

## Journal Pre-proofs

A modelling workflow for quantification of photobioreactor performance

Wenjia Gu, Emile Theau, Amos W. Anderson, David F. Fletcher, John M. Kavanagh, Dale D. McClure

PII: S1385-8947(23)05763-7  
DOI: <https://doi.org/10.1016/j.cej.2023.147032>  
Reference: CEJ 147032

To appear in: *Chemical Engineering Journal*

Received Date: 23 August 2023  
Revised Date: 7 October 2023  
Accepted Date: 28 October 2023

Please cite this article as: W. Gu, E. Theau, A.W. Anderson, D.F. Fletcher, J.M. Kavanagh, D.D. McClure, A modelling workflow for quantification of photobioreactor performance, *Chemical Engineering Journal* (2023), doi: <https://doi.org/10.1016/j.cej.2023.147032>

This is a PDF file of an article that has undergone enhancements after acceptance, such as the addition of a cover page and metadata, and formatting for readability, but it is not yet the definitive version of record. This version will undergo additional copyediting, typesetting and review before it is published in its final form, but we are providing this version to give early visibility of the article. Please note that, during the production process, errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.

© 2023 Published by Elsevier B.V.



# 1 *A modelling workflow for quantification of photobioreactor performance*

2 Wenjia Gu<sup>a</sup>, Emile Theau<sup>a</sup>, Amos W. Anderson<sup>b</sup>, David F. Fletcher<sup>a</sup>, John M. Kavanagh<sup>a</sup> and  
3 Dale D. McClure<sup>a,c1</sup>

4 <sup>a</sup> School of Chemical and Biomolecular Engineering, The University of Sydney, 2006, Australia

5 <sup>b</sup> School of Life and Environmental Sciences, The University of Sydney, Sydney, NSW 2006,  
6 Australia

7 <sup>c</sup> Department of Chemical Engineering, College of Engineering, Design and Physical Sciences,  
8 Brunel University, London, Uxbridge, UB8 3PH, UK.

9

## 10 **Abstract**

11 In this work we have developed a comprehensive modelling workflow for the quantification of  
12 photobioreactor performance. Computational Fluid Dynamics (CFD) modelling combined with  
13 Lagrangian particle tracking was used to characterise the flow field inside the reactor; this  
14 information was combined with a Monte-Carlo model of light attenuation and a kinetic growth  
15 model to predict the performance of the system over the duration of the entire batch. The CFD  
16 model was validated against measurements of the overall hold-up, local hold-up and mixing time  
17 for superficial velocities between 0.6 and 6 cm s<sup>-1</sup> in a pilot-scale bubble column photobioreactor,  
18 with the CFD predictions agreeing with the experimental data. Comparison was also made between  
19 the predicted biomass concentration and experimental measurements using the diatom  
20 *Phaeodactylum tricornutum*, with the model predictions being in good agreement with the  
21 experimental results. The model was used to investigate a range of operating conditions and reactor  
22 designs, with the most promising predicted to give a 40% increase in the biomass productivity.  
23 Results from this work can be used for the *in-silico* design and optimisation of photobioreactor  
24 systems, thereby enabling their wider use as a sustainable production technology.

## 25 **Keywords**

- 26 • Photobioreactor
- 27 • Scale-up
- 28 • CFD
- 29 • Microalgae
- 30 • Particle tracking

---

<sup>1</sup> Corresponding author: Dr Dale McClure (dale.mcclure@brunel.ac.uk) Department of Chemical Engineering, College of Engineering, Design and Physical Sciences, Brunel University, London, Uxbridge, UB8 3PH, UK.

## 31 1. Introduction

32 Photoautotrophic microorganisms (e.g., microalgae and cyanobacteria) can be used for the  
33 sustainable production of a range of compounds including high-value products for the food  
34 industry (e.g., carotenoid pigments, omega-3 fatty acids, vitamins) [1], biofuels [2] and chemicals  
35 [3]. A major advantage of using photoautotrophic microorganisms is their minimal nutrient  
36 requirements; light and carbon dioxide are the primary feedstocks. This may be advantageous from  
37 a sustainability perspective as there is no need for arable land, potable water and also no  
38 competition with food crops [4]. A major challenge in the commercialization of bioprocesses using  
39 photoautotrophic microorganisms is the process economics [1]. The majority of existing processes  
40 utilize open photobioreactors (e.g., ponds and raceways), which have the advantage of lower  
41 capital costs than closed systems (e.g., flat-panel, bubble column and tubular photobioreactors)  
42 [1]. However, open photobioreactors are susceptible to contamination and may not be appropriate  
43 for all organisms (e.g., it may not be suitable to grow engineered organisms in an open system).

44 Closed photobioreactors can generally achieve higher cell densities and biomass productivities  
45 than open systems, and are less susceptible to contamination [5]. The major challenge with such  
46 reactors is efficiently using the available light to achieve high cell densities. As the distance from  
47 the illuminated surface of the Photo Bio Reactor (PBR) increases the light intensity decreases in  
48 an exponential fashion due to absorption and scattering of light by the cells [6]. This can lead to a  
49 situation where the central volume of the reactor is essentially 'dark', with the cells located in this  
50 volume receiving insufficient light for photosynthesis. Under conditions of high illumination (e.g.,  
51 near the walls of the reactor) the capacity of the electron transport chain (the biochemical pathway  
52 involved in absorbing energy from light) can become saturated, which can lead to the production  
53 of reactive oxygen species and in turn damage to the cells [7]. In practice, cells will alternate  
54 between zones of high and low light intensity due to transport by the liquid. The extent to which  
55 this occurs will depend on the reactor design and operating conditions [8, 9]. By optimizing the  
56 frequency at which cells move between zones of light and dark it may be possible to improve the  
57 utilization of the incident light (the flashing light effect). Numerous authors [10-15] have examined  
58 this problem, reporting that light-dark frequencies of the order 10-100 Hz are needed to make  
59 maximum use of the flashing light effect [11, 12, 16], while some benefits may be obtained at  
60 frequencies of the order 0.08 Hz [17]. Understanding the light intensity experienced by cells within  
61 a PBR, the light-dark cycle frequency and how these variables are affected by the reactor design  
62 and operating conditions is key in PBR design and optimization.

63 As previously noted, a major challenge in PBR design is making effective use of the supplied light  
64 to achieve high cell densities. One way in which this can be achieved is to simply reduce the  
65 thickness of the reactor, thereby reducing the optical path length and hence the amount of light  
66 attenuation. However, this has the significant drawback of requiring a higher surface area for the  
67 same liquid volume, thereby increasing the capital cost. Another potentially promising direction is  
68 the use of internal structures (e.g., baffles, static mixers, etc.) which promote mixing within a PBR  
69 and hence potentially increase the light-dark cycle frequency. It is hypothesized that by promoting  
70 mixing between the central 'dark' region of the PBR and the illuminated wall region cells will  
71 experience higher light intensities and this in turn will lead to higher cell densities. Recent work  
72 has found this to be the case, for example installation of baffles was found to increase the biomass  
73 productivity of *Chlorella* cultures by 60-90% [18-22]. Similarly, Ryu et al. demonstrated that the  
74 use of horizontal sieve baffles and slanted baffles led to approximately 40% increases in the

75 biomass concentration of *Chlorella* sp. in 4 cm diameter cylindrical bubble column PBRs [23].  
76 Merchuk et al. showed that the installation of helical flow promoter in cylindrical PBRs lowered  
77 the air flow rate required to achieve the maximum cell density of *Porphyridium* sp. cultures and  
78 thereby reduced the energy expenditure in air compression [24]. Such results demonstrate the  
79 potential of modified reactor designs, while also highlighting the need for tools to better understand  
80 the hydrodynamics of PBRs in order to facilitate the development of optimised reactor designs.

81 Computational Fluid Dynamics (CFD) models are increasingly [25] being used as tools to model  
82 bioprocesses, including photobioreactors [26-30]. In the case of photobioreactors Lagrangian  
83 particle tracking is a particularly useful approach [31]. Particles having the same density and size  
84 as cells are included in the model which tracks their position in the reactor as a function of time.  
85 Using this information it is possible to construct the 'history' or 'lifeline' of a given cell as it moves  
86 throughout the reactor. As the light intensity experienced by a cell is largely a function of its  
87 location it is possible to determine the light intensity experienced by a cell as a function of time  
88 by coupling the particle tracking approach with a model of light attenuation. From this it is possible  
89 to then determine key information like the light-dark cycle frequency and the average light  
90 intensity. By including a large number of cells in the model it is also possible to calculate  
91 population-averaged values. Such an approach based on Lagrangian particle tracking has the  
92 advantage that it is likely to be the most representative of the behavior occurring within the  
93 photobioreactor, and as such may offer the most accurate way of quantifying reactor performance.

94 CFD models have the advantages of providing a high degree of spatial and temporal resolution,  
95 and allowing for multiple reactor configurations to be evaluated *in silico*, thereby minimizing the  
96 need for experimental work. Another key advantage is that models can be used to generate  
97 information which is very difficult to obtain experimentally (e.g., simulating the movement of cells  
98 throughout a PBR). A disadvantage of CFD models is that their high computational demand means  
99 that it is only feasible to simulate a relatively short length of time (typically hundreds of seconds),  
100 while a typical growth cycle in a PBR would last several days/weeks. Hence, it not possible to use  
101 CFD to simulate an algal cultivation from start to finish. To circumvent this limitation, results from  
102 simulations of the fluid dynamics can be combined with light and growth models to give an overall  
103 model which can predict the overall process performance [32, 33]. Such models can be used to  
104 understand the reactor performance and develop optimized designs; however, relatively little work  
105 has been done in this area looking at bubble column photobioreactors.

106 Use of a CFD model to quantify the performance of PBRs obviously relies on having accurate  
107 predictions of the hydrodynamics in order to correctly predict the trajectory of the cells. Hence,  
108 before any CFD model can be used to quantify the performance of the PBR it should be validated  
109 against experimental data to ensure it offers accurate predictions of the hydrodynamics. Our  
110 previous work [34, 35] has focused largely on the modelling of large-scale aerobic bioprocesses,  
111 which typically operate at higher superficial velocities ( $> 0.1 \text{ m s}^{-1}$ ) than PBRs (which typically  
112 operate at superficial velocities below  $0.05 \text{ m s}^{-1}$ ). Therefore, before the model can be used as a  
113 tool to quantify PBR performance there is a need to validate its predictions at conditions found in  
114 PBRs.

115 The aim of this work is to develop and validate a modelling workflow which can be used to  
116 quantify the effect of PBR design and operating conditions on performance, and to use such a  
117 model to identify improved reactor designs. To do this it is firstly necessary to validate the CFD

118 methodology used, and secondly to integrate the CFD model with models of light attenuation and  
 119 growth and determine whether the combined workflow provides accurate predictions of algal  
 120 growth. Once the modelling workflow has been validated it can then be used as a tool to examine  
 121 different designs with the aim of identifying those which will provide improved performance.

## 122 2. Method

### 123 2.1 Experimental measurements

124 Experimental measurements used to validate the CFD model were performed using the bubble  
 125 column configuration without any internals. The bubble column used in this work was 190 mm in  
 126 diameter, 2000 mm in height and it was fabricated from clear acrylic. Air was introduced through  
 127 an L-shaped stainless-steel perforated tube sparger. The sparger had three rows of  $10 \times 2$  mm  
 128 diameter holes. There was a 10 mm spacing between hole centres. A detailed schematic of the  
 129 sparger is shown in Figure 1.

130 Compressed air was sourced from the building supply. The flow rate was measured using a RM  
 131 series rotameter (Dwyer) and corrected to the flow rate at standard conditions (298 K and  
 132 101325 Pa) using measurements of the pressure at the rotameter outlet (typically 14-17 kPa as  
 133 measured using a Dwyer LPG3 series pressure gauge). Volumetric flow rates were converted to  
 134 the superficial velocity by dividing the flow rate at standard conditions by the cross-sectional area  
 135 of the column.

136 Measurements of the liquid height ( $H_L$ ) and the height of the two-phase mixture ( $H_{G+L}$ ) were  
 137 made using a ruler attached to the side of the column and these values were used to calculate the  
 138 overall hold-up ( $\alpha$ ):

$$\alpha = 1 - \frac{H_L}{H_{G+L}} \quad (1)$$

139 Three measurements of the hold-up were made at each superficial velocity. Reported results are  
 140 the average of these three measurements. Error bars denote one standard deviation about the mean  
 141 or the error as calculated using error propagation methodology, whichever was larger.

142 The bubble size distribution (BSD) was measured using two-point needle probes, details of the  
 143 probe design are described in detail elsewhere [36]. The measured chord-length distribution was  
 144 converted to the BSD using the non-parametric transform developed by Liu et al. [37], here it was  
 145 assumed that the bubbles were ellipsoidal in shape with a fixed aspect ratio of 0.6. Probes were  
 146 positioned on the column centerline, facing down at heights of 800 and 1200 mm above the base  
 147 of the column. Measurements of the BSD were made at superficial velocities of 0.6, 1.6, 3.2 and  
 148  $6.0 \text{ cm s}^{-1}$ . Three measurements each 180 s in duration were made at each condition, these data  
 149 were combined for analysis. The error in the reported mean bubble sizes is of the order  $\pm 20\%$ , as  
 150 determined in our previous work [36].

