Self-Triggered Filter Design for A Class of Nonlinear Stochastic Systems with Markovian Jumping Parameters

Hua Yang, Zidong Wang, Yuxuan Shen and Fuad E. Alsaadi

Abstract

This paper is concerned with the self-triggered filtering problem for a class of Markovian jumping nonlinear stochastic systems. The event-triggered mechanism (ETM) is employed between the sensor and the filter to reduce unnecessary measurement transmission. Governed by the ETM, the measurement is transmitted to the filter as long as a predefined condition is satisfied. The purpose of the addressed problem is to synthesize a filter such that the dynamics of the filtering error is bounded in probability (BIP). A sufficient condition is first given to ensure the boundedness in probability of the filtering error dynamics, and the characterization of the desired filter gains is then realized by means of the feasibility of certain matrix inequalities. Furthermore, a self-triggered mechanism is designed to guarantee the filtering error dynamics to be BSP with excluded Zeno phenomenon. In the end, numerical simulation is carried out to illustrate the usefulness of the proposed self-triggered filtering algorithm.

Index Terms

Nonlinear stochastic systems, event-triggered mechanism, self-triggered mechanism, Markovian jumping parameters, boundedness in probability.

I. Introduction

Nonlinearity and stochasticity are commonly known as two inherent yet ubiquitous features of almost all real-world systems that constitute a large degree of the system complexities [1], [2], [38]. In the past few decades, both analysis and synthesis issues for nonlinear stochastic systems (NSSs) have received an ever-increasing research interest from various communities and a great number of excellent results have been published in the literature, see e.g. [8], [12], [21], [24], [32], [36], [42].

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On the other hand, in industrial systems, the true states of the underlying plants are often unavailable and it makes practical sense to reconstruct the system states based on available but possibly contaminated measurements, which gives rise to the so-called filtering problem. In fact, the filtering problem has long been serving as a fundamental research topic in both areas of signal processing and control engineering [5], [11], [13], [19], [20], [44]. Recently, due to its practical significance in engineering, the nonlinear filtering problem for stochastic systems has gained particular research attention and a great many methods have been developed according to the performance specifications of the filtering error dynamics [3], [4], [15], [27], [30].

Recently, Markovian jumping systems (MJSs) have received considerable research attention since many practical systems, whose structures are subject to random yet unpredictable changes, can be modeled as MJSs. In MJSs, the parameters of the system switch between a few prescribed modes, and such a switching phenomenon obeys a Markov process [45]. So far, a surge of research attention has been devoted to the filtering problems for NSSs with Markovian jumping parameters, and some representative results can be seen in [18], [29], [46]. For example, the H_∞ filtering problem has been studied in [46] for NSSs with both time-delays and Markovian jumping parameters, where sufficient conditions have been given to ensure the existence of the desired H_∞ filters by solving a set of Hamilton-Jacobi inequalities. The state estimation problem has been investigated in [18] for NSSs with Markovian jumping parameters, and the almost sure asymptotic stability has been examined on the estimation error dynamics.

In filtering problems for networked systems, the traditional time-triggered mechanism (TTM) has been widely considered under which the data transmission between the sensor and the filter is executed at predetermined time instants. Nevertheless, in the case of limited communication resources, the TTM will lead to unnecessary data transmissions which, in turn, would result in the so-called network-induced phenomena such as time-delays and packet dropouts [26], [47]. Therefore, the event-triggered mechanism (ETM) has been introduced with hope to reduce unnecessary data transmissions. Under the ETM, the data transmission is executed only when a certain prescribed condition is satisfied [39]. Recently, the event-triggered filtering problem has attracted considerable research attention [7], [10], [16], [22], [25], [33], [34], [41], [43], [48]. The event-triggered H_∞ filtering issue has been examined in [48] with packet dropouts, and the filter gain and the triggering condition have been co-designed by resorting to the solution to certain matrix inequalities. In [22], the correlated noises have been taken into account with respect to the event-triggered filtering problem for nonlinear systems, and efforts have been made to acquire sufficient conditions that guarantee the convergence of the filtering error dynamics.

Note that, in filtering problem with ETM, the measurement of the sensor is transmitted to the filter only if the prescribed condition is met and therefore triggered. In order to check whether the triggering condition is satisfied or not, the ETM needs to continuously monitor certain information of the system which still brings terrible burden on the limited communication resource. To overcome this shortcoming, the self-triggered mechanism (STM) has been proposed in [35] under which the next triggering instant is pre-computed at the current triggering instant by exploiting the received information and the knowledge of the plant dynamics. Therefore, such a communication mechanism can avoid the continuously monitoring of the concerned information and further reduce the consumption of the limited communication resource. Until now, the STM has stirred some initial research interests [9], [14], [31], [39], [40]. Nonetheless, the filtering problem for NSSs under STM has not been looked into yet, not to mention the more complicated case where the Markovian jumping parameters are involved as well. Naturally, we are motivated to shorten such a gap in this paper.

The main aim of this paper is to deal with the filter design problem for NSSs with Markovian jumping parameters under the STM. The primary contributions of the current investigation can be outlined below. *1) The self-triggered filtering problem is, as the first-ever research attempt, tackled for nonlinear stochastic systems with Markovian jumping parameters. 2) Sufficient condition is given for the boundedness in probability of the filtering error dynamics under the ETM, and the filter gains are computed by solving certain matrix inequalities. 3) A STM is proposed that ensures the boundedness in probability of the filtering error dynamics while excluding the Zeno phenomenon.* Finally, a numerical example is given to verify the usefulness of the developed self-triggered filtering algorithm.

Notation The notation adopted in this paper is quite standard except. $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ denotes a complete probability space under a filtration $\{\mathscr{F}\}_{t\geq 0}$ meeting the usual conditions (i.e. the filtration is right continuous while containing all P-null sets). $\mathbb{E}\{\cdot\}$ represents the mathematical expectation of the stochastic variable " $\cdot$ ". $P\{\cdot\}$ means the probability of the event " $\cdot$ ". For a real vector x, the symbol $|x|$ is its Euclidean norm. A^T denotes the transpose of a matrix A. For a matrix A, $\lambda_M(A)$ stands for the maximum eigenvalue of A, and $\mathrm{Tr}(A)$ denotes the trace of A. The block-diagonal matrix $\mathrm{diag}\{A_1, A_2, \cdots, A_n\}$ consists of the square matrices A_i being its corresponding main diagonal blocks. $\mathcal{K}$ denotes a class of continuous (strictly) increasing functions μ from $\mathbb{R}_+$ to $\mathbb{R}_+$ with $\mu(0) = 0$. $\mathcal{KL}$ denotes a class of functions $\beta(s,t) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$, which are of $\mathcal{K}$-class for each fixed t and decrease to zero as $t \to \infty$ for each fixed s.

