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Slope stability analysis via Discrete Element Method and Monte Carlo Simulations

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Abstract. It is difficult to consider the heterogeneity and discontinuity of rock and soil in conventional slope stability analysis. This paper proposes an efficient slope stability analysis method based on discrete element method (DEM) and Monte Carlo simulations(MCS). The analysis employed the stacking models with multiple slopes and the element displacement to achieve rapid analysis of slope stability. Using on the high-performance discrete element software MatDEM, a stacking model containing 100 2D slopes and a total of more than 800,000 elements was generated. The element tensile strength F and friction coefficient μ_p were set as random variables to consider the material heterogeneity and uncertainty. We performed the stability analyses of 40,000 slopes for 28 hours. The numerical results show that the slopes were instable when a sudden displacement and penetrating sliding surface occurred. The coefficient of variation COV_{Pr} of slope failure probability is greatly meets the target accuracy requirements of MCS, which shows the method can be applied to the refined analysis of slopes with high-precision requirements and multiple random variables.

Keywords: Slope stability analysis, Monte Carlo, DEM

1 Introduction

Slope stability analysis is an important issue of engineering geology. Since Crawford and Eden (1967) first introduced the reliability theory to slope stability analysis, it has been applied in slope protection designs and stability evaluations[[1]][[2]]. Currently, there are several reliability analysis methods, such as the first order reliability analysis method, Monte-Carlo simulation(MCS) method, response surface method, etc., among which MCS is the most popular one [3].

Conventional slope MCS is often combined with limit equilibrium methods (LEM) and finite element methods (FEM)[4]. Based on the four limit equilibrium theories of Fellenius method, Bishop method, Janbu method and Morgenstern-Price method, Wang et al.[5] conducted MCS of slope stability analysis and contrastively analyzed the failure probability of the slope. Zuo et al.[6]calculated the safety factor of expansive soil to evaluate the slope stability applying the Swedish Circle Method. In practical applications, the LEM requires pre-determination of the shape and position of the sliding surface, which depends on engineering experience with wide deviations[7]. Griffiths et al.[8]used the random finite element method (RFEM) to perform slope MCS and automatically search for the most dangerous sliding surface. Compared with the LEM, the FEM is more sophisticated and suitable for calculating homogeneous models and continuum problems, but difficult to simulate heterogeneous bodies and large deformation failure.



The rock and soil are composed of discrete particles, void spaces and cracks, with complex mechanical characteristics such as nonlinearity, discontinuity, heterogeneity and anisotropy. The discontinuous nature of rock and soil can hardly be captured by conventional slope MCS, while the discrete element method (DEM) provides an alternative approach forward[9]. In the past decades, many researchers have tried to use DEM to simulate slope failure. Gianvito et al.[10] established a 3D thin shell model by MatDEM to simulate the whole evolution process of Xinmocun landslide in Maoxian County.

However, due to high computational cost, the application of DEM in slope stability analysis is mostly limited to the analysis of the initiation and propagation mechanisms of slope failure. Based on our best knowledge, there is no study on slope MCS with DEM. This research uses high-performance discrete element software MatDEM to perform slope stability analysis method based on DEM and MCS[11]. Based on the highly efficient matrix-based discrete element method, a stacking model including multiple slopes is generated and instability criterion based on element displacement is adopted to complete the huge computation of discrete element analysis. By testing the stability of 40000 slope samples, we verified the effectiveness of the proposed method as a new way for slope stability analysis.

2 MCS method based on DEM

2.1 Fundamental theory of DEM

The DEM was first proposed by Cundall[12], which reproduces rock and soil model by stacking and cementing discrete spherical elements with specific mechanical properties. In this paper, the linear elastic contact model is used. The interaction between elements includes the normal force, tangential force and sliding friction. The normal force (F_n) and normal deformation (X_n) between the elements are computed as :

$$F_n = \begin{cases} K_n X_n & X_n < X_b & \text{intact bond } a \\ K_n X_n & X_n < 0 & \text{broken bond } b \\ 0 & X_n > 0 & \text{broken bond } c \end{cases} \quad (1)$$

in Eq. (1), K_n is the normal stiffness; X_n is the normal relative displacement; X_b is the fracture displacement. Originally, the elements are connected with its neighboring elements with the tension or compression interaction force (Eq. 1a). When the normal relative displacement X_n exceeds the fracture displacement X_b , the connection is broken. After that, if the elements are subjected to tensile strength, the relative displacement X_n is greater than 0, and there is no interaction between the elements (Eq. 1c). If the elements return to the compression state, repulsive force is generated between the elements (Eq. 1b).