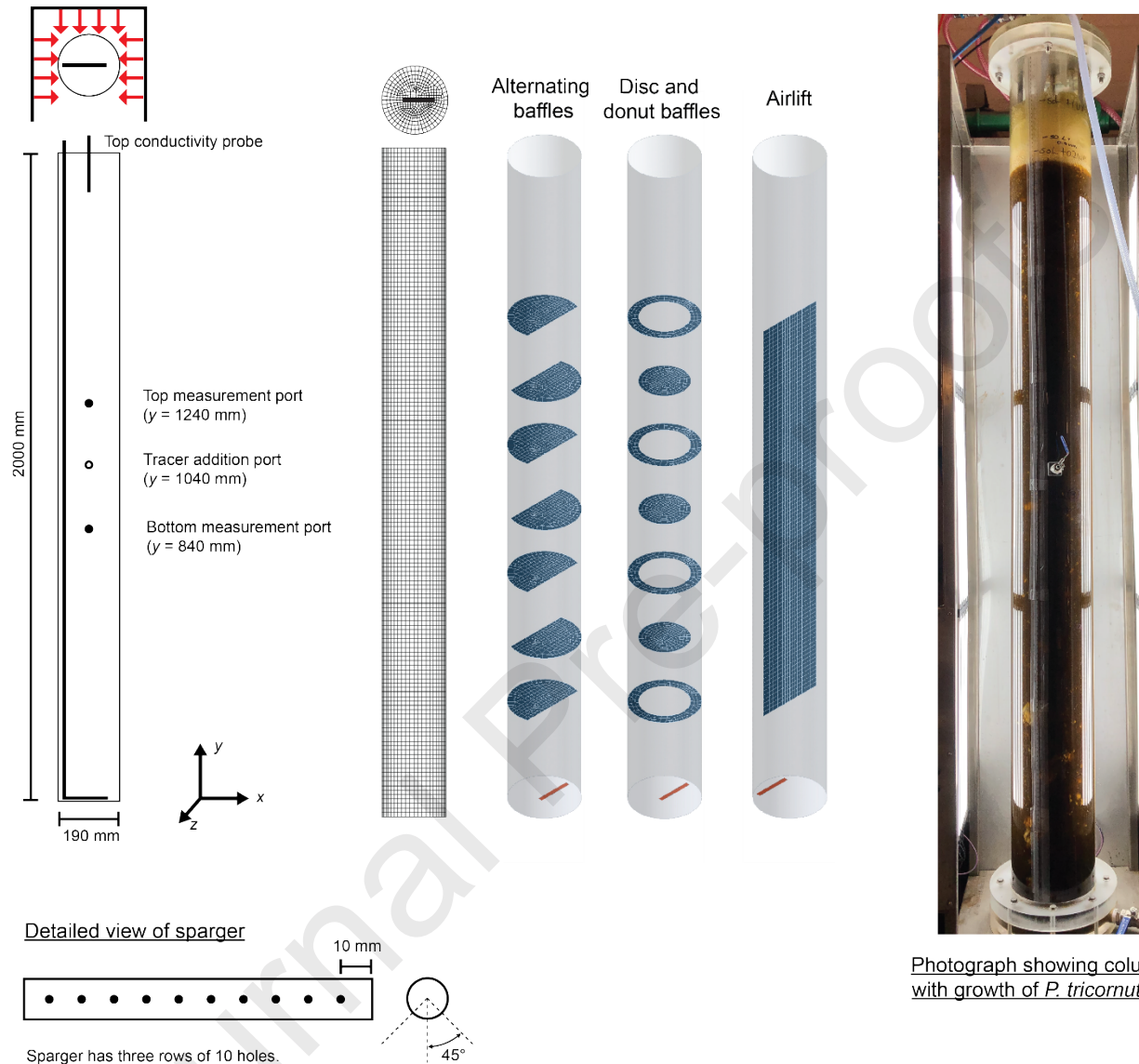
151 Local hold-up measurements were made using single-point needle probes as detailed elsewhere  
152 [36]. Probes were located at heights of 840 and 1240 mm above the base of the column and at  
153 radial locations of 0,  $\pm 30$ ,  $\pm 60$  and 90 mm. Measurements made at a location of  $-90$  mm generally  
154 did not result in a signal, most likely due to the small (5 mm) gap between the column wall and  
155 the probe tip (meaning no bubbles were able to pass through the gap), similar behavior being  
156 observed in our previous work [38]. The local volume fraction was measured for  $5 \times 30$  s at each  
157 point, with reported values being the average, error bars denote one standard deviation about the  
158 mean. It was found that area-averaging the local hold-up profiles gave overall hold-up values less  
159 than the experimentally measured overall hold-up value, suggesting that some bubbles were  
160 'missed' by the probes. To correct for this the measured values were multiplied by the ratio  
161 between the overall hold-up and the area-averaged local hold-up, this being 1.35 for measurements  
162 at a height of 1240 mm above the base of the column and 1.65 for measurements made at a height  
163 of 840 mm above the base of the column. The same correction factor was used for all superficial  
164 velocities examined.

165 Mixing in the column was quantified by measuring the mixing time. This was measured by adding  
166 a salt tracer (4 M NaCl) and measuring the conductivity as a function of time. Conductivity probes  
167 (Real Time Instruments) were positioned at heights of 840, 1240 and 1700 mm above the base of  
168 the column, as shown in Figure 1. The probes located in the middle of the column were positioned  
169 at the centerline. To quantify the effect of the tracer addition location it was added both to the top  
170 of the column (by pouring on to the free surface) or by injecting it into a port 1040 mm above the  
171 base of the column. A volume of 130-150 mL of tracer was used. All measurements were made in  
172 triplicate, the reported values are the average with error bars denoting one standard deviation about  
173 the mean. This approach was used in order to quantify the variability in the mixing time caused by  
174 the inherently transient nature of flow inside bubble columns. After three tracer additions the  
175 column was drained and refilled to minimize the effect of salt addition on the hydrodynamics. The  
176 mixing time was defined as the time required for the tracer concentration to settle within  $\pm 5\%$  of  
177 the final equilibrium value. Further details about the methodology used are presented elsewhere  
178 [39].



### Top view of column

The column is illuminated from three sides (indicated by red arrows)



Photograph showing column with growth of *P. tricornutum*

179

The middle row faces downwards. The centre of the other two rows is  $\pm 45^\circ$  from the centreline of the middle row. The rows have 10 mm spacing between hole centres.

180

**Figure 1 – Schematic of bubble column photo-bioreactor, as well as the mesh used in the CFD modelling.**

181

### 2.2 CFD Modelling

182

In this work we have applied a computational approach we developed previously and validated for bubble columns [38, 40]. In this work the Euler-Euler approach is used to model the two-phase flow. Inter-phase momentum transfer is modelled as the sum of drag and turbulent dispersion. The drag force was calculated using the Grace et al. model for an isolated bubble [41], combined with a volume fraction correction term based on our previous work [42], values of the constants  $n$  and  $b$  were 50 and 0.20, respectively. Bubbles had a fixed size (8 mm), this being the experimentally

183

184

185

186

187

188 measured mean value (see Supplementary Figure S6). Turbulent dispersion was modelled using  
189 the Favre-averaged drag approach outlined by Burns et al. [43]. Liquid phase turbulence was  
190 modelled using the standard  $k$ - $\epsilon$  approach as implemented in Ansys CFX, with the source terms  
191 developed by Yao and Morel [44] being included to account for bubble-induced turbulence. Gas-  
192 phase turbulence was modelled using the dispersed phase zero approach. This approach was used  
193 as it has been shown to provide good agreement with experimental data across a broad range of  
194 column designs and operating conditions [34, 40].

195 A schematic of the mesh used is shown in Figure 1, a hexahedral mesh with 58,800 elements was  
196 used. To ensure the model predictions were independent of the grid size simulations were also  
197 performed with coarse (36,120 elements) and fine (132,480 elements) meshes. As shown in  
198 Supplementary Figure S1 it was found that the results did not depend on the grid size used.

199 The sparger was modelled as an inlet boundary condition on the bottom face of the column  
200 12.5 mm in width and 90 mm in length. The top of the column was modelled as an outlet at  
201 atmospheric pressure, the remaining surfaces, including the column walls and any internals, were  
202 modelled as walls using the no-slip condition for the liquid and free-slip for the gas. Baffles were  
203 spaced 200 mm apart, with the bottom baffle being 300 mm from the base of the column. The  
204 alternating baffles were 125 mm wide, the disc shaped baffles had a diameter of 134 mm, while  
205 the cut-out in the donut baffles was also 134 mm. The baffle for the airlift was located on the  
206 column centerline 300 mm above the base of the column, the height of the baffle was 1200 mm.

207 An initial liquid height of 1.70 m was used, below this height the liquid volume fraction was one,  
208 while above it the initial liquid volume fraction was zero (i.e. the headspace was full of gas as is  
209 physically correct). Densities of  $1.2 \text{ kg m}^{-3}$  and  $1000 \text{ kg m}^{-3}$  were used for the gas and liquid phases  
210 respectively. A value of  $0.072 \text{ N m}^{-1}$  was used for the surface tension. The viscosities of the gas  
211 and liquid phases were  $1.83 \times 10^{-5} \text{ Pa s}$  and  $1 \times 10^{-3} \text{ Pa s}$ , respectively.

212 Ansys CFX 2021R1 was used in this work. The bubble column flow was modelled as a transient  
213 using small timesteps ( $1 \times 10^{-3} \text{ s}$ ) as is required for such two-phase flows. Each simulation was  
214 run for a period of 150 s before averaging, then for an additional 150 s with transient averaging  
215 turned on. Unless stated otherwise all reported results are transient averages. All runs were solved  
216 using double-precision and further details about the numerical methods used are available  
217 elsewhere [40].

218 In order to quantify the predicted mixing time tracers were introduced at the same locations as  
219 those used experimentally (see Figure 1). Tracers were introduced at 151, 161 and 171 s, this being  
220 done to account for any fluctuations in the hydrodynamics which is known [39] to affect the mixing  
221 time. The tracer concentration was calculated at the same locations as used experimentally, and  
222 like the experimental results the reported values are the average of the three repeats, with error  
223 bars denoting one standard deviation about the mean.

### 224 2.3 Coupling of CFD with algal growth kinetics

225 In this work, we have developed an approach that integrates CFD particle tracking with algal  
226 growth kinetics, thereby allowing simulation of the influence of flashing light on biomass  
227 accumulation over time. The workflow used for the model integration is shown in Figure 2.



228 To understand the distribution of algal cells throughout the reactor 2,000 Lagrangian particles were  
 229 introduced at one timestep (i.e.  $1 \times 10^{-3}$  s) at a simulation time of either 150 or 300 s, with their  
 230 location in the column being tracked for 150 s. The particles (representing cells) were introduced  
 231 uniformly throughout the column, using a grid with  $20 \times 10$  evenly spaced particles with 10 radial  
 232 divisions. The particle post-processing was done using a custom script written in Matlab R2022.  
 233 The particle solver in Ansys CFX uses a variable time-step [45]. To simplify the analysis an array  
 234 having a fixed timestep (0.01 s) was generated and the particle  $x$  and  $z$  coordinates generated by  
 235 the CFD model were interpolated onto this array using the one-dimensional spline interpolation  
 236 function implemented in Matlab R2022. Any particle where the track ended before the designated  
 237 time (150 s) was excluded from the analysis; this corresponded to a maximum of 3.6% of the  
 238 particles added.

239 The distribution of local light intensity within the PBR was simulated by adopting a Monte Carlo  
 240 type procedure that tracks the trajectories of numerous photons within the PBR, following the  
 241 method developed by others [32]. This approach accounts for absorption of light, as well as  
 242 changes in the trajectories of the photons due to scattering. Here it was assumed that the light  
 243 intensity was uniform in the vertical ( $y$ ) direction, meaning light attenuation was only modelled in  
 244 two-dimensions (i.e., along the  $x$  and  $z$  coordinates). Other optical phenomena, such as the  
 245 refraction and reflection across/by the PBR wall were omitted. When modelling the configurations  
 246 with internals (i.e., the bubble column with segmented baffles, disc and donut baffles and the  
 247 airlift) any effect of the internals on the light propagation was neglected. Additionally, it was  
 248 assumed that the cells were distributed uniformly throughout the medium, and the effects of the  
 249 bubbles on the light scattering were minimal (in line with results reported elsewhere [28]). No  
 250 wavelength dependent behaviour was considered in this work. Furthermore, it was also assumed  
 251 that the hydrodynamic behavior did not change throughout the course of a batch, something which  
 252 is likely to be true provided the algae do not produce large quantities of extracellular compounds  
 253 which could affect the fluid flow. Hence, the same CFD results were used to represent the  
 254 hydrodynamics (using the Lagrangian particle tracks) for the entirety of the batch.

