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Moving Horizon Estimation of Networked Nonlinear Systems with Random Access Protocol

Lei Zou, Zidong Wang, Qing-Long Han and Donghua Zhou

***Abstract*—This work is concerned with the moving horizon (MH) estimation issue for a type of networked nonlinear systems (NNSs) with the so-called random access (RA) protocol scheduling effects. To handle the signal transmissions between sensor nodes and the MH estimator, a constrained communication channel is employed whose channel constraints implies that, at each time instant, only one sensor node is permitted to access the communication channel and then send its measurement data. The RA protocol, whose scheduling behavior is characterized by a discrete-time Markov chain (DTMC), is utilized to orchestrate the access sequence of sensor nodes. By extending the robust MH estimation method, a novel nonlinear MH estimation scheme and the corresponding approximate MH estimation scheme are developed to cope with the state estimation task. Subsequently, some sufficient conditions are established to guarantee that the estimation error is exponentially ultimately bounded in mean square. Based on that, the main results are further specialized to linear systems with the RA protocol scheduling. Finally, two numerical examples and the corresponding figures are provided to verify the effectiveness/correctness of the developed MH estimation scheme and approximate MH estimation scheme.**

***Index Terms*—Moving horizon estimation, Random access protocol, Networked systems, Nonlinear systems, Recursive estimator.**

I. Introduction

As a hot yet important topic in signal processing and control communities, the state estimation (SE) problem has attracted considerable research interest in the past several decades. The main idea of SE is to generate satisfactory state estimates of a given system via the available measured outputs which are probably corrupted by noises. By now, a rich body of SE strategies have appeared in the literature (e.g. $\mathcal{H}_\infty$ SE [16], [26], [28], [33], Kalman filtering [11], [23] and moving horizon (MH) estimation [10], [30]). The primary idea of the MH estimation is to compute the "best" state estimates by solving a given least squares problem (LSP), which is defined on a sliding window with fixed length (i.e. the horizon length). Since the pioneering works presented in [18], the MH estimation problems have gained a persistent research interest for various systems. Some representative results are discussed as follows. In [20], the MH estimation issue has been considered for constrained linear systems and sufficient conditions have been achieved to ensure the stability of the estimation error (EE). A robust MH estimator has been developed in [1] to cope with the SE issue for a type of uncertain discrete-time (DT) linear systems, where the state estimates have been derived through solving the minimization problem (MP) of a quadratic cost function (QCF) in worst-case scenario. In [8], several partition-based MH (PBMH) estimation algorithms have been presented for DT partitioned large-scale systems. The stability properties of MH estimation have been studied in [20], [21], [24] for linear and nonlinear systems.

On another research hotspot, in response to the prompt development of network communication technique, network-based signal transmission scheme becomes a mainstream communication method for numerous industrial applications. Compared with the conventional point-to-point (PtP) communication technology, network-based communication has superiorities in the cost, installation, maintenance and reliability. Networked systems (NSs) are dynamical systems where the signal transmission among system components (e.g. sensors, estimator) is implemented over the shared communication networks. So far, NSs have been successfully applied in the numerous fields including unmanned vehicles, industrial automation, advanced aircraft, smart grids and distributed/mobile communications [4], [19], [29]. Accordingly, significant research efforts has been directed toward the SE problems for NSs subject to various networked-induced constraints, see [3], [6], [14], [15], [17], [27], [32] and the references therein.

In the context of the MH estimation problem for NSs subject to networked-induced constraints, some recent typical research results are discussed as follows. In [13], by extending the aforementioned robust MH estimation approach, the MH estimation issue has been investigated for NSs with quantized measurements and packet dropouts. The MH estimation issue has been investigated for NSs in [30] with multiple packet dropouts. In [9], a decentralized MH estimator has been designed for navigation-oriented NSs with communication link failures and random parametric uncertainties. The distributed MH estimation issue has been handled in [31] subject to data

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losses and transmission delays.

Among most of the existing results of the SE problems for NSs, an underlying assumption is that all the networked nodes are capable of simultaneously accessing the communication channel and transmitting signals. Unfortunately, in numerous practical NSs, it is almost impossible to implement such a communication scheme since simultaneous multiple accessing the communication network would inevitably result in data collisions. An effective method of "protecting" signal transmissions from data collisions is to orchestrate the data transmissions subject to some predefined "agreements", based on which the network access opportunity would be assigned to one node at each transmission instant. These agreements are known as communication protocols (CPs). There are three widely adopted protocols in NSs, namely the random access (RA) protocol (or stochastic CP) [25], [34]–[36], the Try-Once-Discard (TOD) protocol [37] and the Round-Robin (RR) protocol [38].

Among the aforementioned CPs, the RA protocol is a preferred one in practical engineering. One of the representative RA protocol is the carrier-sense multiple access (CSMA) protocols [25]. Generally speaking, the scheduling behaviors of RA protocol could be described by two kinds of stochastic models, namely the discrete-time Markov chain (DTMC) [7] and the sequence of independent and identically distributed (i.i.d) variables [25]. For the control and filtering problems of NSs, the employment of the RA protocol would generate certain specific protocol-induced effects, which, in turn, complicate the analysis and synthesis issues of the NSs. To this end, a seemingly valuable and interesting research topic is to consider the MH estimation problem for NNSs with certain CP. However, as far as the authors' knowledge goes, such a problem has not received adequate research attention yet and this leads to the primary motivation of our study.

In response to the above discussion, this work is concerned with the MH estimation issue for NNSs with the RA protocol scheduling effects, which is non-trivial due to the following three technical challenges: 1) how to generate the estimates of states based on the MH estimation strategy for NNSs with certain CP scheduling effects? 2) how to handle the boundedness analysis (BA) problem of the EE for NNSs? 3) how to understand the effects of the RA protocol scheduling on the estimation performance? It is, therefore, the primary objective of our study to provide satisfactory answers to the above three questions. *The essential contributions of our work are listed as follows. 1) The MH estimation problem is, for the first time, considered for NNSs with the RA protocol scheduling. 2) A novel robust MH estimation strategy is employed to deal with the nonlinearity of the NSs and the RA protocol scheduling behaviors. 3) Sufficient conditions are obtained for approximate MH estimation to ensure the exponential ultimate boundedness of the EE in mean square.*

The remainder of our work is organized as follows. In Section II, the NNS with the RA protocol scheduling is introduced and the corresponding MH estimation problem is formulated. In Section III, a novel robust MH estimation approach and an approximate MH estimation scheme are proposed to solve the aforementioned MH estimation problem. Then, the BA issue of the EE is studied for the approximate MH estimation scheme. Furthermore, two numerical examples are provided in Section IV to examine the effective of the main results. Finally, the conclusion of this work is presented in Section V.

Notations: The notations utilized in this work are given as follows, which are standard except where otherwise stated. $\mathbb{R}^{m\times n}$ and $\mathbb{R}^n$ stand for, respectively, the set of all $m\times n$ real matrices and n dimensional Euclidean space. $\mathbb{R}^+$ represents the set of positive real scalars. The sets of nonnegative integers, negative integers and integers are represented by $\mathbb{N}^+$, $\mathbb{N}^-$ and $\mathbb{N}$, respectively. Letting Φ_A and Φ_B be two real symmetric matrices, the notation $\Phi_A < \Phi_B$ ($\Phi_A \leq \Phi_B$) denote that the matrix $\Phi_A - \Phi_B$ is negative definite (negative semi-definite). For any real matrix M, the transpose of M is represented by M^T and the Moore-Penrose pseudo inverse of M is denoted by $M^\dagger$. For any matrix $P \in \mathbb{R}^{n\times n}$, $\bar{\sigma}\{P\}$ ($\underline{\sigma}\{P\}$) is the largest (smallest) eigenvalue of P, and $\mathrm{tr}\{P\}$ means the trace of P. Furthermore, $\|P\| = \sqrt{\bar{\sigma}(P^TP)}$ is the spectral norm of P. The zero matrix with compatible dimensions is represented by 0. $\mathbf{1}_N$ stands for an N dimensional row vector with all ones. I is the identity matrix with compatible dimensions. $\mathbb{E}\{u|v\}$ and $\mathbb{E}\{u\}$ denote, respectively, the expectation of u conditional on v and the expectation of u. The shorthand $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix. For any vector $u \in \mathbb{R}^n$, $\|u\|$ means the Euclidean norm of u. Furthermore, for any matrix P satisfying $P \in \mathbb{R}^{n\times n}$ and $P > 0$, $\|u\|_P$ stands for the weighted norm of the vector u (i.e. $\|u\|_P \triangleq \sqrt{u^TPu}$). $\delta(s)$ denotes the Kronecker delta function (i.e. $\delta(s) = \begin{cases} 1, & \text{if } s=1 \\ 0, & \text{otherwise} \end{cases}$). It is assumed that matrices have compatible dimensions if they are not explicitly specified.

II. Problem Formulation and Preliminaries

A. The system model

The SE problem considered in this work is shown in Fig. 1, in which the communication between the state estimator and sensors is executed via a communication network with certain CP.

Fig. 1: State estimation issue for a networked system

Next, we shall consider the plant, communication network and the state estimator in a mathematical way. The underlying plant is a DT nonlinear system of the following form:

$$\begin{cases} x(k+1) = f(k, x(k)) + \omega(k) \\ y(k) = Cx(k) + \nu(k) \end{cases} \tag{1}$$

in which $x(k) \in \mathbb{R}^{n_x}$ denotes the state vector; $y(k) \in \mathbb{R}^{n_y}$ represents the measurement output before transmitted; $\omega(k) \in \mathcal{X} \triangleq \{\ell | \ell^T\ell \leq \sqrt{\omega_{\max}}; \ell \in \mathbb{R}^{n_x}\}$ and $\nu(k) \in \mathcal{Y} \triangleq \{\ell | \ell^T\ell \leq \sqrt{\nu_{\max}}; \ell \in \mathbb{R}^{n_y}\}$ denote the system noise and the measurement noise, respectively. $\omega_{\max}$ and $\nu_{\max}$ are known positive constants. C is constant matrix of proper dimension. $f(\cdot,\cdot)$ is the vector-valued time-varying (TV) nonlinear function.

