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Recursive Bayesian Estimation for Discrete-Time Systems With State-Dependent Packet Dropouts: A Cross-coupled Method

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***Abstract*—In this paper, the recursive Bayesian estimation problem is investigated for a class of linear discrete-time systems subject to state-dependent packet dropouts. During the transmission to a remote estimator, the data packets carrying the local measurements might be dropped if the system state is located within certain occlusion region, and this gives rise to a non-stationary dropout process relying on real system states. In this scenario, due to the exponential growth of the computational cost, it is almost impossible to calculate the exact posterior distribution of the system state for the purpose of optimal state estimation. To address this issue, we propose a novel cross-coupled estimation framework consisting of two interactively working estimators, namely, a region-label estimator and a state estimator, where the former is utilized to obtain the optimal estimates of the region-label sequence in the maximum a posteriori sense, while the latter is adopted to achieve the optimal estimates of the system states in the minimum mean-square error sense. Moreover, a sufficient condition is obtained to ensure the mean-square boundedness of the resultant estimation error. The effectiveness of the proposed cross-coupled estimation framework is verified by a numerical simulation example.**

***Index Terms*—State estimation, Bayesian inference, stochastic systems, state-dependent packet dropouts, Kalman filter.**

I. Introduction

State estimation has been an active research realm over the past few decades due mainly to its significant application insights in a variety of areas such as computer vision, guidance and navigation, econometrics, target tracking, and power systems [7], [14], [35], [36]. A fundamental issue for state estimation is to develop appropriate algorithms capable of restoring system states of interest based on a series of measurements observed over time, and some well-known algorithms have been developed for Kalman filtering, extended Kalman filtering, unscented Kalman filtering, particle filtering and Bayesian filtering [1], [4]–[6], [19], [20], [29], [30], [41]. Among other, the Bayesian filtering method, which views state estimation as a probability inference process, aims to establish the posterior probability density function (PDF) of the interested system state. Such a method has proven to be extremely powerful in dealing with state estimation problems for complicated dynamic systems subject to nonlinear processes and/or non-Gaussian noises. It is noteworthy that most of the aforementioned filtering algorithms can be deduced from the Bayesian framework according to different approximations employed in the computational procedure.

Owing to the rapid development of sensing, processing and communication technologies, the networked systems have recently found widespread applications in control engineering and signal processing [8], [9], [12], [17], [47], [48]. Under the networked configuration, the measurements of sensor nodes are transmitted to a remote estimator for further processing via communication networks [3], [13], [21], [27], [28], [40], [44]. Since the capacity of networks is often limited in practice, measurement transmission suffers inevitably from certain network-induced phenomena including channel congestions, communication delays and signal distortions [24], [42], [43], [45], [49], and this might eventually result in unexpected packet dropouts of transmitted measurements which, if not properly dealt with, could further the jeopardize the estimation performance or even lead to the divergence of the estimation error dynamics.

According to the way it occurs, the phenomenon of the packet dropout can be generally characterized by two main models. The first is the independent and identically distributed (i.i.d.) Bernoulli model where the packet dropout phenomenon is described by a Bernoulli i.i.d. random process [2], [18], [22], [26], [32], [34], [46], and the second is the two-state Gilbert-Elliot channel model where the packet dropout is characterized by using a binary Markov chain [10], [15], [16], [23], [39]. Basically, in comparison with the i.i.d. Bernoulli model, the Gilbert-Elliot channel model has been deemed to be more general because of its capability of capturing the temporal correlation in practical communication channels. So far, considerable research attention has been devoted to the networked state estimation subject to packet dropouts described by the two models.

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For the i.i.d. Bernoulli model, the optimal linear estimation problems have been investigated in [18], [34], where linear minimum variance filters have been designed based on the statistics of Bernoulli variables. The stability issue of the Kalman filter with intermittent measurements has been considered in [22], [26], [32] and a threshold of the packet dropout rate has been critical in ensuring the convergence of the mean covariance. As for the Gilbert-Elliot model, the optimal recursive estimation problems have been fully examined in [15], [23], and the Kalman filtering problem with Markovian packet dropouts has been further addressed in [16] with sufficient conditions established for the stability of the peak covariance process. Moreover, the relationship between the peak-covariance stability and the mean-square stability has been thoroughly discussed in [39].

Up to now, the optimal state estimation problems subject to various kinds of packet dropouts have drawn considerable research attention, and most existing results have been obtained by formulating the packet dropout phenomenon as a Bernoulli or Markov random process. Such a formulation, unfortunately, might be inappropriate in some practical scenarios [25], [38]. For example, consider the scenario of remote target tracking where the target information is first collected by local sensors and then transmitted to an estimation center for further processing. When the target enters certain occlusion regions, data transmission between the sensor and the estimator could blocked, making the sensor observations unavailable to the estimator. In this case, the phenomenon of the packet dropouts turns out to be a non-stationary random process dependent on the real-time target state, and this renders substantial difficulties to the corresponding filter design and stability analysis.

To tackle the state estimation problem related to state-dependent packet dropouts (SDPDs), some initial efforts have been made in [37] where the packet loss has been described by a state-dependent hybrid measurement model and the optimal estimation has been accomplished by using the orthogonal projection approach. The pioneering results presented in [37] have been obtained based on a proposed optimal estimator in the *linear* minimum mean-square error (MMSE) sense. Unfortunately, the optimal filtering problem with SDPDs is effectively a *nonlinear* filtering problem and, therefore, it makes both practical and theoretical sense to improve the existing results by specifically tackling the inherent nonlinearities resulting from the SDPDs. In doing so, the Bayesian inference framework appears to be particularly suitable, and this motivates our current investigation.

Concluding the above-mentioned discussions, we are interested in addressing the optimal estimation problems for a class of discrete-time systems subject to SDPDs based on the Bayesian inference framework. The primary contributions of this paper can be highlighted from the following aspects. *1) To the best of our knowledge, this paper makes one of the first few attempts to deal with the optimal estimation problem subject to SDPDs based on a general Bayesian inference framework; 2) a novel cross-coupled estimation algorithm, which is composed of a region-label estimator and a system state estimator, is proposed to obtain the MMSE estimate of the system state; 3) several approximation methods are utilized such that the obtained MMSE estimates have recursive linear forms, which greatly reduces the computational complexity; and 4) a sufficient condition is provided to guarantee the mean-square boundedness of the estimation error dynamics.*

The remainder of this paper is organized as follows. Section II formulates the state estimation with SDPDs. Section III proposes a novel cross-coupled estimation algorithm consisting of the region-label and state estimators, and then analyzes the mean-square boundedness of the associated error dynamics. Numerical simulation is carried out in Section IV. Finally, some conclusion remarks are made in Section V.

Notation: Throughout the paper, the notations utilized are mostly standard except where otherwise stated. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The superscript T denotes the transpose. $\|\star\|$ denotes the Euclidian norm of real vectors or the spectral norm of real matrices. The indicator function $\mathbb{I}_\Omega$ is equal to 1 if the event Ω occurs and zero otherwise. The PDF of a random vector x is denoted as $p(x)$ and the conditional PDF of x given y is denoted as $p(x|y)$. If $x \in \mathbb{R}^n$ obeys Gaussian distribution, then its PDF is denoted as $p(x) \triangleq \mathcal{N}(x, \mu, \Sigma)$, where

$$\mathcal{N}(x, \mu, \Sigma) \triangleq \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\right).$$

II. Problem Formulation

A. System description

Consider a discrete linear time-invariant system described by the following state-space model:

$$\begin{aligned} x_{k+1} &= Ax_k + Bw_k, \\ z_k &= Cx_k + Dv_k \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ and $z_k \in \mathbb{R}^{n_z}$ are the state vector and the measurement vector, respectively; $w_k \in \mathbb{R}^{n_w}$ and $v_k \in \mathbb{R}^{n_v}$ are sequences of white Gaussian noises with zero mean and covariance matrices $Q > 0$ and $R > 0$, respectively; A, B, C and D are known matrices of appropriate dimensions; and the initial state x_0 obeys a Gaussian distribution with mean μ_0 and covariance matrix $\Sigma_0 > 0$. It is assumed that $BQB^{\mathrm{T}} > 0$.

