STOCK MARKET INDICES AND INTEREST RATES IN THE US AND EUROPE: PERSISTENCE AND LONG-RUN LINKAGES

Guglielmo Maria Caporale, Brunel University London, UK

Luis A. Gil-Alana, University of Navarra, Pamplona, Spain and Universidad Francisco de Vitoria, Madrid, Spain

Eduard Melnicenco, University of Navarra, Pamplona, Spain

Revised, October 2023

Structured Abstract

Purpose: We aim to analyse (i) the persistence of the S&P500 and DAX 30 stock indices as well as of the Fed's Effective Federal Funds rate and of the ECB's Marginal Lending Facility rate, and (ii) the long-run linkages between stock prices and interest rates in the US and Europe respectively.

Methodology: The methodology is based on the concepts of fractional integration and cointegration.

Findings: Using monthly data from January 1999 to December 2022, the results can be summarised as follows. All series examined are nonstationary: stock prices are found to be I(1) while interest rates display orders of integration substantially above 1, which implies a rejection of the hypothesis of mean reversion in all cases examined.

Originality: We use an appropriate econometric framework to obtain new, reliable empirical evidence. All four series are highly persistent, and mean reversion does not occur in any single case. Moreover, the fractional cointegration analysis suggests that stock prices and interest rates are not linked in the long run.

Keywords: Stock market prices; interest rates; persistence; fractional integration; fractional cointegration

JEL Classification: C22: C32: G15

Corresponding author: Professor Guglielmo Maria Caporale, Department of Economics and Finance, Brunel University London, Uxbridge, UB8 3PH, UK. Email: Guglielmo-Maria.Caporale@brunel.ac.uk; https://orcid.org/0000-0002-0144-4135

Luis A. Gil-Alana gratefully acknowledges financial support from the Grant PID2020-113691RB-I00 funded by MCIN/AEI/ 10.13039/501100011033, and from an internal Project from the Universidad Francisco de Vitoria.

Comments from the Editor and two anonymous reviewers are gratefully acknowledged.

1. Introduction

The degree of persistence of interest rates and stock prices is a very important issue for several reasons. Knowledge of the former is essential to assess the effectiveness of monetary policy in controlling inflation and the empirical relevance of alternative theories such as consumption-based asset pricing models and the Fisher effect. As for the latter, examining whether or not stock prices follow a random walk is a key test of market efficiency. Further, establishing whether or not these two variables are linked in the long run can shed light on the extent to which monetary policy can achieve financial stability.

In light of the above, this paper aims to examine (i) the degree of persistence of some representative interest rate and stock price series for the US and Europe; (ii) the possible existence of long-run equilibrium linkages between these two variables in each case. More specifically, the two interest rate series used for the empirical analysis are the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate; the stock indices are the S&P500 and the German DAX 30. The importance of the present study comes from using a fractional integration/cointegration approach. This is more general and flexible than the standard framework based on the I(0) versus I(1) (stationary versus non-stationary) dichotomy used in most previous papers. In particular, it allows for fractional values of the differencing parameter. Therefore it encompasses a much wider range of stochastic processes and of adjustment mechanisms towards the long-run equilibrium. This enables us to shed new light on the issues of interest, with important implications for both monetary authorities and investors.

Of particular importance is the nature of the monetary policy transmission mechanism. This describes how changes to interest rates flow through to economic activity and inflation. It is a complicated process subject to considerable uncertainty concerning the timing and size of the effects of interest rate changes on the economy. It will affect bank and money-market interest rates, expectations, asset prices, saving and investment decisions, and the supply of credit. Note that the Fed's Effective Federal Funds rate is the interest rate charged to banks when they lend money to each other overnight (it is also known as the overnight rate). The ECB's Marginal Lending Facility rate is instead the rate banks pay when they borrow from the ECB overnight (a collateral being required). Therefore in both cases an interest rate rise will decrease profitability by making debt more expensive and thus reducing the capital available for investment. In addition, it will make savings accounts and fixed income securities more attractive to investors, who will become less inclined to invest in equity. For both these reasons, one would expect a negative effect of higher interest rates on stock prices. However, the financial industry (banks, brokerages, mortgage companies, and insurance companies) benefits from an increase in interest rates by being able to charge more for lending. Therefore the total effect on stock prices of higher interest rates could be positive instead if the financial industry dominates. Interestingly, Bernanke and Kuttner (2005) concluded that the effects of unanticipated monetary policy actions on expected excess returns account for the largest part of the response of stock prices.