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Assumption 1: $f(\cdot,\cdot)$ is a sector-bounded nonlinearity satisfying the following condition:

$$\big(f(k,\vartheta_1)-f(k,\vartheta_2)-F_1(\vartheta_1-\vartheta_2)\big)^T\big(f(k,\vartheta_1)-f(k,\vartheta_2)\\ -F_2(\vartheta_1-\vartheta_2)\big)\leq 0, f(k,0)=0, \forall\vartheta_1,\vartheta_2\in\mathbb{R}^{n_x}, \forall k\in\mathbb{N}^+ \tag{2}$$

in which F_1, F_2 are real constant matrices satisfying $F_1 > F_2$.

B. Description of the communication protocol

Let us now discuss the effects of the protocol scheduling. In the underlying NS, the measurement data is transmitted through a shared and constrained communication network subject to the so-called *random access (RA) protocol* scheduling. Without loss of generality, it is assumed that the sensors of the plant are grouped into M ($M > 1$) sensor nodes according to their spatial distribution. For technical analysis, we rewrite the output vector $y(k)$ as follows:

$$y(k)=\begin{bmatrix} y_1^T(k) & y_2^T(k) & \cdots & y_M^T(k)\end{bmatrix}^T \tag{3}$$

where $y_i(k)$ ($i\in\{1,2,\cdots,M\}$) represents the measurement output before transmitted of the i-th sensor node.

In network-based communication schemes, the CPs are developed to assign the network access opportunity for sensor nodes. In the underlying NNS, we suppose that only 1 sensor node is physically selected to access the channel per transmission for the sake of avoiding data collisions. Let the integer variable $1\leq\varrho(k)\leq M$ denote the chosen sensor node assigned with the opportunity accessing the channel at transmission instant k. As described in [7], under the effects of the RA protocol scheduling, $\varrho(k)$ can be characterized by a DTMC, whose occurrence probability of $\varrho(k+1)=j$ ($M\geq j\geq 1$) conditioned on $\varrho(k)=i$ ($\forall i\in\{1,2,\cdots,M\}$) is

$$\text{Prob}\{\varrho(k+1)=j|\varrho(k)=i\}=p_{ij} \tag{4}$$

where $p_{ij}\geq 0$ represents the transition probability (TP) from node i to node j at transmission instant k and $\sum_{i=j}^{M}p_{ij}=1$ ($i\in\{1,2,\cdots,M\}$).

Remark 1: It is worth noting that, in this work, the plant (1) and the scheduling effects (4) share the same time scale (i.e. the k-th scheduling behavior is triggered at the k-th time instant of the plant for any $k\in\mathbb{N}^+$). In fact, the nonlinear system (1) could be regarded as the discretization of a continuous-time nonlinear plant subject to the operation period (or sampling period) of the network. In other words, the plant and communication channel share the same the sampling period in this paper. The results of this paper could be easily extended to the case that the operation of the communication network is faster than the sampling of the plant by applying the method in our previous work [35].

In what follows, we are going to consider the signal received by the state estimator. Let the measurement signal after transmitted over the communication network be $\bar{y}(k)\triangleq\begin{bmatrix}\bar{y}_1^T(k) & \bar{y}_2^T(k) & \cdots & \bar{y}_M^T(k)\end{bmatrix}^T\in\mathbb{R}^{n_y}$ where $\bar{y}_i(k)\in\mathbb{R}^{n_y^i}$ with $\sum_{i=1}^{M}n_y^i=n_y$. The updating rule of $\bar{y}_i(k)$ ($k\in\mathbb{N}^+$, $i\in\{1,2,\cdots,M\}$) with the RA protocol scheduling effects is characterized by

$$\bar{y}_i(k)=\begin{cases} y_i(k), & i=\varrho(k)\\ 0, & \text{otherwise.}\end{cases} \tag{5}$$

According to (5), we have

$$\bar{y}(k)=\digamma(\varrho(k))y(k) \tag{6}$$

in which

$$\digamma(\varrho(k))=\text{diag}\{\bar{\delta}_{1,\varrho(k)}I_{n_y^1},\bar{\delta}_{2,\varrho(k)}I_{n_y^2},\cdots,\bar{\delta}_{M,\varrho(k)}I_{n_y^M}\}$$

and $\bar{\delta}_{i,\varrho(k)}\triangleq\delta(\varrho(k)-i)\in\{0,1\}$ ($i=1,2,\cdots,M$) is the Kronecker delta function.

C. Moving horizon state estimator

In this work, we shall employ the ME estimation strategy to design an estimator for the NNS (1) with the RA protocol scheduling effects described by (6). Specifically, for each time instant $k\geq N$ ($N\geq 0$), we aim to find the estimates for system states $x(k-i)$ ($N\geq i\geq 0$) according to the past measurement data $\{\bar{y}(i)\}_{k-N\leq i\leq k}$ and a prior prediction $\bar{x}(k-N)$ of the state vector $x(k-N)$, where $N+1$ represents the window length or horizon. Let $\hat{x}(i|k)$ ($k\geq i\geq k-N$) be the state estimates of $x(i)$ ($k\geq i\geq k-N$) at time instant k, respectively.

The MH estimation problem considered in this work is presented as follows.

Problem 1: For the received measurement output data $\{\bar{y}(i)\}_{k\geq i\geq k-N}$, the estimates $\hat{x}^*(k-N|k)$ is derived by suppressing the following QCF at each time instant k:

$$\mathscr{I}_k(\hat{x}(k-N|k))=\|\hat{x}(k-N|k)-\bar{x}(k-N)\|_Q^2\\ +\sum_{i=0}^{N}\|\hat{y}(k-i|k)-\bar{y}(k-i)\|^2 \tag{7}$$

subject to

$$\begin{cases}\bar{x}(k-N)=f(k-1-N,\hat{x}^*(k-1-N|k-1))\\ \hat{x}(i|k)=f(i,\hat{x}(i-1|k)), \quad k\geq i\geq 1+k-N\\ \hat{y}(i|k)=\digamma(\varrho(i))C\hat{x}(i|k), \quad k\geq i\geq k-N\end{cases} \tag{8}$$

where the weight matrix $Q>0$ is the estimator parameter.

Roughly speaking, it is quite difficult to minimize the QCF $\mathscr{I}_k(\hat{x}(k-N|k))$ subject to a nonlinearity. The solution of such a MP is always achieved by solving certain nonlinear programming problem on-line and such a task will result in heavy calculations. Next, let us introduce an alternative MH estimation problem by extending the "robust MH estimation" approach studied in [1]. Firstly, let $\vec{f}_k=f(k,\hat{x}(k))-f(k,x(k))-\bar{F}(\hat{x}(k)-x(k))$ where $\bar{F}=\frac{F_1+F_2}{2}$. Then, it follows from Assumption 1 that

$$\big(\vec{f}_k-\hat{F}(\hat{x}(k)-x(k))\big)^T\big(\vec{f}_k+\hat{F}(\hat{x}(k)-x(k))\big)\leq 0$$

where $\hat{F}=\frac{F_1-F_2}{2}$, which implies that $\|\vec{f}_k\|^2\leq\|\hat{F}(\hat{x}(k)-x(k))\|^2$. Similar to [5], it can be concluded that there exists at least a function Θ_k satisfying $\vec{f}_k=\Theta_k(\hat{x}(k)-x(k))$

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and $\Theta_k^T\Theta_k \leq \hat{F}^T\hat{F}$. Letting $\Delta F(k) = \Theta_k\hat{F}^{-1}$, we have $\Delta F^T(k)\Delta F(k) \leq I$. Then, it can be derived that

$$f(k,\hat{x}(k)) = \big(\Delta F(k)\hat{F} + \bar{F}\big)(\hat{x}(k) - x(k)) + f(k, x(k)),\\ \forall x(k), \hat{x}(k) \in \mathbb{R}^{n_x}, \forall k \in \mathbb{N}^+ \tag{9}$$

with the uncertainty $\Delta F(k)$ satisfying the constraint $\|\Delta F(k)\| \leq 1$. Therefore, by denoting $z(k-N|k) = \bar{x}(k-N) - \hat{x}(k-N|k)$, for all $k \geq i \geq k-N$, we have

$$\begin{aligned}&\bar{y}(i) - \hat{y}(i|k)\\ &= s(i) + \digamma(\varrho(i))C\prod_{j=k-i+1}^{N}\big(\Delta F(k-j)\hat{F} + \bar{F}\big)z(k-N|k),\end{aligned}$$

where

$$s(i) = \bar{y}(i) - \digamma(\varrho(i))Cf^{(i-k+N)}(\bar{x}(k-N))$$

with $f^{(j)}(\bar{x}(k-N)) = f(j-1+k-N, f^{(j-1)}(\bar{x}(k-N)))$, $f^{(0)}(\bar{x}(k-N)) = \bar{x}(k-N)$ and $\prod_{j=N+1}^{N}(\cdot) = 1$. Therefore, the QCF $\mathscr{I}_k(\hat{x}(k-N|k))$ can be reformulated as follows:

$$\begin{aligned}&\mathscr{I}_k(\hat{x}(k-N|k))\\ &= \big\|z(k-N|k)\big\|_Q^2 + \sum_{i=k-N}^{k}\Big\|s(i) + \digamma(\varrho(i))C\\ &\quad\times\prod_{j=k-i+1}^{N}\big(\bar{F} + \Delta F(k-j)\hat{F}\big)z(k-N|k)\Big\|^2\\ &= \Big\|\bar{\digamma}_{k-N}^{k}(\bar{\varrho}_k)\big(\Delta\mathcal{F}(k) + \mathcal{F}_{k-N}^k\big)z(k-N|k)\\ &\quad + \bar{s}_{k-N}^k\Big\|^2 + \big\|z(k-N|k)\big\|_Q^2\end{aligned} \tag{10}$$

