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Energy-to-Peak State Estimation with Intermittent Measurement Outliers: The Single-Output Case

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***Abstract*—This paper is concerned with the energy-to-peak state estimation problem for a class of linear discrete-time systems with energy-bounded noises and intermittent measurement outliers (IMOs). In order to capture the intermittent nature, two sequences of step functions are introduced to model the occurrence of the IMOs. Furthermore, two special indices (i.e. minimum and maximum interval lengths) are adopted to describe the "occurrence frequency" of IMOs. Different from the considered energy-bounded noises, the outliers are assumed to have their magnitudes larger than certain thresholds. In order to achieve a satisfactory performance constraint on the energy-to-peak state estimation under the addressed kind of measurement outliers, a novel parameter-dependent (PD) state estimation strategy is developed to guarantee that the measurements contaminated by outliers would be removed in the estimation process. The proposed PD state estimation method is essentially a two-step process, where the first step is to examine the appearing and disappearing moments for each IMO by using a dedicatedly constructed outlier detection scheme, and the second step is to implement the state estimation task according to the outlier detection results. Sufficient conditions are obtained to ensure the existence of the desired estimator, and the gain matrix of the desired estimator is then derived by solving a constrained optimization problem. Finally, a simulation example is presented to illustrate the effectiveness of our developed PD state estimation strategy.**

***Index Terms*—State estimation; Intermittent measurement outliers; Energy-to-peak performance; Parameter-dependent state estimator.**

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I. INTRODUCTION

The past several decades have witnessed successful applications of a variety of state estimation strategies in a wide range of industrial systems including guidance and navigation systems, target tracking systems and monitoring system. Such applications are motivated by the fact that the state information of a system is often *partially* available only despite its great importance in various commitments such as control and fault detection tasks. The essential purpose of the state estimation is to acquire the accurate estimates of the system states through available but possibly partial/noisy measurement data [13], [30], [48].

In order to evaluate the estimation accuracy, different criteria have been put forward according to different types of the disturbances and different performance specifications, thereby leading to different state estimation approaches (e.g. $\mathcal{H}_\infty$ state estimation [20], [23], [44], minimum mean-squared error state estimation [14], [16], [32], [37], energy-to-peak state estimation [33], [36], moving-horizon estimation [49], [50], set-membership state estimation [9], [18], [21] and ultimately bounded state estimation [22], [24], [45]). For example, in [15], the well-known $\mathcal{H}_\infty$ performance index has been adopted to evaluate the estimation accuracy subject to the energy-bounded noises and the corresponding $\mathcal{H}_\infty$ state estimator has been developed by using the linear matrix inequality technique. In [32], the error covariance has been employed to characterize the estimation performance subject to the Gaussian noises and a recursive filter has been designed by solving two Riccati-type optimal equations.

Energy-to-peak state estimation is an effective estimation scheme that has been developed according to the so-called l_2-l_∞ performance (also known as energy-to-peak performance), whose main idea is to guarantee a sufficiently small peak value of the state estimation error in the presence of energy-bounded noises. Such an l_2-l_∞ performance index was originated in 1989 in [40] where the energy-to-peak control problem was investigated. The energy-to-peak state estimation problem has been first considered in [10] where the existence condition of the desired state estimator has been expressed in terms of linear matrix inequalities. So far, the energy-to-peak state estimation problem has gained an ongoing research interest for various systems, see e.g. [19], [35] and the references therein.

In the past decade or so, the energy-to-peak state estimation problem has aroused a great deal of research interest in networked systems subject to several network-induced effects including packet dropouts [46], quantization measurements [34], protocol scheduling effects [36] and event-triggered transmis-

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sions [38]. For example, in [36], a model-dependent estimator has been developed to deal with the energy-to-peak state estimation problem under high-rate communication channel with Round-Robin protocol scheduling effects. Note that the energy-to-peak state estimation acquires satisfactory performance based on the assumption that the external disturbances are *energy-bounded*. In engineering practice, however, system measurements may occasionally suffer from certain large-amplitude disturbances/perturbations, customarily referred to as the *measurement outliers* which, unfortunately, violate the energy-boundedness assumption. As such, the conventional energy-to-peak state estimation methods are incapable of dealing with measurement outliers.

Compared with the widely studied norm-bounded and energy-bounded noises, measurement outliers have their very own characteristics of 1) occasional/intermittent/probabilistic occurrences and 2) unexpectedly large magnitudes. So far, the estimation problem subject to measurement outliers has stirred some initial research attention, see, e.g. [1], [7], [8], [17], [31]. Generally speaking, there are mainly two classes of strategies (i.e., passive robustness-based strategies and active detection-based strategies) that have been developed in the literature. The passive robustness-based strategy is to reduce the sensitivity of the estimation performance to the outliers, and the representative works in this regard include the stubborn state estimation [2] and the outlier-robust Kalman filtering [7]. Concerning the active detection-based strategies, the main idea is to develop certain outlier detection schemes in order to remove "harmful" innovations (i.e. the innovations that might be corrupted by outliers) in the state estimators, see, e.g. the leave-one-out moving-horizon estimation [3] and the attack-detector-based recursive filter [6]. Nevertheless, to the best of the authors' knowledge, the energy-to-peak state estimation problem subject to measurement outliers has not yet gained adequate research attention despite its potential in practical applications, and this gives rise to the main motivation of our research.

In our previous research [51], the set-membership filtering problem has been studied subject to *impulsive* measurement outliers described by a sequence of impulsive signals whose interval length/amplitude are greater than certain known constants/thresholds. In this paper, we consider another kind of measurement outliers, namely, intermittent measurement outliers (IMOs). Different from the impulsive outliers, the IMOs investigated in this paper are defined as the abnormal signals with certain duration lengths and interval lengths. Such kind of IMOs can be found in numerous practical applications including the electronic systems, aerospace systems, mechanical equipment and power systems [43], [47]. Unfortunately, up to now, the state estimation problem subject to IMOs has not been properly investigated yet, let alone the simultaneous consideration of energy-to-peak estimation performance requirement. It is, therefore, the main motivation of this paper to shorten such a gap.

Summarizing the above discussions made thus far, we aim to investigate the energy-to-peak state estimation issue for a class of discrete-time linear systems with IMOs. This is a nontrivial problem as we are going to face the following three inevitable challenges: 1) how to establish a reasonable model for IMOs according to engineering practice? 2) how to discriminate the measurements corrupted by IMOs from those normal measurements? and 3) how to design the energy-to-peak state estimator that prevents the estimation performance from being degraded by IMOs? In this paper, we are set to overcome the above-listed challenges. *The primary contributions of this paper are highlighted as follows: 1) the energy-to-peak state estimation problem is, for the first time, studied for linear systems subject to IMOs; 2) a novel parameter-dependent (PD) state estimator is designed within the active detection-based framework, where a novel outlier detection method is developed to determine whether the received measurement output is contaminated by an outlier; and 3) a particle-swarm-optimization-based algorithm is put forward to calculate gain matrix of the desired state estimator.*

The remainder of this paper is organized as follows. In Section II, the linear systems with IMOs and the corresponding PD state estimator structure are proposed. In Section III, the outlier detection strategy of IMOs is designed, and the desired estimator gain matrix is calculated by using a particle-swarm-optimization-based algorithm. A simulation example is given in Section IV to demonstrate the correctness and effectiveness of our proposed PD state estimation scheme. Finally, we present the conclusion of this work in Section V.

