Event-Triggered State Estimation for Markovian Jumping Neural Networks: On Mode-Dependent Delays and Uncertain Transition Probabilities

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***Abstract*—This paper is concerned with the event-triggered state estimation (ETSE) problem for a class of discrete-time Markovian jumping neural networks with mode-dependent time-delays and uncertain transition probabilities. The parameters of the neural networks experience switches that are characterized by a Markovian chain whose transition probabilities are allowed to be uncertain. The event-triggered mechanism is introduced in the sensor-to-estimator channel to reduce the frequency of signal communication. The aim of this paper is to develop an ETSE scheme such that the estimation error dynamics is exponentially ultimately bounded in the mean square. To achieve the aim, two sufficient conditions are proposed with the first one guaranteeing the existence of the required state estimator, and the second one giving the algorithm for designing the corresponding estimator gain by solving some matrix inequalities. In the end, the validity of the proposed estimation scheme is illustrated by a numerical example.**

***Index Terms*—Artificial neural networks, Markovian jumping parameters, uncertain transition probabilities, event-triggered mechanism, mode-dependent time-delays.**

I. Introduction

Enlightened by the biological neural network, the artificial neural network (ANN) has attracted much attention since it was first proposed in the 1940s [17], [41], [43], [44]. With the continuous development of both the theories and algorithms of ANNs, the application scope of ANNs have been broadened to include a number of subject areas such as crack detection [4], [38] and image processing [1], [11]. In some specific applications, the state information of certain primary neurons plays an important role in some tasks such as optimization and approximation, and such state information is thus required to be known *a priori*. Unfortunately, due to the large size of ANNs and the limited resources available, it is nearly impossible to obtain the full/exact state information. Therefore, an alternative way is to estimate the neuron states by using available (usually partial) network measurements, and such a neuron state estimation problem has received a great deal of research attention, see e.g. [3], [12], [22], [24], [29], [30], [32].

Time-delays serve as an inevitable phenomenon in the ANNs due to parallel signal transmissions among the neurons which may cause oscillation or even divergence of the ANNs. Up to now, numerous research results have been devoted to the problem of state estimation for ANNs with time delays [13], [15], [18], [33], [45]. For example, for static neural networks with time-varying delays, a delay-dependent sufficient condition has been proposed to meet the desired state estimation performance requirement [45]. In [15], the event-triggered H_∞ state estimation problem has been studied for ANNs with mixed time-delays. On the other hand, in real practice, the ANNs may switch among different modes and the switching can be characterized by a Markovian chain. Recently, many researchers have begun to focus on the problem of state estimation for ANNs with Markov jump parameters [2], [35], [42]. For example, for Markovian jumping networks with time-varying delays, a state estimator has been designed in [42]. For fuzzy neural networks with Markovian jumping parameters, the mixed H_∞ and passive filtering problem has been addressed in [35], where a filtering algorithm has been proposed to ensure that the fuzzy neural network is mean-square stable while satisfying a predefined passivity constraint.

In most of the available results concerning the state estimation problem for Markovian jumping ANNs (MJANNs), an implicit assumption that has been made is that the transition probabilities of the Markovian chain are precisely known. Such an assumption is, unfortunately, unrealistic in practical engineering since the transition probabilities may not be exactly identifiable due to the resource/environment constraints. Accordingly, it would be of practical significance to take into account the uncertainties of the transition probabilities in the state estimation problem for MJANNs. To the best of our knowledge, the state estimation problem for MJANNs with uncertain transition probabilities has not been thoroughly investigated yet, and our intension in this paper is therefore to shorten such a gap.

Recently, the event-triggered mechanism (ETM) has become a popular research topic due to its advantages (over the conventional time-triggered mechanisms) [5]–[7], [16], [28], [34], [37], [40] in reducing data transmission rate subject to limited

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resources. Under the ETM, the messages are transmitted only when the predetermined trigger condition is satisfied, and hence the undesired data collisions are largely avoided. By now, the state estimation problems for various systems under ETMs have received an ever-increasing research interest [10], [20], [39]. For example, the H_∞ state estimation problem with event-triggered condition has been investigated for genetic regulatory networks in [14]. For time-delayed complex networks with partially accessible nodes, the state estimation problem has been investigated in [9]. The distributed event-triggered H_∞ filtering problem has been dealt with in [8] over sensor networks. Unfortunately, when it comes to the MJANNs, the ETM-based state estimation issue has received very little research attention despite its theoretical importance and the practical significance, not to mention the case that the transition probabilities are uncertain. As such, we are motivated to further study the event-triggered state estimation (ETSE) problem for MJANNs with uncertain transition probabilities.

In summary, this paper investigates the ETSE problem for MJANNs with uncertain transition probabilities and mode-dependent time-delays. Our aim is to develop a state estimator to guarantee that the estimation error dynamics is exponentially ultimately bounded (EUB) in the mean square. *The major contributions are highlighted as follows: 1) for the first time, the ETSE problem is studied for MJANNs with uncertain transition probabilities; 2) important factors that complicate the ANNs (e.g. Markovian jumping parameters, mode-dependent time-delays, uncertain transition probabilities and ETMs) are simultaneously considered in a unified framework; and 3) a sufficient condition is derived that guarantees the exponentially ultimate boundedness in the mean square for the estimation error dynamics.*

This paper is structured as follows. In Section II, the M-JANNs with uncertain transition probabilities is presented, the ETM is introduced, and the ETM-based estimator is proposed. The main results (including Theorem 1 and Theorem 2) are presented in Section III. A simulation study is conducted in Section IV to show the effectiveness of our estimator design algorithm. Finally, the conclusion is drawn in Section V.

Notation M^T denotes the transpose of the matrix M. $|e|$ represents the Euclidean norm of a vector e. For a symmetric matrix A, $\lambda_m(A)$ ($\lambda_M(A)$) refers to the minimum (maximum) eigenvalue of A. $\mathrm{diag}_n\{z_i\}$ denotes $\mathrm{diag}\{z_1, z_2, \cdots, z_n\}$ and $\mathrm{vec}_n\{x_i\}$ stands for $[x_1^T, x_2^T, \cdots, x_n^T]^T$. $\mathbb{E}\{x\}$ means the expectation of x. $\mathbb{P}\{a\}$ indicates the occurrence probability of the event "a".

