

Adaptive partitioning approach to self-sustained smart grid

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Abstract: Effective network partitioning becomes an essential step to realise self-sustained smart grid, which serves as a prerequisite for 'self-healing' enabled decentralised control. Splitting the power network (PN) into areas is the last resort to avoid the spread of disruption and to maintain as many network survivals as possible. This study aims to resolve the issue of multi-objective PN partitioning by deploying a newly proposed hybrid approach concerning both real power balance and voltage profile. The proposed approach combines the Laplacian spectrum and self-organising map, which adaptively attains self-sustained network partitions on different operating conditions. The resultant partitions are characterised by the minimal intra-area real power imbalance with a healthy voltage profile. The authors experimentally evaluate the partitioning effectiveness and computational efficiency in several case studies including on the New England 39-bus, IEEE 118-bus, and Polish 2383-bus transmission systems.

1 Introduction

Power system security is a major concern at the present day due to the increasing complexities and uncertainties involved in system operation and control. The smart grid vision aims at developing the power system towards a 'self-healing' grid that is well integrated with advanced metering technologies, wide-area communication and automated controls. The benefits include fast decision-making, high controllability and system reliability. In general, a self-healing process comprises three phases [1]. First, system faults are timely and accurately monitored. Second, specific control actions are promptly taken to counteract system disturbance and isolate faulty parts of the network to avoid further spread of disruption. Finally, network reconfiguration and restoration will be in place and tune the entire system towards a stable and secure state. It is obvious that a suitable control scheme governing the whole system is of fundamental importance for achieving self-healing capability to cope with abnormal situations.

Admittedly, hierarchical centralised supervisory controls can be effective in a global perspective to achieve self-healing in small-scale power networks (PNs) but may be less efficient for large-scale systems [2], in which several profound complexities with respect to generation mix, load diversity and interconnected network topologies are involved. In some recent works, decentralised controls have been exploited and proven to be more tangible and manageable for practical applications in smart grids. In [3, 4], it is investigated that system data could be handled by the local agents without the need of centralised data acquisition and processing to achieve system monitoring and distributed control. However, the feasibility of controlling and recovering the network functionalities of large-scale PNs in a distributed manner is critically dependent on effective network partitioning, which still remains an open problem especially for large inter-connected PNs. In light of the above, it is mainly concerned in this paper to pursue self-sustained partitions with multiple optimisation objectives under different operating conditions. This work aims at laying a useful groundwork for smart grid planning and operation control with enabled self-healing capability. The detailed mechanisms of decentralised control and reconfigurations are beyond the scope of this paper.

As distinguished from existing partitioning works with sole objective, our study simultaneously takes into account of network connectivity, intra-area real power balance and voltage profile, which extensively addresses the prerequisites of self-healing capability. In particular, real and reactive power imbalances are jointly considered to formulate a multi-objective PN partitioning problem, of which the optimal partitions is the result of minimising real and reactive power exchange between adjacent areas. In a static sense of power system operation, minimising the intra-area real power imbalance (i.e. intra-area generation-load imbalance) can greatly assist in regulating frequency stability in each islands. In addition, sufficient reactive power supply within islands can avoid voltage instability and prevents the occurrence of undesirable electric components tripping events. Solely considering real or reactive power only may result in ineffective partitioning solutions for practical applications. For example, an optimal partitioning solution reflecting minimal generation-load imbalance do not necessarily guarantee healthy voltage profiles within each island. Nevertheless, solving multi-objective network partitioning problem is challenging and generally NP-hard. In view of the points discussed above, the major contributions of this paper can be summarised as follow:

- A hybrid approach combining specific PN Laplacians and self-organising map (SOM) algorithm is developed to capture eigenstructures directly with respect to multiple objectives.
- Pareto optimality can be achieved, in which multiple partitioning solutions are collectively provided. This can greatly facilitate decision-making of system operators.
- It is efficient to handle large-scale PNs by using the proposed partitioning approach, which is particularly tested on a Polish 2383-bus system.
- The partitions obtained in this paper are mainly based on steady state characteristics (i.e. real and reactive power flows). It is worth noting that the proposed method can be easily extended to fast-time-scale applications by incorporating specific weighted Laplacians with system dynamics.

The rest of this paper is organised as follows. In Section 2, we first review the existing methods of network partitioning for power

system applications. In Sections 3 and 4, Laplacian spectrum with different attributes and detailed mechanism of SOM are introduced, respectively. Furthermore, the proposed heuristic algorithm to achieve Pareto optimality is presented in details. Experimental results are shown in Section 5. Section 6 concludes this paper.

2 Network partitioning of power grids

A power grid is generally composed of several synchronous interconnected sub-systems, which are referred as ‘control areas’. Each of them is normally governed by the respective control entity (e.g. regional transmission organisation). These areas are traditionally formulated by only considering asset ownerships, local policies and regulations. Directly implementing such control areas is unable to fit with the time-varying operating conditions and diverse control requirements in modern power systems. In this regard, smart grid operation and control necessitate adaptive and efficient PN partitions considering specific engineering constraints.

Network partitioning deployed in power engineering is not novel and many research efforts have been made to handle different applications associated with big data and complex topological structures, such as resource adequacy assessments [5], area-based voltage stability assessment and control [6], zonal pricing in electricity market [7], etc. Partitioning methods are also employed in system planning and operational security analysis. For example, Donde *et al.* [8] address generation-load imbalance problems based on steady or quasi-steady state analysis; several studies investigate the synchronisation of complex PNs (e.g. [9]) or slow coherency (e.g. [10]) to identify the dynamic coherent sub-networks based on dynamic analysis. Moreover, Xie *et al.* [11] proposed a useful graph theory based approach to find out the optimal network partitions for the PMU placement. From the perspective of the network topology, this approach is effective to provide the structural information to the system operators. In general, two broad categories of partitioning methods have been reported in power system literature – mathematical modelling based optimisation methods and network Laplacians based spectral methods. The first class of methods typically formulate the network partitioning as a mix-integer linear/non-linear optimisation problem and have it solved through well-established commercial software packages (e.g. AMPL, CPLEX, etc.) or intelligent optimisation algorithms. With advanced global optimisation algorithms involved, this type of methods can normally provide guarantees regarding the optimality. However, the convergence speed is essentially dependent on the modelling complexity. In the existing literature, most of such works were proposed with different levels of modelling simplification to make the problem be tractable. For example, Sun *et al.* [12] proposed a two-step partitioning method for system restoration, in which the network topology is simplified to reduce the computational burden. Ding *et al.* [13] proposed an efficient mix-integer linear programming based partitioning approach for controlled islanding operation, whereas only DC power flow is formulated and voltage constraints are neglected. In considering multiple attributes/partitioning objectives, Cotilla-Sanchez *et al.* [14] proposed an efficient method by combining all attributes into one objective. Partitioning solutions can be easily obtained by employing evolutionary algorithms, but the performance of this kind of methods can be sensitive to single objective formulation and the compatibilities of all attributes remain questionable. Instead of complex modelling of network topologies and engineering constraints, weighted Laplacians based spectral methods (e.g. [15]) can directly yield partitions from eigenstructures of Laplacian matrices. Even though multi-objectives can be effortlessly incorporated into this type of methods by taking into account, e.g. combined weights on each branch [16], partitioning solutions obtained by normal spectral clustering might be semi-optimal and no Pareto sets can be provided. Therefore, it is noted that not every partitioning method is equally good or suitable for a specific engineering application. Moreover, to the authors’ best knowledge, most of the existing methods are less effective to target multi-objective partitioning.