II. Problem Formulation

On the probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$, let $\{r(t),\ t \geq 0\}$ be a right-continuous Markov process taking values in the finite state space $S = \{1, 2, \cdots, N\}$ with generator $\Gamma = \{\gamma_{ij}\}_{N\times N}$ given by

$$P\{r(t+\Delta) = j|r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if} \quad i \neq j, \\ 1 + \gamma_{ii}\Delta + o(\Delta) & \text{if} \quad i = j \end{cases}$$

where $\gamma_{ij} \geq 0$ is the transition rate from mode i to mode j if $i \neq j$, $\gamma_{ii} = -\sum_{j\neq i}\gamma_{ij}$, and $\triangle > 0$.

Consider the following NSS with Markovian jumping parameters:

$$\begin{cases} dx(t) = f(x(t), r(t))dt + g(t, r(t))dw(t) \\ \quad y(t) = h(x(t), r(t)) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^q$ is the measurement output, $f(\cdot,\cdot)$, $h(\cdot,\cdot)$ and $g(\cdot,\cdot) = \begin{bmatrix} g_1(\cdot,\cdot) & g_2(\cdot,\cdot) & \cdots & g_n(\cdot,\cdot) \end{bmatrix}^T$ are measurable nonlinear functions, and $w(t)$ is a scalar Wiener process defined on the probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_t\}_{t\geq 0}, P)$ that is independent of $r(t)$ and satisfies $\mathbb{E}\{dw(t)\} = 0$ and $\mathbb{E}\{dw^2(t)\} = t$.

For convenience of presentation, for each possible mode $r(t) = i$ $(i \in S)$, we denote the matrix $M(r(t))$ as M_i, the scalar $m(r(t))$ as m_i, and the function $\Phi(\cdot, r(t))$ as $\Phi(\cdot, i)$.

Assumption 1: For mode $r(t) = i$ $(i \in S)$, the measurable nonlinear functions $f(\cdot, \cdot)$ and $h(\cdot, \cdot)$ satisfy

$$\begin{aligned} & f(0, i) = 0, \quad x^T(t) f(x(t), i) \leq q_i |x(t)|^2, \\ & |f(x(t), i)|^2 \leq b_{fi}(1 + |x(t)|^2), \\ & |f(x(t) + \delta, i) - f(x(t), i) - A_i \delta| \leq a_i |\delta|, \ \forall \delta \in \mathbb{R}^n \end{aligned} \tag{2}$$

and

$$\begin{aligned} & h(0, i) = 0, d_{Mi} \left| \delta \right| \geq \left| h(x(t) + \delta, i) - h(x(t), i) \right| \geq d_{mi} \left| \delta \right|, \\ & \left| h(x(t) + \delta, i) - h(x(t), i) - C_i \delta \right| \leq c_i \left| \delta \right|, \ \forall \delta \in \mathbb{R}^n \end{aligned} \tag{3}$$

where a_i, b_{fi}, c_i, d_{Mi}, d_{mi}, q_i are known positive scalars and A_i, C_i are known constant matrices with appropriate dimensions.

Assumption 2: For mode $r(t) = i$ $(i \in S)$, the components $g_k(t, i)$ $(k = 1, 2, \ldots, n)$ of the measurable nonlinear function $g(t, i)$ satisfy the following inequalities:

$$-\bar{g}_{ki} \leq g_k(t, i) \leq \bar{g}_{ki}, \ k = 1, 2, \ldots, n \tag{4}$$

where $\bar{g}_{ki}$ $(k = 1, 2, \ldots, n)$ are known positive scalars.

In this paper, in order to reduce unnecessary data transmission, the ETM is employed in the channel between the sensor and the filter. The measurement from the sensor will be transmitted to the filter only if the predefined condition is satisfied. The sequence of triggering time instants of the ETM is denoted as $\{t_k\}_{k \in \mathbb{N}}$ and can be determined iteratively by the following rule:

$$t_{k+1} = \inf\{t | t > t_k, \ |v(t)|^2 - d_e \geq 0\} \tag{5}$$

where $v(t)$ is denoted as

$$v(t) \triangleq y(t_k) - y(t)$$

for $t \in [t_k, \ t_{k+1})$ and d_e is a given positive scalar.

Remark 1: From the event-triggered condition $|v(t)|^2 - d_e \geq 0$, it can be seen that the triggering frequency of the ETM is determined by the threshold d_e. A smaller threshold d_e leads to more frequent data transmission. Moreover, the ETM (5) will reduce to a traditional time-triggered one when $d_e = 0$.

Based on the ETM, the actual measurement $\bar{y}(t)$ received by the filter (with a zero-order holder) from the sensor can be written as

$$\bar{y}(t) = y(t_k), \ \forall t \in [t_k, \ t_{k+1}).$$

In this paper, for mode $i \in S$, the filter of the following structure is adopted:

$$d\hat{x}(t) = f(\hat{x}(t), i)dt + K_i\big(\bar{y}(t) - h(\hat{x}(t), i)\big)dt \tag{6}$$

where $\hat{x}(t)$ is the estimate of the state $x(t)$ and K_i is the filter gain to be designed.

