

Crystal Chemistry and Electronic Structure of the β -AlFeSi Phase from First-Principles

C. M. Fang*, Z. P. Que and Z. Fan

BCAST, Brunel University London, Kingston Lane, Uxbridge, Middlesex, UB8 3PH, UK

*Corresponding author: Changming Fang

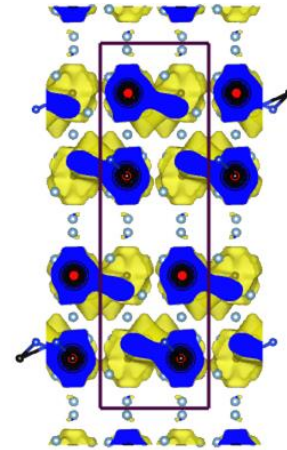
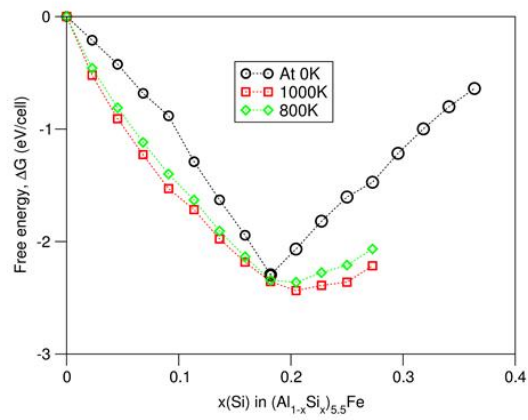
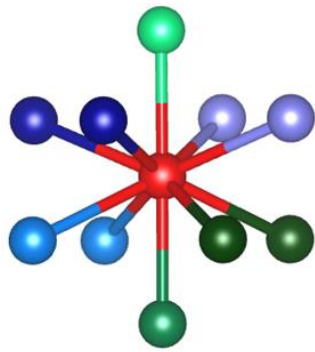
E-mail: Changming.Fang@Brunel.AC.UK

Highlights

- Si prefers to substitute on the Al1 or Al6 site, forming $\beta\text{-Al}_{4.5}(\text{Si},\text{Al})^{\text{I}}\text{Fe}$ or $\beta\text{-Al}_{4.5}(\text{Si},\text{Al})^{\text{V}}\text{Fe}$;
- Stacking of the FeAlSi blocks leads to phase transformations;
- Extra freedom permits more Si solute in the β -phase at the casting temperature;
- Electronically $\beta\text{-Al}_{4.5}\text{SiFe}$ is anisotropic with a narrow pseudo bandgap.

Graphical Abstract

Si solute on the Al1 or Al6 site, forming $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$ or $\beta\text{-Al}_{4.5}\text{Si}^{\text{VI}}\text{Fe}$ at low temperature. At casting temperature, the extra freedom allows more Si solute in the crystal. Stacking of the FeAlSi blocks of the layered $\beta\text{-Al}_{4.5}\text{SiFe}$ phase may lead to structural transformations.



Abstract

β -AlFeSi has a layered structure composed of FeAlSi blocks and exhibits a rich variety of crystal chemistry. Plate-like/rod-like β -AlFeSi particles formed in Al-based alloys have nontrivial influences on the mechanical performance of the cast parts. Here, we investigate the stability, crystal chemistry and electronic structure of the β -phase using the first-principles density-functional theory (DFT) method. We reveal that Si prefers on the Al1 or Al6 sites, forming stable β -Al_{4.5}Si^IFe or β -Al_{4.5}Si^{VI}Fe (the Roman numerals represent the Al sites in the Rømming's labels). This differs from the existing model with a homogeneous Si/Al distribution. Moreover, the calculations also find that stacking of the FeAlSi blocks leads to structural transformations. Electronically β -Al_{4.5}SiFe is anisotropic with a narrow pseudo-band-gap, indicating its unusual physical properties. The obtained information here sheds some light not only on the stability and crystal chemistry of the β -phase as a member of the large family of the Fe-containing intermetallic compounds in Al-based alloys, but also on its potential applications as low-dimensional functional materials.

Key words: Fe-intermetallic Compounds; Silicon Substitution; β -Al_{4.5}SiFe; *Ab-Initio* Calculations; Crystal Chemistry.

1. Introduction

As a family member of the Fe-containing intermetallic compounds (Fe-IMCs), β -AlFeSi has been a topic of intensive study due to both academic curiosity and industrial interest [1-15]. The β -phase has a list of names, $\text{Al}_9\text{Fe}_2\text{Si}_2$, Al_5FeSi , $\text{Al}_{4.5}\text{FeSi}$, β -AlFeSi, *etc.* in the literature [1-19]. Structurally β -AlFeSi is layered and exhibits a rich variety of crystal chemistry [11-15]. Its symmetry and crystal structure have been under discussions [10-15]. Currently it is accepted that the β -phase has a monoclinic lattice with space group $A2/a$ (Nr. 15) [2, 12, 22, 15]. It contains seven crystallographically different atomic species (one Fe and six Al/Si) (Fig. 1). In this structural model the distribution of the Si atoms was treated to be homogeneous at the Al sites [11-15]. This indicates that the local symmetry in the crystal is broken in the atomic picture [23]. This Si/Al homogeneous distribution model has been applied in the structural determinations of the Fe-IMCs [2-6, 10-22]. Thus, knowledge about the accurate Si distribution in β -AlFeSi is helpful to get insight into the accurate structures of the Fe-IMCs, too. The structure of β - $\text{Al}_{4.5}\text{FeSi}$ is composed of building blocks (Fig. 1a). Each block is composed of centered Fe atoms coordinated by 10 Al/Si atoms (FeAlSi blocks) [11-15] (Fig. 1b). Different stackings of the blocks may change the lattice symmetry and form new phases [11-15, 24, 25]. Understanding of the accurate structure of β -AlFeSi sheds light on its phase relations with and phase transformations from/to the other Fe-IMCs during casting of Al-based alloys [6, 16]. Electronically the unique layered structure of β -AlFeSi indicates unusual physical properties with potential applications as low-dimensional functional materials. To reach this goal, knowledge about its accurate crystal structure including the Si distribution is the prerequisite. Metallurgically, both iron and silicon exist unavoidably in commercial Al metals and Al-based alloys as impurities. Iron has a low solid solubility (less than 0.05 percent) at equilibrium in Al [1, 2]. Thus, iron exist in aluminum-based alloys in the form of Fe-IMCs [1-4]. The formed Fe-IMCs particles, including β -AlFeSi exhibit various morphologies and may deteriorate the mechanical properties of cast parts. Information about the stability and structural properties of the harmful β -AlFeSi phase is useful to control/minimize its existence in cast products for various Al ingots including Al-scrap which contains various contents of iron and silicon. The latter is important for our environments and the recycling economy [3, 4, 6].