In our proposed approach, weighted Laplacians represented by real and reactive power distribution are collectively considered to measure self-sustainability of each partitioning area. SOM with an advanced visualisation technique is used as a tool to produce partitions from different eigenstructures. In order to achieve Pareto optimality, a heuristic algorithm to manage and tune the combined eigenspace of weighted Laplacians is proposed in this paper. Detailed mechanisms of the proposed partitioning approach will be illustrated in the forthcoming sections.

3 Laplacian spectrum of PNs

3.1 Preliminaries of spectral representation

The topological properties of a PN can be extracted from the network Laplacian spectrum. Unnormalised Laplacian matrix is expressed as

$$\mathbf{L} = \mathbf{D} - \mathbf{W} \quad (1)$$

where $\mathbf{W} = (\omega_{ij})_{i,j=1,2,\dots,N}$ is the weighted power flow matrix, which can be formulated in terms of the real and reactive power [17, 18]. According to the well-established power flow equations, the weights of the transmission lines are assigned as the real and reactive power flows through them.

$$\omega_{ij}^{\text{MW}} = P_{ij} = U_i U_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \quad (2)$$

$$\omega_{ij}^{\text{MVar}} = Q_{ij} = U_i U_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \quad (3)$$

where P_{ij} and Q_{ij} represent the real and reactive power flow on line (i, j) . U_i is the bus voltage of bus i . θ_{ij} is the phase-angle difference between bus i and j . $Y_{ij} = G_{ij} + jB_{ij}$, Y_{ij} is the admittance of line (i, j) . \mathbf{D} is a diagonal matrix and each of the diagonal element indicates the contribution of the corresponding bus in handling power traffic in the PN [17], which is formulated as

$$\mathbf{D}(\mathcal{G}) = \text{diag}\{d_1, d_2, \dots, d_{|N|}\}, \quad d_i = \sum_{j=1}^N \omega_{ij} \quad (4)$$

There are two forms of normalised Laplacian matrix. Both are closely related to each other.

$$\mathbf{L}_{\text{sym}} = \mathbf{D}^{-1/2} \mathbf{L} \mathbf{D}^{-1/2} = \mathbf{I} - \mathbf{D}^{-1/2} \mathbf{W} \mathbf{D}^{-1/2} \quad (5)$$

$$\mathbf{L}_{\text{rw}} = \mathbf{D}^{-1} \mathbf{L} = \mathbf{I} - \mathbf{D}^{-1} \mathbf{W} \quad (6)$$

where \mathbf{L}_{sym} is a symmetric matrix, and \mathbf{L}_{rw} is closely related to a random walk. \mathbf{L}_{sym} and \mathbf{L}_{rw} are positive semi-definite and have N non-negative real-valued eigenvalues. The multiplicity of zero eigenvalue is equal to the number of connected sub-graphs [19].

Mathematically speaking, the clustering information on the loosely coupled connected components in a graph is spanned over the eigenvectors of Laplacian matrix. Davis–Kahan theorem [20] introduced below reveals the relationship between clustering performance and the eigengap, which is defined as the largest gap between two neighbouring eigenvalues.

Theorem (Davis–Kahan): Given that symmetrical matrices $\mathbf{A}, \mathbf{H} \in \mathbf{R}^{N \times N}$. Let $\mathcal{S} \subset \mathbf{R}$ be an interval. The set of eigenvalues of \mathbf{A} which fall into the interval \mathcal{S} is denoted by $\Lambda_{\mathcal{S}}(\mathbf{A})$. \mathbf{V} is the corresponding eigenspace of $\Lambda_{\mathcal{S}}(\mathbf{A})$. Consider perturbed $\tilde{\mathbf{A}} = \mathbf{A} + \mathbf{H}$, and $\tilde{\mathbf{V}}$ is the analogous quantity for $\tilde{\mathbf{A}}$. The distance between \mathcal{S} and the spectrum of \mathbf{A} outside of \mathcal{S} is

$$\delta = \min \{|\lambda - s|; \lambda \notin \mathcal{S}, s \in \mathcal{S}\} \quad (7)$$

Thus the distance $d(\mathbf{V}, \tilde{\mathbf{V}}) = \|\sin \Theta(\mathbf{V}, \tilde{\mathbf{V}})\|$ is bounded by

$$d(\mathbf{V}, \tilde{\mathbf{V}}) \leq \|\mathbf{H}\|/\delta \quad (8)$$

where $\sin\Theta$ denotes a diagonal matrix with the canonical angles on the diagonal.

Given that $\tilde{L} = L + H$, where L is the Laplacian matrix of a disconnected graph, if both $\Lambda_S(L)$ and $\Lambda_S(\tilde{L})$ are within \mathcal{S} , the eigenspace of L and \tilde{L} are approximately close to each other. Since their distance is bounded by $\|H\|/\delta$ according to the Davis–Kahan theory, hence the degree of such ‘approximation’ is strongly related to the perturbation $\|H\|$. Given that \mathcal{S} is set by $[0, \lambda_k]$, then δ is consistent with the eigengap $|\lambda_{k+1} - \lambda_k|$.

Based on the Davis–Kahan theorem, one can safely draw the conclusion that the larger the eigengap $|\lambda_{k+1} - \lambda_k|$ is, the more clustering information can be extracted from the first k eigenvectors. This conclusion is widely used in classical spectral clustering and forms the theoretical basis in our study for the construction of combined eigenspace, which will be introduced in the forthcoming subsection.

3.2 Spectral properties of a PN

By recalling the partitioning objective as mentioned in Section 2, we aim at approaching the optimal cut which minimises both real and reactive power exchanges among all clusters. It is obvious that less power exchange among different clusters reflects a better ‘self-sufficient’ condition. However, it should be noted that an optimal partition considering only real power flows may lead to an unsatisfactory voltage profile within sufficient reactive power support. Likewise, network partition with respect to reactive power is unable to guarantee a small real power generation-load imbalance.

In this paper, we mathematically formulate the concerning PN partitioning problem as a multi-objective minimal *Ncut* to minimise the inter-cluster similarities and maximise the intra-cluster affinities. *Ncut* (first introduced in [21]) is defined as

$$Ncut(C_1, \dots, C_k) = \sum_{i=1}^k \frac{cut(C_i, \bar{C}_i)}{vol(C_i)} \quad (9)$$

where \bar{C}_i is the complement of C_i

$$cut(C_i, \bar{C}_i) = \sum_{i \in C_i, j \in \bar{C}_i} \omega_{ij}, \quad vol(C_i) = \sum_{i \in C_i} \sum_{j=1}^{|M|} \omega_{ij} \quad (10)$$

In particular, the formulated Min-*Ncut* minimises the inter-cluster real and reactive power flow exchanges and maximises the

intra-cluster self-contained capacities. Such formulation effectively avoids ‘one-single-node cluster’ phenomena, in which normalised Laplacian matrix is selected as the topological representation. Between L_{sym} and L_{rw} , the proposed approach is in favour of L_{rw} due to its better experimental performance than that of L_{sym} . This may be because eigenvectors of L_{sym} are additionally multiplied by $D^{1/2}$ and this undesirably influences its eigenstructure. In addition, there is no computational advantage when using L_{sym} . Hence L_{rw} is selected in our approach.