where

$$\begin{aligned}&\bar{s}_{k-N}^k = \begin{bmatrix} s^T(k-N) & s^T(k-N+1) & \cdots & s^T(k)\end{bmatrix}^T,\\ &\mathscr{F}_{k-N}^k = \Big[C^T \quad (C\tilde{F}_{k-N})^T \quad \cdots \quad \big(C\textstyle\prod_{j=1}^N \tilde{F}_{k-j}\big)^T\Big]^T\\ &\bar{\digamma}_{k-N}^k(\bar{\varrho}_k) = \begin{bmatrix} \digamma(\varrho(k-N)) & 0 & \cdots & 0\\ 0 & \digamma(\varrho(k-N+1)) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \digamma(\varrho(k))\end{bmatrix},\\ &\tilde{F}_i = \Delta F(i)\hat{F} + \bar{F},\\ &\mathcal{F}_{k-N}^k = \begin{bmatrix} C^T & (C\bar{F})^T & \cdots & (C\bar{F}^N)^T\end{bmatrix}^T,\\ &\bar{\varrho}_k = \begin{bmatrix}\varrho^T(k-N) & \varrho^T(k-N+1) & \cdots & \varrho^T(k)\end{bmatrix}^T,\end{aligned}$$

and $\Delta\mathcal{F}(k) = \mathscr{F}_{k-N}^k - \mathcal{F}_{k-N}^k$. For the sake of briefness, we shall write $\bar{\digamma}_{k-N}^k$ instead of $\bar{\digamma}_{k-N}^k(\bar{\varrho}_k)$. Then, as shown in [1], there exist a matrix $\Gamma > 0$ such that

$$\big\|\Delta\bar{\mathcal{F}}(k)\big\| \leq 1. \tag{11}$$

where $\Delta\mathcal{F}(k) \triangleq \Delta\bar{\mathcal{F}}(k)\Gamma^{\frac{1}{2}}$. A typical choice is to define the matrix Γ as $\Gamma \triangleq \gamma^2 I$ with $\gamma = \max_{\Delta F(k)}\|\Delta\mathcal{F}(k)\|$. Then, we can conclude from (10) and (11) that

$$\begin{aligned}&\mathscr{I}_k(\hat{x}(k-N|k)) = \bar{\mathscr{I}}_k(\hat{x}(k-N|k), \Delta\bar{\mathcal{F}}(k))\\ &\triangleq \big\|z(k-N|k)\big\|_Q^2 + \Big\|\bar{s}_{k-N}^k + \big(\bar{\digamma}_{k-N}^k\mathcal{F}_{k-N}^k\\ &\quad + \bar{\digamma}_{k-N}^k\Delta\bar{\mathcal{F}}(k)\Gamma^{\frac{1}{2}}\big)z(k-N|k)\Big\|^2.\end{aligned} \tag{12}$$

Based on the above analysis and manipulations, the following alternative robust MH estimation problem is employed in this work:

Problem 2: Based on the received measurement output data $\{\bar{y}(i)\}_{k\geq i\geq k-N}$, the estimates $\hat{x}(k-N|k)$ is acquired by solving the following optimization problem (OP):

$$\hat{x}^*(k-N|k) = \arg\min_{\hat{x}_{k-N}}\max_{\|\Delta\bar{\mathcal{F}}(k)\|\leq 1}\bar{\mathscr{I}}_k(\hat{x}_{k-N}, \Delta\bar{\mathcal{F}}(k)) \tag{13}$$

subject to the constraint (8).

This paper aims to develop a MH estimator by solving *Problem 2* at each time step. Moreover, we shall handle the BA issue on the EE.

III. Main results

A. Moving-horizon estimator

To state the following results, we shall introduce the following lemma.

Lemma 1: [22] Consider the following robust OP:

$$\min_z\max_{\|S\|\leq 1}\Big\{\|z\|_Q^2 + \big\|(B+\Delta B)z - (D+\Delta D)\big\|_R^2\Big\} \tag{14}$$

where $\Delta B = HSE_b$, $\Delta D = HSE_d$. H, E_b and E_d are known matrices, S is an arbitrary contraction. As such, the unique global minimum z^* of the OP (14) is described by

$$z^* = (\hat{Q} + B^T\hat{R}B)^{-1}\big(B^T\hat{R}D + \lambda^* E_b^T E_d\big) \tag{15}$$

where $\hat{Q} = Q + \lambda^* E_b^T E_b$, $\hat{R} = R + RH(\lambda^* I - H^T RH)^\dagger H^T R$. The value of λ^* is calculated by

$$\begin{aligned}\lambda^* = \arg\min_{\lambda\geq\|H^TRH\|}&\Big\{\lambda\big\|E_b z(\lambda) - E_d\big\|^2 + \big\|z(\lambda)\big\|_Q^2\\ &+ \big\|Bz(\lambda) - D\big\|_{\hat{R}(\lambda)}^2\Big\}\end{aligned} \tag{16}$$

where

$$\begin{aligned}&z(\lambda) = \Big(\hat{Q}(\lambda) + B^T\hat{R}(\lambda)B\Big)^{-1}\Big(B^T\hat{R}(\lambda)D + \lambda E_b^T E_d\Big),\\ &\hat{Q}(\lambda) = Q + \lambda E_b^T E_b,\\ &\hat{R}(\lambda) = R + RH(\lambda I - H^T RH)^\dagger H^T R.\end{aligned}$$

By virtue of Lemma 1, the following theorem is derived.

Theorem 1: Consider the received measurement output data $\{\bar{y}(i)\}_{k\geq i\geq k-N}$ and the MH estimation constraint (8). The solution to *Problem 2* is given by

$$\begin{aligned}&\hat{x}(k-N|k) = \bar{x}(k-N) + \Big(\mathcal{Q}(\lambda^*) + \big(\bar{\digamma}_{k-N}^k\mathcal{F}_{k-N}^k\big)^T\\ &\times\mathcal{R}(\lambda^*)\bar{\digamma}_{k-N}^k\mathcal{F}_{k-N}^k\Big)^{-1}\big(\bar{\digamma}_{k-N}^k\mathcal{F}_{k-N}^k\big)^T\mathcal{R}(\lambda^*)\bar{s}_{k-N}^k\end{aligned} \tag{17}$$

where $\mathcal{R}(\lambda^*) = I + \bar{\digamma}_{k-N}^k\big(\lambda^* I - \bar{\digamma}_{k-N}^k\big)^\dagger\bar{\digamma}_{k-N}^k$, $\mathcal{Q}(\lambda^*) = Q + \lambda^*\Gamma$, and the value of λ^* is derived as follows:

$$\begin{aligned}\lambda^* = \arg\min_{\lambda\geq 1}&\Big\{\big\|z_{k-N}(\lambda)\big\|_Q^2 + \lambda\big\|\Gamma^{\frac{1}{2}}z_{k-N}(\lambda)\big\|^2\\ &+ \big\|\bar{\digamma}_{k-N}^k\mathcal{F}_{k-N}^k z_{k-N}(\lambda) + \bar{s}_{k-N}^k\big\|_{\mathcal{R}(\lambda)}^2\Big\}\end{aligned} \tag{18}$$

with

$$z_{k-N}(\lambda) = -\Big(\big(\bar{\digamma}_{k-N}^k\mathcal{F}_{k-N}^k\big)^T\mathcal{R}(\lambda)\bar{\digamma}_{k-N}^k\mathcal{F}_{k-N}^k$$

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$$+ \mathcal{Q}(\lambda)\Big)^{-1}\big(\bar{F}^{k}_{k-N}\mathcal{F}^{k}_{k-N}\big)^{T}\mathcal{R}(\lambda)\bar{s}^{k}_{k-N}.$$

Proof: The proof of Theorem 1 can be directly obtained from Lemma 1, which is omitted here for conciseness. ■

Remark 2: The moving-horizon estimator proposed in Theorem 1 could be regarded as an extension of the robust MH estimation approach developed in [1]. It can be seen from the estimator (17) that both the information of the nonlinear function $f(\cdot)$ and RA protocol scheduling behaviors has been reflected in the expression of $\hat{x}(k-N|k)$. The uncertainty $\Delta F(k)$, which is generated in 9, could be seen as a kind of "linearization error" of the nonlinear function. Such an uncertainty has also been reselected in the term $\mathcal{Q}(\lambda^*)$.

By means of the above Theorem, the corresponding Moving-Horizon Estimation algorithm is summarized as follows:

Moving-Horizon Estimation algorithm:	
Step 1.	Let the window length $N+1$ and the MH estimator parameter $Q>0$ be given. Set $k=N$ and $\bar{x}(0)=0$.
Step 2.	Calculate the value of Γ as $\Gamma=\gamma^2 I$ where the scalar γ is determined by $\gamma=\max_{\Delta F(k)}\|\Delta\mathcal{F}(k)\|$.
Step 3.	Compute the scalar parameter λ^* by solving the one-dimensional OP (18).
Step 4.	Generate the matrix $\bar{F}^{k}_{k-N}$ and the vector $\bar{s}^{k}_{k-N}$ based on $\{F(\varrho(i))\}_{k\geq i\geq k-N}$ and $\{\bar{y}(i)\}_{k\geq i\geq k-N}$. Then, $\hat{x}(k-N|k)$ can be obtained by (17).
Step 5.	Set $k=k+1$ and compute the value of $\bar{x}(k-N)$ by (8) and go to *Step 3*.

As to the optimization issue in (18), if the boundary point $\lambda = 1$ is excluded, as shown in [22], the pseudoinverse operation of $\mathcal{R}(\lambda^*)$ is solved as follows:

$$\mathcal{R}(\lambda^*) = I + \frac{1}{\lambda^*-1}\bar{F}^{k}_{k-N}$$

and hence, the estimation of (17) is rewritten as follows

$$\hat{x}(k-N|k) = \frac{\lambda^*}{\lambda^*-1}\bigg(\frac{\lambda^*}{\lambda^*-1}\big(\mathcal{F}^{k}_{k-N}\big)^{T}\bar{F}^{k}_{k-N}\mathcal{F}^{k}_{k-N} + Q + \lambda^*\Gamma\bigg)^{-1}\big(\mathcal{F}^{k}_{k-N}\big)^{T}\bar{F}^{k}_{k-N}\bar{s}^{k}_{k-N} + \bar{x}(k-N) \tag{19}$$

In view of the expressions (18) and (19), it can be found that the presented MH estimator is a TY and nonlinear one due mainly to the derivation of the parameter λ^*, which is obtained by solving an OP via certain on-line algorithm. However, it is sometimes difficult to solve such a problem in the required computational time (i.e. the interval between sampling time instants). For the purpose of real-time implementation, we can choose a reasonable approximation to the expression (17) by setting the scalar λ^* as $\lambda^* = \alpha + 1$ where the scalar α can be properly adjusted off-line based on certain numerical simulations, and this gives rise to the approximate solution of *Problem 2* with the following form:

$$\begin{aligned}\hat{x}(k-N|k) = &\big(\alpha^{-1}+1\big)\Big(Q + (1+\alpha)\Gamma + (\alpha^{-1}+1)\\ &\times\big(\mathcal{F}^{k}_{k-N}\big)^{T}\bar{F}^{k}_{k-N}\mathcal{F}^{k}_{k-N}\Big)^{-1}\big(\mathcal{F}^{k}_{k-N}\big)^{T}\\ &\times\bar{F}^{k}_{k-N}\bar{s}^{k}_{k-N} + \bar{x}(k-N).\end{aligned} \tag{20}$$

The approximate MH Estimation algorithm associating with (20) can be easily accessible from the MH estimation algorithm. Hence, we omit the details of the approximate MH estimation algorithm here for conciseness. Obviously, by eliminating the computation of the scalar parameter λ^*, the computational effort of the approximate MH estimation algorithm could be largely reduced compared with the MH estimation algorithm.