Some previous work has already examined the linkages between interest rates and stock prices. For instance, a study by Huang et al. (2016) analysed the impact on US stock indices (S&P500, NASDAQ, etc.) of interest rates, exchange rates and oil prices. They used weekly data from January 3, 2003 to March 27, 2015. They reported that stock prices tend to go up when oil prices increase, due to the expected recovery in the economy, and down when real interest rates increase. Stock prices also decrease when the US dollar appreciates against other important currencies such as the euro or the Swiss franc. In another related study, Hu et al. (2020) instead investigated the relationship between interest rates and stock market prices in China over the period from January 1996 to

December 2016. Surprisingly, they found that the Shanghai Composite Index is positively related to interest rates. Finally, Bats et al. (2020) studied how a negative interest rate period affects the price of bank stocks. They found that during such periods they face a disadvantage compared to general stocks. This is due to the fact that bank deposits are no longer attractive, and therefore investors move their money out. This has a negative impact on the balance sheet of the bank.

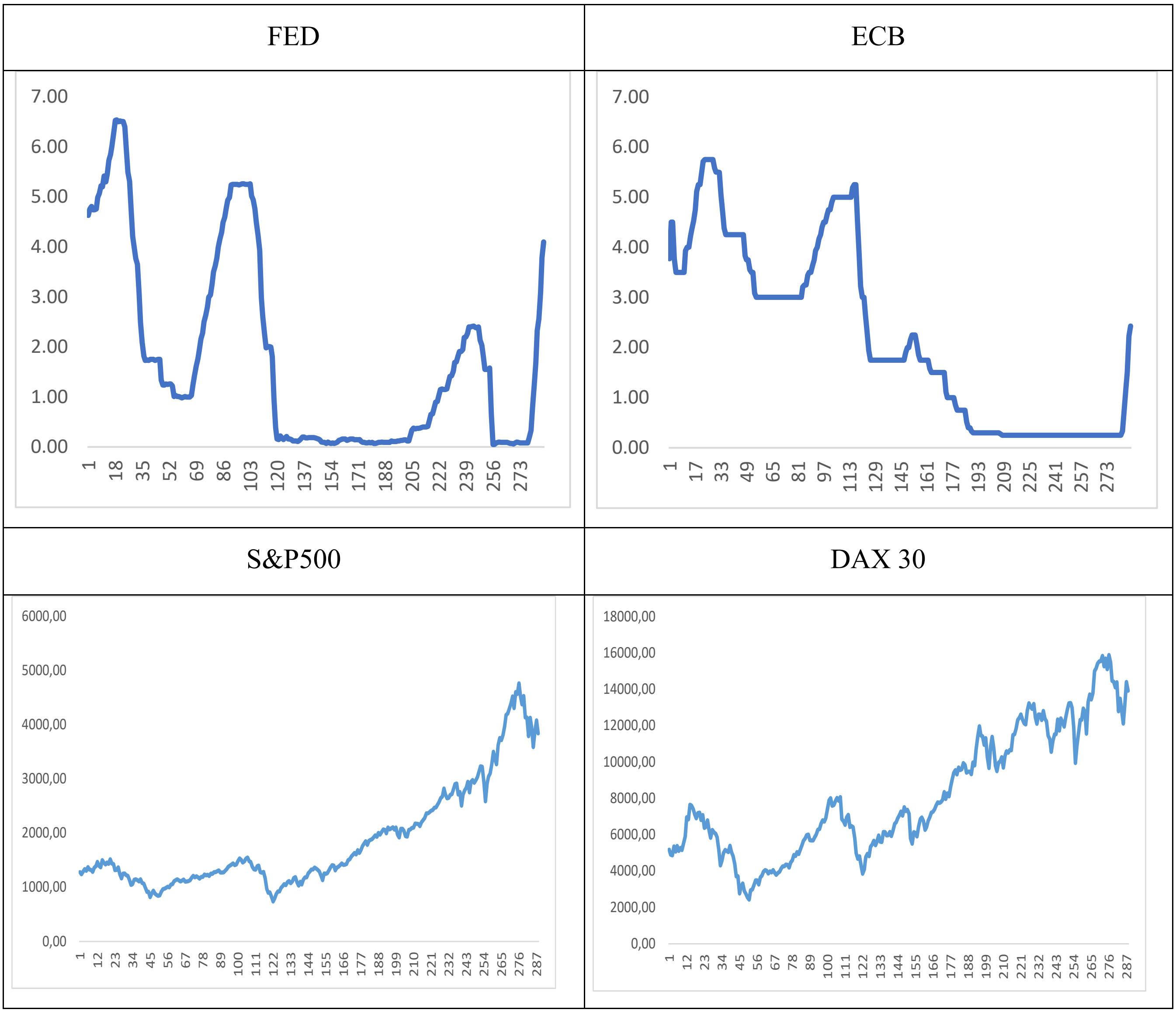
Note that causality could also run in the opposite direction, namely from stock prices to interest rates. For instance, Rigobon and Sack (2003) used an identification method based on heteroscedasticity. They reported that a 5 percent rise (fall) in the S&P 500 index increased the likelihood of a 25 basis point tightening (easing) by the Fed by about a half. Bjørnland and Leitemo (2009) estimated a Vector AutoRegressive (VAR) model and found bidirectional causality between the S&P500 and the Federal Funds rate. As can be gathered from the above discussion of the existing literature, there exists already a body of work analysing the relationship between interest rates and stock prices. However, none of the studies mentioned above uses long-memory techniques to examine both the degree of persistence of the two variables of interest and the possible existence of long-run linkages between the two of them. This is the main contribution of the present paper, which obtains such information by applying fractional integration/cointegration methods, namely by using a very general framework which allows for a much wider range of dynamic processes than those previously used in the literature. To preview our results, we find that exogenous shocks have permanent effects on the series examined. Moreover, stock prices and interest rates do not appear to be linked in the long run. This has important implications for both policy makers and market participants. Specifically, it suggests that monetary authorities should resort to unconventional measures to ensure financial stability, and that investors should rebalance their portfolios permanently in response to shocks affecting financial markets.

