Estimator Design for Complex Networks With Encoding Decoding Mechanism and Buffer-Aided Strategy: A Partial-Nodes Accessible Case

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Abstract

This article focuses on the partial-nodes-based state estimation (PNBSE) issue for a complex network with the encoding-decoding mechanism (EDM) over the unreliable communication channel, where the signals are transmitted in an intermittent manner. A so-called EDM is exploited to convert the transmitted signals into a set of codewords with finite bits so as to facilitate the transmission efficiency between the complex networks and the estimator. To guarantee the state estimation (SE) performance subject to the intermittent communication nature of the channel, a buffer with limited capacity, which stores the recent measurement signals and sends them to the estimator simultaneously, is adopted to improve the utilization rate of the measurement signals in the estimation process. The main objective of the investigated problem is to construct a partial-nodes-based (PNB) estimator to generate the desired state estimates for the underlying complex networks. Considering the intermittent feature of signal transmission, the ultimate boundedness of the SE error under the constructed PNB estimator is discussed, and then, sufficient conditions are derived which ensure that the desired PNB estimator exists. An simulation example is given to confirm the correctness and effectiveness of the proposed estimator design strategy in the end.

Keywords: Unreliable transmission, complex networks, encoding decoding scheme, buffer-aided strategy, partial-nodes-based state estimation.

1. Introduction

A typical complex network (CN) consists of numerous spatially distributed nodes with connections in the form of certain types of topologies, and each node achieves information exchange with its neighbors through those connections. In engineering practice, taking advantage of their network like structure and rich dynamical behavior, CNs have shown powerful ability in modeling dynamical systems (e.g., social networks, power grid networks, the World Wide Web, biological networks, and artificial neural networks, as seen in [1, 6, 20, 21]). Notably, SE plays a fundamental

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role in the research of CNs, since acquiring the state relating information is the precondition for performing engineering missions, such as optimal control, consensus, and fault-tolerant control, as seen in [34, 51]. Unfortunately, the state information of a CN is rarely fully accessible. Rather, only the measurement signals from sensor nodes can be obtained by the users. Consequently, it is necessary to develop SE approach. Now, the SE issue of CNs is a current research topic in both control engineering and computer science communities [13, 30, 48]. In [32], the finite horizon SE problem has been investigated and the resilient SE issue has been analyzed in [40]. In [12], the research status of the the SE problem of CN are summarized.

As is well known, an implied assumption for previous research concerning the SE problems of CNs is that measurement signals from all sensor nodes are accessible. Such an assumption, unfortunately, is so divergent from reality in practical applications for various reasons, such as unreliable communications, harsh working environments, and the large scale of CNs, as seen in [42]. Therefore, it is meanful to perform the SE task based on partial measurements, which is the so-called partial-node-based (PNB) SE. Notably, the PNB estimation scheme would contribute to lower economic costs than the full-node-based estimation scheme, which makes more economic sense when the budget is limited. To date, the PNBSE problem of CNs has been playing as a hot theme in the research field, as seen in [9, 17, 22].

In real-world applications, limited network bandwidth is one of the major concerns for networked systems, especially for CNs which have frequent and large transmission of signals, which necessitates the improvement of transmission efficiency, as seen in [19, 43, 44]. One of the most popular approaches to improve information efficiency is utilizing the encoding-decoding mechanism (EDM). More specifically, with the help of the encoding rule, an encoder is employed to firstly convert the original signals into certain codewords represented by finite length. In this way, signals are compressed, and therefore, transmission efficiency can be improved. Then, the codewords would be transmitted forward. Subsequently, a decoder is utilized to reconstruct the original signals from the received codewords. Finally, the estimator/controller can perform the estimation/control task by means of the reconstructed signals. However, decoding errors are produced during the encoding-decoding process. In other words, the reconstructed signals generated by the decoders are different from the original ones, and correspondingly, the estimation performance would be deteriorated. So far, the control/estimation problems under various EDM have provoked considerable research attention. For example, under a class of uniform quantization based EDM, the tracking control issue has been analyzed in [31] and the set-membership filter designing issue has been researched in [18]. In [41], under a class of security EDM, the remote SE problem has been investigated. Although different kinds EDM has been considered in this paper, different forms of decoding errors are inevitably produced during those encoding decoding process, which would degrade the estimation performance.

In engineering practice, communication constraints, which greatly reduce the reliability of signal transmission, is inevitable in networked systems, as seen in [7, 45, 47]. One of the most common results of communication constraints is intermittent transmission (i.e., signal transmissions would fail at some instants) , as seen in [33, 37]. In this case, estimation/control performance would be degraded or even devastated. Therefore, a buffer is introduced to store the newly generated signals. Then, the stored signals, which include the current instant signals and the historical instants signals, would be sent to the estimator simultaneously at the sent instant (i.e., the current instant). At the same time, the buffer would be cleared up to store the signals generated in the following moments, as seen in [27]. This is the so-called buffer-aided strategy. In this way, the estimators can make the best advantage of measurement signals to achieve desired estimation performance, which greater the resource utilization rate and therefore makes engineering sense. So far, the buffer-related control/estimation problems have stirred some initial research attention, as seen in [26, 49]. For example, in [36] the stability problem for the discrete system with buffer storage has been studied, and a method was proposed to test whether a controller can guarantee the systems stability or not. In [39], the SE issue for Markov jump systems with buffer storage has been analyzed.

In response to the discussion mentioned above, the PNBSE problem has clear engineering insight from the application perspective and economic perspective. EDM and buffer-aided strategy both are popular research topics on signal transmission community, but the influences of EDM and buffer-aided strategy on SE have not been investigated yet. Therefore, investigating the PNBSE problem for the CN with EDM and buffer-aided strategy has significant engineering sense, and this is the main factor which motivates us to implement a research in this paper. This is a nontrivial problem bringing about three challenges:

1. How to quantify the effect of EDM and buffer-aided strategy on the required estimation performance?
2. How to construct a PNB estimator for the CN under EDM and buffer-aided strategy?
3. How to make sure the exponentially ultimate boundedness (EUB) of the SE error under the intermittent transmission case?

The key contributions of this research are

1. *The EUB SE problem is firstly investigated for the PNB CN with EDM subject to unreliable communications.*
2. *The impacts of EDM and buffer-aided strategy on the estimation performance have been investigated, respectively.*
3. *A novel PNB state estimator is put forward to guarantee the EUB of the SE error.*

The rest of this article is arranged as follows: The considered CN model, the communication network, and the PNB state estimator are introduced in detail in Section 2. The requirements which ensure the EUB of the SE error are presented in Section 3. Then, numerical examples are shown in Section 4 to confirm research results. In the end, we give the conclusions of this article in Section 5.

2. Problem Formulation

As Fig. 2 shows, this paper considers a CN in which system components (i.e., sensors and estimators) transmit signals via a network with EDM subject to unreliable communications, and buffer-aided strategy is employed to mitigate the effects of unreliable communications.

2.1. Buffer-Aided Strategy

Consider a CN, signal transmissions of which occur intermittently. A buffer is introduced to store the signal generated at each time instant, and then transmits all the stored signals to the estimator simultaneously at the transmission moment (i.e., the buffer would be cleared up at the transmission moment).

Define y_k as the signal generated at the time instant k, k_t as the $t-$th transmission moment for $t \in \mathbb{N}^+$ with initial value $k_0 = 1$, and h_t as the interval between k_t and k_{t-1}, i.e.,

$$h_t \triangleq \begin{cases} 1, & \text{if } t = 0 \\ k_t - k_{t-1}, & \text{if } t \in \mathbb{N}^+ \end{cases}$$

where $\mathbb{N}^+$ refers to the set of nonnegative integers. Next, the next two assumptions are introduced to further characterize the buffer-aided strategy:

Assumption 1. *(Transmission interval) For $t \in \mathbb{N}^+$, the transmission interval h_t satisfies*

$$h_t \in \mathbb{H} \triangleq \{1, 2, \cdots, H\}$$

where $H \geq 0$ is a known constant (KC) representing the maximum transmission interval.

Assumption 2. *(Buffer capacity) For $t \in \mathbb{N}^+$, at the instant k_t, the number of signals stored in the buffer satisfies*

$$q_t \in \mathbb{M} \triangleq \{1, 2, \cdots, M\}$$

and $q_0 \triangleq 1$, where $M \geq 0$ is a known KC representing the limited capacity of the buffer and $M \leq H$.

Based on the above assumptions, we can easily have

$$q_t \triangleq \min\{M, h_t\}, \ t \in \mathbb{N}^+.$$

Accordingly, at the transmission moment k_t, the signal packet $\mathcal{Y}_{k_t}$ stored in the buffer (containing the signals which would be transmitted at k_t) can be expressed as follows:

$$\mathcal{Y}_{k_t} \triangleq \{y_{k_t}, y_{k_t-1}, ..., y_{k_t-q_t+1}\}.$$

Remark 1. *Assumptions 1 and 2 are fairly reasonable in engineering practice. Assumptions 1 comes from the consideration for the intermittent characteristic of signal transmissions in practical networked systems. It is easy to observed that the transmission intervals of networked systems are upper bounded. Assumption 2 provides some typical characteristics of the buffer. A buffer with limited capacity is preferred in real-world applications for the purpose of saving economic costs and reducing energy consumption. In these cases, it is of practical significance to assume that the number of signals transmitted are bounded.*

Before proceeding further, we give an example to further illustrate the characteristics of buffer-aided strategy. As shown in Fig. 1, at the instant $k_0 = 1$, y_1 is successfully transmitted (i.e., $h_0 = 1$, $q_0 = 1$). At the instant $k_1 = 3$, the signal packet $\mathcal{Y}_3 = \{y_3, y_2\}$ is transmitted to the estimator (i.e., $h_1 = 2$, $q_1 = 2$). Then, $\mathcal{Y}_8 = \{y_8, y_7, y_6\}$ is sent to the estimator at the instant $k_2 = 8$ (i.e., $h_2 = 5$, $q_2 = 3$).

2.2. CN

This article considers a class of CN with time delays with S coupled nodes:

$$x_{i,k+1} = A_i x_{i,k} + \sum_{i=1}^{S} \vartheta_{ij} \mathcal{W} x_{j,k} + G_i x_{i,k-d} + B_i \omega_{i,k} \tag{1}$$

where $x_{i,k} \in \mathbb{R}^{n_x}$ represents the state of the ith node ($i \in \{1, 2, , \cdots, S\}$). $\mathcal{J} = [\vartheta_{ij}]_{S \times S} \in \mathbb{R}^{S \times S}$ is the adjacency matrix of the CN and $\vartheta_{ij} > 0$ ($i \neq j$) if signal transmissions can be achieved between nodes i and node j. $\mathcal{W} = \text{diag}\{\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_{n_x}\} \geq 0$ is the coupling matrix of each node.

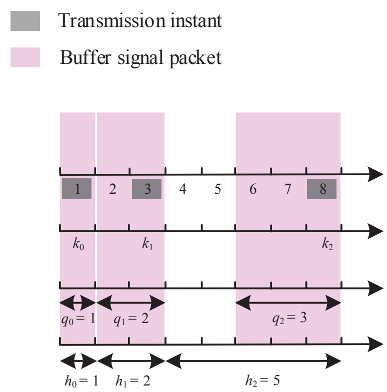

Figure 1: Signal transmission diagram (i.e., $H = 5$ and $M = 3$).

d is a KC which represents the time delay. $\omega_{i,k} \in \mathbb{R}^{n_\omega}$ denotes the process noise (PN). A_i, B_i, and G_i are KC matrices whose dimensions are appropriate.

We assume that the first l_0 nodes are accessible. The measurement is

$$y_{i,k} = C_i x_{i,k} + D_i \upsilon_{i,k}$$

where $(i \in \{1, 2, \cdots, l_0\})$. $\upsilon_{i,k} \in \mathbb{R}^{n_\upsilon}$ denotes the measurement noise (MN). C_i and D_i are KC matrices whose dimensions are appropriate.