Experimental efforts for β -AlFeSi have been made mainly on preparation, phase characterization and structural analysis [2, 5-22, 24]. It is difficult for most diffraction methods to distinguish Si and Al in the Fe-IMCs, as they are neighbors in Periodic Table of Elements and differ from each other with only one-electron. Till now there has been no experimental report on the electronic properties of this layered compound, too. In these aspects, theoretical approaches, especially parameters-free first-principles methods are helpful. First-principles approaches have been successfully applied to investigate the stability and structural and electronic properties of many Fe-containing intermetallic compounds and precipitates [26-29], including Si preference on the atomic sites in θ - $\text{Al}_{13}\text{Fe}_4$ [26, 30]. Here we investigate Si distribution in and electronic structure of β -AlFeSi using a first-principles density-functional theory (DFT) approach. We reveal that the stable structure has a chemical composition of β - $\text{Al}_{4.5}\text{SiFe}$ with Si at specific Al sites. The crystal symmetry is strictly kept in the structural model. At the casting temperature, configuration entrop-

py contribution enables more Si solute in β -Al_{4.5}SiFe. Electronically β -Al_{4.5}SiFe is unusually anisotropic with a narrow pseudo-band-gap.

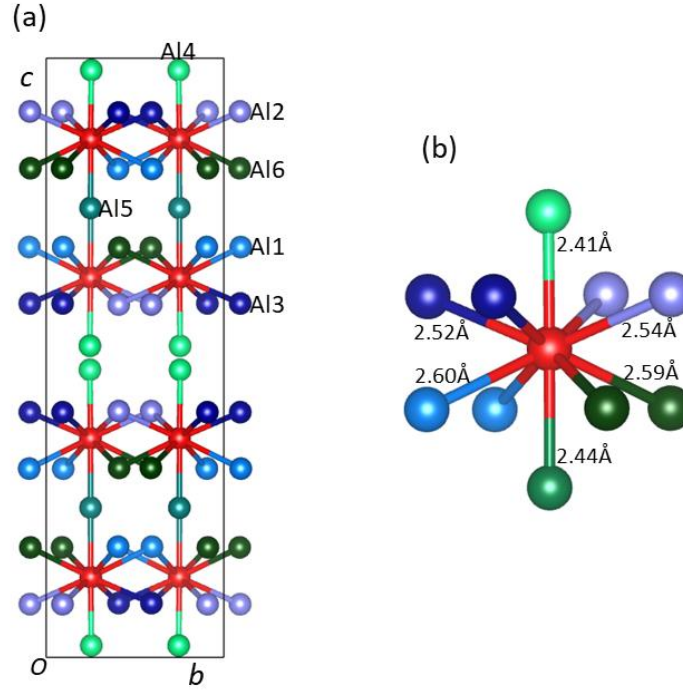


Fig. 1 (Color online) Schematic structure of β -(Al_{4.5}Si)Fe along the [1 0 0] projection (a) [12] and the coordination of Fe by Al/Si atoms (b). The bond lengths in (b) are from the optimized structure for novel β -Al_{5.5}Fe (see text). The red spheres represent Fe, the rest colored spheres represent the six Al (represents here for Al/Si) species [12] as labelled in (a).

2. Details of Calculation Methods

We performed first-principles calculations for the (novel) binary β -Al_{5.5}Fe based on the structural model [12]. The formation energy here is defined as,

$$\Delta E(\text{Al}_{5.5}\text{Fe}) = E(\text{Al}_{5.5}\text{Fe}) - [5.5 E(\text{Al}) + E(\text{Fe})] \quad (1)$$

Here $E(\text{Al}_{5.5}\text{Fe})$, $E(\text{Al})$ and $E(\text{Fe})$ are the total valence electron energies of β -Al_{5.5}Fe, the solid element α -Al and α -Fe, respectively. The unit of ΔE is eV/Fe or eV/f.u., where f.u. represents formula unit.

To have a measure of the relative stability of β -AlFeSi with respect to the parent intermetallic compound, β -Al_{5.5}Fe and the elemental solids (α -Al and Si), the formation energy per cell for Si doping in the compounds is given by:

$$\Delta E_{\text{Si}}[(\text{Al}_{1-x}\text{Si}_x)_{5.5}(\text{Fe})] = E\{(\text{Al}_{1-x}\text{Si}_x)_{5.5}\text{Fe}\} - \{E(\text{Al}_{5.5}\text{Fe}) + 5.5 x[E(\text{Si}) - E(\text{Al})]\} \quad (2)$$

Here, $E\{(\text{Al}_{1-x}\text{Si}_x)_{5.5}\text{Fe}\}$ is the calculated energy of $(\text{Al}_{1-x}\text{Si}_x)_{5.5}\text{Fe}$ in the unit cell. The unit of the formation energy here is eV/cell.

At the temperature $T = 0$ K and the pressure $p = 0$ Pa, the enthalpy difference is equal to the energy difference, $\Delta H = \Delta E$, when the zero-point vibration contribution is not taken into account.

A negative value of the formation energy means that the formation is exothermic and this reaction is favored.

We utilized the first-principles code VASP (Vienna *Ab initio* Simulation Package) [31, 32]. This code employs the Density-Functional Theory (DFT) within the Projector-Augmented Wave (PAW) approach [33, 34]. The Generalized Gradient Approximation (GGA-PBE) [35, 36] was used for the exchange and correlation energy terms. The GGA works better for the transition metals including iron and related compounds than the Local Density Approximation (LDA) [29, 34-36]. We used a cut-off energy of 550 eV for the wave functions and the cut-off energy of 700 eV for the augmentation functions, which are notably higher than the corresponding default values ($E_{\text{MAX}}/E_{\text{AUG}} = 240.3\text{eV}/291.1\text{eV}$ for Al, $267.9\text{eV}/511.4\text{eV}$ for Fe and $245.3\text{eV}/322.1\text{eV}$ for Si, respectively). The electronic wave functions were sampled with a dense, e.g. a $8 \times 8 \times 2$ grid and 50 to 68 k -points in the irreducible Brillouin zone (BZ) of β -AlFeSi and the related compositions depending on the symmetry, using the Monkhorst-Pack method [37]. First-principles structural optimizations were performed for both lattice parameters and atomic coordinates. Different k -meshes and cut-off energies were first tested. The tests showed good convergence ($<1\text{meV}$ per atom).