The layout of the paper is the following: Section 2 describes the data; Section 3 outlines the methodology; Section 4 discusses the empirical results; Section 5 summarises the main findings and offers some concluding remarks.

2. Data

The four series analysed are the Fed's effective Federal Funds rate (FED), the European Central Bank's Marginal Lending Facility rate (ECB), the Standard and Poor's 500 (S&P500) and the Deutscher Aktien Index 30 (DAX 30). The data are monthly closing prices and cover the period from January 1999 to December 2022, for a total of 288 observations. The start date corresponds to the introduction of the euro for accounting purposes and digital transactions, and the end date reflects data availability at the time of the estimation. The source for the S&P500 and the DAX 30 is Yahoo finance; specifically, we use the adjusted closing price (the results are almost the same using the closing price instead). The interest rate series have been obtained from the FRED webpage. All series are displayed in Figure 1 below.

Figure 1: Time series plots



Note: FED stands for the Fed's effective Federal Funds rate; ECB is the European Central Bank's Marginal Lending Facility rate; S&P500 and DAX denote the Standard and Poor's 500 and the Deutscher Aktien Index 30 respectively. The data are monthly closing prices and cover the period from January 1999 to December 2022.

It is noteworthy that the ECB lowered interest rates to stimulate the economy much later than the Fed in the wake of both the DOTCOM and the Global Financial Crisis (GFC), and also kept them at a higher level compared to the Fed. Then, at the onset of the Covid-19 pandemic in 2020, unlike the Fed, it was not able to reduce rates since these had been very close to 0 from 2014. Most recently, in response to a surge in inflation, the Fed increased interest rates in March 2022 whilst the ECB did so in July 2022. At the end of 2022, the ECB's Marginal Lending Facility rate was 2.75% whilst the Fed's Federal Funds Effective Rate was 4.33%. The lower panel plots show that both stock market indices exhibit volatility but have increased significantly since 1999 and peaked in December 2021, before starting to decrease and then to rebound.

Table 1: Descriptive statistics

	S&P 500	DAX 30	ECB rate	FED rate
No. of obs.	288.00	288.00	288.00	288.00
Mean	1852.574	8155.108	2.197	1.783
Std. Dev.	952.79	3444.35	1.802	1.946
Minimum	735.09	2423.87	0.25	0.05
25%	1190.17	5413.28	0.25	0.14
50%	1414.17	7103.23	1.75	1.095
75%	2248.44	11387.09	3.75	2.615
Maximum	4766.18	15884.86	5.75	6.54
ADF test	1.509*	-0.686*	-1.626*	0.06*
P values	0.99	0.85	0.46	0.06

*: Evidence of a unit root at the 95% level.

Table 1 reports some descriptive statistics for the series under examination. The number of observations is 288 in all cases. Regarding the stock indices, it is noteworthy that the DAX 30 has a higher mean and is more volatile than the S&P500. As for the interest rates, the ECB series has a higher mean but lower volatility than the FED one. Most importantly, all four series exhibit a unit root according to the ADF test results.

3. Methodology

For the empirical analysis we use fractional integration methods to model the series as I(d) processes, where d is the order of integration, which can be any real value, including fractional ones, as proposed by Granger (1980, 1981), Granger and Joyeux (1980) and Hosking (1981). Such a process x_t can be represented as follows:

$$(1-L)^d x_t = u_t \qquad t = 1, 2, \ldots, \tag{1}$$

where L is the lag operator, u_t is assumed to be stationary I(0) and d can be a fractional value (see Gil-Alana and Robinson, 1997 for an empirical application to the 14 macroeconomic variables analysed in Nelson and Plosser, 1992). Note that the parameter d can be interpreted as a measure of persistence, since the polynomial on the left-hand side of (1) can be expressed in terms of its Binomial expansion, such that for all real d,

$$(1-L)^d = \sum_{j=0}^{\infty} (d\ j)(-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \ldots, \tag{2}$$

and thus, if d is a fractional value, x_t can be expressed in terms for all its past history, i.e.,

$$x_t = d x_{t-1} + \frac{d(d-1)}{2} x_{t-2} - \ldots + u_t. \tag{3}$$

As already mentioned, the parameter d provides a measure of persistence, higher values of d corresponding to a higher degree of dependence between the observations.

