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Soft Computing in Investment Appraisal

Antoaneta Serguieva
Dept. of Economics&Finance
Brunel University
Uxbridge UB8 3PH, UK
Antoaneta.Serguieva@brunel.ac.uk

John Hunter
Dept. of Economics&Finance
Brunel University
Uxbridge UB8 3PH, UK
John.Hunter@brunel.ac.uk

Tatiana Kalganova
Dept. of Electronic&Computer Eng.
Brunel University
Uxbridge UB8 3PH, UK
Tatiana.Kalganova@brunel.ac.uk

Abstract

Standard financial techniques neglect extreme situations and regards large market shifts as too unlikely to matter. Such approach accounts for what occurs most of the time in the market, but does not reflect the reality, as major events happen in the rest of the time and investors are 'surprised' by 'unexpected' market movements. An alternative fuzzy approach permits fluctuations well beyond the probability type of uncertainty and allows one to make fewer assumptions about the data distribution and market behaviour. Fuzzifying the present value criteria, we suggest a measure of the risk associated with each investment opportunity and estimate the project's robustness towards market uncertainty. The procedure is applied to thirty-five UK companies traded on the London Stock Exchange and a neural network solution to the fuzzy criterion is provided to facilitate the decision-making process. Finally, we suggest a specific evolutionary algorithm to train a fuzzy neural net - the bidirectional incremental evolution will automatically identify the complexity of the problem and correspondingly adapt the parameters of the fuzzy network.

Keywords: Finance, Evaluating fuzzy expressions, Neural networks, Evolutionary algorithms.

1 Introduction

Investment projects are typically chosen on the basis of a restricted information set, while the volatility literature claims that stock prices are too volatile to accord with simple present value models. To approach the problem, we model the restricted information and incorporate price uncertainty into calculations. Uncertain share prices and dividend yields, associated with a family of possible streams of future cash flows, as well as uncertain discount rates are handled by the introduction of fuzzy variables, whose values are restricted by possibility distributions. Alternatively, fuzzy numbers are suggested with corresponding membership functions. Increasing the range of uncertainty modelled by the fuzzified data, we determine the robustness of the investment risk associated with each project. Neural networks yield a mechanism to facilitate the solution of the fuzzy criterion. Once trained to evaluate a project, a neural net provides investors with a simple re-evaluation tool when the information available is subject to change. Finally, as a direction for future research, we suggest a fuzzy neural net, as it will directly handle fuzzy market data, and recommend the bidirectional incremental evolution as an effective training mechanism.

2 Net present value models

NPV evaluations are increasingly being used in the UK [14,17]. The technique is broadly adopted in practice, managers are comfortable with it, and it is reasonable to consider the fuzzy alternative to take account of more general forms of uncertainty. Fig. 1 describes the interrelations between standard and soft computing techniques as well as between theoretical and empirical research, all involved into the process of formulating the fuzzy criterion.[1]

The standard NPV formula has been continuously

[1] For a detailed discussion, see [19].

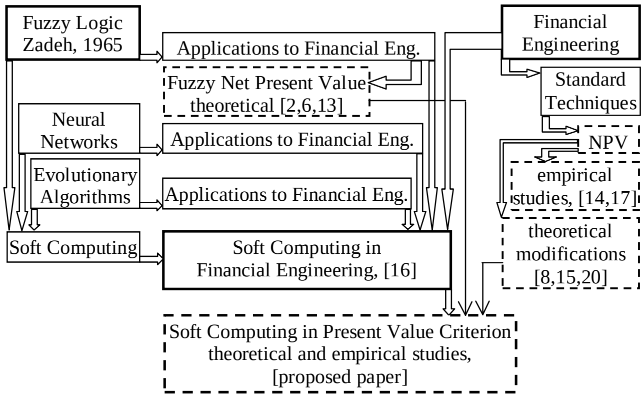

Fig. 1: Interrelations in investment appraisal techniques
▪ standard and soft computing; theoretical and empirical studies