255 In modelling the trajectory of a photon, the photon enters the PBR from one of the illuminated  
 256 surfaces of the PBR, each of which covered a  $180^\circ$  arc. The photon propagates through the medium  
 257 for a distance,  $\Delta l$ , before it is scattered, this scattering changes its direction. The photon then travels  
 258 for another distance ( $\Delta l$ ) before being scattered again. This procedure continues until the photon  
 259 exits the boundary of the PBR, or its intensity reaches a value of  $0.1 \mu\text{mol photons m}^{-2}\text{s}^{-1}$ , at which  
 260 point the tracking process was stopped. The position of a photon at step  $n$  is determined based on  
 261 its position at the previous step ( $n - 1$ ):

$$x_n = x_{n-1} + \Delta l \cdot \cos\theta \quad (2)$$

$$z_n = z_{n-1} + \Delta l \cdot \sin\theta \quad (3)$$

262 The Monte Carlo sampling procedure was used to determine (a) the starting position of a photon,  
 263 (b) the propagation distance ( $\Delta l$ ) at each step and (c) the scattering angle ( $\theta$ ) at each step.

264 As shown in Figure 1, the PBR is illuminated by three light sources; each covered 180° of the PBR  
 265 surface. In determining the starting position of a photon, a value was sampled from a uniform  
 266 distribution across the interval  $[0, \pi]$ , and the distance to the PBR centre was one PBR radius ( $R$ ).  
 267 At this starting position, the photon has an incident intensity ( $I_o$ ) of 360  $\mu\text{mol photons m}^{-2} \text{s}^{-1}$ , this  
 268 being the experimentally measured light intensity on the side of the photobioreactor [46].

269 The propagation distance for the photons ( $\Delta l$ ) was set to be a random number across the uniform  
 270 distribution between 0 and 1 mm. The scattering angle ( $\theta$ ) was determined using the Henyey-  
 271 Greenstein phase function [32]:

$$\cos(\theta) = \frac{1}{2g} \left\{ 1 + g^2 - \left( \frac{1 - g^2}{1 + g(2P - 1)} \right)^2 \right\}$$

272 where  $P$  is a random number drawn from the uniform distribution between 0 and 1. The amount  
 273 of forward and back-scattering is adjusted by the value of the parameter  $g$ , with  $g = 1$   
 274 corresponding for forward scattering only, and  $g = 0$  corresponding to isotropic scattering. Here  
 275 a value of  $g = 0.95$  was used, this being based on the experimental work of Marken et al. [47].

276 The attenuation in the light intensity between two consecutive steps due to absorption was  
 277 accounted for using the Beer-Lambert equation. The local light intensity at each step ( $I_n$ ) was  
 278 expressed as:

$$I_n = I_{n-1} e^{-K_a X \Delta l}$$

279 where  $K_a$  is the attenuation constant. Here we have used a value of 0.35  $\text{L mg}^{-1} \text{m}^{-1}$  for the light  
 280 attenuation constant ( $K_a$ ), this value being based on our previous work [46].  $I_{n-1}$  is the local light  
 281 intensity at the previous step;  $\Delta l$ , is the propagation distance in this step;  $X$  is the biomass  
 282 concentration.

283 The output of the Monte-Carlo procedure was an array containing the position and intensity of  
 284 photons throughout the photobioreactor for each of the light sources. To enable use of these data  
 285 to generate a light profile across the PBR, the horizontal ( $XZ$ ) plane of the PBR was discretized  
 286 onto a two-dimensional mesh containing 6078 elements, this was generated using the Delaunay  
 287 triangulation methodology implemented in Matlab. Photons were allocated to a mesh element  
 288 based on their  $x, z$  coordinates. The light intensity in each mesh element for a given light source  
 289 was calculated by averaging the intensity of all photons allocated to that element. From this the  
 290 total light intensity was determined by taking the sum of the light intensity from each of the three  
 291 sources. This procedure generated a two-dimensional map of the light intensity (see Figure 7).  
 292 Using these data, it was possible to allocate the particles from the CFD model to a mesh element  
 293 at each time point (based on their  $x, z$  coordinates) and hence generate an array containing the light  
 294 intensity for each timepoint for each particle.

295 The growth rates of the cells under flashing light were determined following the findings by Terry  
 296 for *P. tricornutum* [10]. At very high  $L/D$  cycle frequencies (i.e., those much greater than 1 Hz)  
 297 the cells are thought to respond as though the lighting is continuous, and hence the growth rate ( $\mu_{\text{full}}$ )  
 298 ) can be calculated based on the time-averaged light intensity for the particle ( $\bar{I}_p$ ) [10]:

$$\mu_{\text{full}} = \frac{\mu_{\text{max}} \bar{I}_p^k}{\bar{I}_p^k + K_I^k} \quad (6)$$

299 where the  $\mu_{\text{max}}$  is the maximum specific growth rate (2.4 day<sup>-1</sup>),  $K_I$  is the half-saturation constant  
 300 for light (50  $\mu\text{mol photons m}^{-2} \text{s}^{-1}$ ) and  $k$  is a constant (1.9). Values of the constants in the growth  
 301 model are based on our previous experimental work and that of others in the literature [46, 48].  
 302 The time-averaged light intensity for each particle  $\bar{I}_p$  was calculated using the interpolated values:

$$\bar{I}_p = \frac{\sum_{n=1}^{n_{\text{total}}} I_{p,n}}{n_{\text{total}}} \quad (7)$$

303 where  $I_{p,n}$  is the instantaneous light intensity experienced by particle  $p$  at time  $n$  and  $n_{\text{total}}$  is the  
 304 total number of time points (here 15,000).

305 At sufficiently low  $L/D$  cycle frequencies the growth rate of the cell depends on the instantaneous  
 306 light intensity experienced by the particle ( $I_p$ ) meaning there is no light integration. This can be  
 307 used to calculate the specific growth rate:

$$\mu_{\text{no}} = \frac{\mu_{\text{max}} I_p^k}{I_p^k + K_I^k} \quad (8)$$

308 Partial light integration will occur at  $L/D$  cycle frequencies between those where full and no light  
 309 integration occurs; these are the frequencies likely to be found in industrial photobioreactors. Here  
 310 the specific growth rate depends on the  $L/D$  cycle frequency ( $F$ ):

$$\mu = \frac{\Gamma_{\text{max}} F}{K_F + F} (\mu_{\text{full}} - \mu_{\text{no}}) + \mu_{\text{no}} \quad (9)$$

311 where  $\Gamma_{\text{max}}$  and  $K_F$  are constants; Terry [10] determined that the values of  $\Gamma_{\text{max}}$  and  $K_F$  to be 0.972  
 312 and 0.67 Hz, respectively, for *P. tricornutum*.

313 Here we have defined a cell as being in the dark if the instantaneous light intensity was less than  
 314  $5 \mu\text{mol photons m}^{-2} \text{ s}^{-1}$ . This value was selected on the basis that at this light intensity the net  
 315 growth rate is zero (i.e., the light intensity is sufficient for cellular maintenance but not growth)  
 316 [49]. Using the calculated values of the instantaneous light intensity it is possible to determine  
 317 whether or not a particle is in the light or dark zone for each time point. From this it is possible to  
 318 determine the  $L/D$  cycle frequency and thus the specific growth rate ( $\mu$ ) for this cell. The specific  
 319 growth rate was determined for each of the simulated algal cells; the mean of the specific growth  
 320 rates of the cell population was then calculated:

$$\bar{\mu} = \frac{1}{p} \sum_{i=1}^p \mu_i \quad (10)$$

321 where  $p$  is the number of cells evaluated.

322 The Monte Carlo sampling procedure and the subsequent determination of the growth rate for the  
 323 algal cell population were repeated for biomass concentrations ranging between 5 and 2005 mg  
 324  $\text{L}^{-1}$ , thereby generating a set of datapoints providing the population averaged specific growth rate  
 325 ( $\bar{\mu}$ ) for the range of cell densities examined. These data were integrated with our recently  
 326 developed, ODE-based model for simulating the growth of the alga *P. triornutum* [46]. Here, it  
 327 was assumed that light was the sole growth-limiting factor. In the model the specific growth rate  
 328 for a given cell density was found by interpolating onto the  $[X, \bar{\mu}]$  array generated using the  
 329 workflow developed in this paper. The one-dimensional spline interpolation function implemented  
 330 in Matlab R2022 was used to perform the interpolation.

331 To ensure the results were independent of the numerical values used in setting up the simulation a  
 332 range of conditions were investigated, with the full details being available in the Supplementary  
 333 Material. Simulations were performed to investigate the effect of the number of photons per light  
 334 source (100, 300, 500, 1000 and 10,000), the number of mesh elements (570, 1710, 6078 and  
 335 14,286) and the number of Lagrangian particles (200, 500, 1000 and 2000). Based on these results  
 336 all subsequent simulations were performed with 1000 photons per light source, a mesh containing  
 337 6078 elements and 1000 Lagrangian particles.

338

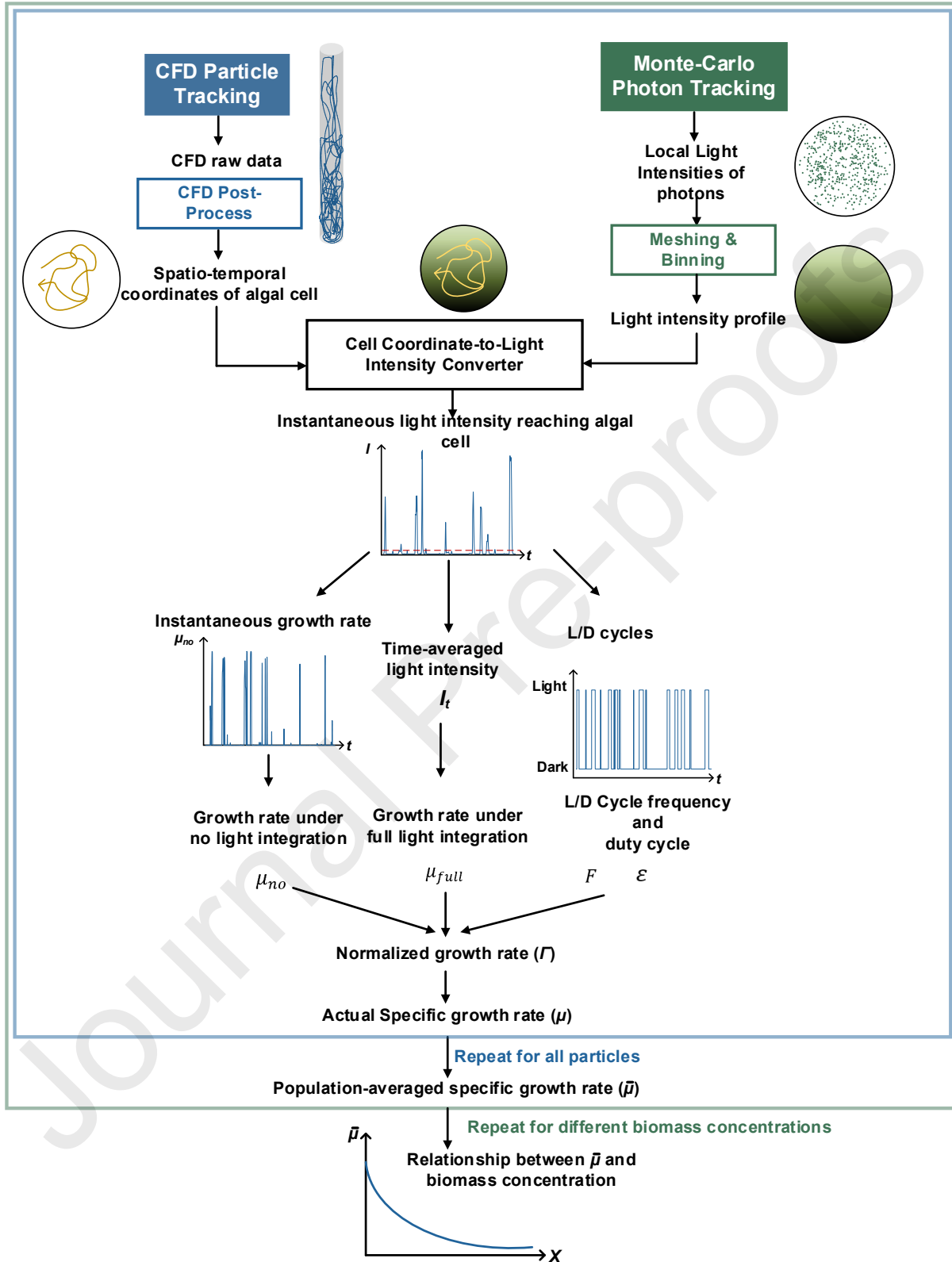
339

#### 340 2.4 Algal cultivation experiments

341 In this work we have focused on the cultivation of *Phaeodactylum triornutum*, a marine diatom  
 342 which can be used for the production of valuable compounds like eicosapentaenoic acid and  
 343 fucoxanthin [50-52]. Cultivations were performed using the bubble column configuration as  
 344 described in Section 2.1. Air (supplemented with 1 % (v/v) carbon dioxide) was introduced into  
 345 the column at a superficial velocity of  $1.3 \text{ cm s}^{-1}$ . Here 1% (v/v) carbon dioxide was used as this  
 346 concentration is sufficient to maintain the pH below 8.5, hence ensuring the growth is not limited  
 347 by the availability of carbon.