B. State-dependent packet dropouts

The remote state estimation problem is investigated in this paper where the measurement vectors z_k are transmitted to a remote estimation center in order to generate an estimate of the state vector x_k. Furthermore, we consider the situation where the transmissions might suffer from SDPDs.

As shown in Fig. 1, there are finite occlusion regions (for the target plant) that are denoted as

$$\mathcal{R}^o_{i,k}, \quad \text{for} \quad i \in \mathbb{Z}_{[1,S]}$$

where $\mathbb{Z}_{[1,S]}$ is a set of positive numbers $\{1, 2, \cdots, S\}$ with S representing the number of the regions. The target measurements cannot be transmitted to the remote estimator once the state variables (i.e., $Ex_k \in \mathbb{R}^{n_e}$) fall into these occlusion

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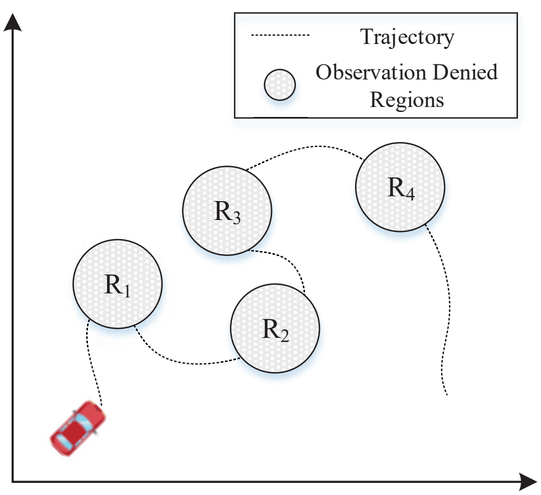

Fig. 1. State-dependent packet dropouts. The remote estimator cannot obtain measurements of the target plant when its trajectory enters the occlusion region.

regions. The location of the occlusion region is defined as follows:

$$\mathcal{R}_{i,k}^o \triangleq \left\{ x \in \mathbb{R}^{n_e} : \|x - \zeta_{i,k}^o\|^2 \leq r_i^2 \right\} \tag{2}$$

where $\zeta_{i,k}^o = \zeta_i - n_k$ with $\zeta_i \in \mathbb{R}^{n_e}$ and $r_i \in \mathbb{R}$ representing the expected center and the radius of the occlusion region, respectively. $n_k \in \mathbb{R}^{n_e}$ is a white zero-mean Gaussian noise process with covariance Ψ_k. Apparently, the occlusion regions $\mathcal{R}_{i,k}^o$ are randomly distributed at every time k because the center $\zeta_{i,k}^o$ is a random variable obeying the Gaussian distribution. Throughout the paper, it is assumed that x_0, w_k, v_k and n_k are i.i.d variables which are independent with each other.

We introduce an auxiliary variable as follows:

$$u_k \triangleq Ex_k + n_k, \tag{3}$$

which implies that $Ex_k \in \mathcal{R}_{i,k}^o$ is equivalent to $u_k \in \mathcal{R}_i$ where $\mathcal{R}_i$ is an auxiliary region defined by

$$\mathcal{R}_i \triangleq \left\{ x \in \mathbb{R}^{n_e} : \|x - \zeta_i\|^2 \leq r_i^2 \right\}.$$

Moreover, let us define the set of the occlusion region and the corresponding auxiliary region (i.e., $\mathcal{R}_k^o$ and $\mathcal{R}$, respectively) as follows

$$\mathcal{R}_k^o \triangleq \{\mathcal{R}_{1,k}^o, \mathcal{R}_{2,k}^o, \cdots, \mathcal{R}_{S,k}^o\},$$

and

$$\mathcal{R} \triangleq \{\mathcal{R}_1, \mathcal{R}_2, \cdots, \mathcal{R}_S\}.$$

As such, the measurements available at the estimator at each sampling instant k can be formulated by

$$y_k = \begin{cases} z_k, & \text{if } u_k \notin \mathcal{R}_i, \\ \phi, & \text{if } u_k \in \mathcal{R}_i, \end{cases}$$

for $i \in \mathbb{Z}_{[1,S]}$, where ϕ represents the empty set. For simplicity of presentation, we abbreviate $u_k \in \mathcal{R}_i$ for $i \in \mathbb{Z}_{[1,S]}$ as $u_k \in \mathcal{R}$. It can be observed that, when $u_k \in \mathcal{R}_i$ (i.e., $Ex_k \in \mathcal{R}_{i,k}^o$), the packet dropout occurs and the estimator will not receive any measurement signal at instant k. Furthermore, we introduce a region-label h_k satisfying $h_k = i$ if $u_k \in \mathcal{R}_i$ and $h_k = 0$ if $u_k \notin \mathcal{R}$. Then, the sequence of received measurements and region-labels are denoted as

$$Y_k \triangleq \{y_0, y_1, \cdots, y_k\} \quad \text{and} \quad H_k \triangleq \{h_0, h_1, \cdots, h_k\}.$$

C. Estimation Objectives

The estimation problem is to recursively calculate a degree of belief in x_k given the information set Y_k by constructing the posterior PDF $p(x_k|Y_k)$. Generally speaking, the posterior PDF can be obtained from the state-observation model in (1) using the prediction-correction steps.

The prediction stage involves the knowledge of the system model to obtain a prior PDF of the state x_k at instant k via the Chapman–Kolmogorov equation

$$p(x_k|Y_{k-1}) = \int p(x_k|x_{k-1})p(x_{k-1}|Y_{k-1})dx_{k-1}. \tag{4}$$

At the correction stage, when a measurement y_k is newly available, it can be exploited to update the prior PDF via Bayes' rule

$$p(x_k|Y_k) = \frac{p(y_k|x_k)p(x_k|Y_{k-1})}{\int f(y_k|x_k)f(x_k|Y_{k-1})dx_k}. \tag{5}$$

If all the observations z_k from instant 0 to instant k are obtained at the estimator, the prior PDF is naturally Gaussian distributed, and its expectation and covariance can be calculated by the Kalman filter. Moreover, if observation z_k at instant k suffers from a packet dropout, which randomly occurs according to an i.i.d Bernoulli process, the correction stage is *invalid* and the posterior would be equal to the prior, i.e., $p(x_k|Y_k) = p(x_k|Y_{k-1})$ (as in [32]). Unfortunately, such an equality apparently no longer holds in this paper since the occurrence of packet dropouts depends explicitly on the real-time system state. In this scenario, calculating the posterior PDF $p(x_k|Y_k)$ involves integrations of nonlinear terms and requires a large amount of computational cost, which makes the concerned optimal filtering problem intractable in general. To handle such an issue, a simplified workaround is developed in the following section via designing a cross-coupled estimation framework consisting of two interactively working state estimators, i.e. a region-label estimator and a state estimator.

In what follows, we briefly describe the framework of this cross-coupled estimation framework. To begin with, at every instant k, let us approximate the PDF $p(x_{k-1}|Y_{k-1})$ as a Gaussian distribution characterized with mean $\hat{x}_{k-1}$ and covariance P_{k-1}. On one hand, if the observation z_k arrives at time k, then the posteriori PDF will also be Gaussian distributed, and therefore the subsequent estimation procedure would be the same as that of the Kalman filter by computing the updated mean $\hat{x}_k$ and covariance P_k. On the other hand, if the observation z_k is dropped while the region-label $h_k = i$ is available, then the knowledge is available about the trajectory

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of the system state located in the ith occlusion region (i.e., $u_k \in \mathcal{R}_i$), which provides additional information for updating the prior PDF. To be specific, we aim to 1) utilize the historical estimation results to evaluate a most possible value for the current region-label h_k in the maximum a posteriori (MAP) sense; and 2) exploit the obtained label to generate an MMSE estimate of the target state.

Motivated by the above discussion, we outline the main objectives of this paper as follows.

i) *MAP estimation of H_k.* The optimal estimate of the region-label sequence is obtained by maximizing the PDF of H_k conditioned on Y_k, i.e.

$$\hat{H}_k = \arg\max_{H_k} p(H_k|Y_k). \tag{6}$$

ii) *MMSE estimation of x_k.* The MMSE estimate of the system state is obtained by calculating the expectation of x_k conditioned on both Y_k and H_k, i.e.

$$\hat{x}_k = \mathbb{E}\{x_k|H_k, Y_k\}. \tag{7}$$

Before proceeding, a useful lemma is provided as follows to benefit the subsequent derivation.