revised. It has been realised that the removal of any of the perfect market assumptions destroys the foundation and reduces the effectiveness of the method. We do not attempt to cope with a specific drawback of the technique but permit into the model structure as much uncertainty as the market could possibly suffer. The calculation based on the standard criterion may be re-optimised due to investment irreversibility [8] and altered because of the effect of capital and labour market rationing [15] or impact of a project on the investor's total risk [20]. Whatever reason one has for modifying the classic result, the allowances provided by the fuzzy criterion will cover these specific circumstances and will include the modified values. The outcome is an effective method under restricted information, uncertain data and market imperfections.

Fuzzy net present value was first introduced in [2], then considered in a broader framework of accumulation and discount models in [6], and recently modified with an alternative fuzzification of the project duration in [13]. All these studies are theoretical. The major difference here is the emphasis we place on the empirical results - we evaluate 35 projects investing in UK companies traded on the LSE. Concurrently, it is the analysis of the empirical results that facilitates the formulation of measures for the investment risk and the project robustness towards market uncertainty, and assists the definition of an alternative ranking technique. Thus the fuzzy criterion evolves into a considerably informative and advantageous to investors method. Further, we build a neural network to solve the fuzzy investment criterion. In result, investors are provided with an effortless instrument for risk re-evaluation, any time they update a project. We also suggest a fuzzy neural net and recommend an effective evolutionary training algorithm. In conclusion, combining the advantages of various soft methodologies, the article balances empirical and theoretical results, with the driving force being the investor's benefit.

3 Investment project evaluation using a fuzzy criterion with a constant discount rate

We apply two methods of evaluating fuzzy algebraic expressions from [3]. First, if positive real triangular fuzzy numbers $\tilde{P}_t$, $\tilde{D}Y_t$ and $\tilde{R}$ are substituted for the share price P_t, dividend yield DY_t and discount rate R, then $\tilde{P}V_{FN}$ is the triangular-shaped fuzzy number providing a set of values that belongs to the present value with various degrees of membership. Form the α-cuts of the price time-series $\tilde{P}(\alpha)=\{\tilde{P}_t(\alpha)\}$ and the dividend-yield time-series $\tilde{D}Y=\{\tilde{D}Y_t\}$ for 1≤t≤N, and find $\Omega_{PV_{FN}}(\alpha)$.

$$\tilde{P}(\alpha)=\prod_{t=1}^{N}\begin{cases}[P_{ta},P_{tc}], & \alpha=0\\ \{x_{Pt}\,|\,\mu(x_{Pt}\,|\,\tilde{P}_t)\ge\alpha\}, & 0<\alpha\le 1\end{cases} \quad (1a)$$

$$\tilde{D}Y(\alpha)=\prod_{t=1}^{N}\begin{cases}[DY_{ta},DY_{tc}], & \alpha=0\\ \{x_{DYt}\,|\,\mu(x_{DYt}\,|\,\tilde{D}Y_t)\ge\alpha\}, & 0<\alpha\le 1\end{cases} \quad (1b)$$

$$\Omega_{PV_{FN}}(\alpha)=\left\{\sum_{t=1}^{N}\frac{P_t DY_t}{(1+R)^t}+\frac{P_N}{(1+R)^N}\,\middle|\,P_{1\times N}\in\tilde{P}(\alpha), DY_{1\times N}\in\tilde{D}Y(\alpha), R\in\tilde{R}(\alpha)\right\} \quad (1c)$$

Then the first solution is defined by $\mu(x_{PV_{FN}}\,|\,\tilde{P}V_{FN})$:

$$\mu(x_{PV_{FN}}\,|\,\tilde{P}V_{FN})=\sup\{\alpha\,|\,x_{PV_{FN}}\in\Omega_{PV_{FN}}(\alpha)\} \quad (1d)$$