II. Problem Formulation and Preliminaries

Let $\varsigma(h) \in S \triangleq \{1, 2, \ldots, N\}$ $(h \geq 0)$ be a Markov chain whose transition probability matrix is given by $\Pi = [\tilde{\pi}_{ij}]_{N\times N}$. Moreover, the transition probability of the Markov chain $\varsigma(h)$ is

$$\mathbb{P}\{\varsigma(h+1) = j | \varsigma(h) = i\} = \tilde{\pi}_{ij}, \ \forall\ i, j \in S$$

where $\tilde{\pi}_{ij} \triangleq \pi_{ij} + \Delta\pi_{ij}$ represents the uncertain transition probability from mode i to mode j satisfying $\tilde{\pi}_{ij} \geq 0$ and $\sum_{j=1}^N \tilde{\pi}_{ij} = 1$. π_{ij} is a known constant and $\Delta\pi_{ij}$ is the uncertainty satisfying

$$|\Delta\pi_{ij}| \leq \kappa_{ij}, \ \kappa_{ij} \geq 0.$$

Consider the following discrete-time n-neuron MJANNs with mode-dependent delays:

$$\begin{cases} x_i(h+1) = a_i(\varsigma(h))x_i(h) + \sum_{j=1}^n b_{ij}^0(\varsigma(h))f_j(x_j(h)) \\ \qquad\qquad + \sum_{j=1}^n b_{ij}^1(\varsigma(h))g_j(x_j(h-\tau_j(h,\varsigma(h)))) \\ \qquad\qquad + d_i(\varsigma(h))\omega_i(h), \ i = 1, 2, \ldots, n \\ x_i(s) = \phi_i(s), \quad s \in [-\tau_M, 0] \end{cases} \tag{1}$$

where $x_i(h) \in \mathbb{R}$ is the state of the ith neuron and $a_i(\varsigma(h))$ is the state feedback coefficient. $f_j(\cdot)$ and $g_j(\cdot)$ are the activation functions of the jth neuron. $b_{ij}^0(\varsigma(h))$ and $b_{ij}^1(\varsigma(h))$ represent the connection weight and the delayed connection weight of the jth neuron on the ith neuron, respectively. $d_i(\varsigma(h))$ is a known constant and $\omega_i(h)$ is a zero mean Gaussian white noise process with $\mathbb{E}\{\omega_i^2(h)\} = 1$. The positive integer $\tau_j(h, \varsigma(h))$ denotes the mode-dependent time-varying delays of the jth neuron satisfying $\tau_m \leq \tau_j(h, \varsigma(h)) \leq \tau_M$, and it is assumed that $\tau_j(0, \varsigma(0)) = \tau_M$. $\phi_i(s)$ is the given initial condition.

The MJANNs (1) can be rewritten in a compact form as follows:

$$\begin{cases} x(h+1) = & A(\varsigma(h))x(h) + B(\varsigma(h))f(x(h)) \\ & + B_d(\varsigma(h))g(x(h-\tau(h,\varsigma(h)))) \\ & + D(\varsigma(h))\omega(h), \\ x(s) = & \phi(s), \quad s \in [-\tau_M, 0] \end{cases} \tag{2}$$

where

$$\begin{aligned} x(h) &\triangleq \mathrm{vec}_n\{x_i(h)\}, \\ f(x(h)) &\triangleq \mathrm{vec}_n\{f_i(x_i(h))\}, \\ g(x(h)) &\triangleq \mathrm{vec}_n\{g_i(x_i(h-\tau_i(h,\varsigma(h))))\}, \\ A(\varsigma(h)) &\triangleq \mathrm{diag}_n\{a_i(\varsigma(h))\}, \\ B(\varsigma(h)) &\triangleq \left[b_{ij}^0(\varsigma(h))\right]_{n\times n}, \\ B_d(\varsigma(h)) &\triangleq \left[b_{ij}^1(\varsigma(h))\right]_{n\times n}, \\ D(\varsigma(h)) &\triangleq \mathrm{diag}_n\{d_i(\varsigma(h))\}, \\ \omega(h) &\triangleq \mathrm{vec}_n\{\omega_i(h)\}, \\ \phi(s) &\triangleq \mathrm{vec}_n\{\phi_i(s)\}. \end{aligned}$$

The following assumption is first made on the activation functions $f_i(\cdot)$ and $g_i(\cdot)$.

Assumption 1: For $1 \leq i \leq n$, the activation functions $f_i(\cdot)$ and $g_i(\cdot)$ satisfy the following conditions:

$$\alpha_i^- \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq \alpha_i^+, \quad s_1, s_2 \in \mathbb{R},$$
$$\beta_i^- \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq \beta_i^+, \quad s_1, s_2 \in \mathbb{R}$$

where $\alpha_i^-, \alpha_i^+, \beta_i^-$ and β_i^+ are known constants.

Remark 1: It should be noted that Assumption 1 has been discussed in [39] where the constants $\alpha_i^-, \alpha_i^+, \beta_i^-$ and β_i^+ are allowed to be set as zero, positive and negative. Therefore, activation functions under this assumption can be nonmonotonic and is more general than the widely used sigmoid and Lipschitz-type functions.

The measurement of the sensor is modeled as

$$y(h) = C(\varsigma(h))x(h) + L(\varsigma(h))v(h) \tag{3}$$

where $y(h) \in \mathbb{R}^m$ is the measurement output, $C(\varsigma(h))$ and $L(\varsigma(h))$ are known matrices, and $v(h) \in \mathbb{R}^p$ is the bounded disturbance satisfying $|v(h)|^2 \le \bar{v}$.

In this paper, we consider the scenario that the transmission of measurements from sensors to state estimator is implemented over a network with limited bandwidth. To economize on the communication resources, an ETM is enforced in the sensor-to-estimator channel as defined by

$$\rho^T(h)\rho(h) - \theta > 0$$

where $\rho(h) \triangleq y(h_i) - y(h)$, $\theta > 0$ is the threshold, and $y(h_i)$ is the measured output at the most recent triggering instant.

Based on the event generator function, the sequence of event-triggering instants is determined by

$$h_{i+1} \triangleq \min\left\{h \in \mathbb{N} | h > h_i,\ \rho^T(h)\rho(h) - \theta > 0\right\}. \tag{4}$$

To estimate the state of the ANNs (2), in this paper, the state estimator is constructed as follows:

$$\begin{cases} \hat{x}(h+1) = & A(\varsigma(h))\hat{x}(h) + B(\varsigma(h))f(\hat{x}(h)) \\ & + B_d(\varsigma(h))g(\hat{x}(h-\tau(h,\varsigma(h)))) \\ & + K(\varsigma(h))\left[y(h_i) - C(\varsigma(h))\hat{x}(h)\right] \\ \hat{x}(s) = & 0, \quad s \in [-\tau_M, 0] \end{cases} \tag{5}$$

where $\hat{x}(h)$ is the estimate of $x(h)$ and $K(\varsigma(h))$ is the matrix gain of the estimator to be designed.

Denoting $e(h) \triangleq x(h) - \hat{x}(h)$, the estimation error dynamics is obtained from (2) and (5) as follows:

$$\begin{cases} e(h+1) = & A(\varsigma(h))e(h) + B(\varsigma(h))\tilde{f}(e(h)) \\ & + B_d(\varsigma(h))\tilde{g}(e(h-\tau(h,\varsigma(h)))) \\ & - K(\varsigma(h))\rho(h) - K(\varsigma(h))L(\varsigma(h))v(h) \\ & - K(\varsigma(h))C(\varsigma(h))e(h) \\ & + D(\varsigma(h))\omega(h) \\ e(s) = & \phi(s), \quad s \in [-\tau_M, 0] \end{cases} \tag{6}$$

where

$$\begin{aligned} \tilde{f}(e(h)) \triangleq & f(x(h)) - f(\hat{x}(h)), \\ \tilde{g}(e(h-\tau(h,\varsigma(h)))) \triangleq & g(x(h-\tau(h,\varsigma(h)))) \\ & - g(\hat{x}(h-\tau(h,\varsigma(h)))). \end{aligned}$$

Definition 1: [31] The estimation error system (6) is said to be EUB in the mean square if there exist constants $0 < \mu < 1$, $\vartheta > 0$ and $\bar{\psi} > 0$ such that

$$\mathbb{E}\left\{|e(h)|^2\right\} \le \mu^h \vartheta + \psi(h) \quad \text{and} \quad \lim_{\mathrm{h} \to +\infty} \psi(\mathrm{h}) = \bar{\psi}. \tag{7}$$

Based on the discussion above, the main objective of this paper is to develop a set of event-triggered-based estimators in the form of (5) such that the estimation error system (6) is EUB in the mean square.