Assumption 3. *The PN $\omega_{i,k}$ and the MN $\upsilon_{i,k}$, which are uncorrelated Gaussian white noise with zero-means and the properties:*

$$\mathbb{E}\{\omega_{i,k}\omega_{i,k}^T\} = \bar{R}_1\bar{R}_1^T,\ \mathbb{E}\{\upsilon_{i,k}\upsilon_{i,k}^T\} = \bar{R}_2\bar{R}_2^T.$$

where $\mathbb{E}\{\cdot\}$ denotes the mathematical expectation (ME) of $\cdot$.

Remark 2. *Although both $\omega_{i,k}$ and $\upsilon_{i,k}$ are disturbance noises, they represent different kinds of signals in practical applications. More specifically, $\omega_{i,k} \in \mathbb{R}^{n_\omega}$ describes the disturbance from external environment and can be regarded as an uncontrollable external input. $\upsilon_{i,k} \in \mathbb{R}^{n_\upsilon}$ reflects the deviation of sensor measurements.*

2.3. Communication Network

As shown in Fig. 2, the signal transmissions are implemented via a communication network with EDM and buffer-aided strategy, can be summed up as follows: 1) the measurement signal is firstly encoded as certain codewords before being transmitted forward; 2) the codewords are stored in the buffer, and at k_t, the buffer would send the stored codewords to the decoders simultaneously; and 3) decoders are adopted to restore the received signals. Finally, the PNB state estimator can receive the restored signal packet for future processing.

1. Quantization based encoding process:

For the ith node whose measurements are accessible (i.e., $i \in \{1, 2, \cdots, l_0\}$), a group of probability quantization based encoders are employed to convert its measurement signal $y_{i,k}$ into a set

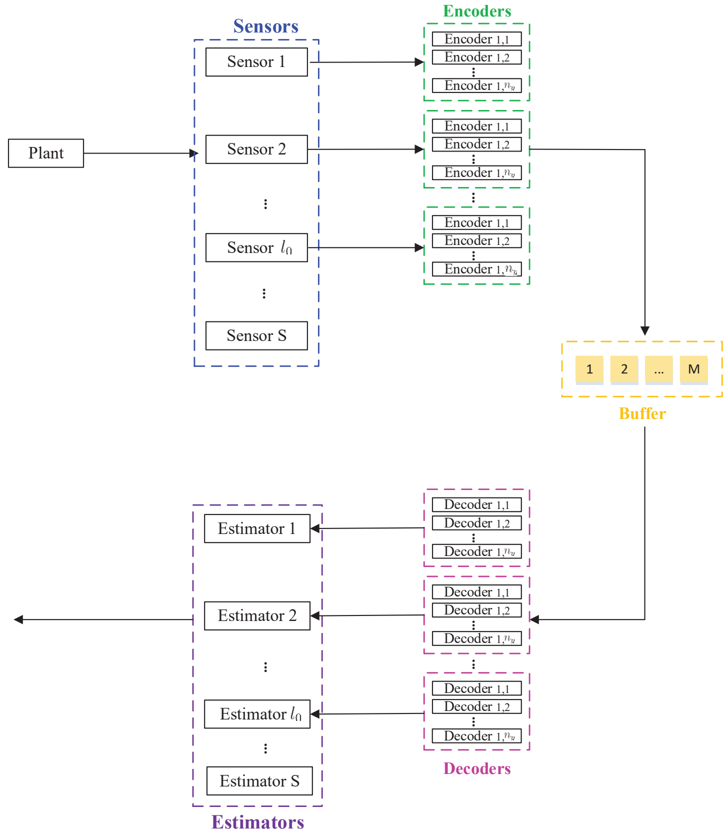

Figure 2: Structure of the CN with EDM and buffer-aided strategy subject to unreliable communications

of codewords. We denote the gth ($g \in \{1, 2, \cdots, n_y\}$) scalar element of $y_{i,k}$ as $y_{i,g,k}$ and the gth encoder of the ith node as $\mathscr{F}_{i,g}(\cdot)$. In particular, the encoding levels set $\mathcal{U}_{i,g}$ of $\mathscr{F}_{i,g}(\cdot)$ has the following form:

$$\mathcal{U}_{i,g} \triangleq \{\mu_{i,g,z}, \mu_{i,g,z} \triangleq z\varphi_{i,g}, z = 0, \pm 1, \pm 2 \cdots\},$$

where $\varphi_{i,g}$ (a known positive scalar) is the encoding interval, and $\mu_{i,g,z}$ is the encoding level, of which the digital expression is z. Define $\breve{y}_{i,g,k}$ is the codeword of $y_{i,g,k}$, i.e., $\breve{y}_{i,g,k} = \mathscr{F}_{i,g}(y_{i,g,k})$. For $\mu_{i,g,z} \leq y_{i,g,k} < \mu_{i,g,z+1}$, the encoding process is governed by

$$\begin{cases} \text{Prob}\{\mathscr{F}_{i,g}(y_{i,g,k}) = z\} = 1 - \varpi_{i,g,k} \\ \text{Prob}\{\mathscr{F}_{i,g}(y_{i,g,k}) = z + 1\} = \varpi_{i,g,k} \end{cases} \tag{2}$$

where $\varpi_{i,g,k} \triangleq (\breve{y}_{i,g,k} - \mu_{i,g,z})/\varphi_{i,g}$, $0 \leq \varpi_{i,g,k} < 1$. To facilitate further design, we introduce the following definition:

$$\breve{y}_k = \{\breve{y}_{1,1,k}, \breve{y}_{1,2,k}, \cdots, \breve{y}_{l_0,n_y,k}\}.$$

where $\breve{y}_k$ is the codeword set of y_k.

2. Buffer-aid strategy:

A buffer whose capacity has restrictions is utilized to store the newly generated codeword set $\breve{y}_k$, and then simultaneously transmits all the stored codeword sets to the decoder at the transmission moment k_t (means that the buffer would be cleared up at k_t). Based on the buffer model introduced in Section 2.1, the buffer codeword packet $\breve{\mathcal{Y}}_{k_t}$ at k_t can be represented as

$$\breve{\mathcal{Y}}_{k_t} = \{\breve{y}_{k_t}, \breve{y}_{k_t-1}, ..., \breve{y}_{k_t-q_t+1}\}.$$

Specifically, the packet $\breve{\mathcal{Y}}_{k_t}$ only contains the partially available measurement signals of q_t instants, since only partial nodes in the CN are accessible (i.e., $i \in \{1, 2, \cdots, l_0\}$) and the buffer has limited capacity (i.e., $q_t \in \{1, 2, \cdots, M\}$).

To characterize the feature of intermittent transmissions, a necessary assumption is introduced

Assumption 4. *The transmission intervals $\{h_t\}_{t\geq 1}$, which taking values from $\mathbb{H} = \{1, 2, ..., H\}$, are a sequence of random variables which are independently and identically distributed, and the occurrence probability of h_t are partially unknown, i.e.,*

$$\begin{cases} \text{Prob}\{h_t = z\} = p^{(z)}, & z \in \mathbb{H}_k \\ \text{Prob}\{h_t = n\} = ?, & n \in \mathbb{H}_{uk} \end{cases}$$

where $0 \leq p^{(\iota)} \leq 1$ and "?" are the known and unknown probability, respectively, and $\sum_{h_t=1}^{H} p^{(h_t)} = 1$. $\mathbb{H} = \{1, 2, ..., H\}$, $\mathbb{H}_k \triangleq \{z \mid p^{(z)}$ is known$\}$, and $\mathbb{H}_{uk} \triangleq \{n \mid p^{(n)}$ is unknown$\}$. $\mathbb{H}_k \cup \mathbb{H}_{uk} = \mathbb{H}$ and $\mathbb{H}_k \cap \mathbb{H}_{uk} = \varnothing$.

Remark 3. *Assumption 4 is very reasonable, as mentioned in [14], taking into consideration the randomly changing state of the communication network, the specific values of the occurrence probabilities are very difficult to obtain, especially for CNs which have a large amount of nodes.*

3. Decoding process:

Based on the received buffer codeword packet $\breve{\mathcal{Y}}_{k_t}$, which contains the partially accessible measurement signals of q_t instants (i.e., $k_t - q_t + 1 \leq k \leq k_t$), a group of decoders are utilized to restore the measurement signals. Let the gth decoder of the ith accessible node be $\mathscr{D}_{i,g}(\cdot)$ with the following decoding rule

$$\bar{y}_{i,g,k} = \mathscr{D}_g(\breve{y}_{i,g,k}) \triangleq \breve{y}_{i,g,k}\varphi_{i,g} \tag{3}$$

where $k_t - q_t + 1 \leq k \leq k_t$ and $\bar{y}_{i,g,k}$ is the signal generated by the decoder corresponding to the codeword $\breve{y}_{i,g,k}$.

Define the decoding error as $\Psi_{i,g,k} \triangleq \bar{y}_{i,g,k} - y_{i,g,k}$. and Considering (2) and (3), we can easily infer that

$$\begin{aligned} \mathbb{E}\{\Psi_{i,g,k}\} =& \text{Prob}\{-\varpi_{i,g,k}\varphi_{i,g}\}(-\varpi_{i,g,k}\varphi_{i,g}) + \text{Prob}\{(1 - \varpi_{i,g,k})\varphi_{i,g}\}(1 - \varpi_{i,g,k})\varphi_{i,g} \\ =& -(1 - \varpi_{i,g,k})\varpi_{i,g,k}\varphi_{i,g} + (1 - \varpi_{i,g,k})\varphi_{i,g}\varpi_{i,g,k} = 0 \end{aligned}$$

and

$$\mathbb{E}\{\Psi_{i,g,k}^2\} = \text{Prob}\{(1 - \varpi_{i,g,k})\varphi_{i,g}\}(1 - \varpi_{i,g,k})^2\varphi_{i,g}^2 + \text{Prob}\{-\varpi_{i,g,k}\varphi_{i,g}\}(-\varpi_{i,g,k}\varphi_{i,g})^2$$

$$
\begin{aligned}
&= (1 - \varpi_{i,g,k})^2 \varphi_{i,g}^2 \varpi_{i,g,k} + (-\varpi_{i,g,k}\varphi_{i,g})^2 (1 - \varpi_{i,g,k}) \\
&= (1 - \varpi_{i,g,k}) \varphi_{i,g}^2 \varpi_{i,g,k},
\end{aligned}
$$

which imply that

$$
\mathbb{E}\{\Psi_{i,g,k}\} = 0, \quad \mathbb{E}\{\Psi_{i,g,k}^2\} \le \varphi_{i,g}^2/4. \tag{4}
$$