Notations: The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and set of all $n\times m$ real matrices. $\mathbb{N}\,(\mathbb{N}^+,\mathbb{N}^-)$ denote, respectively, the set of integers (nonnegative integers, negative integers), and the set of all nonnegative real numbers is denoted by $\mathbb{R}^+$. The notation $X\geq Y$ ($X>Y$), where X and Y are real symmetric matrices, means that $X-Y$ is positive semi-definite (positive definite). M^T represents the transpose of the matrix M. If A is a square matrix, $\lambda_{\max}\{A\}$ ($\lambda_{\min}\{A\}$) stands for the maximum (minimum) eigenvalue of A, $\mathrm{tr}\{A\}$ represents the trace of A, and $\det(A)$ denotes the determinant of A. 0 represents zero matrix of compatible dimensions. $\mathbf{1}_N$ represents an N dimensional row vector with all ones. The n-dimensional identity matrix is denoted as I_n or simply I, if no confusion is caused. The shorthand $\mathrm{diag}\{\cdots\}$ stands for a block-diagonal matrix and the notation $\mathrm{diag}_n\{\bullet\}$ is employed to stand for $\mathrm{diag}\{\underbrace{\bullet,\cdots,\bullet}_{n}\}$. Given a generic vector x, $\|x\|$ describes the Euclidean norm of x. $l_2([0,\infty),\mathbb{R}^n)$ is the space of square summable n-dimensional vector-valued functions. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions. The step function $\Gamma(a)$ is a binary function that equals 1 if $a\geq 0$ and equals 0 otherwise.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. Intermittent measurement outliers

In this paper, we are concerned with the sensors measurements that suffer from outlier-induced-effects. The outliers

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under consideration are a series of special signals with their own appearing and disappearing moments, and the number of outliers is accountable. In this sense, let $\underline{t}_i$ and $\bar{t}_i$ denote the appearing moments and disappearing moments, respectively, which satisfy $\underline{t}_i < \bar{t}_i < \underline{t}_{i+1}$.

Based on the sequences $\{\underline{t}_i\}_{i\geq 1}$ and $\{\bar{t}_i\}_{i\geq 1}$, the measurement outlier $\bar{o}_k$ (i.e. the outlier occurring at the time instant k) can be modeled by the following form of step signals:

$$\bar{o}_k = \sum_{i=1}^{\infty}\big(\Gamma(k-\underline{t}_i) - \Gamma(k-\bar{t}_i)\big)o_k^i \tag{1}$$

where $\Gamma(\cdot)$ is the step function and o_k^i is the magnitude (a vector to be defined later) of the i-th outlier at time instant k. In this work, the measurement outlier $\bar{o}_k$ modeled by (1) is referred to as the *intermittent measurement outlier* (IMO). Furthermore, we define the interval length $\check{T}_i$ and duration length $\hat{T}_i$ as

$$\begin{cases} \check{T}_i \triangleq \underline{t}_i - \bar{t}_{i-1}, & i \geq 2 \\ \hat{T}_i \triangleq \bar{t}_i - \underline{t}_i, & i \geq 1 \end{cases}$$

with initial value $\check{T}_1 = \underline{t}_1$.

Next, let us introduce a justifiable assumption on the proposed IMOs.

Assumption 1: For any $i \geq 1$, the interval length $\check{T}_i$ and duration length $\hat{T}_i$ satisfy $\check{T}_i \geq \underline{T}$ and $\hat{T}_i \leq \bar{T}$, respectively. Here, $\underline{T}$ and $\bar{T}$ are two known positive constants representing the minimum interval length and maximum duration length, respectively. Furthermore, the magnitude o_k^i satisfies $\|o_k^i\| \geq \underline{o}$ where $\underline{o}$ is a known positive scalar.

Remark 1: The outliers are defined as certain anomalous signals that might be caused by various reasons including sensor faults, cyber-attacks or large non-Gaussian noises. In fact, the so-called intermittent sensor faults belong to the category of IMOs. These kinds of anomalous signals, as compared with the conventional disturbances/noises, are likely to occur on an *intermittent* basis with relatively large magnitudes. A typical example of such outliers is the system failure phenomenon. In reliability engineering, two important indices, namely, the time between failures and the time to repair, are commonly utilized to characterize the failure model of a repairable system [4], [5]. Obviously, such failure model falls within the scope of our proposed IMO. In this paper, to model the intermittent nature, a sequence of "shifted gate functions" (i.e. the differences of two sequences of step functions) has been adopted to describe the occurrences of the outliers. In this work, two important concepts (i.e. the interval length and duration length) are introduced to reflect the intermittency property.

B. Problem formulation: plant and state estimator structure

Consider a linear discrete-time system of the form

$$\begin{cases} x_{k+1} = Ax_k + B\omega_k \\ y_k = Cx_k + D\nu_k + \bar{o}_k \\ z_k = Mx_k \end{cases} \tag{2}$$

where $x_k \in \mathbb{R}^n$, $y_k \in \mathbb{R}$ and $z_k \in \mathbb{R}^l$ denote, respectively, the system state, the measurement output and the output vector to be estimated; $\omega_k \in l_2([0,\infty),\mathbb{R}^r)$ and $\nu_k \in l_2([0,\infty),\mathbb{R}^s)$ are the process and measurement noises, respectively; and the parameters A, B, C, D and M are real-valued matrices of appropriate dimensions. Here, the vector $\bar{o}_k \in \mathbb{R}$ is the IMO of the form (1). In this paper, it is assumed that the appearing moment sequence and disappearing moment sequence of the outliers are *completely unknown* under Assumption 1.

The following assumptions are quite standard on system parameters and noises.

Assumption 2: The time-invariant system (2) is observable. In other words, the rank of the following observability matrix

$$F \triangleq \begin{bmatrix} CA^{n-1} \\ CA^{n-2} \\ \vdots \\ C \end{bmatrix}$$

is equal to n.

Assumption 3: The energy-bounded noises ω_k and ν_k satisfy the following condition:

$$\|\omega_k\| \leq \bar{\omega}, \quad \|\nu_k\| \leq \bar{\nu},$$

where $\bar{\omega}$ and $\bar{\nu}$ are two known positive scalars.

Remark 2: In this paper, the noises under consideration are energy-bounded. Obviously, the norm of the energy-bounded noise is also bounded. In order to distinguish the outliers from the noises, we assume that the upper bounds of the noise norms are completely known. In what follows, we focus our attention on the relationship between the external inputs (i.e. ω_k, ν_k and $\bar{o}_k$) and measurements (i.e. y_k) and, with such an established relationship, we would be able to develop suitable outlier detection strategy.

Now, let us consider the state estimator for the plant (2). In order to restrain the estimation performance from being degraded by the IMOs, we adopt the following parameter-dependent (PD) state estimator:

$$\begin{cases} \hat{x}_{k+1} = A\hat{x}_k + L(\theta_k)\big(y_k - C\hat{x}_k\big) \\ L(\theta_k) = (1-\theta_k)K \\ \hat{z}_k = M\hat{x}_k \end{cases} \tag{3}$$

where $\hat{x}_k$ and $\hat{z}_k$ denote, respectively, the estimates of x_k and z_k. The binary function θ_k and the gain matrix K are the estimator parameters to be designed. Note that the structure of (3) is purposely designed to protect the estimation performance from the outlier by setting $\theta_k = 1$ when there is an outlier occurred at time k (i.e. $\sum_{i=1}^{\infty}\big(\Gamma(k-\underline{t}_i) - \Gamma(k-\bar{t}_i)\big) = 1$). In other words, our main objective is to design the function θ_k such that the possible outliers in the innovations $y_k - C\hat{x}_k$ are removed.

Remark 3: In this work, the outliers under consideration are regarded as the abnormal signals whose magnitudes are quite large. As such, such signals would significantly impact the estimation performance by degrading the estimation accuracy. The key point of removing the "harmful" innovations (corrupted by outliers) is to "identify" the appearing and disappearing moments for each IMO. This task is, however, difficult to accomplish by adopting the traditional model-based fault detector (MbFD) because the MbFD-based detection

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result at each time instant is based on the so-called residual, whose value is largely affected by the historical external inputs (including the historical outliers that are unfortunately unknown). In what follows, we focus our attention on the detection method which is capable of accurately distinguishing the measurements with outliers from the normal measurements based on a fixed number of past measurements.

We are now in a position to state the problem addressed in this work as follows. This paper aims to design the binary function θ_k and the PD state estimator gain K such that the following requirements are met simultaneously.

a) The binary function θ_k is designed such that the condition $\theta_k = 1$ holds if and only if $\bar{o}_k \neq 0$.
b) The state estimation error (i.e. $e_k \triangleq x_k - \hat{x}_k$) with $\omega_k = 0$ and $\nu_k = 0$ is asymptotically stable.
c) Under the zero-initial condition, the output estimation error (i.e. $\tilde{z}_k \triangleq z_k - \hat{z}_k$) satisfies

$$\|\tilde{z}_k\|_\infty^2 < \gamma^2 \sum_{k=0}^{\infty}(\|\omega_k\|^2 + \|\nu_k\|^2)$$

for all nonzero ω_k and ν_k, where $\|\tilde{z}_k\|_\infty \triangleq \sup_{k\geq 0}\|\tilde{z}_k\|$ is the peak value of the output estimation error, and $\gamma > 0$ is the given l_2-l_∞ disturbance attenuation level (or the energy-to-peak performance index).

