearly verified. Finally, different from lots of existing versions dependent on the global information of sensor networks, the our scheme is scalable and without the issue of computational complexity with the increased network scale.

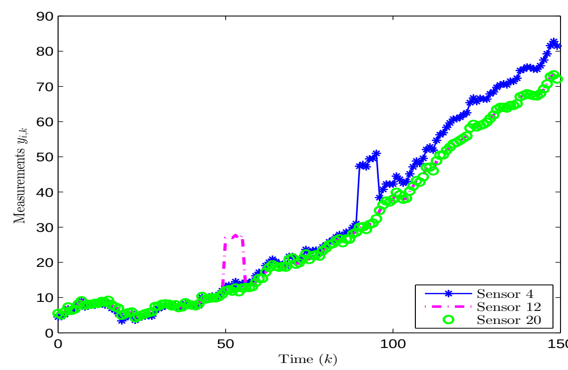

Fig. 5. Measurements subject to attacks

VI. Conclusions

In this paper, a recursive secure filtering has been investigated for a class of discrete-time systems subject to unreliable measurements and communication coming from the inherent security vulnerability of SNs. For the constructed filter with a χ^2 detector, the information adopted to update the predicted state has been made up of the innovation from the individual node itself and the weighted sum of predicted state errors among its neighboring nodes. In addition, the desired filter gains have been obtained by means of the solution to a Riccati-like difference equation. Furthermore, a sufficient condition on the failure and recovery rates of channels has been established to guarantee the boundedness of the sequence of the estimation error covariance. An illustrative example with target tracking background has been provided to show the effectiveness of the proposed method. Further research topics would include the extension of the main results in this paper to more complicated systems with various network-induced phenomena or dynamic topologies [4], [6], [15], [30], [35]–[38].

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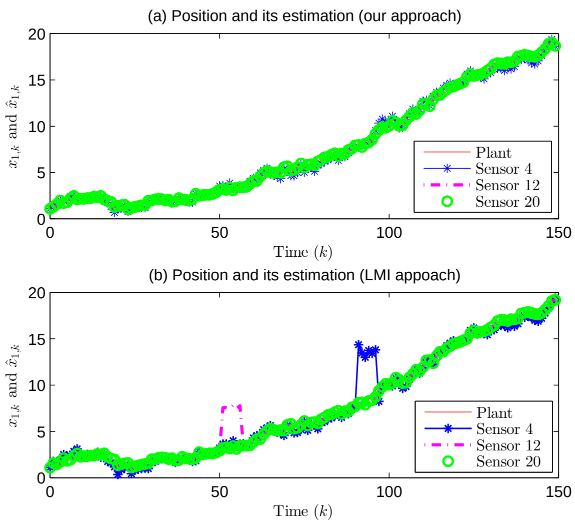

Fig. 6. Position $x_{1,k}$ and its estimation

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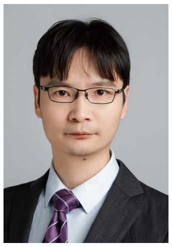

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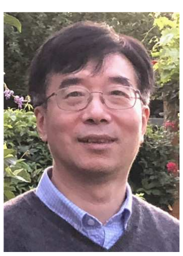

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