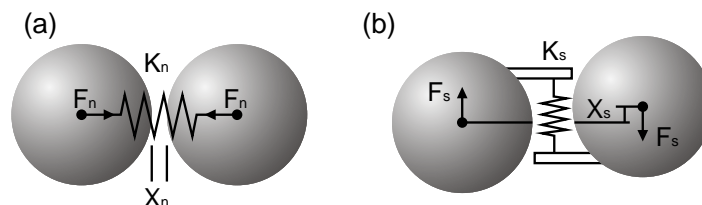


Fig. 1. Inter-particle force. (a) Normal spring force; (b) Shear force.

Similarly, tangential springs are used to simulate the shear force (F_s) and shear deformation (X_s) between elements:

$$F_s = K_s X_s \quad (2)$$

in Eq. (2), K_s is the tangential stiffness coefficient; X_s is the tangential relative displacement. The maximum tangential force $F_{s\max}$ that can withstand between elements is calculated according to the Mohr-Coulomb criterion:

$$F_{s\max} = F_{s0} - \mu_p F_n \quad (3)$$

in Eq. (3), F_{s0} is the initial shear force; μ_p is the friction coefficient; F_n is the normal force. When the inter-particle shearing force F_s exceeds the maximum shearing force $F_{s\max}$, the tangential connection is broken and F_{s0} becomes 0. At this time, there is only sliding friction - $\mu_p F_n$ between the elements, and relative sliding occurs.

2.2 Multil-slope model generation based on matrix discrete elements

Based on the fundamental principles of DEM, a numerical model can be generated. However, MCS evaluates slope stability through probability distribution, which requires the simulation of a large amount of slope samples. Generally, at least 5000~10000 times of MCS are used to meet the statistical requirements, which will requires a huge amount of calculations[13].

In this paper, the secondary development of independent high-performance discrete element software MatDEM (Matrix Discrete Element) was used to generate slope models and perform MCS[14]. MatDEM adopts innovative matrix discrete element calculation method and 3D contact algorithm, which realizes discrete element analysis at the engineering-scale. Based on the complete matrix calculation, MatDEM can complete the force and motion calculations between all elements at one time[11]. As the number of elements increases, the number of element motion calculations completed per second increases almost linearly(see Fig. 2).

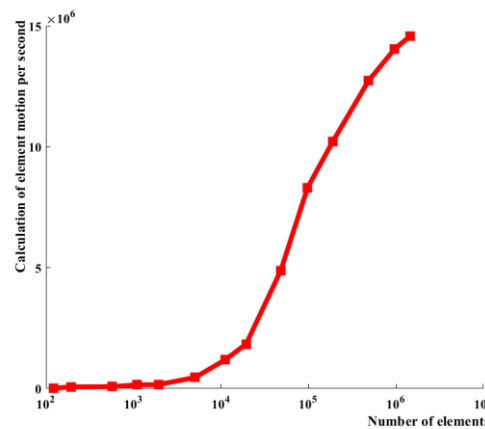


Fig. 2. Calculation speed of GPU with different numbers of elements.

Based on the calculation procedures of MatDEM and the demand for a large number of slope samples, a stacking model containing multilateral identical slopes is generated. In motion calculation, the stacking model is used as the basic calculation unit. That is to say, we will calculate the stability of multiple slopes through one MCS calculation. At the same time, because more elements are included in a single calculation, the calculation speed of MatDEM has also been increased by tens of times. It is worth noting that the slopes are separated by a distance of one element diameter, with no interaction force. The motion calculation is relatively independent.

2.3 Random variable assignment of discrete element MCS

Different from the conventional LEM and FEM, the mechanical parameters of each element in the discrete element slope model can be different, which can fully consider the heterogeneity and spatial variability of the rock and soil.

Therefore, in slope stability analysis based on DEM and MCS, the variation coefficient *Ratio* of random variables is introduced due to the different mechanical properties of each element. The random variable X is assigned *Ratio* times to realize the random change of average mechanical properties (X_0) of the slope model. The value of random variable X is calculated in Eq. (4).

$$X = X_0 * Ratio \quad (4)$$

where X_0 is the original value of the random variable X .