Normalised Laplacian matrices L_{MW} and L_{MVar} are constructed, in which the weights of branches are assigned by the absolute value of real and reactive power flows, respectively. According to the discussion in Section 3.1, eigengaps exist in both L_{MW} and L_{MVar} expose the clustering information contained in the first few eigenvectors. The clustering boundaries could be easily identified with a large eigengap. On the contrary, the existing clusters are overlapping and boundaries are ambiguous.

In considering both real and reactive power flows, a combined eigenspace comprising eigenvectors of L_{MW} and L_{MVar} is constructed

$$V_{comb} := [V_{MW} | V_{MVar}] \quad (11)$$

where V_{MW} and V_{MVar} consist of selective number of eigenvectors of L_{MW} and L_{MVar} , respectively. The combined eigenspace can be adjusted in a heuristic way by containing different clustering information of real and reactive power. k is termed as a *heuristic indicator* in this paper and formulated as

$$k_{MW} = \arg \max \{ |\lambda_{k+1} - \lambda_k|; \lambda \in \Lambda_S(L_{MW}) \} \quad (12)$$

$$k_{MVar} = \arg \max \{ |\lambda_{k+1} - \lambda_k|; \lambda \in \Lambda_S(L_{MVar}) \} \quad (13)$$

where \mathcal{S} is a predefined interval. Algorithm 1 illustrates (see Fig. 1) the procedure of constructing V_{comb}

Clustering the points $(y_i)_{i=1, \dots, |N|} := [x_1^i, \dots, x_{k_{MW}+k_{MVar}}^i]$ obtained from the eigenspace is an essential step to identify the partitioning boundaries. Instead of using k -means algorithm directly, which has been found to be inaccurate and suffer from local minima for large networks, SOM algorithm with advanced visualisation technique is used in our approach to adaptively handle massive network data. Full details of SOM based partitioning algorithm are discussed in Section 4.

Algorithm 1:

inputs: *case data* is the basic operating data of a power system for real and reactive power flow calculation.

\mathcal{S} is a predefined interval

output: combined eigenspace

1. Run AC power flow based on *case data*
 2. Construct the weighted power flow matrices
 ω = the absolute value of real or reactive power flow
 3. Compute the normalised Laplacian matrices L_{MW} and L_{MVar} with Eq.(6)
 4. Compute the heuristic indicators k_{MW} and k_{MVar} with Eqs. (12) and (13)
 5. Compute the first k_{MW} and k_{MVar} eigenvectors of L_{MW} and L_{MVar} respectively
 6. Construct combined eigenspace $V_{comb} \in R^{|N| \times (k_{MW} + k_{MVar})}$
 7. **for** $i=1:|N|$
 8. $y_i =$ the i -th row of V_{comb}
- end for**
-

Fig. 1 Construction of combined eigenspace

4 Proposed methodology of adaptive K -way network partitioning

Multi-objective partitioning problem is known to be NP-complete. Traditional spectral clustering fails to tackle such problems while some of its variations have been proposed to handle multi-objectives, e.g. [22] by using collective weights integrating all concerned aspects. However, this kind of approaches is highly sensitive to predetermined preference factors when forming integrated weights.

The proposed method consists of two parts including automated clustering based on SOM and multi-objective optimisation. The flowchart shown in Fig. 2 illustrates the basic procedures of the proposed approach.

4.1 Automated clustering based on SOM

SOM (also called as Kohonen Map) [23] is a widely used tool for cluster extraction, visualisation and data mining. SOM is an unsupervised, competitive artificial neural network that produces topology-preserving mappings of data spaces utilising a self-organising learning algorithm. This algorithm can project high-dimensional data $\mathcal{Y} \subset \mathbb{R}^d$ onto a low-dimensional (typically one or two-dimensional) fixed lattice of weighted neurons denoted as \mathcal{K} . The classical SOM algorithm is briefly explained as follows.

The weights (denoted as $W_i^{\text{neu}} := [w_1, \dots, w_{|W^{\text{neu}}|}]$) of each neural unit in the Kohonen Map \mathcal{K} are trained through a competitive learning process. The best matching unit is determined by the minimal Euclidean norms for a given data sample $y \in \mathcal{Y}$,

such that

$$\|y - W_{\text{best}}^{\text{neu}}\| \leq \|y - W_i^{\text{neu}}\| \quad \forall i \in \mathcal{K} \quad (14)$$

The weights of each neuron are updated based on the following rule

$$W^{\text{neu}}(t+1) = W^{\text{neu}}(t) + \eta(t)\Theta[\sigma(t)](y - W^{\text{neu}}) \quad (15)$$

where $\eta(t)$ is learning rate which exponentially decreases with t . $\Theta(\cdot)$ is the neighbourhood function and $\sigma(t)$ represents the neighbourhood size. The weight vector of each neuron is the centroid of its receptive field (RF).

SOM has high capability of extracting data structures on the condition of cluster-overlaps. In our work, a hierarchical agglomerative clustering algorithm based on SOM is applied to the multi-objective partitioning problem. Geometric representations (provided by Laplacian spectrum) of PNs serve as the inputs for SOM training. An effective visualisation technique (i.e. CONNvis [24]) is utilised to achieve automated clustering for large dataset and diverse cluster sizes. CONNvis is advantageous over other visualisation methods (e.g. Euclidean distance between quantisation neurons [25], neighbourhood of SOM [26], etc.) to handle overlapping clusters. This automated clustering algorithm is summarised in (see Fig. 3).

(i) Construct combined eigenspace V_{comb} . Each row of V_{comb} serves as an input for SOM training.

(ii) A $t \times t$ Kohonen map is constructed and trained through a competitive training process. Selecting the number of neurons in Kohonen map follows the rule that t^2 should be significantly less than the number of data samples whilst greater than the expected number of clusters [24].

(iii) Connectivity matrix contains two important clustering information, namely neighbourhood relations in datasets and detailed data distribution [27]. It can be expressed as

$$\text{CONN}(i, j) = |RF_{ij}| + |RF_{ji}| \quad (16)$$

where $RF_{ij} = \{y \in RF_i : \|y - W_j\| \leq \|y - W_i\| \quad \forall l \neq i\}$, and $|RF_{ij}|$ indicates the number of data vectors in RF_{ij} .

(iv) Similarities between neurons are determined by the local data distribution. Mathematically speaking, similarity matrix is initialised as connectivity matrix. $S = \text{CONN}$.