III. MAIN RESULTS

A. Design of the function θ_k

Let us start by developing an effective algorithm on detecting whether the received measurement contains an outlier or not. In doing so, we first introduce the input-output model of the plant (2). Considering the characteristic polynomial of the square matrix A subject to the variable $z \in \mathbb{R}$, we have

$$\det(zI - A) \triangleq z^n + \sum_{i=0}^{n-1}\alpha_i z^i$$

where $\{\alpha_i\}_{0\leq i\leq n-1}$ are the coefficients of the characteristic polynomial. Define the transformation matrix as follows:

$$Q \triangleq \begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \vdots\\ \varepsilon_n\end{bmatrix} = \begin{bmatrix}1 & \alpha_{n-1} & \alpha_{n-2} & \cdots & \alpha_2 & \alpha_1\\ 0 & 1 & \alpha_{n-1} & \cdots & \alpha_3 & \alpha_2\\ 0 & 0 & 1 & \cdots & \alpha_3 & \alpha_2\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 1 & \alpha_{n-1}\\ 0 & 0 & 0 & \cdots & 0 & 1\end{bmatrix} F$$

where the matrix F is given in Assumption 2. Then, it follows from the plant (2) that

$$\begin{cases}\vec{x}_{k+1} = \vec{A}\vec{x}_k + \vec{B}\omega_k\\ y_k = \vec{C}\vec{x}_k + D\nu_k + \bar{o}_k\\ \bar{o}_k = \displaystyle\sum_{i=1}^{\infty}\big(\Gamma(k-\underline{t}_i) - \Gamma(k-\bar{t}_i)\big)o_k^i\\ z_k = \vec{M}\vec{x}_k\end{cases} \tag{4}$$

where

$$\vec{x}_k \triangleq Qx_k, \quad \vec{C} \triangleq CQ^{-1}, \quad \vec{A} \triangleq QAQ^{-1},$$

$$\vec{B} \triangleq QB \triangleq \begin{bmatrix}b_1^T & b_2^T & \cdots & b_n^T\end{bmatrix}^T.$$

According to (4), we have the following proposition.

Proposition 1: Consider the dynamical system (4). For any $j \geq 0$, the output measurement sequence $\{y_k\}_{k\geq 0}$ satisfies the following equality

$$y_{k+j} + \sum_{i=0}^{n-1}\alpha_i^{(j)}y_{k-n+i} = \vec{\nu}_{k+j}^{(j)} + \vec{o}_{k+j}^{(j)} + \sum_{i=0}^{n+j-1}b_{i+1}^{(j)}\omega_{k-n+i},$$
$$k = n, n+1, n+2, \cdots \tag{5}$$

where

$$\vec{\nu}_{k+j}^{(j)} \triangleq D\nu_{k+j} + \sum_{i=0}^{n-1}\alpha_i^{(j)}D\nu_{k-n+i},$$
$$\vec{o}_{k+j}^{(j)} \triangleq \bar{o}_{k+j} + \sum_{i=0}^{n-1}\alpha_i^{(j)}\bar{o}_{k-n+i},$$

and the parameters $\alpha_i^{(j)}$ and $b_{i+1}^{(j)}$ are computed recursively by

$$\alpha_i^{(j)} \triangleq \begin{cases}\alpha_{i-1}^{(j-1)} - \alpha_{n-1}^{(j-1)}\alpha_i, & 1\leq i\leq n-1\\ -\alpha_{n-1}^{(j-1)}\alpha_0, & i=0\end{cases}, \quad j = 1, 2, \cdots$$
$$b_{i+1}^{(j)} \triangleq \begin{cases}b_i^{(j-1)} - \alpha_{n-1}^{(j-1)}b_{i+1}, & 1\leq i\leq n-1\\ -\alpha_{n-1}^{(j-1)}b_1, & i=0\\ b_i^{(j-1)}, & n+j-1\geq i\geq n\end{cases}, \quad j = 1, 2, \cdots$$

with the initial variables $\alpha_i^{(0)} \triangleq \alpha_i$ and $b_{i+1}^{(0)} \triangleq b_{i+1}$.

Proof: First, let us consider the structure of $\vec{A}$. It follows from the definition of $\vec{A}$ that

$$\vec{A}Q = QA = \begin{bmatrix}(\varepsilon_1 A)^T & (\varepsilon_2 A)^T & \cdots & (\varepsilon_n A)^T\end{bmatrix}^T$$
$$= \begin{bmatrix}1 & \alpha_{n-1} & \alpha_{n-2} & \cdots & \alpha_2 & \alpha_1\\ 0 & 1 & \alpha_{n-1} & \cdots & \alpha_3 & \alpha_2\\ 0 & 0 & 1 & \cdots & \alpha_4 & \alpha_3\\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 1 & \alpha_{n-1}\\ 0 & 0 & 0 & \cdots & 0 & 1\end{bmatrix}\begin{bmatrix}CA^n\\ CA^{n-1}\\ \vdots\\ CA^2\\ CA\end{bmatrix} \tag{6}$$

By applying the Cayley-Hamilton theorem, we have

$$\begin{aligned}\varepsilon_1 A &= C\big(A^n + \alpha_{n-1}A^{n-1} + \cdots + \alpha_1 A + \alpha_0 I\big) - \alpha_0 C\\ &= -\alpha_0\varepsilon_n\\ \varepsilon_2 A &= C\big(A^{n-1} + \alpha_{n-1}A^{n-2} + \cdots + \alpha_2 A + \alpha_1 I\big) - \alpha_1 C\\ &= \varepsilon_1 - \alpha_1\varepsilon_n\\ &\ \ \vdots\\ \varepsilon_{n-1}A &= CA^2 + \alpha_{n-1}CA + \alpha_{n-2}C - \alpha_{n-2}C\\ &= \varepsilon_{n-2} - \alpha_{n-2}\varepsilon_n\\ \varepsilon_n A &= CA + \alpha_{n-1}C - \alpha_{n-1}C = \varepsilon_{n-1} - \alpha_{n-1}\varepsilon_n\end{aligned} \tag{7}$$

Hence, it follows from (6) and (7) that

$$\vec{A}Q = \begin{bmatrix}0 & 0 & \cdots & 0 & -\alpha_0\\ 1 & 0 & \cdots & 0 & -\alpha_1\\ 0 & 1 & \cdots & 0 & -\alpha_2\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & -\alpha_{n-1}\end{bmatrix}\begin{bmatrix}\varepsilon_1\\ \varepsilon_2\\ \vdots\\ \varepsilon_n\end{bmatrix} \tag{8}$$

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Then, it is easy to conclude from (8) and the definition of Q that

$$\vec{A}=\begin{bmatrix}0 & 0 & \cdots & 0 & -\alpha_0\\ 1 & 0 & \cdots & 0 & -\alpha_1\\ 0 & 1 & \cdots & 0 & -\alpha_2\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & 1 & -\alpha_{n-1}\end{bmatrix} \tag{9}$$

Similarly, we can obtain that $\vec{C}=\begin{bmatrix}0 & 0 & \cdots & 0 & 1\end{bmatrix}$, which implies that $y_k=\vec{x}_k^{(n)}+D\nu_k+\bar{o}_k$ where $\vec{x}_k^{(n)}$ denotes the n-th entry of the vector $\vec{x}_k$. Then, one can infer from (4) and (9) that

$$\begin{cases}\vec{x}_{k+1}^{(1)}=-\alpha_0\vec{x}_k^{(n)}+b_1\omega_k,\\ \vec{x}_{k+1}^{(2)}=-\sum_{i=0}^{1}\alpha_i\vec{x}_{k-1+i}^{(n)}+\sum_{i=0}^{1}b_{i+1}\omega_{k-1+i},\\ \quad\vdots\\ \vec{x}_{k+1}^{(n)}=-\sum_{i=0}^{n-1}\alpha_i\vec{x}_{k-n+1+i}^{(n)}+\sum_{i=0}^{n-1}b_{i+1}\omega_{k-n+1+i}.\end{cases} \tag{10}$$