The estimated model is the following:

$$y_t = \alpha + \beta t + x_t, \quad t = 1, 2, \ldots, \tag{4}$$

where α is a constant, β is the slope coefficient, and x_t is the error that follows the process given by equation (1). Combining equations (1) and (2) one obtains the following framework:

$$y_t = \alpha + \beta t + x_t, \quad (1-L)^d x_t = u_t, \quad t = 1, 2, \ldots, \tag{5}$$

to be estimated from the observed data. In particular, the parameter d is estimated under three different assumptions for the errors: White Noise, Bloomfield-type and Seasonal MA(1) errors. In the first case no time dependence structure is imposed; in the second the adopted (non-parametric) specification is used to approximate ARMA structures; in the third, given the monthly nature of the data, a seasonal MA(1) process is assumed which can be represented as:

$$u_t = \rho u_{t-12} + \varepsilon_t, \qquad t = 1, 2, \ldots \qquad (6)$$

where ρ is a (seasonal) AR parameter and ε_t is NID(0, σ^2). In each of those three cases, three model specifications are estimated:

i) without either a constant or a trend, i.e., imposing $\alpha = \beta = 0$ in equation (5).

ii) with a constant but without a trend, i.e., with $\beta = 0$ a priori in equation (5).

iii) with a constant and a (linear) time trend

Note that if there exists a linear combination of two (fractionally integrated) variables that displays an order of integration smaller than that of the individual series these are said to be (fractionally) cointegrated. Specifically, we follow the two-step approach originally developed by Engle and Granger (1982), testing first

i) If x_{1t} (stock prices) and x_{2t} (interest rates) are both integrated of a given order, say d, and then

ii) Regressing each stock price series on the corresponding interest rate series,

$$x_{1t} = \delta + \gamma x_{2t} + \varepsilon_t, \qquad t = 1, 2, \ldots$$

and testing if the estimated residuals are integrated of a smaller order, i.e., d – b, with b > 0, which would imply cointegration (see Engle and Granger, 1987, and more recently Cheung and Lai, 1993, and Gil-Alana, 2003) of a certain degree.

4. Empirical Results

As reported earlier, the four series are non-stationary according to the ADF test (other unit root tests such as Phillips and Perron, 1988, and Elliot et al., 1996, produce essentially the same results, but these are not reported to save space). However, it is well known that these tests have very low power against fractional alternatives (see Diebold and Rudebusch, 1991; Hassler and Wolters, 1993; Lee and Schmidt, 1996). This motivates the fractional integration approach we adopt to estimate the differencing parameter d using the three previously mentioned specifications for the error term: white noise (Table 1), Bloomfield-type errors (Table 2) and seasonal AR (Table 3). Each table reports the estimated values of d (and the corresponding 95% confidence intervals) for the three cases of no deterministic terms (2nd column), a constant only (3rd column), and both a constant and a linear trend (last column) in the regression model. The coefficients in bold are those from the specification selected on the basis of the statistical significance of the regressors.

Table 2 shows that for the DAX 30 the estimated value of d is 0.96 with a confidence interval of (0.88, 1.06), whilst the corresponding value for the S&P500 is 0.94 with a confidence interval of (0.88 and 1.01). For the logged series the corresponding estimates are 1.02 and 1.01 respectively, and the confidence intervals still contain 1. Therefore the null of d = 1 cannot be rejected, which represents evidence in favour of the Efficient Market Hypothesis (EMH). For the ECB rate the estimated value of d is 1.45 with a confidence interval of (1.36, 1.57), and for the Fed rate it is 1.56 with a confidence interval of (1.48, 1.66), and thus the null of d = 1 is decisively rejected for both interest rate series.