(v) In each iterative step, the most similar pair of clusters C_i and C_j (which may include one or more neurons) is merged into a new one which is referred as cluster p . $S(p, q)$ is formulated as

$$S(p, q) = [|C_i|/(|C_i| + |C_j|)]S(i, q) + [|C_j|/(|C_i| + |C_j|)]S(j, q) \quad (17)$$

The corresponding rows and columns of the most similar pair in S are omitted accordingly. After $t^2 - 1$ iterations, all neurons are merged into one cluster. CONN linkage is calculated in each step, which is formulated as

$$\text{conn_index} = \text{intra_conn} \times (1 - \text{inter_conn}) \quad (18)$$

where

$$\text{intra_conn} = \frac{1}{R} \sum_{r=1}^R \frac{\sum_{i,j \in C_r} |RF_{ij}|}{\sum_{i \in C_r} |RF_i|}$$

$$\text{inter_conn} = \frac{1}{R} \sum_{r=1}^R \max_{i \in C_r, j \in C_l} \frac{|RF_{ij}|}{\sum_{i \in B_{rl}} \text{CONN}(i, j)}$$

R is the number of clusters. B_{rl} is the set of neurons in cluster r that are directly connecting to the neurons in cluster l .

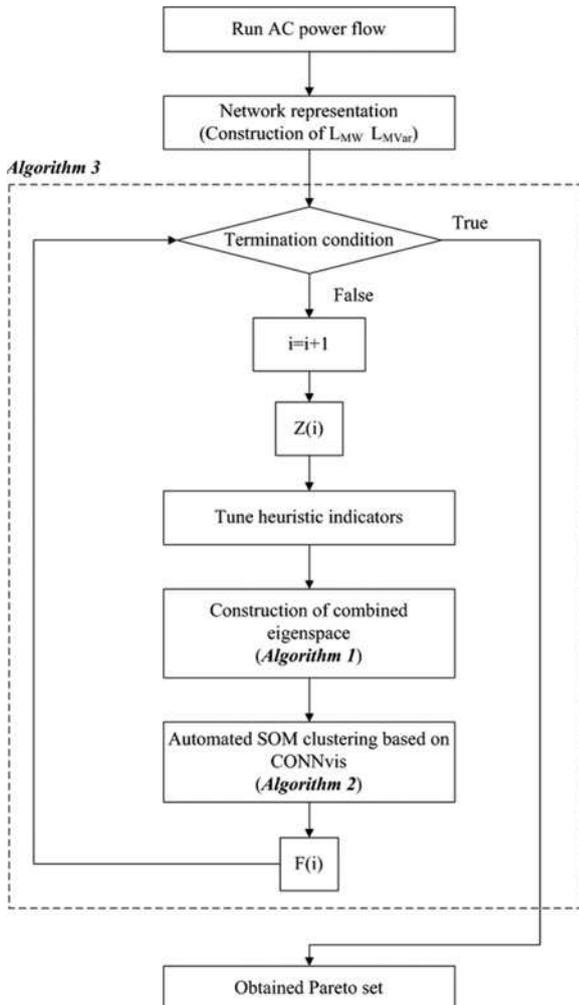


Fig. 2 Flowchart of the proposed partitioning approach

Algorithm 2:**inputs:** V_{comb} is the combined Laplacian eigenspace**output:** Clustering results

1. Construct V_{comb}
 2. Train SOM with $t \times t$ neurons
 3. Construct connectivity matrix $CONN$
 4. Initialise similarity matrix S
 5. (Update similarity matrix iteratively)
 $c=1$
 while $c < t^2$
 $c=c+1$
 merge the most similar pair of clusters into one
 update similarity matrix S
 compute $CONN$ linkage
 end while
 6. Determine the number of clusters based on $CONN$ linkage and produce clustering results
-

Fig. 3 Automated clustering based on SOM

(vi) The maximum value of $conn_index$ is 1, which indicates that all clusters are clearly separated. Based on the analysis in [24], higher $conn_index$ value reflects better clustering results. In the proposed study, the number of clusters is determined according to the maximum $conn_index$ value. Since $R=1$ is meaningless for the proposed partitioning problem, R is bounded within $[2, R_{max}]$, where R_{max} is the maximum number of acceptable clusters in a specific PN.

This algorithm has merits that no prior knowledge of cardinality of clusters is needed and obtained clusters can be of unequal sizes [28]. Since training set of SOM can be adjusted by k_{MW} and k_{MVar} , this algorithm is capable of approaching a trade-off partitioning by considering both real and reactive power flows.

4.2 Pareto optimality

As discussed in previous sections, combined eigenspace constructed by different pairs of k_{MW} and k_{MVar} can significantly influence the clustering results. Therefore, k_{MW} and k_{MVar} can be used to adjust the clustering results regarding real and reactive power. For example, if more effective clustering information based on reactive power flow is contained in V_{comb} , clustering boundaries would be 'dragged' towards optimal reactive power partition. As a result of continuous adjustments of k_{MW} and k_{MVar} , one enclosing with more clustering information would 'prevail' over another.

Based on the analysis above, a heuristic method is proposed to solve the following bi-objective optimisation problem

$$\min \left\{ f_1: \sum_{r=1}^R |g_r - l_r|, f_2: \sum_{r=1}^R |\Delta v_r| \right\} \quad (19)$$

where g_r and l_r are the total real power generation and load in cluster r , respectively. Δv_r denotes the voltage violation in cluster r . In practice, voltage of each bus should be maintained within tolerance (i.e. ± 0.05 pu). In this paper, the voltage issue is simplified based on the assumption that only limited reactive power compensation is locally available on each bus.

The proposed approach aims to find out solutions of Pareto-optimal front and maintain a good spread of solutions on this front. Inspired by NSGA-II framework [29], a tailor-made algorithm for network partitioning is developed based on *fast non-dominated sorting* and *crowding distance* [29], which is illustrated in Fig. 4.

(i) $U(\bullet)$ represents a string of m individuals. Each individual is a pair of heuristic indicators.

$$U(\cdot) = [u_1, u_2, \dots, u_m], \quad \text{where } u = \begin{bmatrix} k_{MW} \\ k_{MVar} \end{bmatrix} \quad (20)$$

Algorithm 3: heuristic partitioning algorithm**inputs:** L_{MW}, L_{MVar} **output:** Pareto front of partitioning solutions

1. Initialization. $U(0)$ and $U(1)$ are initialized respectively including m pair-wise heuristic indicators.
 $i=0$
 $Z(1)=U(0) \cup U(1)$
 2. Calculate crowding distance and fast nondominated sorting: $Z(1) \rightarrow F(1)$
 3. **while** (termination condition is not true)
 $i=i+1$
 split $F(i-1) \rightarrow F_{best}(i-1) \cup F_{rest}(i-1)$
 4. update $F_{rest}(i-1) \rightarrow U(i)$
 form $Z(i)=U(i) \cup F_{best}(i-1)$
 calculate crowding distance and fast nondominated sorting:
 $Z(i) \rightarrow F(i)$
-
- end while**
-