For notation simplicity, we let $\vec{x}_k^{(n)}=\chi_k$. Then, it follows from (10) that

$$\chi_{k+j}+\alpha_{n-1}\chi_{k+j-1}+\alpha_{n-2}\chi_{k+j-2}+\cdots+\alpha_0\chi_{k+j-n} =\sum_{i=0}^{n-1}b_{i+1}\omega_{k+j-n+i}.$$

Noting that

$$\chi_{k+j-1}=-\sum_{i=0}^{n-1}\alpha_i\chi_{k+j-1-n+i}+\sum_{i=0}^{n-1}\omega_{k+j-1-n+i},$$

we have

$$\chi_{k+j}+\alpha_{n-1}^{(1)}\chi_{k+j-2}+\alpha_{n-2}^{(1)}\chi_{k+j-3}+\cdots+\alpha_0^{(1)}\chi_{k+j-1-n} =\sum_{i=0}^{n}b_{i+1}^{(1)}\omega_{k+j-1-n+i}.$$

where

$$\alpha_i^{(1)}\triangleq\begin{cases}\alpha_{i-1}-\alpha_{n-1}\alpha_i, & 1\le i\le n-1\\ -\alpha_{n-1}\alpha_0, & i=0\end{cases},$$

$$b_{i+1}^{(1)}\triangleq\begin{cases}b_i-\alpha_{n-1}b_{i+1}, & 1\le i\le n-1\\ -\alpha_{n-1}b_1, & i=0\\ b_i, & i\ge n\end{cases}.$$

Along the similar lines of the aforementioned calculations, we finally arrive at

$$\chi_{k+j}+\alpha_{n-1}^{(j)}\chi_{k-1}+\alpha_{n-2}^{(j)}\chi_{k-2}+\cdots+\alpha_0^{(j)}\chi_{k-n} =\sum_{i=0}^{n+j-1}b_{i+1}^{(j)}\omega_{k-n+i}. \tag{11}$$

Since $\chi_k\triangleq\vec{x}_k^{(n)}=y_k-D\nu_k-\bar{o}_k$, it follows from (11) that

$$y_{k+j}+\sum_{i=0}^{n-1}\alpha_i^{(j)}y_{k-n+i}=\vec{\nu}_k^{(j)}+\vec{o}_k^{(j)}+\sum_{i=0}^{n+j-1}b_{i+1}^{(j)}\omega_{k-n+i} \tag{12}$$

which is equivalent to (5). The proof is now complete. ■

Remark 4: By now, we have built up a special input-output model (5) by using the observable canonical form of the plant (2). The method developed in Proposition 1 can be extended to deal with the multi-output case by using the observable canonical form for multi-input-multi-output systems (e.g. the well-known Wonham type canonical form [41]).

Next, let us consider the design problem of the binary function θ_k on the detection of $\underline{t}_i$ and $\bar{t}_i$. In the following, we present a proposition to detect the appearing and disappearing moments for IMOs.

Proposition 2: Let Assumptions 1-3 hold and suppose that $\underline{T}\ge n$. Define the sequences of $\{\underline{\tau}_i\}_{i\ge1}$ and $\{\bar{\tau}_i\}_{i\ge1}$ as follows:

$$\begin{cases}\underline{\tau}_i\triangleq\min\limits_k\{k|k\ge\bar{\tau}_{i-1}+n, f_0(k)>\bar{f}\}\\ \bar{\tau}_i\triangleq\min\limits_j\{j+\underline{\tau}_i|j>0, f_j(\underline{\tau}_i)\le\bar{f}\}\end{cases} \tag{13}$$

where

$$f_j(k)\triangleq\left|y_{k+j}+\sum_{i=0}^{n-1}\alpha_i^{(j)}y_{k-n+i}\right|,\ \bar{\alpha}\triangleq\max_{0\le j\le\bar{T}}\{\bar{\alpha}_j\},$$

$$\bar{f}\triangleq\bar{\alpha}\|D\|(n+1)\bar{\nu}+\bar{b}(n+\bar{T})\bar{\omega},\ \bar{\tau}_0\triangleq0,\ \bar{b}\triangleq\max_{0\le j\le\bar{T}}\{\bar{b}_j\},$$

$$\bar{\alpha}_j\triangleq\max\{1,\max_{0\le i\le n-1}\{|\alpha_i^{(j)}|\}\},\ \bar{b}_j\triangleq\max_{0\le i\le n+j-1}\{\|b_{i+1}^{(j)}\|\}.$$

if $\underline{o}>2\bar{f}$, then the conditions $\underline{t}_i=\underline{\tau}_i$ and $\bar{t}_i=\bar{\tau}_i$ hold for all $i\ge1$.

Proof: The proof of this proposition is performed by mathematical induction.

<u>The initial step.</u> For $i=1$, it is easy to see that

$$f_0(k)\triangleq\left|\vec{\nu}_k^{(0)}+\vec{o}_k^{(0)}+\sum_{i=0}^{n-1}b_{i+1}^{(0)}\omega_{k-n+i}\right| \tag{14}$$

Noting that

$$\left|\vec{\nu}_k^{(0)}\right|\le\bar{\alpha}\|D\|\left|\sum_{i=0}^{n}\nu_{k-n+i}\right|\le\bar{\alpha}\|D\|(n+1)\bar{\nu}$$

and

$$\left|\sum_{i=0}^{n-1}b_{i+1}^{(0)}\omega_{k-n+i}\right|\le\left|\sum_{i=0}^{n+j-1}b_{i+1}^{(0)}\omega_{k-n+i}\right|\le\bar{b}(n+\bar{T})\bar{\omega},$$

we have from (14) that

$$\begin{cases}f_0(k)\le\bar{\alpha}\|D\|(n+1)\bar{\nu}+\bar{b}(n+\bar{T})\bar{\omega}=\bar{f}, & k<\underline{t}_1\\ f_0(k)\ge|\vec{o}_k^{(0)}|-\left|\vec{\nu}_k^{(0)}+\sum_{i=0}^{n+j-1}b_{i+1}^{(0)}\omega_{k-n+i}\right|, & k=\underline{t}_1\end{cases} \tag{15}$$

Since $\vec{o}_{\underline{t}_1}^{(0)}=\bar{o}_{\underline{t}_1}=o_{\underline{t}_1}$, we have that

$$f_0(\underline{t}_1)>\bar{f},$$

which implies that

$$\underline{\tau}_1\triangleq\min_k\{k|k\ge n, f_0(k)>\bar{f}\}=\underline{t}_1. \tag{16}$$

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Similarly, for any $0 \leq j \leq \bar{T}$, one can infer from (5) that

$$\begin{cases} f_j(\underline{\tau}_i) \triangleq \left| \vec{\nu}^{(j)}_{\underline{\tau}_i+j} + \vec{o}^{(j)}_{\underline{\tau}_i+j} + \sum_{i=0}^{n+j-1} b^{(j)}_{i+1}\omega_{\underline{\tau}_i-n+i} \right| \\ |\vec{\nu}^{(j)}_{\underline{\tau}_i+j}| \leq \bar{\alpha}\|D\|(n+1)\bar{\nu} \\ \left| \sum_{i=0}^{n+j-1} b^{(j)}_{i+1}\omega_{\underline{\tau}_i-n+i} \right| \leq \bar{b}(n+\bar{T})\bar{\omega} \end{cases} \tag{17}$$

Noticing that $\underline{\tau}_1 \triangleq \underline{t}_1$ and

$$\vec{o}^{(j)}_{\underline{\tau}_1+j} \triangleq \begin{cases} \bar{o}_{t_1+j} = o_{t_1+j}, & \underline{t}_1 \leq \underline{\tau}_1 + j < \bar{t}_1 \\ 0, & \underline{\tau}_i + j = \bar{t}_1 \end{cases}$$

we obtain from (17) that

$$\begin{cases} f_j(\underline{\tau}_1) \geq |o_{\underline{t}_1+j}| - \left| \vec{\nu}^{(j)}_{\underline{\tau}_1+j} + \sum_{i=0}^{n+j-1} b^{(j)}_{i+1}\omega_{\underline{\tau}_1-n+i} \right| \\ \qquad > \bar{f}, \qquad \underline{t}_1 \leq \underline{\tau}_1 + j < \bar{t}_1 \\ f_j(\underline{\tau}_1) \leq \left| \vec{\nu}^{(j)}_{\underline{\tau}_1+j} + \sum_{i=0}^{n+j-1} b^{(j)}_{i+1}\omega_{\underline{\tau}_1-n+i} \right| \leq \bar{f}, \ \underline{\tau}_1 + j = \bar{t}_1 \end{cases}$$

which implies that

$$\bar{\tau}_1 \triangleq \min_j\{j + \underline{\tau}_1 | j > 0, f_j(\underline{\tau}_1) \leq \bar{f}\} = \bar{t}_1 \tag{18}$$

As such, it is immediately known from (13) that the conditions $\underline{t}_i = \underline{\tau}_i$ and $\bar{t}_i = \bar{\tau}_i$ hold for $i = 1$.

