Distributed State and Fault Estimation Over Sensor Networks With Probabilistic Quantizations: The Dynamic Event-Triggered Case ⋆

Qi Li [a], Zidong Wang [b], Jun Hu [c], Weiguo Sheng [a,*]

[a] *School of Information Science and Engineering, Hangzhou Normal University, Hangzhou 311121, China.*
[b] *Department of Computer Science, Brunel University London, Uxbridge, Middlesex, UB8 3PH, United Kingdom.*
[c] *Department of Mathematics, Harbin University of Science and Technology, Harbin 150080, China.*

Abstract

In this paper, the distributed state and fault estimation problem is discussed for a class of nonlinear time-varying systems with probabilistic quantizations and dynamic event-triggered mechanisms. To reduce resource consumption, a dynamic event-triggered strategy is exploited to schedule the data communication among sensor nodes. In addition, the measurement signals are quantized and then transmitted through the network, where the probabilistic quantizations are taken into consideration. Attention is focused on the problem of constructing a distributed estimator such that both the plant state and the fault signal are estimated simultaneously. By using the matrix difference equation method, certain upper bound on the estimation error covariance is first guaranteed and then minimized at each iteration by properly designing the estimator parameters. Subsequently, for the proposed distributed estimation algorithm, the estimator performance is analyzed and a sufficient condition is established to guarantee that the estimation error is exponentially bounded in mean-square sense. Finally, an illustrative example is provided to verify the usefulness of the developed estimation scheme.

Key words: Sensor networks; state and fault estimation; distributed estimation; dynamic event-triggered mechanisms; probabilistic quantizations.

1 Introduction

Sensor networks (SNs) have found extensive applications in various practical areas including military (e.g. battle damage assessment), environment (e.g. forest fire detection), health (e.g. drug administration) and home (e.g. home automation), see e.g. [10, 12, 29, 35, 37, 38]. Typically, a SN contains plenty of smart sensor nodes that are distributed in a pre-determined region to achieve certain goals such as target tracking. These nodes are equipped with transceivers and therefore have the capabilities to share the information with each other via wireless communication channels [8]. As one of core issues with SNs, the distributed state estimation problem aims to obtain the local estimate by using the measured outputs from the sensor itself and its neighbors, and such a problem has drawn particular research attention in the past decade. Accordingly, a large number of distributed estimation/filtering algorithms have been developed with respect to various performance indices, see e.g. [6, 42, 43] for the distributed H_∞ state estimation methods and [3, 5, 34, 36] for the distributed Kalman filtering approaches.

As is well known, even a small fault in the system could result in deterioration or even divergence of the system performance [1]. As such, the fault detection (FD) problem, whose purpose is to detect whether an undesired fault occurs or not in a timely way, has recently become a critical issue and attracted an increasing research interest, see e.g. [30,45]. It should be noticed that it is generally difficult to obtain sufficiently accurate characteristics (e.g. the shape and the size) of the faults by using the FD technique only. Therefore, it becomes a vitally important problem as how to acquire the detailed information of the fault itself in the area of fault tolerant control, which brings about the so-called *fault estimation* problem. Up till now, great effort has been made to develop the fault estimation algorithms for various types of systems including networked control systems [39], complex networks [11, 26], 2-D systems [25] and multi-agent systems [27]. For example, a new fault estimation method has been proposed in [11] for a class of time-varying stochastic complex networks, where the influences from stochastic inner couplings, randomly varying topologies as well as missing measurements have been well examined. Nevertheless, when it comes to the SNs, the corresponding distributed fault estimation is still an open problem, and this constitutes one of the motivations of this paper.

In a networked environment, it is a fairly ubiquitous phenomenon that the signal is quantized before being transmitted through bandwidth-constrained channels, where the quantization is implemented by a quantizer that converts a real-valued signal into a piecewise constant one taking values on a finite set [22, 44]. Obviously, this conversion process would inevitably introduce certain quantization errors that might degrade the system per-

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* Corresponding author.
Email address: weiguouk@hotmail.com (Weiguo Sheng).

formance or even result in the instability of the overall dynamical systems [41]. As such, in the past few years, there has been an ever-increasing research interest on the control/estimation problems subject to signal quantization effects, see e.g. [15,21,23,47]. Roughly speaking, the existing quantization models can be classified into two types, namely, the deterministic quantization model and the probabilistic quantization (PQ) model. With regard to the PQs, the component of the quantized signal would be a random variable whose expectation is equal to the real value of the signal, where the unbiasedness of the quantization is ensured. So far, a few preliminary research results have been reported on the investigation of PQs [4, 21, 46].

For the SNs, a noticeable fact is that a typical sensor can only be equipped with an energy-limited battery and the energy consumption is primarily caused by the data communications. For energy-saving purposes, it is naturally desirable to have appropriate communication strategies with aim to avoid unnecessary data exchanges between sensor nodes. In this regard, many effective communication scheduling strategies have been developed to determine whether certain data transmission is necessary or not [48]. Some representative communication strategies include the Round-Robin protocol, static event-triggered mechanisms (SETMs) and dynamic event-triggered mechanisms (DETMs), see e.g. [9, 17, 18, 20, 33, 40, 49, 50]. Among these strategies, the DETMs stand out for the additionally introduced dynamic variable in the triggering rules, and have aroused a particular research interest for their capability of providing an adequate trade-off between the communication burden and the system performance [14]. Very recently, the distributed set-membership estimation scheme has been proposed in [13] for a class of time-varying dynamical systems over SNs under DETMs. Nevertheless, in case that the PQs are concerned, the corresponding distributed state and fault estimation problem with DETMs has not been studied yet, and another motivation of the current research is therefore to shorten such a gap.

Inspired by the above discussions, the aim of this paper is to tackle the recursive distributed state and fault estimation problem over SNs with PQs and DETMs. In doing so, we are confronted with the following difficulties/challenges: 1) how to design a suitable recursive distributed estimator such that an acceptable upper bound is firstly ensured on the estimation error covariance (EEC) and subsequently minimized; 2) how to examine the impacts from DETMs and PQs on the estimation performance; and 3) how to establish a sufficient condition under which the estimation error is exponentially bounded in mean square sense. In this sense, the main purpose of this paper is to overcome the above-mentioned difficulties/challenges through developing dedicated distributed estimators. *Accordingly, the main contributions of this paper are summarized as follows: 1) a new joint state and fault estimator design problem is investigated for SNs with PQs and DETMs; 2) an effective algorithm is proposed to obtain the desired estimator parameters by solving a recursive matrix equation; and 3) a sufficient condition is provided to ensure the exponentially mean-square boundedness of the estimation error.*

Notation $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. The superscript "T" means the matrix transposition and $\|\cdot\|$ stands for the Euclidean norm. Let I be an identity matrix and diag$\{\cdots\}$ be a block-diagonal matrix. Pr$\{\cdot\}$ denotes the probabilities of "$\cdot$". $\mathbb{E}\{x\}$ is the expectation of the random variable x. tr$\{A\}$ represents the trace of A. For symmetric matrices X and Y, $X \geq Y$ ($X > Y$) means that $X - Y$ is positive semi-definite (positive definite).

2 Problem Formulation

Consider the following nonlinear time-varying target plant:

$$x_{k+1} = g_k(x_k) + A_k f_k + B_k w_k \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the plant state; A_k and B_k are known time-varying matrices; the process noise $w_k \in \mathbb{R}^{n_w}$ is a Gaussian white-noise sequence with covariance $R_k > 0$. The mean and covariance of the initial state x_0 are, respectively, $\bar{x}_0$ and $P_0 > 0$. $f_k \in \mathbb{R}^{n_f}$ is the fault signal whose dynamic characteristics are described by

$$f_{k+1} = F_k f_k. \tag{2}$$

Here, F_k is a known matrix and the initial value of the fault f_0 has the mean $\bar{f}_0$ and covariance $Z_0 > 0$.