3. Results

Structure optimizations and total energy calculations were performed for the elemental solids using above settings. The calculations produced the lattice parameter, $a = 4.039\text{\AA}$ (experimental value 4.04325\AA at 0K [38]) for α -Al, $a = 2.831\text{\AA}$ (experimental value 2.86072\AA at 0K [38]) for α -Fe and $a = 5.468\text{\AA}$ (experimental value 5.43298\AA at 0K [38]) for the cubic Si. The calculations reproduced well the experimental values at 0K.

3.1. Energies and structural properties of Si solute in the β -phase

We first performed structural optimizations for the novel binary β -Al_{5.5}Fe. The calculated formation energy is -1.267eV/Fe for β -Al_{5.5}Fe according to Eq. 1. This energy is notably higher than that for θ -Al₁₃Fe₄ (-1.403eV/Fe) [26], indicating that β -Al_{5.5}Fe is less stable than θ -Al₁₃Fe₄.

Next, we investigated the energies of intrinsic defects and Si solute, Fe or Al vacancies, replacements of Al by Fe, Fe by Al and Fe by Si in β -Al_{5.5}Fe. The definitions of the formation energies are from our previous work [26]. The calculations revealed high costs for formation of the intrinsic defects with respect to β -Al_{5.5}Fe and the elemental solids: $+1.244\text{eV}$ for an Fe vacancy, $+1.240$ to $+1.1724\text{eV}$ to create a vacancy at an Al site. To replace one Fe by Al costs $+1.170\text{eV}$ and replacing one Al by Fe costs $+1.546$ to $+2.084\text{eV}$. Moreover, replacement of one Fe by Si costs $+1.407\text{eV}$. Such high energy costs indicate highly unlikely of formation of these defects. This agrees with the previous work for the defects in θ -Al₁₃Fe₄ [26, 39, 40].

The calculations were performed for one Si substitution on the six Al sites. The obtained formation energies according to Eq. 2 are shown in Fig. 2 and in the supplementary materials, Table S-I.

The calculations showed that the formation energy of one Si substitution on Al1 is equal to that

on Al6 (Al1/Al6) and that at Al2 to that at Al3. The formation energies at the six Al sites can be classified into four groups: Al/Al6, Al2/Al3, Al4, Al5. The Si preference has the series (from high to low): Al1/Al6 > Al5 > Al2/Al3 > Al4. Each Al4 atom has one Fe neighbors whereas at the rest sites each Al has two Fe neighbors. This indicates that Si prefers the Al sites of more Fe neighbors. The lengths of the axis of cell volume decrease with Si solute at the Al sites, whereas the angle increases slightly (Table S-I).

The distinct preference of Si substitution at the Al sites except Al4 suggested additions of more Si atoms. The obtained dependences of the formation energies on Si content are shown in Fig. 2. We also performed structural optimizations and total energy calculations for full Si occupations on the Al sites. The results (formation energy, lattice parameters and important interatomic distances) are listed in Table I for the highly-stable ones and Table S-II for all the configurations.

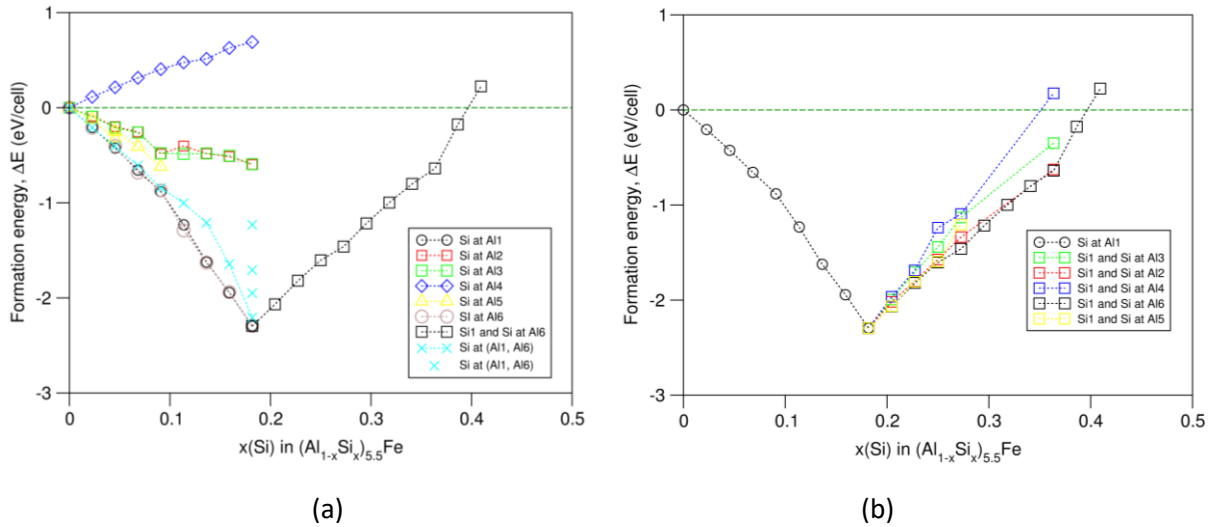


Fig. 2 (Color online) Formation energies of the β -phase with Si solutes at the Al sites from first-principles DFT-GGA calculations.

From the calculations we obtained the following conclusions:

- 1) The formation energies decrease with Si content on Al1/Al6, Al5, Al2/Al3 until each Al site is fully occupied by Si, whereas Si solute at the Al4 site is unfavored (Fig. 2a).
- 2) The configurations of the same Si content have the following order series (from high to low): Si(Al1/Al6) > Si(Al5) > Si(Al2/Al3) >> Si(Al4) (Fig. 2a).
- 3) At lower Si content ($x < 4/44$ in $(Al_{1-x}Si_x)_{5.5}Fe$), Si atoms occupy the Al1/Al6 sites randomly. At higher Si content, ordered Al1/Al6 configurations become more favored, especially for the configurations with $x = 8/44$.
- 4) The formation energy reaches its minimum for Si full occupation at the (Al1 or Al6), Al5 and (Al2 or Al3) sites, whereas it reaches a maximum for Si full occupation at the Al4 site (Fig. 2) and Table S-II. The most stable configurations are β - $Al_{4.5}Si^1Fe$ with Si at the Al1 sites and

β -Al_{4.5}Si^{VI}Fe with Si at the Al6 sites. They are referred as β -Al_{4.5}SiFe. The calculated lattice parameters are in good agreement with the experimental values (<1%) (Table 1)

The calculations revealed that addition of extra Si into β -Al_{4.5}SiFe increases the formation energy at 0K (Fig. 2b). Low Si addition into the Al sites in β -Al_{4.5}SiFe has similar formation energies, whereas addition of more Si cause diversity of the formation (Fig. 2b). Interestingly, configurations with a full occupation of (Al1 and Al3), and (Al6 and Al2) are more stable than those of (Al1 and Al2), (Al6 and Al3). This is due to their local symmetry [12, 15].