By noting $y(t_k) = v(t) + y(t)$, we have

$$d\hat{x}(t) = f(\hat{x}(t), i)dt + K_i\big(y(t) + v(t) - h(\hat{x}(t), i)\big)dt. \tag{7}$$

Denote the filtering error as $e(t) \triangleq x(t) - \hat{x}(t)$. Then, combining (1) and (7), we have

$$\begin{aligned} de(t) =& (A_i - K_iC_i)e(t)dt - K_in_i(t)dt + l_i(t)dt \\ &- K_iv(t)dt + g(t,i)dw(t) \end{aligned} \tag{8}$$

where

$$\begin{aligned} l_i(t) &= f(x(t), i) - f(\hat{x}(t), i) - A_ie(t), \\ n_i(t) &= h(x(t), i) - h(\hat{x}(t), i) - C_ie(t). \end{aligned} \tag{9}$$

Definition 1: [28] The filtering error dynamics (8) is said to be bounded in probability (BIP) if, for any given scalar $\varepsilon > 0$, there exist a function $\beta \in \mathcal{KL}$, a function $\gamma \in \mathcal{K}$, and a nonnegative scalar d such that the solution $e(t) = e(t; e_0, r_0)$ satisfies

$$P\{|e(t)| < \beta(|e_0|, t) + \gamma(d)\} \geq 1 - \varepsilon, \quad \forall t \geq 0$$

for any initial conditions $e(0) = e_0$ and $r(0) = r_0$.

The purpose of this paper is to: 1) design a filter in the form of (6) such that the filtering error dynamics (8) is BIP; and 2) design a STM that ensures the boundedness in probability of the filtering error dynamics and excludes the Zeno phenomenon.

III. Filter Design

In this section, the boundedness in probability is first analyzed for system (8) and the filter design problem is then investigated. Before presenting our results, the following infinitesimal generator is first introduced.

Let $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times S; \mathbb{R}_+)$ denote the family of all nonnegative functions that are twice continuously differentiable in e and once in t. For $V(e,t,i) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+ \times S; \mathbb{R}_+)$, an infinitesimal generator $\mathcal{L}$ $(\mathbb{R}^n \times \mathbb{R}_+ \times S \to \mathbb{R})$ of (8) is defined as

$$\begin{aligned} &\mathcal{L}V(e,t,i) \\ &= V_t(e,t,i) + V_e(e,t,i)\bar{f}(e,t,i) \\ &+ \frac{1}{2}\mathrm{Tr}\left\{g^T(t,i)V_{ee}(e,t,i)g(t,i)\right\} + \sum_{j=1}^{N}\gamma_{ij}V(e,t,j) \end{aligned} \tag{10}$$

where

$$\begin{aligned} V_t(e,t,i) &= \frac{\partial V(e,t,i)}{\partial t}, V_{ee}(e,t,i) = \left(\frac{\partial^2 V(e,t,i)}{\partial e_j \partial e_k}\right)_{n\times n}, \\ V_e(e,t,i) &= \left(\frac{\partial V(e,t,i)}{\partial e_1}, \cdots, \frac{\partial V(e,t,i)}{\partial e_n}\right), \end{aligned}$$

$$\bar{f}(e,t,i) = (A_i - K_iC_i)e(t) - K_iv(t) + l_i(t) - K_in_i(t).$$

Lemma 1: [28] Consider the following system:

$$dx = f(x,t)dt + g(x,t)dw(t),$$

where $x(t) \in \mathbb{R}^n$ is the state, w is an r-dimensional standard Brownian (Wiener) process, $f(\cdot,\cdot)$ and $g(\cdot,\cdot)$ are measurable nonlinear functions.

If there exists a positive-definite, radially unbounded, twice continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}$, a constant $c \geq 0$, and a positive-definite and radially unbounded function $W(x)$ such that

$$\mathcal{L}V \leq -W(x) + c,$$

then the solution process is BIP.

In the following theorem, a sufficient condition is given under which the filtering error dynamics (8) is BIP.

Theorem 1: Let the filter gains K_i $(i \in S)$ be given. Under the ETM (5), the filtering error dynamics (8) is BIP if there exist positive definite matrices P_i $(i \in S)$ and positive scalars ε_{1i}, ε_{2i} and ε_{3i} $(i \in S)$ such that

$$\begin{aligned}\Pi_i =&(A_i - K_iC_i)^TP_i + P_i(A_i - K_iC_i) + \sum_{j=1}^{N}\gamma_{ij}P_j \\ &+ P_i\left[(\varepsilon_{1i} + \varepsilon_{3i})K_iK_i^T + \varepsilon_{2i}I\right]P_i \\ &+ (\varepsilon_{2i}^{-1}a_i^2 + \varepsilon_{3i}^{-1}c_i^2)I < 0.\end{aligned} \tag{11}$$

Proof: Choose the Lyapunov function for system (8) as

$$V(e(t),i) = e^T(t)P_ie(t). \tag{12}$$

Then, the infinitesimal generator $\mathcal{L}V(e(t),i)$ for system (8) can be obtained as

$$\begin{aligned}&\mathcal{L}V(e(t),i) \\ =&e^T(t)\left[(A_i - K_iC_i)^TP_i + P_i(A_i - K_iC_i)\right]e(t) \\ &- e^T(t)P_iK_iv(t) - v^T(t)K_i^TP_ie(t) + g^T(t,i)P_ig(t,i) \\ &+ e^T(t)P_i\left[l_i(t) - K_in_i(t)\right] + \left[l_i(t) - K_in_i(t)\right]^TP_ie(t) \\ &+ e^T(t)\sum_{j=1}^{N}\gamma_{ij}P_je(t).\end{aligned}$$

It is easily known from $2\eta^T\xi \leq \eta^T\eta + \xi^T\xi$ (where η and ξ are vectors of appropriate dimensions) that

$$\begin{aligned}&-e^T(t)P_iK_iv(t) - v^T(t)K_i^TP_ie(t) \\ &\qquad\leq\varepsilon_{1i}e^T(t)P_iK_iK_i^TP_ie(t) + \varepsilon_{1i}^{-1}v^T(t)v(t).\end{aligned}$$

Moreover, from (2) and (3), we have $l_i^T(t)l_i(t) \leq a_i^2 e^T(t)e(t)$ and $n_i^T(t)n_i(t) \leq c_i^2 e^T(t)e(t)$, which derive

$$\begin{aligned} & e^T(t)P_i l_i(t) + l_i^T(t)P_i e(t) \\ & \qquad\qquad \leq e^T(t)\left(\varepsilon_{2i}P_i^2 + \varepsilon_{2i}^{-1}a_i^2 I\right)e(t), \\ & -e^T(t)P_i K_i n_i(t) - n_i^T(t)K_i^T P_i e(t) \\ & \qquad\qquad \leq e^T(t)\left(\varepsilon_{3i}P_i K_i K_i^T P_i + \varepsilon_{3i}^{-1}c_i^2 I\right)e(t). \end{aligned}$$

Then, it can be obtained that

$$\begin{aligned} \mathcal{L}V(e(t),i) \leq & e^T(t)\Pi_i e(t) + \varepsilon_{1i}^{-1}v^T(t)v(t) + g^T(t,i)P_i g(t,i) \\ \leq & -e^T(t)(-\Pi_i)e(t) + \varepsilon_{1i}^{-1}d_e + \lambda_M(P_i)\bar{g}_i \\ = & -e^T(t)(-\Pi_i)e(t) + h_i \end{aligned} \tag{13}$$

where $\bar{g}_i \triangleq \sum_{k=1}^n \bar{g}_{ki}^2$, $h_i \triangleq \varepsilon_{1i}^{-1}d_e + \lambda_M(P_i)\bar{g}_i$.