Lemma 1 ([31]): Given two Gaussians $\mathcal{N}(x, \mu_1, \Sigma_1)$ and $\mathcal{N}(\mu_2, Ax, \Sigma_2)$, letting

$$\begin{aligned} \mu_c &\triangleq \Sigma_c\Sigma_1^{-1}\mu_1 + \Sigma_c A^{\mathrm{T}}\Sigma_2^{-1}\mu_2 \\ c_c &\triangleq \mathcal{N}(\mu_2, A\mu_1, A\Sigma_1 A^{\mathrm{T}} + \Sigma_2) \\ \Sigma_c &\triangleq (\Sigma_1^{-1} + A^{\mathrm{T}}\Sigma_2^{-1}A)^{-1} \end{aligned}$$

the following equality holds

$$\mathcal{N}(x, \mu_1, \Sigma_1)\mathcal{N}(\mu_2, Ax, \Sigma_2) = c_c\mathcal{N}(x, \mu_c, \Sigma_c).$$

III. Main results

In this section, we are dedicated to establishing a cross-coupled estimation algorithm composed of a region-label estimator and a state estimator for networked systems subject to SDPDs. A sufficient condition for the mean-square boundedness of the estimation error dynamics will be further obtained.

A. MAP estimation of the region-label

From the previous section, we know that a precise recognition of the current region-labels is critical to the development of the subsequent state estimation procedure as well as the enhancement of the state estimation performance. As such, a recursive estimator will be firstly developed to obtain the MAP estimate of the region-label sequence.

The propagation of the conditional PDF $p(H_k|Y_k)$ can be derived by applying Bayes' rule as follows:

$$\begin{aligned} &p(H_k|Y_k) \\ &= \frac{p(y_k|H_k, Y_{k-1})p(H_k|Y_{k-1})}{p(y_k|Y_{k-1})} \\ &= \frac{p(y_k|H_k, Y_{k-1})p(h_k|H_{k-1}, Y_{k-1})}{p(y_k|Y_{k-1})}p(H_{k-1}|Y_{k-1}) \end{aligned}$$

where the last step follows from the fact that

$$p(H_k|Y_{k-1}) = p(h_k|H_{k-1}, Y_{k-1})p(H_{k-1}|Y_{k-1}).$$

From the above equality, it is not difficult to see that the MAP estimate of H_k is, in fact, intractable because the computational cost of the optimization problem (6) increases exponentially as time goes on. Such a high computational cost is inherent to the augmented dimension of H_k as k increases, and this motivates the following approximation:

$$p(H_k|Y_k) \approx p\left(h_k, \hat{H}_{k-1}|Y_k\right)$$

where

$$\hat{H}_{k-1} = \arg\max_{H_{k-1}} p(H_{k-1}|Y_{k-1}).$$

Using such an approximation, the conditional PDF $p(H_k|Y_k)$ can be rewritten as

$$\begin{aligned} &p(H_k|Y_k) \\ &= \frac{p(y_k|h_k, \hat{H}_{k-1}, Y_{k-1})p(h_k|\hat{H}_{k-1}, Y_{k-1})}{p(y_k|Y_{k-1})}p(\hat{H}_{k-1}|Y_{k-1}). \end{aligned}$$

Given the previous estimate of the region-label sequence at instant $k-1$, we can then reformulate the MAP estimate of the region-label h_k as

$$\hat{h}_k = \arg\max_{h_k} p\left(h_k, \hat{H}_{k-1}|Y_k\right). \tag{8}$$

Once the current estimate $\hat{h}_k$ is acquired, the approximated MAP estimate of sequence $\hat{H}_k$ can be finally obtained by augmenting $\hat{h}_k$ with the previous estimate $\hat{H}_{k-1}$ as

$$\hat{H}_k = \begin{bmatrix} \hat{H}_{k-1} & \hat{h}_k \end{bmatrix}.$$

In the sequel, we will turn our attention to the optimization problem (8) (instead of the optimization problem (6)). Note that a proportional counterpart of $p(h_k, \hat{H}_{k-1}|Y_k)$ has the following form:

$$\begin{aligned} &p\left(h_k, \hat{H}_{k-1}|Y_k\right) \\ &\qquad \propto p\left(y_k|h_k, \hat{H}_{k-1}, Y_{k-1}\right)p\left(h_k|\hat{H}_{k-1}, Y_{k-1}\right), \end{aligned}$$

which implies

$$\hat{h}_k = \arg\max_{h_k} p\left(y_k|h_k, \hat{H}_{k-1}, Y_{k-1}\right)p\left(h_k|\hat{H}_{k-1}, Y_{k-1}\right).$$

To proceed further, the probability distributions $p(y_k|h_k, \hat{H}_{k-1}, Y_{k-1})$ and $p(h_k|\hat{H}_{k-1}, Y_{k-1})$ shall be evaluated. Obviously, the following relationship

$$p\left(y_k \neq \phi|h_k = i, \hat{H}_{k-1}, Y_{k-1}\right) = 0$$

is true for $i \in \mathbb{Z}_{[1,S]}$. Consequently, whenever an observation z_k arrives, an optimal estimate of current label h_k is set to be $\hat{h}_k = 0$. In this case, the major challenge we are encountering is to estimate the region-label h_k for the case $y_k = \phi$. Moreover, we have

$$\begin{aligned} &p\left(h_k|\hat{H}_{k-1}, Y_{k-1}\right) \\ &= \int p\left(x_k|\hat{H}_{k-1}, Y_{k-1}\right)p\left(h_k|x_k, \hat{H}_{k-1}, Y_{k-1}\right)dx_k \\ &= \int p\left(x_k|\hat{H}_{k-1}, Y_{k-1}\right)p(h_k|x_k)dx_k. \end{aligned} \tag{9}$$

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Taking advantage of the Gaussian approximation

$$p(x_{k-1}|\hat{H}_{k-1}, Y_{k-1}) \triangleq \mathcal{N}(x_{k-1}, \hat{x}_{k-1}, P_{k-1})$$

and the Chapman-Kolmogorov equation, the predicted PDF $p(x_k|\hat{H}_{k-1}, Y_{k-1})$ can be obtained as

$$\begin{aligned} &p(x_k|\hat{H}_{k-1}, Y_{k-1}) \\ &= \int p(x_k|x_{k-1}) p(x_{k-1}|\hat{H}_{k-1}, Y_{k-1}) dx_{k-1} \\ &= \int \mathcal{N}(x_k, Ax_{k-1}, BQB^{\mathrm{T}}) \mathcal{N}(x_{k-1}, \hat{x}_{k-1}, P_{k-1}) dx_{k-1}. \end{aligned}$$

Applying Lemma 1 to the above equation yields

$$p\Big(x_k|\hat{H}_{k-1}, Y_{k-1}\Big) = \mathcal{N}\big(x_k, \hat{x}_{k|k-1}, P_{k|k-1}\big) \tag{10}$$

where

$$\begin{aligned} \hat{x}_{k|k-1} &= A\hat{x}_{k-1}, \\ P_{k|k-1} &= AP_{k-1}A^{\mathrm{T}} + BQB^{\mathrm{T}}. \end{aligned} \tag{11}$$