Table 2: Estimates of d. White noise errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
	i) Stock market prices		
DAX 30	0.96 (0.88, 1.05)	**0.96 (0.88, 1.06)**	0.96 (0.88, 1.06)
S&P500	0.91 (0.85, 0.98)	**0.94 (0.88, 1.01)**	0.93 (0.87, 1.00)
Log DAX 30	0.98 (0.91, 1.08)	**1.02 (0.94, 1.12)**	1.02 (0.94, 1.12)
Log S&P500	0.98 (0.91, 1.07)	**1.01 (0.94, 1.10)**	1.01 (0.94, 1.14)

ii) Interest rates			
ECB	1.27 (1.19, 1.38)	**1.45 (1.36, 1.57)**	1.45 (1.36, 1.59)
FED	1.25 (1.18, 1.33)	**1.56 (1.48, 1.66)**	1.56 (1.48, 1.66)

Note: We report the estimates of the differencing parameter for the three cases of i) no deterministic terms (in column 2); with an intercept (column 3) and with an intercept and a linear time trend (column 4). The values in parenthesis are the 95% confidence bands. In bold, the selected specification for each series.

Under the assumption of Bloomfield-type errors (Table 3) the estimated value of of d is 0.88 with a confidence interval of (0.77, 1.05) for the DAX 30, and 1.03 with a confidence interval of (0.94, 1.17) for the S&P500. Both of them are higher than in the previous case but are still within the I(1) interval. The corresponding estimates for the logged series are 0.97 with a confidence interval of (0.84, 1.13) for the DAX 30, and a 1 with a confidence interval of (0.89, 1.14) for the S&P500. Those for the ECB and Fed rates are 1.23 and 1.45 with corresponding confidence intervals of (1.06, 1.40) and (1.31, 1.60) respectively. These values are lower than in previous case, but still above the unit root.

Table 3: Estimates of d. Bloomfield errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
i) Stock market prices			
DAX 30	0.96 (0.83, 1.20)	**0.88 (0.77, 1.05)**	0.89 (0.77, 1.05)
S&P500	1.05 (0.93, 1.22)	**1.03 (0.94, 1.17)**	1.03 (0.94, 1.17)
Log DAX 30	0.97 (0.86, 1.14)	**0.97 (0.84, 1.13)**	0.97 (0.83, 1.13)
Log S&P500	0.98 (0.85, 1.15)	**1.00 (0.89, 1.14)**	1.00 (0.89, 1.15)
ii) Interest rates			
ECB	1.17 (1.07, 1.37)	**1.23 (1.06, 1.40)**	1.23 (1.06, 1.40)
FED	1.35 (1.22, 1.52)	**1.45 (1.31, 1.60)**	1.45 (1.31, 1.60)

Note: We report the estimates of the differencing parameter for the three cases of i) no deterministic terms (in column 2); with an intercept (column 3) and with an intercept and a linear time trend (column 4). The values in parenthesis are the 95% confidence bands. In bold, the selected specification for each series.

As can be seen, the estimates under the assumption of MA(1) errors (Table 4) are almost the same as those in the case of white noise errors. This again supports the I(1) specification for stock prices but rejects it in favour of higher values of d for the interest rates. Similar results were obtained when using other parametric (Sowell, 1992) or semiparametric (Shimotsu and Phillips, 2001) methods, all of them supporting the I(1) specification in all cases examined.

Table 4: Estimates of d. Seasonal MA(1) errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
	i) Stock market prices		
DAX 30	0.96 (0.88, 1.05)	**0.96 (0.88, 1.05)**	0.95 (0.88, 1.05)
S&P500	0.91 (0.86, 0.99)	**0.93 (0.88, 1.01)**	0.93 (0.87, 1.00)
Log DAX 30	0.98 (0.90, 1.08)	**1.02 (0.94, 1.12)**	1.02 (0.94, 1.12)
Log S&P500	0.98 (0.90, 1.08)	**1.01 (0.94, 1.10)**	1.01 (0.94, 1.10)
	ii) Interest rates		
ECB	1.27 (1.19, 1.38)	**1.45 (1.36, 1.57)**	1.45 (1.36, 1.59)
FED	1.25 (1.18, 1.33)	**1.56 (1.48, 1.66)**	1.56 (1.48, 1.66)

Note: We report the estimates of the differencing parameter for the three cases of i) no deterministic terms (in column 2); with an intercept (column 3) and with an intercept and a linear time trend (column 4). The values in parenthesis are the 95% confidence bands. In bold, the selected specification for each series.

The next step is to check for the existence of a long-run relationship between the S&P500 and the Fed rate, as well as between the DAX 30 and the ECB rate. For this purpose we use the cointegration approach of Engle and Granger (1987). Table 5 displays the OLS estimates of α and β for these two regressions. Both intercepts are positive, whilst both slope coefficients are negative, and all of them are statistically significant.

Table 5: Estimates of the coefficients in the regression model

Regression model	Intercept (t-value)	Regr. Coefficient (t-value)
S&P500 / FED	3.2575 (212.67)	-0.0209 (-3.59)
DAX 30 / ECB	4.0340 (316.99)	-0.0741 (-16.54)

Note: Estimates of the intercept and the slope (with their corresponding t-values) in the OLS regression of stock market prices on interest rates.