348 Cultures were illuminated for 16 hours per day using the LED lights mounted on three sides of the  
349 column (Figure 1). The lighting consisted of 9 W cool-white (6000 K color temperature) LED bars  
350 (Jaycar Australia), these were arranged in a  $3 \times 5$  grid (vertical  $\times$  horizontal) on each of the  
351 illuminated sides of the PBR. To quantify the biomass density samples (typically 50-60 mL) were  
352 taken and filtered using pre-weighed glass fibre filters (Advantec GA-55, Toyo Roshi Kaisha Ltd,  
353 Tokyo Japan). Samples were washed with three volumes of 0.5 M ammonium bicarbonate before  
354 being dried at 105°C overnight. After drying the samples were cooled and then weighed to  
355 determine the dry cell weight. Further details about the cultivation conditions and the analytical  
356 methods are available in our previous work [46].





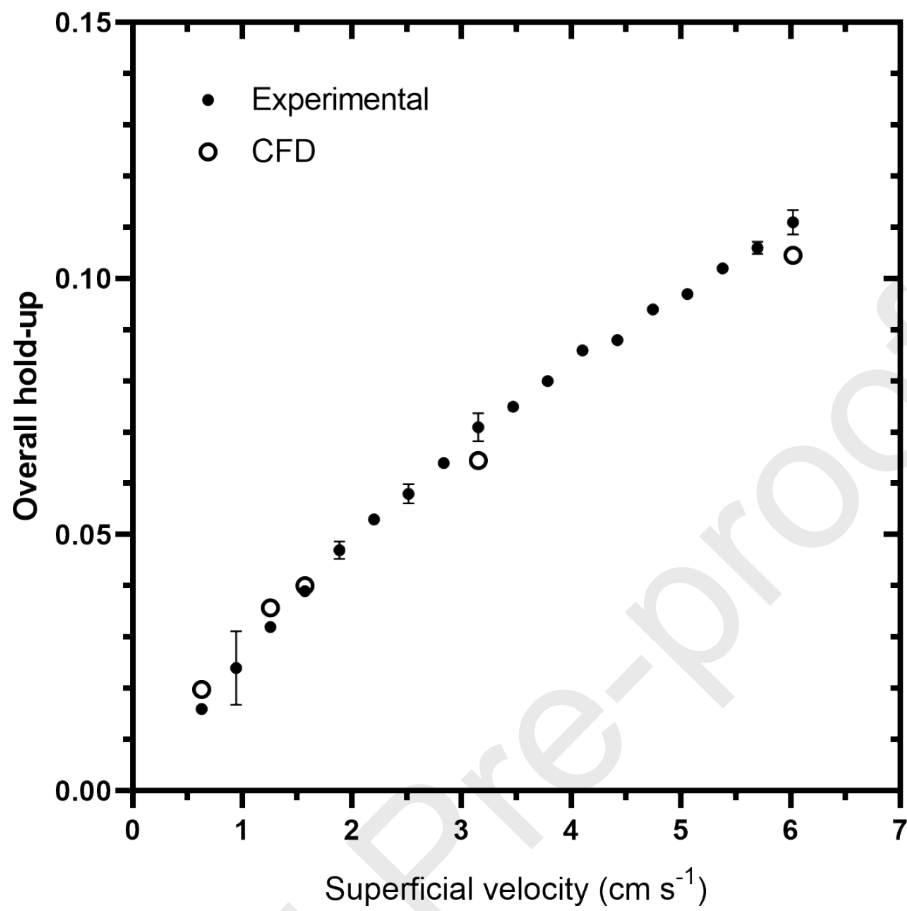
357

358 Figure 2 – Schematic showing the workflow used in the model.

### 359 3. Results and Discussion

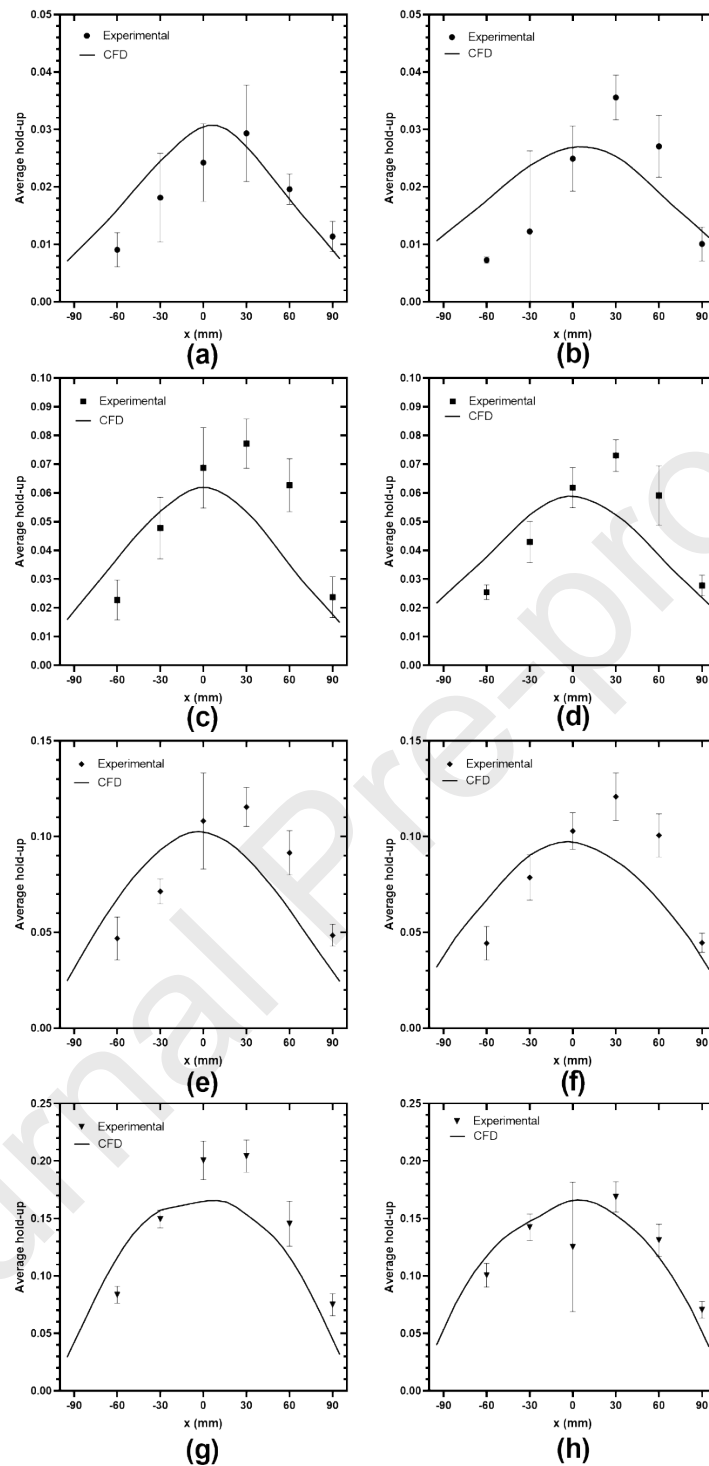
#### 360 3.1. CFD model validation

361 As the performance of the model relies upon accurate predictions of the hydrodynamics within the  
362 reactor it is necessary to validate the CFD model against experimental data. Figure 3 gives a  
363 comparison between the experimentally measured overall hold-up values and those predicted by  
364 the CFD model, while a more detailed comparison of the local-hold-up profiles is given in Figure  
365 4. It was found that there was good agreement between the experimental measurements and the  
366 model predictions, with the model slightly under-predicting the hold-up. Interestingly the  
367 maximum values for the experimentally measured hold-up values were found to occur at  $x =$   
368 30 mm and not at the column centerline (as was found for the CFD predictions). Such results could  
369 be because the CFD model over-predicts the extent to which the bubble plume becomes  
370 symmetrical. This could be caused by the model over-predicting the magnitude of the turbulent  
371 dispersion force, or over-predicting the eddy viscosity which would damp out oscillations. Equally  
372 such results could be due to the fact that the sparger is not located perfectly perpendicular to the  
373 measurement location, thereby introducing a degree of asymmetry into the results. While every  
374 attempt was made to ensure that the sparger was located on the centerline of the column it was  
375 possible that it could have moved by a small amount (of the order 10 mm). Interestingly, it appears  
376 that at higher superficial velocities the observed asymmetry is reduced (Figure 4 (h)). Inclusion of  
377 the lift force, or modification of the coefficient in the turbulent dispersion model may lead to  
378 improved agreement with the experimentally measured hold-up profiles. However, this was not  
379 pursued as the model gives reasonable agreement with the overall and local hold-up, as well as the  
380 mixing time (Figure 5).



381

382 Figure 3 – Plot showing comparison between experimentally measured overall hold-up values and those  
383 predicted by the CFD model.



384

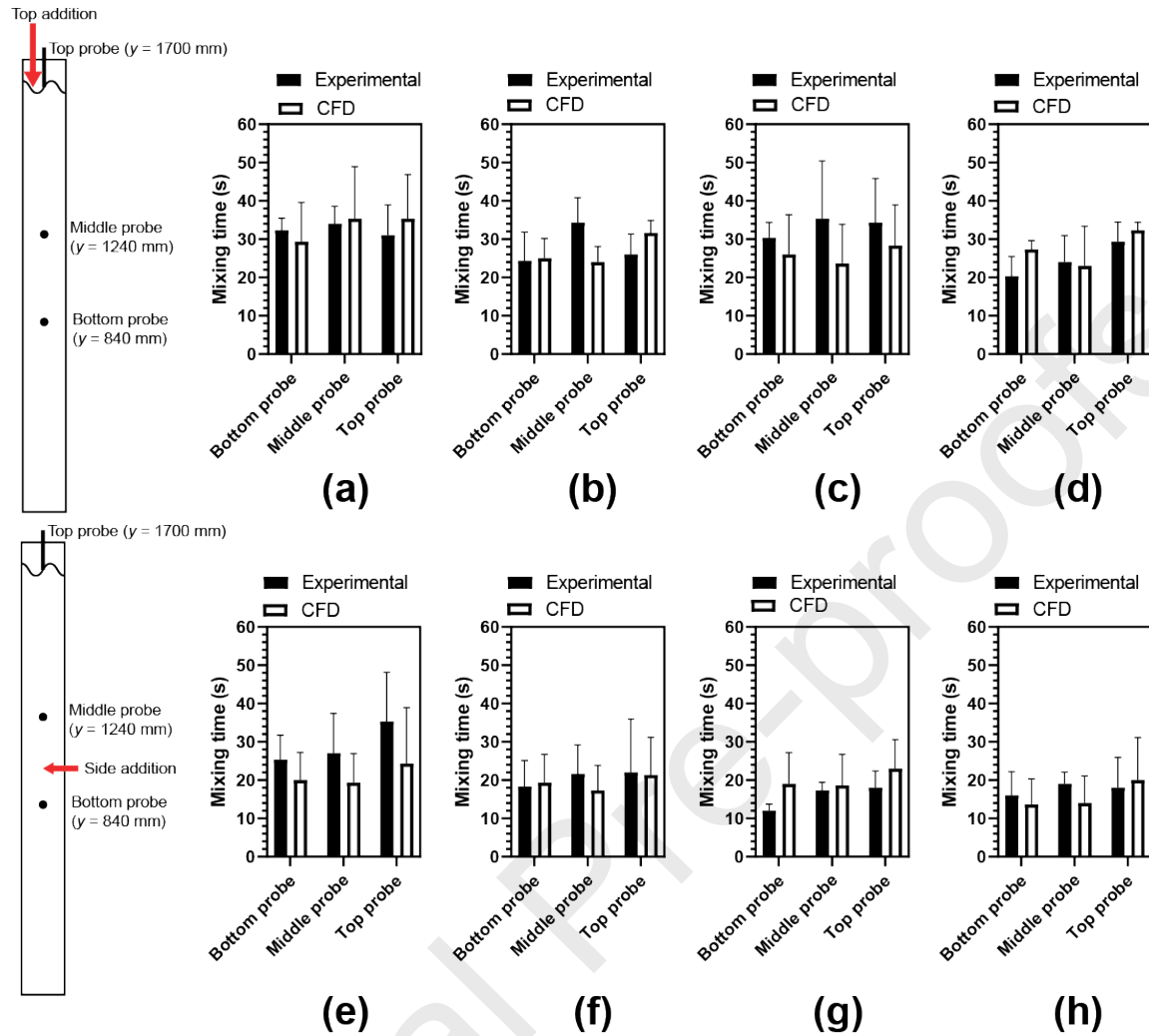
385 Figure 4 - Plot showing comparison between experimentally measured local hold-up profiles and those  
 386 predicted by the CFD model. Results on the first row (a) and (b) are for a superficial velocity of  $0.6 \text{ cm s}^{-1}$ , those  
 387 on the second row (c) and (d) are for a superficial velocity of  $1.6 \text{ cm s}^{-1}$ , those on the third row (e) and (f) are  
 388 for a superficial velocity of  $3.2 \text{ cm s}^{-1}$  and those on the final row (g) and (h) are for a superficial velocity of  $6.0$   
 389  $\text{cm s}^{-1}$ . Plots in the first column (a), (c), (e) and (g) are for a height of 840 mm above the base of the column,  
 390 while those in the second column, (b), (d), (f) and (h) are for a height of 1240 mm above the base of the column.