III. Main Results

In this section, we discuss the ETSE problem for MJANNs with mode-dependent delays and uncertain transition probabilities. First, we present sufficient conditions under which the estimation error system (6) is EUB in the mean square, and then we will characterize the desired estimator gain.

In the following, for presentation convenience, we denote the matrix $M(\varsigma(t))$ as M_i, the scalar $m(\varsigma(t))$ as m_i or m^i, and the function $\varrho(\cdot, \varsigma(t))$ as $\varrho(\cdot, i)$ for $\varsigma(t) = i$ $(i \in S)$.

Theorem 1: Let the estimator gains K_i $(i \in S)$ be given. The estimation error system (6) is EUB in the mean square if there exist a set of positive definite matrices $P_i > 0$ $(i \in S)$, a positive definite matrix $Q > 0$, three sets of diagonal matrices $\Lambda_i \triangleq \mathrm{diag}\{\lambda_1^i, \lambda_2^i, \cdots, \lambda_n^i\} > 0$, $\Gamma_i \triangleq \mathrm{diag}\{\gamma_1^i, \gamma_2^i, \cdots, \gamma_n^i\} > 0$, $\Sigma_i \triangleq \mathrm{diag}\{\sigma_1^i, \sigma_2^i, \cdots, \sigma_n^i\} > 0$ and positive scalars ε_1, ε_2 such that

$$\Phi_i \triangleq \begin{bmatrix} \Pi_{11}^i & 0 & \Pi_{13}^i & \Pi_{14}^i & 0 & 0 & 0 & \Pi_{18}^i \\ * & \Pi_{22}^i & 0 & 0 & \Pi_{25}^i & 0 & 0 & 0 \\ * & * & \Pi_{33}^i & 0 & 0 & 0 & 0 & \Pi_{38}^i \\ * & * & * & \Pi_{44}^i & 0 & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55}^i & 0 & 0 & \Pi_{58}^i \\ * & * & * & * & * & \Pi_{66}^i & 0 & \Pi_{68}^i \\ * & * & * & * & * & * & \Pi_{77}^i & \Pi_{78}^i \\ * & * & * & * & * & * & * & \Pi_{88}^i \end{bmatrix} < 0 \tag{8}$$

where

$$\begin{aligned} \Pi_{11}^i \triangleq & -P_i - \Lambda_i\tilde{\alpha}_1 - \Gamma_i\tilde{\beta}_1, \\ \Pi_{13}^i \triangleq & -\Lambda_i\tilde{\alpha}_2, \\ \Pi_{14}^i \triangleq & -\Gamma_i\tilde{\beta}_2, \\ \Pi_{18}^i \triangleq & A_i^T\bar{P}_i - C_i^T K_i^T \bar{P}_i, \\ \Pi_{22}^i \triangleq & -\Sigma_i\hat{\beta}_1, \\ \Pi_{25}^i \triangleq & -\Sigma_i\hat{\beta}_2, \\ \Pi_{33}^i \triangleq & -\Lambda_i, \\ \Pi_{38}^i \triangleq & B_i^T\bar{P}_i, \\ \Pi_{44}^i \triangleq & \epsilon Q - \Gamma_i, \\ \Pi_{55}^i \triangleq & -(\bar{\kappa} - \pi_{ii})Q - \Sigma_i, \\ \Pi_{58}^i \triangleq & B_{di}^T\bar{P}_i, \\ \Pi_{66}^i \triangleq & -\varepsilon_1 I, \\ \Pi_{68}^i \triangleq & -K_i^T\bar{P}_i, \\ \Pi_{77}^i \triangleq & -\varepsilon_2 I, \\ \Pi_{78}^i \triangleq & -L_i^T K_i^T \bar{P}_i, \\ \Pi_{88}^i \triangleq & -\bar{P}_i, \\ \bar{P}_i \triangleq & \sum_{j=1}^{n}(\pi_{ij} + \kappa_{ij})P_j, \\ \underline{\pi} \triangleq & \min\left\{\pi_{ii} | i \in S\right\}, \\ \epsilon \triangleq & (1 + \bar{\kappa} - \underline{\pi})(\tau_M - \tau_m) + 1, \\ \tilde{\alpha}_1 \triangleq & \frac{1}{2}\left(\alpha_+^T\alpha_- + \alpha_-^T\alpha_+\right), \\ \tilde{\alpha}_2 \triangleq & -\frac{1}{2}\left(\alpha_+^T + \alpha_-^T\right), \end{aligned}$$

$$\begin{aligned}
\alpha_+ \triangleq&\mathrm{diag}_n\left\{\alpha_i^+\right\},\\
\alpha_- \triangleq&\mathrm{diag}_n\left\{\alpha_i^-\right\},\\
\tilde{\beta}_1 \triangleq&\frac{1}{2}\left(\beta_+^T\beta_- + \beta_-^T\beta_+\right),\\
\tilde{\beta}_2 \triangleq&-\frac{1}{2}\left(\beta_+^T + \beta_-^T\right),\\
\beta_+ \triangleq&\mathrm{diag}_n\left\{\beta_i^+\right\},\\
\beta_- \triangleq&\mathrm{diag}_n\left\{\beta_i^-\right\},\\
\hat{\beta}_1 \triangleq&\frac{1}{2}\left(\hat{\beta}_+^T\hat{\beta}_- + \hat{\beta}_-^T\hat{\beta}_+\right),\\
\hat{\beta}_2 \triangleq&-\frac{1}{2}\left(\hat{\beta}_+^T + \hat{\beta}_-^T\right),\\
\hat{\beta}_+ \triangleq&I_n\otimes\beta_+,\\
\hat{\beta}_- \triangleq&I_n\otimes\beta_-,\\
\bar{\kappa} \triangleq&\max\left\{\kappa_{ii}|i\in S\right\}.
\end{aligned}$$

Proof: Construct a Lyapunov-Krasovskii functional as follows:

$$V(e(h),h,i) = V_1(e(h),h,i) + V_2(e(h),h,i) + V_3(e(h))$$

where

$$\begin{aligned}
V_1(e(h),h,i) \triangleq& e^T(h)P_ie(h),\\
V_2(e(h),h,i) \triangleq& \sum_{s=h-\tau(h,i)}^{h-1}\tilde{g}^T(e(s))Q\tilde{g}(e(s)),\\
V_3(e(h)) \triangleq& \sum_{\iota=\tau_m}^{\tau_M-1}\sum_{s=h-\iota}^{h-1}\tilde{g}^T(e(s))\bar{Q}\tilde{g}(e(s))
\end{aligned}$$

with $\bar{Q}\triangleq(1+\bar{\kappa}-\underline{\pi})Q$.