From the definition of $V(e(t),i)$ in (12) and the condition of (11), we know that $V(e(t),i)$ is a positive-definite, radially unbounded, twice continuously differentiable function and $e^T(t)(-\Pi_i)e(t)$ is a positive-definite and radially unbounded function. Thus, according to Lemma 1, we know that the filtering error dynamics (8) is BIP. The proof is complete. ■

Now, we are ready to derive the solution to the event-triggered filtering problem for nonlinear stochastic systems with Markovian jumping parameters.

Theorem 2: For nonlinear stochastic system (1) with the filter (6) under the ETM (5), the filtering error dynamics (8) is BIP if there exist positive definite matrices P_i $(i \in S)$, matrices Y_i $(i \in S)$ and positive scalars ε_{1i}, ε_{2i} and ε_{3i} $(i \in S)$ such that

$$\begin{bmatrix} \Upsilon_i & \Omega_i \\ * & -\Xi_i \end{bmatrix} < 0, \quad i \in S \tag{14}$$

where

$$\begin{aligned} \Upsilon_i &= A_i^T P_i + P_i A_i - C_i^T Y_i^T - Y_i C_i + \sum_{j=1}^N \gamma_{ij}P_j, \\ \Omega_i &= \begin{bmatrix} Y_i & Y_i & P_i & \varepsilon_{2i}^{-1}a_i I & \varepsilon_{3i}^{-1}c_i I \end{bmatrix}, \\ \Xi_i &= \mathrm{diag}\{\varepsilon_{1i}^{-1}I, \varepsilon_{3i}^{-1}I, \varepsilon_{2i}^{-1}I, \varepsilon_{2i}^{-1}I, \varepsilon_{3i}^{-1}I\}. \end{aligned}$$

Furthermore, if the inequalities (14) are feasible, the desired filter gains are given as

$$K_i = P_i^{-1}Y_i, \ i \in S. \tag{15}$$

Proof: For $i \in S$, pre- and post-multiplying the inequality (14) by $\mathrm{diag}\{I, \varepsilon_{1i}^{1/2}I, \varepsilon_{3i}^{1/2}I, \varepsilon_{2i}^{1/2}I, \varepsilon_{2i}^{1/2}I, \varepsilon_{3i}^{1/2}I\}$ and letting $Y_i = P_i K_i$, the inequality (11) can be easily obtained from (14) by using the Schur Complement Lemma. Therefore, the rest of the proof follows immediately from Theorem 1. ■

Remark 2: It is obvious that the conditions in Theorem 2 are in the form of certain LMIs. As is well known, the computation complexity of the standard LMI system is bounded by $O(\mathcal{X}\mathcal{Y}^3)$, where $\mathcal{X}$ is the total row size of the LMIs and $\mathcal{Y}$ is the total number of the scalar decision variables. From (14), we know that $\mathcal{X} = 6Nn$ and $\mathcal{Y} = Nn^2 + Nnq + 3N$. Obviously, the computation complexity of the conditions in Theorem 2 depends polynomially on the size of the system.

Until now, the event-triggered filter has been designed such that the filtering error dynamics is BIP. In the following section, a STM is proposed based on the obtained event-triggered filter.

IV. Self-Triggered Mechanism

As discussed in the introduction, to reduce the consumption of the communication resource in continuously monitoring the measurement output, a STM is proposed under which the filtering error dynamics (8) is BIP and the Zeno phenomenon is excluded.

Lemma 2: Suppose that the condition of Theorem 1 is satisfied and the STM is triggered at $t = t_k$. Then, for $t \in [t_k,\ t_{k+1})$ and each $i \in S$, $|v(t)|^2$ satisfies

$$\mathbb{E}\{|v(t)|^2\} \leq \varphi\left(|y(t_k)|, t - t_k, i\right) \tag{16}$$

where

$$\begin{aligned}
&\varphi\big(|y(t_k)|, t - t_k, i\big) \\
&\qquad\qquad \triangleq \frac{s_{3i}|y(t_k)|^2 + s_{4i}}{s_{2i}} \left(e^{s_{2i}(t-t_k)} - 1\right), \\
&s_{1i} \triangleq d_{Mi}^2 a^2 b_{fi}, \quad s_{2i} \triangleq \frac{d_{Mi}^2 + s_{1i}a^2\left(1+b^2\right)}{a^2 d_{mi}^2} \\
&s_{3i} \triangleq \frac{s_{1i}\left(1+b^2\right)}{b^2 d_{mi}^2}
\end{aligned}$$

with $a > 0, b > 0$.

Proof: For $t \in [t_k,\ t_{k+1})$, we know from (3) that there exists a positive scalar $d_{ki} \in [d_{mi}, d_{Mi}]$ such that

$$|v(t)|^2 = d_{ki}^2 \left|e_x(t)\right|^2 \tag{17}$$

where $e_x(t) \triangleq x(t_k) - x(t)$.