Fig. 4 Heuristic partitioning algorithm

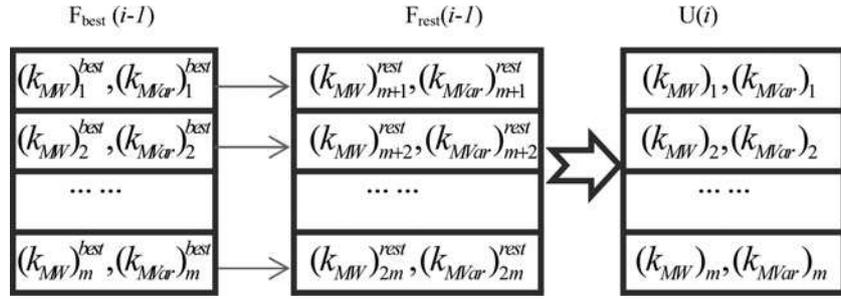


Fig. 5 Schematic diagram of updating process

The heuristic indicator is randomly initialised in a given acceptable range $[1, k_{\max}]$, which can be formulated as

$$k = \lfloor 1 + k_{\max} \cdot \gamma \rfloor \quad (21)$$

where γ is a random number within $[0,1]$. $U(0)$ and $U(1)$ are randomly initialised without duplicate individuals. $Z(1)$ is a combination of $U(0)$ and $U(1)$.

(ii) In this step, all individuals in $Z(1)$ are evaluated through the partitioning algorithm proposed in Section 4.1. Each partitioning solution is associated with a pair of *heuristic indicators*. $Z(1)$ is first sorted by *fast non-dominated sorting* algorithm with total complexity of $O(2m^2)$. *Crowding distance* is calculated between neighbouring non-dominated solutions. All of such solutions are subsequently re-sorted and re-arranged according to crowding distance ranking. High-ranking ones will lead to update the offspring populations.

(iii) In our approach, the termination criterion is $i \leq (k_{MW}k_{MVar})_{\max}/2m$. $F(i)$ is obtained by sorting $Z(i)$. $F(i)$ is truncated into two parts with equal size. The first part consists of best individuals, which is denoted as $F_{\text{best}}(i)$. The rest is denoted as $F_{\text{rest}}(i)$. In the present application, it is defined that the heuristic algorithm converges as long as the rate of change of fitness values reaches a predefined tolerance (e.g. 2% in case studies in Section 5).

iv. The schematic diagram of updating process is shown in Fig. 5.

The updating operator is formulated as

$$k_j = \begin{cases} \lfloor k_{m+j}^{\text{rest}} + (k_j^{\text{best}} - k_{m+j}^{\text{rest}} + 2) \cdot \gamma \rfloor, & k_j^{\text{best}} \geq k_{m+j}^{\text{rest}} \\ \lfloor k_{m+j}^{\text{rest}} + (k_j^{\text{best}} - k_{m+j}^{\text{rest}} - 2) \cdot \gamma \rfloor, & k_j^{\text{best}} < k_{m+j}^{\text{rest}} \end{cases} \quad (22)$$

where γ is a random number ranged within $[0,1]$.

k_j is allowed to be assigned with a random value ranged within $[k_{m+j}^{\text{rest}}, k_j^{\text{best}} + 1]$ or $[k_{m+j}^{\text{rest}} - 1, k_j^{\text{best}}]$. This enables the proposed algorithm to explore and exploit the search space in an efficient way. $Z(i)$ incorporates $U(i)$ and $F_{\text{best}}(i-1)$ to strengthen the elitism throughout the whole process. $F(i)$ is obtained by rearranging $Z(i)$ according to *crowding distance* and *fast non-dominated sorting*.

4.3 Fuzzy decision-making

Given no prior knowledge for the selection of candidate partitions in Pareto set, un-weighted fuzzy logic decision-making strategy [30] is employed to yield the best trade-off solution. It is worth noting that other decision-making criterions can also be considered with particular preferences.

In this paper, we assume that the preferences of minimising voltage violation and generation-load imbalance are unbiased. Fuzzy logic decision-making is formulated as follows.

Given a Pareto set $\mathcal{S}_{\text{pareto}}$ for pair-wise objectives, the fuzzy membership is defined as

$$\mu_i = \frac{f_i^{\max} - f_i}{f_i^{\max} - f_i^{\min}}, \quad i = 1, 2 \quad (23)$$

The normalised membership for each solution is expressed as

$$\mu(j) = \frac{\sum_{i=1}^2 \mu_i(j)}{\sum_{j=1}^{|\mathcal{S}_{\text{pareto}}|} \sum_{i=1}^2 \mu_i(j)} \quad (24)$$

According to the fuzzy logic principle [30], the trade-off (most satisfactory) solution is selected with the maximum fuzzy membership value.

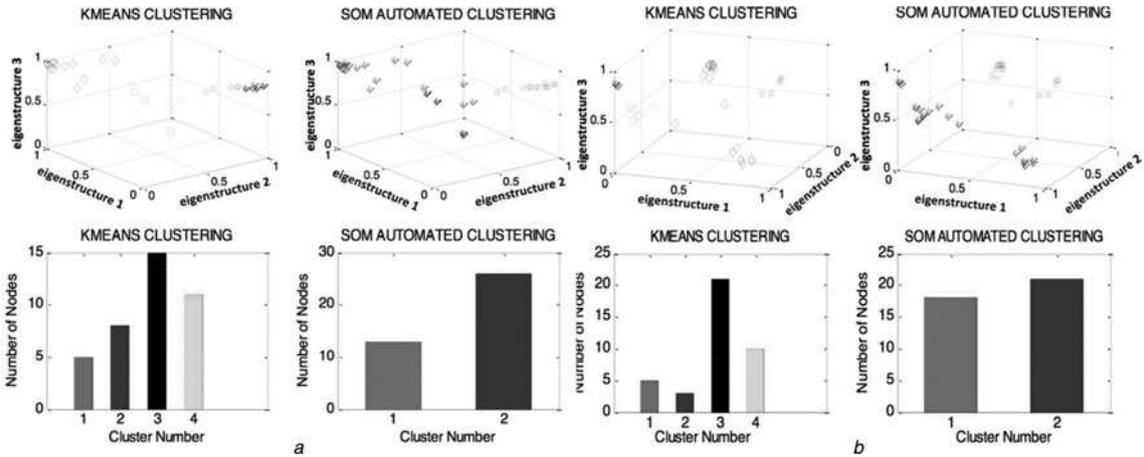


Fig. 6 Comparative experiment results

a Clustering results based on SOM ($k_{MW} = 1, k_{MVar} = 2$) and k -means in normal operating condition
b Clustering results based on SOM ($k_{MW} = 3, k_{MVar} = 2$) and k -means in normal operating condition

Table 1 Clustering performance based on SOM and k -means (New England 39-bus System)

		Real power imbalance, MW	Voltage violation, p.u	Buses (abnormal voltage)
SOM	$k_{MW} = 1$	147.259	0.0036	36
	$k_{MVar} = 2$			
	$k_{MW} = 3$	4.2	0.0212	4,7,8,36
k -means	$k_{MVar} = 2$	170.759	0.0288	4,15,24,36
	$k_{MW} = 1$			
	$k_{MVar} = 3$	195.1	0.0246	4,7,8,24,36