The inductive step. We know that the assertion of this proposition is true for $i = 1$. Now, letting this assertion is true for $i = N$ (i.e. $\underline{t}_N = \underline{\tau}_N$ and $\bar{t}_N = \bar{\tau}_N$), it remains to show that the same assertion is true for $i = N + 1$.

Along the similar lines of the initial step, it is concluded that

$$\begin{cases} f_0(k) \leq \bar{f}, & \bar{t}_N + n \leq k < \underline{t}_{N+1} \\ f_0(k) > \bar{f}, & k = \underline{t}_{N+1} \end{cases}$$

which means that

$$\underline{\tau}_{N+1} \triangleq \min_k\{k | k \geq \bar{\tau}_N + n, f_0(k) > \bar{f}\} = \underline{t}_{N+1} \tag{19}$$

Similar to the proof in the initial step, it follows from (19) that

$$\begin{cases} f_j(\underline{\tau}_{N+1}) > \bar{f}, & \underline{t}_{N+1} \leq \underline{\tau}_1 + j < \bar{t}_{N+1} \\ f_j(\underline{\tau}_{N+1}) \leq \bar{f}, \ \underline{\tau}_{N+1} + j = \bar{t}_{N+1} \end{cases}$$

which indicates that

$$\bar{\tau}_{N+1} \triangleq \min_j\{j + \underline{\tau}_{N+1} | j > 0, f_j(\underline{\tau}_{N+1}) \leq \bar{f}\} = \bar{t}_{N+1} \tag{20}$$

To this end, by the induction, it is concluded that conditions $\underline{t}_i = \underline{\tau}_i$ and $\bar{t}_i = \bar{\tau}_i$ hold for all $i \geq 1$. The proof is now complete. ∎

Remark 5: In this paper, the outliers under consideration are assumed to take place on an intermittent basis for which the duration length is not more than a certain given constant. It is worth noting that, the "detection" method proposed in Proposition 2 can be easily applied to the case where the measurement outputs are corrupted by impulsive outliers through simply setting the upper bound of the duration length as 1 (i.e. $\bar{T} = 1$).

Based on the derived sequences $\{\underline{\tau}_i\}_{i\geq 1}$ and $\{\bar{\tau}_i\}_{i\geq 1}$, the binary function θ_k is designed as follows:

$$\theta_k \triangleq \begin{cases} 1, & \text{if } \{i | \underline{\tau}_i \leq k < \bar{\tau}_i\} \neq \emptyset \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

Then, it is seen from Proposition 2 that

$$\theta_k = \sum_{i=1}^{\infty} \big(\Gamma(k - \underline{t}_i) - \Gamma(k - \bar{t}_i)\big). \tag{22}$$

According to (22), it is easy to see that

$$(1-\theta_k)\bar{o}_k = \left(1 - \sum_{i=1}^{\infty} \big(\Gamma(k - \underline{t}_i) - \Gamma(k - \bar{t}_i)\big)\right)\bar{o}_k.$$

Noting that

$$\sum_{i=1}^{\infty} \big(\Gamma(k - \underline{t}_i) - \Gamma(k - \bar{t}_i)\big) \sum_{i=1}^{\infty} \big(\Gamma(k - \underline{t}_i) - \Gamma(k - \bar{t}_i)\big)$$
$$= \sum_{i=1}^{\infty} \big(\Gamma(k - \underline{t}_i) - \Gamma(k - \bar{t}_i)\big),$$

which implies that

$$(1-\theta_k)\bar{o}_k = \bar{o}_k - \bar{o}_k = 0.$$

Hence, we have

$$\begin{aligned} & L(\theta_k)(y_k - C\hat{x}_k) \\ =& (1-\theta_k)K(Cx_k + D\nu_k - C\hat{x}_k) + (1-\theta_k)K\bar{o}_k \\ =& (1-\theta_k)K(Cx_k + D\nu_k - C\hat{x}_k). \end{aligned}$$

Obviously, according to the PD state estimator (3), it is easy to find that the rejection of the outliers has been ensured based on our designed binary function θ_k.

Remark 6: In Proposition 2, we have established a method to derive the appearing moment sequence and disappearing moment sequence of the outliers based on the input-output model (5). It is easy to find that the method proposed in Proposition 2 is effective for "large outliers" (i.e. the outliers satisfying $\underline{o} > 2\bar{f}$). In practical applications, the "small outliers" can be regarded as a class of norm-bounded noises. Obviously, the corresponding measurements contaminated by such small outliers would not dramatically deteriorate the estimation performance even if these measurements are involved in the estimation process, which implies that the constructed state estimator (3) is still effective to guarantee the desired estimation performance by selecting the suitable estimation parameter.

B. Design of the estimator parameter K

Now, we are ready to consider the error dynamics of the state estimation. Let the state estimation error and output estimation error be $e_k \triangleq x_k - \hat{x}_k$ and $\tilde{z}_k \triangleq z_k - \hat{z}_k$. According

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to the proposed PD state estimator (3) and Proposition 2, the estimation error dynamics is described as follows:

$$\begin{cases} e_{k+1} = Ae_k + B\omega_k - (1-\theta_k)K(y_k - C\hat{x}_k) \\ \qquad = \begin{cases} (A-KC)e_k + B\omega_k - KD\nu_k, & \text{if } \theta_k = 0 \\ Ae_k + B\omega_k, & \text{if } \theta_k = 1 \end{cases} \\ \tilde{z}_k = Me_k \end{cases} \tag{23}$$

In light of the estimation error dynamics (23), sufficient conditions are derived in the following theorem to ensure the desired estimation performance specified in this paper.

Theorem 1: Consider the estimation error dynamics (23). Let Assumptions 1-3 and $\underline{T} \geq n$ hold. For a given estimator gain matrix K, if there exist a positive definite matrix $P \in \mathbb{R}^{n\times n}$ and two positive constants μ_i $(i = 1,2)$ satisfying

$$\Upsilon_1 \triangleq \begin{bmatrix} \Upsilon_1^{11} & \Upsilon_1^{12} & \Upsilon_1^{13} \\ * & \Upsilon_1^{22} & \Upsilon_1^{23} \\ * & * & \Upsilon_1^{33} \end{bmatrix} < 0 \tag{24}$$

$$\Upsilon_2 \triangleq \begin{bmatrix} \Upsilon_2^{11} & \Upsilon_2^{12} \\ * & \Upsilon_2^{22} \end{bmatrix} < 0 \tag{25}$$

$$\gamma^2 P \geq (1+\mu_2)^{\bar{T}} M^T M \tag{26}$$

$$(1+\mu_2)^{\bar{T}}(1-\mu_1)^{\underline{T}} < 1 \tag{27}$$

where

$$\begin{aligned} &\Upsilon_1^{11} \triangleq (A-KC)^T P(A-KC) - (1-\mu_1)P, \\ &\Upsilon_1^{12} \triangleq (A-KC)^T PB, \quad \Upsilon_1^{13} \triangleq -(A-KC)^T PKD, \\ &\Upsilon_1^{22} \triangleq B^T PB - I, \quad \Upsilon_1^{33} \triangleq D^T K^T PKD - I, \\ &\Upsilon_1^{23} \triangleq -B^T PKD, \quad \Upsilon_2^{11} \triangleq A^T PA - (1+\mu_2)P, \\ &\Upsilon_2^{12} \triangleq A^T PB, \quad \Upsilon_2^{22} \triangleq B^T PB - (1+\mu_2)I, \end{aligned}$$

then the dynamics of the estimation error system (23) is asymptotically stable with a prescribed energy-to-peak performance index γ.