For notation simplification, in the sequel, we denote

$$\begin{aligned}
\xi(h,i) \triangleq& \big[\ e^T(h) \quad e_{\tau hi}^T(h) \quad \tilde{f}^T(e(h)) \quad \tilde{g}^T(e(h))\\
&\quad \tilde{g}_{\tau hi}^T(e(h)) \quad \rho^T(h) \quad v^T(h)\ \big]^T,\\
\digamma_i \triangleq& \left[A_i - K_iC_i \quad 0 \quad B_i \quad 0 \quad B_{di} \quad -K_i \quad -K_iL_i\right]
\end{aligned}$$

where $e_{\tau hi}(h)\triangleq e(h-\tau(h,i))$ and $\tilde{g}_{\tau hi}(e(h))\triangleq\tilde{g}(e(h-\tau(h,i)))$.

Calculating the differences of $V_1(e(h),h,i)$, $V_2(e(h),h,i)$, and $V_3(e(h))$, one has

$$\begin{aligned}
&\mathbb{E}\{V_1(e(h+1),h+1,\varsigma(h+1))|e(h),\varsigma(h)=i\}\\
&\quad - V_1(e(h),h,i))\\
=&\sum_{j=1}^{N}\tilde{\pi}_{ij}\xi^T(h,i)\digamma_i^TP_j\digamma_i\xi(h,i)\\
&+\sum_{j=1}^{N}\tilde{\pi}_{ij}\mathbb{E}\{\omega^T(h)D_i^TP_jD_i\omega(h)\}\\
&-e^T(h)P_ie(h)\\
\leq&\xi^T(h,i)\digamma_i^T\bar{P}_i\digamma_i\xi(h,i) + n\lambda_{\max}\{D_i^T\bar{P}_iD_i\}\\
&-e^T(h)P_ie(h),
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}\{V_2(e(h+1),h+1,\varsigma(h+1))|e(h),\varsigma(h)=i\}\\
&\quad - V_2(e(h),h,i)\\
=&\sum_{j=1}^{N}\tilde{\pi}_{ij}\sum_{s=h-\tau(h,j)+1}^{h}\tilde{g}^T(e(s))Q\tilde{g}(e(s))\\
&-\sum_{s=h-\tau(h,i)}^{h-1}\tilde{g}^T(e(s))Q\tilde{g}(e(s))\\
=&\tilde{\pi}_{ii}\left[\sum_{s=h-\tau(h,i)+1}^{h-1}-\sum_{s=h-\tau(h,i)}^{h-1}\right]\tilde{g}^T(e(s))Q\tilde{g}(e(s))\\
&+\tilde{g}^T(e(h))Q\tilde{g}(e(h)) + \sum_{j\neq i}\tilde{\pi}_{ij}\left[\sum_{s=h-\tau(h,j)+1}^{h-1}\right.\\
&\left.-\sum_{s=h-\tau(h,i)}^{h-1}\right]\tilde{g}^T(e(s))Q\tilde{g}(e(s))\\
\leq&\tilde{g}^T(e(h))Q\tilde{g}(e(h)) - \tilde{g}_{\tau hi}^T(e(h))Q\tilde{g}_{\tau hi}(e(h))\\
&\times(\bar{\kappa}-\pi_{ii}) + \sum_{s=h-\tau_M+1}^{h-\tau_m}\tilde{g}^T(e(s))\bar{Q}\tilde{g}(e(s)),
\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{E}\{V_3(e(h+1)) - V_3(e(h))\}\\
&=\sum_{s=\tau_m}^{\tau_M-1}\tilde{g}^T(e(h))\bar{Q}\tilde{g}(e(h))\\
&\quad-\sum_{s=\tau_m}^{\tau_M-1}\tilde{g}^T(e(h-s))\bar{Q}\tilde{g}(e(h-s))\\
&=(1+\bar{\kappa}-\underline{\pi})(\tau_M-\tau_m)\tilde{g}^T(e(h))Q\tilde{g}(e(h))\\
&\quad-\sum_{s=h-\tau_M+1}^{h-\tau_m}\tilde{g}^T(e(s))\bar{Q}\tilde{g}(e(s)).
\end{aligned}$$

Then, it is derived that

$$\begin{aligned}
&\mathbb{E}\{V(e(h+1),h+1,\varsigma(h+1))|e(h),\varsigma(h)=i\}\\
&\quad - V(e(h),h,i)\\
&\leq\xi^T(h,i)\digamma_i^T\bar{P}_i\digamma_i\xi(h,i) - e^T(h)P_ie(h)\\
&\quad + n\lambda_{\max}\{D_i^T\bar{P}_iD_i\} + \epsilon\tilde{g}^T(e(h))Q\tilde{g}(e(h))\\
&\quad - (\bar{\kappa}-\pi_{ii})\tilde{g}_{\tau hi}^T(e(h))Q\tilde{g}_{\tau hi}(e(h)).
\end{aligned}$$

From Assumption 1, it is easily known that

$$\begin{bmatrix} e(h)\\ \tilde{f}(e(h))\end{bmatrix}^T\begin{bmatrix}\tilde{\alpha}_1 & \tilde{\alpha}_2\\ * & I\end{bmatrix}\begin{bmatrix} e(h)\\ \tilde{f}(e(h))\end{bmatrix}\leq 0 \tag{9}$$

and

$$\begin{bmatrix} e(h)\\ \tilde{g}(e(h))\end{bmatrix}^T\begin{bmatrix}\tilde{\beta}_1 & \tilde{\beta}_2\\ * & I\end{bmatrix}\begin{bmatrix} e(h)\\ \tilde{g}(e(h))\end{bmatrix}\leq 0, \tag{10}$$

which further indicates

$$\begin{bmatrix} e_{\tau hi}(h)\\ \tilde{g}_{\tau hi}(e(h))\end{bmatrix}^T\begin{bmatrix}\hat{\beta}_1 & \hat{\beta}_2\\ * & I\end{bmatrix}\begin{bmatrix} e_{\tau hi}(h)\\ \tilde{g}_{\tau hi}(e(h))\end{bmatrix}\leq 0. \tag{11}$$

Taking into account both the event-triggering condition (4) and the constraint on the bounded disturbance $v(h)$, we obtain

$$\begin{aligned}
\Xi_1 \triangleq& \rho^T(h)\rho(h) - \theta \leq 0,\\
\Xi_2 \triangleq& v^T(h)v(h) - \bar{v} \leq 0.
\end{aligned}$$