Second, if $\breve{P}_t$, $\breve{D}Y_t$ and $\breve{R}$ are positive real fuzzy variables with triangular possibility distributions Poss[$\breve{P}_t=x_{Pt}$]=$\mu(x_{Pt}|\tilde{P}_t)$, Poss[$\breve{D}Y_t=x_{DYt}$]=$\mu(x_{DYt}|\tilde{D}Y_t)$ and Poss[$\breve{R}=x_R$]=$\mu(x_R|\tilde{R})$, correspondingly, then $\breve{P}V_{FV}$ is the fuzzy variable, whose values are possible solutions to the fuzzy present value criteria. Form the joint possibility distribution of share-price time-series and dividend-yield time-series

$$\pi_P=\min_{1\le t\le N}\{\text{Poss}[\breve{P}_t=x_{Pt}]\}=\min_{1\le t\le N}\mu(x_{Pt}|\tilde{P}_t)\} \quad (2a)$$

$$\pi_{DY}=\min_{1\le t\le N}\{\text{Poss}[\breve{D}Y_t=x_{DYt}]\}=\min_{1\le t\le N}\mu(x_{DYt}|\tilde{D}Y_t)\} \quad (2b)$$

and find the joint possibility distribution $\pi_{PV_{FV}}$,

$$\pi_{PV_{FV}}=\min\{\pi_P,\pi_{DY},\text{Poss}[\breve{R}=x_R]\} \quad (2c)$$

assuming that $\breve{P}_t$, $\breve{D}Y_t$ and $\breve{R}$ are non-interactive. Then, the possibility distribution of the second solution is

$$\text{Poss}[\breve{P}V_{FV}=x_{PV_{FV}}]=\sup\{\pi_{PV_{FV}}\,|\,x_{PV_{FV}}=\sum_{t=1}^{N}\frac{P_t DY_t}{(1+R)^t}+\frac{P_N}{(1+R)^N}\} \quad (2d)$$

The two solutions are identical but we present them both, as the first provides the computational algorithm and the second justifies the uncertainty-modelling technique.

We use monthly (DataStream) data on share prices and dividend yields covering 35 UK companies for Jan.'75-Jan.'00. The discount rate equals the average 3-month UK treasury-bill rate. The support of the triangular membership function of each $\tilde{P}_t$, $\tilde{DY}_t$, and $\tilde{R}$, is 2.5% wider than the 99% normal-distribution confidence interval. The calculations involve relation (3) and Fig. 2 presents the graphics for BBA GROUP.[2]

$$\Omega_{PV_{FN}}(\alpha)=\left[\underline{PV_{FN}}(\alpha),\overline{PV_{FN}}(\alpha)\right]= =\left[\sum_{t=1}^{N}\frac{\underline{P_t}(\alpha)\underline{DY_t}(\alpha)}{\left(1+\overline{R}(\alpha)\right)^t}+\frac{\underline{P_N}(\alpha)}{\left(1+\overline{R}(\alpha)\right)^N},\sum_{t=1}^{N}\frac{\overline{P_t}(\alpha)\overline{DY_t}(\alpha)}{\left(1+\underline{R}(\alpha)\right)^t}+\frac{\overline{P_N}(\alpha)}{\left(1+\underline{R}(\alpha)\right)^N}\right] \quad (3)$$

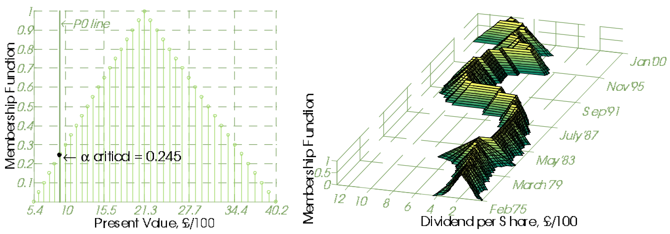

Figure 2: Results for BBA GROUP under N-calibration
▪ fuzzy present value and fuzzy cash-flow