Before processing, let us build up a compact form of the CN to facilitate further design. By defining

$$
\begin{aligned}
&A \triangleq \mathrm{diag}\{A_1, A_2, \cdots, A_S\} \in \mathbb{R}^{Sn_x \times Sn_x},\ G \triangleq \mathrm{diag}\{G_1, G_2, \cdots, G_S\} \in \mathbb{R}^{Sn_x \times Sn_x}, \\
&B \triangleq \mathrm{diag}\{B_1, B_2, \cdots, B_S\} \in \mathbb{R}^{Sn_x \times Sn_\omega},\ C \triangleq \mathrm{diag}\{C_1, C_2, \cdots, C_{l_0}\} \in \mathbb{R}^{l_0 n_y \times l_0 n_x}, \\
&D \triangleq \mathrm{diag}\{D_1, D_2, \cdots, D_{l_0}\} \in \mathbb{R}^{l_0 n_\upsilon \times l_0 n_\upsilon},\ x_k \triangleq \begin{bmatrix} x_{1,k}^T & x_{2,k}^T & \cdots & x_{S,k}^T \end{bmatrix}^T \in \mathbb{R}^{Sn_x}, \\
&\omega_k \triangleq \begin{bmatrix} \omega_{1,k}^T & \omega_{2,k}^T & \cdots & \omega_{S,k}^T \end{bmatrix}^T \in \mathbb{R}^{Sn_\omega},\ \grave{y}_k \triangleq \begin{bmatrix} \bar{y}_{1,1,k}^T & \bar{y}_{1,2,k}^T & \cdots & \bar{y}_{l_0,n_y,k}^T \end{bmatrix}^T \in \mathbb{R}^{l_0 n_y}, \\
&\bar{\upsilon}_k \triangleq \begin{bmatrix} \upsilon_{1,k}^T & \upsilon_{2,k}^T & \cdots & \upsilon_{l_0,k}^T \end{bmatrix}^T \in \mathbb{R}^{l_0 n_\upsilon},\ \bar{C} \triangleq \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{Sn_y \times Sn_x}, \\
&\grave{\Psi}_k \triangleq \begin{bmatrix} \Psi_{1,k}^T & \Psi_{2,k}^T & \cdots & \Psi_{l_0,n_y,k}^T \end{bmatrix}^T \in \mathbb{R}^{l_0 n_y},\ \bar{D} \triangleq \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{Sn_y \times Sn_\upsilon}, \\
&\bar{y}_k \triangleq \begin{bmatrix} \grave{y}_k^T & 0 \end{bmatrix}^T \in \mathbb{R}^{Sn_y},\ \upsilon_k \triangleq \begin{bmatrix} \bar{\upsilon}_k^T & 0 \end{bmatrix}^T \in \mathbb{R}^{Sn_\upsilon},\ \Psi_k \triangleq \begin{bmatrix} \grave{\Psi}_k^T & 0 \end{bmatrix}^T \in \mathbb{R}^{Sn_y}, \\
&\tilde{y}_k \triangleq \begin{bmatrix} y_{1,1,k}^T & y_{1,2,k}^T & \cdots & y_{l_0,n_y,k}^T \end{bmatrix}^T \in \mathbb{R}^{l_0 n_y},\ y_k \triangleq \begin{bmatrix} \tilde{y}_k^T & 0 \end{bmatrix}^T \in \mathbb{R}^{Sn_y},
\end{aligned}
$$

where $\mathrm{diag}\{\cdots\}$ represents the block-diagonal matrix, and 0 and I, are the zero and identity matrix, respectively. Then, we have

$$
\begin{cases}
x_{k+1} = Ax_k + (\mathcal{J} \otimes \mathcal{W})x_k + Gx_{k-d} + B\omega_k \\
\quad y_k = \bar{C}x_k + \bar{D}\upsilon_k
\end{cases} \tag{5}
$$

where $\otimes$ represents the Kronecker product, and we also have

$$
\Psi_k = \bar{y}_k - y_k \tag{6}
$$

where Ψ_k is the decoding error of the whole CN.

Moreover, according to (4) and (6), one has

$$
\mathbb{E}\{\Psi_k\} = 0, \quad \mathbb{E}\{\Psi_k^T \Psi_k\} \le \bar{\Psi},
$$

where $\bar{\Psi} \triangleq \sum_{i=1}^{l_0} \sum_{g=1}^{n_y} \varphi_{i,g}^2/4$.

2.4. PNB State estimator

The intermittent transmission case of a class of CNs with partially accessible nodes, EDM, and buffer-aid strategy is considered in this paper, which means that 1) only the measurement signals from partial nodes are accessible, 2) only the measurement signals of some instants are available, and 3) the measurement signals of some instants would be transmitted simultaneously at k_t. Considering the above discussion, we propose the following PNB estimator:

<u>Case 1:</u> $k \neq k_t$

$$\begin{cases} \hat{x}_{k+1|k} = A\hat{x}_{k|k} + (\mathcal{J} \otimes \mathcal{W})\hat{x}_{k|k} + G\hat{x}_{k-d|k-d} \\ \hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} \end{cases}$$

Case 2: $k = k_t$ (7)

$$\begin{cases} \hat{x}_{k+1|k-q_t+1} = (A + (\mathcal{J} \otimes \mathcal{W}) - \bar{L}^{(h_t)}\bar{C})^{q_t}\hat{x}_{k-q_t+1|k-q_t+1} + \sum_{\psi=0}^{q_t-1}(A + (\mathcal{J} \otimes \mathcal{W}))^{\psi}G\hat{x}_{k-\psi-d|k-\psi-d} \\ \qquad + \sum_{\psi=0}^{q_t-1}(A + (\mathcal{J} \otimes \mathcal{W}))^{\psi}\bar{L}^{(h_t)}(y_{k-\psi} + \Psi_{k-\psi}) \\ \hat{x}_{k+1|k+1} = \hat{x}_{k+1|k-q_t+1} \end{cases}$$

where at the time instant k, $\hat{x}_{k|k}$, $\hat{x}_{k+1|k}$, and $\hat{x}_{k+1|k-q_t+1}$ represent the estimate of x_k with $\hat{x}_{0|0}$, the one-step prediction, and the q_t-step prediction, respectively. $L_i^{(h_t)} \in \mathbb{R}^{n_x \times n_y}$ is the gain of the ith node. $L^{(h_t)} \triangleq \text{diag}\{L_1^{(h_t)}, L_2^{(h_t)}, \cdots, L_{l_0}^{(h_t)}\} \in \mathbb{R}^{l_0 n_x \times l_0 n_y}$ and $\bar{L}^{(h_t)} \triangleq \text{diag}\{L^{(h_t)}, 0\} \in \mathbb{R}^{Sn_x \times Sn_y}$.

Remark 4. *As shown in equation (7), influenced by the intermittent transmission, although the measurement signals can not be sent to the estimator when $k \neq k_t$, this estimator still can utilize the state estimate of the current instant to generate the state estimate of the next instant. Morover, at the instant k_t, once the transmission occurs, the decoded signal packet $\bar{\mathcal{Y}}_{k_t} = \{\bar{y}_{k_t}, \bar{y}_{k_t-1}, \cdots, \bar{y}_{k_t-q_t+1}\}$ would be sent to the estimator. Subsequently, with the help of the decoded signal packet, the states from $k_t - q_t + 2$ to k_t would be re-estimated, and then, the state of $k_t + 1$ would be estimated. In other words, the estimation of x_{k_t+1} (i.e., $\hat{x}_{k_t+1}$) is rely on the decoded signal packet $\bar{\mathcal{Y}}_{k_t}$ and the historical estimation $\hat{x}_{k_t-q_t+1}$.*

Furthermore, we denote the one-step prediction error as $e_{k+1|k} = x_{k+1} - \hat{x}_{k+1|k}$, the q_t-step prediction error as $e_{k+1|k-q_t+1} = x_{k+1} - \hat{x}_{k+1|k-q_t+1}$, and the estimation error as $e_{k+1|k+1} = x_{k+1} - \hat{x}_{k+1|k+1}$. Then, the dynamics of the prediction error is determined by:

Case 1: $k \neq k_t$

$$\begin{cases} e_{k+1|k} = (A + \mathcal{J} \otimes \mathcal{W})e_{k|k} + Ge_{k-d|k-d} + B\omega_k \\ e_{k+1|k+1} = e_{k+1|k} \end{cases}$$

Case 2: $k = k_t$ (8)

$$\begin{cases} e_{k+1|k-q_t+1} = (A + (\mathcal{J} \otimes \mathcal{W}) - \bar{L}^{(h_t)}\bar{C})^{q_t}e_{k-q_t+1|k-q_t+1} + \sum_{\psi=0}^{q_t-1}(A + (\mathcal{J} \otimes \mathcal{W}) - \bar{L}^{(h_t)}\bar{C})^{\psi} \\ \qquad \times (B\omega_{k-\psi} - \bar{L}^{(h_t)}D\upsilon_{k-\psi} - \bar{L}^{(h_t)}\Psi_{k-\psi}) + \sum_{\psi=0}^{q_t-1}(A + (\mathcal{J} \otimes \mathcal{W}) - \bar{L}^{(h_t)}\bar{C})^{\psi} \\ \qquad \times Ge_{k-\psi-d|k-\psi-d} \\ e_{k+1|k+1} = e_{k+1|k-q_t+1} \end{cases}$$

Before going any further, Let us introduce the desired performance index.

Definition 1. *[28] The SE error (8) of the CNs (5) is said to be EUB in mean square (MS) if there exist constants $Z > 0$, $0 \leq m < 1$ and $\mathscr{S} > 0$ such that*

$$\mathbb{E}[\|e_{k|k}\|^2] \leq m^{\theta}Z + \mathscr{S}$$

The main purpose of this paper is to derive the gain parameters of the PNB state estimator to guarantee the EUB of the SE error for a class of CNs with EDM and buffer-aid strategy.

3. Main Results

Theorem 1. *For the CN(5), if there exist positive definite matrices P_i $(i = 1, 2, \cdots, S)$, Q_i $(i = 1, 2, \cdots, S), \Upsilon_1 \in \mathbb{R}^{Sn_\omega \times Sn_\omega}, \Upsilon_2 \in \mathbb{R}^{Sn_\upsilon \times Sn_\upsilon}, \Upsilon_3 \in \mathbb{R}^{Sn_y \times Sn_y}$, two positive scalars $\alpha_\xi(\xi = 1, 2)$, and $l_0 H$ estimation gain matrices $L_i^{(\iota)}(i = 1, 2, \cdots, l_0)(\iota = 1, 2, \cdots, H)$ satisfying the following conditions:*

$$\mathcal{R}_1 \triangleq \begin{bmatrix} \mathcal{R}_{1,11} & \mathcal{R}_{1,12} & 0 & 0 \\ * & \mathcal{R}_{1,22} & 0 & 0 \\ * & * & \mathcal{R}_{1,33} & 0 \\ * & * & * & \mathcal{R}_{1,44} \end{bmatrix} < 0 \tag{9}$$

$$\mathcal{R}_2 \triangleq \begin{bmatrix} \mathcal{R}_{2,11} & 0 & \mathcal{R}_{2,13} & 0 \\ * & \mathcal{R}_{2,22} & 0 & 0 \\ * & * & \mathcal{R}_{2,33} & 0 \\ * & * & * & \mathcal{R}_{2,44} \end{bmatrix} < 0 \tag{10}$$

$$\beta \triangleq \sum_{\gamma=1}^{H} \bar{p}^{(\gamma)} (1 - \alpha_2)^{\min\{M,\gamma\}} (1 + \alpha_1)^{\gamma - \min\{M,\gamma\}} < 1 \tag{11}$$

where

$$\mathcal{R}_{1,11} \triangleq (A + \mathcal{J} \otimes \mathcal{W})^T P (A + \mathcal{J} \otimes \mathcal{W}) - (1 + \alpha_1) P + dQ,\ \mathcal{R}_{1,13} \triangleq (A + \mathcal{J} \otimes \mathcal{W})^T PG,$$
$$\mathcal{R}_{1,22} \triangleq \mathrm{diag}\{-(1 + d\alpha_1)Q, \cdots, -(1 + 2\alpha_1)Q\},\ \mathcal{R}_{1,33} \triangleq G^T PG - (1 + \alpha_1)Q,$$
$$\mathcal{R}_{1,44} \triangleq B^T PB - \Upsilon_1,\ \mathcal{R}_{2,11} \triangleq (A + \mathcal{J} \otimes \mathcal{W} - \tilde{L}^{(\gamma)}\bar{C})^T P (A + \mathcal{J} \otimes \mathcal{W} - \tilde{L}^{(\gamma)}\bar{C}) - (1 - \alpha_2) P + dQ,$$
$$\mathcal{R}_{2,13} \triangleq (A + \mathcal{J} \otimes \mathcal{W} - \tilde{L}^{(\gamma)}\bar{C})^T PG,\ \mathcal{R}_{2,22} \triangleq \mathrm{diag}\{-(1 - d\alpha_2)Q, \cdots, -(1 - 2\alpha_2)Q\},$$
$$\mathcal{R}_{2,33} \triangleq G^T PG - (1 - \alpha_2)Q,\ \mathcal{R}_{2,44} \triangleq \mathrm{diag}\{\mathcal{R}_{2,44,1}, \mathcal{R}_{2,44,2}, \mathcal{R}_{2,44,3}\},\ \mathcal{R}_{2,44,1} \triangleq B^T PB - \Upsilon_1,$$
$$\mathcal{R}_{2,44,2} \triangleq (\tilde{L}^{(\gamma)} D)^T P \tilde{L}^{(\gamma)} D - \Upsilon_2,\ \mathcal{R}_{2,44,3} \triangleq (\tilde{L}^{(\gamma)})^T P \tilde{L}^{(\gamma)} - \Upsilon_3,\ \tilde{L}^{(\gamma)} \triangleq \sum_{s=1}^{H} \bar{p}^{(s)} \bar{L}^{(\gamma)},$$

$$\bar{p}^{(\gamma)} \triangleq \begin{cases} p^{(\gamma)}, & \text{if } \gamma \in \mathbb{H}_k \\ 1 - \sum_{j \in \mathbb{H}_k} p^{(j)}, & \text{if } \gamma \in \mathbb{H}_{uk} \end{cases} \tag{12}$$

Then, the error dynamics (8) is EUB in MS under the influences of the disturbance noise ω_k ,υ_k, and the decoding error Ψ_k.