Summarizing the above inequalities, we have

$$\begin{aligned}
&\mathbb{E}\{V(e(h+1),h+1,\varsigma(h+1))|e(h),\varsigma(h)=i\}\\
&\quad -V(e(h),h,i)\\
&\le\xi^T(h,i)\digamma_i^T\bar{P}_i\digamma_i\xi(h,i)-e^T(h)P_ie(h)\\
&\quad +\epsilon\tilde{g}^T(e(h))Q\tilde{g}(e(h))+n\lambda_{\max}\{D_i^T\bar{P}_iD_i\}\\
&\quad -(\bar{\kappa}-\pi_{ii})\tilde{g}_{\tau hi}(e(h))Q\tilde{g}_{\tau hi}(e(h))-\varepsilon_1\Xi_1-\varepsilon_2\Xi_2\\
&\quad -\begin{bmatrix} e(h)\\ \tilde{f}(e(h))\end{bmatrix}^T\begin{bmatrix}\Lambda_i\tilde{\alpha}_1 & \Lambda_i\tilde{\alpha}_2\\ * & \Lambda_i\end{bmatrix}\begin{bmatrix} e(h)\\ \tilde{f}(e(h))\end{bmatrix}\\
&\quad -\begin{bmatrix} e(h)\\ \tilde{g}(e(h))\end{bmatrix}^T\begin{bmatrix}\Gamma_i\tilde{\beta}_1 & \Gamma_i\tilde{\beta}_2\\ * & \Gamma_i\end{bmatrix}\begin{bmatrix} e(h)\\ \tilde{g}(e(h))\end{bmatrix}\\
&\quad -\begin{bmatrix} e_{\tau hi}(h)\\ \tilde{g}_{\tau hi}(e(h))\end{bmatrix}^T\begin{bmatrix}\Sigma_i\hat{\beta}_1 & \Sigma_i\hat{\beta}_2\\ * & \Sigma_i\end{bmatrix}\begin{bmatrix} e_{\tau hi}(h)\\ \tilde{g}_{\tau hi}(e(h))\end{bmatrix}\\
&=\xi^T(h,i)\left(\tilde{\Phi}_i+\digamma_i^T\bar{P}_i\digamma_i\right)\xi(h,i)+\delta_i
\end{aligned}$$

where $\delta_i\triangleq n\lambda_{\max}\{D_i^T\bar{P}_iD_i\}+\varepsilon_1\theta+\varepsilon_2\bar{v}$ and

$$\tilde{\Phi}_i=\begin{bmatrix}
\Pi_{11}^i & 0 & \Pi_{13}^i & \Pi_{14}^i & 0 & 0 & 0\\
* & \Pi_{22}^i & 0 & 0 & \Pi_{25}^i & 0 & 0\\
* & * & \Pi_{33}^i & 0 & 0 & 0 & 0\\
* & * & * & \Pi_{44}^i & 0 & 0 & 0\\
* & * & * & * & \Pi_{55}^i & 0 & 0\\
* & * & * & * & * & \Pi_{66}^i & 0\\
* & * & * & * & * & * & \Pi_{77}^i
\end{bmatrix}.$$

With the help of the Schur Complement Lemma, it follows from (8) that

$$\hat{\Phi}_i\triangleq\tilde{\Phi}_i+\digamma_i^T\bar{P}_i\digamma_i<0$$

and, accordingly,

$$\begin{aligned}
&\mathbb{E}\{V(e(h+1),h+1,\varsigma(h+1))|e(h),\varsigma(h)=i\}\\
&\quad -V(e(h),h,i)\\
&\le-\lambda_{\min}(-\hat{\Phi}_i)\xi^T(h,i)\xi(h,i)+\delta_i\\
&\le-\lambda_{\min}(-\hat{\Phi}_i)|e(h)|^2+\delta_i.
\end{aligned}$$

From Assumption 1 and the definition of $e(h)$, we know that there exists a positive constant h satisfying

$$\tilde{g}^T(e(h))\tilde{g}(e(h))\le h|e(h)|^2.$$

Recalling the definition of $V(e(h),h,i)$, it is readily seen that

$$\begin{aligned}
&V(e(h),h,i)\\
&\le\lambda_{\max}(P_i)|e(h)|^2+\lambda_{\max}(Q)h\sum_{s=h-\tau(h,i)}^{h-1}|e(s)|^2\\
&\quad +\lambda_{\max}(\bar{Q})h\sum_{\iota=\tau_m}^{\tau_M-1}\sum_{s=h-\iota}^{h-1}|e(s)|^2.
\end{aligned}$$

For a scalar $\alpha>1$, we have

$$\begin{aligned}
&\alpha^{h+1}\mathbb{E}\{V(e(h+1),h+1,\varsigma(h+1))|e(h),\varsigma(h)=i\}\\
&\quad -\alpha^hV(e(h),h,i)\\
&=\alpha^{h+1}\mathbb{E}\{(V(e(h+1),h+1,\varsigma(h+1))|e(h),\varsigma(h)=i)\\
&\quad -V(e(h),h,i)\}+\alpha^h(\alpha-1)V(e(h),h,i)\\
&\le\alpha^h\phi(\alpha)|e(h)|^2+\alpha^h\varphi_1(\alpha)\sum_{s=h-\tau(h,i)}^{h-1}|e(s)|^2\\
&\quad +\alpha^{h+1}\delta_i+\alpha^h\varphi_2(\alpha)\sum_{\iota=\tau_m}^{\tau_M-1}\sum_{s=h-\iota}^{h-1}|e(s)|^2 \qquad (12)
\end{aligned}$$

where

$$\begin{aligned}
\phi(\alpha)&\triangleq-\alpha\lambda_{\min}(-\hat{\Phi}_i)+(\alpha-1)\lambda_{\max}(P_i),\\
\varphi_1(\alpha)&\triangleq(\alpha-1)h\lambda_{\max}(Q),\\
\varphi_2(\alpha)&\triangleq(\alpha-1)h\lambda_{\max}(\bar{Q}).
\end{aligned}$$

For any integer $T\ge\tau_M+1$, summing up both sides of (12) from 0 to $T-1$ with respect to h, we have

$$\begin{aligned}
&\alpha^T\mathbb{E}\{V(e(T),T,\varsigma(T))|e(T-1),\varsigma(T-1)\}\\
&\quad -\mathbb{E}\{V(e(0),0,\varsigma(0))\}\\
&\le\phi(\alpha)\sum_{h=0}^{T-1}\alpha^h\mathbb{E}\left\{|e(h)|^2\right\}+\frac{\alpha(1-\alpha^T)}{1-\alpha}\delta_i\\
&\quad +\varphi_1(\alpha)\sum_{h=0}^{T-1}\sum_{s=h-\tau(h,\varsigma(h))}^{h-1}\alpha^h\mathbb{E}\left\{|e(s)|^2\right\}\\
&\quad +\varphi_2(\alpha)\sum_{h=0}^{T-1}\sum_{\iota=\tau_m}^{\tau_M-1}\sum_{s=h-\iota}^{h-1}\alpha^h\mathbb{E}\left\{|e(s)|^2\right\}. \qquad (13)
\end{aligned}$$