Proof: First, we consider the asymptotic stability of the estimation error system (23) under the conditions in Theorem 1. Construct the Lyapunov function as follows:

$$V_k \triangleq e_k^T P e_k$$

Then, for any $\bar{t}_i < k \leq \underline{t}_{i+1}$ $(i \in \mathbb{N}^+)$, we have $\theta_{k-1} = 0$, which indicates that

$$\begin{aligned} &\Delta V_k = V_k - V_{k-1} \\ &= \big((A-KC)e_{k-1} + B\omega_{k-1} - KD\nu_{k-1}\big)^T P\big((A \\ &\quad - KC)e_{k-1} + B\omega_{k-1} - KD\nu_{k-1}\big) - e_{k-1}^T P e_{k-1} \\ &= \begin{bmatrix} e_{k-1} \\ \omega_{k-1} \\ \nu_{k-1} \end{bmatrix}^T \Upsilon_1 \begin{bmatrix} e_{k-1} \\ \omega_{k-1} \\ \nu_{k-1} \end{bmatrix} - \mu_1 V_{k-1} + \begin{bmatrix} \omega_{k-1} \\ \nu_{k-1} \end{bmatrix}^T \begin{bmatrix} \omega_{k-1} \\ \nu_{k-1} \end{bmatrix} \\ &\leq -\mu_1 V_{k-1} + \begin{bmatrix} \omega_{k-1} \\ \nu_{k-1} \end{bmatrix}^T \begin{bmatrix} \omega_{k-1} \\ \nu_{k-1} \end{bmatrix}, \end{aligned} \tag{28}$$

from which we have

$$\begin{aligned} V_k &\leq (1-\mu_1)V_{k-1} + \begin{bmatrix} \omega_{k-1} \\ \nu_{k-1} \end{bmatrix}^T \begin{bmatrix} \omega_{k-1} \\ \nu_{k-1} \end{bmatrix} \\ &\leq (1-\mu_1)^2 V_{k-2} + \sum_{j=k-2}^{k-1} (1-\mu_1)^{k-1-j} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \\ &\quad \vdots \\ &\leq (1-\mu_1)^{k-\bar{t}_i} V_{\bar{t}_i} + \sum_{j=\bar{t}_i}^{k-1} (1-\mu_1)^{k-1-j} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \\ &\leq (1-\mu_1)^{k-\bar{t}_i} V_{\bar{t}_i} + \sum_{j=\bar{t}_i}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \end{aligned} \tag{29}$$

On the other hand, for any $\underline{t}_{i+1} < k \leq \bar{t}_{i+1}$, we have $\theta_{k-1} = 1$, which means that

$$\begin{aligned} &\Delta V_k = V_k - V_{k-1} \\ &= \big(Ae_{k-1} + B\omega_{k-1}\big)^T P\big(Ae_{k-1} + B\omega_{k-1}\big) - e_{k-1}^T P e_{k-1} \\ &= \begin{bmatrix} e_{k-1} \\ \omega_{k-1} \end{bmatrix}^T \Upsilon_2 \begin{bmatrix} e_{k-1} \\ \omega_{k-1} \end{bmatrix} + \mu_2 V_{k-1} + (1+\mu_2)\|\omega_{k-1}\|^2 \\ &\leq \mu_2 V_{k-1} + (1+\mu_2)\|\omega_{k-1}\|^2 \end{aligned}$$

Similarly, it follows that

$$\begin{aligned} V_k &\leq (1+\mu_2)V_{k-1} + (1+\mu_2)\|\omega_{k-1}\|^2 \leq \cdots \\ &\leq (1+\mu_2)^{k-\underline{t}_{i+1}} V_{\underline{t}_{i+1}} + \sum_{j=\underline{t}_{i+1}}^{k-1} (1+\mu_2)^{k-j} \|\omega_j\|^2 \\ &\leq (1+\mu_2)^{k-\underline{t}_{i+1}} V_{\underline{t}_{i+1}} + (1+\mu_2)^{\bar{T}} \sum_{j=\underline{t}_{i+1}}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \end{aligned} \tag{30}$$

Hence, for any $\underline{t}_{i+1} < k \leq \bar{t}_{i+1}$, it follows from (29) and (30) that

$$\begin{aligned} V_k &\leq (1+\mu_2)^{k-\underline{t}_{i+1}} V_{\underline{t}_{i+1}} + (1+\mu_2)^{\bar{T}} \sum_{j=\underline{t}_{i+1}}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \\ &\leq (1+\mu_2)^{\bar{T}} \left((1-\mu_1)^{\underline{T}} V_{\bar{t}_i} + \sum_{j=\bar{t}_i}^{\underline{t}_{i+1}-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \right) \\ &\quad + (1+\mu_2)^{\bar{T}} \sum_{j=\underline{t}_{i+1}}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \\ &\leq \beta V_{\bar{t}_i} + (1+\mu_2)^{\bar{T}} \sum_{j=\bar{t}_i}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \end{aligned} \tag{31}$$

where $\beta \triangleq (1+\mu_2)^{\bar{T}}(1-\mu_1)^{\underline{T}} < 1$. Then, it is seen from (29) and (31) that the inequality

$$V_k \leq \beta V_{\bar{t}_i} + (1+\mu_2)^{\bar{T}} \sum_{j=\bar{t}_i}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \tag{32}$$

holds for all $\bar{t}_i < k \leq \bar{t}_{i+1}$ $(i \in \mathbb{N}^+)$. As such, when assuming zero disturbances (i.e. $\omega_k = 0$ and $\nu_k = 0$ for all $k \geq 0$), we have that

$$V_{\bar{t}_{i+1}} \leq \beta^i V_{\bar{t}_1},$$

which implies that $\lim_{i\to\infty} V_{\bar{t}_{i+1}} = 0$. Noting that $V_k \leq \beta V_{\bar{t}_i}$ where $\bar{t}_i \triangleq \max\{\bar{t}_i | k > \bar{t}_i\}$, it follows that $\lim_{k\to\infty} V_k = 0$.

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Therefore, we conclude that the estimation error system (23) is asymptotically stable with zero disturbances.

Next, let us consider the energy-to-peak performance of the estimation error system (23). Similarly, we can obtain that

$$V_k < \beta V_0 + (1+\mu_2)^{\bar{T}} \sum_{j=0}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2$$

for all $0 < k \le \bar{t}_1$. This, together with (32), implies that the following is true for all $k > 0$:

$$V_k < V_0 + (1+\mu_2)^{\bar{T}} \sum_{j=0}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 \tag{33}$$

Consider the zero initial condition, it follows from (26) that $\|\tilde{z}_k\|^2 \le \gamma^2 (1+\mu_2)^{-\bar{T}} V_k$, which implies that

$$\|\tilde{z}_k\|^2 < \gamma^2 \sum_{j=0}^{k-1} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2 < \gamma^2 \sum_{j=0}^{\infty} \left\| \begin{bmatrix} \omega_j \\ \nu_j \end{bmatrix} \right\|^2$$

Finally, taking the supremum of $\|\tilde{z}_k\|^2$ over time k gives rise to $\|\tilde{z}_k\|_\infty^2 < \gamma^2 \sum_{k=0}^{\infty} (\|\omega_k\|^2 + \|\nu_k\|^2)$ for any non-zero $\omega_k \in l_2([0,\infty), \mathbb{R}^r)$ and $\nu_k \in l_2([0,\infty), \mathbb{R}^s)$. The proof is now complete. ■

Remark 7: So far, we have designed the binary function θ_k and also obtained sufficient conditions that guarantee the asymptotic stability as well as the prescribed energy-to-peak performance of the estimation error dynamics. Based on the derived input-output model (5), we can identify the appearing and disappearing moments for each IMO exactly via a fixed number of past measurements. Apparently, it can be found from Proposition 2 and Theorem 1 that all the important factors contributing to the system complexity have been reflected in the main results. These factors include the system parameters, noise information (upper bounds), energy-to-peak performance index (l_2-l_∞ disturbance attenuation level) and the outlier information (the upper bound of the duration length, the lower bound of the interval length, the smallest magnitude of the outliers). In addition, when it comes to the algorithm implementation, we make the following observations according to Theorem 1.