Assumption 1 *[28] $g_k(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ is a continuous nonlinear function that satisfies $g_k(0) = 0$ and the following constraint:*

$$\|g_k(u) - g_k(v) - E_k(u - v)\| \leq \mu_k \|u - v\| \tag{3}$$

for all $u, v \in \mathbb{R}^{n_x}$, where E_k is a known matrix and μ_k is a nonnegative scalar.

Remark 1 *In this paper, the distributed state and fault estimation problem is addressed for the time-varying system (1), where the additive fault is taken into account. In order to reflect the engineering practice, we adopt the model (2) that is capable of depicting the dynamical change of the fault and such a model has been widely used in the literature [11]. Note that one of the main differences between the fault estimation problem in this paper and disturbance estimation problem considered in many papers is that the fault has its own dynamics and the disturbance is generally unknown and bounded. Moreover, it is easy to see that the fault would reduce to a constant one when $F_k = I$. Consequently, the fault in (2) is more general that covers the frequently investigated constant fault as a special case. Such kind of fault is quite common in real-world systems such as three-tank systems [2] and wind turbine condition monitoring systems [31].*

In this paper, there are n sensor nodes in the network and the network topology structure is described by $\mathcal{G} = (\mathcal{V}, \mathcal{Q}, \mathcal{W})$ with the node set $\mathcal{V} = \{1, 2, \ldots, n\}$, the edge set $\mathcal{Q} \subseteq \mathcal{V} \times \mathcal{V}$, and the weighted adjacency matrix $\mathcal{W} = [\omega_{ij}]_{n\times n}$. The edge $(i, j) \in \mathcal{Q}$ means that node i has access to the information from node j. If $(i, j) \in \mathcal{Q}$, we assume that the corresponding element ω_{ij} in $\mathcal{W}$ satisfies $\omega_{ij} = 1$, otherwise $\omega_{ij} = 0$. For node i, denote by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{Q}\}$ the set of its neighbors plus the node itself.

For the ith ($i = 1, 2, \ldots, n$) node, the measurement output is expressed as follows:

$$y_{i,k} = C_{i,k} x_k + D_{i,k} v_k, \tag{4}$$

where $y_{i,k} \in \mathbb{R}^{n_y}$ is the measurement output of the ith node, $C_{i,k}$ and $D_{i,k}$ are known time-varying matrices, the measurement noise $v_k \in \mathbb{R}^{n_v}$ is a Gaussian white-noise sequence with covariance $Q_k > 0$.

Assumption 2 *All the mentioned random variables x_0, f_0, w_k and v_k are mutually independent.*

In order to save the communication cost, for the sensor node i, a DETM is exploited to judge when the information is sent to its neighboring nodes. Let $0 \le t_0^i < t_1^i < \cdots < t_l^i < \cdots$ represent the triggering instants determined by:

$$\begin{cases} t_{l+1}^i = \min\Big\{k|k > t_l^i, \frac{1}{\delta_i}\hbar_{i,k} + \sigma_i - \|\vartheta_{i,k}\| \le 0\Big\}, \\ \hbar_{i,k+1} = \lambda_i \hbar_{i,k} + \sigma_i - \|\vartheta_{i,k}\|, \ \hbar_{i,0} = \hbar_0^i. \end{cases} \tag{5}$$

Here, λ_i, σ_i and δ_i are given positive scalars, $\vartheta_{i,k}$ is defined by $\vartheta_{i,k} \triangleq y_{i,k} - y_{i,t_l^i}$ with the latest broadcast measurement y_{i,t_l^i}, and $\hbar_0^i \ge 0$ is the given initial condition. Assume that the parameters λ_i and δ_i satisfy $\lambda_i\delta_i \ge 1$, which implies that the variable $\hbar_{i,k}$ satisfies $\hbar_{i,k} \ge 0$ for all time instants. For notation simplicity, we set $\bar{y}_{i,k} \triangleq y_{i,t_l^i}$ when $k \in [t_l^i, t_{l+1}^i)$.

In the sequel, we consider the case that the triggered measurement $\bar{y}_{i,k}$ is quantized before being transmitted. Define the quantized measurement as follows:

$$q(\bar{y}_{i,k}) \triangleq [q_1(\bar{y}_{i,k}^1) \ q_2(\bar{y}_{i,k}^2) \ \ldots \ q_{n_y}(\bar{y}_{i,k}^{n_y})]^T, \tag{6}$$

where $\bar{y}_{i,k}^j$ ($j = 1, 2, \ldots, n_y$) is the jth element of $\bar{y}_{i,k}$, $q_j(\cdot) : \mathbb{R} \to \mathcal{U}_j$ is the probabilistic quantizer that maps a real value to the following quantization level set $\mathcal{U}_j$:

$$\mathcal{U}_j \triangleq \Big\{\varrho_{i,\iota}^j | \varrho_{i,\iota}^j \triangleq \iota\tau_i^j, \ \iota = 0, \pm1, \pm2, \ldots\Big\}, \ \tau_i^j > 0. \tag{7}$$

When $\bar{y}_{i,k}^j \in [\varrho_{i,\iota}^j, \varrho_{i,\iota+1}^j]$, the signal $\bar{y}_{i,k}^j$ is quantized in the following probabilistic manner:

$$\Pr\{q_j(\bar{y}_{i,k}^j) = \varrho_{i,\iota}^j | \alpha_i^j\} = 1 - \alpha_i^j,$$
$$\Pr\{q_j(\bar{y}_{i,k}^j) = \varrho_{i,\iota+1}^j | \alpha_i^j\} = \alpha_i^j$$

with $\alpha_i^j \triangleq \frac{\bar{y}_{i,k}^j - \varrho_{i,\iota}^j}{\tau_i^j} \in [0, 1]$.

Defining $\varphi_{i,k}^j \triangleq q_j(\bar{y}_{i,k}^j) - \bar{y}_{i,k}^j$ as the quantization error, from [21], we have

$$\mathbb{E}\{\varphi_{i,k}^j\} = 0, \quad \mathbb{E}\{(\varphi_{i,k}^j)^2\} \le (\tau_i^j)^2/4,$$
$$\mathbb{E}\{\varphi_{i,k}^j \varphi_{i,k}^l\} = 0 \text{ (for } j \ne l). \tag{8}$$

By setting $\vec{x}_k \triangleq [x_k^T \ f_k^T]^T$ and combining (1)-(2) with (4), the following discrete time-varying system can be obtained:

$$\begin{aligned} \vec{x}_{k+1} =& \vec{g}_k(\vec{x}_k) + \vec{A}_k\vec{x}_k + \vec{B}_k w_k \\ y_{i,k} =& \vec{C}_{i,k}\vec{x}_k + D_{i,k}v_k, \end{aligned} \tag{9}$$

where

$$\vec{g}_k(\vec{x}_k) \triangleq \begin{bmatrix} g_k^T(x_k) & 0 \end{bmatrix}^T, \ \vec{C}_{i,k} \triangleq \begin{bmatrix} C_{i,k} & 0 \end{bmatrix},$$
$$\vec{B}_k \triangleq \begin{bmatrix} B_k \\ 0 \end{bmatrix}, \ \vec{A}_k \triangleq \begin{bmatrix} 0 & A_k \\ 0 & F_k \end{bmatrix}.$$

For the node i ($i = 1, 2, \ldots, n$), the following distributed estimator is constructed:

$$\hat{x}_{i,k+1} = K_{i,k}\hat{x}_{i,k} + \sum_{j\in\mathcal{N}_i} \omega_{ij} G_{ij,k} q(\bar{y}_{j,k}), \tag{10}$$

where $\hat{x}_{i,k}$ denotes the estimate of $\vec{x}_k$ with $\hat{x}_{i,0} = \begin{bmatrix}\bar{x}_0^T & \bar{f}_0^T\end{bmatrix}^T$, and $K_{i,k}$ and $G_{ij,k}$ are the estimator gains to be designed.