Moreover, for the last two terms in (13), one has

$$\begin{aligned}
&\sum_{h=0}^{T-1}\sum_{s=h-\tau(h,\varsigma(h))}^{h-1}\alpha^h\mathbb{E}\left\{|e(s)|^2\right\}\\
&\le\left(\sum_{s=-\tau_M}^{-1}\sum_{h=0}^{s+\tau_M}+\sum_{s=0}^{T-\tau_M-1}\sum_{h=s+1}^{s+\tau_M}\right.\\
&\quad\left.+\sum_{s=T-\tau_M}^{T-1}\sum_{h=s+1}^{T-1}\right)\alpha^h\mathbb{E}\left\{|e(s)|^2\right\}\\
&\le\frac{\alpha^{\tau_M}-1}{\alpha-1}\sum_{s=-\tau_M}^{-1}\mathbb{E}\left\{|e(s)|^2\right\}\\
&\quad +\frac{\alpha(\alpha^{\tau_M}-1)}{\alpha-1}\sum_{s=0}^{T-1}\alpha^s\mathbb{E}\left\{|e(s)|^2\right\}\\
&\quad +\frac{\alpha(\alpha^{\tau_M-1}-1)}{\alpha-1}\sum_{s=0}^{T-1}\alpha^s\mathbb{E}\left\{|e(s)|^2\right\} \qquad (14)
\end{aligned}$$

and

$$\begin{aligned}
&\sum_{h=0}^{T-1}\sum_{\iota=\tau_m}^{\tau_M-1}\sum_{s=h-\iota}^{h-1}\alpha^h\mathbb{E}\left\{|e(s)|^2\right\}\\
&\le(\tau_M-\tau_m)\left(\sum_{s=-\tau_M}^{-1}\sum_{h=0}^{s+\tau_M}+\sum_{s=0}^{T-\tau_M-1}\sum_{h=s+1}^{s+\tau_M}\right.\\
&\quad\left.+\sum_{s=T-\tau_M}^{T-1}\sum_{h=s+1}^{T-1}\right)\alpha^h\mathbb{E}\left\{|e(s)|^2\right\}\\
&\le\frac{(\alpha^{\tau_M}-1)(\tau_M-\tau_m)}{\alpha-1}\sum_{s=-\tau_M}^{-1}\mathbb{E}\left\{|e(s)|^2\right\}
\end{aligned}$$

$$+\frac{\alpha(\alpha^{\tau_M}-1)(\tau_M-\tau_m)}{\alpha-1}\sum_{s=0}^{T-1}\alpha^s\mathbb{E}\left\{|e(s)|^2\right\}$$
$$+\frac{\alpha(\alpha^{\tau_M-1}-1)(\tau_M-\tau_m)}{\alpha-1}\sum_{s=0}^{T-1}\alpha^s\mathbb{E}\left\{|e(s)|^2\right\}. \quad (15)$$

From (14) and (15), it is easily known that

$$\begin{aligned}&\alpha^T\mathbb{E}\left\{V(e(T),T,\varsigma(T))|e(T-1),\varsigma(T-1)\right\}\\&\quad-\mathbb{E}\left\{V(e(0),0,\varsigma(0))\right\}\\&\leq\phi(\alpha)\sum_{h=0}^{T-1}\alpha^h\mathbb{E}\left\{|e(h)|^2\right\}+\frac{\alpha(1-\alpha^T)}{1-\alpha}\delta_i\\&\quad+(\varphi_1(\alpha)+\varphi_2(\alpha)(\bar{\tau}-\underline{\tau}))(\frac{\alpha^{\tau_M}-1}{\alpha-1}\sum_{s=-\tau_M}^{-1}\mathbb{E}\left\{|e(s)|^2\right\}\\&\quad+\frac{\alpha(\alpha^{\tau_M}-1)}{\alpha-1}\sum_{s=0}^{T-1}\alpha^s\mathbb{E}\left\{|e(s)|^2\right\}\\&\quad+\frac{\alpha(\alpha^{\tau_M-1}-1)}{\alpha-1}\sum_{s=0}^{T-1}\alpha^s\mathbb{E}\left\{|e(s)|^2\right\})\\&\leq\zeta(\alpha)\sum_{h=0}^{T-1}\alpha^h\mathbb{E}\left\{|e(h)|^2\right\}+\frac{\alpha(1-\alpha^T)}{1-\alpha}\delta_i\\&\quad+\frac{(\varphi_1(\alpha)+\varphi_2(\alpha)(\tau_M-\tau_m))(\alpha^{\tau_M}-1)}{\alpha-1}\\&\quad\times\max_{-\tau_M\leq s\leq 0}\mathbb{E}\left\{|e(s)|^2\right\}\end{aligned} \quad (16)$$

where

$$\begin{aligned}\zeta(\alpha)\triangleq&\phi(\alpha)+\frac{\varphi_1(\alpha)+\varphi_2(\alpha)(\tau_M-\tau_m)}{\alpha-1}\\&\times\left(\alpha^{\tau_M+1}+\alpha^{\tau_M}-2\alpha\right).\end{aligned}$$

Noting that $\zeta(1)=-\lambda_{\min}(-\Phi_i)<0$ and $\lim_{\alpha\to\infty}\zeta(\alpha)=+\infty$, there exists a scalar $\alpha_0>1$ such that $\zeta(\alpha_0)=0$. Then, it is inferred from (16) that

$$\begin{aligned}&\alpha_0^T\mathbb{E}\left\{V(e(T),T,\varsigma(T))|e(T-1),\varsigma(T-1)\right\}\\&\quad-\mathbb{E}\left\{V(e(0),0,\varsigma(0))\right\}\\&\leq\frac{(\varphi_1(\alpha_0)+\varphi_2(\alpha_0)(\tau_M-\tau_m))(\alpha_0^{\tau_M}-1)}{\alpha_0-1}\\&\quad\times\max_{-\tau_M\leq s\leq 0}\mathbb{E}\left\{|e(s)|^2\right\}+\frac{\alpha_0(1-\alpha_0^T)}{1-\alpha}\delta_i.\end{aligned}$$

Noting

$$\begin{aligned}&\mathbb{E}\left\{V(e(0),0,\varsigma(0))\right\}\\&\leq\lambda_{\max}\left(P_{\varsigma(0)}\right)\mathbb{E}\left\{|e(0)|^2\right\}+\lambda_{\max}(Q)h\sum_{-\tau_M}^{-1}\mathbb{E}\left\{|e(0)|^2\right\}\\&\quad+(\tau_M-\tau_m)\lambda_{\max}(\bar{Q})h\sum_{-\tau_M}^{-1}\mathbb{E}\left\{|e(0)|^2\right\}\\&\leq\max\left(\lambda_{\max}\left(P_{\varsigma(0)}\right)+(\lambda_{\max}(Q)h+(\tau_M-\tau_m)\right.\\&\quad\left.\times\lambda_{\max}(\bar{Q})h\right)\tau_M\Big)\times\max_{-\tau_M\leq s\leq 0}\mathbb{E}\left\{|e(0)|^2\right\}\end{aligned}$$

and

$$\alpha_0^T\mathbb{E}\left\{V(e(T),T,\varsigma(T))\right\}\geq\lambda_{\min}(P_i)\alpha_T^T\mathbb{E}\left\{|e(0)|^2\right\},$$

we have

$$\mathbb{E}\left\{|e(T)|^2\right\}\leq\frac{(\alpha_0^T-1)\delta_i}{\alpha_0^{T-1}(\alpha_0-1)\lambda_{\min}(P_i)}+\frac{\varpi(\alpha_0)}{\alpha_0^T\lambda_{\min}(P_i)}$$

where

$$\begin{aligned}&\varpi(\alpha_0)\\&\triangleq\Big(\frac{(\varphi_1(\alpha)+\varphi_2(\alpha)(\tau_M-\tau_m))(\alpha^{\tau_M}-1)}{\alpha-1}\\&\quad+\max(\lambda_{\max}(P_{\varsigma(0)})+(\lambda_{\max}(Q)h+(\tau_M-\tau_m)\\&\quad\times\lambda_{\max}(\bar{Q})h)\tau_M)\Big)\max_{-\tau_M\leq s\leq 0}\mathbb{E}\{|e(0)|^2\}.\end{aligned}$$