2.4 Instability criterion and failure probability based on element displacement

In conventional MCS of slope stability, the factor of safety is usually set as the slope state function in LEM or FEM. The instability criterion is set as the factor of safety, Z , is greater than 1. Zheng[15] pointed out that one of the criteria for soil failure is the indefinite soil movement. When the soil is broken, the strain and displacement of the soil at the sliding surface have abrupt changes and develop indefinitely. In the discrete element analysis, the elements will move individually when the bond between the elements is broken. When a large number of elements move independently, it appears as a sudden change in the slope displacement on the macroscopic level. At this time, the slope is destroyed. Therefore, in this paper, we use the element displacement of a large number of particles as instability criterion to determine whether the slope is instability or not.

In the discrete element analysis of landslides, it is usually necessary to simulate the whole process of landslide initiation, propagation and deposition, which usually takes tens of seconds to several minutes and requires a lot of time and computational cost. In the slope stability analysis, only the initiation process needs to be simulated. Because a lot of simulation tests show that: usually within 2 seconds of the simulation, it can be determined whether the main body of the slope has moved, and then according to element displacement to determine the slope state.

According to the law of large numbers, when the number of simulations, N , is large enough, the frequency of the slope state at this time can be approximated as a probability[16]. Thus, among the N slope samples, if M slopes are instable and fail, the slope failure probability is:

$$P_f = M / N \quad (5)$$

where M is the number of slopes judged to be failure; N is the total number of slope samples.

3 Numerical programme

3.1 Model configuration

This study takes a slope with a weak layer as an example to verify the effectiveness of proposed method. The slope is 42 m high and has three layers. The macro-mechanical properties of each soil layer are shown in Table 1. Complex effects such as groundwater and earthquakes are not considered in the simulation. Using the secondary development function of MatDEM, we build a slope stacking model through the following steps:

(1) Create an initial geometric model. A 2D numerical model box with a scale of 60 m*60 m was first created, within which particles are generated at random locations with an average radius of 0.2 m and a dispersion coefficient of 0.2. Then, gravity is applied to the particles to simulate particle deposition. Perform gravity compaction to achieve a compact deposition process. Finally, we get initial geometric model consisting of 23451 particles;

(2) Build a slope stacking model. The slope elevation data in Fig. 3a was used to cut the initial geometric model and the 2D slope model in Fig. 3b was obtained. Then, the 2D slope model is copied and arranged 100 times in the Y direction with a space of one element diameter. Then we get a stacking model with 100 2D slopes, which contains 853,400 particles in total(Fig. 3c).

(3) Set the initial material parameter. Based on the macro-mechanical properties of the slope and the macro-micro conversion formula proposed by Liu et al.(2013), MatDEM can automatically construct rock-soil materials with specific mechanical properties. The average micromechanical parameters of the slope elements are shown in Table 2. Note that the normal stiffness (k_n) and tangential stiffness (k_s) in Table 2 are element stiffnesses, which are equal to twice the normal stiffness (K_n) and tangential stiffness (K_s) of the connection between elements[11].

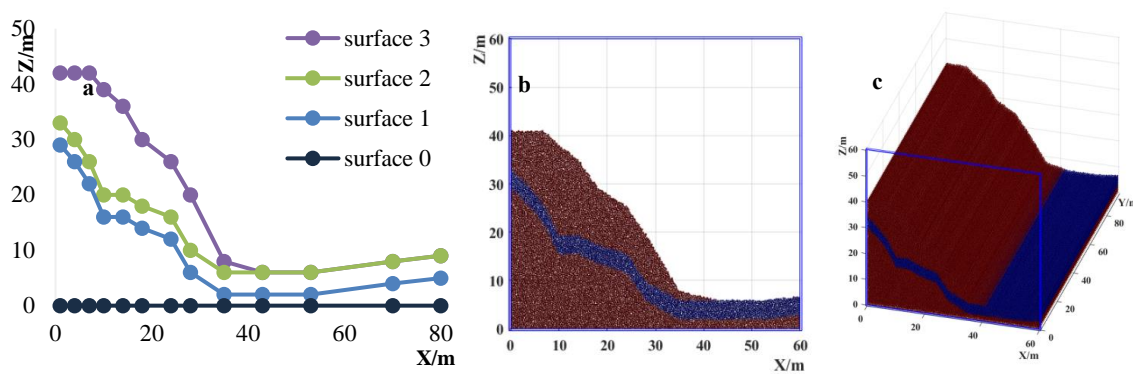


Fig. 3. Slope model with a weak interlayer.