Proof. In order to facilitate the understanding of how the SE $\hat{x}_{k_{t+1}+1}$ is generated, for $k \in \{k_{t-1} + 1, ..., k_t\}$, a virtual iterative procedure is introduced as follows:

$$\begin{aligned}
&\underline{\text{Case 1:}}\ k_{t-1}+1 \le\ k <\ k_t - q_t + 1 \\
&\begin{cases}
\tilde{x}_{k+1|k} = \hat{x}_{k+1|k} \\
\tilde{x}_{k+1|k+1} = \tilde{x}_{k+1|k}
\end{cases} \\
&\underline{\text{Case 2:}}\ k_t - q_t + 1 \le\ k \le\ k_t \\
&\begin{cases}
\tilde{x}_{k_t-q_t+2|k_t-q_t+1} = (A + \mathcal{J} \otimes \mathcal{W})\tilde{x}_{k_t-q_t+1|k_t-q_t+1} + G\tilde{x}_{k_t-q_t+1-d_{k_t-q_t+1}|k_t-q_t+1-d_{k_t-q_t+1}} \\
\qquad + \bar{L}^{(h_t)}(y_{k_t-q_t+1} + \Psi_{k_t-q_t+1}) - \bar{L}^{(h_t)}\bar{C}\tilde{x}_{k_t-q_t+1|k_t-q_t+1} \\
\qquad \vdots \\
\tilde{x}_{k_t+1|k_t} = (A + \mathcal{J} \otimes \mathcal{W})\tilde{x}_{k_t|k_t} + G\tilde{x}_{k_t-d_{k_t}|k_t-d_{k_t}} + \bar{L}^{(h_t)}(y_{k_t} + \Psi_{k_t}) - \bar{L}^{(h_t)}\bar{C}\tilde{x}_{k_t|k_t} \\
\tilde{x}_{k+1|k+1} = \tilde{x}_{k+1|k}
\end{cases}
\end{aligned} \tag{13}$$

where $\tilde{x}_{k|k}$ and $\tilde{x}_{k+1|k}$ represent the virtual estimation of x_k with $\tilde{x}_{0|0}$ and the virtual one-step prediction at the time instant k, respectively. At the end of this virtual period, $\hat{x}_{k_t+1|k_t+1} = \tilde{x}_{k_t+1|k_t+1}$.

Then, the PNB estimation error system (8) can be rewritten as:

$$\begin{aligned}
&\underline{\text{Case 1:}}\ k_{t-1}+1 \le\ k <\ k_t - q_t + 1 \\
&\begin{cases}
e_{k+1|k} = (A + \mathcal{J} \otimes \mathcal{W})e_{k|k} + Ge_{k-d|k-d} + B\omega_k \\
e_{k+1|k+1} = e_{k+1|k}
\end{cases} \\
&\underline{\text{Case 2:}}\ k_t - q_t + 1 \le\ k \le\ k_t \\
&\begin{cases}
e_{k_t-q_t+2|k_t-q_t+1} = Ge_{k_t-q_t+1-d|k_t-q_t+1-d} \\
\qquad + (A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(h_t)}\bar{C})e_{k_t-q_t+1|k_t-q_t+1} + B\omega_{k_t-q_t+1} - \bar{L}^{(h_t)}D\upsilon_{k_t-q_t+1} \\
\qquad - \bar{L}^{(h_t)}\Psi_{k_t-q_t+1} \\
\qquad \vdots \\
e_{k_t+1|k_t} = (A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(h_t)}\bar{C})e_{k_t|k_t} - \bar{L}^{(h_t)}\Psi_{k_t} + Ge_{k_t-d|k_t-d} - \bar{L}^{(h_t)}D\upsilon_{k_t} + B\omega_{k_t}
\end{cases}
\end{aligned} \tag{14}$$

For $k_t + 1 \le k < k_{t+1} + 1$, we define the following Lyapunov-like functional:

$$V_k \triangleq V_k^{(1)} + V_k^{(2)} \tag{15}$$

where

$$\begin{aligned}
V_k^{(1)} &\triangleq e_{k|k}^T P e_{k|k}, \\
V_k^{(2)} &\triangleq \sum_{j=0}^{d-1} \sum_{\psi=k-d+j}^{k-1} e_{\psi|\psi}^T Q e_{\psi|\psi}. \\
P &\triangleq \operatorname{diag}\{P_1, P_2, ..., P_S\}, \\
Q &\triangleq \operatorname{diag}\{Q_1, Q_2, ..., Q_S\}.
\end{aligned}$$

Case 1: $k_t+1 \le k < k_{t+1}-q_{t+1}+1$

Under this case, the estimator has no measurement signals to utilize. In other words, y_k ($k \in \{k_t+1, \cdots, k_{t+1}-q_{t+1}\}$) are not included in the decoded signal packet $\bar{\mathcal{Y}}_{k_t} = \{\bar{y}_{k_t}, \bar{y}_{k_t-1}, \cdots, \bar{y}_{k_t-q_t+1}\}$ which are restored from the buffer signal packet $\breve{\mathcal{Y}}_{k_t} = \{\breve{y}_{k_t}, \breve{y}_{k_t-1}, ..., \breve{y}_{k_t-q_t+1}\}$. Obviously, one has

$$\begin{aligned}
&V_{k+1}^{(1)} - V_k^{(1)} - \alpha_1 V_k^{(1)} \\
=& e_{k|k}^T (A+\mathcal{J}\otimes\mathcal{W})^T P (A+\mathcal{J}\otimes\mathcal{W}) e_{k|k} + e_{k-d|k-d}^T G^T P G e_{k-d|k-d} + 2 e_{k|k}^T (A+\mathcal{J}\otimes\mathcal{W})^T \\
&\times PGe_{k-d|k-d} + \omega_k^T (B^T P B - \Upsilon_1 I)\omega_k + \omega_k^T \Upsilon_1 \omega_k - (1+\alpha_1) e_{k|k}^T P e_{k|k} \\
=& (\eta_k^{(1)})^T \tilde{\mathcal{R}}_1 \eta_k^{(1)} + \omega_k^T \Upsilon_1 \omega_k
\end{aligned} \tag{16}$$

where

$$\begin{aligned}
\tilde{\mathcal{R}}_1 &\triangleq \begin{bmatrix} \tilde{\mathcal{R}}_{1,11} & \tilde{\mathcal{R}}_{1,12} & 0 \\ * & \tilde{\mathcal{R}}_{1,22} & 0 \\ * & * & \tilde{\mathcal{R}}_{1,33} \end{bmatrix}, \\
\eta_k^{(1)} &\triangleq \begin{bmatrix} e_{k|k}^T & e_{k|k-d}^T & \omega_k^T \end{bmatrix}^T, \\
\tilde{\mathcal{R}}_{1,11} &\triangleq (A+\mathcal{J}\otimes\mathcal{W})^T P (A+\mathcal{J}\otimes\mathcal{W}) - (1+\alpha_1)P, \\
\tilde{\mathcal{R}}_{1,12} &\triangleq (A+\mathcal{J}\otimes\mathcal{W})^T PG, \ \tilde{\mathcal{R}}_{(1,22)} \triangleq G^T P G, \\
\tilde{\mathcal{R}}_{1,33} &\triangleq B^T P B - \Upsilon_1,
\end{aligned}$$

and

$$\begin{aligned}
&V_{k+1}^{(2)} - V_k^{(2)} - \alpha_1 V_k^{(2)} \\
=& \sum_{j=0}^{d-1} \sum_{\psi=k+1-d+j}^{k} e_{\psi|\psi}^T Q e_{\psi|\psi} - \sum_{j=0}^{d-1} \sum_{\psi=k-d+j}^{k-1} e_{\psi|\psi}^T Q e_{\psi|\psi} - \alpha_1 V_k^{(2)} \\
=& -\sum_{j=1}^{d} (1+(d+1-j)\alpha_1) e_{k-j|k-j}^T Q e_{k-j|k-j} + d e_{k|k}^T Q e_{k|k} \\
=& \bar{\eta}_k^T \mathcal{R}_1 \bar{\eta}_k
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\bar{\mathcal{R}}_1 &\triangleq \begin{bmatrix} \bar{\mathcal{R}}_{1,11} & 0 & 0 & 0 \\ * & \bar{\mathcal{R}}_{1,22} & 0 & 0 \\ * & * & \bar{\mathcal{R}}_{1,33} & 0 \\ * & * & * & 0 \end{bmatrix}, \\
\bar{\eta}_k^{(1)} &\triangleq \begin{bmatrix} e_{k|k}^T & \breve{e}_{k|k}^T & e_{k-d|k-d}^T & \omega_k^T \end{bmatrix}^T, \\
e_{\breve{k|k}}^{\ T} &\triangleq \begin{bmatrix} e_{k-1|k-1}^T & e_{k-2|k-2}^T & \cdots & e_{k-d+1|k-d+1}^T \end{bmatrix}^T, \\
\bar{\mathcal{R}}_{1,22} &\triangleq \mathrm{diag}\{-(1+d\alpha_1)Q, \cdots, -(1+2\alpha_1)Q\},
\end{aligned}$$

$$\bar{\mathcal{R}}_{1,11} \triangleq dQ,\ \bar{\mathcal{R}}_{1,33} \triangleq -(1+\alpha_1)Q.$$

According to (16) and (17), we have

$$\begin{aligned}
&V_{k+1} - V_k - \alpha_1 V_k \\
=&(\eta_k^{(1)})^T \tilde{\mathcal{R}}_1 \eta_k^{(1)} + (\bar{\eta}_k^{(1)})^T \bar{\mathcal{R}}_1 \bar{\eta}_k^{(1)} + \omega_k^T \Upsilon_1 \omega_k \\
=&(\bar{\eta}_k^{(1)})^T \mathcal{R}_1 \bar{\eta}_k^{(1)} + d_1
\end{aligned}$$

where $d_1 = \omega_k^T \Upsilon_1 \omega_k$.