Table 2 Partitioning results (New England 39-bus system)

Heuristic partitioning	Partitioning alternatives (with cluster sizes)	MW imbalance, MW	Voltage violation, p.u	μ
Normal operating condition				
SOM	Cluster 1:14	0	0.0036	0.9241*
	Cluster 2:25			
k -means	Cluster 1:7	46.6	0.003	0.0759
	Cluster 2:32			
	Cluster 1:15	4.2	0.02117	0.5044
	Cluster 2:24			
	Cluster 1:6	420	0.007012	0.4063
	Cluster 2:13			
	Cluster 3:14			
Heavy load operating condition	Cluster 4:6			
	Cluster 1:7	543.6	0.0036	0.0894
	Cluster 2:13			
	Cluster 3:14			
	Cluster 4:5			
SOM	Cluster 1:13	309.5	0.0036	0.8926*
	Cluster 2:26			
k -means	Cluster 1:12	353.5	0.0027	0.1074
	Cluster 2:27			
	Cluster 1:19	378.3	0.2777	0.4195
	Cluster 2:20			
	Cluster 1:13	587.1	0.1877	0.3608
	Cluster 2:13			
	Cluster 3:13			
k -means	Cluster 1:9	1081	0.0036	0.2198
	Cluster 2:16			
	Cluster 3:14			

4.4 Computational efficiency analysis

As compared with *global search* partitioning methods, our proposed approach turns out to be more efficient by significantly decreasing

the problem dimension. With the employment of network Laplacian spectrum, the search space of full network dataset is converted into pair-wise *heuristic indicator* (associated with a semi-optimal partition) including $(k_{MW}k_{MVar})_{\max}/2$ times of evaluations. In order to achieve Pareto optimality, the typical procedure for k -means clustering approach is repeatedly evaluating the partitioning solutions by using cluster validity indices (e.g. Davies–Bouldin index (DBI) based on Davies–Bouldin criterion [31]). The optimal clustering is determined through constantly testing predefined cardinalities from 2 to $\lfloor \sqrt{n} \rfloor$, where n is the number of data samples. Distinguished from k -means algorithm, a well-trained SOM can efficiently produce network partitions with specific visualisation techniques.

Overall, the major computational task of the proposed approach includes construction of eigenspace, SOM based clustering and heuristic optimisation. For the first part, computing eigenvectors of L_{MW} and L_{MVar} normally takes $O(|N|^3)$ time. The time complexity can still be reduced by utilising sparse techniques for Laplacian matrix. Second, the total time complexity of training SOM is nearly $O((T+1)I)$, where T is the number of neurons and I is the number of iterations. In addition, automated clustering takes $O(T)$ time. Thus, the time complexity for the second part is $O((T+2)I)$. In conjunction with heuristic optimisation, the total time complexity is $O((T+2)Ik_{MW}k_{MVar})_{\max}/2$. Such degree of time complexity makes the proposed partitioning method promising and competitive for practical applications.

5 Experiments and performance

In this section, the proposed approach is tested on three different systems i.e. New England 39-bus, IEEE 118-bus and Polish 2383-bus transmission systems. Comparative experiments are carried out with classical k -means algorithm. MATLAB SOM toolbox is used for SOM training.

5.1 New England 39-bus system

This test system consists of 39 buses, 10 generators and 46 branches. In this case, two operating conditions are considered including normal-loading and heavy-loading.

5.1.1 Parameter setting: In normal operating condition, the total generation is 6297.87 MW and 1274.94 MVar, and the total load is 6254.23 MW and 1387.1 MVar. In heavy loading condition, the system is operated under a total generation of 7044.46 MW and 2020.11 MVar, and a total load of 7004.23 MW and 2136.10 MVar. Power flow solution is calculated based on the typical New England 39-bus system datasets [32].

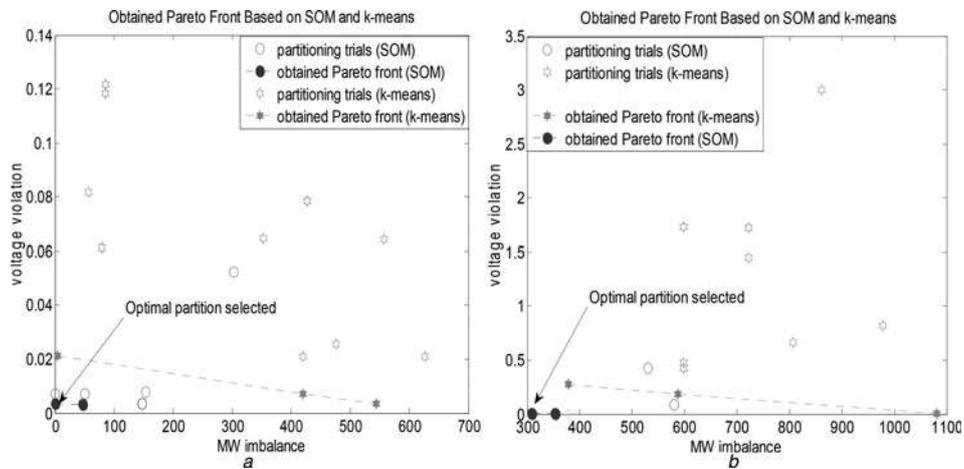


Fig. 7 Partitioning candidates

- a Heuristic partitioning results based on SOM and k -means in normal operating condition
- b Heuristic partitioning results based on SOM and k -means in heavy load operating condition

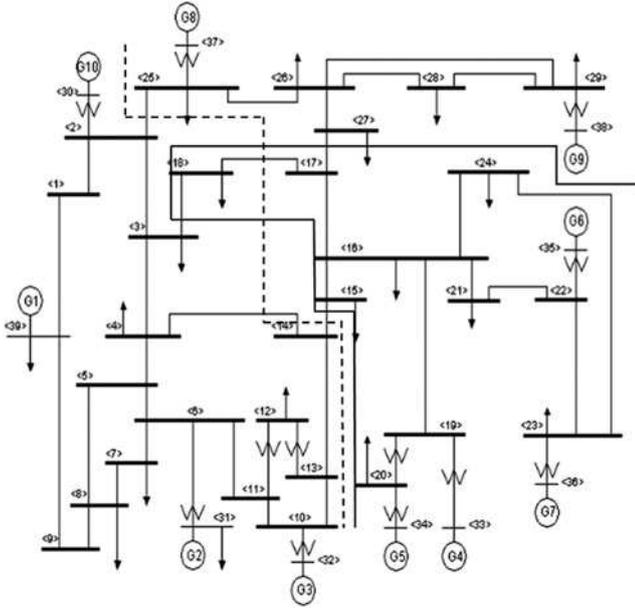


Fig. 8 New England 39-bus system partitioning boundaries on normal (solid line) and heavy load (dash line) operating conditions

In this case, one trains a 3×3 hexagonal Kohonen map by using MATLAB SOM toolbox (with a Gaussian neighbourhood function). As a comparative experiment, k -means algorithm (in MATLAB Statistics Toolbox) is used to cluster combined eigenspace. The clustering performance is evaluated by DBI. The optimal cardinality is selected with minimum DBI.