Let $e_{i,k} \triangleq \vec{x}_k - \hat{x}_{i,k}$ be the estimation error. Recalling the definitions of $q(\bar{y}_{i,k})$ and $\bar{y}_{i,k}$, one has

$$\begin{aligned} e_{i,k+1} =& \vec{g}_k(\vec{x}_k) + (\vec{A}_k - K_{i,k})\vec{x}_k + K_{i,k}e_{i,k} + \vec{B}_k w_k \\ &- G_{i,k}W_i y_k + G_{i,k}W_i\vartheta_k - G_{i,k}W_i\varphi_k, \end{aligned} \tag{11}$$

where

$$G_{i,k} \triangleq [G_{i1,k} \ \ldots \ G_{in,k}], \ W_i \triangleq \mathrm{diag}\{\omega_{i1}I, \ldots, \omega_{in}I\},$$
$$y_k \triangleq [y_{1,k}^T \ \ldots \ y_{n,k}^T]^T, \ \vartheta_k \triangleq [\vartheta_{1,k} \ \ldots \ \vartheta_{n,k}]^T,$$
$$\varphi_k \triangleq [\varphi_{1,k}^1 \ \ldots \ \varphi_{1,k}^{n_y} \ \varphi_{2,k}^1 \ \ldots \ \varphi_{n,k}^{n_y}]^T.$$

To end this section, let us state the main purpose of this paper. We are interested in designing a distributed estimator of the form (10) for each senor node such that, in the presence of DETMs and PQs, an upper bound on the EEC $P_{i,k} \triangleq \mathbb{E}\{e_{i,k}e_{i,k}^T\}$ is guaranteed and, moreover, such an upper bound is locally minimized at each time instant by properly designing the estimator gain matrices $K_{i,k}$ and $G_{ij,k}$.

3 Main Results

3.1 Estimation Algorithm Design

Lemma 1 *For any real-valued matrices H_1 and H_2, the following inequality*

$$\mathsf{H}_1\mathsf{H}_2^T + \mathsf{H}_2\mathsf{H}_1^T \le \mathsf{a}\mathsf{H}_1\mathsf{H}_1^T + \mathsf{a}^{-1}\mathsf{H}_2\mathsf{H}_2^T$$

holds for any scalar $\mathsf{a} > 0$.

Lemma 2 *Let the positive scalars a_k, $b_{i,k}$ and $c_{i,k}$ be given. Assume that there exist two sets of real-valued matrices $\bar{X}_k$ and $\bar{Y}_{i,k}$ satisfying*

$$\begin{aligned} \bar{X}_{k+1} \triangleq& (1 + a_k)\mu_k^2 tr\{I_1\bar{X}_k I_1^T\}I + (1 + a_k^{-1})\big(\vec{E}_k + \vec{A}_k\big) \\ &\times \bar{X}_k\big(\vec{E}_k + \vec{A}_k\big)^T + \vec{B}_k R_k \vec{B}_k^T \end{aligned} \tag{12}$$

and

$$\begin{aligned} \bar{Y}_{i,k+1} \triangleq& \Big((1 + b_{i,k})(1 + c_{i,k})\lambda_i^2 + (1 + \delta_i) \\ &\times (1 + b_{i,k}^{-1})/\delta_i^2\Big)\bar{Y}_{i,k} + \Big((1 + b_{i,k})(1 + c_{i,k}^{-1}) \\ &+ (1 + b_{i,k}^{-1})(1 + \delta_i^{-1})\Big)\sigma_i^2 \end{aligned} \tag{13}$$

with the initial conditions $\bar{X}_0 = \mathrm{diag}\{P_0 + \bar{x}_0\bar{x}_0^T, Z_0 + \bar{f}_0\bar{f}_0^T\}$ and $\bar{Y}_{i,0} = (\hbar_0^i)^2$, where $\vec{E}_k \triangleq \mathrm{diag}\{E_k, 0\}$ and $I_1 \triangleq \begin{bmatrix} I & 0 \end{bmatrix}$. Then, the covariances $X_k \triangleq \mathbb{E}\{\vec{x}_k\vec{x}_k^T\}$ and $Y_{i,k} \triangleq \mathbb{E}\{\hbar_{i,k}^2\}$ satisfy $X_k \le \bar{X}_k$ and $Y_{i,k} \le \bar{Y}_{i,k}$, respectively.

Proof: By applying Lemma 1, one obtains from (3) and (9) that

$$\begin{aligned} X_{k+1} =& \mathbb{E}\{\vec{x}_{k+1}\vec{x}_{k+1}^T\} \\ =& \mathbb{E}\Big\{\big[\vec{g}_k(\vec{x}_k) + \vec{A}_k\vec{x}_k + \vec{B}_k w_k\big] \end{aligned}$$

$$\times \left[\vec{g}_k(\vec{x}_k) + \vec{A}_k\vec{x}_k + \vec{B}_k w_k\right]^T \Big\}$$
$$\leq (1+a_k)\mu_k^2 \text{tr}\{I_1 X_k I_1^T\} I + (1+a_k^{-1})\big(\vec{E}_k + \vec{A}_k\big) \times X_k\big(\vec{E}_k + \vec{A}_k\big)^T + \vec{B}_k R_k \vec{B}_k^T. \quad (14)$$

Then, it is easy to derive $X_{k+1} \leq \bar{X}_{k+1}$ by means of an induction method.

On the other hand, one has from (5) that

$$\vartheta_{i,k}^T \vartheta_{i,k} \leq \left(\frac{1}{\delta_i}\hbar_{i,k} + \sigma_i\right)^2 \leq (1+\delta_i)\hbar_{i,k}^2/\delta_i^2 + (1+\delta_i{}^{-1})\sigma_i^2. \quad (15)$$

Using the similar techniques in [17], we obtain $Y_{i,k+1} \leq \bar{Y}_{i,k+1}$ readily and the proof is thus complete. □

Theorem 1 *Let the positive scalars $d_{i,k}$, $g_{i,k}$, $h_{i,k}$ and $m_{i,k}$ be given. Assume that there exists a set of real-valued matrices $\Sigma_{i,k}$ with initial constraint $\Sigma_{i,0} = \text{diag}\{P_0, Z_0\}$ satisfying the following recursive equation:*

$$\begin{aligned}
&\Sigma_{i,k+1} \\
&\triangleq (1+d_{i,k})(1+h_{i,k})\mu_k^2 tr\{I_1 \bar{X}_k I_1^T\} I + (1+d_{i,k}) \\
&\quad \times (1+h_{i,k}^{-1})\big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)\bar{X}_k \\
&\quad \times \big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)^T + (1+d_{i,k}^{-1}) \\
&\quad \times (1+m_{i,k})K_{i,k}\Sigma_{i,k}K_{i,k}^T + \vec{B}_k R_k \vec{B}_k^T + \Big[g_{i,k}^{-1} \\
&\quad + (1+d_{i,k}^{-1})(1+m_{i,k}^{-1})\Big]\sum_{s=1}^{n} G_{i,k}W_i\Delta_{s,k}W_i G_{i,k}^T \\
&\quad + G_{i,k}W_i\Big[\Upsilon + (1+g_{i,k})\vec{D}_k Q_k \vec{D}_k^T\Big]W_i G_{i,k}^T, \quad (16)
\end{aligned}$$

where

$$\vec{C}_k \triangleq \Big[\vec{C}_{1,k}^T\ \dots\ \vec{C}_{n,k}^T\Big]^T,\ \vec{D}_k \triangleq \Big[D_{1,k}^T\ \dots\ D_{n,k}^T\Big]^T,$$
$$\Upsilon \triangleq \text{diag}\Big\{(\tau_1^1)^2/4, \dots, (\tau_1^{n_y})^2/4, \dots, (\tau_n^{n_y})^2/4\Big\},$$
$$\Delta_{s,k} \triangleq \Big[(1+\delta_s)\bar{Y}_{s,k}/\delta_s^2 + (1+\delta_s{}^{-1})\sigma_s^2\Big]I.$$