Then, for any $k_t + 1 \leq k < k_{t+1} - q_{t+1} + 1$ and positive scalar ε, it can be derived that

$$\begin{aligned}
&\varepsilon^{k+1} V_{k+1} - \varepsilon^k V_k \\
=&\varepsilon^{k+1}(V_{k+1} - V_k) + \varepsilon^k(\varepsilon - 1)V_k \\
\leq&\varepsilon^k(\varepsilon + \alpha_1\varepsilon - 1)V_k + \varepsilon^{k+1} d_1.
\end{aligned} \tag{18}$$

We define $\bar{\varepsilon} \triangleq \frac{1}{1+\alpha_1}$ and sum up both sides of (18) with respect to k ($k_t + 1 \leq\ k \leq\ k_{t+1} - q_{t+1} + 1$). It is obvious that

$$\begin{aligned}
&\bar{\varepsilon}^{\ k_{t+1}-q_{t+1}+1} V_{k_{t+1}-q_{t+1}+1} - \bar{\varepsilon}^{\ k_t+1} V_{k_t+1} \\
\leq& d_1 \sum_{s=k_t+2}^{k_{t+1}-q_{t+1}+1} \bar{\varepsilon}^s =\ d_1 \frac{\bar{\varepsilon}^{\ k_t+2} - \bar{\varepsilon}^{\ k_{t+1}-q_{t+1}+2}}{1-\bar{\varepsilon}}.
\end{aligned}$$

which yields

$$V_{\ k_{t+1}-q_{t+1}+1} \leq\ \bar{\varepsilon}^{\ q_{t+1}-h_{t+1}} V_{k_t+1} + d_1 \frac{\bar{\varepsilon}^{\ q_{t+1}-h_{t+1}+1} - \bar{\varepsilon}}{1-\bar{\varepsilon}}. \tag{19}$$

Subsequently, since $0 \leq p^{(\phi)} \leq 1 - \sum_{\vartheta \in \mathbb{H}_k} p^{(\vartheta)}$, for $\nu \in \mathbb{H}_{uk}$, we have

$$\mathbb{E}\{\bar{\varepsilon}^{\ q_{t+1}-h_{t+1}}\} = \sum_{\gamma=1}^{H} p^{(\gamma)} \bar{\varepsilon}^{\ \min\{M,\gamma\}-\gamma} \leq \sum_{\gamma=1}^{H} \bar{p}^{(\gamma)} \bar{\varepsilon}^{\ \min\{M,\gamma\}-\gamma}. \tag{20}$$

In addition, it can be observed from Assumption (3) that

$$\mathbb{E}\{\omega_k^T \Upsilon_1 \omega_k\} = S\mathrm{tr}\{\bar{R}_1^T \Upsilon_1 \bar{R}_1\}$$

where $\mathrm{tr}\{\bar{R}_1^T \Upsilon_1 \bar{R}_1\}$ denotes the trace of matrix $\bar{R}_1^T \Upsilon_1 \bar{R}_1$. Calculating the conditional expectation of (19), we have

$$\mathbb{E}\{V_{k_{t+1}-q_{t+1}+1} | k_t\} \leq\ \hat{\varepsilon}\mathbb{E}\{V_{k_t+1} | k_t\} + \tilde{d}_1, \tag{21}$$

where

$$\hat{\varepsilon} \triangleq \sum_{\gamma=1}^{H} \bar{p}^{(\gamma)} \bar{\varepsilon}^{\ \min\{M,\gamma\}-\gamma},\ \tilde{d}_1 \triangleq \frac{\bar{\varepsilon}\hat{\varepsilon} - \bar{\varepsilon}}{1-\bar{\varepsilon}} \bar{d}_1,\ \bar{d}_1 \triangleq\ S\mathrm{tr}\{\bar{R}_1^T \Upsilon_1 \bar{R}_1\}.$$

Calculating the ME of (21), one has

$$\mathbb{E}\{V_{k_{t+1}-q_{t+1}+1}\} \leq \hat{\varepsilon}\mathbb{E}\{V_{k_t+1}\} + \tilde{d}_1, \tag{22}$$

<u>Case 2:</u> $k_{t+1} - q_{t+1} + 1 \leq k < k_{t+1} + 1$

At k_{t+1}, the decoding signals packet $\bar{\mathcal{Y}}_{k_{t+1}} = \{\bar{y}_{k_{t+1}}, \bar{y}_{k_{t+1}-1}, \cdots, \bar{y}_{k_{t+1}-q_{t+1}+1}\}$ is sent to the estimator. By means of $\bar{\mathcal{Y}}_{k_{t+1}}$, the states from $k_{t+1} - q_{t+1} + 2$ to k_{t+1} would be re-estimated, and then, the state of $k_t + 1$ would be estimated (as the introduced virtual estimation process (13)).

We set $\gamma \triangleq h_{t+1}$. For $k_{t+1} - q_{t+1} + 1 \leq k < k_{t+1}$, the difference between $V^{(1)}_{k+1}$ and $V^{(1)}_k$ can be calculated as follows:

$$\begin{aligned}
&V^{(1)}_{k+1} - V^{(1)}_k \\
=&((A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C})e_{k|k} + Ge_{k-d|k-d} + B\omega_k - \bar{L}^{(\gamma)}D\upsilon_k - \bar{L}^{(\gamma)}\Psi_k)^T P \\
&\times ((A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C})e_{k|k} + Ge_{k-d|k-d} + B\omega_k - \bar{L}^{(\gamma)}D\upsilon_k - \bar{L}^{(\gamma)}\Psi_k) - e^T_{k|k}Pe_{k|k} \\
=&e^T_{k|k}((A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C})^T P((A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C})e_{k|k} + 2e^T_{k|k}(A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C})^T P \\
&\times Ge_{k-d|k-d} + e^T_{k-d|k-d}G^T PGe_{k-d|k-d} - (1 - \alpha_2)e^T_{k|k}Pe_{k|k} + \omega^T_k(B^T PB - \Upsilon_1 I)\omega_k \\
&+ \upsilon^T_k((\bar{L}^{(\gamma)}D)^T P\bar{L}^{(\gamma)}D - \Upsilon_2 I)\upsilon_k + \Psi^T_k((\bar{L}^{(\gamma)})^T P\bar{L}^{(\gamma)} - \Upsilon_3 I)\Psi_k \\
&- \alpha_2 e^T_{k|k}Pe_{k|k} + \omega^T_k\Upsilon_1\omega_k + \upsilon^T_k\Upsilon_2\upsilon_k + \Psi^T_k\Upsilon_3\Psi_k \\
=&(\eta^{(2)}_k)^T\tilde{\mathcal{R}}_2\eta^{(2)}_k - \alpha_2 e^T_{k|k}Pe_{k|k} + \omega^T_k\Upsilon_1\omega_k + \upsilon^T_k\Upsilon_2\upsilon_k + \Psi^T_k\Upsilon_3\Psi_k
\end{aligned} \tag{23}$$

where

$$\begin{aligned}
\tilde{\mathcal{R}}_2 \triangleq &\begin{bmatrix} \tilde{\mathcal{R}}_{2,11} & \tilde{\mathcal{R}}_{2,12} & 0 \\ * & \tilde{\mathcal{R}}_{2,22} & 0 \\ * & * & \tilde{\mathcal{R}}_{2,33} \end{bmatrix}, \ \eta^{(2)}_k \triangleq \begin{bmatrix} e^T_{k|k} & e^T_{k|k-d} & \omega^T_k & \upsilon^T_k & \Psi^T_k \end{bmatrix}^T \\
\tilde{\mathcal{R}}_{2,11} \triangleq &(A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C})^T P(A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C}) - (1 - \alpha_2)P, \\
\tilde{\mathcal{R}}_{2,12} \triangleq &((A + \mathcal{J} \otimes \mathcal{W} - \bar{L}^{(\gamma)}\bar{C})^T PG, \ \tilde{\mathcal{R}}_{2,22} \triangleq G^T PG, \\
\tilde{\mathcal{R}}_{2,33} \triangleq &\mathrm{diag}\{\tilde{\mathcal{R}}_{2,33,1}, \tilde{\mathcal{R}}_{2,33,2}, \tilde{\mathcal{R}}_{2,33,3}\}, \ \tilde{\mathcal{R}}_{2,33,1} \triangleq B^T PB - \Upsilon_1, \\
\tilde{\mathcal{R}}_{2,33,2} \triangleq &(\bar{L}^{(\gamma)}D)^T P\bar{L}^{(\gamma)}D - \Upsilon_2, \ \tilde{\mathcal{R}}_{2,33,3} \triangleq (\bar{L}^{(\gamma)})^T P\bar{L}^{(\gamma)} - \Upsilon_3.
\end{aligned}$$

By calculating the difference of $V^{(2)}_{k+1}$ and $V^{(2)}_k$, we have

$$\begin{aligned}
&V^{(2)}_{k+1} - V^{(2)}_k \\
=&\sum_{j=0}^{d-1}\sum_{\psi=k+1-d+j}^{k} e_{\psi|\psi}Qe_{\psi|\psi} - \sum_{j=0}^{d-1}\sum_{\psi=k-d+j}^{k-1} e_{\psi|\psi}Qe_{\psi|\psi} \\
=&\sum_{j=1}^{d}(e^T_{k|k}Qe_{k|k} - e^T_{k-j|k-j}Qe_{k-j|k-j})) - \alpha_2 V^{(2)}_k + \alpha_2 V^{(2)}_k \\
=& -\sum_{j=1}^{d}(e^T_{k-j|k-j}(1 - (d + 1 - j)\alpha_2)Qe_{k-j|k-j}) + de^T_{k|k}Qe_{k|k} - \alpha_2 V^{(2)}_k \\
=&(\bar{\eta}^{(2)}_k)^T\bar{\mathcal{R}}_2\bar{\eta}^{(2)}_k
\end{aligned} \tag{24}$$

where

$$\bar{\mathcal{R}}_2 \triangleq \begin{bmatrix} \bar{\mathcal{R}}_{2,11} & 0 & 0 & 0 \\ * & \bar{\mathcal{R}}_{2,22} & 0 & 0 \\ * & * & \bar{\mathcal{R}}_{2,33} & 0 \\ * & * & * & \bar{\mathcal{R}}_{2,44} \end{bmatrix},$$

$$\bar{\eta}_k^{(2)} \triangleq \begin{bmatrix} e_{k|k}^T & \breve{e}_{k|k}^T & e_{k-d|k-d}^T & \omega_k^T & \upsilon_k^T & \Psi_k^T \end{bmatrix}^T,$$

$$\bar{\mathcal{R}}_{2,11} \triangleq dQ,\ \bar{\mathcal{R}}_{2,22} \triangleq \text{diag}\{-(1-d\alpha_2)Q, \cdots, -(1-2\alpha_2)Q\},$$

$$\bar{\mathcal{R}}_{2,33} \triangleq -(1-\alpha_2)Q,\ \bar{\mathcal{R}}_{2,44} \triangleq \text{diag}\{0,0,0\}.$$

Up to now, we can easily imply from (23) and (24) that

$$\begin{aligned} & V_{k+1} - V_k \\ = & (\eta_k^{(2)})^T \tilde{\mathcal{R}}_2 \eta_k^{(2)} + (\bar{\eta}_k^{(2)})^T \bar{\mathcal{R}}_2 \bar{\eta}_k^{(2)} - \alpha_2 V_k + \omega_k^T \Upsilon_1 \omega_k + \upsilon_k^T \Upsilon_2 \upsilon_k + \Psi_k^T \Upsilon_3 \Psi_k \\ = & (\bar{\eta}_k^{(2)})^T \mathcal{R}_2 \bar{\eta}_k^{(2)} - \alpha_2 V_k + d_2 \end{aligned}$$

where $d_2 = \omega_k^T \Upsilon_1 \omega_k + \upsilon_k^T \Upsilon_2 \upsilon_k + \bar{\Psi}\Upsilon_3$.