391 Experimentally measured bubble size distributions are shown in Figure S6 in the supplementary  
392 data. It was found that the superficial velocity had a relatively small effect on the bubble size  
393 distribution, while the distance from the sparger had a larger effect. Measured mean bubble sizes  
394 were of the order 10-12 mm at a height of 800 mm above the base of the column and 8-9 mm for  
395 a height of 1200 mm above the base of the column. The correlation developed by Akita and  
396 Yoshida [53] predicts initial bubble sizes between 9 and 19 mm for the superficial velocities  
397 examined in this work. This, combined with the experimental measurements is consistent with the  
398 idea that the bubbles produced by the sparger undergo break-up as they rise through the column.

399 Figure 5 gives a comparison between the experimentally measured mixing times and those  
400 predicted by the CFD model. Both the experimental measurements and CFD predictions showed  
401 a considerable amount of variation, this is due to the transient nature of the flow inside the bubble  
402 column, where the instantaneous flow pattern at the time of tracer addition impacts upon the  
403 mixing time [39]. Generally speaking, the model predictions were in good agreement with the  
404 experimental measurements for the range of superficial velocities ( $0.6 - 6.0 \text{ cm s}^{-1}$ ) and tracer  
405 addition and measurement locations examined. It was observed that increasing the superficial  
406 velocity led to a reduction in the mixing time, as expected, with the measured values being less  
407 than those predicted (46 – 108 s) using correlations from the literature [54, 55]. Interestingly, it  
408 was found that the measurement location did not have a large impact on the mixing time with the  
409 values being similar for the three points examined. However, it was found that the tracer addition  
410 location did have an impact on the mixing time, with the side addition point generally resulting in  
411 lower mixing times than when the tracer was introduced to the top of the column, the reduction  
412 being of the order 40-80%. This can be most likely be explained by the fact that when the tracer is  
413 added to the top of the column it has to travel a greater distance to be uniformly mixed throughout  
414 the column, thereby resulting in a longer mixing time.

415 As shown in Figure 3-5 the CFD model offers a good prediction of the hydrodynamics within the  
416 bubble column, at a range of superficial velocities and measurement locations. Hence, the CFD  
417 model is suitable to be used as a basis for quantifying photobioreactor performance.





418

419 **Figure 5 – Comparison between experimentally measured values of the mixing time and CFD predictions.**  
 420 **Results are shown for two tracer addition locations and three measurement locations. All results are the average**  
 421 **of three tracer additions, with error bars denoting one standard deviation about the mean. Results on the top**  
 422 **row (a)-(d) are for tracer addition to the top of the column, results on the bottom row (e)-(h) are for tracer**  
 423 **addition in the middle of the column. The first column (a) and (e) are for a superficial velocity of  $0.6 \text{ cm s}^{-1}$ , the**  
 424 **second column (b) and (f) are for a superficial velocity of  $1.6 \text{ cm s}^{-1}$ , the third column (c) and (g) is for a**  
 425 **superficial velocity of  $3.2 \text{ cm s}^{-1}$  and the last column (d) and (h) are for a superficial velocity of  $6.0 \text{ cm s}^{-1}$ .**

426

427

428

429

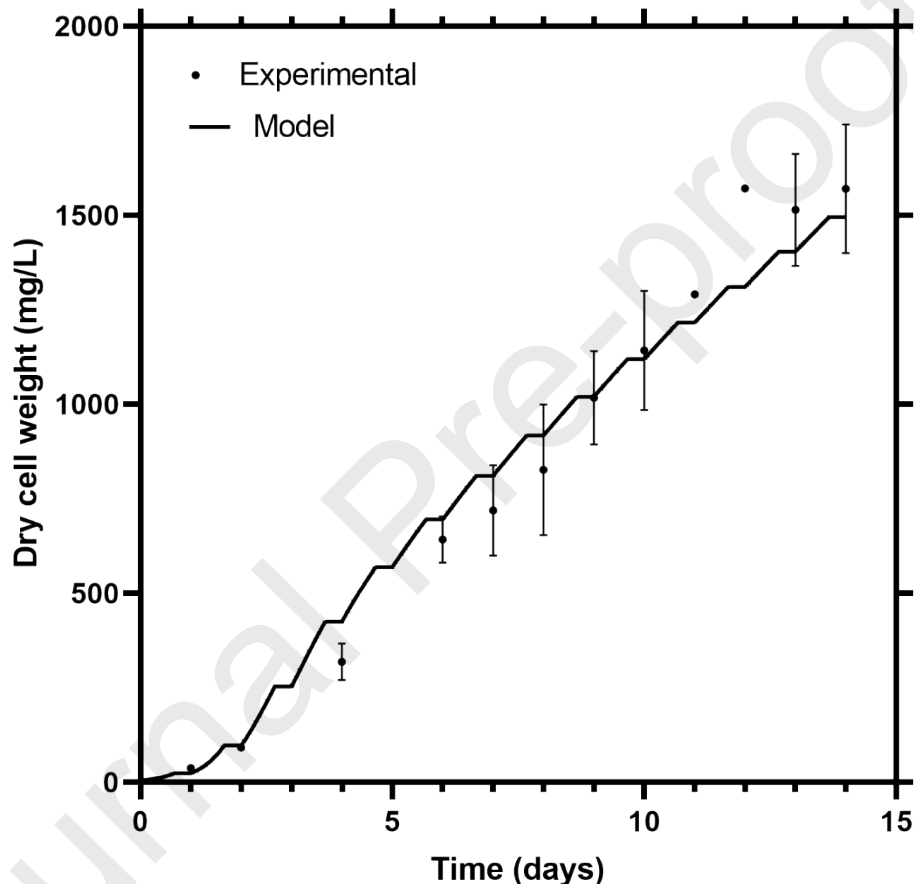
430

431

432

## 433 3.2. Validation of modelling workflow

434 The next step in the model validation process was to compare the predictions of the cell growth  
 435 generated by the model workflow with experimental data. This comparison is given in Figure 6  
 436 for *P. tricornutum*. It was found that the model was in good agreement with the experimental  
 437 predictions, demonstrating that the workflow developed was able to offer predictions in line with  
 438 experimental measurements.



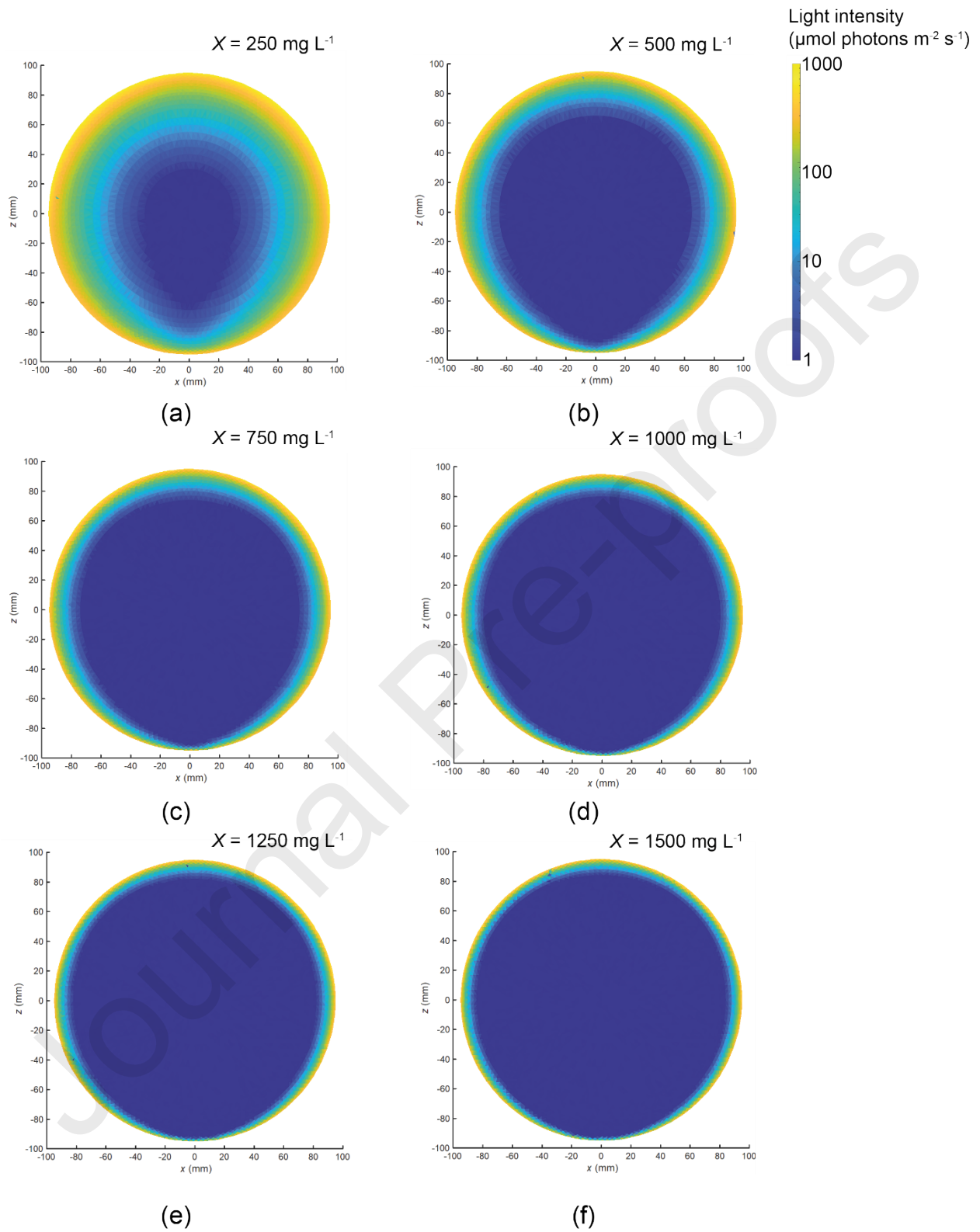
439

440 **Figure 6 – Comparison between experimental results and model predictions for growth of *P. tricornutum* in**  
 441 **50 L bubble column bioreactors. Experiments were performed at a superficial velocity of 1.3 cm s<sup>-1</sup>; the same**  
 442 **conditions were used in the modelling. Results are shown for three runs, with error bars denoting one standard**  
 443 **deviation about the mean.**

444 To further understand the model behaviour, plots of the calculated light field for cell densities from  
 445 250 – 1500 mg L<sup>-1</sup> were generated and are shown in Figure 7. As expected the illumination within  
 446 the photobioreactor is not uniform due to the fact that light is being provided from three sides of  
 447 the reactor. Similarly, it can be observed that the central portion of the reactor is essentially ‘dark’,  
 448 while the edges are illuminated, and that the size of the dark zone increases with cell density (due  
 449 to greater attenuation of the light). These results demonstrate that the best way to improve the

450 growth of the algal culture would be to ensure cells are not ‘trapped’ in the central, ‘dark’ area of  
451 the column.

Journal Pre-proofs



452

(e)

(f)

453  
454

Figure 7 – Plot showing calculated light profiles in the PBR. Values have been calculated at cell densities of (a) 250  $\text{mg L}^{-1}$ , (b) 500  $\text{mg L}^{-1}$ , (c) 750  $\text{mg L}^{-1}$ , (d) 1000  $\text{mg L}^{-1}$ , (e) 1250  $\text{mg L}^{-1}$  and (f) 1500  $\text{mg L}^{-1}$ .