Table 6: Estimates of d for the regression errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
i) White noise errors			
S&P500 / FED	**1.08 (1.01, 1.17)**	1.08 (1.01, 1.16)	1.08 (1.01, 1.16)
DAX 30 / ECB	**1.12 (1.04, 1.23)**	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)
ii) Bloomfield (autocorrelated) errors			
S&P500 / FED	**1.07 (0.95, 1.20)**	1.09 (0.98, 1.24)	1.09 (0.98, 1.24)
DAX 30 / ECB	**1.13 (0.96, 1.29)**	1.13 (0.98, 1.33)	1.13 (0.98, 1.33)

Note: We report the estimates of the differencing parameter for the three cases of i) no deterministic terms (in column 2); with an intercept (column 3) and with an intercept and a linear time trend (column 4). The values in parenthesis are the 95% confidence band. In bold, the selected specification for each series.

Table 6 reports the estimates d based on the errors in the above regression models. For cointegration to hold it is necessary that d = 0. Again three model specifications are used (with $\alpha = \beta = 0$, $\beta = 0$, α and β different from 0 respectively). The intercept and the time trend coefficients are found to be statistically insignificant and the estimates of d are above 1 in all four cases. When assuming white noise errors the estimates of d are significantly higher than 1. Under the assumption of autocorrelation the unit root null hypothesis cannot be rejected. The hypothesis of mean reversion ($d < 1$) is rejected in all four cases, thus no evidence of cointegration has been found so far.

Since the residuals are clearly nonstationary, least squares and generalized least squares estimates will be inconsistent (see Robinson and Hidalgo, 1997). Robinson (1994) proposed a semi-parametric NBFDLS estimator which uses OLS on a degenerated band of frequencies around the origin. An improved version of the test for the stationary case is given in Christensen and Nielsen (2006).

In the two-variable case, the NBFDLS estimator proposed in Robinson (1994) is given by:

$$\hat{\beta} = \left\{ \frac{1}{m} \sum_{j=1}^{m} \mathrm{Re}\left[I_{y_1 y_1}\left(\lambda_j\right) \right] \right\}^{-1} \times \frac{1}{m} \sum_{j=1}^{m} \mathrm{Re}\left[I_{y_1 y_1}\left(\lambda_j\right) \right] \tag{5}$$

which is asymptotically distributed as:

$$\sqrt{m}\lambda_m^{d_e - d}\left(\hat{\beta} - \beta_0\right) \xrightarrow{D} N\left[0, \frac{g_e\left(1-2d\right)^2}{2g_{y1}\left(1-2d-2d_e\right)}\right], \tag{6}$$

where g_{y1} and g_e are the elements of a G diagonal 2×2 matrix. From (6), normality is ensured as long as $d + d_e < 0.5$ (Christensen and Nielsen, 2006). Note that this estimator crucially depends on the value of the bandwidth parameter m.

Table 7: Estimates of d in the regression errors

Series	No deterministic terms	An intercept	An intercept and a linear time trend
S&P500 / FED			
i) White noise errors			
m = 0.5	**0.99 (0.91, 1.08)**	1.05 (0.99, 1.13)	1.05 (0.99, 1.14)
m = 0.6	**0.97 (0.90, 1.06)**	0.95 (0.89, 1.05)	0.95 (0.88, 1.05)
m = 0.7	**0.99 (0.91, 1.08)**	1.05 (0.98, 1.13)	1.05 (0.98, 1.13)
ii) Bloomfield (autocorrelated) errors			
m = 0.5	**1.00 (0.86, 1.17)**	1.08 (0.96, 1.22)	1.09 (0.96, 1.23)
m = 0.6	**1.00 (0.86, 1.14)**	1.08 (0.96, 1.11)	1.09 (0.97, 1.11)
m = 0.7	**1.00 (0.86, 1.17)**	1.09 (0.97, 1.22)	1.09 (0.97, 1.23)
DAX 30 / ECB			
i) White noise errors			

m = 0.5	**1.00 (0.93, 1.10)**	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)
m = 0.6	**0.96 (0.89, 1.06)**	1.04 (0.96, 1.14)	1.04 (0.96, 1.14)
m = 0.7	**1.00 (0.93, 1.10)**	1.13 (1.05, 1.22)	1.13 (1.05, 1.22)
ii) Bloomfield (autocorrelated) errors			
m = 0.5	**0.99 (0.87, 1.15)**	1.13 (0.98, 1.33)	1.13 (0.98, 1.32)
m = 0.6	**0.95 (0.84, 1.13)**	0.96 (0.84, 1.12)	0.97 (0.83, 1.12)
m = 0.7	**0.99 (0.87, 1.16)**	1.13 (0.98, 1.33)	1.13 (0.98, 1.33)

Note: We report the estimates of the differencing parameter for the three cases of i) no deterministic terms (in column 2); with an intercept (column 3) and with an intercept and a linear time trend (column 4). The values in parenthesis are the 95% confidence bands. In bold, the selected specification for each series.