1) $\mu_1 > 0$ should be selected to satisfy the condition $0 < \mu_1 < 1$ in order to guarantee the feasibility of the matrix inequality (24).
2) The feasibility of the matrix inequality constraints (24)-(27) is affected by the values of $\bar{T}$ and $\underline{T}$. Obviously, decreasing the value of $\bar{T}$ and increasing the value of $\underline{T}$ would enhance the feasibility of the constraints in Theorem 1, which means that a small upper bound of the duration length and a large lower bound of the interval length would help improve the desired estimation performance.
3) Note that the designed algorithm for θ_k and the obtained sufficient conditions in Theorem 1 are independent of the magnitude of the outlier o_k^i. As such, our developed estimation scheme is applicable in handling unbounded measurement outliers.

In Theorem 1, sufficient conditions have been obtained to guarantee the asymptotic stability and the prescribed energy-to-peak performance of the estimation error dynamics subject to the disturbances and the IMOs. Note that the derived conditions in Theorem 1 are described by several nonlinear matrix inequalities, which are quite hard to solve. In what follows, we are going to develop an algorithm to deal with the design of the estimator gain K.

Before presenting the algorithm, let us first give the following corollary which will be employed in the algorithm.

Corollary 1: Consider the estimation error dynamics (23). Suppose that Assumptions 1-3 hold and $\underline{T} \ge n$. Let two given scalars $0 < \mu_1 < 1$ and $\mu_2 > 0$ satisfy the constraint (27). Assume that there exist a positive definite matrix P, a matrix $\bar{K} \in \mathbb{R}^{n\times m}$ and a positive scalar $\bar{\gamma}$ satisfying the constraint (25) and the following matrix inequalities

$$\bar{\Upsilon}_1 = \begin{bmatrix} \bar{\Upsilon}_1^{11} & 0 & 0 & \bar{\Upsilon}_1^{14} \\ * & \bar{\Upsilon}_1^{22} & 0 & \bar{\Upsilon}_1^{24} \\ * & * & \bar{\Upsilon}_1^{33} & \bar{\Upsilon}_1^{34} \\ * & * & * & \bar{\Upsilon}_1^{44} \end{bmatrix} < 0 \tag{34}$$

$$\bar{\gamma}(1+\mu_2)^{\bar{T}} M^T M \le P \tag{35}$$

where

$$\bar{\Upsilon}_1^{11} = -(1-\mu_1)P,\ \bar{\Upsilon}_1^{14} = A^T P - C^T \bar{K}^T,\ \bar{\Upsilon}_1^{22} = -I,$$
$$\bar{\Upsilon}_1^{24} = B^T P,\ \bar{\Upsilon}_1^{33} = -I,\ \bar{\Upsilon}_1^{34} = D^T \bar{K}^T,\ \bar{\Upsilon}_1^{44} = -P.$$

Then, the estimation error dynamics (23) is asymptotically stable with a prescribed energy-to-peak performance index $\gamma = \bar{\gamma}^{-0.5}$. Furthermore, the minimum energy-to-peak performance index can be derived by solving the following optimization problem:

$$\max\{\bar{\gamma}\} \tag{36}$$

subject to the matrix inequality constraints (25), (34) and (35). An admissible estimator in the form of (3) is determined by the following gain matrix:

$$K = P^{-1}\bar{K} \tag{37}$$

Proof: The proof is straightforward based on Theorem 1 and Schur complement lemma, and is therefore omitted here for space saving. ■

By means of Corollary 1, we propose a Particle-Swarm-Optimization-based Estimator Parameter Design (PSObEPD) algorithm as follows.

Remark 8: In this paper, we have investigated the energy-to-peak state estimation problem for time-invariant systems with the measurements corrupted by intermittent outliers. The distinctive novelty of this work lies on the following three aspects: 1) a special detection approach has been developed, based on the observable canonical form of the plant, to distinguish the measurement outputs corrupted by outliers from the normal measurements; 2) a PD state estimator has been designed to ensure the “rejection” of the IMOs; and 3) the energy-to-peak state estimation performance has been achieved by selecting the estimator gain matrix according to a special algorithm (the particle-swarm-optimization-based estimator parameter design algorithm).

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Algorithm PSObEPD:

Step 1. Initialization: let $X_0^i \triangleq \begin{bmatrix}\mu_1 & \mu_2\end{bmatrix}^T$ be the location of the at the i-th particle initial step. Generate s particles under which the matrix inequalities (25), (27), (34) and (35) are feasible. The velocity of the i-th particle is set to be S_0^i. Let the maximum number of iterations be $k_{\max}$.

Step 2. Let the local best location of the i-th particle and the global best location at step k be $P_L^i = X_0^i$ and $P_G^k = 0$, respectively. Let the iteration step k be 0.

Step 3. Update the values of P_L^i and P_G^k: for the i-th particle, solve the optimization problem (36) subject to (25), (34) and (35), and let the solution to the optimization problem (36) be $\eta(X_k^i)$. Then, update the values of P_L^i and P_G^k by $P_L^i = \max_k \{\eta(X_k^i)\}$ and $P_G^k = \max_i \{P_L^i\}$, respectively. If $\|P_G^k - P_G^{k-1}\| < \xi$ where ξ is a given small positive scalar representing the computation accuracy, go to *Step 7*.

Step 4. Update the values of S_k^i and X_k^i as follows:
$S_{k+1}^i = \varphi S_k^i + \upsilon_1 r_i(P_L^i - X_k^i) + \upsilon_2 r_g(P_G^k - X_k^i)$
$X_{k+1}^i = X_k^i + S_{k+1}^i$
where φ, υ_1 and υ_2 are the given inertia parameter and two momentum parameters, respectively. r_i and r_g are the random numbers between $(0, 1)$. Let $k = k + 1$.

Step 5. Based on the values of μ_1^i and μ_2^i according to the i-th particle X_k^i, if the matrix inequalities (25), (27), (34) and (35) are infeasible, let $X_k^i = X_{k-1}^i$ and $S_k^i = S_{k-1}^i$.

Step 6. If $k < k_{\max}$, go back to *Step 3*, else update the values of P_L^i and P_G^k by $P_L^i = \max_k \{\eta(X_k^i)\}$ and $P_G^k = \max_i \{P_L^i\}$, respectively, and turn to the next step.

Step 7. Solve the optimization problem (36) subject to (25), (34) and (35) according to P_G^k. Calculate the desired estimator parameter by $K = P^{-1}\bar{K}$. Stop.

IV. AN ILLUSTRATIVE EXAMPLE

In order to verify the effectiveness and correctness of the our developed PD state estimation design scheme, in this section, we shall provide a numerical example.

To render our simulation convincing, we adopt an *unstable* linear system of the form (2) with the following parameters:

$$A = \begin{bmatrix} 0.67 & 0.42 \\ 0.33 & 0.62 \end{bmatrix}, \quad B = \begin{bmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}, \quad C = \begin{bmatrix} 0.9 & 0.6 \end{bmatrix},$$
$$D = 1, \quad M = 0.35I.$$

According to the above matrices, it is observed that (2) is observable. The process noise and measurement noise are chosen as follows:

$$\omega_k = \begin{cases} \dfrac{\bar{\omega}}{\sqrt{2}} \begin{bmatrix} \sin(10r_1(k)) \\ \sin(10r_2(k)) \end{bmatrix}, & \text{if } 0 \le k \le 150 \\ 0 & , \text{otherwise} \end{cases}$$
$$\nu_k = \begin{cases} \bar{\nu}\cos(5r_3(k)), & \text{if } 0 \le k \le 150 \\ 0 & , \text{otherwise} \end{cases}$$

where $\{r_i(k)\}_{i=1,2,3}$ are three random numbers at time instant k satisfying $r_i(k) \in [0, 1]$, $\bar{\omega} = 0.4$ and $\bar{\nu} = 0.3$. Obviously, we have $\|\omega_k\|^2 \le \bar{\omega}^2$ and $\|\nu_k\|^2 \le \bar{\nu}^2$, which means that the energy-bounded noises ω_k and ν_k satisfy the condition in Assumption 2.