Then, $\Sigma_{i,k+1}$ is an upper bound on the EEC $P_{i,k+1}$, i.e.,

$$P_{i,k+1} \leq \Sigma_{i,k+1}.$$

Proof: The EEC $P_{i,k+1}$ is calculated as

$$\begin{aligned}
&P_{i,k+1} \\
&= \mathbb{E}\Big\{\Big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k + \big(\vec{A}_k - K_{i,k} + \vec{E}_k \\
&\quad - G_{i,k}W_i\vec{C}_k\big)\vec{x}_k + K_{i,k}e_{i,k} - G_{i,k}W_i\vec{D}_k v_k \\
&\quad + G_{i,k}W_i\vartheta_k - G_{i,k}W_i\varphi_k\Big]\Big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k \\
&\quad + \big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)\vec{x}_k + K_{i,k}e_{i,k} \\
&\quad - G_{i,k}W_i\vec{D}_k v_k + G_{i,k}W_i\vartheta_k - G_{i,k}W_i\varphi_k\Big]^T\Big\} \\
&\quad + \vec{B}_k R_k \vec{B}_k^T. \quad (17)
\end{aligned}$$

Noticing the following facts

$$\mathbb{E}\{\varphi_k\} = 0,\ \mathbb{E}\{\varphi_k\varphi_k^T\} \leq \Upsilon,\ \mathbb{E}\{\varphi_k e_{i,k}^T\} = 0,$$
$$\mathbb{E}\{\varphi_k\vec{x}_k^T\} = 0,\ \mathbb{E}\{\varphi_k\vartheta_k^T\} = 0,\ \mathbb{E}\{\varphi_k v_k^T\} = 0,$$

we have

$$\begin{aligned}
&P_{i,k+1} \\
&\leq \mathbb{E}\Big\{\Big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k + \big(\vec{A}_k - K_{i,k} - G_{i,k}W_i\vec{C}_k \\
&\quad + \vec{E}_k\big)\vec{x}_k + K_{i,k}e_{i,k} + G_{i,k}W_i\vartheta_k\Big]\Big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k \\
&\quad + \big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)\vec{x}_k + K_{i,k}e_{i,k} \\
&\quad + G_{i,k}W_i\vartheta_k\Big]^T\Big\} + G_{i,k}W_i\Big(\Upsilon + \vec{D}_k Q_k \vec{D}_k^T\Big)W_i G_{i,k}^T \\
&\quad + \vec{B}_k R_k \vec{B}_k^T - G_{i,k}W_i\vec{D}_k\mathbb{E}\Big\{v_k\vartheta_k^T\Big\}W_i G_{i,k}^T \\
&\quad - G_{i,k}W_i\mathbb{E}\Big\{\vartheta_k v_k^T\Big\}\vec{D}_k^T W_i G_{i,k}^T. \quad (18)
\end{aligned}$$

With the help of Lemma 1, it follows further from (18) that

$$\begin{aligned}
&P_{i,k+1} \\
&\leq (1+d_{i,k})(1+h_{i,k})\mu_k^2 \text{tr}\{I_1 X_k I_1^T\} I + (1+d_{i,k}) \\
&\quad \times (1+h_{i,k}^{-1})\Big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\Big)X_k \\
&\quad \times \Big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\Big)^T + (1+d_{i,k}^{-1}) \\
&\quad \times (1+m_{i,k})K_{i,k}P_{i,k}K_{i,k}^T + \vec{B}_k R_k \vec{B}_k^T + \Big[g_{i,k}^{-1} \\
&\quad + (1+d_{i,k}^{-1})(1+m_{i,k}^{-1})\Big]G_{i,k}W_i\mathbb{E}\Big\{\vartheta_k\vartheta_k^T\Big\}W_i G_{i,k}^T \\
&\quad + G_{i,k}W_i\Big[\Upsilon + (1+g_{i,k})\vec{D}_k Q_k \vec{D}_k^T\Big]W_i G_{i,k}^T \quad (19)
\end{aligned}$$

where $d_{i,k}$, $g_{i,k}$, $h_{i,k}$ and $m_{i,k}$ are positive scalars.

In addition, it follows from (15) that

$$\vartheta_k\vartheta_k^T \leq \vartheta_k^T\vartheta_k I = \sum_{s=1}^{n}\vartheta_{s,k}^T\vartheta_{s,k} I \leq \sum_{s=1}^{n}\Big[(1+\delta_s)\hbar_{s,k}^2/\delta_s^2 + (1+\delta_s{}^{-1})\sigma_s^2\Big]I, \quad (20)$$

which leads to

$$\mathbb{E}\Big\{\vartheta_k\vartheta_k^T\Big\} \leq \sum_{s=1}^{n}\Big[(1+\delta_s)\bar{Y}_{s,k}/\delta_s^2 + (1+\delta_s{}^{-1})\sigma_s^2\Big]I, \quad (21)$$

where Lemma 2 has been utilized. Substituting (21) into (19) yields

$$\begin{aligned}
&P_{i,k+1} \\
&\leq (1+d_{i,k})(1+h_{i,k})\mu_k^2 \text{tr}\{I_1 \bar{X}_k I_1^T\} I + (1+d_{i,k}) \\
&\quad \times (1+h_{i,k}^{-1})\Big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\Big)\bar{X}_k \\
&\quad \times \Big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\Big)^T + (1+d_{i,k}^{-1}) \\
&\quad \times (1+m_{i,k})K_{i,k}P_{i,k}K_{i,k}^T + \vec{B}_k R_k \vec{B}_k^T + \Big[g_{i,k}^{-1} \\
&\quad + (1+d_{i,k}^{-1})(1+m_{i,k}^{-1})\Big]\sum_{s=1}^{n} G_{i,k}W_i\Delta_{s,k}W_i G_{i,k}^T \\
&\quad + G_{i,k}W_i\Big[\Upsilon + (1+g_{i,k})\vec{D}_k Q_k \vec{D}_k^T\Big]W_i G_{i,k}^T. \quad (22)
\end{aligned}$$

Finally, it follows from (16) that

$$P_{i,k+1} \leq \Sigma_{i,k+1}, \tag{23}$$

which ends the proof. □

In the following theorem, an effective algorithm is provided to parameterize the estimator gains which ensure that the upper bound derived in Theorem 1 is minimized.

Theorem 2 *For $1 \leq i \leq n$, the upper bound $\Sigma_{i,k}$ obtained in Theorem 1 achieves its minimum*

$$\begin{aligned}
&\Sigma_{i,k+1}\\
=&\big(G_{i,k}W_i - \Theta_{5i,k}\Theta_{4i,k}^{-1}\big)\Theta_{4i,k}\big(G_{i,k}W_i - \Theta_{5i,k}\Theta_{4i,k}^{-1}\big)^T\\
&- \Theta_{5i,k}\Theta_{4i,k}^{-1}\Theta_{5i,k}^T - \Theta_{1i,k}\Theta_{2i,k}^{-1}\Theta_{1i,k}^T + (1+d_{i,k})\\
&\times (1+h_{i,k}^{-1})\big(\vec{A}_k + \vec{E}_k\big)\bar{X}_k\big(\vec{A}_k + \vec{E}_k\big)^T + \vec{B}_k R_k \vec{B}_k^T\\
&+ (1+d_{i,k})(1+h_{i,k})\mu_k^2 tr\{I_1\bar{X}_k I_1^T\}I
\end{aligned} \tag{24}$$