Next, let us study the boundedness of the EE according to the obtained approximate MH estimation scheme (20).

B. Boundedness analysis issue of the estimation error

In what follows, we shall investigate the boundedness properties for the EE. Let us first consider the dynamics of the EE. For the sake of clarity of exposition, by defining

$$\begin{aligned}\Psi_\alpha(\bar{\varrho}_k) \triangleq &\Big(1+\frac{1}{\alpha}\Big)\Big(Q + (1+\alpha)\Gamma + \Big(1+\frac{1}{\alpha}\Big)\\ &\times\big(\mathcal{F}^{k}_{k-N}\big)^{T}\bar{F}^{k}_{k-N}\mathcal{F}^{k}_{k-N}\Big)^{-1}\big(\mathcal{F}^{k}_{k-N}\big)^{T}\bar{F}^{k}_{k-N},\end{aligned}$$

it follows from (20) that

$$\hat{x}(k-N|k) - \bar{x}(k-N) = \Psi_\alpha(\bar{\varrho}_k)\bar{s}^{k}_{k-N}. \tag{21}$$

On the other hand, by defining $e(i-N) \triangleq x(i-N) - \hat{x}(i-N|i)$, it can be derived from the definition of $s(i)$ and the expression of $\bar{x}(k-N)$ that

$$\begin{cases} s(k-N) = F(\varrho(k-N))\Big(C\tilde{F}_{k-N-1}e(k-N-1)\\ \qquad + \nu(k-N) + C\omega(k-N-1)\Big)\\ \qquad\vdots\\ s(k-i) = F(\varrho(k-i))\Big(C\prod_{j=i+1}^{N+1}\tilde{F}_{k-j}e(k-N-1)\\ \qquad + \nu(k-i) + C\sum_{j=i+1}^{N+1}\prod_{t=i+1}^{j-1}\tilde{F}_{k-t}\omega(k-j)\Big),\\ \qquad (N-1\geq i\geq 0)\end{cases}$$

Hence, we have

$$\begin{aligned}\bar{s}^{k}_{k-N} = &\bar{F}^{k}_{k-N}\mathscr{F}^{k}_{k-N}\tilde{F}_{k-N-1}e(k-N-1)\\ &+ \bar{F}^{k}_{k-N}\mathscr{G}_k\bar{\omega}(k-1) + \bar{F}^{k}_{k-N}\bar{\nu}(k)\end{aligned} \tag{22}$$

where

$$\mathscr{G}_k = \begin{bmatrix} C & 0 & \cdots & 0\\ C\tilde{F}_{k-N} & C & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ C\prod_{j=1}^{N}\tilde{F}_{k-j} & C\prod_{j=1}^{N-1}\tilde{F}_{k-j} & \cdots & C\end{bmatrix},$$

$$\bar{\nu}(k) = \begin{bmatrix}\nu(k-N)\\ \nu(k-N+1)\\ \vdots\\ \nu(k)\end{bmatrix},\quad \bar{\omega}(k-1) = \begin{bmatrix}\omega(k-N-1)\\ \omega(k-N)\\ \vdots\\ \omega(k-1)\end{bmatrix},$$

$$\mathcal{C} = \mathrm{diag}_{N+1}\{C\}.$$

By taking (21) and (22) into account, we have

$$e(k-N)$$

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$$\begin{aligned}&= \Big(\tilde{F}_{k-N-1} - \Psi_\alpha(\bar{\varrho}_k)\mathscr{F}^k_{k-N}\tilde{F}_{k-N-1}\Big)e(k-N-1)\\&\quad + \Big(\mathscr{I} - \Psi_\alpha(\bar{\varrho}_k)\mathscr{G}_k\Big)\bar{\omega}(k-1) - \Psi_\alpha(\bar{\varrho}_k)\bar{\nu}(k) \end{aligned}\tag{23}$$

where $\mathscr{I} = \begin{bmatrix} I & 0 & 0 & \cdots & 0\end{bmatrix}$.

Similar to the same technique in [1], we first rewrite the EE dynamics (23) as follows:

$$\begin{aligned}e(k-N) =& \Big(\Delta\mathcal{A}(\bar{\varrho}_k) + \mathcal{A}(\bar{\varrho}_k)\Big)e(k-N-1)\\&+ \Big(\Delta\mathcal{B}(\bar{\varrho}_k) + \mathcal{B}(\bar{\varrho}_k)\Big)\tilde{\omega}(k)\end{aligned}\tag{24}$$

where

$$\begin{aligned}&\mathcal{B}(\bar{\varrho}_k) = \begin{bmatrix}\mathscr{I} - \Psi_\alpha(\bar{\varrho}_k)\mathcal{G}_k & -\Psi_\alpha(\bar{\varrho}_k)\end{bmatrix},\\&\mathcal{A}(\bar{\varrho}_k) = \bar{F} - \Psi_\alpha(\bar{\varrho}_k)\mathcal{F}^k_{k-N}\bar{F},\\&\Delta\mathcal{A}(\bar{\varrho}_k) = \Delta F(k-1-N)\hat{F} - \Psi_\alpha(\bar{\varrho}_k)\\&\qquad\qquad \times \big(\mathscr{F}^k_{k-N}\tilde{F}_{k-1-N} - \mathcal{F}^k_{k-N}\bar{F}\big),\\&\Delta\mathcal{B}(\bar{\varrho}_k) = \begin{bmatrix}-\Psi_\alpha(\bar{\varrho}_k)\big(\mathscr{G}_k - \mathcal{G}_k\big) & 0\end{bmatrix},\\&\mathcal{G}_k = \begin{bmatrix} C & 0 & \cdots & 0\\ C\bar{F} & C & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ C\bar{F}^N & C\bar{F}^{N-1} & \cdots & C\end{bmatrix},\ \tilde{\omega}(k) = \begin{bmatrix}\bar{\omega}(k-1)\\ \bar{\nu}(k)\end{bmatrix}.\end{aligned}$$

with the matrix inequalities $\Delta\mathcal{A}^T(\bar{\varrho}_k)\Delta\mathcal{A}(\bar{\varrho}_k) \le \Gamma_A(\bar{\varrho}_k)$ and $\Delta\mathcal{B}^T(\bar{\varrho}_k)\Delta\mathcal{B}(\bar{\varrho}_k) \le \Gamma_B(\bar{\varrho}_k)$ in which $\Gamma_A(\bar{\varrho}_k)$ and $\Gamma_B(\bar{\varrho}_k)$ represent two known positive definite matrices. Trivial yet conservative choices for $\Gamma_A(\bar{\varrho}_k)$ and $\Gamma_B(\bar{\varrho}_k)$ can be given by $\Gamma_A(\bar{\varrho}_k) = \gamma_A^2(\bar{\varrho}_k)I$ and $\Gamma_B(\bar{\varrho}_k) = \gamma_B^2(\bar{\varrho}_k)I$ where $\gamma_A(\bar{\varrho}_k) = \max_{\{\Delta F(i)\}_{k-N\le i\le k}} \|\Delta\mathcal{A}(\bar{\varrho}_k)\|$, $\gamma_B(\bar{\varrho}_k) = \max_{\{\Delta F(i)\}_{k-N\le i\le k}} \|\Delta\mathcal{B}(\bar{\varrho}_k)\|$.

So far, we have obtained the dynamics of the EE. Next, let us reformulating the EE system (24) by mapping the sequence $\varrho(k-i)$ $(i = 0, 1, \cdots, N)$ to one stochastic process.

Proposition 1: Map the RA protocol scheduling behavior governed by $\{\varrho(k-i)\}_{0\le i\le N}$ to the new variable $\aleph(k) \in \mathfrak{Q} \triangleq \{1, 2, \cdots, M^{N+1}\}$ according to the following mapping function:

$$\aleph(k) = \mathscr{R}(\bar{\varrho}_k) \triangleq \sum_{i=0}^{N} M^i\big(\varrho(k-i) - 1\big) + 1. \tag{25}$$

Then, for the given value of $\aleph(k)$, the values of $\varrho(k-i)$ $(0 \le i \le N)$ can be calculated by the function $\phi_i(\aleph(k))$ $(N \ge i \ge 0)$:

$$\varrho(k-i) = \phi_i(\aleph(k)) \triangleq \mathrm{mod}\left(\left\lfloor\frac{\aleph(k)-1}{M^i}\right\rfloor, M\right) + 1. \tag{26}$$

Proof: Firstly, it is easy to see that the value of $\aleph(k)$ obtained by (25) satisfies $\aleph(k) \in \mathfrak{Q}$. Next, we shall prove that $\varrho(k-i)$ derived in (26) is correct. For any given $\aleph(k) \in \mathfrak{Q}$, we have

$$\begin{aligned}\phi_i(\aleph(k)) &= \mathrm{mod}\left(\left\lfloor\frac{\aleph(k)-1}{M^i}\right\rfloor, M\right) + 1\\&= \mathrm{mod}\left(\sum_{j=i}^{N} M^{j-i}\big(\varrho(k-j)-1\big), M\right) + 1\\&= \varrho(k-i) - 1 + 1 = \varrho(k-i).\end{aligned}$$

The proof is complete. ■

Obviously, $\aleph(k)$ is a random variable and the corresponding characteristics are given in the following proposition.

