

Universal attraction force-inspired freeform surface modeling for 3D sketching

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Abstract: - This paper presents a novel freeform surface modeling method to construct a freeform surface from 3D sketch. The approach is inspired by Newton's universal attraction force law to construct a surface model from rough boundary curves and unorganized interior characteristic curves which may cross the boundary curves or not. Based on these unorganized curves, an initial surface can be obtained for conceptual design and it can be improved later in a commercial package. The approach has been tested with examples and it is capable of dealing with unorganized design curves for surface modeling.

Key-Words: - Conceptual design, Unorganized curves, sketch, surface modeling

1 Introduction

Surface design plays a very important role in product development. Currently, there are a variety of commercial CAD systems to support interactive surface design through creating a well-organized curve network or virtual sculpting via 3D haptic devices [1]. However, at the early stage of form design, styling or industrial designers prefer a more intuitive design way by which they can obtain more sensory feedback during the design process, e.g., making 3D physical mockups or clay models [2] and 3D virtual sketching [3,4]. From the 3D physical models, reverse engineering techniques [5] and systems such as rapidFromTM, can be used to construct surface from unorganized 3D measured points; while for 3D sketching, there are some challenges in constructing surface from unorganized curves. In [6], we have demonstrated a motion-based 3D sketch system to support large-sized freeform surface design. With an optical motion capture system [7], artists or designers wearing reflective motion marks on their hands can sketch out 3D design splines when moving their body and hand in space.

By nature of freehand sketching, 3D sketched strokes may be not well-connected and organized. For example, initial sketches may consist of rough boundary and interior curves. The rough boundary curves may be closed (intersected with together) or open. The interior curves that are assumed on the design surface may have various positions and orientations; they may not form a regular or an irregular curve network with the boundary curves.

This brings a surface modeling challenge in dealing with unorganized curves because traditional surface construction methods are based on regular and well-organized curves. On the other hand, the number of all sampled points on sketch curves is not big enough and the distribution of the points may be not good enough for using reverse engineering software to construct a surface.

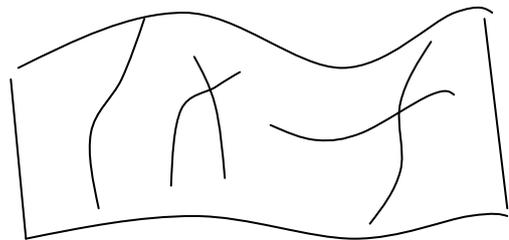


Fig. 1. Surface from unorganized sketch curves

In this paper, we present a new surface modeling technique for constructing surface from rough boundary curves and unorganized interior curves with arbitrary positions and orientations (Fig 1). Inspired by Newton's universal attraction force law, we create interpolation points from the unorganized curves (points) by blending their normalized attractions.

The rest of the paper is organized as follows. After the review of related work in Section 2, a new surface interpolation scheme based on unorganized curves is

presented in Section 3. This scheme is used for creating an initial conceptual surface (interpolated point-based curves) directly from rough boundary curves and unorganized interior curves. Application examples are shown in Section 4 and final conclusions are drawn in Section 5.

2 Related Work

It is well known that traditional surface construction methods are based on regular and well-organized curves [8][9]. They have difficulties in sketch-based surface modeling application with unorganized curves [10]. Related works can be found in two categories: surface modeling from an irregular mesh (requiring boundary and interior curves connected) and surface deformation (interior curves may not connected to the boundary). For an irregular mesh, a Gregory surface patch [11,12] is often used to generate surfaces. The irregular curves are assumed to intersect with each other. The basic Gregory technique [11] allows the design surface to be filled by free-form surface patches, which join together to make an over-all surface that is tangent plane continuous. Kuriyama [13] developed a curve mesh-based method for surface modeling with an irregular network of curves via sweeping and blending. The surfaces generated from the network of intersected curves are represented by multisided patches defined on a multivariate coordinate system. The over-all surface is a resulting interpolation from all intersected curves. These methods only support organized curves, and are not easy to be adapted for unorganized curves.