with the estimator parameters given by

$$K_{i,k} = \Theta_{1i,k}\Theta_{2i,k}^{-1} - G_{i,k}W_i\Theta_{3i,k}\Theta_{2i,k}^{-1}, \tag{25}$$

and

$$G_{ij,k} = \begin{cases} \bar{G}_{ij,k}\omega_{ij}^{-1} & \text{if } \omega_{ij} \neq 0,\\ 0, & \text{if } \omega_{ij} = 0 \end{cases} \tag{26}$$

where

$$\begin{aligned}
\bar{G}_{i,k} &\triangleq \big[\bar{G}_{i1,k}\ \bar{G}_{i2,k}\ \dots\ \bar{G}_{in,k}\big] = \Theta_{5i,k}\Theta_{4i,k}^{-1},\\
\Theta_{1i,k} &\triangleq (1+d_{i,k})(1+h_{i,k}^{-1})\big(\vec{A}_k + \vec{E}_k\big)\bar{X}_k,\\
\Theta_{2i,k} &\triangleq (1+d_{i,k})(1+h_{i,k}^{-1})\bar{X}_k + (1+d_{i,k}^{-1})(1+m_{i,k})\Sigma_{i,k},\\
\Theta_{3i,k} &\triangleq (1+d_{i,k})(1+h_{i,k}^{-1})\vec{C}_k\bar{X}_k,\\
\Theta_{4i,k} &\triangleq (1+d_{i,k})(1+h_{i,k}^{-1})\vec{C}_k\bar{X}_k\vec{C}_k^T + \big[g_{i,k}^{-1} + (1+d_{i,k}^{-1})\\
&\quad \times (1+m_{i,k}^{-1})\big]\sum_{s=1}^{n}\Delta_{s,k} + (1+g_{i,k})\vec{D}_k Q_k \vec{D}_k^T\\
&\quad + \Upsilon - \Theta_{3i,k}\Theta_{2i,k}^{-1}\Theta_{3i,k}^T,\\
\Theta_{5i,k} &\triangleq \Theta_{1i,k}\vec{C}_k^T - \Theta_{1i,k}\Theta_{2i,k}^{-1}\Theta_{3i,k}^T.
\end{aligned} \tag{27}$$

Proof: By using the "completing the square" technique, it is deduced from (16) that

$$\begin{aligned}
&\Sigma_{i,k+1}\\
=&\big[K_{i,k} - (\Theta_{1i,k} - G_{i,k}W_i\Theta_{3i,k})\Theta_{2i,k}^{-1}\big]\Theta_{2i,k}\big[K_{i,k}\\
&- (\Theta_{1i,k} - G_{i,k}W_i\Theta_{3i,k})\Theta_{2i,k}^{-1}\big]^T - \Theta_{1i,k}\Theta_{2i,k}^{-1}\Theta_{1i,k}^T\\
&+ \Theta_{1i,k}\Theta_{2i,k}^{-1}\Theta_{3i,k}^T W_i G_{i,k}^T + G_{i,k}W_i\Theta_{3i,k}\Theta_{2i,k}^{-1}\Theta_{1i,k}^T\\
&+ G_{i,k}W_i\Theta_{4i,k}W_iG_{i,k}^T + (1+d_{i,k})(1+h_{i,k}^{-1})\\
&\times \big[(\vec{A}_k + \vec{E}_k)\bar{X}_k(\vec{A}_k + \vec{E}_k)^T - (\vec{A}_k + \vec{E}_k)\\
&\times \bar{X}_k\vec{C}_k^T W_i G_{i,k}^T - G_{i,k}W_i\vec{C}_k\bar{X}_k(\vec{A}_k + \vec{E}_k)^T\big]\\
&+ (1+d_{i,k})(1+h_{i,k})\mu_k^2 \mathrm{tr}\{I_1\bar{X}_k I_1^T\}I + \vec{B}_k R_k \vec{B}_k^T.
\end{aligned} \tag{28}$$

Then, it is easy to see that $\Sigma_{i,k+1}$ is minimized if the estimator gain $K_{i,k}$ is selected as (25).

Next, (28) is further converted to

$$\begin{aligned}
&\Sigma_{i,k+1}\\
=&\big(G_{i,k}W_i - \Theta_{5i,k}\Theta_{4i,k}^{-1}\big)\Theta_{4i,k}\big(G_{i,k}W_i - \Theta_{5i,k}\Theta_{4i,k}^{-1}\big)^T\\
&- \Theta_{5i,k}\Theta_{4i,k}^{-1}\Theta_{5i,k}^T - \Theta_{1i,k}\Theta_{2i,k}^{-1}\Theta_{1i,k}^T + (1+d_{i,k})\\
&\times (1+h_{i,k}^{-1})\big(\vec{A}_k + \vec{E}_k\big)\bar{X}_k\big(\vec{A}_k + \vec{E}_k\big)^T + (1+d_{i,k})\\
&\times (1+h_{i,k})\mu_k^2 \mathrm{tr}\{I_1\bar{X}_k I_1^T\}I + \vec{B}_k R_k \vec{B}_k^T.
\end{aligned} \tag{29}$$

In general, we have the relationship of $G_{i,k}W_i = \Theta_{5i,k}\Theta_{4i,k}^{-1}$. By recalling the definition of W_i, it is obvious that W_i might be non-invertible. Therefore, the estimator gain matrix $G_{i,k}$ cannot be directly obtained from $G_{i,k}W_i = \Theta_{5i,k}\Theta_{4i,k}^{-1}$. In this context, an alternative yet effective way for acquiring the estimator gain $G_{i,k}$ is to select $G_{i,k}$ as in (26). Then, the minimum of the upper bound can be expressed in the form of (24), which ends the proof. □

3.2 Boundedness Analysis

Lemma 3 *[32] If there exist a stochastic process $\mathsf{V}_k(\zeta_k)$ as well as positive scalars $\bar{\alpha}$, $\underline{\alpha}$, ℓ and $0 < \rho_o < 1$ such that*

$$\underline{\alpha}\|\zeta_k\|^2 \leq \mathsf{V}_k(\zeta_k) \leq \bar{\alpha}\|\zeta_k\|^2 \tag{30}$$

and

$$\mathbb{E}\big\{\mathsf{V}_k(\zeta_k)|\zeta_{k-1}\big\} \leq (1-\rho_o)\mathsf{V}_{k-1}(\zeta_{k-1}) + \ell, \tag{31}$$

then ζ_k is exponentially bounded in the mean-square sense, i.e.,

$$\mathbb{E}\big\{\|\zeta_k\|^2\big\} \leq \frac{\bar{\alpha}}{\underline{\alpha}}\mathbb{E}\big\{\|\zeta_0\|^2\big\}(1-\rho_o)^k + \frac{\ell}{\underline{\alpha}}\sum_{i=1}^{k}(1-\rho_o)^i.$$

Lemma 4 *[16] Let $\mathsf{A}, \mathsf{B}, \mathsf{C} \in \mathbb{R}^{n\times n}$ with $\mathsf{B} > 0$ and $\mathsf{C} > 0$. Then, $\mathsf{B}^{-1} - \mathsf{A}^T\mathsf{C}^{-1}\mathsf{A} > 0$ if $\mathsf{C} - \mathsf{A}\mathsf{B}\mathsf{A}^T > 0$.*

Theorem 3 *Consider the discrete-time nonlinear target plant described by (1)-(2) with estimator (10). Assume that there exist real positive scalars $\bar{a}$, $\bar{e}$, $\bar{c}$, $\bar{r}$, $\underline{q}$, $\bar{q}$, τ, $\underline{x}$, $\bar{x}$, μ, ν, $\underline{\sigma}$ and $\bar{\sigma}$ such that*

$$\begin{aligned}
&\|\vec{A}_k\| \leq \bar{a},\ \|\vec{E}_k\| \leq \bar{e},\ \|\vec{C}_k\| \leq \bar{c},\ \vec{B}_k R_k \vec{B}_k^T \leq \bar{r}I,\\
&\underline{q}I \leq \vec{D}_k Q_k \vec{D}_k^T \leq \bar{q}I,\ \tau_i^j = \tau,\ \mu_k = \mu,\ \underline{x}I \leq \bar{X}_k \leq \bar{x}I,\\
&\nu = d_{i,k} = g_{i,k} = h_{i,k} = m_{i,k},\ \underline{\sigma}I \leq \Sigma_{i,k} \leq \bar{\sigma}I.
\end{aligned} \tag{32}$$

Then, the estimation error $e_{i,k}$ $(i = 1, 2, \dots, n)$ is exponentially bounded in mean square.