Moreover, according to Assumption 1, the differential of $\mathbb{E}\{|v(t)|^2\}$ can be calculated as

$$\begin{aligned}
&\frac{d}{dt}\mathbb{E}\left\{|v(t)|^2\right\} \\
&=d_{ki}^2\frac{d}{dt}\mathbb{E}\left\{|e_x(t)|^2\right\} \\
&=d_{ki}^2\frac{d}{dt}\mathbb{E}\left\{|x(t_k) - x(t)|^2\right\} \\
&=-2d_{ki}^2\mathbb{E}\left\{e_x^T(t)f(x(t),i)\right\} + d_{ki}^2\mathbb{E}\left\{|g(t,i)|^2\right\} \\
&\leq\frac{d_{ki}^2}{a^2}\mathbb{E}\left\{|e_x(t)|^2\right\} + d_{ki}^2a^2\mathbb{E}\left\{|f(x(t),i)|^2\right\} + d_{ki}^2\bar{g}_i \\
&\leq\frac{d_{ki}^2}{a^2}\mathbb{E}\left\{|e_x(t)|^2\right\} + d_{ki}^2a^2b_{fi}(1+\mathbb{E}\left\{|(x(t)|^2\right\}) + d_{ki}^2\bar{g}_i
\end{aligned}$$

$$
\begin{aligned}
\leq & \frac{d_{Mi}^2}{a^2}\mathbb{E}\left\{|e_x(t)|^2\right\} + d_{Mi}^2 a^2 b_{fi} + d_{Mi}^2 \bar{g}_i \\
& + d_{Mi}^2 a^2 b_{fi}\mathbb{E}\left\{|x(t_k) - e_x(t)|^2\right\} \\
\leq & \left(\frac{d_{Mi}^2}{a^2} + s_{1i}\left(1+b^2\right)\right)\mathbb{E}\left\{|e_x(t)|^2\right\} + s_{4i} \\
& + s_{1i}\left(1+\frac{1}{b^2}\right)|x\left(t_k\right)|^2 \\
\leq & s_{2i}\mathbb{E}\left\{|v(t)|^2\right\} + s_{3i}|y(t_k)|^2 + s_{4i}.
\end{aligned}
$$

Applying the Comparison Lemma, along with the fact that $v(t_k) = 0$, we have

$$
\mathbb{E}\left\{|v(t)|^2\right\} \leq \frac{s_{3i}|y(t_k)|^2 + s_{4i}}{s_{2i}}\left(e^{s_{2i}(t-t_k)} - 1\right). \tag{18}
$$

The proof is thus completed. ■

Theorem 3: Suppose that the condition of Theorem 1 is satisfied and the filter (6) updated at time $t = t_k$ with $y(t_k)$. If the time interval ς_k between two consecutive execution time instants t_k and t_{k+1}, i.e. $\varsigma_k = t_{k+1} - t_k$, satisfies

$$
\varphi\left(|y(t_k)|, \varsigma_k, i\right) \leq d_e \tag{19}
$$

where

$$
\varphi\left(|y(t_k)|, \varsigma_k, i\right) \triangleq \frac{s_{3i}|y(t_k)|^2 + s_{4i}}{s_{2i}}\left(e^{s_{2i}\varsigma_k} - 1\right),
$$

then the filtering error dynamics (8) is BIP. Furthermore, for all initial value e_0, the inter-execution times will not reach an accumulation point, that is, there exists a positive constant τ_k^* such that $\tau_k^* \leq \varsigma_k$ with

$$
\tau_k^* \triangleq \min_{i\in S}\tau_k(i) \tag{20}
$$

where $\tau_k(i)$ can be obtained by

$$
\tau_k(i) = \frac{1}{s_{2i}}\ln\frac{z_{1i}}{z_{2i}} \tag{21}
$$

with

$$
\begin{aligned}
z_{1i} &\triangleq d_e s_{2i} + s_{3i}|y(t_k)|^2 + s_{4i}, \\
z_{2i} &\triangleq s_{3i}|y(t_k)|^2 + s_{4i}.
\end{aligned}
$$

Proof: As the proof of Theorem 1, it is easily known from (19) that the filtering error dynamics is BIP. The first part of the proof is thus completed.

On the other hand, based on Lemma 2, we have that for any $t \in [t_k, t_{k+1})$ and each $i \in S$

$$
\varphi\left(|y(t_k)|, t - t_k, i\right) = \frac{s_{3i}|y(t_k)|^2 + s_{4i}}{s_{2i}}\left(e^{s_{2i}(t-t_k)} - 1\right) \tag{22}
$$

with $\varphi(|y(t_k)|, 0, i) = 0$.

Let $\phi(t) = \varphi(|y(t_k)|, t - t_k, i)$, it is easy to see that $\phi(t)$ is an increasing function with $\phi(0) = 0$. Owing to the monotonicity of function $\phi(t)$ and $d_e > 0$, we see that the following equation

$$
\varphi\left(|y(t_k)|, \varsigma_k, i\right) \geq d_e \tag{23}
$$

has a unique positive solution $\tau_k(i)$ being given by (21) such that $\varsigma_k \geq \tau_k^* = \min_{i\in S}\tau_k(i)$. The proof is completed. ■

Remark 3: In this paper, we aim to design a self-triggered filter for nonlinear stochastic systems with Markovian jumping parameters. In Theorem 1, sufficient condition is presented that ensures the boundedness in probability of the filtering error dynamics. Furthermore, in Theorem 2, the filter gains are characterized by means of solving certain linear matrix inequalities. Finally, in Theorem 3, a STM is designed such that the filtering error dynamics is BIP and the Zeno phenomenon is excluded.

Remark 4: Until now, a self-triggered filtering scheme has been proposed for nonlinear stochastic systems with Markovian jumping parameters. The advantages of the proposed self-triggered filtering scheme can be summarized as follows: 1) the system model under consideration is quite general that takes the nonlinearity, the stochasticity and the Markovian jumping parameters into account; and 2) a STM is proposed which no longer needs to continuously monitor the information of the system and reduces the communication resource consumption.

V. Numerical Example

In this section, a numerical example is presented to illustrate the validity of the developed self-triggered filtering scheme.