Table I. The calculated results (lattice parameters and formation energies) for the novel binary β -Al_{5.5}Fe and β -Al_{4.5}SiFe configurations of high stability with comparison with the experimental data in the literature. XRD represent the X-Ray Diffraction (technique); SAED Selected Area Electron Diffraction, EBED Convergent Beam Electron Diffraction. *This formation energy is per primitive unit cell.

Phase	lattice parameters(Å) $a(\text{Å}), b(\text{Å}), c(\text{Å}), \beta(^{\circ}), V_{\text{cell}}(\text{Å}^3)$	$\Delta E(\text{eV}/\text{cell})$ or expl. technique
β -Al _{5.5} Fe	6.240, 6.240, 21.087, 90.34, 821.13	-
β -Al _{4.5} Si ^I Fe	6.164, 6.165, 20.768, 91.39, 788.91	-2.295
β -Al _{4.5} Si ^{VI} Fe	6.165, 6.165, 20.766, 91.43, 788.98	-2.297
Experiment [12]	6.161, 6.175, 20.813, 90.42, 791.79	XRD, SAED
[13]	6.1676, 6.1661, 20.8093, 91, 791.26	SAED, CBED
β -Al _{4.5} SiFe (C_ii)	6.162, 6.169, 20.779, 90, 789.83	-2.220
Experiment [14]	6.18, 6.20, 20.8, 90, 796.97	SAED, CBED
[11]	6.184, 6.250, 20.69, 90, 799.67	CBED
β -Al _{4.5} SiFe (C_iii_1)	6.163, 6.166, 41.570, 90.15, 1579.63	-2.229
Experiment [41, 42]	6.12, 6.12, 41.5, 91, 1554.12	XRD
β -Al _{4.5} SiFe (C_iii_2)	6.164, 6.164, 41.579, 90, 1579.99	-2.228
Experiment [42]	6.18, 6.18, 42.5, 90, 1617.92	XRD

Fig. 1 shows that along its c -axis, the β -Al_{4.5}SiFe unit cell consists of four FeAlSi blocks. Each block contains two Fe, two Si atoms and nine Al atoms, named as q -FeAlSi. The calculations showed the configurations with the Si atoms occupying both Al1 and Al6 sites in one q -FeAlSi block have formation energies over 1eV higher than that of the stable β -Al_{4.5}SiFe. It is also possible that one block has both Si atom either at Al1 (q -FeAlSi^I) or at Al6 (q -FeAlSi^{VI}). Then we investigate the formation energy and structure of different stackings by such blocks: Configurations i) (C_i) has a stacking of $-(q\text{-FeAlSi}^{\text{I}})-(q\text{-FeAlSi}^{\text{VI}})-(q\text{-FeAlSi}^{\text{I}})-(q\text{-FeAlSi}^{\text{VI}})$ -series; C_ii) has a stacking series $-2(q\text{-FeAlSi}^{\text{I}})-2(q\text{-FeAlSi}^{\text{VI}})$ - in the unit cell; and C_iii) has a $-4(q\text{-FeAlSi}^{\text{I}})-4(q\text{-FeAlSi}^{\text{VI}})$ - in a $1a_0 \times 1b_0 \times 2c_0$ supercell, where a_0, b_0, c_0 are the lattice parameters of the convention cell of the β -phase.

The calculations showed that for C_i) the lattice becomes orthorhombic with $a = 6.1522\text{Å}$, $b = 6.1554\text{Å}$, $c = 20.9332\text{Å}$. However, the formation energy is 1.034eV higher than the most stable ones, indicating this configuration is unlikely.

For C_ii) the optimized lattice becomes orthorhombic with parameters, $a = 6.1616\text{Å}$, $a = 6.1692\text{Å}$, $c = 20.7786\text{Å}$. Moreover, the formation energy (-2.220eV/cell) is slightly higher than

that of the most stable configuration (energy difference of 0.076eV/cell, Fig. 2a)), indicating that this type of stackings is highly likely. The obtained lattice parameters are close to those experimental observations [11, 14].

As shown in Table for the supercell, the structural optimizations produced two replacements of almost identical energy: C_iii_1) is a monoclinic lattice with lattice parameters: $a = 6.163\text{\AA}$, $b = 6.166\text{\AA}$ $c = 41.570\text{\AA}$ and $\beta = 90.15^\circ$; C_ii_2): a tetragonal lattice (fixed) with $a = 6.164\text{\AA}$, $c = 41.579\text{\AA}$. There are slight anisotropic pressures along its a -, b - and c -axis with values -0.47kB, +0.25kB, -0.53kB, respectively. As shown in Table I, the early experiments produced scattering values. The small length-differences of the in-plane-axis and the small angle deviation from 90° might be not accurately determined in the early experiments [41, 42].

The calculations showed that the building unit is the double-(q -FeAlSi) slab which is connected to the neighboring slabs via Al5 atoms (Fig. 1). The changes of lattice symmetry for the configurations with stackings of the double-(q -FeAlSi) blocks help to understand the reported structural models [11-15, 41, 42].

Next we address the dependences of lattice parameters of the highly stable configurations of the β -phase on Si content. The dependences of the lattice parameters and cell volume for the stable configurations are plotted in Fig. S-1.

The lengths of the in-plane axis, a and b decreases with increasing Si concentrations. The cell volume decreases almost linearly with Si concentration. Meanwhile, the length of c -axis and the angle have unusual dependences. The length of c -axis decreases with Si concentration and reaches its minimum at $x = 18.18\%$ and then increases with further addition of Si, whereas the angle behaves oppositely: with Si content, it increases first and reaches a maximum at $x = 18.18\%$ and then decreases.

In brief, the first-principles calculations revealed Si preference at the Al1 or Al6 sites, forming stable β -Al_{4.5}Si^IFe or β -Al_{4.5}Si^{VI}Fe. The calculated lattice parameters agree with the experimental values. The stackings of the double-(q -FeAlSi) blocks result in highly-stable configurations of orthorhombic or tetragonal lattices.

3.2. Chemical bonding in β -Al_{4.5}SiFe

Here we analyze the structure of and chemical bonding in β -Al_{4.5}SiFe in detail. The lattice parameters were listed in Table 1. The atomic coordinates and important interatomic distances for β -Al_{5.5}Fe and β -Al_{4.5}Fe^ISi and β -Al_{4.5}Fe^{VI}Si are listed in Table II and Table III, respectively.