5.1.2 Network partitioning: According to the methodology presented in Section 4, different SOMs are first trained under two operating conditions. To demonstrate the automated clustering of SOM neurons, two given SOMs, which are trained, respectively, by eigenspace with $(k_{MW}=1, k_{MVA_r}=2)$ and $(k_{MW}=3, k_{MVA_r}=2)$ in normal operating condition, are clustered based on CONN linkage. Comparative experiment results are shown in Fig. 6 and Table 1.

Each point plotted in Figs. 6a and b represents the corresponding bus in this system, whose coordinates are determined by the first three eigenvectors in the combined eigenspace. Apparently, clustering results of SOM and k -means are fairly different. As for the k -means algorithm, the number of clusters is determined by DBI, which has a preference of four in this case. Based on the

proposed approach, two clusters with the highest CONN_index are found.

As reported in Table 1, clusters obtained by the proposed approach have less real power imbalance and total voltage violation. Moreover, when more clustering information of real power are considered (i.e. $k_{MW}=3, k_{MVA_r}=2$), the resultant clustering has a bias for less real power imbalance based on the proposed approach. In contrast, the k -means based clustering fails to capture such data features since larger MW imbalance is undesirably obtained under the condition of $k_{MW}=3, k_{MVA_r}=2$. Obviously, SOM based clustering outperforms k -means because SOM can find an optimal distribution of the neuron vectors in the combined eigenspace (which is an adaptive vector quantisation process) such that the density distribution of the clustering information for real and reactive power can be best approximated. Therefore, SOM is more suitable for the proposed heuristic partitioning approach.

Based on the proposed heuristic algorithm, experimental comparisons of SOM based automated clustering and k -means are reported in Table 2. Partitioning candidates are shown in Fig. 7, where the obtained Pareto fronts are marked by dash lines.

As shown in Figs. 7a and b, SOM based heuristic partitioning is more effective than k -means because less partitioning trials are exploited and the obtained Pareto front dominates the one obtained by k -means. This is reasonable since k -means based clustering fails to capture the data topology which combines information of real and reactive power (this has been proven through the whole experiment and an example is shown in Table 1). Among all partitioning alternatives (reported in Table 2), optimal partitions are selected with maximum fuzzy membership values, which are marked by asterisks. Partitioning boundaries are marked in Fig. 8.

5.2 IEEE 118-bus system

The IEEE 118-bus test system is extracted from the Midwestern US power grid in 1962, which consists of 54 generators and 186 branches. In this case, three operating conditions with different loading levels are considered.

5.2.1 Parameter setting: Three operating conditions with total load (4242 MW, 1438 MVA_r), (5302.5 MW, 1840.6 MVA_r), and (6151 MW, 2157 MVA_r) are taken into account. Power flow solutions are produced based on typical IEEE 118-bus system datasets [24].

In this case, a 4×4 hexagonal Kohonen map is trained by using MATLAB SOM toolbox (with a Gaussian neighbourhood function), the k -means algorithm (in MATLAB Statistics Toolbox) is applied to compare the clustering results with the proposed approach.

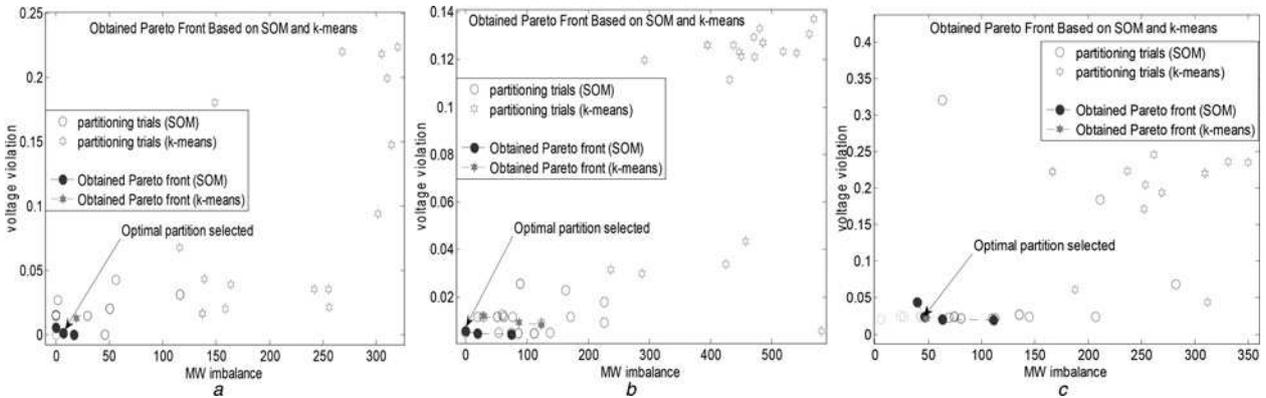


Fig. 9 Heuristic partitioning results based on SOM and k -means with total load

- a 4242 MW, 1438 MVA_r
- b 5302.5 MW, 1840.6 MVA_r
- c 6151 MW, 2157 MVA_r

Table 3 Partitioning results (IEEE 118-bus system)

Heuristic partitioning	Partitioning alternatives (with cluster sizes)	MW imbalance, MW	Voltage violation, p. u	μ
Total load (4242 MW, 1438 MVA _r)				
SOM	Cluster 1:62	0	0.005653	0.3312
	Cluster 2:56			
	Cluster 1:65	7	0.001025	0.3399*
	Cluster 2:53			
	Cluster 1:29	17	0	0.3290
k-means	Cluster 2:89			
	Cluster 1:35	19	0.01284	1
	Cluster 2:36			
	Cluster 3:18			
Total load (5302.5 MW, 1840.6 MVA _r)				
SOM	Cluster 1:15	0	0.005658	0.3522*
	Cluster 2:46			
	Cluster 3:57			
	Cluster 1:12	19.85	0.004657	0.3449
	Cluster 2:46			
	Cluster 3:60			
	Cluster 1:12	74.02	0.004478	0.3029
	Cluster 2:37			
	Cluster 3:69			
	Cluster 1:45	28.46	0.0118	0.3443
k-means	Cluster 2:40			
	Cluster 3:33			
	Cluster 1:27	87.43	0.009168	0.3320
	Cluster 2:38			
	Cluster 3:43			
	Cluster 4:10			
	Cluster 1:47	122.9	0.008321	0.3236
	Cluster 2:33			
	Cluster 3:15			
	Cluster 4:23			
Total load (6151 MW, 2157 MVA _r)				
SOM	Cluster 1:27	39.76	0.04368	0.2598
	Cluster 2:15			
	Cluster 3:46			
	Cluster 4:30			
	Cluster 1:34	46.2	0.02394	0.2675*
	Cluster 2:47			
	Cluster 3:37			
	Cluster 1:35	63.31	0.0206	0.2557
	Cluster 2:46			
	Cluster 3:37			
k-means	Cluster 1:5	111.2	0.01978	0.2170
	Cluster 2:33			
	Cluster 3:43			
	Cluster 4:37			
	Cluster 1:40	47.73	0.02142	1
	Cluster 2:45			
	Cluster 3:33			

5.2.2 Network partitioning: In this case, along with the increasing of network size, the complexity of data topology (i.e. combined eigenspace) is significantly increased. Experimental results are shown in Fig. 9 and Table 3.