As such, for any $k_{t+1} - q_{t+1} + 1 \leq k < k_{t+1}$ and positive scalar σ, one has

$$\begin{aligned} & \sigma^{k+1} V_{k+1} - \sigma^k V_k \\ \leq & \sigma^k (\sigma - \alpha_1 \sigma - 1) V_k + \sigma^{k+1} d_2. \end{aligned} \tag{25}$$

We set $\bar{\sigma} \triangleq \frac{1}{1-\alpha_2}$ and calculate the summation of both sides of (25) with respect to k ($k_{t+1} - q_{t+1} + 1 \leq k < k_{t+1} + 1$), we have

$$\begin{aligned} & \bar{\sigma}^{k_{t+1}+1} V_{k_{t+1}} - \bar{\sigma}^{k_{t+1}-q_{t+1}+1} V_{k_{t+1}-q_{t+1}+1} \\ \leq & d_2 \sum_{s=k_{t+1}-q_{t+1}+2}^{k_{t+1}+1} \bar{\sigma}^s = d_2 \frac{\bar{\sigma}^{k_{t+1}-q_{t+1}+2} - \bar{\sigma}^{k_{t+1}+2}}{1-\bar{\sigma}}, \end{aligned}$$

which implies

$$V_{k_{t+1}} - \bar{\sigma}^{-q_{t+1}} V_{k_{t+1}-q_{t+1}+1} \leq d_2 \frac{\bar{\sigma}^{-q_{t+1}+1} - \bar{\sigma}}{1-\bar{\sigma}} \tag{26}$$

Note that

$$\mathbb{E}\{\bar{\sigma}^{-q_t}\} = \sum_{\iota=1}^{H} p^{(\iota)} \bar{\sigma}^{-\min\{M,\iota\}} \leq \sum_{\vartheta=1}^{H} \bar{p}^{(\vartheta)} \bar{\sigma}^{-\min\{M,\vartheta\}}.$$

and

$$\mathbb{E}\{\omega_k^T \Upsilon_1 \omega_k + \upsilon_k^T \Upsilon_2 \upsilon_k + \Psi_k^T \Upsilon_3 \Psi_k\} = S\text{Tr}\{\bar{R}_1^T \Upsilon_1 \bar{R}_1\} + L_0 \text{Tr}\{\bar{R}_2^T \Upsilon_2 \bar{R}_2\} + \text{Tr}\{\bar{\Psi}\Upsilon_3\}$$

Calculating the conditional expectation of (26), one has

$$\mathbb{E}\{V_{k_{t+1}+1} | k_t\} \leq \hat{\sigma} \mathbb{E}\{V_{k_{t+1}-q_{t+1}+1} | k_t\} + \tilde{d}_2. \tag{27}$$

where

$$\hat{\sigma} \triangleq \sum_{\vartheta=1}^{S} \bar{p}^{(\vartheta)} \bar{\sigma}^{-\min\{M,\gamma\}},\ \tilde{d}_2 \triangleq \frac{\bar{\sigma}\hat{\sigma}-\bar{\sigma}}{1-\bar{\sigma}}\bar{d}_2,$$
$$\bar{d}_2 \triangleq S\mathrm{Tr}\{\bar{R}_1^T \Upsilon_1 \bar{R}_1\} + L_0 \mathrm{Tr}\{\bar{R}_2^T \Upsilon_2 \bar{R}_2\} + \mathrm{Tr}\{\bar{\Psi}\Upsilon_3\}$$

Taking the ME on the inequity(27), we have

$$\mathbb{E}\{V_{k_{t+1}+1}\} \leq \hat{\sigma}\mathbb{E}\{V_{k_{t+1}-q_{t+1}+1}\} + \tilde{d}_2. \tag{28}$$

Up to now, according to (22) and (28), it is obvious

$$\mathbb{E}\{V_{k_{t+1}+1}\} \leq \beta\mathbb{E}\{V_{k_t+1}\} + d$$

where $d \triangleq \hat{\sigma}\tilde{d}_1 + \tilde{d}_2$ and $\beta \triangleq \hat{\sigma}\hat{\varepsilon}$. Then, for any positive scalars $\bar{\zeta}$, one has

$$\bar{\zeta}^{t+1}\mathbb{E}\{V_{k_{t+1}+1}\} - \bar{\zeta}^t\mathbb{E}\{V_{k_t+1}\} \leq \bar{\zeta}^t(\bar{\zeta} - \bar{\zeta}(1-\beta) - 1)\mathbb{E}\{V_{k_t+1}\} + \bar{\zeta}^{t+1}d. \tag{29}$$

Letting $\bar{\zeta} \triangleq \frac{1}{\beta}$ and calculation the summation of(29) from k_0 to k_s with respect to t, we obtain

$$\bar{\zeta}^s\mathbb{E}\{V_{k_s+1}\} - \mathbb{E}\{V_{k_0+1}\} \leq \frac{d(\bar{\zeta}-\bar{\zeta}^{s+1})}{1-\bar{\zeta}} = \frac{d(1-\beta^s)}{\beta^s(1-\beta)}.$$

which implies that

$$\mathbb{E}\{V_{k_s+1}\} \leq \beta^s\mathbb{E}\{V_{k_0+1}\} + d\frac{(1-\beta^s)}{(1-\beta)} < \beta^s((1-\alpha_2)\mathbb{E}\{V_{k_0}\} + \bar{d}_2 - \frac{1}{(1-\beta)}) + \frac{d}{(1-\beta)}.$$

Hence, the dynamics of $\mathbb{E}\{V_{k_s+1}\}$ is EUB, that is

$$\lim_{s\to+\infty}\mathbb{E}\{V_{k_s+1}\} = \frac{d}{(1-\beta)} < +\infty.$$

Next, for any $k_s+1 \leq k < k_{s+1}+1$, one has

$$\begin{aligned}
\mathbb{E}\{V_k\} \leq& \mathbb{E}\{V_{k_s+H}\} \\
\leq& (1+\alpha_1)^{H-1}\mathbb{E}\{V_{k_s+1}\} + \bar{d}_1\frac{1-(1+\alpha_1)^{H-1}}{1-(1+\alpha_1)} \\
\leq& (1+\alpha_1)^{H-1}\frac{d}{(1-\beta)} + \bar{d}_1\frac{1-(1+\alpha_1)^{H-1}}{1-(1+\alpha_1)} \\
\leq& d\frac{\bar{\varepsilon}^{H-1}}{(1-\beta)} + \bar{d}_1\frac{1-\bar{\varepsilon}^{H-1}}{1-\bar{\varepsilon}}
\end{aligned}$$

where $\rho_2 \triangleq \lambda_{\min}(\sum_{h_t=1}^{H} p_{h_t}P_{h_t})$ where $\lambda_{min}(K)$ refers to the minimum eigenvalue of K, and this implies that the error dynamics is EUB in MS. In the end, we come to the conclusion that

$$\lim_{s\to+\infty}\mathbb{E}\{\|e_{k|k}\|^2\} \leq d\frac{\bar{\varepsilon}^{H-1}}{\rho_2(1-\beta)} + \bar{d}_1\frac{1-\bar{\varepsilon}^{H-1}}{\rho_2(1-\bar{\varepsilon})}$$

where $\rho_2 \triangleq \lambda_{\min}(P)$. $\square$

Theorem 2. *For the CN(5), assume that there exist two scalars $\alpha_2 > 1$ and $0 < \alpha_2 < 1$, positive definite matrices P_i $(i = 1, 2, \cdots, S)$, Q_i $(i = 1, 2, \cdots, S)$, $\Upsilon_1 \in \mathbb{R}^{Sn_\omega \times Sn_\omega}$, $\Upsilon_2 \in \mathbb{R}^{Sn_\upsilon \times Sn_\upsilon}$, $\Upsilon_3 \in \mathbb{R}^{Sn_y \times Sn_y}$, and $l_0 H$ gain matrices $L_i^{(\iota)}(i = 1, 2, \cdots, l_0)(\gamma = 1, 2, \cdots, H)$ satisfying (9), (11), and the following condition:*

$$\check{\mathcal{R}}_2 = \begin{bmatrix} \check{\mathcal{R}}_{2,11} & \check{\mathcal{R}}_{2,12} & \check{\mathcal{R}}_{2,13} & \check{\mathcal{R}}_{2,14} \\ * & \check{\mathcal{R}}_{2,22} & 0 & 0 \\ * & * & \check{\mathcal{R}}_{2,33} & 0 \\ * & * & * & \check{\mathcal{R}}_{2,44} \end{bmatrix} < 0 \tag{30}$$

where

$$\begin{aligned}
\check{\mathcal{R}}_{2,11} &\triangleq \mathrm{diag}\{\check{\mathcal{R}}_{2,11,1}, \check{\mathcal{R}}_{2,11,2}, \check{\mathcal{R}}_{2,11,3}\},\ \check{\mathcal{R}}_{2,11,1} \triangleq -(1-\alpha_2)P + dQ, \\
\check{\mathcal{R}}_{2,11,2} &\triangleq \mathrm{diag}\{-(1-d\alpha_2)Q, \cdots, -(1-2\alpha_2)Q\}, \\
\check{\mathcal{R}}_{2,11,3} &\triangleq \mathrm{diag}\{-(1-\alpha_2)Q, B^T P B - \Upsilon_1 I, -\Upsilon_2 I, -\Upsilon_3 I\}, \\
\check{\mathcal{R}}_{2,12} &\triangleq \begin{bmatrix} (A + \mathcal{J} \otimes \mathcal{W} - \tilde{L}^{(\gamma)} \bar{C}) & 0 & G^T & 0 & 0 & 0 \end{bmatrix}^T, \\
\check{\mathcal{R}}_{2,13} &\triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & \tilde{L}^{(\gamma)} D & 0 \end{bmatrix}^T, \\
\check{\mathcal{R}}_{2,14} &\triangleq \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \tilde{L}^{(\gamma)} \end{bmatrix}^T, \\
\check{\mathcal{R}}_{2,22} &\triangleq P - 2I,\ \check{\mathcal{R}}_{2,33} \triangleq P - 2I,\ \check{\mathcal{R}}_{2,44} \triangleq P - 2I, \\
\tilde{L}^{(\gamma)} &\triangleq \sum_{\gamma=1}^{H} \bar{p}^{(\gamma)} \bar{L}^{(\gamma)},\ \bar{p}^{(\gamma)} \triangleq \begin{cases} p^{(\gamma)}, & \text{if } \gamma \in \mathbb{H}_k \\ 1 - \sum_{j \in \mathbb{H}_k} p^{(j)}, & \text{if } \gamma \in \mathbb{H}_{uk} \end{cases}
\end{aligned}$$

Then, for CN(5), the dynamics of the PNBSE error system(8) is EUB subject to the disturbance noise ω_k ,υ_k, and the decoding error Ψ_k.

Proof. The proof follows directly from Theorem1 by using the Schur Complement. Now we have completed the proof.

Remark 5. *Compared with existing results, the core research highlights of this paper are listed as follows: 1) the PNBSE problem is a novel problem in the context of CNs with EDM subject to unreliable communications; 2) a novel SE tactic has been proposed to solve the EUB estimation problem in presence of EDM and intermittent transmission case; 3) recurring to the measurement signals from partial accessible nodes and partial instants, the proposed PNB estimator ensures the EUB of the SE error; and 4) considering the unreliable communications (i.e., intermittent transmissions), a buffer-aided strategy is employed to provide more measurement signals to the estimator, and the influence of the buffer-aided strategy on the estimation performance has been analyzed.*

□

4. A Simulation Example

Assume that there 4 nodes in the CN. The measurement outputs of the first 2 nodes are accessible, i.e., $S = 4, l_0 = 2$. The maximum transmission interval and the limited capacity of the buffer are chosen as $H = 6, M = 4$, respectively, and the other parameters are listed as follows:

$$A_1 = A_2 = \begin{bmatrix} 0.9 & 0.4 & 0 \\ 0.3 & 0.7 & 0.2 \\ 0 & -0.8 & 0 \end{bmatrix}, \; A_3 = A_4 = \begin{bmatrix} 0.5 & 0.2 & 0 \\ 0.25 & 0.65 & 0.1 \\ 0 & -0.2 & 0.4 \end{bmatrix}, \; \mathcal{W} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$G_1 = G_2 = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}, \; G_3 = G_4 = \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.1 \end{bmatrix},$$

$$\mathcal{J} = \begin{bmatrix} -0.4 & 0.15 & 0.15 & 0.1 \\ 0.2 & -0.6 & 0.2 & 0.2 \\ 0.15 & 0.15 & -0.5 & 0.2 \\ 0.2 & 0.2 & 0.2 & -0.6 \end{bmatrix}, \; B_1 = B_2 = B_3 = B_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$C_1 = C_2 = \begin{bmatrix} 0.3 & 0 & 0 \\ 0 & 0.3 & 0 \end{bmatrix}, \; D_1 = D_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \; \alpha_1 = 0.225, \; \alpha_2 = 0.1878, \; d = 2.$$

Moreover, the variance of the PN and MN are $R_1 = 0.2025I$ and $R_2 = 0.16I$, respectively. The transmission intervals h_t take values in the set $\mathbb{H} \triangleq \{1, 2, 3, 4, 5, 6\}$, with the following occurrence probability:

$$p^{(1)} = 0.1, p^{(2)} = 0.2, p^{(3)} = 0.4,$$
$$p^{(4)} = 0.1, p^{(5)} = 0.2, p^{(6)} = 0.2.$$

Then, by utilizing MATLAB Linear Matrix Inequality Toolbox, we obtain estimator parameter matrices

Simulation results are given in Figs. $3-8$. Fig. 3 and Fig. 4 show the state trajectories of the CN, from which we can find that each nodes in the whole CN are unstable. Fig. 5 and Fig. 6 plot the estimate estimation error evolution, which indicate that the state estimator achieves good estimation performance both for the accessible nodes and unaccessible nodes. Furthermore, even if successful signal transmissions can not occur all the time and the estimation error would increase at some instants, the estimation error still enters an ultimately bounded region eventually. Therefore, the simulation results have verified our analysis on the EUB estimation problem.