Moreover, noting that $p(h_k = i|x_k)$ ($i \in \mathbb{Z}_{[1,S]}$) represents the probability of the event that the system trajectory is in the ith occlusion region given the condition of current state vector x_k, we have

$$\begin{aligned} p(h_k = i|x_k) &= p(u_k \in \mathcal{R}_i|x_k) \\ &= |\mathcal{R}_i| \int \Lambda_{\mathcal{R}_i}(u_k) p(u_k|x_k) du_k \end{aligned} \tag{12}$$

where $\Lambda_{\mathcal{R}_i}(u_k)$ is denoted as $\Lambda_{\mathcal{R}_i}(u_k) = 0$ if $u_k \notin \mathcal{R}_i$ and $\Lambda_{\mathcal{R}_i}(u_k) = |\mathcal{R}_i|^{-1}$ if $u_k \in \mathcal{R}_i$ with $|\mathcal{R}_i|$ representing the Lebesgue measure of $\mathcal{R}_i$. Note that it is analytically impossible to calculate $p(h_k = i|x_k)$ due to its non-Gaussian distribution. As such, a sum of Gaussian densities is employed to provide an approximation on $\Lambda_{\mathcal{R}_i}(u_k)$ (see [31], [33]):

$$\Lambda_{\mathcal{R}_i}(u_k) \approx \frac{1}{N} \sum_{s=1}^{N} \mathcal{N}\big(u_k, \tilde{u}_{ik}^s, V_{ik}^s\big)$$

where the mean and covariance of the sth Gaussian distribution are denoted as $\tilde{u}_{ik}^s$ and V_{ik}^s, respectively. For brevity, $\tilde{u}_{ik}^s$ are chosen by equidistantly sampling the region $\mathcal{R}_i$, and V_{ik}^s are chosen to have the same value V_k. Moreover, by introducing a new variable

$$\Sigma_k = V_k + \Psi_k,$$

we have

$$\begin{aligned} &\int \mathcal{N}\big(u_k, \tilde{u}_{ik}^s, V_k\big) p(u_k|x_k) du_k \\ &= \int \mathcal{N}\big(u_k, \tilde{u}_{ik}^s, V_k\big) \mathcal{N}\big(u_k, Ex_k, \Psi_k\big) du_k \\ &= \mathcal{N}\big(\tilde{u}_{ik}^s, Ex_k, \Sigma_k\big), \end{aligned}$$

where the last equality follows from Lemma 1. Substituting the above equation into (12) yields

$$p(h_k = i|x_k) = \frac{1}{N} |\mathcal{R}_i| \sum_{s=1}^{N} \mathcal{N}\big(\tilde{u}_{ik}^s, Ex_k, \Sigma_k\big). \tag{13}$$

Combining (10) and (13) and using Lemma 1 once again, we have

$$\begin{aligned} &p\left(x_k|\hat{H}_{k-1}, Y_{k-1}\right) p(h_k = i|x_k) \\ &= \frac{1}{N} |\mathcal{R}_i| \sum_{s=1}^{N} \mathcal{N}\big(\tilde{u}_{ik}^s, Ex_k, \Sigma_k\big) \mathcal{N}\big(x_k, \hat{x}_{k|k-1}, P_{k|k-1}\big) \\ &= \frac{1}{N} \sum_{s=1}^{N} \omega_{ik}^s \mathcal{N}\big(x_k, \theta_{ik}^s, \Theta_k\big) \end{aligned} \tag{14}$$

where

$$\begin{aligned} \Theta_k &= \left(P_{k|k-1}^{-1} + E^{\mathrm{T}} \Sigma_k^{-1} E\right)^{-1}, \\ \theta_{ik}^s &= \Theta_k \left(P_{k|k-1}^{-1} \hat{x}_{k|k-1} + E^{\mathrm{T}} \Sigma_k^{-1} \tilde{u}_{ik}^s\right), \\ \omega_{ik}^s &= |\mathcal{R}_i|\ \mathcal{N}\big(\tilde{u}_{ik}^s, E\hat{x}_{k|k-1}, EP_{k|k-1}E^{\mathrm{T}} + \Sigma_k\big). \end{aligned} \tag{15}$$

By noting that the integral of the Gaussian distribution over the state is equal to 1, an explicit expression of (9) can be derived as

$$\begin{aligned} &p\Big(h_k = i|\hat{H}_{k-1}, Y_{k-1}\Big) \\ &= \int p\Big(x_k|\hat{H}_{k-1}, Y_{k-1}\Big) p(h_k = i|x_k) dx_k \\ &= \frac{1}{N} \int \sum_{s=1}^{N} \omega_{ik}^s \mathcal{N}\left(x_k, \theta_{ik}^s, \Theta_k\right) dx_k \\ &= \frac{1}{N} \sum_{s=1}^{N} \omega_{ik}^s, \end{aligned} \tag{16}$$

for $i \in \mathbb{Z}_{[1,S]}$. It is acknowledged that, when the system trajectory is in the occlusion region, the transmission would suffer from packet dropouts, and thus

$$p\Big(y_k = \phi|h_k = i, \hat{H}_{k-1}, Y_{k-1}\Big) = 1, \text{ for } i \in \mathbb{Z}_{[1,S]}.$$

Therefore, the MAP estimate of the current region-label can be given as follows:

$$\hat{h}_k = \begin{cases} 0, & \text{if } y_k = z_k, \\ \arg \max\limits_{i \in \mathbb{Z}_{[1,S]}} \sum_{s=1}^{N} \omega_{ik}^s, & \text{if } y_k = \phi. \end{cases} \tag{17}$$

In summary, the proposed MAP estimation procedure of the current region-label h_k is outlined in Algorithm 1. It is worth pointing out that the proposed algorithm works in a recursive manner, where the label sequence H_{k-1} and state estimate $\hat{x}_{k-1}$ at instant $k-1$ are required to generate an MAP estimate of the region-label h_k at instant k.

B. MMSE estimation of the system state

Given the MAP estimate of the region-label sequence H_k, we are now in the position to compute the approximated MMSE estimate of the state vector in this subsection.

According to the arrivals of the measurements, the posterior PDF $p(x_k|H_k, Y_k)$ can be divided into two situations. Firstly, we consider that the measurements are able to be received by

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Algorithm 1 MAP estimate of the region-label H_k.

$$\hat{H}_k = \mathrm{MAPH}\,[\hat{x}_{k-1}, P_{k-1}, \hat{H}_{k-1}, z_k]$$

Input: $\hat{x}_{k-1}$, P_{k-1}, $\hat{H}_{k-1}$, z_k.
Output: $\hat{H}_k$.

1: calculate the one-step prediction
$$\hat{x}_{k|k-1} = A\hat{x}_{k-1}.$$
2: calculate the parameters
$$\omega_{ik}^s = |\mathcal{R}_i|\mathcal{N}\big(\tilde{u}_{ik}^s, E\hat{x}_{k|k-1}, EP_{k|k-1}E^{\mathrm{T}} + \Sigma_k\big).$$
3: **if** the observation arrives, i.e., $y_k = z_k$, **then**
4: set $\hat{h}_k = 0$.
5: **else if** the observation is missing, i.e., $y_k = \phi$ **then**
6: calculate $\hat{h}_k$ by solving
$$\hat{h}_k = \arg\max_{i\in\mathbb{Z}_{[0,S]}} \sum_{s=1}^{N} \omega_{ik}^s.$$
7: augment $\hat{H}_{k-1}$ and $\hat{h}_k$ as $\hat{H}_k = \left[\begin{array}{cc} \hat{H}_{k-1} & \hat{h}_k \end{array}\right]$.
8: **return** $\hat{H}_k$.

the estimator at instant k (i.e., $h_k = 0$). Then, the prior PDF can be updated via the Bayes' rule as follows:

$$\begin{aligned}
&p(x_k|H_k, Y_k)\\
&= \frac{p(z_k, h_k|x_k, H_{k-1}, Y_{k-1})p(x_k|H_{k-1}, Y_{k-1})}{p(z_k, h_k|H_{k-1}, Y_{k-1})}\\
&= \frac{p(z_k, h_k|x_k)p(x_k|H_{k-1}, Y_{k-1})}{p(z_k, h_k|H_{k-1}, Y_{k-1})}
\end{aligned}$$

where the last equality exploits the fact that x_k has the sufficient information to determined z_k which makes the information from H_{k-1} and Y_{k-1} redundant. Moreover, the normalizing constant $p(z_k, h_k|H_{k-1}, Y_{k-1})$ can be written as

$$\begin{aligned}
&p(z_k, h_k|H_{k-1}, Y_{k-1})\\
&= \int p(z_k, h_k|x_k)p(x_k|H_{k-1}, Y_{k-1})dx_k.
\end{aligned}$$

As has been shown in (10), under the Gaussian approximation of the conditional PDF $p(x_{k-1}|H_{k-1}, Y_{k-1})$, it is not difficult to see that the prior PDF of the state vector at time k has the following structure:

$$p(x_k|H_{k-1}, Y_{k-1}) = \mathcal{N}(x_k, \hat{x}_{k|k-1}, P_{k|k-1}).$$

Moreover, it is apparent that the likelihood function

$$p(z_k, h_k|x_k) = \mathcal{N}(z_k, Cx_k, DRD^{\mathrm{T}})$$

is Gaussian. In light of Lemma 1 and the matrix inversion lemma, the posterior is also a Gaussian distributed PDF of the form:

$$p(x_k|H_k, Y_k) = \mathcal{N}(x_k, \hat{x}_k, P_k)$$

with mean and covariance given by

$$\begin{aligned}
\hat{x}_k &= \hat{x}_{k|k-1} + P_{k|k-1}C^{\mathrm{T}}\Omega_k^{-1}(z_k - C\hat{x}_{k|k-1}),\\
P_k &= P_{k|k-1} - P_{k|k-1}C^{\mathrm{T}}\Omega_k^{-1}CP_{k|k-1},\\
\Omega_k &= CP_{k|k-1}C^{\mathrm{T}} + DRD^{\mathrm{T}}.
\end{aligned} \tag{18}$$

According to the properties of Gaussian distributions, it can be seen that the MMSE estimate of the state x_k is $\hat{x}_k$ given in (18).