For surface deformation, the goal is to deform a surface according to given curves with known pre-images in the domain of the surface, such that attached constraints like “incidence of a curve on the surface” are satisfied and the new surface exhibits a change in consistence with the given curves [10,14]. This approach employs the “curve on the surface” concept and uses Least-squares techniques to isolate the control vertices relevant to a curve placed on or near a surface so that motion of the curve displaces the control points, which in turn changes the surface. For surface patches with a low density of control points, changing a surface by deforming control points can suffer from aliasing artefacts [14]. In addition, the determination of the pre-image of an interior curve is not straightway. Maekawa and Ko [10] subdivided the input curve into sub-segments so that the pre-image of each segment is a straight line in the parameter space.

In general, the surface deformation method has difficulty in efficiency and numerical stability. The algorithm is quite complex and difficult in supporting interactive design and local modifications.

3 New surface interpolation scheme from unorganized curves

In form design application, characteristic curves from 3D sketches are not always intersected to form a regular or irregular curve network. Even for boundary curves, they may not form a closed curve network too by a sketched input. The existing researches still have difficulty to cope with them. In order to construct a rough conceptual design surface from characteristic curves, a new surface representation scheme has been developed.

Our surface creation strategy is to create more isoparametric curves by directly interpolating both unorganised boundary and interior curves. Resulting isoparametric curves will form a regular curve network and then used for generating a surface. Alternatively, the points on the resultant curves can be fed into reverse engineering software for approximating a surface.

Our new surface interpolation method is inspired by Newton's law of universal gravitation. It states that all objects attract each other with a force of gravitational attraction. This force of gravitational attraction is directly dependent upon the masses of both objects and inversely proportional to the square ($1/d^2$) of the distance d which separates their centers. If we regard each sketch point as an object with unity mass, a new interpolation point i (target object with unity mass) will be pulled towards the sketch point; the movement should be proportional to their universal attraction, i.e. ($1/d^2$). If we have n sketch points, the final position P_i of the interpolation point will depend on the sum of the movements corresponding to each sketch point position V_j and the distance d_{ij} between them. That is

$$P_i = (1/d_{i1})^2 \cdot V_1 + (1/d_{i2})^2 \cdot V_2 + \dots + (1/d_{in})^2 \cdot V_n = \sum_{j=1}^n (1/d_{ij}^2) \cdot V_j \quad (1)$$

The formula (1) can be rationalised by the sum of each inverse square of the distance d_{ij} as follows.

$$W_i = \sum_{j=1}^n (1/d_{ij}^2) \quad (2)$$

$$S_i = \frac{1}{W_i} \sum_{j=1}^n (1/d_{ij})^2 V_j = \sum_{j=1}^n \frac{(1/d_{ij})^2}{W_i} V_j = \sum_{j=1}^n R_{ij} V_j \quad (3)$$

$$R_{ij} = \frac{(1/d_{ij})^2}{W_i} \quad (4)$$

Where W_i is the rational denominator, S_i is the interpolated position from n sketch points; each has its interpolation blending function R_{ij} . The interpolation functions have properties similar to rational Bézier blending functions [11]. They are as follows:

- Nonnegativity: all R_{ij} are larger than zero;
- Partition of unity: $\sum R_{ij} = 1$;
- When $d_{ij}=0$, $R_{ij}=1$;
- One maximum: each interpolation function attains exactly one maximum on the interval $[0,1]$.

Actually, the design surface can be regarded as a soft skin (sheet) touching to the sketched rigid frames. The final shape of surface should depend on not only the frames but also the softness of the skin material. Therefore, we can employ a parameter τ to replace the fixed attraction factor 2. It can be used for creating different surfaces through interpolating the same frames (sketch curves), reflecting the softness of the skin surface. Thus, the above equations (2), (3) and (4) can be rewritten as

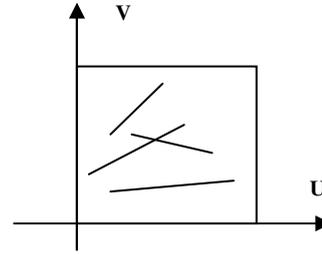
$$W_i = \sum_{j=1}^n (1/d_{ij})^\tau \quad (5)$$

$$S_i = \frac{1}{W_i} \sum_{j=1}^n (1/d_{ij})^\tau V_j = \sum_{j=1}^n \frac{(1/d_{ij})^\tau}{W_i} V_j = \sum_{j=1}^n R_{ij} V_j \quad (6)$$