(a) Actual slope elevation;(b) 2D slope model;(c) stacking model consisting of 100 2D slopes.

Table 1. Macro-mechanical parameter values of soil mass in slope formation

	Young's modulus E/GPa	Poisson's ratio λ	Tensile strength T_u/MPa	Compressive strength C_u/MPa	Coefficient of internal friction μ_i
Layer1,3	0.02	0.14	0.2	2	0.8
Layer2	0.01	0.18	0.06	0.6	0.6

Table 2. Micro-mechanical parameter values of particle in slope formation

	Normal stiffness $k_n/(\text{MN.m}^{-1})$	Tangential stiffness $k_s/(\text{MN.m}^{-1})$	Fracture displacement x_b/mm	Shearing resistance f_{s0}/KN	Friction coefficient μ_p
Layer1,3	19.7	7.79	2.8	195	0.309
Layer2	10.7	2.63	1.6	75.7	0.192

3.2 Monte Carlo simulations

The most important factors affecting the slope stability are soil cohesion (c) and internal friction angle (φ), which can be determined by the micro-mechanical parameter tensile strength F and the friction coefficient μ_p [18]. Therefore, this paper takes F and μ_p as random variables, and assigns values through Eq. (4). It is assumed that the variation coefficient *Ratio* of F and μ_p obey the uniform distribution in the range of (0.2, 1).

The numerical simulation uses a server containing 4 Tesla P100 GPUs and enables 4 MatDEM simulation processes to run simultaneously. Each MatDEM process runs on an independent GPU, which ensures the smooth

running of the simulation process. Each calculation model contains 100 2D slopes, and the slope stability calculation loop is iterated 100 times. In a single calculation and different loop iterations, different random variables are assigned to different slopes. Finally, after 100 iterations of the loop, 40,000 slope sample data are obtained. The single slip calculation simulates 1.78 s in the real world, that is, only the movement within 1.78 s of the slope sliding is calculated. The slope simulation took 28 hours totally, and the stability calculation of single slope was completed every 2.5 s on average.

4 Results and Discussion

4.1 Displacement instability criterion

From the simulation results, we can obtain the tensile strength and friction coefficient of each slope, as well as the displacement of each element in the slope. In the strongly cemented discrete element system, it can be considered that when the displacement of two local adjacent contact elements is greater than their own diameter, the bond between them breaks and the local failure of the material occurs. Therefore, we regard the displacement of a certain amount of elements larger than their own diameter as the criterion. At the same time, in the process of model loading, when the pressure is too large or the element stiffness is too small, the elements will be too deformed and cross and fly out of the boundary. Therefore, combined with the microscopic characteristics of the elements, and in order to prevent the escape of the elements from the slope surface from affecting the calculation results, it is assumed that when the displacement of more than 1% of the elements in the slope is greater than the element diameter, the slope model is considered to produce a penetrating sliding surface, and slope failure is determined. In order to verify the effectiveness of the proposed criterion, we draw the displacement field diagrams of the slope with the largest mechanical properties judged as failure (Fig. 4a) and the slope with the smallest mechanical properties judged as stability (Fig. 4b) respectively. It can be found that the slope judged as failure produces a through sliding surface along the weak interlayer, which meets the soil failure criterion [15], while the slope judged to be stable does not produce a through sliding surface, which proves the validity of the proposed criterion.