As for the situation where the measurements are dropped at instant k, we know that the system trajectory is in the ith occlusion region (i.e., $h_k = i$, for $i \in \mathbb{Z}_{[0,S]}$), and therefore the posterior PDFs $p(x_k|H_k, Y_k)$ can be obtained via the following Bayes' rule:

$$\begin{aligned}
&p(x_k|H_k, Y_k)\\
&= \frac{p(y_k = \phi, h_k = i|x_k)p(x_k|H_{k-1}, Y_{k-1})}{p(y_k = \phi, h_k = i|H_{k-1}, Y_{k-1})}
\end{aligned} \tag{19}$$

where the likelihood function can be determined from (2) and (3) as follows:

$$\begin{aligned}
&p(y_k = \phi, h_k = i|x_k) = p(u_k \in \mathcal{R}_i|x_k)\\
&= \int p(u_k \in \mathcal{R}_i)p(u_k|x_k)du_k.
\end{aligned} \tag{20}$$

It is trivial to see that

$$p(y_k = \phi, h_k = i|x_k) = p(h_k = i|x_k),$$

and

$$p(y_k = \phi, h_k = i|H_{k-1}, Y_{k-1}) = p(h_k = i|H_{k-1}, Y_{k-1}),$$

for $i \in \mathbb{Z}_{[0,S]}$. Substituting (14) and (16) into (19), we have the following Gaussian sum approximation to the true posterior PDF:

$$p(x_k|Y_k, H_k) = \sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N}\omega_{ik}^s}\mathcal{N}(x_k, \theta_{ik}^s, \Theta_k)$$

Note that the above posterior PDF (approximated by a sum of Gaussians PDFs) could be further approximated by a single Gaussian PDF $\mathcal{N}(x_k, \hat{x}_k, P_k)$ whose structure is similar to that of the prior probability distribution $p(x_{k-1}|Y_{k-1}, H_{k-1})$. By doing so, the approximated posterior PDF can be calculated recursively in each step. Next, we need to determine the mean and covariance $\hat{x}_k$ and P_k so as to obtain the best approximation. To achieve this goal, the Kullback-Leibler divergence is introduced as follows to measure the difference between two probability distributions $f_1(x)$ and $f_2(x)$ over the same variable x:

$$\mathcal{D}\left(f_1(x), f_2(x)\right) \triangleq \int f_1(x)\log\frac{f_1(x)}{f_2(x)}dx.$$

Then, the best mean and covariance that minimize the Kullback-Leibler divergence between

$$\sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N}\omega_{ik}^s}\mathcal{N}(x_k, \theta_{ik}^s, \Theta_k)$$

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and $\mathcal{N}(x_k, \hat{x}_k, P_k)$ can be obtained by solving the following optimization problem:

$$\min_{\hat{x}_k, P_k} \mathcal{D}\left(\sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N} \omega_{ik}^s} \mathcal{N}(x_k, \theta_{ik}^s, \Theta_k), \mathcal{N}(x_k, \hat{x}_k, P_k)\right).$$

whose unique solution is

$$\begin{aligned} \hat{x}_k &= \sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N} \omega_{ik}^s} \theta_{ik}^s, \\ P_k &= \sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N} \omega_{ik}^s} \left(\Theta_k + (\hat{x}_k - \theta_{ik}^s)(\hat{x}_k - \theta_{ik}^s)^{\mathrm{T}}\right), \end{aligned} \tag{21}$$

which is optimal in the MMSE sense. To sum up, the procedure of the proposed MMSE estimation scheme can be outlined in Algorithm 2.

Algorithm 2 MMSE estimate of the system state x_k.

$$[\hat{x}_k, P_k] = \mathrm{MAPX}\ [\hat{x}_{k-1}, P_{k-1}, \hat{H}_k, z_k]$$

Input: $\hat{x}_{k-1}$, P_{k-1}, $\hat{H}_k$, z_k.
Output: $\hat{x}_k$, P_k.

1: calculate $\hat{x}_{k|k-1}$ and $P_{k|k-1}$ by

$$\begin{aligned} \hat{x}_{k|k-1} &= A\hat{x}_{k-1}, \\ P_{k|k-1} &= AP_{k-1}A^{\mathrm{T}} + BQB^{\mathrm{T}}. \end{aligned}$$

2: **if** $\hat{h}_k = 0$, **then**
3: calculate $\hat{x}_k$ and P_k by

$$\begin{aligned} \hat{x}_k &= \hat{x}_{k|k-1} + P_{k|k-1}C^{\mathrm{T}}\Omega_k^{-1}(z_k - C\hat{x}_{k|k-1}), \\ P_k &= P_{k|k-1} - P_{k|k-1}C^{\mathrm{T}}\Omega_k^{-1}CP_{k|k-1}, \\ \Omega_k &= CP_{k|k-1}C^{\mathrm{T}} + DRD^{\mathrm{T}}. \end{aligned}$$

4: **else if** $\hat{h}_k = i$, for $i = 1, 2, \cdots, S$, **then**
5: calculate the parameters Θ_k, ω_{ik}^s, and θ_{ik}^s by

$$\begin{aligned} \Theta_k &= \left(P_{k|k-1}^{-1} + E^{\mathrm{T}}\Sigma_k^{-1}E\right)^{-1}, \\ \theta_{ik}^s &= \Theta_k\left(P_{k|k-1}^{-1}\hat{x}_{k|k-1} + E^{\mathrm{T}}\Sigma_k^{-1}\tilde{u}_{ik}^s\right), \\ \omega_{ik}^s &= |\mathcal{R}_i|\mathcal{N}\big(\tilde{u}_{ik}^s, E\hat{x}_{k|k-1}, EP_{k|k-1}E^{\mathrm{T}} + \Sigma_k\big). \end{aligned}$$

6: calculate $\hat{x}_k$ and P_k by

$$\begin{aligned} \hat{x}_k &= \sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N} \omega_{ik}^s} \theta_{ik}^s, \\ P_k &= \sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N} \omega_{ik}^s} \left(\Theta_k + (\hat{x}_k - \theta_{ik}^s)(\hat{x}_k - \theta_{ik}^s)^{\mathrm{T}}\right). \end{aligned}$$

7: **return** $\hat{x}_k$, P_k.

Up to now, the MAP estimation of the region-labels and the MMSE estimation of the target state have been presented in Algorithms 1-2. The region-label estimator is able to generate the MAP estimate of the region-label when the target plant is in the occlusion region, and the state estimator is capable of generating the MMSE estimate of the real-time system state. It should be further explained that, at instant k, to implement Algorithm 1, one requires the knowledge of $\hat{x}_{k-1}$ and P_{k-1} provided by Algorithm 2. Also, when implementing Algorithm 2, one requires the knowledge of $\hat{h}_k$ provided by Algorithm 1. In this sense, with the initial information $\hat{x}_0$ and P_0, the MMSE estimate of the system state shall be obtained using both Algorithms 1-2 in a *cross-coupled* manner as shown in Fig. 2.

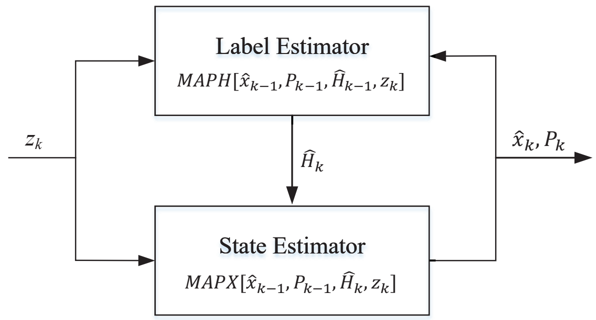

Fig. 2. Cross-coupled estimation algorithm

C. Boundedness analysis

In what follows, we aim to discuss the mean-square boundedness of the proposed cross-coupled estimator. For simplification, we consider the case of $E = I$. The following preliminary assumptions are made before proceeding.