Proposition 2: The sequence $\{\aleph(k)\}_{k\ge 0}$ is a DTMC with the TP matrix $\bar{\mathbb{P}}(k) \triangleq \big[\breve{p}_{ij}\big]_{M^N\times M^N}$ given as follows:

$$\begin{aligned}\breve{p}_{ij} &= \mathrm{Prob}\big(\aleph(k+1) = j | \aleph(k) = i\big)\\&= \begin{cases} 0, & \breve{\pi}(i,j) > M\\ p_{\phi_0(i)\phi_0(j)}, & \text{otherwise}\end{cases}\end{aligned}\tag{27}$$

in which $\breve{\pi}(i,j) = j - M\Big(i - 1 - M^N\big(\phi_N(i) - 1\big)\Big)$ and $p_{ij}(\cdot)$ has been defined in (4).

Proof: According to Proposition 1, we can obtain that

$$\begin{aligned}\breve{p}_{ij} &= \mathrm{Prob}\big(\aleph(k+1) = j | \aleph(k) = i\big)\\&= \mathrm{Prob}\big(\bar{\varrho}_{k+1} = \bar{\phi}_{k+1}(j) | \bar{\varrho}_k = \bar{\phi}_k(i)\big)\end{aligned}$$

where $\bar{\phi}_k(i) = \begin{bmatrix}\phi_N^T(i) & \phi_{N-1}^T(i) & \cdots & \phi_0^T(i)\end{bmatrix}^T$. Let $\bar{\phi}_k^t(i)$ be the t-th element of the vector $\bar{\phi}_k(i)$. Note that $\bar{\phi}_k^t(i) = \bar{\phi}_{k+1}^{t-1}(j)$, which implies that

$$j - \phi_0(j) = M\big(i - 1 - M^N(\phi_N(i) - 1)\big).$$

As such, it can be concluded that $\breve{p}_{ij} = 0$ if $\breve{\pi}(i,j) > M$. Moreover, if the $\breve{\pi}(i,j) \le M$, we have

$$\begin{aligned}\breve{p}_{ij} &= \mathrm{Prob}\big(\bar{\varrho}_{k+1} = \bar{\phi}_{k+1}(j) | \bar{\varrho}_k = \bar{\phi}_k(i)\big)\\&= \mathrm{Prob}\big(\varrho(k+1) = \phi_0(j) | \varrho(k) = \phi_0(i)\big)\\&= p_{\phi_0(i)\phi_0(j)}.\end{aligned}$$

The proof is complete. ■

According to Propositions 1 and 2, the EE dynamical system (24) is reformulated as follows

$$\begin{aligned}e(k-N) =& \big(\mathcal{A}_{\aleph(k)} + \Delta\mathcal{A}_{\aleph(k)}\big)e(k-N-1) + \big(\mathcal{B}_{\aleph(k)}\\&+ \Delta\mathcal{B}_{\aleph(k)}\big)\tilde{\omega}(k)\end{aligned}\tag{28}$$

with $\mathcal{A}_{\aleph(k)} = \mathcal{A}(\bar{\varrho}_k)$, $\Delta\mathcal{A}_{\aleph(k)} = \Delta\mathcal{A}(\bar{\varrho}_k)$, $\mathcal{B}_{\aleph(k)} = \mathcal{B}(\bar{\varrho}_k)$, $\Delta\mathcal{B}_{\aleph(k)} = \Delta\mathcal{B}(\bar{\varrho}_k)$, $\Gamma_{A,\aleph(k)} = \Gamma_A(\bar{\varrho}_k)$ and $\Gamma_{B,\aleph(k)} = \Gamma_B(\bar{\varrho}_k)$.

Remark 3: Due to the RA protocol scheduling and the sector-bounded nonlinearity, the EE system (24) is described by a dynamical system with a DTMC $\aleph(k)$ and the norm-bounded uncertainties. Due to the TV nature of the scalar parameter λ^*, it is difficult to analyze the dynamical behaviors of the EE. Moreover, the value of λ^* is computed by solving an one-dimensional OP and thereby increasing the on-line computational effort of the MH estimation approach. In what follows, we shall focus our attention on the EE dynamics resulting from the approximate MH estimation approach based on the stochastic analysis technique.

Before deriving further results, we firstly give the definition about the exponential ultimate boundedness in mean square.

Definition 1: Consider the EE dynamics (24). Assume that there exist 3 constants $\mu_1 > 0$, $\mu_2 > 0$ and $\mu_3 > 0$ satisfying the following constraint

$$\mathbb{E}\{\|e(k)\|^2 | e(0)\} < \mu_1^k\mu_2 + \mu_3 \tag{29}$$

where $\mu_1 \in [0, 1)$. Then, the dynamics of $e(k)$ is said to be exponentially ultimately bounded (EUB) in mean square. The

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parameters μ_3 and μ_1 are denoted as the asymptotic upper bound (AUB) and the decay rate of $\mathbb{E}\{\|e(k)\|^2\}$, respectively.

Theorem 2: Consider the NNS (1) under the RA protocol scheduling effects. Let the TP matrix of the DTMC $\varrho(k)$ and the updating rule (5) be given. It is supposed that there exist $2M^N+1$ positive definite matrices P_i, $\vec{P}_i$ $(i=1,2\cdots,M^N)$, $\Upsilon=\mathrm{diag}\{r_1I,r_2I,\cdots,r_{2N+2}I\}$, $2M$ positive scalars θ_i $(i=1,2\cdots,M)$ and ϵ_i satisfying

$$\bar{\Omega}_i=\begin{bmatrix}\bar{\Omega}_i^{11} & \bar{\Omega}_i^{12} & \bar{\Omega}_i^{13} & \bar{\Omega}_i^{14}\\ * & \bar{\Omega}_i^{22} & \bar{\Omega}_i^{23} & \bar{\Omega}_i^{24}\\ * & * & \bar{\Omega}_i^{33} & \bar{\Omega}_i^{34}\\ * & * & * & \bar{\Omega}_i^{44}\end{bmatrix}<0 \tag{30}$$

where

$$\begin{aligned}&\bar{P}_i=\sum_{j=1}^{M^N}\check{p}_{ij}P_j,\ \bar{\Omega}_i^{11}=\mathcal{A}_i^T\bar{P}_i\mathcal{A}_i-P_i+\vec{P}_i+\theta_i\Gamma_{A,i},\\ &\bar{\Omega}_i^{12}=\mathcal{A}_i^T\bar{P}_i,\ \bar{\Omega}_i^{13}=\mathcal{A}_i^T\bar{P}_i\mathcal{B}_i,\ \bar{\Omega}_i^{14}=\mathcal{A}_i^T\bar{P}_i,\ \bar{\Omega}_i^{24}=\bar{P}_i,\\ &\bar{\Omega}_i^{22}=\bar{P}_i-\theta_iI,\quad \bar{\Omega}_i^{23}=\bar{P}_i\mathcal{B}_i,\quad \bar{\Omega}_i^{44}=\bar{P}_i-\epsilon_iI,\\ &\bar{\Omega}_i^{33}=\mathcal{B}_i^T\bar{P}_i\mathcal{B}_i+\epsilon_i\Gamma_{B,i}-\Upsilon,\quad \bar{\Omega}_i^{34}=\mathcal{B}_i^T\bar{P}_i.\end{aligned}$$

Then, the EE dynamics (28) is EUB in mean square with the AUB $\frac{\varepsilon}{\rho\min_{1\le i\le M}\{\underline{\sigma}(P_i)\}}$ where $\rho=\frac{\underline{\sigma}(\vec{P}_i)}{\bar{\sigma}(P_i)}$ and $\varepsilon=\big(\sum_{i=N+2}^{2N+2}r_i\nu_{\max}^2+\sum_{i=1}^{N+1}r_i\omega_{\max}^2\big)$.

Proof: For the purpose of studying the ultimately boundedness issue of the EE $e(k)$, choose the Lyapunov-like function as follows:

$$\mathscr{M}(k)=e^T(k-N-1)P_{\aleph(k)}e(k-N-1). \tag{31}$$

The difference of $\mathscr{M}(k)$ (i.e. $\Delta\mathscr{M}(k)\triangleq\mathscr{M}(k+1)-\mathscr{M}(k)$) along the trajectory of the EE dynamics (28) is computed as follows:

$$\begin{aligned}\Delta\mathscr{M}(k)=&\,e^T(k-N)P_{\aleph(k+1)}e(k-N)\\ &-e^T(k-N-1)P_{\aleph(k)}e(k-N-1)\\ =&\Big(\big(\mathcal{A}_{\aleph(k)}+\Delta\mathcal{A}_{\aleph(k)}\big)e(k-N-1)+\big(\mathcal{B}_{\aleph(k)}\\ &+\Delta\mathcal{B}_{\aleph(k)}\big)\tilde{\omega}(k)\Big)^TP_{\aleph(k+1)}\Big(\big(\mathcal{A}_{\aleph(k)}+\Delta\mathcal{A}_{\aleph(k)}\\ &\times\big)e(k-N-1)+\big(\Delta\mathcal{B}_{\aleph(k)}+\mathcal{B}_{\aleph(k)}\big)\tilde{\omega}(k)\Big)\\ &-e^T(k-N-1)P_{\aleph(k)}e(k-N-1)\end{aligned} \tag{32}$$