In this example, the lower bound of the interval length and upper bound of the duration length for each IMO are set as $\underline{T} = 2$ and $\bar{T} = 3$, respectively. Based on the results in Proposition 2, the values of $\alpha_i^{(j)}$ $(i = 0, 1,\ j = 0, 1, 2, 3)$ and $b_{i+1}^{(j)}$ $(j = 0, 1, 2, 3,\ 0 \le i \le j+1)$ are calculated and the "threshold" $\bar{f}$ is given as follows:

$$\bar{f} = \bar{\alpha}\|D\|(n+1)\bar{\nu} + \bar{b}(n+\bar{T})\bar{\omega} = 3.616.$$

Let the lower bound of the measurement outlier be $\underline{o} = 2.1\bar{f}$. Then, based on the design approach of the binary function θ_k described in Proposition 2, the values of $\{\theta_k\}_{k\ge 0}$ are shown in Fig. 1. It is easy to see that our designed "detection" strategy is capable of exactly "identifying" the appearing and disappearing moments of each IMO.

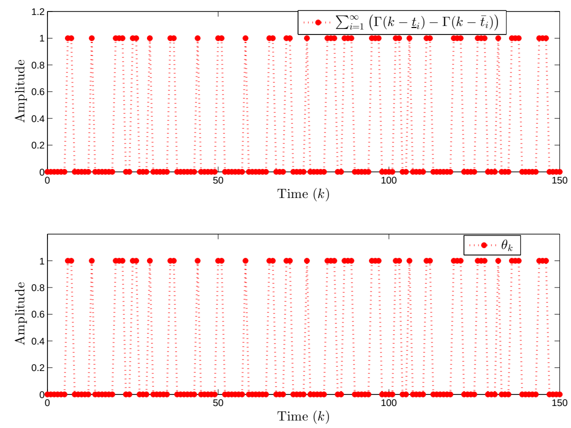

Fig. 1: The appearing moments and disappearing moments of IMOs as well as the values of $\{\theta_k\}_{k\ge 0}$

Next, we shall deal with the design of the gain matrix K of the PD state estimator by applying the developed Algorithm PSObEPD. Set the inertia parameter and two momentum parameters as $\varphi = 0.6$, $\upsilon_1 = 0.7$ and $\upsilon_2 = 0.7$, respectively. Then, the global best location derived by the algorithm is given as follows:

$$\mu_1 = 0.635, \quad \mu_2 = 0.573.$$

The corresponding minimum energy-to-peak performance index is calculated as $\gamma = 0.95$, and the estimator gain matrix K is given by

$$K = \begin{bmatrix} 0.623 \\ 0.525 \end{bmatrix}. \tag{38}$$

Based on the developed estimator parameter and the binary function θ_k, numerical simulation results are shown in Figs. 2-3. All the simulation results confirm that the performance of our developed PD state estimator is well achieved.

In order to show the superiority of our developed PD state estimation scheme, let us now compare it with the traditional Luenberger-observer-based estimation method. In this simulation example, the gain matrix of the Luenberger observer is set to be K, which is exactly the same parameter of our PD estimator when $\theta_k = 0$. The trajectories of $\|\tilde{z}_k\|$ under different estimation schemes are shown in Fig. 4. It is confirmed from Fig. 4 that our developed PD state estimation scheme outperforms the traditional Luenberger-observer-based estimation, which is simply because our developed state estimation algorithm caters for the "rejection" of the IMOs in the estimation process. Table I shows the values of $\|\tilde{z}_k\|_\infty^2$ subject to different estimation schemes (our PD estimation

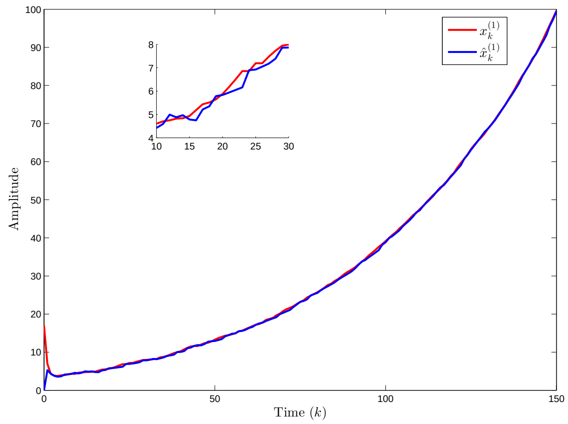

Fig. 2: The state trajectories of $x_k^{(1)}$ and $\hat{x}_k^{(1)}$

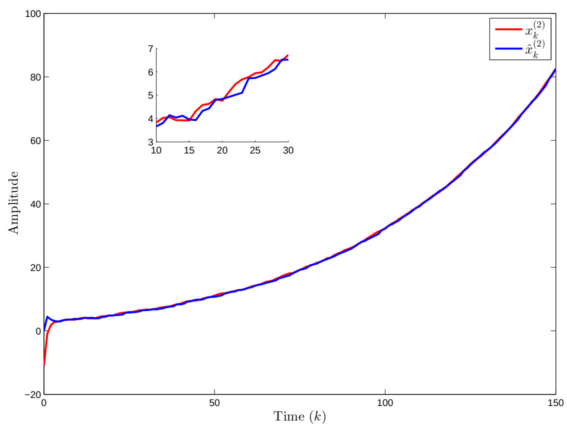

Fig. 3: The state trajectories of $x_k^{(2)}$ and $\hat{x}_k^{(2)}$

and the Luenberger-observer-based estimation), respectively. Obviously, our developed PD estimation scheme performs better by achieving a much smaller peak value of $\|\tilde{z}_k\|$.

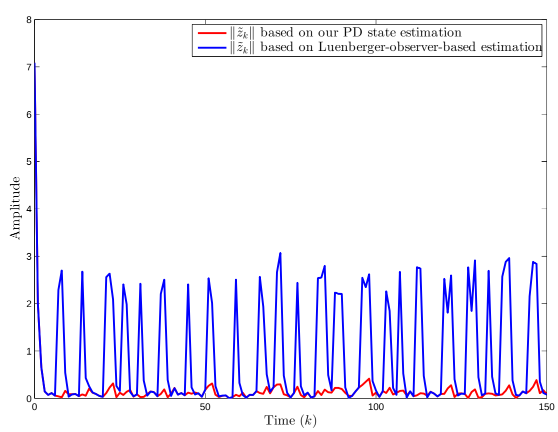

Fig. 4: The trajectories of $\|\tilde{z}_k\|$ under different estimation approaches

V. Conclusion

In this paper, the energy-to-peak state estimation problem has been studied for a class of discrete-time linear systems subject to IMOs. In order to illustrate the intermittent nature of the outliers, the occurrences of the measurement outliers have been characterized by a sequence of shifted gate functions. Furthermore, the norm of the outlier is assumed to be larger than certain known scalar. A novel PD state estimator has been constructed to deal with the estimation task based on an active detection-based framework, under which the "harmful" measurements (i.e. the measurements corrupted by outliers) would be discarded. A special outlier detection strategy has been developed to distinguish the measurements corrupted by outliers from those normal measurements. Then, sufficient conditions have been derived to guarantee the asymptotic stability and energy-to-peak performance requirement of the estimation error dynamics. A Particle-Swarm-Optimization-based algorithm has been utilized to compute the desired estimator parameter. Finally, a numerical simulation example has been used to demonstrate the effectiveness of our developed PD state estimation scheme. Further research topics include the extension of the main results to 1) the distributed state estimation problem for discrete-time systems with IMOs [11]; 2) the set-membership state estimation problem for linear time-varying systems subject to IMOs [39], [42]; 3) state estimation for time-delayed systems with gain variations subject to IMOs [12]; 4) state estimation for neural networks subject to IMOs [27]–[29]; and 5) the improvement of the state estimation performance by using some latest optimization algorithms [25], [26].

TABLE I: The values of $\|\tilde{z}_k\|_\infty^2$ subject to different estimation schemes

	$\|\tilde{z}_k\|_\infty^2$
PD estimation	0.13728
Luenberger-observer-based estimation	8.74303

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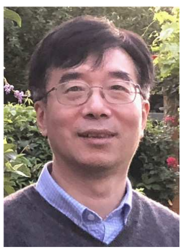

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