Table II. Calculated atomic coordinates and charges at the atomic sites for β -Al_{5.5}Fe and ternary β -Al_{4.5}Fe^ISi and β -Al_{4.5}Fe^{VI}Si with comparison with the available experimental data [12]. The corresponding lattice parameters are listed in Table I. # represent site occupied by Si and * the Si/Al are homogeneously distributed at the Al sites [12].

	β -Al _{5.5} Fe	β -Al _{4.5} Si ^I Fe	β -Al _{4.5} Si ^{VI} Fe	Exptl. [12] β -(Al _{4.5} Si)Fe
Species	Coordinates /charge (e/atom)	Coordinates /charge (e/atom)	Coordinates /charge (e/atom)	Coordinates
Fe, 8f	0.5021, 0.2507, 0.1341 /-3.65	0.4937, 0.2559, 0.1367 /-3.17	0.5063, 0.2441, 0.1367 /-3.16	0.5024, 0.2605, 0.1367
Al1, 8f	0.3458, 0.5957, 0.1848 /0.63	0.3389, 0.5943, 0.1811# /-1.43	0.3642, 0.6134, 0.1899 /1.27	0.3583, 0.6062, 0.1863*
Al2, 8f	0.3432, 0.9083, 0.0889 /0.85	0.3306, 0.9187, 0.0930 /1.11	0.3419, 0.9093, 0.0890 /1.18	0.3387, 0.9116, 0.0897*
Al3, 8f	0.1593, 0.4075, 0.0907 /0.87	0.1576, 0.4087, 0.0891 /1.19	0.1692, 0.4183, 0.0933 /1.12	0.1669, 0.4167, 0.0908*

Al4 8f,	0.4992, 0.2539, 0.0200 /0.24	0.5093, 0.2455, 0.0187 /0.30	0.4906, 0.2562, 0.0186 /0.29	0.4972, 0.2666, 0.0181*
Al5,4d	0.5000, 0.2500, 0.2500 /0.85	0.5000, 0.2500, 0.2500 /1.44	0.5000, 0.2500, 0.2500 /1.44	0.5000, 0.2500, 0.2500*
Al6, 8f	0.1603, 0.0893, 0.1837 /0.66	0.1359, 0.1127, 0.1898 /1.27	0.1612, 0.0939, 0.1810 [#] /-1.44	0.1526, 0.1000, 0.1836*

As shown in Table II, the calculated lattice parameters for the β -Al_{4.5}Si^IFe are identical to those of β -Al_{4.5}Si^{VI}Fe in the numerical error, except for the exchange of Si1/Si6 sites. There is an excellent agreement (within 0.5%) between the calculations and experimental values. For the same species the atomic coordinates in the different configurations/phases in Tale II are generally close each other. The atomic coordinates for Al6 in β -Al_{4.5}Si^IFe and Al11 in β -Al_{4.5}Si^{VI}Fe display more pronounced differences than those in other configurations. This is due to the Si solutes.

Table III. The interatomic distances within 3.0Å in β -Al_{5.5}Fe and β -Al_{4.5}Fe^ISi and β -Al_{4.5}Fe^{VI}Si with comparison with the available experimental data [12]. *In [12] the Si atoms are homogeneously distributed at the Al sites.

	β -Al _{5.5} Fe	β -Al _{4.5} Fe ^I Si	β -Al _{4.5} Fe ^{VI} Si	Exp.[12]*
Fe	-10Al: 2.14, 2.44, 2.52(×2), 2.54(×2), 2.56, 2.58, 2.59, 2.60	-8Al: 2.35, 2.45, 2.49(×3), 2.50, 2.60, 2.68 -2Si: 2.46, 2.52	-8Al:2.35, 2.45, 2.49(×2), 2.50(×2), 2.60, 2.69 -2Si:2.46, 2.52	-10Al:2.358, 2.434, 2.467, 2.468, 2.531, 2.534,2.553, 2.572, 2.596,2.598
Al1 (Si1)	-2Fe: 2.58, 2.60 -7Al: 2.58, 2.73, 2.74, 2.77, 2.78, 2.80, 2.81	(Si1)-2Fe: 2.46, 2.52 -7Al: 2.51, 2.58, 2.68, 2.69, 2.71(×2), 2.76	-2Fe: 2.60, 2.69 -5Al: 2.61, 2.69, 2.73, 2.78, 2.79 -2Si: 2.58, 2.69	-2Fe: 2.534, 2.553 -7Al: 2.566, 2.583, 2.708, 2.709, 2.731, 2.758, 2.764
Al2	-2Fe: 2.54(×2) -7Al: 2.57, 2.70, 2.78(×2), 2.79, 2.80, 2.81	-2Fe: 2.49(×2) -6Al: 2.61, 2.70, 2.74, 2.75, 2.79, 2.85 -Si: 2.71	-2Fe: 2.50(×2) -5A: 2.68, 2.76, 2.77, 2.78, 2.85. -2Si: 2.51, 2.71	-2Fe: 2.531, 2.596 -7Al: 2.553, 2.702, 2.740, 2.758, 2.795, 2.829, 2.863
Al3	-2Fe: 2.52(×2) -7Al: 2.58, 2.70, 2.77(×2), 2.79(×2), 2.80	-2Fe: 2.49, 2.50 -5Al: 2.68, 2.75, 2.77, 2.78, 2.85 -2Si: 2.51, 2.71	-2Fe: 2.49(×2) -6Al: 2.61, 2.69, 2.74, 2.75, 2.79, 2.85 -1Si: 2.71	-2Fe: 2.434, 2.467 -6Al: 2.583, 2.642, 2.680, 2.708, 2.750, 2.764
Al4	-1Fe: 2.41 -7Al: 2.70(×2), 2.77(×2), 2.78(×2), 3.18	-Fe: 2.45 -6Al: 2.68, 2.70, 2.74, 2.75(×2), 2.77	-1Fe: 2.45 -6Al: 2.68, 2.69, 2.74, 2.75, 2.76, 2.77	-1Fe: 2.468 -7Al: 2.642, 2.680, 2.702, 2.708, 2.795, 2.829, 2.979
Al5	-2Fe: 2.44(×2) -8Al: 2.72(×2), 2.73(×4), 2.74(×2)	-2Fe: 2.35(×2) -4Al: 2.69(×2), 2.73(×2) -4Si: 2.68(×2), 2.76(×2)	-2Fe: 2.35(×2) -4Al: 2.69(×2), 2.73(×2) -4Si: 2.68(×2), 2.76(×2)	-Fe: 2.358(×2) -8Al: 2.702(×2), 2.709(×2), 2.731(×2), 2.735(×2)
Al6 (Si6)	-2Fe: 2.56, 2.59 -7Al: 2.57, 2.72, 2.73, 2.77, 2.78, 2.79, 2.80	-2Fe: 2.60, 2.68 -5Al: 2.61, 2.69, 2.73, 2.78, 2.79 -2Si: 2.58, 2.69	(Si6)-2Fe: 2.46, 2.52 -7Al: 2.51, 2.58, 2.68, 2.69, 2.71(×2), 2.76	-2Fe: 2.572, 2.598 -7Al: 2.553, 2.566, 2.702, 2.708, 2.735, 2.740, 2.750,