Proof: First, we select the quadratic function of the following form:

$$V_k(e_{i,k}) = e_{i,k}^T\Sigma_{i,k}^{-1}e_{i,k}. \tag{33}$$

Then, it is easy to obtain from (32) that

$$\bar{\sigma}^{-1}\|e_{i,k}\|^2 \leq V_k(e_{i,k}) \leq \underline{\sigma}^{-1}\|e_{i,k}\|^2, \tag{34}$$

which means that $V_k(e_{i,k})$ satisfies the condition (30).

Next, we aim to find an upper bound on $\mathbb{E}\big\{V_{k+1}(e_{i,k+1})|e_{i,k}\big\}$ satisfying the condition (31). According to (11) and (17)-(21), one obtains

$$\begin{aligned}
&\mathbb{E}\big\{V_{k+1}(e_{i,k+1})|e_{i,k}\big\}\\
=&\mathbb{E}\Big\{\Big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k + \big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)\\
&\times \vec{x}_k + K_{i,k}e_{i,k} - G_{i,k}W_i\vec{D}_k v_k + G_{i,k}W_i\vartheta_k + \vec{B}_k w_k\\
&- G_{i,k}W_i\varphi_k\Big]^T\Sigma_{i,k+1}^{-1}\Big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k + \big(\vec{A}_k - K_{i,k}\\
&+ \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)\vec{x}_k + K_{i,k}e_{i,k} - G_{i,k}W_i\vec{D}_k v_k\\
&+ G_{i,k}W_i\vartheta_k - G_{i,k}W_i\varphi_k + \vec{B}_k w_k\Big]|e_{i,k}\Big\}
\end{aligned}$$

$$
\begin{aligned}
\leq&(1+d_{i,k}^{-1})(1+m_{i,k})e_{i,k}^T K_{i,k}^T \Sigma_{i,k+1}^{-1} K_{i,k} e_{i,k} + \mathbb{E}\Big\{(1\\
&+ d_{i,k})(1+h_{i,k})\big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k\big]^T \Sigma_{i,k+1}^{-1}\big[\vec{g}_k(\vec{x}_k)\\
&- \vec{E}_k\vec{x}_k\big] + (1+d_{i,k})(1+h_{i,k}^{-1})\vec{x}_k^T\big(\vec{A}_k - K_{i,k}\\
&+ \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)^T \Sigma_{i,k+1}^{-1}\big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}\\
&\times W_i\vec{C}_k\big)\vec{x}_k + \Big[(1+m_{i,k}^{-1})(1+d_{i,k}^{-1}) + g_{i,k}^{-1}\Big]\vartheta_k^T W_i\\
&\times G_{i,k}^T \Sigma_{i,k+1}^{-1} G_{i,k} W_i \vartheta_k + (1+g_{i,k})v_k^T (G_{i,k}W_i\vec{D}_k)^T\\
&\times \Sigma_{i,k+1}^{-1}(G_{i,k}W_i\vec{D}_k)v_k + \varphi_k^T W_i G_{i,k}^T \Sigma_{i,k+1}^{-1} G_{i,k} W_i \varphi_k\\
&+ w_k^T \vec{B}_k^T \Sigma_{i,k+1}^{-1}\vec{B}_k w_k\Big\}. \qquad (35)
\end{aligned}
$$

Based on (27) and (32), we obtain

$$
\begin{aligned}
&\|\bar{G}_{i,k}\| = \|\Theta_{5i,k}\Theta_{4i,k}^{-1}\|\\
\leq& \frac{(1+\nu)(1+\nu^{-1})(\bar{a}+\bar{e})\bar{x}\bar{c}\big(1+\bar{x}/(\underline{x}+\underline{\sigma})\big)}{\underline{\Delta} + (1+\nu)\underline{q} + \tau^2/4} \triangleq \bar{g} \qquad (36)
\end{aligned}
$$

with $\underline{\Delta} \triangleq \big(\nu^{-1} + (1+\nu^{-1})^2\big)\sum_{s=1}^{n}(1+\delta_s^{-1})\sigma_s^2$.

By noticing $\|G_{i,k}W_i\| \leq \|\bar{G}_{i,k}\|$, the following can be obtained:

$$
\begin{aligned}
\|K_{i,k}\| =& \|\Theta_{1i,k}\Theta_{2i,k}^{-1} - G_{i,k}W_i\Theta_{3i,k}\Theta_{2i,k}^{-1}\|\\
\leq& \frac{(\bar{a}+\bar{e}+\bar{g}\bar{c})\bar{x}}{\underline{x}+\underline{\sigma}} \triangleq \bar{k}. \qquad (37)
\end{aligned}
$$

From (16) and (37), it is clear that

$$
\begin{aligned}
\Sigma_{i,k+1} \geq& (1+d_{i,k}^{-1})(1+m_{i,k})K_{i,k}\Sigma_{i,k}K_{i,k}^T\\
&+ (1+d_{i,k})(1+h_{i,k})\mu_k^2 \mathrm{tr}\{I_1\bar{X}_k I_1^T\} I\\
>& \Big[1 + \frac{(1+\nu)\mu^2\underline{x}n_x}{2(1+\nu^{-1})\bar{k}^2\bar{\sigma}}\Big](1+d_{i,k}^{-1})\\
&\times (1+m_{i,k})K_{i,k}\Sigma_{i,k}K_{i,k}^T \qquad (38)
\end{aligned}
$$

which, in terms of Lemma 4, indicates that

$$
(1+d_{i,k}^{-1})(1+m_{i,k})K_{i,k}^T\Sigma_{i,k+1}^{-1}K_{i,k} < (1-\epsilon)\Sigma_{i,k}^{-1} \qquad (39)
$$

with

$$
\epsilon \triangleq 1 - \Big[1 + \frac{(1+\nu)\mu^2\underline{x}n_x}{2(1+\nu^{-1})\bar{k}^2\bar{\sigma}}\Big]^{-1}.
$$

It is easily seen that $0 < \epsilon < 1$. By taking (39) into account, we obtain

$$
\begin{aligned}
&(1+d_{i,k}^{-1})(1+m_{i,k})e_{i,k}^T K_{i,k}^T \Sigma_{i,k+1}^{-1} K_{i,k} e_{i,k}\\
\leq& (1-\epsilon)e_{i,k}^T\Sigma_{i,k}^{-1}e_{i,k} = (1-\epsilon)V_k(e_{i,k}). \qquad (40)
\end{aligned}
$$