Then, take the conditional mathematical expectation on the equation (32) and we have

$$\begin{aligned}&\mathbb{E}\big\{\Delta\mathscr{M}(k)\big|\aleph(k)=i\big\}\\ =&\,\mathbb{E}\Big\{\Big(\big(\mathcal{A}_i+\Delta\mathcal{A}_i\big)e(k-1-N)+\big(\mathcal{B}_i+\Delta\mathcal{B}_i\big)\tilde{\omega}(k)\Big)^T\bar{P}_i\\ &\times\Big(\big(\mathcal{A}_i+\Delta\mathcal{A}_i\big)e(k-1-N)+\big(\mathcal{B}_i+\Delta\mathcal{B}_i\big)\tilde{\omega}(k)\Big)\\ &-e^T(k-1-N)P_ie(k-1-N)\Big\}\\ =&\,\mathbb{E}\big\{\varpi^T(k)\Omega_i\varpi(k)\big|\aleph(k)=i\big\}\end{aligned} \tag{33}$$

where

$$\varpi(k)=\begin{bmatrix}e(k-1-N)\\ \Delta\mathcal{A}_ie(k-1-N)\\ \tilde{\omega}(k)\\ \Delta\mathcal{B}_i\tilde{\omega}(k)\end{bmatrix},$$

$$\Omega_i=\begin{bmatrix}\mathcal{A}_i^T\bar{P}_i\mathcal{A}_i-P_i & \mathcal{A}_i^T\bar{P}_i & \mathcal{A}_i^T\bar{P}_i\mathcal{B}_i & \mathcal{A}_i^T\bar{P}_i\\ * & \bar{P}_i & \bar{P}_i\mathcal{B}_i & \bar{P}_i\\ * & * & \mathcal{B}_i^T\bar{P}_i\mathcal{B}_i & \mathcal{B}_i^T\bar{P}_i\\ * & * & * & \bar{P}_i\end{bmatrix}.$$

Adding the following zero term

$$\begin{aligned}0=&\,\rho\mathscr{M}(k)+\theta_ie^T(k-1-N)\Delta\mathcal{A}_i^T\Delta\mathcal{A}_ie(k-1-N)\\ &-\theta_ie^T(k-1-N)\Delta\mathcal{A}_i^T\Delta\mathcal{A}_ie(k-1-N)\\ &+\epsilon_i\tilde{\omega}^T(k)\Delta\mathcal{B}_i^T\Delta\mathcal{B}_i\tilde{\omega}(k)-\epsilon_i\tilde{\omega}^T(k)\Delta\mathcal{B}_i^T\Delta\mathcal{B}_i\tilde{\omega}(k)\\ &+\tilde{\omega}^T(k)\Upsilon\tilde{\omega}(k)-\rho\mathscr{M}(k)-\tilde{\omega}^T(k)\Upsilon\tilde{\omega}(k)\end{aligned} \tag{34}$$

to the right-hand side of (33), we have

$$\begin{aligned}&\mathbb{E}\big\{\Delta\mathscr{M}(k)\big|\aleph(k)=i\big\}\\ =&\,\mathbb{E}\big\{\rho\mathscr{M}(k)+\varpi^T(k)\Omega_i\varpi(k)+\theta_ie^T(k-1-N)\Delta\mathcal{A}_i^T\\ &\times\Delta\mathcal{A}_ie(k-N-1)+\epsilon_i\tilde{\omega}^T(k)\Delta\mathcal{B}_i^T\Delta\mathcal{B}_i\tilde{\omega}(k)\\ &-\rho\mathscr{M}(k)-\theta_ie^T(k-N-1)\Delta\mathcal{A}_i^T\Delta\mathcal{A}_ie(k-1-N)\\ &-\epsilon_i\tilde{\omega}^T(k)\Delta\mathcal{B}_i^T\Delta\mathcal{B}_i\tilde{\omega}(k)+\tilde{\omega}^T(k)\Upsilon\tilde{\omega}(k)\\ &-\tilde{\omega}^T(k)\Upsilon\tilde{\omega}(k)\big|\aleph(k)=i\big\}\\ \le&\,\mathbb{E}\big\{\epsilon_i\tilde{\omega}^T(k)\Gamma_{B,i}\tilde{\omega}(k)+\varpi^T(k)\Omega_i\varpi(k)\\ &+\theta_ie^T(k-N-1)\Gamma_{A,i}e(k-1-N)\\ &+\rho\mathscr{M}(k)-\theta_ie^T(k-1-N)\Delta\mathcal{A}_i^T\Delta\mathcal{A}_ie(k-N-1)\\ &+\tilde{\omega}^T(k)\Upsilon\tilde{\omega}(k)-\epsilon_i\tilde{\omega}^T(k)\Delta\mathcal{B}_i^T\Delta\mathcal{B}_i\tilde{\omega}(k)\\ &-\tilde{\omega}^T(k)\Upsilon\tilde{\omega}(k)-\rho\mathscr{M}(k)\big|\aleph(k)=i\big\}\\ \le&\,\mathbb{E}\big\{\varpi^T(k)\bar{\Omega}_i\varpi(k)\big|\aleph(k)=i\big\}-\rho\mathbb{E}\{\mathscr{M}(k)|\aleph(k)=i\}\\ &+\tilde{\omega}^T(k)\Upsilon\tilde{\omega}(k)\\ <&-\rho\mathbb{E}\{\mathscr{M}(k)|\aleph(k)=i\}+\varepsilon\end{aligned} \tag{35}$$

where $\varepsilon=\sum_{i=N+2}^{2N+2}r_i\nu_{\max}^2+\sum_{i=1}^{N+1}r_i\omega_{\max}^2$. Then, for any positive constant $\varsigma>0$, it can be derived that

$$\begin{aligned}&\mathbb{E}\big\{\varsigma^{1+k}\mathscr{M}(1+k)\big|\aleph(k)=i\big\}-\mathbb{E}\big\{\varsigma^k\mathscr{M}(k)\big|\aleph(k)=i\big\}\\ =&\,\varsigma^{1+k}\big(\mathbb{E}\{\mathscr{M}(1+k)|\aleph(k)=i\}-\mathbb{E}\{\mathscr{M}(k)|\aleph(k)=i\}\big)\\ &+\varsigma^k(\varsigma-1)\mathbb{E}\{\mathscr{M}(k)|\aleph(k)=i\}\\ <&\,\varsigma^k\big(\varsigma-\rho\varsigma-1\big)\mathbb{E}\{\mathscr{M}(k)|\aleph(k)=i\}+\varsigma^{k+1}\varepsilon\end{aligned} \tag{36}$$

Letting $\varsigma=\varsigma_*=\frac{1}{1-\rho}$ and summing up both sides of (36) from 0 to $t-1$ with respect to k, we have

$$-\mathbb{E}\big\{\mathscr{M}(0)\big\}+\mathbb{E}\big\{\varsigma_*^t\mathscr{M}(t)\big|r(t-1)=i\big\}<\frac{\varsigma_*(1-\varsigma_*^t)}{1-\varsigma_*}\varepsilon \tag{37}$$

which implies that

$$\begin{aligned}&\mathbb{E}\{\mathscr{M}(k)|\aleph(k)=i\}\\ <&\,\varsigma_*^{-k}\Big(-\frac{\varsigma_*}{\varsigma_*-1}\varepsilon+\mathbb{E}\big\{\mathscr{M}(0)\big\}\Big)-\frac{\varsigma_*}{1-\varsigma_*}\varepsilon\\ =&\,(1-\rho)^k\Big(-\frac{\varepsilon}{\rho}+\mathbb{E}\big\{\mathscr{M}(0)\big\}\Big)+\frac{\varepsilon}{\rho}.\end{aligned} \tag{38}$$

Furthermore, it is easy to see that

$$\mathbb{E}\{\|e(k-1-N)\|^2\}\le\frac{\mathbb{E}\{\mathscr{M}(k)|\aleph(k)=i\}}{\min_{1\le i\le M}\{\underline{\sigma}(P_i)\}}.$$

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Hence, it is finally concluded from Definition 1 that the EE dynamics (28) is EUB in mean square with the AUB $\frac{\varepsilon}{\rho \min_{1\leq i\leq M}\{\underline{\sigma}(P_i)\}}$. The proof is complete now. ■

The inequality (30) is a typical linear matrix inequality (LMI). The feasibility of LMIs is a P (i.e. polynomial) problem, which could be easily confirmed by certain well-known algorithms such as interior-point methods. In this paper, we shall solve the LMIs by using the LMI toolbox in Matlab.

Remark 4: In the above theorem, the BA issue of the EE for the nonlinear system (1) with RA protocol scheduling based on the approximate MH estimation scheme (20) is considered. It is worth mentioning that, the results obtained in Theorems 1 and 2 contain all the information reflecting the system complexities (e.g. the sector-bounded nonlinearity, RA protocol scheduling constraints, bounds of noises and the length of the moving estimation window). The scheduling behavior of the RA protocol does have an impact on the calculation of the estimates and the boundedness analysis of the estimation error. More specifically, we have defined the QCF (4) according to the protocol scheduling behavior by using the scheduling matrix $\digamma(\varrho(i))$. The state estimate is derived by applying Theorem 1 based on the scheduling matrix $\bar{\digamma}^k_{k-N}(\bar{\varrho}_k)$. Then, the boundedness of the estimation error is analyzed based on the occurrence probabilities about the scheduling behavior. Compared with the existing research works, this work possesses the following 3 distinguishing features: 1) this work is one of the first attempts to address the MH estimation problem for a class of NNSs under certain CP scheduling; 2) a novel robust MH estimation strategy is employed to deal with the nonlinearity of the systems and the RA protocol scheduling behaviors; and 3) some sufficient conditions are obtained for handling the BA issue of the EE in mean square under the approximate MH estimation approach.

C. The linear case

In this subsection, we are going to cope with the MH estimation issue for linear systems with RA protocol scheduling, which means that the TV nonlinear function $f(\cdot,\cdot)$ is specialized to the following form:

$$f(k, x(k)) = \bar{F}x(k).$$

In this case, the corresponding MH estimator is designed by the following proposition.