Table III shows that the Fe atoms in the β -phase have eight Al and two Si neighbors, or 10 Al/Si as in Fig. 1b. This coordination number (CN) is between that of α -Fe (CN = 8) and α -Al (CN = 12). The interatomic Fe-Al distances in the binary are in the range between 2.14Å and 2.60Å, which is smaller than that in the ternary configurations 2.35Å and 2.69Å. The latter is closer to the experimentally observed values from 2.36Å and 2.60Å. The Fe-Si distances are about 2.5 Å, between those of the Fe-Al bonds. It is also noted that in the binary β -Al_{5.5}Fe, the two Fe-Al bonds have the same length for Al2, Al3 and Al5, whereas they have different lengths for Al1 and Al6. This may relate to the lower formation energy for β -Al_{4.5}Si^IFe and for β -Al_{4.5}Si^{VI}Fe as compared with for β -Al_{4.5}Si^{II}Fe, β -Al_{4.5}Si^{III}Fe and for β -Al_{4.5}Si^VFe.

3.3. Electronic properties of the β -Al_{4.5}Si^IFe

Electronic structure calculations were performed for both $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$ and $\beta\text{-Al}_{4.5}\text{Si}^{\text{VI}}\text{Fe}$. The obtained electron density distribution in $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$, and related partial density of states of the atoms (pDOS) and total density of states (tDOS) for $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$ are shown in Fig. 3a and Fig. 3b, respectively. Fig. 4 shows the dispersion curves near the Fermi level (at 0eV) along the high symmetry lines in the Brillouin zone (BZ) of $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$.

Fig. 3a shows clear layered characteristics of $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$. In plane the Fe-Si form clusters which connected with each other, indicating covalent nature between them. Fig. 3a also showed that along the z -axis, the FeAlSi blocks in the first half of the cell are closer with each other, forming a slab which is well-separated from the other slab in the other half of the cell. This corresponds to the calculations for the highly-stable configurations with the stackings of the double(q -FeAlSi) blocks in Table I.

The Si/Al 3s, 3p states are all-over the valence and conduction bands. The Si 3s states are mainly at -11.6eV to -7.6eV. The upper part of the valence band is dominated by Al/Si 3p and Fe 3d states. Fig. 3b shows that the Fe 3d states dominates the upper part valence band with high densities range from -2.0eV to -0.5eV (Fig. 3b). This agrees with states with small dispersion in this energy range as shown in Fig. 4.

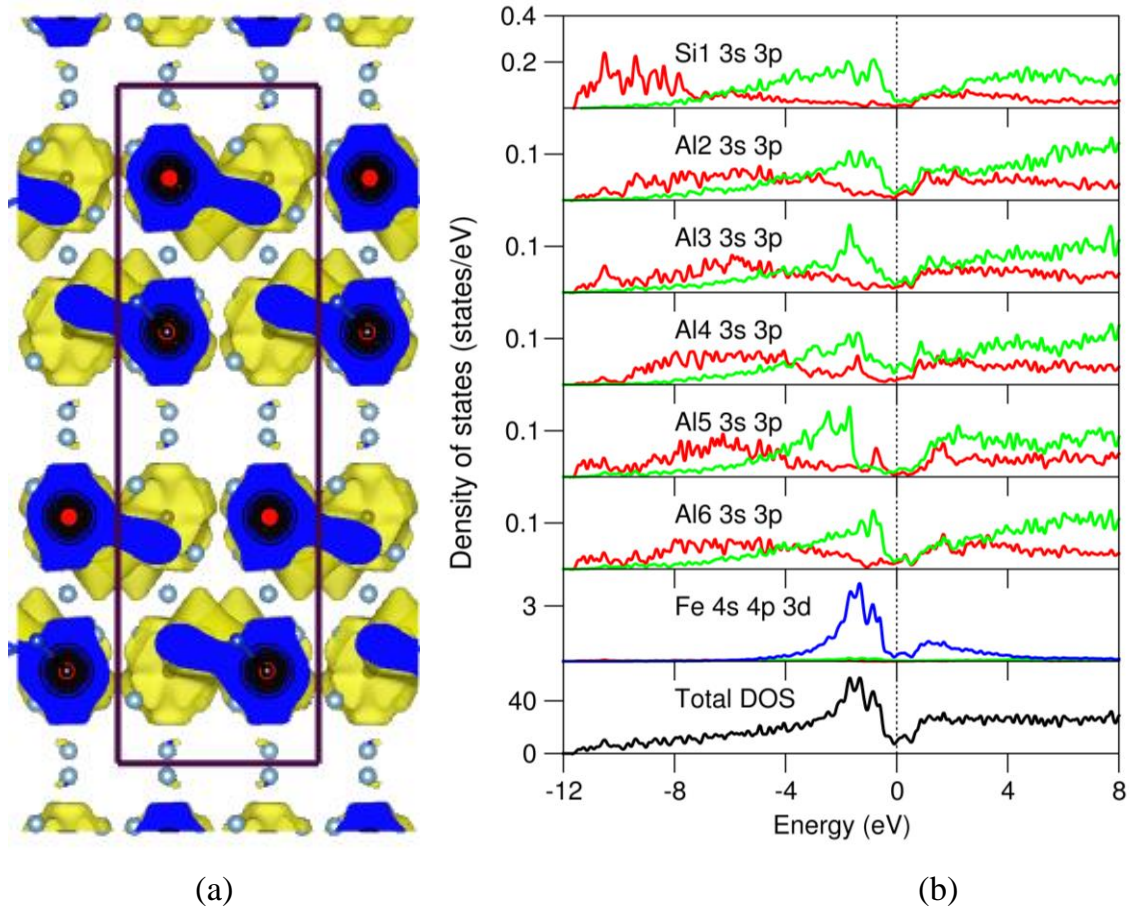


Fig. 3 (Color online) The electron density distribution with iso-surface density ($\rho_0(r) = 0.04e/\text{\AA}^3$) (a) and partial and total density of states (b) of $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$. The latter was compared with those of $\beta\text{-Al}_{4.5}\text{Si}^{\text{VI}}\text{Fe}$

in Fig. S-2. In (a) the red/black-regions represent the atomic cores of Fe, the yellow clouds represent the iso-surfaces, the blue regions have higher valence-electrons; and the silvery spheres represent Al.