Together with (16), one obtains

$$
\begin{aligned}
\Sigma_{i,k+1} \geq& (1+d_{i,k})(1+h_{i,k}^{-1})\big(\vec{A}_k - K_{i,k} - G_{i,k}W_i\vec{C}_k\\
&+ \vec{E}_k\big)\bar{X}_k\big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)^T\\
&+ (1+d_{i,k})(1+h_{i,k})\mu_k^2\mathrm{tr}\{I_1\bar{X}_k I_1^T\}I\\
&+ \Big[g_{i,k}^{-1} + (1+d_{i,k}^{-1})(1+m_{i,k}^{-1})\Big]\\
&\times \sum_{s=1}^{n} G_{i,k}W_i\Delta_{s,k}W_iG_{i,k}^T, \qquad (41)
\end{aligned}
$$

which further guarantees that

$$
\begin{aligned}
&\mathbb{E}\Big\{(1+d_{i,k})(1+h_{i,k}^{-1})\vec{x}_k^T\big(\vec{A}_k - K_{i,k} - G_{i,k}W_i\vec{C}_k\\
&+ \vec{E}_k\big)^T\Sigma_{i,k+1}^{-1}\big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)\vec{x}_k\\
&+ (1+d_{i,k})(1+h_{i,k})\big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k\big]^T\\
&\times \Sigma_{i,k+1}^{-1}\big[\vec{g}_k(\vec{x}_k) - \vec{E}_k\vec{x}_k\big]\\
&+ \big[(1+d_{i,k}^{-1})(1+m_{i,k}^{-1}) + g_{i,k}^{-1}\big]\vartheta_k^T W_i G_{i,k}^T\\
&\times \Sigma_{i,k+1}^{-1}G_{i,k}W_i\vartheta_k\Big\}\\
\leq& \mathrm{tr}\Big\{\Sigma_{i,k+1}^{-\frac{1}{2}}\Big[(1+d_{i,k})(1+h_{i,k}^{-1})\big(\vec{A}_k - K_{i,k} + \vec{E}_k\\
&- G_{i,k}W_i\vec{C}_k\big)\bar{X}_k\big(\vec{A}_k - K_{i,k} + \vec{E}_k - G_{i,k}W_i\vec{C}_k\big)^T\\
&+ (1+d_{i,k})(1+h_{i,k})\mu_k^2\mathrm{tr}\{I_1\bar{X}_kI_1^T\}I\\
&+ \big[g_{i,k}^{-1} + (1+d_{i,k}^{-1})(1+m_{i,k}^{-1})\big]\\
&\times \sum_{s=1}^{n} G_{i,k}W_i\Delta_{s,k}W_iG_{i,k}^T\Big]\Sigma_{i,k+1}^{-\frac{1}{2}}\Big\}\\
\leq& \mathrm{tr}\{I\} = n_x + n_f. \qquad (42)
\end{aligned}
$$

Furthermore, it is not difficult to obtain that

$$
\begin{aligned}
&\mathbb{E}\Big\{(1+g_{i,k})v_k^T(G_{i,k}W_i\vec{D}_k)^T\Sigma_{i,k+1}^{-1}(G_{i,k}W_i\vec{D}_k)v_k\\
&+ \varphi_k^T W_i G_{i,k}^T \Sigma_{i,k+1}^{-1} G_{i,k}W_i\varphi_k + w_k^T\vec{B}_k^T\Sigma_{i,k+1}^{-1}\vec{B}_k w_k\Big\}\\
\leq& \frac{(1+\nu)\bar{g}^2}{\underline{\sigma}}\mathbb{E}\Big\{v_k^T\vec{D}_k^T\vec{D}_k v_k\Big\} + \frac{\bar{g}^2}{\underline{\sigma}}\mathbb{E}\Big\{\varphi_k^T\varphi_k\Big\}\\
&+ \frac{1}{\underline{\sigma}}\mathbb{E}\Big\{w_k^T\vec{B}_k^T\vec{B}_k w_k\Big\}\\
\leq& \frac{(1+\nu)\bar{g}^2\bar{q} + \bar{g}^2\tau^2/4}{\underline{\sigma}}nn_y + \frac{\bar{r}}{\underline{\sigma}}(n_x+n_f). \qquad (43)
\end{aligned}
$$

Denote $\iota \triangleq \frac{(1+\nu)\bar{g}^2\bar{q}+\bar{g}^2\tau^2/4}{\underline{\sigma}}nn_y + (\frac{\bar{r}}{\underline{\sigma}}+1)(n_x+n_f)$. Substituting (40), (42)-(43) into (35) implies

$$
\mathbb{E}\big\{V_{k+1}(e_{i,k+1})|e_{i,k}\big\} \leq (1-\epsilon)V_k(e_{i,k}) + \iota. \qquad (44)
$$

According to Lemma 3, it follows from (34) and (44) that the stochastic process $e_{i,k}$ is exponentially bounded in mean-square sense. □

Remark 2 *In the literature, there has been a rich body of results reported on the distributed state/fault estimation problem over various SNs. In comparison with the existing literature, the main results obtained in this paper distinguish themselves in the following aspects: 1) the addressed joint state and fault estimator design problem is new in the sense that the SNs are subject to PQs and DETMs; 2) the developed algorithm is appealing as it calculates the desired estimator parameters in a recursive way; and 3) the derived exponential boundedness condition is new in the sense of mean-square, which quantifies the performance of the proposed state/fault estimator.*

4 An Illustrative Example

4.1 Demonstrations of Results

Consider the target plant (1)-(2) with

$$
g_k(x_k) = E_kx_k + \bar{f}_k(x_k),\ A_k = \begin{bmatrix}0.3\\0.2\end{bmatrix},\ B_k = \begin{bmatrix}0.5\\0.4\end{bmatrix},
$$

$$
F_k = 1.68\sin(0.5k)
$$

where

$$E_k = \begin{bmatrix} 0.8 & -0.1+0.01\cos(k) \\ 0.15 & 0.8 \end{bmatrix}, \bar{f}_k(x_k) = \begin{bmatrix} 0.01\sin(x_k^1) \\ 0.01\sin(x_k^2) \end{bmatrix}.$$

It can be easily seen that the nonlinear function $g_k(x_k)$ satisfies (3) with $\mu_k = 0.01$.
The SN consists of 4 sensor nodes whose topology can be represented by $\mathcal{G} = (\mathcal{V}, \mathcal{Q}, \mathcal{W})$, where $\mathcal{V} = \{1, 2, 3, 4\}$, $\mathcal{Q} = \{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 3), (4, 1), (4, 4)\}$, and $\omega_{ij} = 1$ for $(i, j) \in \mathcal{Q}$. The measurement output (4) is considered with the following parameters:

$$C_{1,k} = \begin{bmatrix} 1.8 & 1.3+0.01\cos(k) \end{bmatrix}, \ D_{1,k} = 0.6,$$
$$C_{2,k} = \begin{bmatrix} 1.2+0.01\sin(k) & 0.7 \end{bmatrix}, \ D_{2,k} = 0.5,$$
$$C_{3,k} = \begin{bmatrix} 1+0.01\sin(k) & 0.9+0.01\cos(k) \end{bmatrix},$$
$$C_{4,k} = \begin{bmatrix} 1.5 & 0.8 \end{bmatrix}, \ D_{3,k} = -0.4, \ D_{4,k} = 0.6.$$

For node i ($i = 1, 2, 3, 4$), we set the parameters in the DETMs (5) as $\lambda_i = 0.2$, $\delta_i = 10$, $\sigma_i = 0.4$, $\hbar_0^i = 1$ and select the quantization level as $\tau_i^1 = 0.2$. The covariances of the noise w_k and v_k are given by $R_k = 0.6$ and $Q_k = 0.6$, respectively. The initial values of the plant state and the fault are zero-mean Gaussian variables with covariances $P_0 = \text{diag}\{0.1, 0.1\}$ and $Z_0 = 0.1$. Furthermore, set $d_{i,k} = g_{i,k} = h_{i,k} = m_{i,k} = 1$. Based on the above parameters, the minimized upper bound on the EEC and the estimator gains can be calculated recursively at each time instant from (24) and (25)-(26).
Figs. 1-5 are the simulation results. Among them, Figs. 1-2 plot the first and second state trajectories and their corresponding estimates, respectively. In Fig. 3, the fault signal and its estimate are displayed. Fig. 4 depicts the trace of the minimal upper bound $\Sigma_{i,k}$ and the mean square error (MSE) defined by

$$\text{MSE}_{i,k} \triangleq \frac{1}{M} \sum_{t=1}^{M} \sum_{s=1}^{3} (\vec{x}_{i,k}^s - \hat{x}_{i,k}^s)^2$$

with $M = 300$, which verifies that $\text{MSE}_{i,k}$ always stays below its upper bound $\Sigma_{i,k}$. In Fig. 5, the broadcast instants of each sensor node determined by the DETM are shown. All simulation results have shown the feasibility of the estimation algorithm developed in this paper.