The character of the fuzzy cash-flow stream affects the form of the fuzzy present value. There exists a unique for each company $\alpha_{critical}$ where the initial-outlay line crosses the present-value membership function. Then the critical level of uncertainty is $u_{critical}=1-\alpha_{critical}$. A project is profitable at any $u<u_{critical}$, while at $u\geq u_{critical}$ there is a chance of being unprofitable. Table 1 presents part of the results.[3]

Table 1: Constant discount rate with normal calibration - critical values

company	BBA	BOC	BP AMOCO	DIXONS
$\alpha_{Ncritical}$	0.245	0.000	0.000	0.656
company	**GOODWIN**	**HANSON**	**N. FOODS**	**WOLSELEY**
$\alpha_{Ncritical}$	0.421	0.507	0.339	0.000

4 Measuring the investment risk and its robustness

The results in section 3 show that a critical level of uncertainty is associated with each project. The solution procedure applied allows for finding the set of the project's present values that corresponds to all the share prices, dividend yields and discount rates possible at some level of uncertainty. This set is situated at the same level of uncertainty. Therefore, there is a critical level of uncertainty, $u_{critica}$, embodied in the market data we use to evaluate the project and this level delimits the project's investment risk. We suggest $1-u_{critical}=\alpha_{critical}\in[0,1]$ as a risk measure. The following reasons support the suggestion. The lower the critical level of uncertainty at which there is a chance for the project being unprofitable, the higher the investment risk. Second, $\alpha_{critical}$ is the membership level of the fuzzy present value, below and at which it is certain that the solution includes values below or equal to the initial outlay, and above which the project is definitely profitable.

Further, evaluating the same projects under increased uncertainty of the market environment and comparing the resultant critical values, we derive estimates of the investment risk robustness, $\Delta\alpha=\alpha_{t6critical}-\alpha_{Ncritical}$. To model increased market uncertainty, a calibration procedure is applied based on the 95% t6-confidence interval rather than the N-interval, thus producing fatter-tail possibility distributions. Fig. 3 illustrates how this affects the results for BP AMOCO.[2] Some of the values are shown in Table 2.[3]

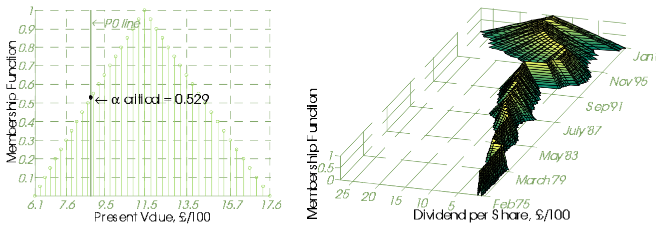

Figure 3: Results for BP AMOCO under t6-calibration
▪ fuzzy present value and fuzzy cash-flow

Raising the market uncertainty, we logically obtain decreased critical levels $u_{critical}$. For all the projects, the relation $u_{t6critical}\leq u_{Ncritical}$ holds, which indicates that the chance of a project being unprofitable now occurs at lower levels of uncertainty embodied in the data. Simultaneously, the relation $\alpha_{t6critical}\geq\alpha_{Ncritical}$ reveals increased investment risk. Now we formulate an alternative rating procedure. First, the companies are ordered according to their risk in the increased-uncertainty case, $\alpha_{t6critical}$. Then, the more robust investments are preferred when choosing between projects with close risks. Thus the companies are accordingly reordered. For example, HANSON is preferable to NORTHERN FOODS, BBA GROUP and BP AMOCO, because the project is more robust, although with slightly higher investment risk. By analogy, NORTHERN FOODS and GOODWIN

[2] See [18] for the complete set of graphics.
[3] See [19] for the full table of results.

are preferable to BBA GROUP and BP AMOCO.