Consider stochastic nonlinear system (1) with two modes and the generator Γ being

$$\Gamma = \{\gamma_{ij}\}_{2\times 2} = \begin{bmatrix} -3 & 3 \\ 1.5 & -1.5 \end{bmatrix}.$$

The nonlinear functions $f(x(t),r(t))$, $g(t,r(t))$ and $h(x(t),r(t))$ are given as

$$f(x(t),1) = \begin{bmatrix} -x_1(t) - 0.2x_2(t) + 0.2\sin(x_1(t)+x_2(t)) \\ 1.3x_1(t) - 1.1x_2(t) + 0.3\sin x_1(t) \end{bmatrix},$$
$$f(x(t),2) = \begin{bmatrix} -0.5x_1(t) - 1.1x_2(t) + 0.3\sin x_2(t) \\ 0.2x_1(t) - 0.4x_2(t) + 0.4\sin x_1(t) \end{bmatrix},$$
$$g(t,1) = \begin{bmatrix} 0.2\sin(t) \\ 0.2\cos(t) \end{bmatrix}, \quad g(t,2) = \begin{bmatrix} 0.1\sin(t) \\ 0.1\cos(t) \end{bmatrix},$$
$$h(x(t),1) = \begin{bmatrix} 0.7x_1(t) + 0.2\sin x_2(t) \\ 0.7x_2(t) + 0.1\sin x_1(t) \end{bmatrix},$$
$$h(x(t),2) = \begin{bmatrix} 0.8x_1(t) + 0.1\sin x_1(t) \\ 0.8x_2(t) + 0.2\sin x_2(t) \end{bmatrix}.$$

Then, the parameters A_i, C_i, a_i, b_{fi}, c_i, d_{Mi}, d_{mi} and q_i can be obtained as

$$A_1 = \begin{bmatrix} -1 & -0.2 \\ 1.3 & -1.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.5 & -1.1 \\ 0.2 & -0.4 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.7 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix},$$

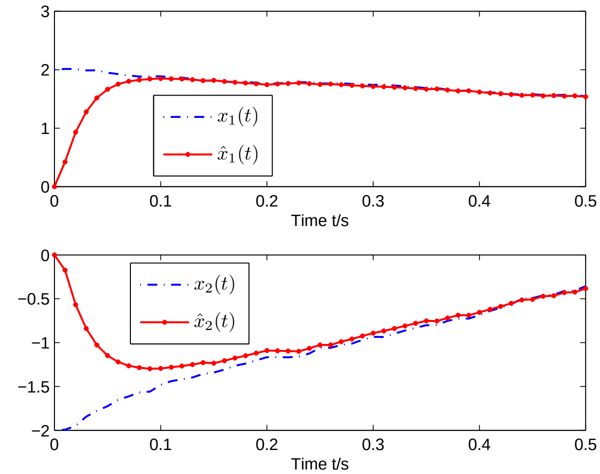

Fig. 1: The trajectory of the state $x(t)$ and its estimate

$$a_1 = 0.4123, \quad a_2 = 0.5, \quad b_{f1} = 4.13, \quad b_{f2} = 2.26,$$
$$c_1 = 0.2449, \quad c_2 = 0.2236, \quad d_{M1} = 0.0.96, \quad d_{M2} = 1.29,$$
$$d_{m1} = 0.5, \quad d_{m2} = 0.65, \quad q_1 = 0.2, \quad q_2 = 0.6.$$

Moreover, the threshold d_e is chosen as $d_e = 1.1$.

With the above parameters, by using the MATLAB LMI toolbox, the filter gains are obtained as

$$K_1 = \begin{bmatrix} 0.4230 & -0.0890 \\ -0.0868 & 0.2669 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.2602 & 0.0134 \\ 0.0107 & 0.1207 \end{bmatrix}.$$

The simulation results are given in Figs. 1-3. Fig. 1 depicts the trajectory of the state $x(t)$ and its estimate. Fig. 2 shows the filtering error $e(t)$. Fig. 3 plots the triggering time instants of the STM and the corresponding $|y(t_k)|^2$. The simulation results have verified that the proposed self-triggered filtering scheme is indeed effective for the NSSs with Markovian jumping parameters.

Remark 5: It can be seen from Fig. 3 that the data transmission rate under the STM is 62%. Compared with the TTM, the STM is able to significantly reduce the data transmissions and save the network resource. On the other hand, as discussed in the introduction, the STM can avoid the continuously monitoring of the concerned information and further save the network resource. Therefore, the STM is indeed effective in reducing the resource consumption.

VI. Conclusion

In this paper, the self-triggered filtering problem has been studied for nonlinear stochastic systems with Markovian jumping parameters. The ETM has been implemented in the sensor-to-filter channel to

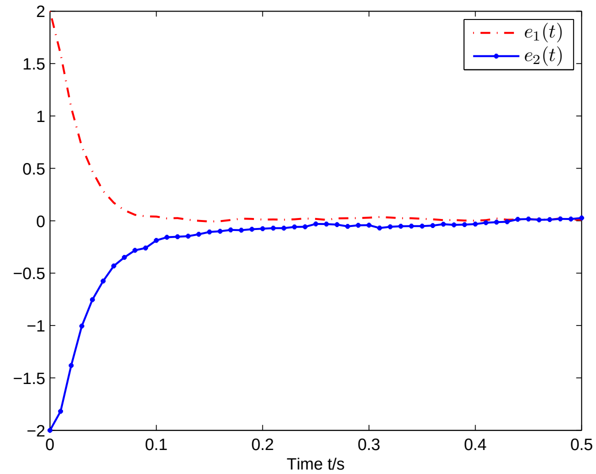

Fig. 2: The filtering error $e(t)$

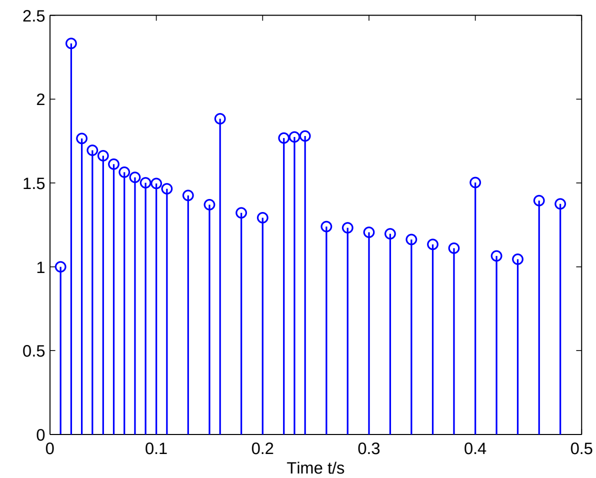

Fig. 3: The triggering time instants and the corresponding $|y(t_k)|^2$

reduce unnecessary data transmission. Sufficient condition has been first given such that the filtering error dynamics of the event-triggered filter is BIP. Then, the filter gains have been acquired by solving certain linear matrix inequalities. Furthermore, a STM has been proposed under which the filtering error dynamics of the obtained filter is BIP with excluded Zeno phenomenon. Finally, a numerical example has been presented to demonstrate the effectiveness of the proposed self-triggered filtering scheme. In our further research, we will study the self-triggered filtering problem for systems with Markovian jumping parameters where the transition probabilities are uncertain or unknown [17]. On the other hand, the sampled-data-based event-triggered mechanism is an interesting topic [6] and the corresponding filtering problem is worth investigating.