By taking $\mu=1/\alpha_0$, $\psi(T)=(\alpha_0^T-1)\delta_i/(\alpha_0^{T-1}(\alpha_0-1)\lambda_{\min}(P_i))$, and $\vartheta=\varpi(\alpha_0)/\lambda_{\min}(P_i)$, it is readily seen from Definition 1 that the estimation error system (6) is EUB in the mean-square with

$$\bar{\psi}=\lim_{T\to\infty}\psi(T)=\frac{\alpha_0\delta_i}{(\alpha_0-1)\lambda_{\min}(P_i)}.$$

The proof is complete. ■

Theorem 1 provides a sufficient condition guaranteeing that the estimation error dynamics is EUB in the mean square sense. In the following theorem, we are going to characterize the desired estimator gain.

Theorem 2: The estimation error system (6) is EUB in the mean square if there exist a set of matrices $P_i>0$ $(i\in S)$, a set of matrices Y_i $(i\in S)$, a matrix $Q>0$, three sets of diagonal matrices $\Lambda_i=\mathrm{diag}\{\lambda_1^i,\lambda_2^i,\cdots,\lambda_n^i\}>0$, $\Delta_i=\mathrm{diag}\{\delta_1^i,\delta_2^i,\cdots,\delta_n^i\}>0$, $\Sigma_i=\mathrm{diag}\{\sigma_1^i,\sigma_2^i,\cdots,\sigma_n^i\}>0$ and positive scalars $\varepsilon_1,\varepsilon_2$ such that

$$\begin{bmatrix}\Pi_{11}^i&0&\Pi_{13}^i&\Pi_{14}^i&0&0&0&\Theta_{18}^i\\ *&\Pi_{22}^i&0&0&\Pi_{25}^i&0&0&0\\ *&*&\Pi_{33}^i&0&0&0&0&\Pi_{38}^i\\ *&*&*&\Pi_{44}^i&0&0&0&0\\ *&*&*&*&\Pi_{55}^i&0&0&\Pi_{58}^i\\ *&*&*&*&*&\Pi_{66}^i&0&\Theta_{68}^i\\ *&*&*&*&*&*&\Pi_{77}^i&\Theta_{78}^i\\ *&*&*&*&*&*&*&\Pi_{88}^i\end{bmatrix}<0 \quad (17)$$

where

$$\Theta_{18}^i=\bar{A}_i^T\bar{P}_i-C_i^TY_i^T,\ \Theta_{68}^i=-Y_i^T,\ \Theta_{78}^T=-L_i^TY_i^T.$$

Moreover, the estimator gain matrices can be calculated by $K_i=\bar{P}_i^{-1}Y_i$ $(i\in S)$.

Proof: The proof of this theorem follows directly from that of Theorem 1, and is therefore omitted. ■

Remark 2: The purpose of this paper is to develop an event-based state estimator for a class of MJANNs with uncertain transition probabilities and mode-dependent delays. To achieve this purpose, the existence of the estimator is first ensured by a sufficient condition, which also guarantees that the estimation error dynamics is EUB in the mean square. Then, a sufficient condition in terms of certain matrix inequalities is given in Theorem 2 to design the estimator gain.

Remark 3: Until now, an ETSE scheme has been designed for a class of MJANNs with uncertain transition probabilities and mode-dependent delays. The major contribution of this paper is outlined in twofold as follows: 1) the considered system model is general that accounts for the factors including the Markovian jumping parameters, the uncertain transition probabilities, the ETM and the mode-dependent time-delays; and 2) the effects of the above factors on state estimation are all reflected in the main results.

IV. Numerical Simulation

A numerical simulation is carried out to demonstrate the validity of the proposed ETSE scheme for a class of discrete-time MJANNs with mode-dependent delays and uncertain transition probabilities.

Let the ANNs (1) have three neurons with the following parameters:

$$A_1 = \begin{bmatrix} 0.7 & 0 & 0 \\ 0 & -0.4 & 0 \\ 0 & 0 & 0.6 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & 0.7 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.2 & 0.6 & 0.2 \\ 0.2 & -0.2 & 0 \\ 0.2 & -0.1 & -0.1 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0.2 & 0.5 & 0.3 \\ 0.2 & -0.4 & 0 \\ 0.3 & -0.1 & -0.2 \end{bmatrix},$$
$$B_{d1} = \begin{bmatrix} 0.05 & 0.04 & -0.03 \\ 0.05 & 0.1 & 0.01 \\ 0.05 & 0.01 & 0.05 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.25 \\ 0.2 \\ 0.18 \end{bmatrix},$$
$$B_{d2} = \begin{bmatrix} 0.08 & -0.03 & 0.03 \\ 0.01 & 0.2 & 0.02 \\ -0.06 & 0.03 & 0.02 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.25 \end{bmatrix}$$

and other parameters are chosen as $\tau_M = 3$, $\tau_m = 2$ and $\theta = 0.15$.

The activation functions are selected as

$$f(x(s)) = \begin{bmatrix} \tanh(0.5x_1(s)) & \tanh(0.4x_2(s)) & \tanh(0.6x_3(s)) \end{bmatrix}^T,$$
$$g(x(s)) = \begin{bmatrix} \tanh(0.3x_1(s)) & \tanh(0.4x_2(s)) & \tanh(0.2x_3(s)) \end{bmatrix}^T,$$

from which we conclude that $\tilde{\alpha}_1 = \tilde{\beta}_1 = 0$, $\tilde{\alpha}_2 = \mathrm{diag}\{-0.2, -0.25 - 0.1\}$ and $\tilde{\beta}_2 = \mathrm{diag}\{-0.15, -0.2, -0.1\}$.

The measurement output of the sensor is modeled by (3) with the following parameters:

$$C_1 = \begin{bmatrix} 2.2 & 0.5 & 1.8 \\ 0.3 & 1.2 & 0.2 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1.2 & 0.8 & 1.2 \\ 0.6 & 1.9 & 0.6 \end{bmatrix},$$
$$L_1 = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 0.6 \\ 0.5 \end{bmatrix}.$$

The uncertain transition probability matrix is denoted by

$$\Pi = \begin{bmatrix} 0.4 + \Delta\pi_{11} & 0.3 + \Delta\pi_{12} \\ 0.35 + \Delta\pi_{21} & 0.35 + \Delta\pi_{22} \end{bmatrix}$$

with $\kappa_{11} = 0.15$, $\kappa_{12} = 0.15$, $\kappa_{21} = 0.1$, and $\kappa_{22} = 0.2$.