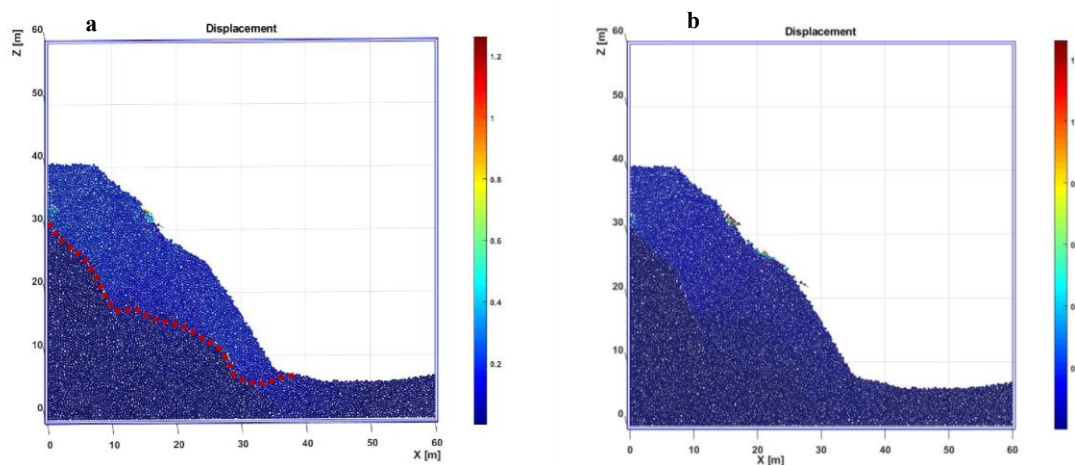


Fig. 4. Displacement field diagram of the slope with the largest mechanical property judged as failure(a) and the slope with the smallest mechanical property judged as stability(b).

Among them, (a) the Ratio of F is 0.42 and Ratio of μ_p is 0.25; (b) the Ratio of F is 0.45 and Ratio of μ_p is 0.21.

4.2 Displacement field distribution

Fig. 5 shows the displacement field (Fig. 5a) and velocity field diagram (Fig. 5b) of a failed 2D slope. As shown in Fig. 5a, the displacement of elements at the leading edge is slightly greater than that of the trailing edge elements, which reflects that during the landslide initiation stage, the elements at the trailing edge moves downward to squeeze the leading edge elements and trigger the landslide. The slope velocity field diagram shown in Fig. 5b corresponds to the displacement field diagram. Along the sliding surface downwards, the velocity field of the sliding element gradually increases from the trailing edge to the leading edge and reaches the maximum at the landslide shear exit.

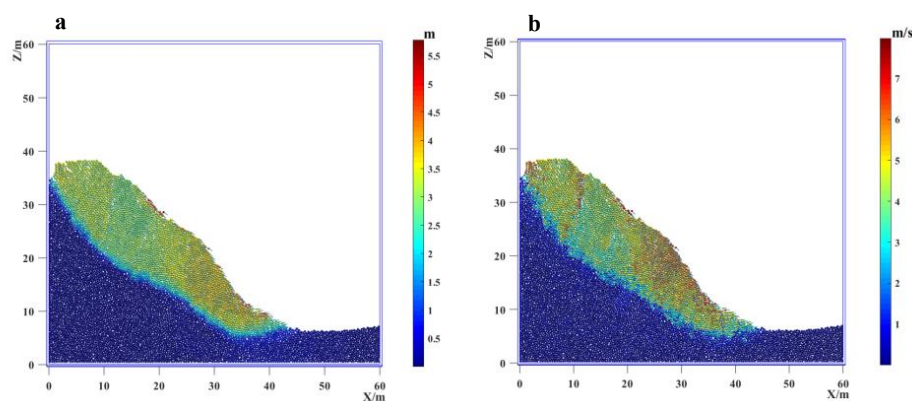


Fig.5. Displacement field diagram (a) and velocity field diagram (b) of a 2D slope.

Fig.6 shows the displacements of the slope stacking model with 4 MatDEM processes in one calculation. Due to the random distribution of tensile strength F , the 100 2D slopes have different displacements. Comparing and analyzing these four displacement field diagrams, it is found that the overall displacement of the 100 slopes in Fig.6a is significantly larger than other slopes, and the displacement of a single slope in Fig.6a is significantly increased. This is because the $\mu_p Rate$ of the slope model in Fig. 6a is the smallest (0.3325). It shows that under the premise of random distribution of tensile strength, as the friction coefficient decreases, the slope displacement increases and the probability of instability failure increases.