Table 7 reports the results based on this estimator, again for the three cases of no regressors, an intercept only, and an intercept as well as a time trend, for three different bandwidth parameters, m = 0.5, 0.6 and 0.7.[1] In all cases the estimates are again very close to 1 and the unit root null hypothesis cannot be rejected, which again provides evidence against (fractional) cointegration.

In the cointegration analysis it is implicitly assumed that all variables are stochastic. In what follows we depart from this assumption by imposing exogeneity of the interest rates. Therefore we estimate the following regressions with lagged rates:

$$S\&P\ 500_t = \alpha + \beta IR_{t-k} + x_t \qquad (7)$$

$$DAX_t = \alpha + \beta IR_{t-k} + x_t, \qquad (8)$$

where k is the lag index, and x_t is assumed again to be an I(d) process as in equation (1).

Table 8: Estimates in a regression of SP500(t) on FED(t-k)

K	d (95% band)	a (t-value)	b (t-value)
k = 1	1.00 (0.88, 1.14)	7.117 (157.79)	0.0006 (0.60)
k = 2	1.02 (0.90, 1.15)	7.159 (161.04)	0.0038 (0.38)
k = 3	1.01 (0.90, 1.18)	7.200 (160.37)	0.0043 (0.42)
k = 4	1.01 (0.90, 1.16)	7.173 (163.10)	0.0070 (0.60)

k = 5	1.01 (0.90, 1.14)	7.225 (165.08)	0.0007 (0.70)
k = 6	1.00 (0.90, 1.17)	7.192 (165.44)	0.0006 (0.68)
k = 7	0.93 (0.89, 1.15)	7.186 (166.27)	0.0068 (0.70)
k = 8	0.99 (0.88, 1.15)	7.157 (165.58)	0.0168 (0.70)
k = 9	0.99 (0.89, 1.16)	7.217 (166.68)	0.0068 (0.70)
k = 10	0.99 (0.90, 1.17)	7.733 (161.14)	0.0064 (0.66)
k = 11	1.00 (0.89, 1.18)	7.292 (170.71)	0.0061 (0.63)
k = 12	0.99 (0.90 1.17)	7.242 (170.24)	0.0062 (0.64)

Note: The values in column 2 are the estimated values of the differencing parameter (and the corresponding 95% confidence bands) in the model given by Eq. (7) where x_t is I(d). Columns 3 and 4 report the estimates of
the intercept and the slope with their corresponding t-values.

Table 8 reports the estimated values of d, α and β for the regression of S&P500 on the Fed rate. The estimates of d are very close for all values of k, and the confidence intervals contain 1, therefore the hypothesis d = 1 cannot be rejected. Note that the estimates of α, but not those of β, are statistically significant.

Table 9: Estimates in a regression of DAX 30(t) on ECB(t-k)

K	d (95% band)	a (t-value)	b (t-value)
k = 1	0.96 (0.85, 1.14)	8.492 (137.77)	0.0095 (0.70)
k = 2	0.99 (0.85, 1.17)	8.491 (140.23)	0.0055 (0.40)
k = 3	0.99 (0.84, 1.15)	8.490 (141.06)	0.0077 (0.56)
k = 4	0.97 (0.84, 1.15)	8.535 (142.09)	0.0087 (0.64)
k = 5	0.97 (0.84, 1.14)	8.590 (143.44)	0.0097 (0.72)
k = 6	0.96 (0.83, 1.16)	8.540 (142.98)	0.0098 (0.73)
k = 7	0.95 (0.84, 1.13)	8.373 (144.48)	0.0100 (0.75)
k = 8	0.96 (0.83, 1.14)	8.552 (143.80)	0.0100 (0.73)
k = 9	0.95 (0.83, 1.14)	8.624 (145.47)	0.0098 (0.74)
k = 10	0.95 (0.85, 1.13)	8.686 (145.44)	0.0098 (0.62)
k = 11	0.96 (0.86, 1.14)	8.846 (150.91)	0.0083 (0.55)
k = 12	0.96 (0.85, 1.14)	8.832 (150.52)	0.0072 (0.55)

Note: The values in column 2 are the estimated values of the differencing parameter (and the corresponding 95% confidence bands) in the model given by Eq. (7) where x_t is I(d). Columns 3 and 4 report the estimates of the intercept and the slope with their corresponding t-values.

Table 9 reports the corresponding results for the regression of the DAX 30 index on the ECB rate. The estimates of d are slightly below 1 but once more the unit root null hypothesis cannot be rejected. Similarly to the previous case, only the intercepts are statistically significant.