Assumption 1: The pair (A, C) is detectable.

Assumption 2: There are positive real numbers $\bar{a}, \bar{t}, \underline{b}, \bar{b}, \underline{q}, \bar{q}, \underline{\chi}, \bar{\chi} > 0$ such that the following inequalities

$$\begin{aligned} \|A\| &\leq \bar{a}, \\ \|\Psi_k\| &\leq \bar{t}, \\ \underline{b} \leq \|B\| &\leq \bar{b}, \\ \underline{q} \leq \|Q\| &\leq \bar{q}, \\ \underline{\chi} \leq \|\Sigma_k\| &\leq \bar{\chi}, \end{aligned} \tag{22}$$

are satisfied for every $k > 0$.

Assumption 3: There are positive real numbers $\bar{r}, \bar{\zeta}, \bar{\varphi} > 0$ such that $r_i < \bar{r}$, $\|\zeta_i\| < \bar{\zeta}$, and $\|\zeta_i - \zeta_j\| < \bar{\varphi}$, $\forall i, j \in \{1, 2, \cdots, S\}$.

The inequalities (22) in Assumption 2 impose some constraints on the system parameters, and Assumption 3 supposes that the radius and the spatial locations of the occlusion region are upper bounded. These assumptions are in general quite mild for practical systems with numerically calculated parameters.

Theorem 1: Under Assumptions 1-3, if the inequality $\bar{a}\underline{b}^2\bar{\chi}\bar{q} < 1$ is satisfied, then the estimation error $e_k = x_k - \hat{x}_k$ is bounded in the mean-square sense, i.e.,

$$\sup_{\forall k} \mathbb{E}\{\|e_k\|^2\} < +\infty.$$

Proof: The proof of this theorem is divided into two situations. Firstly, we consider the situation where there exists

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an instant k_0 such that, for every instant $k > k_0$, the target would never enter the occlusion region. In other words, the measurements will be always received, and Algorithm 2 is thus degraded to the standard Kalman filtering algorithm. Since (A, C) is detectable and $BQB > 0$, it is straightforward to see that, given any initial state $\hat{x}_{k_0}$ and covariance P_{k_0}, the resultant covariance will always converge, and this proves $\sup_{\forall k} \mathbb{E}\{\|e_k\|^2\} < +\infty$.

The second situation considers that, for instant $k > k_0$, the target would re-enter the occlusion region after a finite interval k_1. The packet dropouts occur at instant $k = k_0 + k_1$, and the corresponding label estimate given by Algorithm 1 is $\hat{h}_k = i$. From (21), we have

$$e_k = x_k - \sum_{s=1}^{N} \frac{\omega_{ik}^s}{\sum_{s=1}^{N} \omega_{ik}^s} \theta_{ik}^s. \tag{23}$$

Denote

$$\alpha_{ik}^s = \frac{\omega_{ik}^s}{\sum_{s=1}^{N} \omega_{ik}^s}.$$

Then, one has $0 \leq \alpha_{ik}^s \leq 1$ and $\sum_{s=1}^{N} \alpha_{ik}^s = 1$. Inserting (15) into (23) yields

$$\begin{aligned} e_k &= x_k - \sum_{s=1}^{N} \alpha_{ik}^s \Theta_k \left(P_{k|k-1}^{-1} \hat{x}_{k|k-1} + \Sigma_k^{-1} \tilde{u}_{ik}^s \right) \\ &= x_k - \Theta_k P_{k|k-1}^{-1} \hat{x}_{k|k-1} - \sum_{s=1}^{N} \alpha_{ik}^s \Theta_k \Sigma_k^{-1} \tilde{u}_{ik}^s. \end{aligned} \tag{24}$$

Utilizing (11) and recalling

$$\Theta_k^{-1} = P_{k|k-1}^{-1} + \Sigma_k^{-1},$$

it is not difficult to establish

$$\Theta_k P_{k|k-1}^{-1} \hat{x}_{k|k-1} = (I - \Theta_k \Sigma_k^{-1}) A \hat{x}_{k-1|k-1}.$$

Substituting the above equality into (24) leads to

$$\begin{aligned} e_k = \Theta_k P_{k|k-1}^{-1} A e_{k-1} - \sum_{s=1}^{N} \alpha_{ik}^s \Theta_k \Sigma_k^{-1} \tilde{u}_{ik}^s \\ + \Theta_k \Sigma_k^{-1} x_k + \Theta_k P_{k|k-1}^{-1} B w_{k-1}. \end{aligned}$$

Adding the zero terms

$$\Theta_k \Sigma_k^{-1} A(n_k - \zeta_i) - \Theta_k \Sigma_k^{-1} A(n_k - \zeta_i)$$

and

$$\sum_{s=1}^{N} \alpha_{ik}^s \Theta_k \Sigma_k^{-1} \zeta_i - \sum_{s=1}^{N} \alpha_{ik}^s \Theta_k \Sigma_k^{-1} \zeta_i$$

to both sides of the above equation results in

$$\begin{aligned} e_k = \Theta_k P_{k|k-1}^{-1} A e_{k-1} - \sum_{s=1}^{N} \alpha_{ik}^s \Theta_k \Sigma_k^{-1} (\tilde{u}_{ik}^s - \zeta_i) \\ + \Theta_k \Sigma_k^{-1} (x_k + n_k - \zeta_i) + \Theta_k P_{k|k-1}^{-1} B w_{k-1} \\ - \Theta_k \Sigma_k^{-1} (n_k - \zeta_i) - \Theta_k \Sigma_k^{-1} \zeta_i. \end{aligned}$$

Taking norms on both sides of the above equation yields that

$$\begin{aligned} \|e_k\| \leq \|\Theta_k\| \|P_{k|k-1}^{-1}\| \left(\|A\| \|e_{k-1}\| + \|B\| \|w_{k-1}\| \right) \\ + \|\Theta_k\| \|\Sigma_k^{-1}\| \left(\|x_k + n_k - \zeta_i\| + \|n_k\| + 2\|\zeta_i\| \right) \\ + \sum_{s=1}^{N} \alpha_{ik}^s \|\Theta_k\| \|\Sigma_k^{-1}\| \|\tilde{u}_{ik}^s - \zeta_i\|. \end{aligned} \tag{25}$$

From the fact $BQB^{\mathrm{T}} > 0$, it is readily obtained that

$$P_{k-1|k} = A P_{k-1|k-1} A^{\mathrm{T}} + BQB^{\mathrm{T}} > 0.$$

As such, one has

$$0 < \|P_{k-1|k}^{-1}\| < \underline{b}^2 \underline{q}.$$

Moreover, we have

$$\Theta_k = \left(P_{k|k-1}^{-1} + \Sigma_k^{-1} \right)^{-1} \leq \Sigma_k.$$

Since the remote estimator does not receive the transmitted signals, one knows that the target state is in certain occlusion region. Although the label estimate given by Algorithm 1 is $\hat{h}_k = i$, the actual target state could be in the jth region (i.e., $u_k \in \mathcal{R}_j$) accounting for the possible estimation error, and this indicates that

$$\begin{aligned} \|x_k + n_k - \zeta_i\|^2 &\leq \|x_k + n_k - \zeta_j\|^2 + \|\zeta_j - \zeta_i\|^2 \\ &\quad + 2\|x_k + n_k - \zeta_j\| \|\zeta_j - \zeta_i\| \\ &\leq (\bar{r} + \bar{\varphi})^2 \end{aligned}$$

for all $i \in \mathbb{Z}_{[0,S]}$. Moreover, since $\tilde{u}_{ik}^s$ are chosen by equidistantly sampling the region $\mathcal{R}_i$, it is direct to see that $\tilde{u}_{ik}^s \in \mathcal{R}_i$, and therefore we have