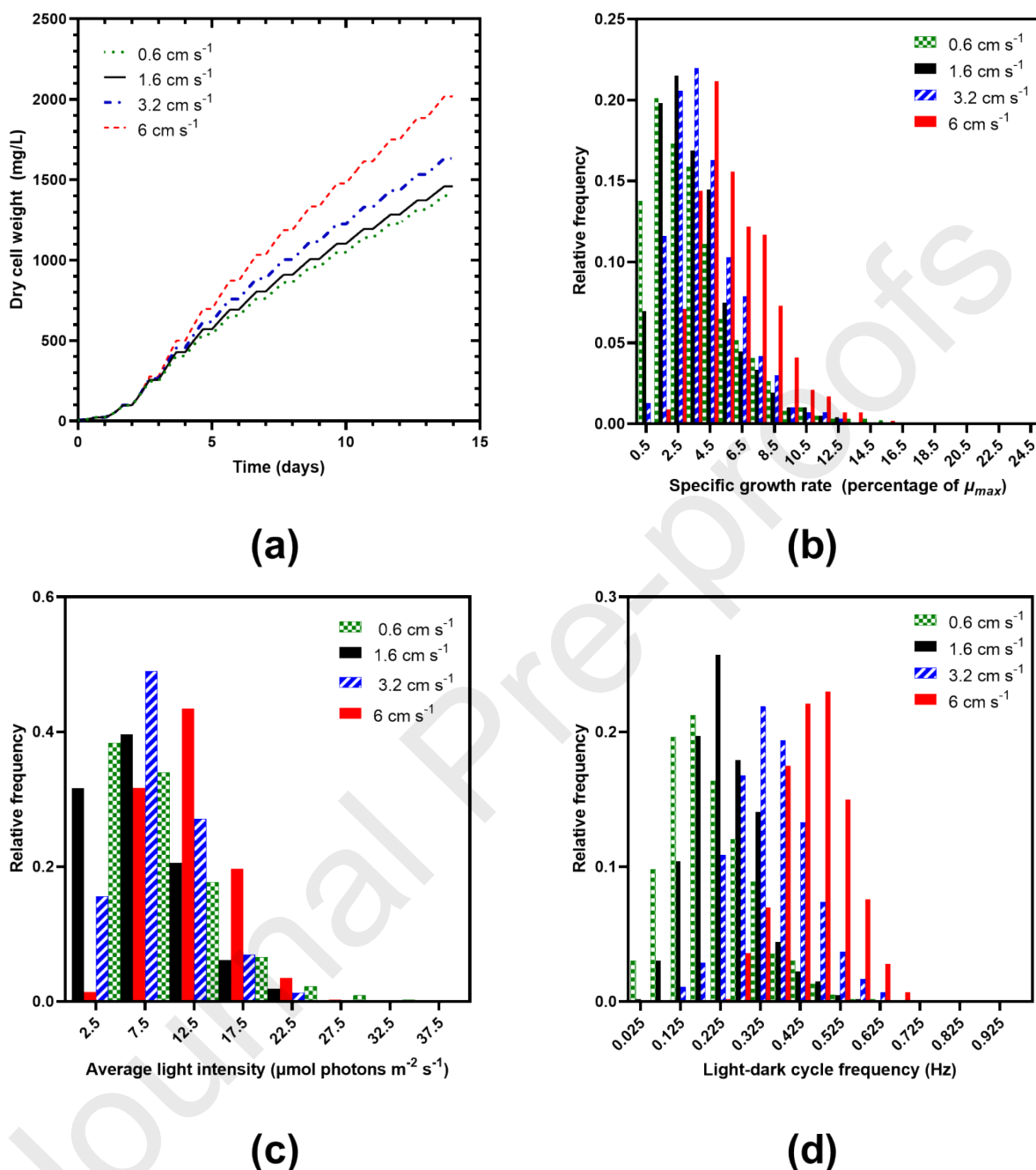
455 As part of the model validation the sensitivity of the model to various input parameters was  
456 examined. Unsurprisingly, the chosen value of the attenuation coefficient ( $K_a$ ) had a large impact  
457 on the model predictions, with this effect being most pronounced as the culture density increased  
458 (i.e., towards the end of the batch). Our previous work [46] had shown that the value of  $K_a$  changed  
459 depending on the availability of nitrate, and the chosen value ( $0.35 \text{ L mg}^{-1} \text{ m}^{-1}$ ) was representative  
460 of conditions from day 5 onwards. In this work we have used the Beer-Lambert law to model light  
461 attenuation; this approach being selected on the basis of its simplicity. However, it may be  
462 desirable to replace this approach with a more complex model of light attenuation [56, 57],  
463 something which can be done in a relatively straightforward manner using the current workflow.  
464 Similarly, it may be desirable to consider wavelength dependent behaviour when modelling  
465 absorption and scattering within the culture. Such an approach would introduce considerable  
466 additional complexity to the model. Given it was possible to achieve good agreement between the  
467 model predictions and experimental results (see Figure 6) without accounting for wavelength  
468 dependent behaviour it remains an open question as to whether the increase in model accuracy  
469 justifies the additional complexity.

### 470 3.3. Evaluation of alternative PBR designs and operating conditions

471 Previous work [58] using a PBR of similar size showed that increasing the superficial velocity led  
472 to increased biomass productivity for *P. tricornutum*. It was hypothesized that the improvement in  
473 productivity at higher superficial velocities was due to an increase in the  $L/D$  frequency. However,  
474 this was not experimentally quantified, as previously noted making such measurements is very  
475 challenging. By using the modelling approach developed in this work we are able to quantify the  
476 effect of the superficial velocity on the  $L/D$  cycle frequency, and hence the growth of the cultures.

477 Figure 8 shows the effect of the superficial velocity on the predicted performance of the bubble  
478 column PBR. It was found that increasing the superficial velocity led to an improvement in the  
479 predicted biomass concentration, particularly for a superficial velocity of  $6 \text{ cm s}^{-1}$ . Increasing the  
480 superficial velocity will lead to an increase in the liquid velocity within the column, and hence  
481 improved mixing (as shown in Figure 5). This will also lead to the cells being transported more  
482 rapidly between the column walls (i.e., the 'light' portion of the PBR) and the centre of the column  
483 (the 'dark' portion of the PBR). Such behaviour is observed in Figure 8, where increasing the  
484 superficial velocity leads to an increase in the  $L/D$  cycle frequency, the average light intensity and  
485 hence the average specific growth rate. The predicted biomass productivity at a superficial velocity  
486 of  $6 \text{ cm s}^{-1}$  was  $144 \text{ mg L}^{-1} \text{ day}^{-1}$ , this being approximately 40% higher than the value at  $1.6 \text{ cm s}^{-1}$   
487 ( $104 \text{ mg L}^{-1} \text{ day}^{-1}$ ). These results are in line with previously published results [58] for a similar  
488 PBR design growing *P. tricornutum*. Based on these results increasing the superficial velocity may  
489 be an easy way to improve the performance of the PBR. However, there are two potential  
490 drawbacks to this approach. Firstly, there is the obvious increase in energy required to supply the  
491 higher flow rate of air. Secondly, the increase in superficial velocity may lead to damage to the  
492 cells [58] which would obviously make this approach unfeasible. An advantage of the modelling  
493 approach developed here is that it is possible to quantify the trade-off between the increase in  
494 biomass concentration and energy demand, allowing systematic process design and optimisation.





495

496 **Figure 8** – Plots showing the effect of the superficial velocity on the performance of the PBR. (a) shows the  
 497 effect of the superficial velocity on the predicted dry cell weight for the course of the cultivation. Plots (b), (c)  
 498 and (d) have been calculated for a fixed cell density of 1500 mg L<sup>-1</sup> and show the distribution of the specific  
 499 growth rate (b), the time-averaged light intensity (c), and the light-dark cycle frequency (d) for the population  
 500 of particles.

501 As previously noted, a range of authors have reported that modifying the PBR design to include  
 502 baffles or other internal structures led to increases in the biomass productivity [18-22]. Such  
 503 increases were again attributed to increased mixing along the light gradient, which led to an

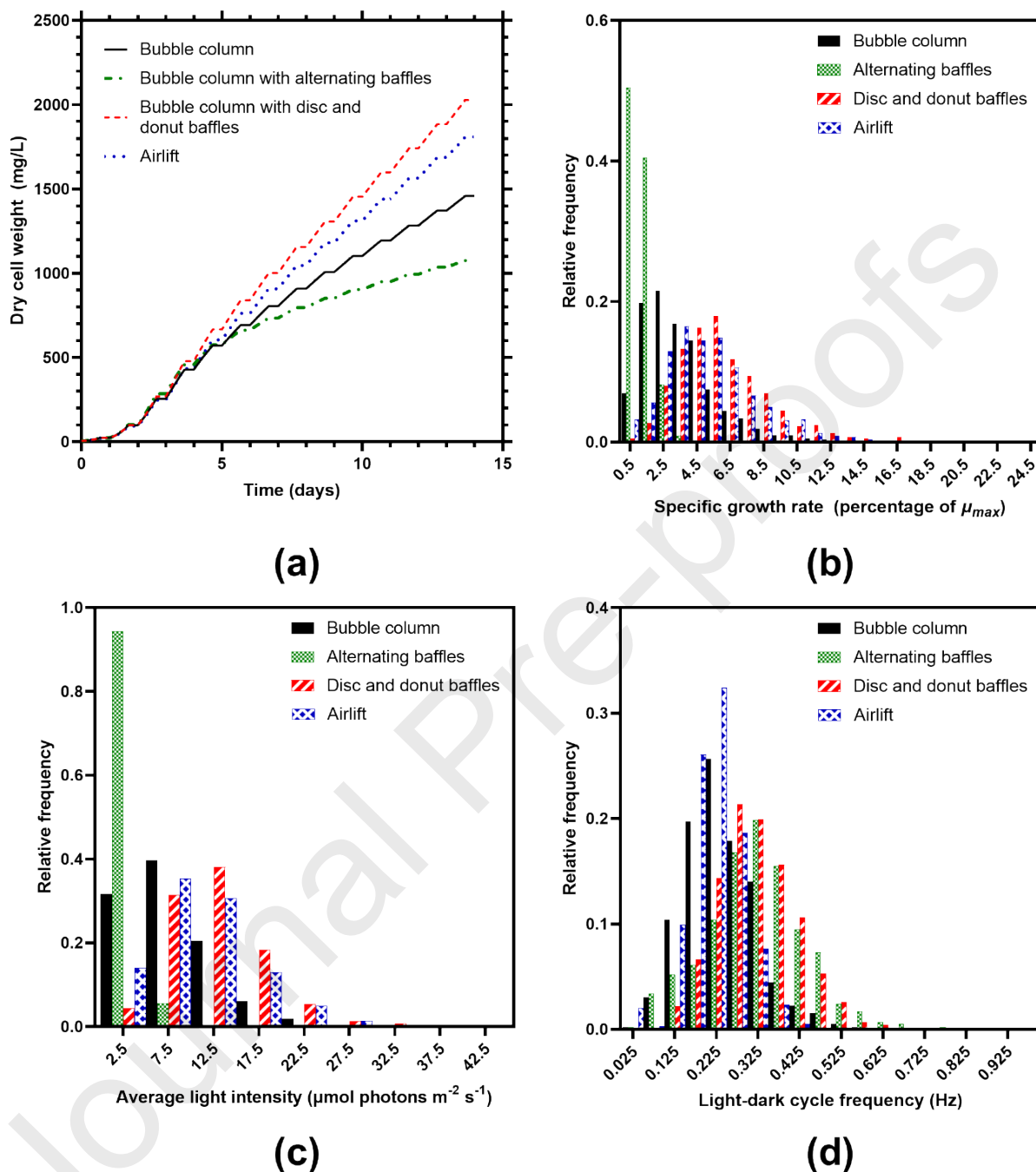
504 increase in the  $L/D$  cycle frequency and in turn an improvement in the biomass productivity. Using  
505 the modelling workflow developed here it is possible to systematically evaluate new PBR designs  
506 *in silico* to determine their performance.

507 Figure 9 shows the predicted performance for a range of PBR designs. Of the configurations  
508 examined it was found that the airlift and disc and donut baffles were predicted to give improved  
509 performance, while including the alternating baffles worsened the performance. Interestingly, the  
510 alternating baffles were also predicted to increase the light-dark cycle frequency, something which  
511 is thought to lead to improved performance. However, this configuration also led to a substantial  
512 reduction in the average light intensity experienced by the cells (Figure 9 (c)). This is caused by  
513 the flow 'confining' the cells in the central portion of the column, away from the illuminated  
514 surface. Preliminary experimental work (shown in Supplementary Figure S7) indicated that the  
515 model predictions agreed with the experimental data, with the performance of the column with the  
516 alternating baffles being similar to or worse than the standard bubble column without any internals.

517 Of the configurations evaluated the disc and donut baffles were predicted to offer the most  
518 improved performance, with the predicted biomass productivity being  $145 \text{ mg L}^{-1} \text{ day}^{-1}$ , this being  
519 an approximately 40% improvement when compared with the bubble column without internals.  
520 Interestingly, the airlift configuration offered a similar increase in performance, while not  
521 improving the light-dark cycle frequency to the same extent. This may be explained by the fact  
522 that the baffle in the airlift confines the cells closer to the walls of the PBR where they are more  
523 likely to experience a higher light intensity. These results highlight the potential advantages of  
524 installing internals in PBRs, as they can lead to substantially improved performance. However,  
525 this must also be weighed against any increases in capital cost, as well as any additional operational  
526 challenges (e.g., making cleaning more difficult).

527 The results shown in Figure 9 suggest that the key metric in optimising the system is the average  
528 light intensity experienced by the cells, and that the light-dark cycle frequency cannot be  
529 considered in isolation.

530 These results also show the advantage of the approach developed in this work, as it is possible to  
531 simultaneously evaluate multiple designs *in-silico*. To illustrate this point, each algal cultivation  
532 performed in this work took approximately two weeks, with an additional 1-2 days being needed  
533 for cleaning and set-up of the PBRs. In the same amount of time it was possible to perform all of  
534 the CFD simulations used in this work in parallel. This demonstrates that once the model has been  
535 set-up and validated it can be used to examine a range of conditions, with the aim of identifying  
536 the most promising for experimental evaluation.