Proposition 3: The solution to the MH estimation problem for the following linear system

$$\begin{cases} x(k+1) = \bar{F}x(k) + \omega(k) \\ y(k) = Cx(k) + \nu(k) \end{cases} \tag{39}$$

with the RA protocol scheduling associating with (4) is given by

$$\begin{cases} \bar{x}(k-N) = \bar{F}\hat{x}(k-N-1|k-1) \\ \hat{x}(k-N|k) = \Big(\big(\bar{\digamma}^k_{k-N}\mathcal{F}^k_{k-N}\big)^T\bar{\digamma}^k_{k-N}\mathcal{F}^k_{k-N} \\ \qquad + Q\Big)^{-1}\big(\bar{\digamma}^k_{k-N}\mathcal{F}^k_{k-N}\big)^T\tilde{s}^k_{k-N} + \bar{x}(k-N) \end{cases} \tag{40}$$

where

$$\tilde{s}^k_{k-N} = \begin{bmatrix}\hat{s}^T(k-N) & \hat{s}^T(k-N+1) & \cdots & \hat{s}^T(k)\end{bmatrix}^T,$$
$$\hat{s}(i) = \bar{y}(i) - \digamma(\varrho(i))C\bar{F}^{i-k+N}\bar{x}(k-N).$$

Proof: Consider the linear system (39), the corresponding QCF is given by

$$\begin{aligned} \mathscr{I}_k(\hat{x}(k-N|k)) &= \|\hat{x}(k-N|k) - \bar{x}(k-N)\|^2_Q + \Big\|\tilde{s}^k_{k-N} \\ &+ \bar{\digamma}^k_{k-N}(\bar{\varrho}_k)\mathcal{F}^k_{k-N}\big(\bar{x}(k-N) - \hat{x}(k-N|k)\big)\Big\|^2 \end{aligned} \tag{41}$$

For the minimization of the QCF (41), we have

$$\begin{aligned} &\frac{\partial\mathscr{I}_k(\hat{x}(k-N|k))}{\partial\hat{x}(k-N|k)} = 2Q\big(\hat{x}(k-N|k) - \bar{x}(k-N)\big) \\ &- 2\big(\bar{\digamma}^k_{k-N}(\bar{\varrho}_k)\mathcal{F}^k_{k-N}\big)^T\big(\tilde{s}^k_{k-N} + \bar{\digamma}^k_{k-N}(\bar{\varrho}_k)\mathcal{F}^k_{k-N} \\ &\times\big(\bar{x}(k-N) - \hat{x}(k-N|k)\big)\big) = 0 \end{aligned} \tag{42}$$

which is equivalent to

$$\begin{aligned} &\Big(\big(\bar{\digamma}^k_{k-N}\mathcal{F}^k_{k-N}\big)^T\bar{\digamma}^k_{k-N}\mathcal{F}^k_{k-N} + Q\Big)\big(-\bar{x}(k-N) \\ &+ \hat{x}(k-N|k)\big) = \big(\bar{\digamma}^k_{k-N}(\bar{\varrho}_k)\mathcal{F}^k_{k-N}\big)^T\tilde{s}^k_{k-N}. \end{aligned} \tag{43}$$

Then, it is easy to see that the solution to (43) is (40). The proof is completed. ■

Based on the estimator (40), we have the following EE system:

$$e(k-N) = \bar{\mathcal{A}}_{\aleph(k)}e(k-N-1) + \bar{\mathcal{B}}_{\aleph(k)}\tilde{\omega}(k) \tag{44}$$

where

$$\begin{aligned} \bar{\mathcal{A}}_{\aleph(k)} &= \bar{F} - \bar{\Psi}_{\aleph(k)}\mathcal{F}^k_{k-N}\bar{F}, \\ \bar{\mathcal{B}}_{\aleph(k)} &= \begin{bmatrix}\mathscr{I} - \bar{\Psi}_{\aleph(k)}\mathcal{G}_k & -\bar{\Psi}_{\aleph(k)}\end{bmatrix}, \\ \bar{\Psi}_{\aleph(k)} &= \Big(Q + \big(\mathcal{F}^k_{k-N}\big)^T\bar{\digamma}^k_{k-N}\mathcal{F}^k_{k-N}\Big)^{-1}\big(\mathcal{F}^k_{k-N}\big)^T\bar{\digamma}^k_{k-N}. \end{aligned}$$

Theorem 3: For the linear NS (39) with the RA protocol scheduling governed by the DTMC $\varrho(k)$ associating with (4) and the updating rule (6), it is supposed that there exist $2M^N + 1$ positive definite matrices $\hat{P}_i$, $\tilde{P}_i$ ($i = 1, 2\cdots, M^N$) and $\hat{\Upsilon} = \mathrm{diag}\{\hat{r}_1 I, \hat{r}_2 I, \cdots, \hat{r}_{2N+2}I\}$ satisfying

$$\hat{\Omega}_i = \begin{bmatrix}\bar{\mathcal{A}}_i^T\check{P}_i\bar{\mathcal{A}}_i - \hat{P}_i + \tilde{P}_i & \bar{\mathcal{A}}_i^T\check{P}_i\bar{\mathcal{B}}_i \\ * & \bar{\mathcal{B}}_i^T\check{P}_i\bar{\mathcal{B}}_i - \hat{\Upsilon}\end{bmatrix} < 0 \tag{45}$$

where

$$\check{P}_i = \sum_{j=1}^{M^N}\check{p}_{ij}\hat{P}_j.$$

Then, the EE dynamics $\|e(k)\|$ is EUB in mean square with the AUB $\frac{\varepsilon}{\rho\min_{1\leq i\leq M}\{\underline{\sigma}(P_i)\}}$ where $\rho = \frac{\underline{\sigma}(\tilde{P}_i)}{\bar{\sigma}(\hat{P}_i)}$ and $\varepsilon = \big(\sum_{i=1}^{N+1}\hat{r}_i\omega^2_{\max} + \sum_{i=N+2}^{2N+2}\hat{r}_i\nu^2_{\max}\big)$.

Proof: The proof is similar to that of Theorem 2 and is therefore omitted for the conciseness. ■

Remark 5: In Proposition 3 and Theorem 3, the MH estimator has been proposed for the linear system with the RA protocol scheduling, and sufficient conditions have been obtained to ensure the ultimately boundedness of the EE in mean square. Obviously, the MH estimator (40) is a special

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case of (17). By setting $\mathcal{Q}(\lambda^*) = Q$ and $\mathcal{R}(\lambda^*) = I$ in (17), the MH estimator (17) for nonlinear systems could be degraded to the MH estimator (40) for linear systems.

Remark 6: It is easy to see that the MH estimation scheme obtained in this work could be easily extended to the MH estimation issue for NSs under the *RR protocol scheduling* effects. More specifically, with the MH estimator given by Theorem 1 (or (40)), the results in Theorem 2 (or Theorem 3) still hold true by setting $\bar{P}_i = P_{i-1}$ (or $\check{P}_i = \hat{P}_{i-1}$) for $i = 2, 3, \cdots, M^N$ and $\bar{P}_1 = P_{M^N}$ (or $\check{P}_1 = \hat{P}_{M^N}$).

Remark 7: The results obtained in Theorems 2 and 3 are achieved based on linear matrix inequalities (LMIs), and the corresponding algorithm dealing with LMIs has a polynomial time complexity. Specifically, the number $\mathcal{N}(\varepsilon)$ of flops needed to compute an ε-accurate solution is bounded by $O(\mathcal{M}\mathcal{N}^3 \log(\mathcal{V}/\varepsilon))$, where $\mathcal{M}$ is the total row size of the LMI system, $\mathcal{N}$ is the total number of scalar decision variables, $\mathcal{V}$ is a data-dependent scaling factor, and ε is relative accuracy set for algorithm. As such, the computational complexities of the established results in Theorems 2 and 3 could be represented as $O((3n_x + (N+1)(n_x+n_y))(M^N(n_x^2+n_x)+2N+2)^3 \log(\mathcal{V}/\varepsilon))$ and $O((n_x + (N+1)(n_x+n_y))(M^N(n_x^2 + n_x)+2N+2)^3 \log(\mathcal{V}/\varepsilon))$, respectively. Obviously, such two computational complexities depend not only on the variable dimensions, but also on the number of sensor nodes. This is mainly due to the fact that the number of LMIs is determined by the number of sensor nodes. On the other hand, it is worth noting that the computation complexity of the moving horizon estimation algorithm is independent of the number of sensor nodes. It can be find from Theorem 1 that such a computation complexity depends largely on the window length of the moving horizon estimation algorithm since the estimate is determined by the measurement output data $\{\bar{y}(i)\}_{k\geq i\geq k-N}$, whose dimension depends on the window length.

IV. TWO ILLUSTRATIVE EXAMPLES

In what follows, we would like to provide two numerical examples to verify the effectiveness and correctness of the developed MH estimation scheme and approximate MH estimation scheme.

Example 1: To make our simulation nontrivial, consider an *unstable* system of the following form:

$$\begin{cases} x(k+1) = 0.5\big(\sin(k)(\gimel_1 - \gimel_2)x(k) + (\gimel_1 \\ \qquad\qquad + \gimel_2)x(k)\big) + \omega(k) \\ \qquad y(k) = Cx(k) + \nu(k) \end{cases}$$

in which

$$\gimel_1 = \begin{bmatrix} 0.86 & 0.1 & 0 \\ 0.1 & 0.98 & 0 \\ 0 & 0 & 1.04 \end{bmatrix}, \ \gimel_2 = \begin{bmatrix} 0.82 & 0.1 & 0 \\ 0.1 & 0.92 & 0 \\ 0 & 0 & 0.98 \end{bmatrix},$$
$$C = I, \quad \omega(k) = 0.6\cos(0.4k)\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T,$$
$$\nu(k) = 0.4\sin(0.3k)\begin{bmatrix} 1 & 1 \end{bmatrix}^T.$$

In this example, assume that there are three sensor nodes of the system and the TP matrix of the RA protocol is

$$\mathcal{P} = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}.$$

The weight matrix Q is set to be $Q = I$. We choose the window length as $N + 1 = 6$. Then, by applying Theorem 1, we can obtain the corresponding moving-horizon estimator of the form (17). For the purpose of dealing with the one-dimensional OP (18), we adopt a Particle Swarm optimization (PSO) algorithm to search the best solution for λ^* at each step. On the other hand, by choosing a reasonable approximation of λ^* as $\lambda^* = 1.5$, we can obtain an approximate moving-horizon estimator of the form (20).

Set the state initial value be $x(0) = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}^T$. Based on the above obtained estimators, the tracking performance is shown in Figs. 2-4. Fig. 5 depicts the response of the EE (i.e. $\|e(k)\|^2$). Fig. 6 plots the sensor node obtaining access to the communication network. The simulation result has verified that both the MH estimation approach and the approximate MH estimation method are indeed effective for the addressed NNSs under the RA protocol scheduling effects.