$$\|\tilde{u}_{ik}^s - \zeta_i\| < r_i.$$

Taking expectation on both sides of (25) yields

$$\begin{aligned} \mathbb{E}\{\|e_k\|\} \leq \bar{\chi} \underline{b}^2 \bar{q} \Big(\bar{a} \mathbb{E}\{\|e_{k-1}\|\} \\ + \bar{b} \sqrt{n_w \bar{q}} \Big) + \bar{\chi} \underline{\chi}^{-1} \Big(4\bar{r} + \bar{\varphi} + \sqrt{n_x \bar{t}} \Big) \end{aligned}$$

where we have used the facts that $\mathbb{E}\{w_{k-1}^{\mathrm{T}} w_{k-1}\} = \mathrm{tr}(Q) \leq n_w \bar{q}$ and $\mathbb{E}\{n_k^{\mathrm{T}} n_k\} = \mathrm{tr}(\Psi_k) \leq n_x \bar{t}$. Since k_1 is a finite positive number, during the interval $[k_0, k_1]$ when the observation is available, it follows from the properties of the Kalman filter that $\|e_{k-1}\|$ should have an upper bound. Moreover, notice $\bar{\chi} \underline{b}^2 \bar{q} \bar{a} < 1$ and

$$\begin{aligned} \mathbb{E}\{\|e_{k+d}\|\} \leq \Big(\bar{a} \underline{b}^2 \bar{\chi} \bar{q} \Big)^{d+1} \mathbb{E}\{\|e_{k-1}\|\} \\ + \bar{a} \underline{b}^2 \bar{\chi} \bar{q} (1 - \epsilon^{d+1}) / (1 - \epsilon) \end{aligned}$$

where $\epsilon = \bar{\chi} \underline{b}^2 \bar{b} \bar{q} \sqrt{n_w \bar{q}} + \bar{\chi} \underline{\chi}^{-1} \Big(4\bar{r} + \bar{\varphi} + \sqrt{n_x \bar{t}} \Big)$. Then, we see that for any bounded initial condition $\mathbb{E}\{\|e_{k-1}\|\}$ (as $d \to \infty$), even if the target state always stays in the occlusion region, $\mathbb{E}\{\|e_{k+d}\|\}$ will finally converge to $\bar{a} \underline{b}^2 \bar{\chi} \bar{q} / (1 - \epsilon)$. From the above discussions, we have $\sup_{\forall k} \mathbb{E}\{\|e_k\|\} < +\infty$, which completes the proof. ■

As has been discussed, Assumptions 1-3 natually hold for many practical systems, and therefore, the mean-square boundedness of the estimation error can be easily verified by checking the condition $\bar{a}\underline{b}^2\bar{\chi}\bar{q} < 1$.

Remark 1: This paper investigates the remote state estimation problem subject to SDPDs. A cross-coupled estimation algorithm is proposed, which is composed of a region-label estimator and a system state estimator. Although a linear MMSE estimator has been initially designed in [37] for the same problem, such an estimator is not optimal due to the fact that the state estimation problem with SDPDs is essentially a nonlinear filtering problem. In particular, when the state keeps fluctuating over the occlusion region, the linear MMSE estimator in [37] might diverge as the update terms in the Riccati equation would be very small. Different from the linear MMSE estimation scheme in [37], the Bayesian inference approach has been utilized to solve the concerned nonlinear filtering where the region information has been exploited to estimate the target state according to (19), which could constrain the state in a desired interval. Moreover, a sufficient condition is established to guarantee the mean-square boundedness of the resultant estimation error.

IV. A NUMERICAL EXAMPLE

In this section, a numerical example is presented to verify the effectiveness of the proposed cross-coupled estimation algorithm for systems with SDPDs.

The state propagation of the target plant is given by (1) with transition matrix

$$A = \begin{bmatrix} 1.01 & 0.1 & 0 \\ 0 & 1.01 & 0.1 \\ 0 & 0 & -1.02 \end{bmatrix}$$

and measurement matrix

$$C = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 0.98 \end{bmatrix}.$$

The covariances of the process and measurement noises w_k and v_k are assumed to be $Q = 0.3I_3$ and $R = 0.2I_2$, respectively. The initial value of the state x_0 obeys a Gaussian distribution with mean $\mu_0 = \begin{bmatrix} 5 & 5 & 5 \end{bmatrix}^{\mathrm{T}}$ and covariance $\Sigma_0 = 4I_3$.

Let

$$x_k = \begin{bmatrix} x_{1,k} & x_{2,k} & x_{3,k} \end{bmatrix}^{\mathrm{T}}$$

with $x_{i,k}$ being the ith component of state vector x_k. Suppose that the packet dropout occurs when the first component of the state vector enters the occlusion region, i.e., $x_{1,k} \in \mathcal{R}^o_{i,k}$. The occlusion region $\mathcal{R}^o_{i,k}$ is given as

$$\mathcal{R}^o_{i,k} = \{x \in \mathbb{R} : \|x - \zeta^o_{i,k}\|^2 \leq r_i^2\}$$

where $\zeta^o_{i,k} = \zeta_i - n_k$. The expected center and radius of these regions are given by $\zeta_i = 20i$, for $i \in \{1, 2, 3, 4\}$ and $r_i = 5$. $n_k \in \mathbb{R}$ is a white Gaussian noise sequence with zero mean and covariance $\Psi_k = 0.1$. It is obvious that matrix E in (3) is $E = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ and $x_{1,k} \in \mathcal{R}^o_{i,k}$ is equivalent to $u_k \in \mathcal{R}_i$ with

$$\mathcal{R}_i = \{x \in \mathbb{R} : \|x - \zeta_i\|^2 \leq r_i^2\}.$$

TABLE I
THE KULLBACK-LEIBLER DIVERGENCE

σ_i^2	0.10	0.21	0.31	0.50	0.80
$\mathcal{D}(\cdot,\cdot)$	83.4136	10.3434	7.1173	8.1292	10.5365

In this case, we have

$$\Lambda_{\mathcal{R}_i}(u_k) = \frac{1}{2r_i}\mathbb{I}_{\{\zeta_i - r_i \leq u_k \leq \zeta_i + r_i\}}.$$

According to [33], the above uniform density function can be approximated by Gaussian sums

$$\Lambda_{\mathcal{R}_i}(u_k) \approx \frac{1}{N}\sum_{s=1}^{N}\mathcal{N}(u_k, \tilde{u}^s_{ik}, V^s_{ik}).$$

The number of Gaussian distributions are chosen to be $N = 15$, the mean value of each Gaussian distribution is chosen to be $\tilde{u}^s_{ik} \forall j \in \mathbb{Z}_{[1,N]}$ such that the densities are equally spaced on $[\zeta_i - r_i, \zeta_i + r_i]$, and the variance of each Gaussian distribution is set to be the same (i.e., $V^s_{ik} = \sigma_i^2, \forall j \in \mathbb{Z}_{[1,N]}$). The satisfactory mean and variance can be acquired by solving the following Kullback-Leibler divergence:

$$\min_{\sigma_i} \mathcal{D}\left(\Lambda_{\mathcal{R}_i}(x), \frac{1}{N}\sum_{s=1}^{N}\mathcal{N}(x, \tilde{u}^s_{ik}, \sigma_i)\right).$$

The Kullback-Leibler divergence of these two distributions under different variances are listed in Table I. Via numerical simulations, it is not difficult to find that the best variance is $\sigma_i^2 = 0.31$. The actual uniform density function $\Lambda_{\mathcal{R}_1}(x)$ and its Gaussian sum approximation $\frac{1}{N}\sum_{s=1}^{N}\mathcal{N}(x, \hat{u}^s_{1k}, \sigma_1^2)$ are presented in Fig. 3. The results show that the Gaussian sums approximate the uniform density function quite well, and if the number of Gaussian distributions is increased, the approximation accuracy will be further improved.

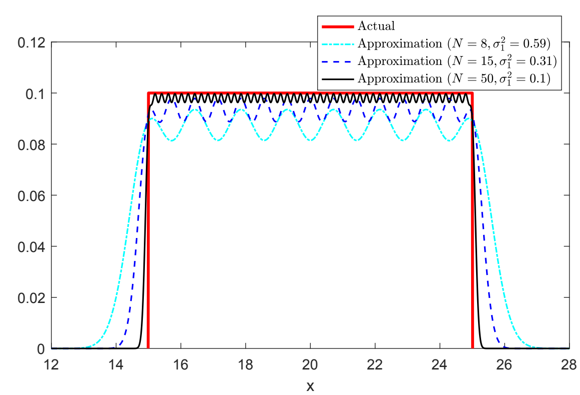

Fig. 3. The uniform density function $\Lambda_{\mathcal{R}_1}(x)$ and its Gaussian sum approximation $\frac{1}{N}\sum_{s=1}^{N}\mathcal{N}(x, \hat{u}^s_{1k}, \sigma_1^2)$ with different parameters.