4.2 Comparisons of Results

I. Comparisons with and without quantization effects

In order to reveal the quantization effects on the estimation performance, the traditional estimation approach without considering quantization effects is realized simultaneously under same parameter settings in the simulation. For the purpose of comparison, the differences between the estimate error of the proposed estimator (with quantization effects) and that of the traditional approach (without quantization effects) are computed for nodes 1, 2, 3 and 4, and the simulation result is shown in Fig. 6. It is easily seen that, during the most of the time, the estimate errors of the traditional estimation are bigger than the ones of our proposed approach. This is because the traditional estimation approach is sensitive to the quantization errors in measurements while the proposed estimation algorithm takes the quantization errors into account, which therefore performs better.

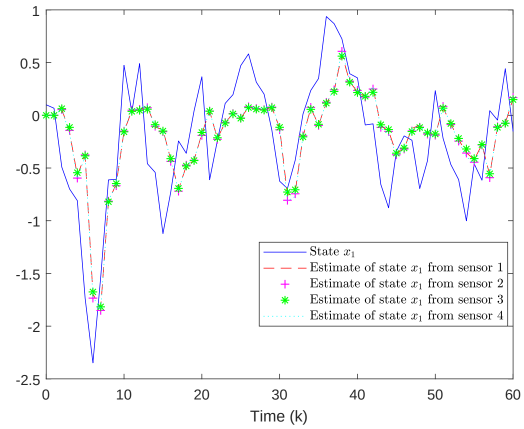

Fig. 1. State x_1 and its estimates

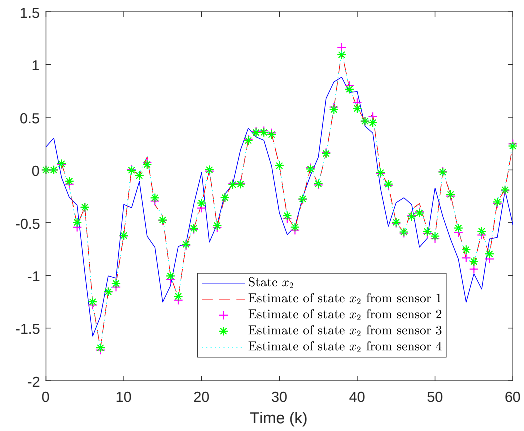

Fig. 2. State x_2 and its estimates

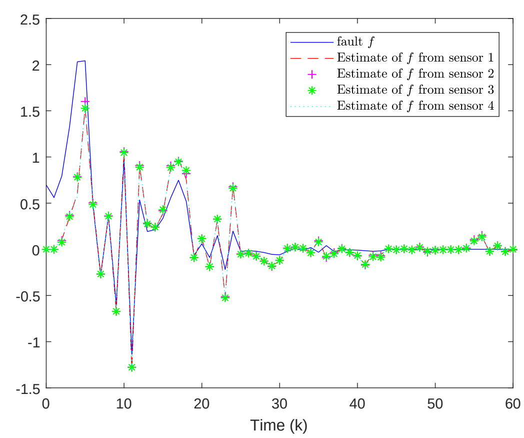

Fig. 3. Fault f and its estimates

II. Comparisons with and without DETMs

Consider the case that the measurements are transmitted through the network at each time instant (i.e. the periodic sampling scheme). In this case, it is obvious that the transmission rates of measurement outputs for sensor nodes 1, 2, 3 and 4 are all 100%. However, by

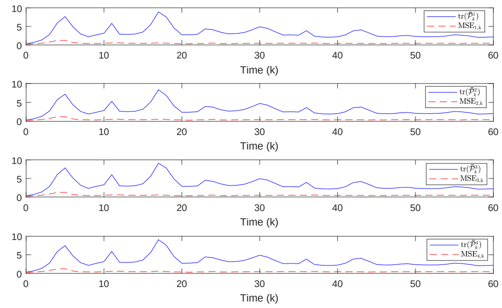

Fig. 4. Trace of state estimation error variance and its upper bound for nodes 1, 2, 3 and 4.

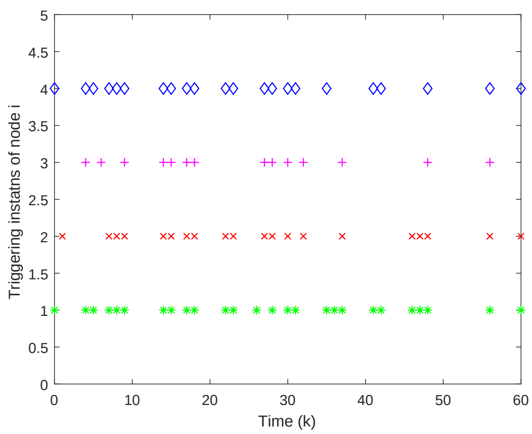

Fig. 5. The broadcast instants.

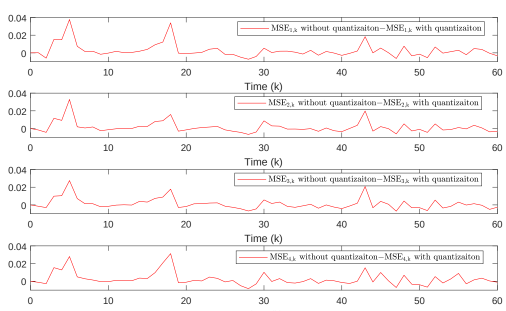

Fig. 6. Estimate error differences.

using our proposed DETM, the event-triggered release instants are shown in Fig. 5, from which, it is easily calculated that the transmission rates for nodes 1, 2, 3 and 4 are 43.3%, 33.3%, 23% and 36.7%, respectively. Therefore, it can be concluded that the proposed DETM is more efficient in alleviating communication burden compared with the periodic sampling scheme. On the other hand, the differences between the estimate errors under DETMs and that under periodic sampling case are plotted in Fig. 7. It appears that estimate errors under DETMs are almost bigger than the ones under periodic sampling case, which means that the estimation performance may deteriorate to a certain extent owing to

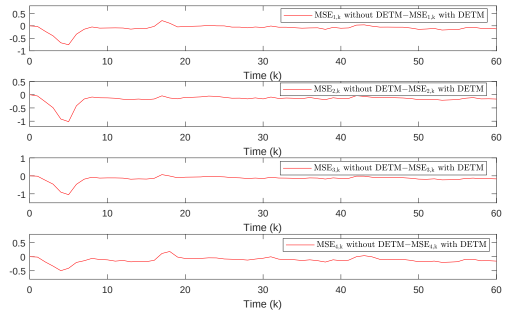

Fig. 7. Estimate error differences.

the introduction of DETMs. Consequently, we can conclude that the proposed DETM can effectively reduce the communication burden at the cost of preserving certain estimation performance.

5 Conclusions

In this paper, the distributed state and fault estimation problem has been dealt with for a class of nonlinear time-varying systems with PQs and DETMs. A SN has been deployed to collect the measurements, where each sensor node exchanges local quantized measurements with its adjacent nodes. To reduce the resource consumption, the DETMs have been employed to schedule the communication between sensor nodes. By using the matrix difference equation method, an upper bound on the EEC has been found, which has been minimized at each iteration by properly designing the estimator parameters. In our upcoming research, the present estimator design results would be extended to SNs subject to fading channels [24] and Markovian jumping systems [7, 19].

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