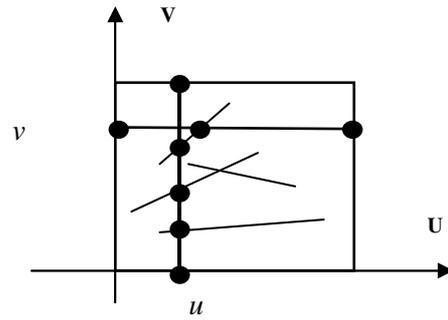
$$R_{ij} = \frac{(1/d_{ij})^\tau}{W_i} \quad (7)$$

The equations (5), (6) and (7) can give a global interpolation scheme; each new interpolated point can be associated with all sketch points. They can also be used for local interpolation, from only related sketch points. Let us say, we knew a network of arbitrary curves $C_k(u,v)$, $k=1, \dots, G$. Their parameter space can be mapped into a unity square. The curves are not

restricted as isoparametric curves. Their preimages can be any 2D line segments (Fig. 2 (a)).



(a) Preimages of characteristic curves



(b) Finding related points for interpolation

Fig. 2. General surface interpolation over unorganized curves

In order to obtain a surface point $P(u, v)$, we first find related points and then interpolate them based on Equations (5), (6) and (7). We start by drawing two straight lines at $U=u$, and $V=v$ and then check intersection points of the u and v lines with the preimages of the curves (Fig. 2b). Each intersected point corresponds to a 3D point on a curve. All these 3D points are then used for the interpolation. For example, in Fig. 2 (b), there are 5 intersection points (black dots) on the u line and 3 on the v line. These 8 sketch points will be interpolated to create a surface point at (u, v) . In general, we may have m intersection points on the u line, and n on the v line. As a result, two sets of parameter data $\{u_1, \dots, u_{m-1}, u_m\}$ and $\{v_1, \dots, v_{n-1}, v_n\}$ can be obtained, as well as two sets of corresponding parametric distances between the parameter point (u, v) and intersection points in the u direction $U_{di}=|u-u_{di}|$, $i=1, \dots, m$ and the v direction $V_{dj}=|v-v_{dj}|$, $j=1, \dots, n$. Finally, we can use these 2D parameter distances in Equations (5),(6) and (7) for interpolation because we don't know the 3D distances to begin with.

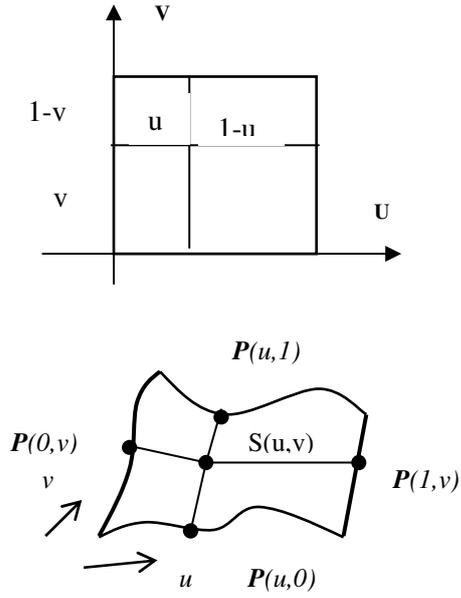


Fig. 3. Surface interpolation from four boundary curves: (a) top; (b) bottom

In order to study the blending function in Equation (7), we have explored surface interpolation from four boundary curves (Fig 3). The four boundary curves have a preimage as shown in Fig 3a. A surface point $S(u,v)$ will relate to four boundary points $P(0,v)$, $P(1,v)$, $P(u,0)$ and $P(u,1)$. The distances between the surface point to the four boundary points are u , $(1-u)$, v , and $(1-v)$ correspondingly. From the Equation (5), we can compute the rational denominator as

$$W(u,v) = \frac{1}{u^\tau} + \frac{1}{(1-u)^\tau} + \frac{1}{v^\tau} + \frac{1}{(1-v)^\tau} = \frac{(1-u)^\tau(1-v)^\tau v^\tau + u^\tau(1-v)^\tau v^\tau + (1-v)^\tau(1-u)^\tau u^\tau + v^\tau(1-u)^\tau u^\tau}{u^\tau v^\tau (1-u)^\tau (1-v)^\tau}$$