The one-trial simulation results of the proposed algorithm are presented in Figs. 4-6. Fig. 4 shows that the true trajectories of the target plant (red line) and the estimated trajectories (blue line), and Fig. 5 plots the trajectories of variables u_k as well as regions $\mathcal{R}_i$. The labels h_k (that characterize the occlusion regions u_k) and their corresponding estimates $\hat{h}_k$ are

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given in Fig. 6. These results demonstrate that the proposed cross-coupled estimation algorithm can estimate the occlusion regions and track the system trajectories well.

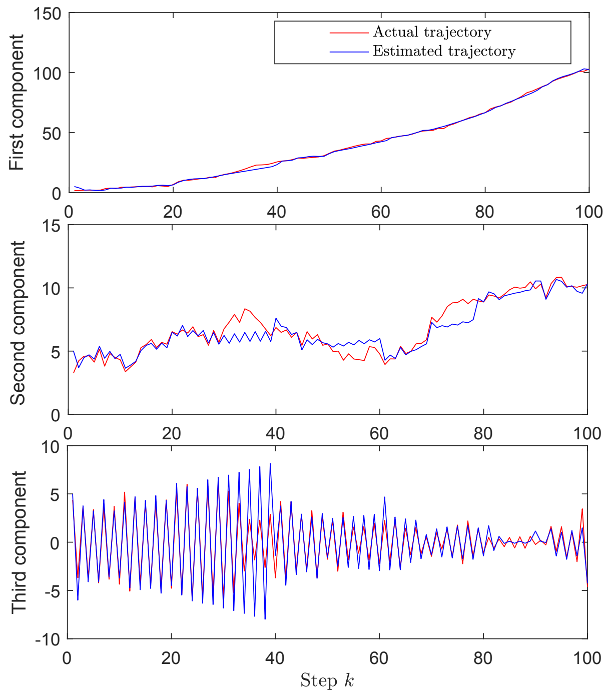

Fig. 4. The true and estimated trajectories of the target plant, x_k and $\hat{x}_k$

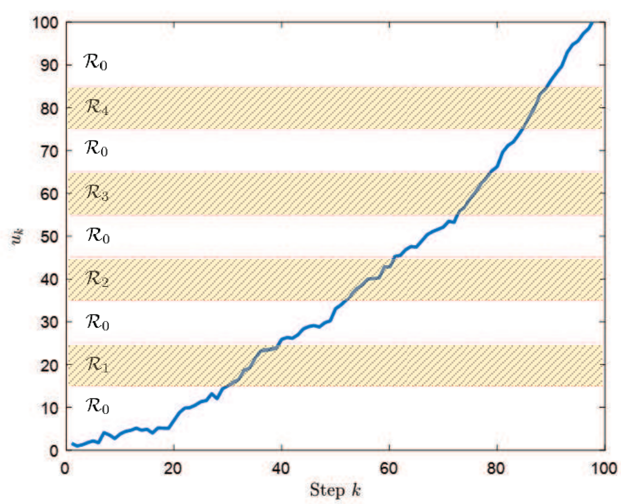

Fig. 5. The trajectories of variable u_k and regions $\mathcal{R}_i$.

Next, we compare the performance of the proposed cross-coupled estimator with that of the intermittent Kalman filter [32], and the linear MMSE estimator [37] (abbreviated as CCE, IKF, and KF-IO, respectively henceforth). To make the IKF applicable to the problem under consideration, the packet arrival sequence is estimated utilizing the label estimator given in Algorithm 1. When a packet arrives, the Kalman filter is adopted, otherwise, the prediction is used. The mean-square error (MSE), which reflects the estimation accuracy, is utilized to evaluate the performance of these estimation algorithms. Since the actual MSE of the proposed algorithm versus time cannot be analytically calculated, the empirical value is obtained through $N_{MC} = 5000$ independent repeated experiments as follows:

$$\mathrm{MSE}(k) = \frac{1}{N_{MC}} \sum_{n=1}^{N_{MC}} (x_{n,k} - \hat{x}_{n,k})^{\mathrm{T}} (x_{n,k} - \hat{x}_{n,k})$$

where $x_{n,k}$ and $\hat{x}_{n,k}$ are the actual and estimated values of x_k in the nth run, respectively. The MSEs of the CCE, IKF, and KF-IO are plotted in Fig. 7, from which it can be seen that the CCE is more effective than the IKF and KF-IO to handle state estimation problems with SDPDs. This is because although we equip the IKF with the label estimator, the proposed CCE further considers the information of occlusion regions, and thus is capable of compensating the estimation performance even when the packet is completely missing. Moreover, KF-IO in fact is a robust linear optimal estimation based on the statistics of the packet dropouts, and thus inevitably induces uncertain terms with respect to the state into the evolution of error covariance, which would lead to divergence for unstable systems.

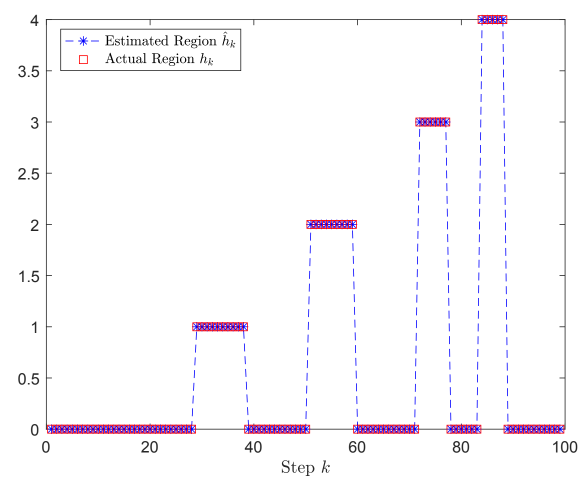

Fig. 6. The labels h_k that characterize the regions u_k belonging to and their corresponding estimates $\hat{h}_k$.

V. CONCLUSION

In this paper, the recursive Bayesian filtering problems have been investigated for linear discrete-time systems subject to packet dropouts. Unlike the widely adopted packet dropouts whose occurrence is described by the Bernoulli or Markov process, in this paper, the sate-dependent packet dropout has been considered to cover the case where the target state enters certain occlusion regions. For the sake of obtaining the MMSE estimate of the state vector, a cross-coupled estimation algorithm has been proposed which is composed of a region-label estimator and a state estimator. Based on the Gaussian sum approximations, the MAP estimate of the region-label has been first obtained by using the observation information

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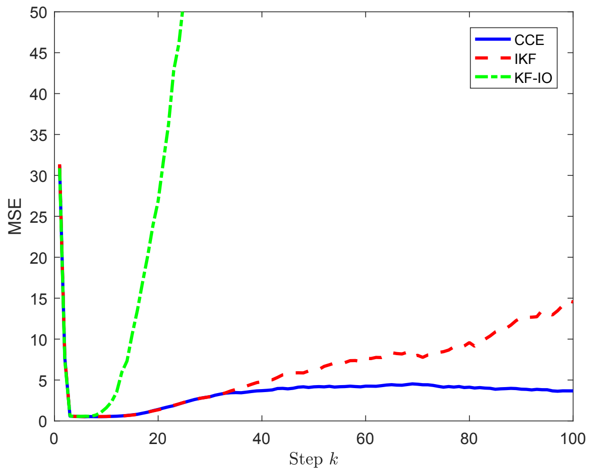

Fig. 7. The respective MSEs of CCE, IKF and KF-IO.

as well as the previous state estimate, and this has supplied the state estimator with region information to generate an MMSE estimate of the system state. Furthermore, we have established a sufficient condition for the mean-square boundedness of the estimation error dynamics. The effectiveness of the proposed estimation algorithm has been demonstrated via a numerical example.

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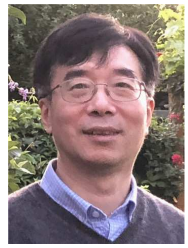

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