537

538 **Figure 9 – Plot showing the effect of changing the PBR design on its performance. The predicted growth curves**  
 539 **are shown in (a), while plot (b) shows the distribution of the specific growth rate for the modelled particle**  
 540 **population, (c) shows the time-averaged light intensity and (d) shows the light-dark cycle frequency. Plots (b),**  
 541 **(c) and (d) have been calculated for a cell density of 1500 mg L<sup>-1</sup>. All simulations were performed at a superficial**  
 542 **velocity of 1.6 cm s<sup>-1</sup>.**

543

544

545 **4. Conclusions**

546 In this work we have developed and validated a modelling approach which synthesises CFD,  
547 Monte-Carlo modelling and kinetic models to enable the detailed characterisation of PBR  
548 performance. The approach developed in this work enables the effect of different reactor designs  
549 and operating conditions to be characterised throughout the course of an entire batch. This enables  
550 *in-silico* evaluation of different reactor configurations, potentially reducing the time and risk  
551 involved in the scale-up process.

552 The CFD model used in this work was based on our previous research into models for bubble  
553 column bioreactors [38, 40] and in this work we have extensively validated it against experimental  
554 data across the range of superficial velocities likely to be used in bubble column PBRs. Results  
555 from the CFD were then combined with illumination and kinetic models to develop a workflow  
556 which can be used to characterise the effect of different reactor designs and operating conditions.  
557 It was found that the model predictions were in good agreement with the experimental data for the  
558 widely cultivated diatom *P. tricornutum*. Using the model, it was possible to evaluate the effect of  
559 different reactor designs and operating conditions. For example, increasing the superficial velocity  
560 from 1.6 cm s<sup>-1</sup> to 6 cm s<sup>-1</sup> was predicted to lead to an approximately 40% increase in the biomass  
561 productivity. The model can also be used to examine a range of different internal designs, it was  
562 found that some designs led to worse performance, while others were predicted to improve the  
563 biomass productivity. Of the configurations examined, the disc and donut baffles were predicted  
564 to increase the biomass productivity by a factor of approximately 40% at a superficial velocity of  
565 1.6 cm s<sup>-1</sup>. Interestingly, it was found the key metric in the reactor design was the average light  
566 intensity experienced by the cells, and not the light/dark cycle frequency.

567 An advantage of the modelling approach used in this work is that can be readily extended to model  
568 any reactor design, an obvious area for future work would be to evaluate additional alternative  
569 internal configurations and then experimentally test the most promising. The aim of this work  
570 would be to increase the cell density/biomass productivity of photoautotrophic systems, a key  
571 factor in the overall process economics. Development of a reliable, accurate *in silico* method for  
572 screening reactor designs could potentially considerably simplify the photobioreactor design  
573 process, thereby facilitating scale-up. Similarly, being able to predict the performance of different  
574 reactor designs would be useful in performing techno-economic analyses.

575 An advantage of the workflow developed here is that it is possible to change the sub-models in a  
576 relatively straightforward way. For example, the model of light attenuation could be modified to  
577 be wavelength dependent, without requiring changes in the CFD or growth models. Potential  
578 further avenues for investigation could include looking at the growth of other species, as well as  
579 modifying the light model to be representative of sunlight in order to model outdoor cultures.

580 In conclusion, the approach outlined here can be used for the comprehensive characterisation of  
581 PBR performance and hence the development of optimised designs. This is a topic of considerable  
582 importance in increasing the productivity of photoautotrophic production systems and thereby  
583 enabling their wider deployment.

## 584 Nomenclature

Symbol	Units	Description
$F$	$[s^{-1}]$	Light/dark cycle frequency
$g$	$[-]$	Scattering constant
$H_L$	$[m]$	Liquid height
$H_{G+L}$	$[m]$	Height of two-phase mixture
$I$	$[\mu\text{mol photons m}^{-2} \text{s}^{-1}]$	Light intensity
$I_0$	$[\mu\text{mol photons m}^{-2} \text{s}^{-1}]$	Initial light intensity
$I_n$	$[\mu\text{mol photons m}^{-2} \text{s}^{-1}]$	Light intensity at point $n$ for a photon
$\bar{I}_p$	$[\mu\text{mol photons m}^{-2} \text{s}^{-1}]$	Time-averaged light intensity for photon $p$
$k$	$[-]$	Constant in growth model
$K_a$	$[m^2 \text{kg}^{-1}]$	Attenuation constant
$K_F$	$[s^{-1}]$	Constant in growth model
$K_I$	$[\mu\text{mol photons m}^{-2} \text{s}^{-1}]$	Half saturation constant
$n$	$[-]$	Constant in volume fraction correction term model
$n_{\text{total}}$	$[-]$	Total number of time points used in averaging procedure
$p$	$[-]$	Number of cells

$P$	[-]	Random number
$R$	[m]	Photobioreactor radius
$x$	[m]	Distance in $x$ direction
$X$	[kg m <sup>-3</sup> ]	Cell density
$y$	[m]	Distance in $y$ direction
$z$	[m]	Distance in $z$ direction
$\alpha$	[-]	Gas volume fraction
$\Gamma_{\max}$	[-]	Constant in growth model
$\Delta l$	[m]	Propagation distance of photon
$\theta$	[radians]	Scattering angle for photon
$\mu$	[s <sup>-1</sup> ]	Specific growth rate
$\bar{\mu}$	[s <sup>-1</sup> ]	Population-averaged specific growth rate
$\mu_{\text{full}}$	[s <sup>-1</sup> ]	Specific growth rate with full light integration
$\mu_{\max}$	[s <sup>-1</sup> ]	Maximum specific growth rate
$\mu_{\text{no}}$	[s <sup>-1</sup> ]	Specific growth rate with no light integration
$\rho_G$	[kg m <sup>-3</sup> ]	Gas density

$\rho_L$  [kg m<sup>-3</sup>] Liquid density

$\sigma$  [kg s<sup>-2</sup>] Surface tension

585

586 **Acknowledgements**

587 The authors acknowledge the Sydney Informatics Hub and the University of Sydney's high-  
588 performance computing cluster, Artemis, for providing the computing resources that have  
589 contributed to the results reported herein.



590 **References**

- 591 [1] M.A. Borowitzka, High-value products from microalgae—their development and commercialisation, *J. Appl.*  
592 *Phycol.* 25(3) (2013) 743-756.
- 593 [2] Y. Chisti, Biodiesel from microalgae, *Biotechnology Advances* 25(3) (2007) 294-306.  
594 <https://doi.org/http://dx.doi.org/10.1016/j.biotechadv.2007.02.001>.
- 595 [3] N.J. Oliver, C.A. Rabinovitch-Deere, A.L. Carroll, N.E. Nozzi, A.E. Case, S. Atsumi, Cyanobacterial metabolic  
596 engineering for biofuel and chemical production, *Current Opinion in Chemical Biology* 35(Supplement C) (2016) 43-  
597 50. <https://doi.org/https://doi.org/10.1016/j.cbpa.2016.08.023>.
- 598 [4] M.J. Barbosa, M. Janssen, C. Südfeld, S. D'Adamo, R.H. Wijffels, Hypes, hopes, and the way forward for  
599 microalgal biotechnology, *Trends in Biotechnology* 41(3) (2023) 452-471.  
600 <https://doi.org/https://doi.org/10.1016/j.tibtech.2022.12.017>.
- 601 [5] C. Posten, Design principles of photo-bioreactors for cultivation of microalgae, *Engineering in Life Sciences* 9(3)  
602 (2009) 165-177. <https://doi.org/10.1002/elsc.200900003>.
- 603 [6] M. Janssen, Chapter Four - Microalgal Photosynthesis and Growth in Mass Culture, in: L. Jack (Ed.), *Advances*  
604 *in Chemical Engineering*, Academic Press 2016, pp. 185-256.  
605 <https://doi.org/http://dx.doi.org/10.1016/bs.ache.2015.11.001>.
- 606 [7] P.S.C. Schulze, R. Guerra, H. Pereira, L.M. Schüler, J.C.S. Varela, Flashing LEDs for Microalgal Production,  
607 *Trends in Biotechnology* 35(11) (2017) 1088-1101. <https://doi.org/https://doi.org/10.1016/j.tibtech.2017.07.011>.
- 608 [8] L.S. Sabri, A.J. Sultan, M.H. Al-Dahhan, Split internal-loop photobioreactor for *Scenedesmus* sp. microalgae:  
609 Culturing and hydrodynamics, *Chinese Journal of Chemical Engineering* (2020).  
610 <https://doi.org/https://doi.org/10.1016/j.cjche.2020.07.058>.
- 611 [9] L.S. Sabri, A.J. Sultan, M.H. Al-Dahhan, Investigating the cross-sectional gas holdup distribution in a split  
612 internal-loop photobioreactor during microalgae culturing using a sophisticated computed tomography (CT)  
613 technique, *Chemical Engineering Research and Design* 149 (2019) 13-33.  
614 <https://doi.org/https://doi.org/10.1016/j.cherd.2019.06.017>.
- 615 [10] K.L. Terry, Photosynthesis in modulated light: Quantitative dependence of photosynthetic enhancement on  
616 flashing rate, *Biotechnology and Bioengineering* 28(7) (1986) 988-995.  
617 <https://doi.org/https://doi.org/10.1002/bit.260280709>.
- 618 [11] M. Yoshioka, T. Yago, Y. Yoshie-Stark, H. Arakawa, T. Morinaga, Effect of high frequency of intermittent light  
619 on the growth and fatty acid profile of *Isochrysis galbana*, *Aquaculture* 338 (2012) 111-117.
- 620 [12] C. Vejrazka, M. Janssen, M. Streefland, R.H. Wijffels, Photosynthetic efficiency of *Chlamydomonas reinhardtii*  
621 in flashing light, *Biotechnology and Bioengineering* 108(12) (2011) 2905-2913.  
622 <https://doi.org/https://doi.org/10.1002/bit.23270>.
- 623 [13] M. Janssen, L. de Bresser, T. Baijens, J. Tramper, L.R. Mur, J.F.H. Snel, R.H. Wijffels, Scale-up aspects of  
624 photobioreactors: effects of mixing-induced light/dark cycles, *J Appl Phycol* 12(3) (2000) 225-237.  
625 <https://doi.org/10.1023/A:1008151526680>.
- 626 [14] P.C. Schulze, C. Brindley, J.M. Fernandez, R. Rautenberger, H. Pereira, R.H. Wijffels, V. Kiron, Flashing light  
627 does not improve photosynthetic performance and growth of green microalgae, *Bioresource Technology Reports*  
628 (2019) 100367. <https://doi.org/https://doi.org/10.1016/j.biteb.2019.100367>.