Considering the parameters mentioned above, the gain matrices of the desired estimator are obtained by using the MATLAB LMI toolbox and outlined as

$$K_1 = \begin{bmatrix} 0.4509 & -0.1083 \\ 0.0762 & -0.4683 \\ 0.1012 & 0.0018 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 0.4927 & -0.1509 \\ 0.1472 & -0.3055 \\ 0.2054 & -0.0431 \end{bmatrix}.$$

The initial values of the states are set to be $x_1(s) = 0.1$, $x_2(s) = -0.1$, and $x_3(s) = 0.2$ for $s \in [-3, 0]$. The corresponding results are shown in Figs. 1-4, where Fig. 1, Fig. 2 and Fig. 3 depict the states of $x_1(h)$, $x_2(h)$, $x_3(h)$, the corresponding estimates and the estimation errors $e_1(h)$, $e_2(h)$ and $e_3(h)$, respectively. The execution status of the ETM is described in Fig. 4. From Figs. 1-4, it is readily seen that the designed estimator performs well in estimating the state trajectory of the system, which visually implies the effectiveness of our design scheme.

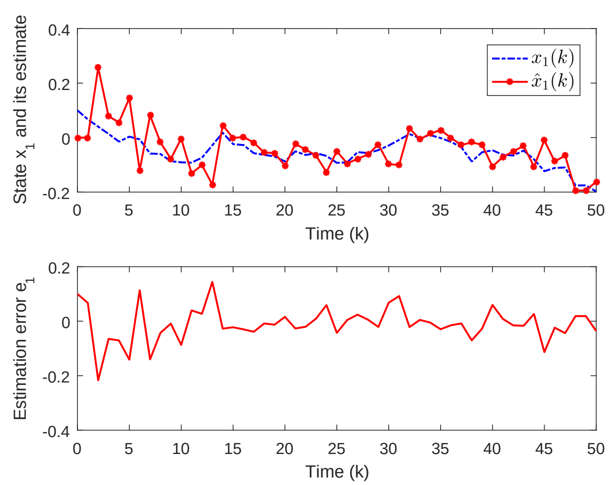

Fig. 1: State $x_1(h)$, estimate $\hat{x}_1(h)$ and estimation error $e_1(h)$.

V. Conclusion

In this paper, the ETSE problem has been studied for a class of discrete-time MJANNs with uncertain transition probabilities and mode-dependent delays. The switching of the ANNs has been characterized by a Markovian chain

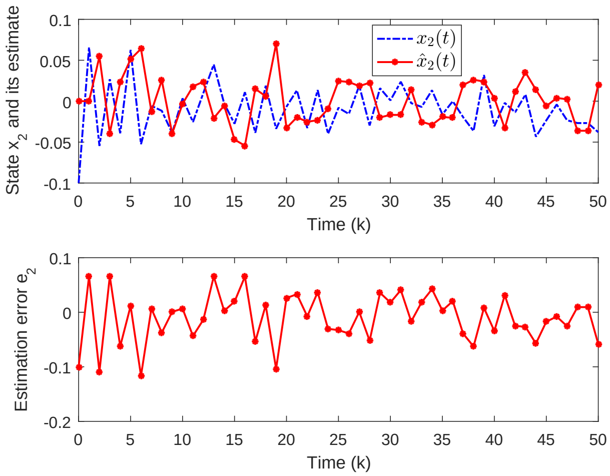

Fig. 2: State $x_2(h)$, estimate $\hat{x}_2(h)$ and estimation error $e_2(h)$.

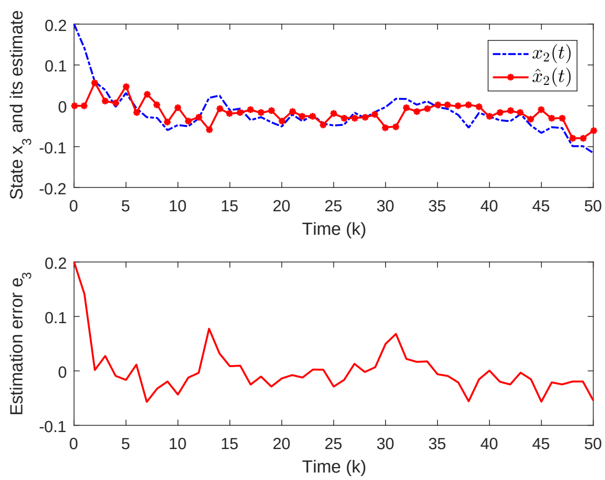

Fig. 3: State $x_3(h)$, estimate $\hat{x}_3(h)$ and estimation error $e_3(h)$.

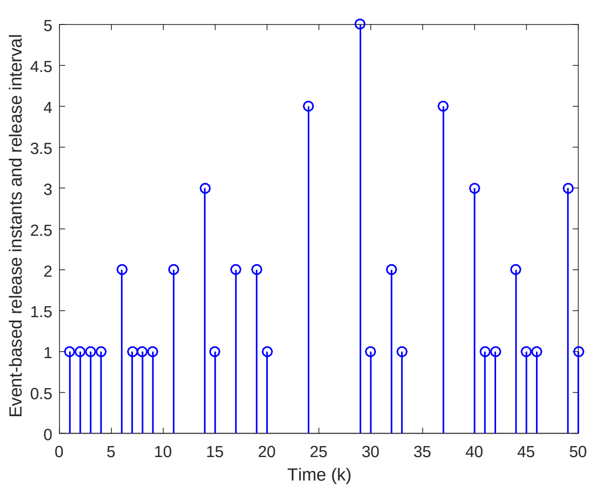

Fig. 4: Event-based release instants and release intervals.

whose transition probabilities are allowed to be uncertain. The ETM has been used to regulate the data transmissions of the sensor. Based on the proposed ETM, a sufficient condition that guarantees that the estimation error system is EUB in the mean square has been derived. Then, the gain of the estimator has been derived by solving certain matrix inequalities proposed in Theorem 2. Finally, a numerical case is offered to demonstrate the validity of the proposed ETSE.

Further research topics would include the extension of the main results to 1) the moving-horizon estimation problem for discrete-time MJANNs with uncertain transition probabilities [19], [46]–[48] ; 2) the reliable H_∞ state estimation problem of MJANNs with missing measurements [21], [27], [36] and mixed mode-dependent time-delays [23]; and 3) the improvement of the state estimation performance by using some latest optimization algorithms [25], [26]. Note that some multi-objective optimization approaches have been proposed in [25], [26] with successful applications to the minimization of energy consumption in crude oil pipeline transportation system and large-scale oil-gas gathering system, and these excellent algorithms are well suited to be applied to the further reduction of the state estimation errors in the problem addressed in this paper.

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