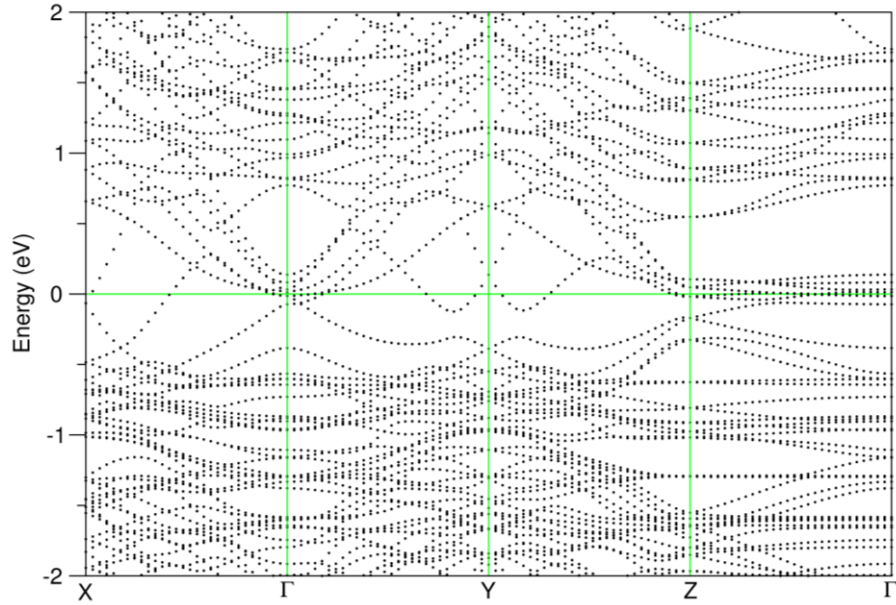


Fig. 4 (Color online) The dispersion curves along the high symmetry lines in the Brillouin zone of β - $\text{Al}_{4.5}\text{Si}^1\text{Fe}$, which can be regarded as pseudo-orthorhombic. The curves with a broad energy window is shown in Fig. S-3. The Fermi level is at 0eV.

As shown in Fig. 4, the dispersion curves along the in-plane directions (X- Γ -Y) show large dispersions with 1eV to 2eV, whereas the dispersion along the z-axis (Z- Γ) is small, typically about 0.1 to 0.2eV. Even taking into the ratio of axis lengths ($c/a \sim 3.37$) in account, this anisotropy is still to be about 3:1. Thus, β - $\text{Al}_{4.5}\text{Si}^1\text{Fe}$ is anisotropic of the electrical and thermal transport properties. Analysis showed that the states at the Fermi level at Γ in the Brillouin Zone are dominated by Al/Si p and Fe d characters.

There is a fall of total DOS at the Fermi level with low density of states at -0.5 to 0.5eV (Fig. 3b). The band structure (Fig. 4) also show few states in this energy range. The lower part of conduction band is dominated by Al/Si 3p and some Fe 3d states. We refer such a unusual low DOS at the Fermi level as a pseudo-gap [43]. Such narrow pseudo band-gap indicates unusual electronic properties and potential applications such as thermoelectric materials [43].

To get some direct impression about chemical bonding, we performed charge analysis using Bader's model [44, 45], in which the borders between one atom and its neighbors is defined by the 'zero-flux' surfaces of the gradient electron density. The obtained charges at the atomic sites for the most stable configurations are listed in Table II.

Charge transfer occurs from Al to Fe in β - $\text{Al}_{5.5}\text{Fe}$ (Table II). The amount of charging at the Al sites vary: about 0.65e/Al at Al1 and Al6, 0.86e at Al2 and Al3. The Al5 atoms loss the largest, 0.85e/Al whereas Al4 atoms loss just 0.24e/Al. The latter has less Fe neighbor (Table I).

The Si atoms derive electrons from Fe and Al: $\text{Fe}^{-3.65}$ in β - $\text{Al}_{5.5}\text{Fe}$ to $\text{Fe}^{-3.17}$ in β - $\text{Al}_{4.5}\text{SiFe}$. Correspondingly the Al atoms in β - $\text{Al}_{4.5}\text{SiFe}$ lose more electrons with comparing to those in β -

Al_{5.5}Fe, particularly the Al5. This indicates that the Al5 sites have higher attraction for Si, agreeing with the calculations in Fig. 2b. Such charge transfer at the atomic spheres is in lines with the electronegativity values of the elements (1.90 for Si, 1.83 for Fe and 1.60 for Al in the Pauling's scale).

4. Discussion

As shown in Figs. 1 and 3, β -Al_{4.5}SiFe has a layered structure composed of q -FeAlSi blocks. Each Fe is coordinated by eight Al and two Si, being sandwiched by two layers of the Al(Si)1/Al(Si)6 and Al2/Al3 atoms and topped/bottomed by an Al5 and Al4. The calculations showed that the Si prefers the Al1 or Al6 sites, forming β -Al_{4.5}SiFe. Such substitutions are partially due to the chemical pressure effects [46] as Si has a smaller atomic size than that of Al. The electronic band-structure calculations showed that the electronic structure exhibits low-dimensional character with a unusual narrow pseudo band-gap. This indicates that β -Al_{4.5}SiFe has unusual physical properties and can be used as low-dimensional functional materials [43, 47].

The structural flexibility of β -Al_{4.5}SiFe permits different stackings of the building a double-(q -FeAlSi) (half of the cell) slabs. Different stacking produces highly stable structures of an orthorhombic even tetragonal unit cell. This result helps us get into insight the variety of the lattices of this compound in the literature [12-15, 41, 42]. This deserves further investigation.

For most Al metals and Alloys casting occurs at typically 550°C to 700°C. At such temperature thermodynamics, including extra freedom or configurational entropy contribution plays a role in the formation and stability of the Fe-IMCs. The crucial role of configurational entropy contribution to the stability of alloys compounds at elevated temperature was discussed in the metals-alloyed mono-oxides, e.g. (Mg_{1-x}A_x)O (A = Cd, Zn) [48, 49]. Our calculations showed that structural openness of the β -phase that there are two configurations for the most stable at 0K. According to thermodynamics, the free energy of a system at elevated temperature is: $\Delta G = \Delta H - T \Delta S_{\text{conf.}}$, here $\Delta H = \Delta E$ is the formation energy with ignorance of the zero-point energy, and $\Delta S_{\text{conf.}} = R \ln w$ (R is the Boltzmann's constant, w the number of configurations) is the configuration energy entropy here. To assess the number of freedom for one composition, we consider the configurations with energy different smaller than 0.3eV/cell based on the first-principles calculations. Details of the obtained results are listed in Table S-III. The dependences of free energies at 0K, 800K and 1000K are plotted in Fig. 5.