From Equation (6), we can obtain the equation (8) as follows:

$$S(u,v) = \frac{\left(\frac{1}{u^\tau}\right)}{w(u,v)} * P(0,v) + \frac{\frac{1}{(1-u)^\tau}}{w(u,v)} * P(1,v) + \frac{\frac{1}{v^\tau}}{w(u,v)} * P(u,0) + \frac{\frac{1}{(1-v)^\tau}}{w(u,v)} * P(u,1)$$

$$S(u,v) = r_{0v} * P(0,v) + r_{1v} * P(1,v) + r_{u0} * P(u,0) + r_{u1} * P(u,1)$$

$$r_{0v} = \frac{(1-u)^\tau(1-v)^\tau v^\tau}{k(u,v)}$$

$$r_{1v} = \frac{u^\tau(1-v)^\tau v^\tau}{k(u,v)}$$

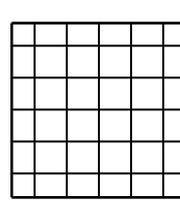
$$r_{u0} = \frac{(1-v)^\tau(1-u)^\tau u^\tau}{k(u,v)}$$

$$r_{u1} = \frac{v^\tau(1-u)^\tau u^\tau}{k(u,v)}$$

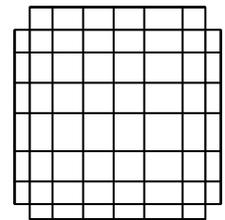
$$k(u,v) = (1-u)^\tau(1-v)^\tau v^\tau + u^\tau(1-v)^\tau v^\tau + (1-v)^\tau(1-u)^\tau u^\tau + v^\tau(1-u)^\tau u^\tau$$

$$0 < u, v < 1$$

Where, the r_{0v} , r_{1v} , r_{u0} , and r_{u1} are rational blending functions for corresponding points $P(0,v)$, $P(1,v)$, $P(u,0)$ and $P(u,1)$. Note that when $u=0$, the r_{0v} equals 1. This means the surface passes through the boundary curve $P(0,v)$. When $u=1$, the r_{1v} values 1. The surface crosses the curve $P(1,v)$. Similarly, while $v=0$ or 1, the surface patch will pass through the curves $P(u,0)$ or $P(u,1)$. Theoretically speaking, the surface $S(u,v)$ plus boundary curves is a full surface domain. If the four curves meet together to form four corners (a closed boundary), the resulting surface will have a topological structure in Fig. 4a. Otherwise, it may look like an open structure (Fig.4b).



(a) Close Corners

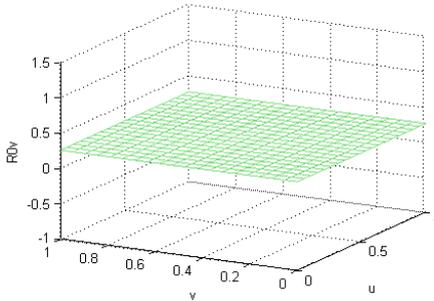


(b) Open corners

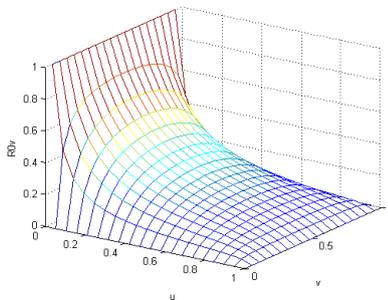
Fig. 4. Surface representation scheme

From the Equation (8), we chose the blending function r_{0v} to demonstrate how the point $P(0,v)$ will affect the interpolation over the preimage (Fig. 5). Fig 5a shows that when $\tau=0$, the blending function has a constant value of 0.25, which means that it will makes 25% contributions to all interpolated points. when $\tau=1$ or 2, the blending function has a value between 0 to 1. The distribution is symmetric to the middle of the curve, which means that middle part of a curve will

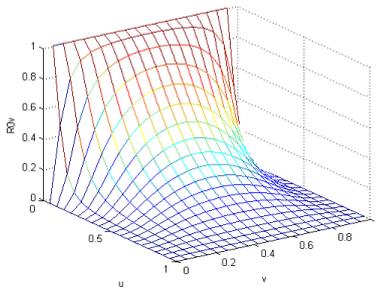
make more contribution to the surface than the two end parts.