As shown in Fig. 9 and Table 3, the proposed approach outperforms *k*-means based clustering. Obviously, more partitioning trials are needed for *k*-means based clustering. With the increasing size of PN, the proposed approach still performs well with less partitioning trials and better obtained Pareto front.

5.3 Polish 2383-bus transmission system

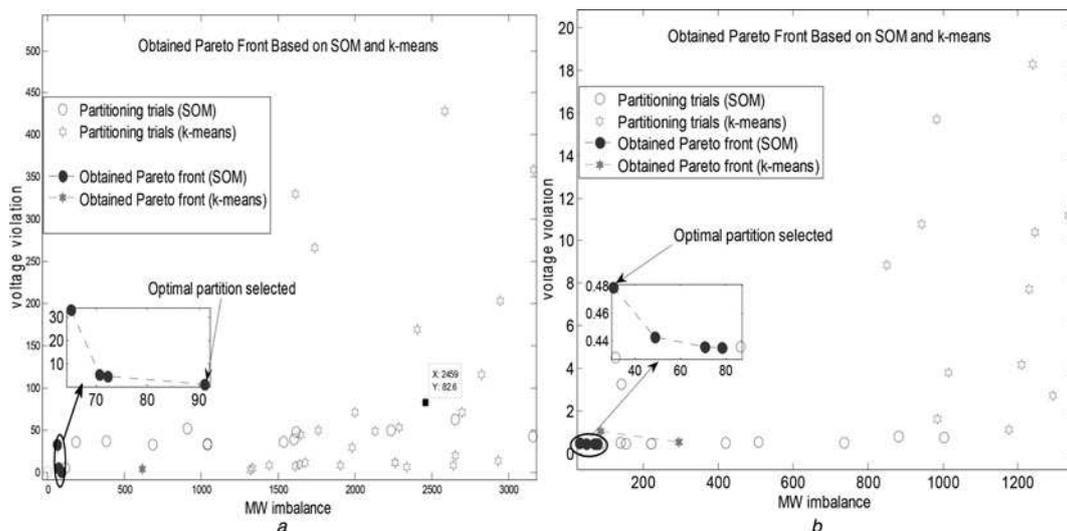
To provide further testing of the proposed approach, a realistic and relatively large transmission system, the 2383-bus transmission network data from Poland grid is used and two realistic loading profiles are considered in this case. This transmission network consists of 327 generators and 2896 branches.

5.3.1 Parameter setting: Two operating conditions with different load profiles are considered, i.e. (i) total generation of 25,280.97 MW, 8810.45 MVA_r, and total load of 24,558.38 MW, 8143.92 MVA_r; and (ii) total generation of 18,961 MW, 6607.8 MVA_r, and total load of 18,419 MW, 6108 MVA_r.

A 6 × 6 hexagonal Kohonen map is trained by using MATLAB SOM toolbox (with a Gaussian neighbourhood function), *k*-means algorithm (in MATLAB Statistics Toolbox) is also applied on this transmission network to compare the clustering results.

5.3.2 Network partitioning: As shown in Fig. 10, with the complexity of data topology considerably increased, the proposed approach incorporating SOM based automated clustering is substantially more effective than *k*-means. As reported in Table 4, it is obvious that *k*-means based clustering with employment of DBI is incapable of achieving satisfactory clustering boundaries that meet the objectives of minimising real power imbalance and voltage violation. As shown in Figs. 10a and b, also reported in Table 4, the Pareto optimal solutions produced by the proposed approach have completely dominated the ones obtained by *k*-means.

5.4 Elapsed time: All case studies were repeated for 50 times on the workstation (Dell Precision T7600, Intel Xeon CPU E5-2667 0 @2.90 GHz, 2 processors). The average computing time for each case is reported in Table 5. Such level of elapsed time is competitive, which shows high potential for practical applications.

**Fig. 10** Heuristic partitioning results based on SOM and *k*-means with total load

a 24558.38 MW, 8143.92 MVA_r
b 18,419 MW, 6108 MVA_r

Table 4 Partitioning results (Polish 2383-bus system)

Heuristic partitioning	Partitioning alternatives (with cluster sizes)	MW imbalance, MW	Voltage violation, p. u	μ	
SOM	Total load (24558.38 MW, 8143.92 MVar)				
	Cluster 1:24	65.21	32.89	0.2146	
	Cluster 2:1894				
	Cluster 3:465				
	Cluster 1:82	70.73	5.315	0.2595	
	Cluster 2:464				
	Cluster 3:1153				
	Cluster 4:684				
	Cluster 1:465	72.31	4.56	0.2607	
	Cluster 2:1153				
	Cluster 3:765				
	Cluster 1:466	91.09	1.237	0.2653*	
Cluster 2:1917					
k-means	Cluster 1:521	619	3.795	1	
	Cluster 2:70				
	Cluster 3:723				
	Cluster 4:1069				
SOM	Total load (18,419 MW, 6108 MVar)				
	Cluster 1:58	30.62	0.479	0.2521*	
	Cluster 2:2325				
	Cluster 1:347	48.72	0.4427	0.2513	
	Cluster 2:2036				
	Cluster 1:350	70.94	0.4352	0.2488	
	Cluster 2:2033				
	Cluster 1:429	78.37	0.4348	0.2478	
	Cluster 2:1944				
	k-means	Cluster 1:852	86.11	1.032	0.5183
		Cluster 2:1277			
		Cluster 3:254			
Cluster 1:395		295.9	0.5156	0.4817	
Cluster 2:960					
Cluster 3:1028					

Table 5 Elapsed time in different cases

Test case	Load profiles	Averaging elapsed time, s
IEEE 39-bus system	6254.23 MW 1387.1 MVar	6.241
	7044.46 MW 2020.11 MVar	6.140
IEEE 118-bus system	4242 MW, 1438 MVar	8.126
	5302.5 MW, 1840.6 MVar	8.233
	6151 MW, 2157 MVar	8.569
Polish 2383-bus system	24,558.38 MW, 8143.92 MVar	89.324
	18,419 MW, 6108 MVar	93.605

6 Conclusions

An adaptive hybrid partitioning approach is proposed in this paper, which combines Laplacian spectrum of PNs and SOM based automated clustering. A tailored heuristic optimisation algorithm is also developed to solve the bi-objective partitioning problem. Case studies are carried out based on the New England 39-bus, IEEE 118-bus and Polish 2383-bus transmission systems, which demonstrate the effectiveness and computational efficiency of the proposed approach, in producing optimal self-sufficient network partitions with small real power imbalance and satisfactory voltage profiles.

This hybrid partitioning approach can layout a useful groundwork for the development of self-healing smart grid. With the advantages of SOM, the approach can be easily extended by taking other operating factors (e.g. generator coherency and voltage angle) in to considerations. The promising partitioning results obtained in this paper have demonstrate the effectiveness of SOM and other relevant learning algorithms for applications in smart grid planning and operation control.

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