Table 2: Constant discount rate - investment risk robustness and project rating

company	$\alpha_{t6critical}$	rating	$\Delta\alpha$	robustness	new rating
BBA	0.527	3rd	0.282	low	6th
BOC	0.000	1st	0.000	high	1st
BP AMOCO	0.529	4th	0.529	none	7th
DIXONS	0.713	8th	0.057	high	8th
GOODWIN	0.575	7th	0.154	medium	5th
HANSON	0.573	6th	0.066	high	3rd
N. FOODS	0.543	5th	0.204	medium	4th
WOLSELEY	0.000	1st	0.000	high	1st

Remember that the standard method will not quite distinguish between the above projects, as they all have a positive crisp net present value. The generally accepted technique will not tell us how to choose between projects with close crisp net present values. It will not reveal whether projects with higher NPV are less robust and less preferable. The standard results are less informative and can be misleading.

5 Investment project evaluation using a fuzzy criterion with a time-varying discount rate

The assumption of time-varying returns transforms the price-dividend relation into nonlinear and a loglinear approximation is required. We equate the log present value lpv with the log share-price estimation $\hat{p}$ at t=0. Then a project is profitable if $lpv_0=\hat{p}_0>p_0$. The real fuzzy numbers to be substituted for the log share-price p_t, log dividend-yield dy_t and log discount-rate r_t, are correspondingly $\tilde{p}_t$, $\tilde{dy}_t$, $\tilde{r}_t$. The level-log data transformation causes triangular-shaped rather then triangular membership functions for $\tilde{p}_t$, $\tilde{dy}_t$ and $\tilde{r}_t$. Now find the α-cut

$$\Omega_{lpv_{fn}}(\alpha)=\Big\{\sum_{t=1}^{N}\rho^{t-1}[(1-\rho)(dy_t+p_t)+k-r_t]+\rho^N p_N \mid \\ \mid p_{1\times N}\in\tilde{p}(\alpha), dy_{1\times N}\in\tilde{dy}(\alpha), r_{1\times N}\in\tilde{r}(\alpha)\Big\} \quad (4a)$$

Then the first solution for the fuzzy log present value is defined by its membership function,

$$\mu(x_{lpv_{fn}}\mid l\tilde{p}v_{fn})=\sup\{\alpha \mid x_{lpv_{fn}}\in\Omega_{lpv_{fn}}(\alpha)\} \quad (4b)$$

By analogy with section 3, the triangular-shaped possibility distribution of the second solution is

$$Poss[\,l\breve{p}v_{fv}=x_{lpv_{fv}}\,]=\sup\{\pi_{lpv_{fv}}=\min\{\pi_p,\pi_{dy},\pi_r\}\mid \\ \mid x_{lpv_{fv}}=\sum_{t=1}^{N}\rho^{t-1}[(1-\rho)(dy_t+p_t)+k-r_t]+\rho^N p_N\} \quad (5)$$

where the possibility distributions of the real fuzzy variables $\breve{p}_t$, $\breve{dy}_t$ and $\breve{r}_t$ are described by $Poss[\breve{p}_t=x_{pt}]=\mu(x_{pt}\mid\tilde{p}_t)$, $Poss[\breve{dy}_t=x_{dyt}]=\mu(x_{dyt}\mid\tilde{dy}_t)$ and $Poss[\breve{r}_t=x_{rt}]=\mu(x_{rt}\mid\tilde{r}_t)$, respectively. The two solutions are identical, as

$$l\tilde{p}v_{fn}(\alpha)=\Omega_{lpv_{fn}}(\alpha)=\{x_{lpv_{fv}}\mid Poss[\,l\breve{p}v_{fv}=x_{lpv_{fv}}\,]\ge\alpha\}, 0\le\alpha\le 1$$

and the calculations include

$$\Omega_{lpv_{fn}}(\alpha)=\left[\underline{lpv_{fn}(\alpha)},\overline{lpv_{fn}(\alpha)}\right]=\Big[\sum_{i=1}^{N}\rho^{t-1}\big[(1-\rho)\big(\underline{dy_t(\alpha)}+\underline{p_t(\alpha)}\big)+k-\overline{r_t(\alpha)}\big] \\ +\rho^N\underline{p_N(\alpha)},\sum_{t=1}^{N}\rho^{t-1}\big[(1-\rho)\big(\overline{dy_t(\alpha)}+\overline{p_t(\alpha)}\big)+k-\underline{r_t(\alpha)}\big]+\rho^N\overline{p_N(\alpha)}\Big]$$