References

[1] M. V. Basin, P. C. R. Ramirez and F. Guerra-Avellaneda, Continuous fixed-time controller design for mechatronic systems with incomplete measurements, *IEEE-ASME Transactions on Mechatronics*, vol. 23, no. 1, pp. 57–67, Feb. 2018.

[2] M. V. Basin, P. Yu and Y. B. Shtessel, Hypersonic missile adaptive sliding mode control using finite- and fixed-time observers, *IEEE Transactions on Industrial Electronics*, vol. 65, no. 1, pp. 930–941, Jan. 2018.

[3] R. Caballero-Águila, A. Hermoso-Carazo and J. Linares-Pérez, Distributed fusion filters from uncertain measured outputs in sensor networks with random packet losses, *Information Fusion*, vol. 34, pp. 70–79, Mar. 2017.

[4] R. Caballero-Águila, A. Hermoso-Carazo and J. Linares-Pérez, New distributed fusion filtering algorithm based on covariances over sensor networks with random packet dropouts, *International Journal of Systems Science*, vol. 48, no. 9, pp. 1805–1817, Jul. 2017.

[5] D. Ciuonzo, A. Aubry and V. Carotenuto, Rician MIMO channel- and jamming-aware decision fusion, *IEEE Transactions on Signal Processing*, vol. 65, no. 15, pp. 3866–3880, Aug. 2017.

[6] L. Ding, Q.-L. Han and X.-M. Zhang, Distributed secondary control for active power sharing and frequency regulation in islanded microgrids using an event-triggered communication mechanism, *IEEE Transactions on Industrial Informatics*, vol. 15, no. 7, pp. 3910–3922, Jul. 2019.

[7] L. Ding, L. Y. Wang, G. Yin, W. X. Zheng and Q.-L. Han, Distributed energy management for smart grids with an event-triggered communication scheme, *IEEE Transactions on Control Systems Technology*, Vol. 27, No. 5, pp. 1950–1961, 2019.

[8] H. Dong, Z. Wang, B. Shen and D. Ding, Variance-constrained H_∞ control for a class of nonlinear stochastic discrete time-varying systems: the event-triggered design, *IEEE Transactions on Industrial Informatics*, Vol. 72, pp. 28–36, 2016.

[9] Y. Fan, C. Zhang and C. Song, Sampling-based self-triggered coordination control for multi-agent systems with application to distributed generators, *International Journal of Systems Science*, Vol. 49, No. 15, pp. 3048–3062, 2018.

[10] X. Ge, Q.-L. Han and Z. Wang, A dynamic event-triggered transmission scheme for distributed set-membership estimation over wireless sensor networks, *IEEE Transactions on Cybernetics*, Vol. 49, No. 1, pp. 171–183, 2019.

[11] F. Han, G. Wei, D. Ding and Y. Song, Finite-horizon bounded H_∞ synchronisation and state estimation for discrete-time complex networks: Local performance analysis, *IET Control Theory* & *Applications*, vol. 11, no. 6, pp. 827–837, Apr. 2017.

[12] M. He and J. Li, Event-triggered adaptive tracking control for a class of uncertain stochastic nonlinear systems with Markov jumping parameters, *International Journal of Adaptive Control and Signal Processing*, Vol. 32, No. 12, pp. 1655–1674, 2018.

[13] S. Hu, D. Yue and J. Liu, H_∞ filtering for networked systems with partly known distribution transmission delays, *Information Sciences*, vol. 194, pp. 270–282, Jul. 2012.

[14] Y. Hu, J. Zhan and X. Li, Self-triggered distributed model predictive control for flocking of multi-agent systems, *IET Control Theory & Applications*, Vol. 12, No. 18, pp. 2441–2448, 2018.

[15] A. Jenabzadeh and B. Safarinejadian, A Lyapunov-based distributed consensus filter for a class of nonlinear stochastic systems, *Automatica*, Vol. 86, pp. 53–62, 2017.

[16] H. Ji, H. Zhang and B. Cui, Event-triggered H_∞ filtering control for a class of distributed parameter systems with Markovian switching topology, *Journal of the Franklin Institute-Engineering and Applied Mathematics*, Vol. 355, No. 13, pp. 5928–5956, 2018.

[17] B. Jiang and H. R. Karimi, Further criterion for stochastic stability analysis of semi-Markovian jump linear systems, *International Journal of Robust and Nonlinear Control*, Vol. 30, No. 7, pp. 2689–2700, 2020.

[18] X. Kan, H. Shu and Z. Li, Almost sure state estimation for nonlinear stochastic systems with Markovian switching, *Nonlinear Dynamics*, Vol. 76, No. 2, pp. 1591–1602, 2014.

[19] H. R. Karimi, Robust delay-dependent H_∞ control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters, *IEEE Transactions on Circuits and Systems I-Regular Papers*, vol. 58, no. 8, pp. 1910–1923, Aug. 2010.

[20] H. R. Karimi, and H. Gao, New delay-dependent exponential H_∞ synchronization for uncertain neural networks with mixed time delays, *IEEE Transactions on Systems, Man, and Cybernetics, Part B-Cybernetics*, vol. 40, no. 1, pp. 173–185, Feb. 2010.

[21] Y. Li, C. Hua and G. Liu, Adaptive state feedback control for switched stochastic high-order nonlinear systems under arbitrary switchings, *International Journal of Robust and Nonlinear Control*, Vol. 28, No. 6, pp. 2047–2063, 2018.

[22] L. Li, M. Niu and H. Yang, Event-triggered nonlinear filtering for networked systems with correlated noises, *Journal of the Franklin Institute-Engineering and Applied Mathematics*, Vol. 355, No. 13, pp. 5811–5829, 2018.

[23] B. Li, Z. Wang, L. Ma and H. Liu, Observer-based event-triggered control for nonlinear systems with mixed delays and disturbances: the input-to-state stability, *IEEE Transactions on Cybernetics*, in press, DOI: 10.1109/TCYB.2018.2837626.