From Fig. 5, we can find that the MH estimation approach

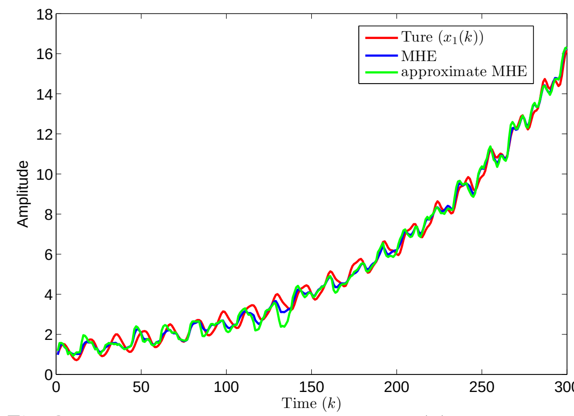

Fig. 2: Example 1 — States trajectories of $x_1(k)$ and its estimates

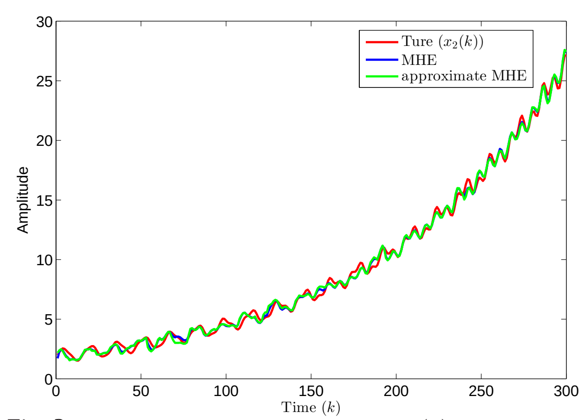

Fig. 3: Example 1 — States trajectories of $x_2(k)$ and its estimates

performs better than the approximate MH estimation approach which is mainly due to the real-time computation of λ^*. On the other hand, such a computation process would largely increase

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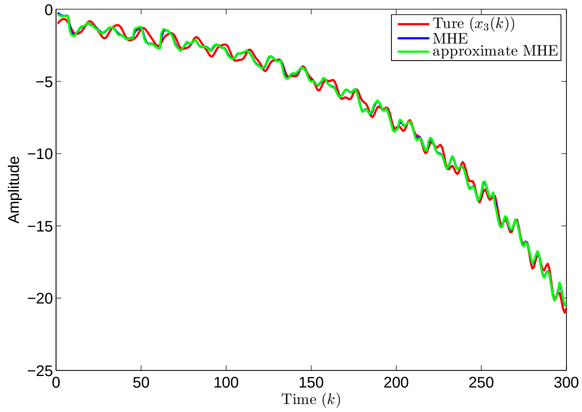

Fig. 4: Example 1 — States trajectories of $x_3(k)$ and its estimates

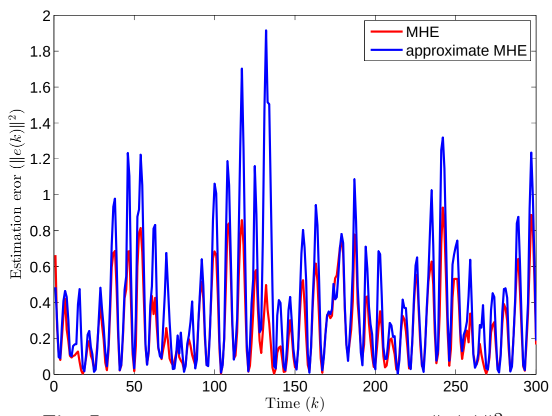

Fig. 5: Example 1 — The estimation error $\|e(k)\|^2$

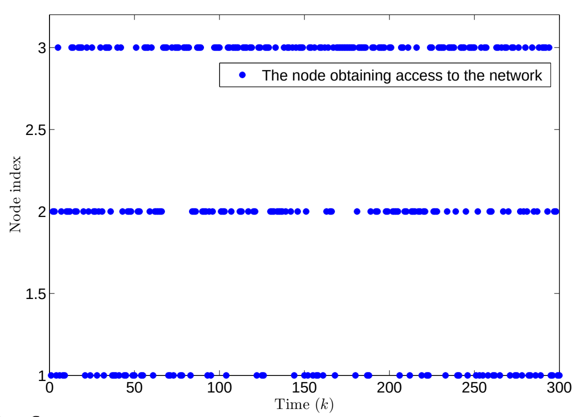

Fig. 6: Example 1 — The sensor node obtaining access to the network

the computational effort. The average CPU time on solving the MH estimation problem and the approximate MH estimation problem at each step is 6.72×10^{-2}s and 4.82×10^{-4}s, respectively, on a standard personal computer (CPU: Intel (R) Core(TM) i7-4720HQ; RAM: 8GB; Operating System: Windows 8.1).

Example 2: Consider the isothermal continuous stirred tank reactor (CSTR) studied in [2], [12]. A discretized and linearized model is obtained as follows:

$$\begin{cases} x(k+1) = \begin{bmatrix} 0.6472 & 0 \\ 0.2135 & 0.7202 \end{bmatrix} x(k) + \omega(k) \\ y(k) = x(k) + \nu(k) \end{cases}$$

where $\nu(k) \in \mathbb{R}^2$ and $\omega(k) \in \mathbb{R}^2$ are bounded noise with the upper bounds $\omega_{\max} = 0.2$ and $\nu_{\max} = 0.1$, respectively. We suppose that the two sensors belong to different sensor nodes and the corresponding TP matrix of the RA protocol is $\mathcal{P} = \begin{bmatrix} 0.1 & 0.9 \\ 0.9 & 0.1 \end{bmatrix}$. Choosing the weight matrix $Q = I$ and the window length $N+1 = 2$, we can obtain the approximate moving-horizon estimator of the form (40). Simulation results are presented in Figs. 6-8.

Next, let us consider the AUB of the EE. According to Proposition 1, we can easily verify that $\aleph(k) \in \mho \triangleq \{1, 2, 3, 4\}$. Then, by applying Theorem 3, the positive definite matrices can be obtained as follows:

$$\hat{P}_1 = \begin{bmatrix} 3.2059 & 1.3849 \\ 1.3849 & 4.5197 \end{bmatrix}, \ \hat{P}_2 = \begin{bmatrix} 1.7855 & -0.0476 \\ -0.0476 & 1.0032 \end{bmatrix},$$
$$\hat{P}_3 = \begin{bmatrix} 1.0595 & 0.0473 \\ 0.0473 & 1.9109 \end{bmatrix}, \ \hat{P}_4 = \begin{bmatrix} 3.4585 & -1.0390 \\ -1.0390 & 2.2146 \end{bmatrix}.$$

Furthermore, based on the derived results in Theorem 3, we obtain the AUB of the $\|e(k)\|$: 0.31317. The simulation results have confirmed the MH estimation performance and our theoretical analysis on the AUB of the EE.

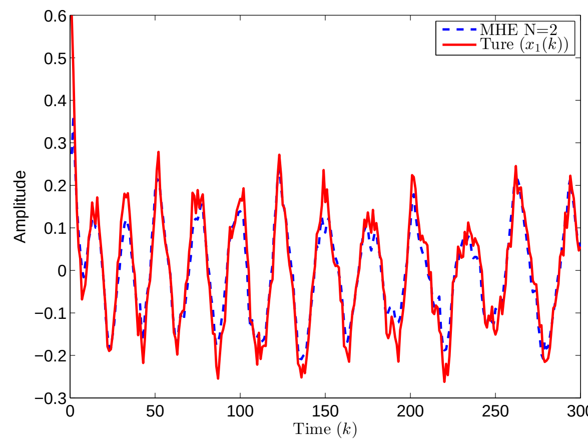

Fig. 7: Example 2 — States trajectories of $x_1(k)$ and its estimates

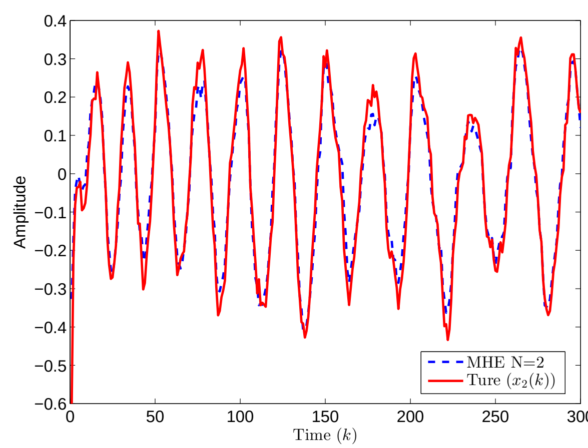

Fig. 8: Example 2 — States trajectories of $x_2(k)$ and its estimates

V. CONCLUSION

In this work, a nonlinear moving-horizon (MH) estimator has been constructed for a type of NNSs under the so-called

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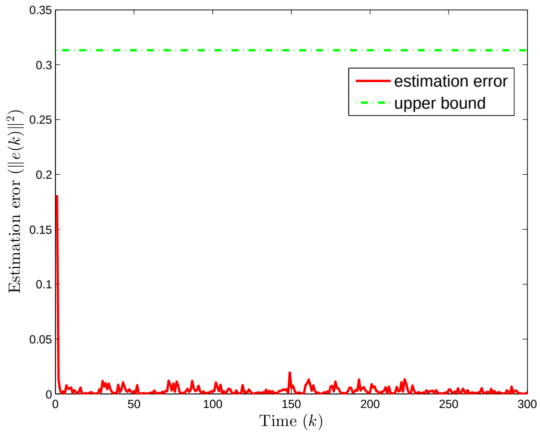

Fig. 9: Example 2 — The estimation error $\|e(k)\|^2$ and the upper bound

random access (RA) protocol scheduling effects. A DTMC with known TP matrix has been introduced to model the scheduling behaviors of the RA protocol. The corresponding MH estimator and approximate MH estimator have been developed to provide the state estimates by extending the robust MH estimation scheme. By using the stochastic analysis technology combined with the mapping approach, some sufficient conditions have been obtained to handle the BA issue of the EE dynamics in mean square under the approximate MH estimation scheme. Moreover, the main results have been further specialized to linear NSs with the RA protocol scheduling. Finally, two illustrative examples have been provided to verify the correctness and effectiveness of our derived results.

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