Furthermore, for the purpose of showing the effects of various buffer capacities and maximum transmission intervals on estimation performance, we provide some results for comparison in Fig. 7 and Fig. 8. As shown in in Fig. 7, when the maximum transmission interval is selected as the same value, the larger the buffer capacity is, the better the estimator performs. On the other hand, Fig. 8 plots the influence of the maximum transmission intervals, from which we can see that a shorter transmission interval contributes to an improvement on estimation performance if the buffer capacity is the same.

5. Conclusion

In this paper, by utilizing the measurements from a fraction of nodes, the SE problem has been addressed for a class of CNs with EDM subject to unreliable communications. For the purpose of improving transmission efficiency, EDM is considered in this paper to achieve signal compression. The buffer-aided strategy has been adopted in the communication network to provide more measurement signals to the estimator. conditions which are sufficient have been derived to guarantee

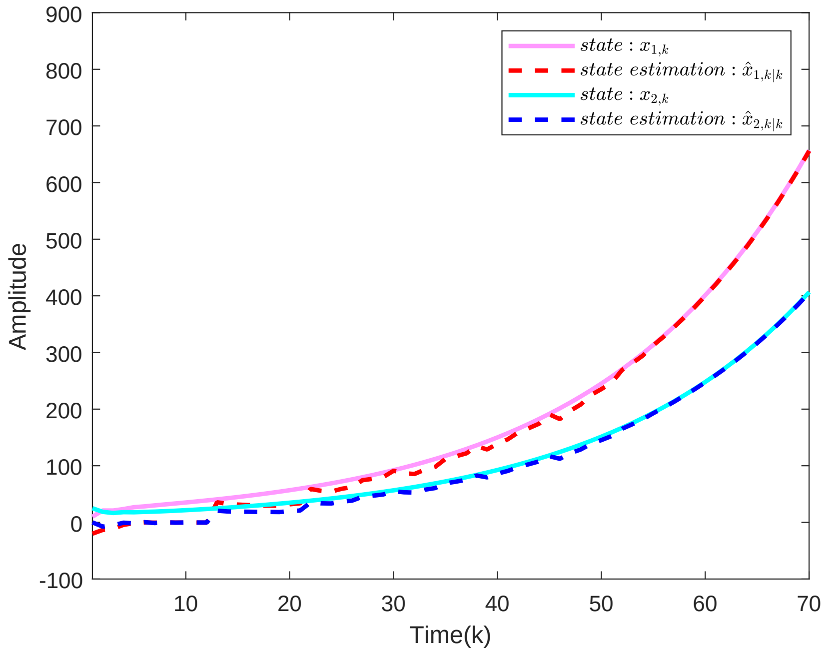

Figure 3: State trajectories of the accessible nodes.

the EUB of the estimation error system, and the desired estimator gains have also been calculated. Simulation has verified the effectiveness and correctness of the proposed estimator designing strategy. In the coming years, we will focus on the research of the control/filtering problem for systems with buffer-aided strategy and other phenomena in networked systems which can be seen in [25, 46], including packet loss seen in [10], actuator failures seen in [15, 52], fading channels seen in [38], communication protocols seen in [24, 50], cyber-attacks [3, 8], and so on, as seen in [2, 4, 5, 11, 23, 28, 29].

References

[1] Albert R, Jeong H, Barabasi A.-L. Diameter of the world-wide Web. Nature 1999; 401: 130-1.

[2] Chen S, Yang R, Zhong M. Graph-based semi-supervised random forest for rotating machinery gearbox fault diagnosis. Control Engineering Practice 2021; 117:art. no. 104952.

[3] Ding D, Wang Z, Han Q-L, Wei G. Security control for discrete-time stochastic nonlinear systems subject to deception attacks. IEEE Transactions on Systems, Man, and Cybernetics: Systems 2018; 48(5): 779-89.

[4] Ding D, Han Q-L, Wang Z, Ge X. A survey on model-based distributed control and filtering for industrial cyber-physical systems. IEEE Transactions on Industrial Informatics 2019; 15(5): 2483-99.

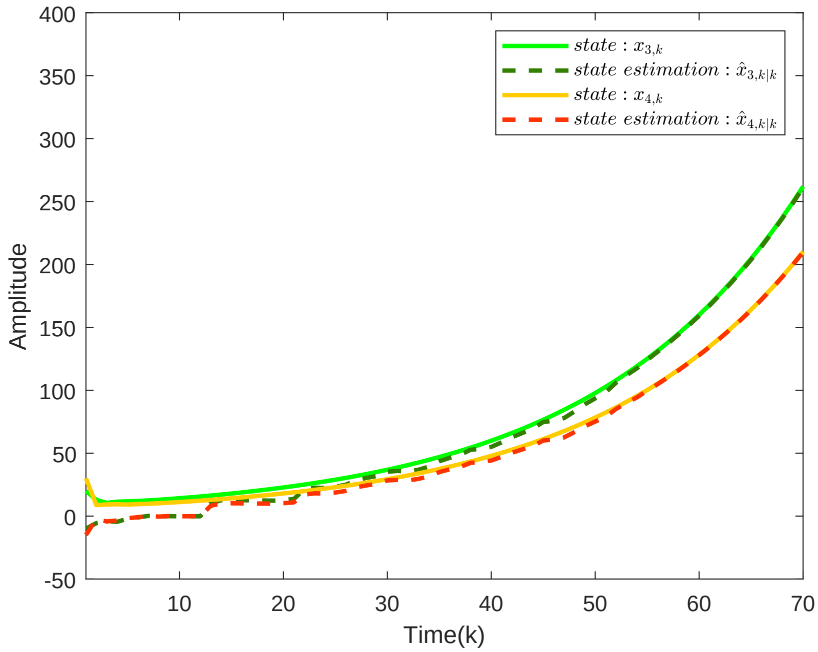

Figure 4: State trajectories of the inaccessible nodes.

[5] Ding D, Wang Z, and Han Q-L. A set-membership approach to event-triggered filtering for general nonlinear systems over sensor networks. IEEE Transactions on Automatic Control 2020; 65(4): 1792-99.

[6] Eagle N, Pentland A S. Sensing complex social systems. Personal and Ubiquitous Computing 2006; 10(4): 255-68.

[7] Geng H, Liu H, Ma L, Yi X. Multi-sensor filtering fusion meets censored measurements under a constrained network environment: Advances, challenges and prospects. International Journal of Systems Science 2021; 52(16): 3410-36.

[8] He W, Mo Z, Han Q-L, Qian F. Secure impulsive synchronization in lipschitz-type multi-agent systems subject to deception attacks. IEEE/CAA J. Automatica Sinica 2020; 7 (5): 1326-34.

[9] Hou N, Dong H, Zhang W, Liu Y, Alsaadi F E. Event-triggered state estimation for time-delayed complex networks with gain variations based on partial nodes, International Journal of General Systems 2018; 47(5): 477-90.

[10] Hu J, Wang Z, Liu G-P, Jia C, Williams J. Event-triggered recursive state estimation for dynamical networks under randomly switching topologies and multiple missing measurements. Automatica 2020; 115: art. no. 108908.

[11] Hu J, Zhang H, Liu H, Yu X. A survey on sliding mode control for networked control systems. International Journal of Systems Science 2021; 52(6): 1129-47.

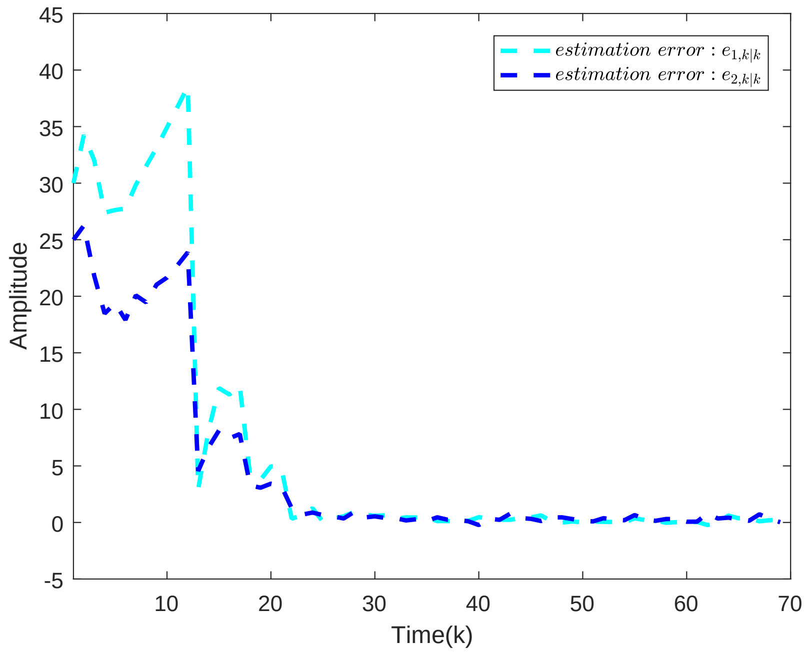

Figure 5: Estimation error of the accessible nodes.

[12] Hu J, Jia C, Liu H, Yi X, Liu Y. A survey on state estimation of complex dynamical networks. International Journal of Systems Science 2021; 52: 3351-67.

[13] Hu J, Wang Z, Liu G. Delay compensation-based state estimation for time-varying complex networks with incomplete observations and dynamical bias. IEEE Transactions on Cybernetics 2021.

[14] Jain R, Routhier S. Packet trains–Measurements and a new model for computer network traffic. IEEE Journal on Selected Areas in Communications 1986; 4(6): 986-95.

[15] Ju Y, Tian X, Liu H, Ma L. Fault detection of networked dynamical systems: A survey of trends and techniques. International Journal of Systems Science 2021; 52(16): 3390-409.

[16] Leung H, Seneviratne C, Xu M. A novel statistical model for distributed estimation in wireless sensor networks. IEEE Signal Processing 2015; 63(12): 3154-64.

[17] Liu D, Liu Y, and Alsaadi F E. Recursive state estimation based-on the outputs of partial nodes for discrete-time stochastic complex networks with switched topology. Journal of the Franklin Institute 2018; 355(11): 4686-707.

[18] Liu L, Ma L, Guo J, Zhang J, Bo Y. Distributed set-membership filtering for time-varying systems: A coding-decoding-based approach. Automatica 2021; 129: art. no. 109684.