[24] Z. Liu, J. Wang and C. L. Philip Chen, Event trigger fuzzy adaptive compensation control of uncertain stochastic nonlinear systems with actuator failures, *IEEE Transactions on Fuzzy Systems*, Vol. 26, No. 6, pp. 3770–3781, 2018.

[25] Q. Liu, Z. Wang and X. He, Event-triggered resilient filtering with measurement quantization and random sensor failures: monotonicity and convergence, *Automatica*, Vol. 94, pp. 458–464, 2018.

[26] Q. Liu, Z. Wang, X. He and D. Zhou, Event-based distributed filtering with stochastic measurement fading, *IEEE Transactions on Industrial Informatics*, Vol. 11, No. 6, pp. 1643–1652, 2015.

[27] S. Liu, G. Wei and Y. Song, Extended Kalman filtering for stochastic nonlinear systems with randomly occurring cyber attacks, *Neurocomputing*, Vol. 207, pp. 708–716, 2016.

[28] S. Liu, J. Zhang and Z. Jiang, A notion of stochastic input-to-state stability and its application to stability of cascaded stochastic nonlinear, *Acta Mathematicae Applicatae Sinica, English Series*, Vol. 24, No. 1, pp. 141–156, 2008.

[29] Y. Long and G. Yang, Fault detection for a class of nonlinear stochastic systems with Markovian switching and mixed time-delays, *International Journal of Systems Science*, Vol. 45, No. 3, pp. 215–231, 2014.

[30] J. Mao, D. Ding and Y. Song, Event-based recursive filtering for time-delayed stochastic nonlinear systems with missing measurements, *Signal Processing*, Vol. 134, pp. 158–165, 2017.

[31] X. Mi, Y. Zou, S. Li and H. R. Karimi, Self-triggered DMPC design for cooperative multiagent systems, *IEEE Transactions on Industrial Electronics*, Vol. 67, No. 1, pp. 512–520, 2020.

[32] H. Min, S. Xu and B. Zhang, Output-feedback control for stochastic nonlinear systems subject to input saturation and time-varying delay, *IEEE Transactions on Automatic Control*, Vol. 64, No. 1, pp. 359–364, 2019.

[33] Y. Pan and G. H. Yang, Event-triggered fault detection filter design for nonlinear networked systems, *IEEE Transactions on Systems, Man, and Cybernetics-Systems*, Vol. 48, No. 11, pp. 1851–1862, 2018.

[34] B. Qiao, X. Su and R. Jia, Event-triggered fault detection filtering for discrete-time Markovian jump systems, *Signal Processing*, Vol. 152, pp. 384–391, 2018.

[35] M. Velasco, P. Marti and J. Fuertes, The self triggered task model for real-time control systems, *Work-in-Progress Session of the 24th IEEE Real-Time Systems Symposium (RTSS' 03)*, Vol. 384, 2003.

[36] J. Wang, Z. Liu and C. L. Philip Chen, Fuzzy adaptive compensation control of uncertain stochastic nonlinear systems with actuator failures and input hysteresis, *IEEE Transactions on Cybernetics*, Vol. 49, No. 1, pp. 2–13, 2019.

[37] M. Wang, Z. Wang, Y. Chen and W. Sheng, Event-based adaptive neural tracking control for discrete-time stochastic nonlinear systems: a triggering-threshold compensation strategy, *IEEE Transactions on Neural Networks and Learning Systems*, 2019, doi:10.1109/TNNLS.2019. 2927595.

[38] M. Wang, Z. Wang, Y. Chen and W. Sheng, Observer-based fuzzy output-feedback control for discrete-time strict-feedback nonlinear systems with stochastic noises, *IEEE Transactions on Cybernetics*, 2019, doi:10.1109/TCYB. 2019.2902520.

[39] X. Wang and H. Su, Self-triggered leader-following consensus of multi-agent systems with input time delay, *Neurocomputing*, Vol. 330, pp. 70–77, 2019.

[40] Y. Wang, Z. Jia and Z. Zou, Dynamic event-triggered and self-triggered output feedback control of networked switched linear systems, *Neurocomputing*, Vol. 314, pp. 39–47, 2018.

[41] Y. Wang, L. Zheng and H. Zhang, Event-triggered fault detection filter design for nonlinear networked systems via fuzzy Lyapunov functions, *Journal of the Franklin Institute-Engineering and Applied Mathematics*, Vol. 355, No. 17, pp. 8392–8411, 2018.

[42] M. Xia and T. Zhang, Adaptive neural dynamic surface control for full state constrained stochastic nonlinear systems with unmodeled dynamics, *Journal of the Franklin Institute-Engineering and Applied Mathematics*, Vol. 356, No. 1, pp. 129–146, 2019.

[43] W. Xu, D. W. C. Ho, J. Zhong and B. Chen, Event/self-triggered control for leader-following consensus over unreliable network with DoS attacks, *IEEE Transactions on Neural Networks and Learning Systems*, Vol. 30, No. 10, pp. 3137–3149, 2019.

[44] Y. Xu, R. Lu, P. Shi, H. Li and S. Xie, Finite-time distributed state estimation over sensor networks with round-robin protocol and fading channels, *IEEE Transactions on Cybernetics*, vol. 48, no. 1, pp. 336–345, Jan. 2018.

[45] C. Yuan and X. Mao, Robust stability and controllability of stochastic differential delay equations with Markovian switching, *Automatica*, Vol. 40, No. 3, pp. 343–354, 2004.

[46] W. Zhang, Z. Pan and Y. Li, H_∞ filtering for general delayed nonlinear stochastic systems with Markov jumps, *International Journal of Fuzzy Systems*, Vol. 19, No. 6, pp. 1989–2002, 2017.

[47] X.-M. Zhang, Q.-L. Han, X. Ge, D. Ding, L. Ding, D. Yue and C. Peng, Networked control systems: A survey of trends and techniques, *IEEE/CAA Journal of Automatica Sinica*, in press, DOI:10.1109/JAS.2019.1911651.

[48] L. Zou, R. Pan and X. Xiao, Event-triggered H_∞ filtering for discrete-time systems over unreliable networks with package dropouts, *Neurocomputing*, Vol. 218, pp. 346–353, 2016.