We consider the t6-calibration and the assumption of a time-varying discount rate enforces the employment of N fuzzy numbers $\tilde{r}_t$, 1≤t≤N. Table 3 includes part of the results. We have not further increased the uncertainty modelled in the t6-data, only introduced a variable discount rate. But a comparison between the t6 constant and time-varying results reveals characteristics similar to increased market uncertainty: $u_{logcritical}\le u_{t6critical}\le u_{Ncritical}$ and $\alpha_{logcritical}\ge\alpha_{t6critical}\ge\alpha_{Ncritical}$. Further, when projects are assessed under t6 calibration and a time-varying discount rate, real market conditions are approached, and this allows an improved evaluation of the investment risk and its robustness. Table 3 repeats the ranking procedure from the previous section with the new results. The projects are first ordered according to the risk $\alpha_{logcritical}$, then the order is refined corresponding to the robustness indicator $\Delta\alpha_{log}=\alpha_{logcritical}-\alpha_{Ncritical}$.

Table 3: Time-varying discount rate - risk robustness and project rating

company	$\alpha_{logcritical}$	rating	$\Delta\alpha_{log}$	robustness	new rating
BBA	0.696	5th	0.451	low	6th
BOC	0.000	1st	0.000	high	1st
BP AMOCO	0.904	6th	0.904	none	7th
DIXONS	1.000	u n p r o f i t a b l e			
GOODWIN	0.925	7th	0.504	low	6th
HANSON	0.673	4th	0.166	medium	3rd
N. FOODS	0.656	3rd	0.317	low	4th
WOLSELEY	0.092	2nd	0.092	high	2st

6 Using neural networks to evaluate the fuzzy criterion

In this section, we apply a technique for evaluating fuzzy expressions suggested in [4,5] and train a neural network to evaluate investment projects according to the fuzzy present value criterion. The time-varying-rate case is considered and the three-layer feedforward neural net in fig. 4 is employed. For bias terms θ_j, sigmoidal transfer functions $g(x)=(1+e^{-x})^{-1}$ and weights w_{ji}, u_{ji}, z_{ji}, v_j, its output is

$$lpv_{nn}=\sum_{j=1}^{m}v_j g\left(\sum_{i=1}^{N}\left(w_{ji}p_i+u_{ji}r_i+z_{ji}dy_i\right)+\theta_j\right). \quad (6a)$$

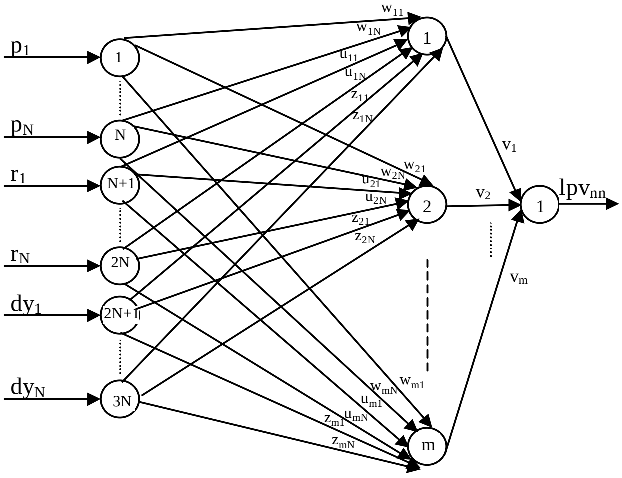

Figure 4: Neural net architecture to solve the fuzzy log present value problem

Let us imply that the network is trained to approximate the crisp log-present value:

$$lpv_{nn} \approx \sum_{i=1}^{N} \rho^{i-1} \left[(1-\rho)(dy_i+p_i)+k-r_i\right]+\rho^n p_n . \qquad (6b)$$