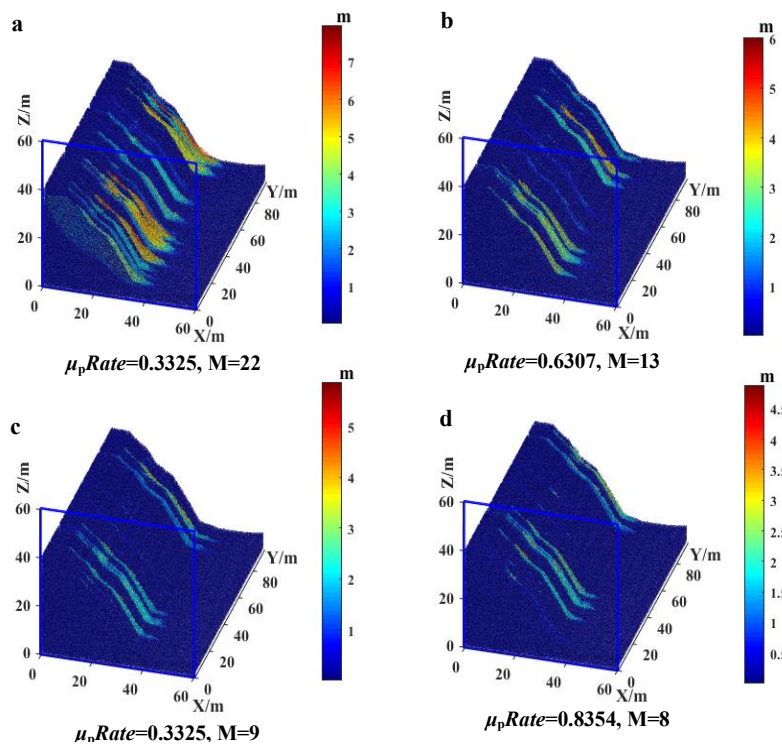


Fig. 6. Slope displacement field in one calculation. Among them, F is randomly distributed.

4.3 Comparative of slope failure probability

According to the proposed criterion and the basic principles of MCS, Table 3 shows the slope failure probability of 4 independent MatDEM processes.

Table 3. Calculation of slope failure probability

Reliability analysis method	Total number of samples N	Number of destroyed samples M	Failure probability P_f
MCS based on DEM	10000	1709	17.09%
	10000	1634	16.34%
	10000	1589	15.89%
	10000	1660	16.60%
	40000	6592	16.48%

As shown in Table 3, in different independent analyses, the slope failure probability calculated by the proposed method is almost equal, and the error is within 2%. When the sample data is 40,000, the average slope failure probability is 16.48%.

According to Eq. 6[19], the coefficient of variation COV_{Pf} of slope failure probability is calculated. When the sample data is 40,000, the COV_{Pf} is computed as 0.011, which greatly meets the target accuracy requirements of MCS[20]. This shows that the method can be applied to the refined analysis of slopes with high-precision

requirements and multiple random variables.

$$COV_{P_f} = \sqrt{\frac{1-P_f}{NP_f}} \quad (6)$$

5 Conclusions

This paper explores a slope stability analysis method based on DEM and MCS with the software MatDEM. In view of the huge amount of calculation, the generation of stacking models including multiple slopes and adoption of instability criterion based on element displacement are proposed to improve the calculation efficiency. The effectiveness of the method is verified by an example of a slope with a weak layer. The following conclusions were reached:

(1) By generating discrete element numerical models of multiple slopes by MatDEM, the stability of hundreds of slopes can be analyzed simultaneously. 40,000 simulations were completed through 100 simulations of stacking models, and the stability analysis of one landslide is completed at every 2.5 s, which can meet the requirement of MCS.

(2) The proposed of instability criterion based on element displacement significantly improves the calculation efficiency. Only the 1.78 s is simulated in DEM, which avoids the simulation of whole sliding process. And the slope state is determined by the element displacement, which avoids the complicated process of solving the safety factor.

(3) Based on the proposed method, the analysis of a slope with a weak layer is carried out. According to the proposed instability criterion, the slope judged to be failed have a sudden displacement and a through sliding surface, which meets the soil failure criterion. The calculation result of failure probability shows that the method can be applied to the refined analysis of slopes with high-precision requirements and multiple random variables.

The MatDEM software and source code of numerical simulation used in this paper can be downloaded from <http://matdem.com>. The method presented in this paper is suitable for the analysis of reliability problems with high accuracy and multiple random variables. In further studies, more complex studies can be carried out in combination with actual slope data, such as adding random variables and considering the influence of groundwater.

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