- 629 [15] C. Combe, P. Hartmann, S. Rabouille, A. Talec, O. Bernard, A. Sciandra, Long-term adaptive response to high-  
630 frequency light signals in the unicellular photosynthetic eukaryote *Dunaliella salina*, *Biotechnology and*  
631 *Bioengineering* 112(6) (2015) 1111-1121. <https://doi.org/https://doi.org/10.1002/bit.25526>.
- 632 [16] E. Sforza, D. Simionato, G.M. Giacometti, A. Bertucco, T. Morosinotto, Adjusted Light and Dark Cycles Can  
633 Optimize Photosynthetic Efficiency in Algae Growing in Photobioreactors, *PLOS ONE* 7(6) (2012) e38975.  
634 <https://doi.org/10.1371/journal.pone.0038975>.
- 635 [17] H. Takache, J. Pruvost, H. Marec, Investigation of light/dark cycles effects on the photosynthetic growth of  
636 *Chlamydomonas reinhardtii* in conditions representative of photobioreactor cultivation, *Algal Research* 8 (2015) 192-  
637 204. <https://doi.org/https://doi.org/10.1016/j.algal.2015.02.009>.
- 638 [18] J. Degen, A. Uebele, A. Retze, U. Schmid-Staiger, W. Trösch, A novel airlift photobioreactor with baffles for  
639 improved light utilization through the flashing light effect, *Journal of Biotechnology* 92(2) (2001) 89-94.
- 640 [19] L.-l. Wang, Y. Tao, X.-z. Mao, A novel flat plate algal bioreactor with horizontal baffles: structural optimization  
641 and cultivation performance, *Bioresource Technology* 164 (2014) 20-27.
- 642 [20] J. Huang, Y. Li, M. Wan, Y. Yan, F. Feng, X. Qu, J. Wang, G. Shen, W. Li, J. Fan, Novel flat-plate  
643 photobioreactors for microalgae cultivation with special mixers to promote mixing along the light gradient,  
644 *Bioresource Technology* 159 (2014) 8-16.
- 645 [21] J. Huang, F. Feng, M. Wan, J. Ying, Y. Li, X. Qu, R. Pan, G. Shen, W. Li, Improving performance of flat-plate  
646 photobioreactors by installation of novel internal mixers optimized with computational fluid dynamics, *Bioresource*  
647 *Technology* 182 (2015) 151-159.
- 648 [22] Z. Yang, J. Cheng, X. Xu, J. Zhou, K. Cen, Enhanced solution velocity between dark and light areas with  
649 horizontal tubes and triangular prism baffles to improve microalgal growth in a flat-panel photo-bioreactor,  
650 *Bioresource Technology* 211 (2016) 519-526. <https://doi.org/https://doi.org/10.1016/j.biortech.2016.03.145>.
- 651 [23] H.J. Ryu, K.K. Oh, Y.S. Kim, Optimization of the influential factors for the improvement of CO<sub>2</sub> utilization  
652 efficiency and CO<sub>2</sub> mass transfer rate, *Journal of Industrial and Engineering Chemistry* 15(4) (2009) 471-475.  
653 <https://doi.org/https://doi.org/10.1016/j.jiec.2008.12.012>.
- 654 [24] J.C. Merchuk, M. Gluz, I. Mukmenev, Comparison of photobioreactors for cultivation of the red microalga  
655 *Porphyridium* sp, *Journal of Chemical Technology & Biotechnology: International Research in Process,*  
656 *Environmental & Clean Technology* 75(12) (2000) 1119-1126.
- 657 [25] G. Nadal-Rey, D.D. McClure, J.M. Kavanagh, S. Cornelissen, D.F. Fletcher, K.V. Gernaey, Understanding  
658 gradients in industrial bioreactors, *Biotechnology Advances* 46 (2021) 107660.  
659 <https://doi.org/https://doi.org/10.1016/j.biotechadv.2020.107660>.
- 660 [26] J.P. Bitog, I.B. Lee, C.G. Lee, K.S. Kim, H.S. Hwang, S.W. Hong, I.H. Seo, K.S. Kwon, E. Mostafa, Application  
661 of computational fluid dynamics for modeling and designing photobioreactors for microalgae production: A review,  
662 *Computers and Electronics in Agriculture* 76(2) (2011) 131-147.  
663 <https://doi.org/http://dx.doi.org/10.1016/j.compag.2011.01.015>.
- 664 [27] B.A. Cho, R.W.M. Pott, The development of a thermosiphon photobioreactor and analysis using Computational  
665 Fluid Dynamics (CFD), *Chemical Engineering Journal* 363 (2019) 141-154.  
666 <https://doi.org/https://doi.org/10.1016/j.cej.2019.01.104>.
- 667 [28] C. McHardy, G. Luzi, C. Lindenberger, J.R. Agudo, A. Delgado, C. Rauh, Numerical analysis of the effects of  
668 air on light distribution in a bubble column photobioreactor, *Algal Research* 31 (2018) 311-325.  
669 <https://doi.org/https://doi.org/10.1016/j.algal.2018.02.016>.

- 670 [29] P. Ranganathan, A.K. Pandey, R. Sirohi, A. Tuan Hoang, S.-H. Kim, Recent advances in computational fluid  
671 dynamics (CFD) modelling of photobioreactors: Design and applications, *Bioresource Technology* 350 (2022)  
672 126920. <https://doi.org/https://doi.org/10.1016/j.biortech.2022.126920>.
- 673 [30] J.C.M. Pires, M.C.M. Alvim-Ferraz, F.G. Martins, Photobioreactor design for microalgae production through  
674 computational fluid dynamics: A review, *Renewable and Sustainable Energy Reviews* 79 (2017) 248-254.  
675 <https://doi.org/https://doi.org/10.1016/j.rser.2017.05.064>.
- 676 [31] G. Luzi, C. McHardy, C. Lindenberger, C. Rauh, A. Delgado, Comparison between different strategies for the  
677 realization of flashing-light effects – Pneumatic mixing and flashing illumination, *Algal Research* 38 (2019) 101404.  
678 <https://doi.org/https://doi.org/10.1016/j.algal.2018.101404>.
- 679 [32] R. Laifa, J. Morchain, L. Barna, P. Guiraud, A numerical framework to predict the performances of a tubular  
680 photobioreactor from operating and sunlight conditions, *Algal Research* 60 (2021) 102550.  
681 <https://doi.org/https://doi.org/10.1016/j.algal.2021.102550>.
- 682 [33] L. Li, Z.M.H. Mohd Shafie, T. Huang, R. Lau, C.-H. Wang, Multiphysics simulations of concentric-tube internal  
683 loop airlift photobioreactors for microalgae cultivation, *Chemical Engineering Journal* 457 (2023) 141342.  
684 <https://doi.org/https://doi.org/10.1016/j.cej.2023.141342>.
- 685 [34] E. Ertekin, J.M. Kavanagh, D.F. Fletcher, D.D. McClure, Validation studies to assist in the development of scale  
686 and system independent CFD models for industrial bubble columns, *Chemical Engineering Research and Design* 171  
687 (2021) 1-12. <https://doi.org/https://doi.org/10.1016/j.cherd.2021.04.023>.
- 688 [35] G. Nadal-Rey, J.M. Kavanagh, B. Cassells, S. Cornelissen, D.F. Fletcher, K.V. Gernaey, D.D. McClure,  
689 Modelling of industrial-scale bioreactors using the particle lifeline approach, *Biochemical Engineering Journal* 198  
690 (2023) 108989. <https://doi.org/https://doi.org/10.1016/j.bej.2023.108989>.
- 691 [36] D.D. McClure, J.M. Kavanagh, D.F. Fletcher, G.W. Barton, Development of a CFD Model of Bubble Column  
692 Bioreactors: Part One – A Detailed Experimental Study, *Chemical Engineering & Technology* 36(12) (2013) 2065-  
693 2070. <https://doi.org/10.1002/ceat.201300544>.
- 694 [37] W. Liu, N.N. Clark, A.I. Karamavruç, Relationship between bubble size distributions and chord-length  
695 distribution in heterogeneously bubbling systems, *Chemical Engineering Science* 53(6) (1998) 1267-1276.  
696 [https://doi.org/10.1016/S0009-2509\(97\)00426-0](https://doi.org/10.1016/S0009-2509(97)00426-0).
- 697 [38] D.D. McClure, H. Norris, J.M. Kavanagh, D.F. Fletcher, G.W. Barton, Validation of a Computationally Efficient  
698 Computational Fluid Dynamics (CFD) Model for Industrial Bubble Column Bioreactors, *Industrial & Engineering  
699 Chemistry Research* 53(37) (2014) 14526-14543. <https://doi.org/10.1021/ie501105m>.
- 700 [39] D.D. McClure, N. Aboudha, J.M. Kavanagh, D.F. Fletcher, G.W. Barton, Mixing in bubble column reactors:  
701 Experimental study and CFD modeling, *Chemical Engineering Journal* 264 (2015) 291-301.  
702 <https://doi.org/http://dx.doi.org/10.1016/j.cej.2014.11.090>.
- 703 [40] D.F. Fletcher, D.D. McClure, J.M. Kavanagh, G.W. Barton, CFD simulation of industrial bubble columns:  
704 Numerical challenges and model validation successes, *Applied Mathematical Modelling* 44 (2017) 25-42.  
705 <https://doi.org/https://doi.org/10.1016/j.apm.2016.08.033>.
- 706 [41] R. Clift, J.R. Grace, M.E. Weber, *Bubbles, Drops and Particles*, Academic Press, New York, 1978.
- 707 [42] D.D. McClure, J.M. Kavanagh, D.F. Fletcher, G.W. Barton, Experimental investigation into the drag volume  
708 fraction correction term for gas-liquid bubbly flows, *Chemical Engineering Science* 170 (2017) 91-97.  
709 <https://doi.org/http://dx.doi.org/10.1016/j.ces.2016.12.066>.

- 710 [43] A.D. Burns, T. Frank, I. Hamill, J.-M. Shi, The Favre Averaged Drag Model for Turbulent Dispersion in Eulerian  
711 Multi-Phase Flows, 5th International Conference on Multiphase Flow, Yokohama, Japan, 2004.
- 712 [44] W. Yao, C. Morel, Volumetric interfacial area prediction in upward bubbly two-phase flow, International Journal  
713 of Heat and Mass Transfer 47(2) (2004) 307-328.  
714 <https://doi.org/http://dx.doi.org/10.1016/j.ijheatmasstransfer.2003.06.004>.
- 715 [45] Ansys, CFX User's Manual, 2021.
- 716 [46] W. Gu, J.M. Kavanagh, D.D. McClure, A scalable model for EPA and fatty acid production by *Phaeodactylum*  
717 *tricornutum*, Frontiers in Bioengineering and Biotechnology 10 (2022). <https://doi.org/10.3389/fbioe.2022.1011570>.
- 718 [47] E. Marken, N. Ssebiyonga, J.K. Lotsberg, J.J. Stamnes, B. Hamre, Ø. Frette, A.S. Kristoffersen, S.R. Erga,  
719 Measurement and modeling of volume scattering functions for phytoplankton from Norwegian coastal waters, Journal  
720 of Marine Research 75(5) (2017) 579-603. <https://doi.org/10.1357/002224017822109514>.
- 721 [48] A. San Pedro, C.V. González-López, F.G. Acién, E. Molina-Grima, Marine microalgae selection and culture  
722 conditions optimization for biodiesel production, Bioresource Technology 134 (2013) 353-361.  
723 <https://doi.org/https://doi.org/10.1016/j.biortech.2013.02.032>.
- 724 [49] K.L. Terry, J. Hirata, E.A. Laws, Light-limited growth of two strains of the marine diatom *Phaeodactylum*  
725 *tricornutum* Bohlin: Chemical composition, carbon partitioning and the diel periodicity of physiological processes,  
726 Journal of Experimental Marine Biology and Ecology 68(3) (1983) 209-227.  
727 [https://doi.org/https://doi.org/10.1016/0022-0981\(83\)90054-0](https://doi.org/https://doi.org/10.1016/0022-0981(83)90054-0).
- 728 [50] V. Patil, T. Källqvist, E. Olsen, G. Vogt, H.R. Gislerød, Fatty acid composition of 12 microalgae for possible use  
729 in aquaculture feed, Aquaculture International 15(1) (2006) 1-9. <https://doi.org/10.1007/s10499-006-9060-3>.
- 730 [51] W. Gu, J.M. Kavanagh, D.D. McClure, Photoautotrophic production of eicosapentaenoic acid, Critical Reviews  
731 in Biotechnology (2021) 1-18. <https://doi.org/10.1080/07388551.2021.1888065>.
- 732 [52] S. Kim, Y.-J. Jung, O.-N. Kwon, K. Cha, B.-H. Um, D. Chung, C.-H. Pan, A Potential Commercial Source of  
733 Fucoxanthin Extracted from the Microalga *Phaeodactylum tricornutum*, Applied Biochemistry and Biotechnology  
734 166(7) (2012) 1843-1855. <https://doi.org/10.1007/s12010-012-9602-2>.
- 735 [53] K. Akita, F. Yoshida, Bubble Size, Interfacial Area, and Liquid-Phase Mass Transfer Coefficient in Bubble  
736 Columns, Industrial & Engineering Chemistry Process Design and Development 13(1) (1974) 84-91.  
737 <https://doi.org/10.1021/i260049a016>.
- 738 [54] Y. Kawase, M. Moo-Young, Mixing time in bioreactors, Journal of Chemical Technology & Biotechnology 44(1)  
739 (1989) 63-75. <https://doi.org/https://doi.org/10.1002/jctb.280440107>.
- 740 [55] J. Tramper, K. van't Riet, Basic Bioreactor Design, M.Dekker, New York, 1991.
- 741 [56] Y.S. Yun, J. Park, Attenuation of monochromatic and polychromatic lights in *Chlorella vulgaris* suspensions,  
742 Applied Microbiology and Biotechnology 55(6) (2001) 765-770. <https://doi.org/10.1007/s002530100639>.
- 743 [57] J.F. Cornet, C.G. Dussap, G. Dubertret, A structured model for simulation of cultures of the cyanobacterium  
744 *Spirulina platensis* in photobioreactors: I. Coupling between light transfer and growth kinetics, Biotechnology and  
745 Bioengineering 40(7) (1992) 817-825. <https://doi.org/10.1002/bit.260400709>.
- 746 [58] A.S. Mirón, M.C.C. García, A.C. Gómez, F.G.a. Camacho, E.M. Grima, Y. Chisti, Shear stress tolerance and  
747 biochemical characterization of *Phaeodactylum tricornutum* in quasi steady-state continuous culture in outdoor  
748 photobioreactors, Biochemical Engineering Journal 16(3) (2003) 287-297.  
749 [https://doi.org/https://doi.org/10.1016/S1369-703X\(03\)00072-X](https://doi.org/https://doi.org/10.1016/S1369-703X(03)00072-X).

750

751 **Declaration of interests**

752

753  The authors declare that they have no known competing financial interests or personal relationships  
754 that could have appeared to influence the work reported in this paper.

755

756  The authors declare the following financial interests/personal relationships which may be considered  
757 as potential competing interests:

758

759

760

761

762

763

764

- 765 • Comprehensive workflow for PBR modelling developed
- 766 • CFD model validated against comprehensive experimental data-set
- 767 • Best designs and operating conditions examined give 40% improvement in productivity
- 768 • Modelling workflow can be used for *in silico* design of photobioreactors

769