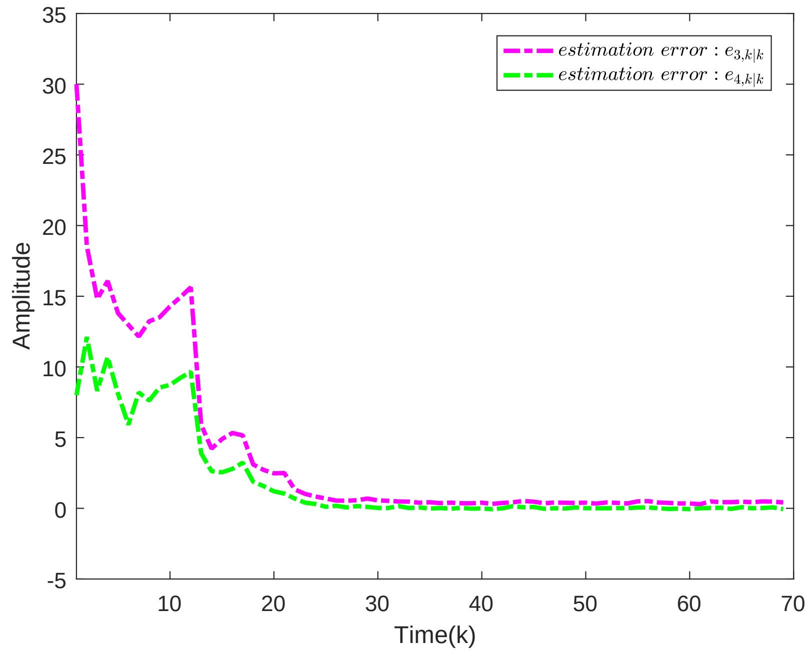

Figure 6: Estimation error of the inaccessible nodes.

[19] Liu S, Song Y, Wei G, Huang X. RMPC-based security problem for polytopic uncertain system subject to deception attacks and persistent disturbances. IET Control Theory Applications 2017; 11(10): 1611-18.

[20] Liu X, Chen T. Synchronization of complex networks via aperiodically intermittent pinning control. IEEE Transactions on Automatic Control 2015; 60(12): 3316-21.

[21] Liu Y, Liu W, Obaid M A, Abbas I A. Exponential stability of markovian jumping CohenC-Grossberg neural networks with mixed mode-dependent time-delays. Neurocomputing 2016; 177: 409-15.

[22] Liu Y, Wang Z, Ma L, Alsaadi F E. A partial-nodes-based information fusion approach to state estimation for discrete-time delayed stochastic complex networks. Information Fusion 2019; 49: 240-8.

[23] Ma L, Wang Z, Liu Y, Alsaadi F E. Distributed filtering for nonlinear time-delay systems over sensor networks subject to multiplicative link noises and switching topology. International Journal of Robust and Nonlinear Control 2019; 29(10): 2941-59.

[24] Mao J, Sun Y, Yi X, Liu H, Ding D. Recursive filtering of networked nonlinear systems: A survey. International Journal of Systems Science 2021; 52(6): 1110-28.

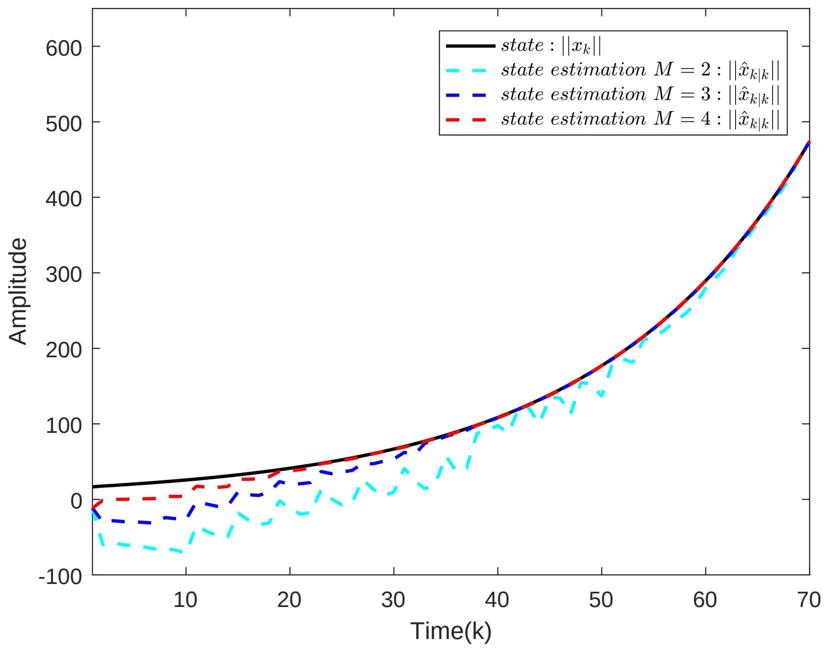

Figure 7: Norm of the state and its estimates when $H = 5$

[25] Ning B, Han Q-L, Ding L. Distributed finite-time secondary frequency and voltage control for islanded microgrids with communication delays and switching topologies. IEEE Transactions on Cybernetics 2021; 51(8): 3988-999.

[26] Ozcelikkale A, McKelvey T, Viberg M. Remote estimation of correlated sources under energy harvesting constraints. IEEE Transactions on Wireless Communications 2018; 17(8): 5300-13.

[27] Robinson C L, Kumar P R. Sending the most recent observation is not optimal in networked control: Linear temporal coding and towards the design of a control specific transport protocol. IEEE Conference on Decision and Control 2007; New Orleans, LA, USA: 334-9.

[28] Shen B, Wang Z, Qiao H. Event-triggered state estimation for discrete-time multidelayed neural networks with stochastic parameters and incomplete measurements. IEEE Transactions on Neural Networks and Learning Systems 2017; 28(5): 1152-63.

[29] Shen B, Wang Z, Wang D, Luo J, Pu H, Peng Y. Finite-horizon filtering for a class of nonlinear time-delayed systems with an energy harvesting sensor. Automatica 2019; 100: 144-52.

[30] Shen B, Wang Z, Wang D, Li Q. State-saturated recursive filter design for stochastic time-varying nonlinear complex networks under deception attacks. IEEE Transactions on Neural Networks and Learning Systems 2020; 31(10): 3788-800.

[31] Shen D, Zhang C. Zero-error tracking control under unified quantized iterative learning framework via encoding-decoding method. IEEE Transactions on Cybernetics 2020.

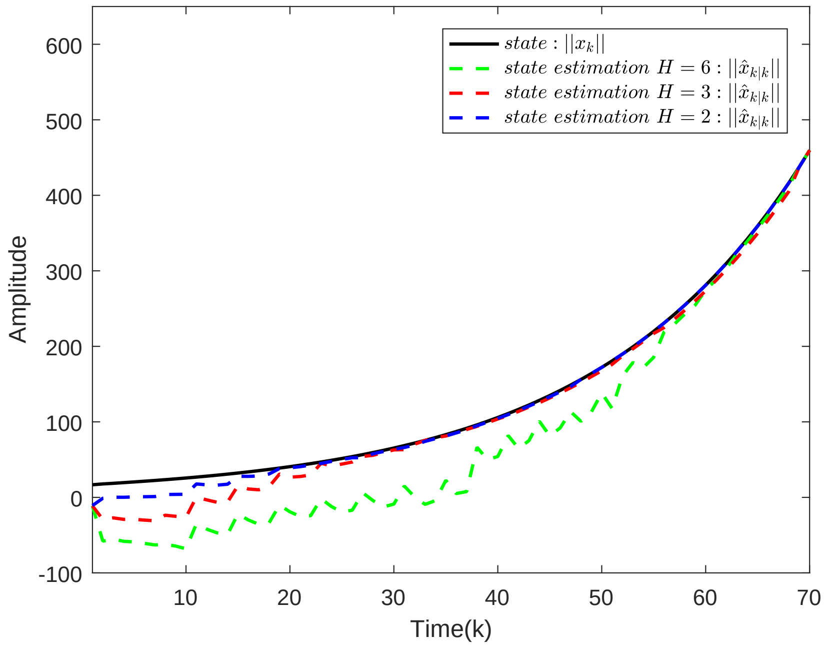

Figure 8: Norm of the state and its estimates when $M = 2$

[32] Sheng L, Zou Y, Liu Y, Alsaadi F E. Finite horizon state estimation for time-varying complex networks with random coupling strengths under Round-Robin protocol. Journal of the Franklin Institute 2018; 355(15): 7417-42.

[33] Shi D, Chen T, Shi L. Event-triggered maximum likelihood state estimation. Automatica 2014; 50(1): 247-54.

[34] Tang Y, Gao H, Zou W, Kurths J. Distributed synchronization in networks of agent systems with nonlinearities and random switchings. IEEE Transactions on Cybernetics 2013; 48(1): 358–370.

[35] Ugrinovskii V, Fridman E. A Round-Robin type protocol for distributed estimation with H_∞ consensus. Systems and Control Letters 2014; 69: 103-10.

[36] Wu D, Wu J, Chen S, Chu J. Stability of networked control systems with polytopic uncertainty and buffer constraint. IEEE Transactions on Automatic Control 2010; 55(5): 1202-8.

[37] Xiao S, Ge X, Han Q-L, Zhang Y. Secure distributed adaptive platooning control of automated vehicles over vehicular Ad-Hoc networks under denial-of-service attacks. IEEE Transactions on Cybernetics 2021.

[38] Xu Y, Lu R, Shi P, Li H, Xie S. Finite-time distributed state estimation over sensor networks with Round-Robin protocol and fading channels. IEEE Transactions on Cybernetics 2016; 48(1): 336-45.

[39] Xu Y, Yang L, Wang Z, Rao H, Lu R. State estimation for networked systems with Markov driven transmission and buffer constraint. IEEE Transactions on Systems, Man, and Cybernetics: Systems 2021.

[40] Yang N, Chen D, Ji D, Wu Z. Resilient state estimation for nonlinear complex networks with time-delay under stochastic communication protocol. Neurocomputing 2019; 346: 38-47.

[41] Yang W, Li D, Zhang H, Tang Y, Zheng W X. An encoding mechanism for secrecy of remote state estimation. Automatica 2020; 120: art. no. 109116.

[42] Yuan Y, Yuan H, Guo L, Yang H, Sun S. Resilient control of networked control system under DoS attacks: A unified game approach. IEEE Transactions on Industrial Informatics 2016; 12(5): 1786-94.

[43] Yuan Y, Zhang P, Guo L, Yang H. Towards quantifying the impact of randomly occurred attacks on a class of networked control systems. Journal of the Franklin Institute-Engineering and Applied Mathematics 2017; 354(12): 4966-88.

[44] Zhan X S, Guan Z H, Zhang X H, Yuan F S. Optimal tracking performance of MIMO control systems with communication constraints and a code scheme. International Journal of Systems Science 2015; 46(3): 464-73.

[45] Zhang D, Han Q-L, Zhang X-M. Network-based modeling and proportional-integral control for direct-drive-wheel systems in wireless network environments. IEEE Transactions on Cybernetics 2020; 50(6): 2462-74.

[46] Zhang X-M, Han Q-L, Ge X. Sufficient conditions for a class of matrix-valued polynomial inequalities on closed intervals and application to H_∞ filtering for linear systems with time-varying delays. Automatica 2021; 125: art. no. 109390.

[47] Zhang X-M, Han Q-L, Ge X. A novel approach to H_∞ performance analysis of discrete-time networked systems subject to network-induced delays and malicious packet dropouts. Automatica 2022; 136: art. no. 110010.

[48] Zhang H, Hu J, Liu H, Yu X, Liu F. Recursive state estimation for time-varying complex networks subject to missing measurements and stochastic inner coupling under random access protocol. Neurocomputing 2019; 346: 48-57.

[49] Zhang Q, Chakhchoukh Y, Vittal V, Heydt G, Logic N, Sturgill S. Impact of PMU measurement buffer length on state estimation and its optimization. IEEE Transactionson Power Systems 2013; 28(2): 1657-65.

[50] Zhao Y, He X, Ma L, Liu H. Unbiasedness-constrained least squares state estimation for time-varying systems with missing measurements under round-robin protocol. International Journal of Systems Science 2022.

[51] Zhao Y-P, He P, Nik H S, Ren J. Robust adaptive synchronization of uncertain complex networks with multiple time-varying coupled delays. Complexity 2015; 20(6): 62-73.

[52] Zhu X, Liu Y, Fang J, Zhong M. Fault detection for a class of linear systems with integral measurements. Science China Information Sciences 2021; 64(3): art. no. 132207.