If we input the α-cuts of $\tilde{p}_i$, $\tilde{r}_i$, $\tilde{dy}_i$, and perform interval arithmetic within the net to get the corresponding α-cut of the fuzzy output $l\tilde{p}v_{nn}$, the result is

$$\left[\underline{l\tilde{p}v_{nn}}(\alpha), \overline{l\tilde{p}v_{nn}}(\alpha)\right] = \sum_{j=1}^{m} v_j g * \left(\sum_{i=1}^{N} \left(w_{ji}\left[\underline{\tilde{p}_i}(\alpha), \overline{\tilde{p}_i}(\alpha)\right] + u_{ji}\left[\underline{\tilde{r}_i}(\alpha), \overline{\tilde{r}_i}(\alpha)\right] + z_{ji}\left[\underline{\tilde{dy}_i}(\alpha), \overline{\tilde{dy}_i}(\alpha)\right]\right) + \theta_j \right) = \left[\sum_{j=1}^{m} v_j g\left(\sum_{i=1}^{N} \left(w_{ji}\underline{\tilde{p}_i}(\alpha) + u_{ji}\overline{\tilde{r}_i}(\alpha) + z_{ji}\underline{\tilde{dy}_i}(\alpha) + \theta_j \right)\right), \sum_{j=1}^{m} v_j g\left(\sum_{i=1}^{N} \left(w_{ji}\overline{\tilde{p}_i}(\alpha) + u_{ji}\underline{\tilde{r}_i}(\alpha) + z_{ji}\overline{\tilde{dy}_i}(\alpha) + \theta_j \right)\right) \right]$$

In order $l\tilde{p}v_{nn}$ to be an approximation to the solution $l\tilde{p}v_{fn}$ described earlier and to provide that $l\tilde{p}v_{nn}>0$ for $l\tilde{p}v_{fn}>0$, the following sign restrictions are introduced on the weights.

$$w_{ji} \geq 0,\ u_{ji}<0,\ z_{ji} \geq 0,\ v_j \geq 0,\ 1 \leq i \leq N,\ 1 \leq j \leq m, \qquad (6c)$$

Then, if the net is trained to approximate (6b) under (6c), we get

$$\left[\underline{l\tilde{p}v_{nn}}(\alpha), \overline{l\tilde{p}v_{nn}}(\alpha)\right] \approx \left[\sum_{i=1}^{N} \rho^{i-1} \left\{ (1-\rho)\left(\underline{\tilde{dy}_i}(\alpha) + \underline{\tilde{p}_i}(\alpha)\right) + k - \overline{\tilde{r}_i}(\alpha) \right\} + \rho^N \underline{\tilde{p}_N}(\alpha), \sum_{i=1}^{N} \rho^{i-1} \left\{ (1-\rho)\left(\overline{\tilde{dy}_i}(\alpha) + \overline{\tilde{p}_i}(\alpha)\right) + k - \underline{\tilde{r}_i}(\alpha) \right\} + \rho^N \overline{\tilde{p}_N}(\alpha) \right] \qquad (6d)$$

The problem is programmed using the neural network toolbox of Matlab[4]. We choose the training function *trainlm* based on the Levenberg-Marquart technique, as it is the fastest backpropagation algorithm available. The toolbox allows function customisation giving the user control over the initialising, simulating and training algorithms. We have modified *trainlm* to provide the satisfaction of the sign constraints. After training, the net is simulated using test vectors for each project, while no element of the training set is included in the test set. For all companies, $\max_s \left|net_s - target_s\right| \leq 0.021$ and for most of them $\max_s \left|net_s - target_s\right| \leq 0.01$, where s stands for the s-th element of the test set. It is a good approximation and concludes that $l\tilde{p}v_{nn} \approx l\tilde{p}v_{fn}$.