(a) $\tau=0$



(b) $\tau=1$



(c) $\tau=2.0$

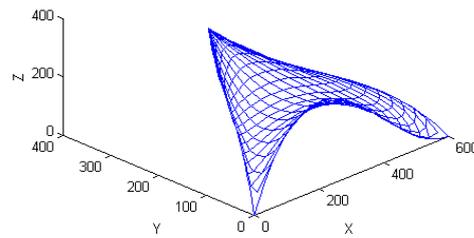
Fig. 5. Blending function of r_{0v}

The blending functions change their shapes on the power of τ . When $\tau=1$, the shape changes sharply near to the boundary (Fig. 5(b)). This is not a good property because the surface will change its shape dramatically around the boundary. However, when $\tau=2$, the shape changes quite smoothly (Fig. 5(c)). Thus we let τ equal 2 in our modeling scheme.

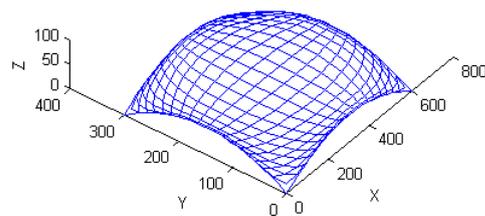
4 Application Examples

After receiving a set of 3D sketches, we first project these curves onto a plane such as (X-Y) according to dimensions of curves and then create a pre-image based on the projections. All boundary curves on the projection plane will be treated to form a unity square image and all interior curves will then be parameterized accordingly.

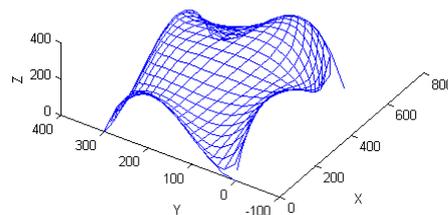
This surface interpolation scheme can support surface modeling from unorganized curves. Figures 6a and 6b illustrate a surface from 3 and 4 boundary curves respectively. Figure 6c gives an example of modeling a surface from 4 open boundary curves (the two lower corners are open). In Figures 6d and 6e, the surface is directly modeled from 4 boundary curves and 2 interior curves. In order to check the surface quality, it has been exchanged into the Alias Studio software. Figure 6d shows its wireframe model and sectional curves. Figure 6e gives the corresponding shaded model.



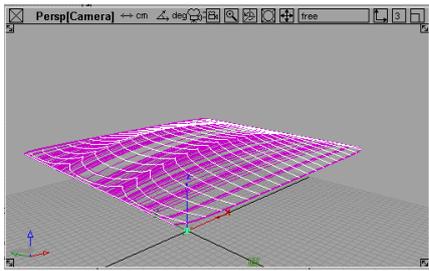
(a) 3-sided surface



(b) 4-sided surface



(c) Surface from an open boundary



(d)Surface from 4 boundary curves and 2 interior curves

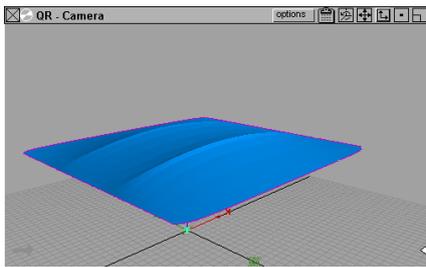


Figure 6. Examples of Surface Modelling

From the examples above, it can be seen that the surface interpolation scheme can interpolate a surface from a set of unorganized curves. It can generate a surface not only from closed boundary curves but also from open boundary curves. Even when initial boundary curves are open, resultant surfaces still can be obtained, and give a good approximation of the initial boundary curves. However, surfaces are not very smooth where the interior curves occur (see cusps and creases in Fig 6 (d) and (e)). Therefore, this surface interpolation scheme is good for some applications such as architectural design where surfaces have no requirements of cross-boundary continuity. But it may be not good enough for directly generating engineering surface. Nevertheless, if resultant points are fed into reverse engineering software, the initial surface may be refined (refitted) with high quality surface.

5 Conclusion

In this paper, we present a novel surface modeling approach for supporting sketch applications in conceptual design. Based on rough boundary curves and unorganized interior curves, an initial surface can be directly interpolated and used as a conceptual surface or an interim surface to be further refined in a commercial software. The proposed surface modeling approach and algorithm has been tested with

examples. It is capable of dealing with unorganized design curves for surface modeling.

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