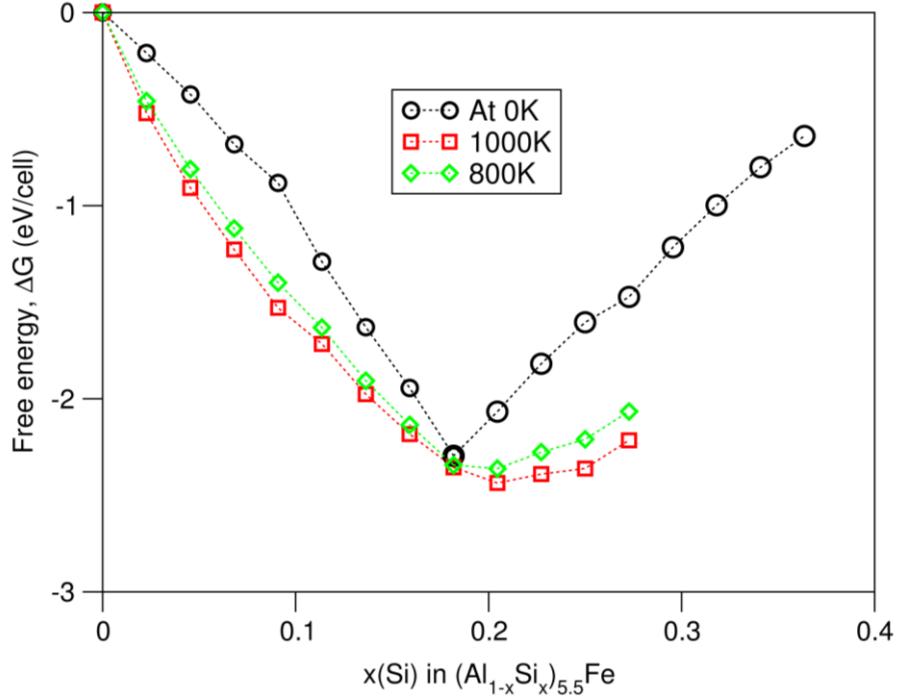


Fig. 5 (Color online) Dependences of free energies of the β -phase on Si contents at 0K, 800K and 1000K (Table S-III).

The Si content range in the β -phase depends pronouncedly on the temperature. Apparently at low temperature, the β -phase has composition formula $\text{Al}_{4.5}\text{SiFe}$. At 1000K, the Si content range from $x = 0.182$ to 0.250. Meanwhile at 800K the Si content range becomes smaller ($x = 0.182$ to 0.215) with the minimum of free energy at $x = 0.205$ as shown in Fig. 5.

The experiments showed that during the casting of Al-Fe-Si alloys, the θ -phase particles form first [16]. Then, the β -phase or α -phase forms, depending on the chemical composition and casting conditions. Experiments also showed influences of Si contents on the related reactions [16]. To get insight into these reactions, we investigate the formation energies of the Al-rich Fe-IMCs, Al_6Fe , $\theta\text{-Al}_{13}\text{Fe}_4$ and the β -phase with/without Si solute. The calculated formation energies with respect to the elemental solids, $\alpha\text{-Al}$, $\alpha\text{-Fe}$ and FCC Si are listed in Table IV. Table IV also includes the formation energies of dilute Si and Fe in $\alpha\text{-Al}$.

Table IV. The comparison of the formation energies per Fe in the related phases with/without Si solute. The data are from this work except of those labelled with references.

Formula	$\Delta E(\text{eV/Fe})$	Formula	$\Delta E(\text{eV/Fe})$
Si solute in $\alpha\text{-Al}$	+0.431eV/Si [26]	-	
Fe solute in $\alpha\text{-Al}$	-0.457 [26, 36]	-	-
Al_6Fe	-1.373 [39]	$(\text{Al}_{0.983}\text{Si}_{0.0417})_6\text{Fe}$	-1.286
$\beta\text{-Al}_{5.5}\text{Fe}$	-1.267	$\beta\text{-Al}_{4.5}\text{FeSi}$	-1.554
$\theta\text{-Al}_{13}\text{Fe}_4$	-1.403	$\theta\text{-(Al}_{0.949}\text{Si}_{0.051})_{13}\text{Fe}_4$ [26]	-1.410

The calculations showed that Si solute in Al_6Fe decreases its stability (Table IV) and thus, is unlikely. Si substitution at the Al sites in both $\theta\text{-Al}_{13}\text{Fe}_4$ and the β -phase enhances their stability. Table IV shows that the binary (novel) $\beta\text{-Al}_{5.5}\text{Fe}$ structure is notably less stable than Al_6Fe and θ -

$\text{Al}_{13}\text{Fe}_4$. The high Si substitution in β -phase (15.38at.%) strongly enhances the stability of this phase that it becomes significantly more stable than the θ -phase with about 3.9at.%.

The higher stability of the θ -phase with small amount of Si enables it form first in Al-Fe-Si alloys. Lower temperature drives more Si into Fe-IMCs due to the energy cost of Si in solid Al (Table IV). This stabilizes other Fe-IMCs, such as the β -phase as the observed reaction: $\text{L} + \theta\text{-phase} \rightarrow \beta\text{-phase}$ [16].

The obtained information about the accurate crystal structure, formation energies of $\beta\text{-Al}_{4.5}\text{SiFe}$ is helpful to get insight into the phase formation and phase transformation in the casting of Al-based alloys. This study also enhances our understanding of the thermodynamics and crystal structures of complex Fe-IMCs formed on solidification of Al-based alloys [1, 16, 19-21, 39].

5. Conclusions

In this study, we investigated the structure chemistry and electronic properties of the $\beta\text{-AlSiFe}$ phase of layered structure. The study revealed that i) Si prefers substitution on the Al^I or Al^{VI} sites, forming stable $\beta\text{-Al}_{4.5}\text{Si}^{\text{I}}\text{Fe}$ or $\beta\text{-Al}_{4.5}\text{Si}^{\text{VI}}\text{Fe}$, which updated the widely used model with a homogeneous Si/Al distribution; ii) Analysis showed that extra freedom permits higher Si contents in $\beta\text{-Al}_{4.5}\text{SiFe}$ at the casting temperature, in line with the variation of Si contents in the crystallites in cast Al-alloys in the literature; iii) Stacking of the building blocks (slabs) produces highly stable structures of different symmetry; iv) The electronic band structure of $\beta\text{-Al}_{4.5}\text{SiFe}$ is anisotropic and has a pseudo band gap, indicating unusual physical properties.

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