If an investment decision has to be taken within a period of time, we can first fuzzify the data using the information available at the beginning of the period and then train a neural network to approximate the fuzzy log present value of the project. The decision-maker will be provided with the trained network and at any moment he or she acquires new information, the net will be simulated with modified inputs.

7 Future research

If one fuzzify the 3N-m-1 network structure from Figure 4, it will handle fuzzy signals - fuzzy market data - at once instead of α-cut by α-cut. The fuzzy network takes fuzzy weights and fuzzy shift terms. The error of the approximation E is a distance measure D between the fuzzy log-present value $l\tilde{p}v_{fn}$ and the fuzzy neural net output $l\tilde{p}v_{fnn}$.

$$E = \frac{1}{L}\sum_{i=1}^{N}\left(D\left(l\tilde{p}v_{fn}, l\tilde{p}v_{fnn}\right)\right)^2 = \frac{1}{L}\sum_{i=1}^{N}\left(D\left(\sum_{i=1}^{N} \rho^{i-1}\left[(1-\rho)\left(\tilde{dy}_i + \tilde{p}_i\right) + k - \tilde{r}_i\right] + \rho^N \tilde{p}_N, fnn\left(\tilde{p}_{1,\ldots,}\tilde{p}_{N,}\tilde{dy}_{1,\ldots,}\tilde{dy}_{N,}\tilde{r}_{1,\ldots,}\tilde{r}_N\right)\right)\right)^2$$

Evolutionary algorithms are the most promising tool in training FNN - they are well capable of searching for the optimal weights and shits while minimising E. As N is very large, a scalability problem occurs. To solve it, we suggest a specific algorithm, bidirectional incremental evolution. The incremental evolution is applied in training neural networks in [9], where a control problem in evolutionary robotics is approached. Unfortunately, it requires advanced knowledge of the complexity of the problem. The bidirectional incremental evolution allows us to overtake this problem. Its major advantages are the automatic identification of the complexity of the task, and the automatic change in the parameters so that the system adapts to the complexity. Bidirectional incremental evolution has been already applied in evolvable hardware [12] and the experimental results have proved it to be a powerful technique for tuning systems automatically.

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[4] All programmes in Sections 3, 4, 5, 6 are written in Matlab.

7 Future research

Our effort lies on the bridge towards a new paradigm of investment selection, where the perception of concepts inherent or surrounding the investment process, whose character is not principally measurable, is best handled by 'nonnumeric' mathematics. [1] We presented some preliminary ideas in [11] and further develop here the technique, introducing fuzzy dividend yields and a second type of calibration. We suggest measures of the investment risk and its robustness. Also neural network solution is worked out and the number of companies is considerably extended. Finally a promising direction for future research is outlined.

The results reveal that there is a critical level of uncertainty, $u_{critical}$, embodied in the market data we use to assess a project and this level delimits the project's investment risk, $\alpha_{critical}$. Evaluating the same project under increased uncertainty of the market environment, we derive an estimate of the risk robustness $\Delta\alpha$. Investment opportunities are first rated in correspondence with their risk and then the order is revised according to their robustness. The more robust investments are preferable when choosing between projects with close risks, and this suggests an alternative ranking technique. It is important for investors to pick out projects having not only a small but also a highly robust investment risk. The fuzzy present value provides them with the necessary information and facilitates their decision, while the crisp technique is less informative and even misleading. Further, empirical tests have convinced financial analysts that stock returns are time-varying rather than constant. In response, we introduce fuzzy log present value. Finally, a trained neural network provides investors with an effortless instrument for risk revaluation, any time they need to update and reconsider a project.

The mathematics underlying the standard financial techniques neglects extreme situations and regards large market shifts as too unlikely to matter. Such techniques may account for what occurs most of the time in the market, but the picture they present does not reflect the reality as major events happen in the rest of the time. The soft computing approach allows for market fluctuations well beyond the probability type of uncertainty, does not impose predefined data or market behaviour, and efficiently works out a solution, producing better investment appraisal and allowing project revaluation.

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