5. Conclusions

This paper has used fractional integration/cointegration methods to analyse (i) the persistence of the S&P500 and DAX 30 stock indices as well as of the Fed's Effective Federal Funds rate and the ECB's Marginal Lending Facility rate, and (ii) the long-run linkages between stock prices and interest rates in both the US and Europe. The data are monthly and the sample period goes from January 1999 to December 2022.

The results can be summarised as follows. From a statistical point of view, we find that all series examined are nonstationary. Stock prices are found to be I(1) while interest rates display orders of integration substantially above 1. Therefore all four series are highly persistent, and mean reversion does not occur in any case. Moreover, the fractional cointegration analysis suggests that stock prices and interest rates are not linked in the long run. Regarding the economic interpretation of our findings, it would appear that shocks to both stock prices and interest rates have permanent effects. This suggests the need for active policies to counteract them. However, our empirical evidence indicates that conventional monetary policy does not affect stock markets in the long run. Therefore, as shown by the global financial crisis of 2007-8, it might be necessary for monetary authorities to adopt unconventional measures such as quantitative easing to achieve their objectives such as financial stability. Further, investors should respond to shocks by making permanent adjustment to their portfolios since their effects will not die away.

Future work should extend the analysis in two ways. First, a multivariate model including other relevant variables such as inflation, money supply, exchange rates etc. should be estimated to shed further light on the linkages between interest rates and stock prices. Second, expectations and announcement effects should be incorporated into the model. It is well known that stock prices can react to anticipated interest rate changes or monetary announcements even before these take place. Because investors have already discounted those changes, the observed correction at the time of their implementation will then be smaller, and so will be the estimated impact. Therefore, not allowing for expectations and announcement effects could result in underestimating the strength of the linkages between monetary policy and stock markets.

Endnotes

1. These are values usually employed in the literature.

References

Bats, J., Giuliodori, M. and Houben, A. (2020) 'Monetary policy effects in times of negative interest rates: What do bank stock prices tell us?', *SSRN Electronic Journal* [Preprint]. doi:10.2139/ssrn.3720459**.**

Bjørnland, H.C., and K. Leitemo, (2009). Identifying the interdependence between US monetary policy and the stock market. Journal of Monetary Economics, 56(2), 275-282. doi:10.1016/j.jmoneco.2008.12.001

Bernanke, B. S. and K.N. Kuttner (2005). What explains the stock market's reaction to federal reserve policy? Journal of Finance, 60(3), 1221-1257. doi:10.1111/j.1540-6261.2005.00760.x

Cheung, Y.W. and K. Lai (1993) A fractional cointegration analysis of purchasing power parity, Journal of Business and Economic Statistics 11, 103-112. doi:10.1080/07350015.1993.10509936

Christensen, B. J. and M. Ø. Nielsen, (2006), 'Asymptotic normality of narrow-band least squares in the stationary fractional cointegration model and volatility forecasting', Journal of Econometrics 133(1), 343-371.
doi:10.1016/j.jeconom.2005.03.018

Dickey, D. A., and W. A. Fuller, (1979). Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association, 74(366), 427-431. doi:10.1080/01621459.1979.10482531

Dickey, D.A. and S.G. Pantula, (1987), Determining the order of differencing in autoregressive processes, Journal of Business and Economic Statistics 5(4), 455-461. doi:10.1080/07350015.1987.10509614

Diebold, F. X. and G.D. Rudebusch, 1991: On the power of Dickey-Fuller tests against fractional alternatives. *Economics Letters,* 35, 155-160.
https://doi.org/10.1016/0165
1765(91)90163-F

Elliot, G., Rothenberg, T.J., and Stock, J.H. (1996). Efficient Tests for an Autoregressive Unit Root, *Econometrica*, 64, 813-836.
doi:10.2307/2171846

Engle, R., and Granger, C.W.J. (1987). Cointegration and error correction. Representation, estimation and testing, Econometrica 55, 251-276.
doi:10.2307/1913236

Gil-Alana, L.A. (2003). Testing of fractional cointegration in macroeconomic time series, Oxford Bulletin of Economics and Statistics 65, 4, 517-529.
doi:10.1111/1468-0084.t01-1-00048

Gil-Alana, L. A., and P.M. Robinson, (1997). Testing of unit root and other nonstationary hypotheses in macroeconomic time series. Journal of Econometrics, 80(2), 241-268. doi:10.1016/S0304-4076(97)00038-9

Granger, C.W.J. (1986). Developments in the study of cointegrated economic variables. Oxford Bulletin of Economics and Statistics, 48(3), 213-228. doi:10.1111/j.1468-0084.1986.mp48003002.x

Granger, C.W.J., (1980), Long memory relationships and the aggregation of dynamic models. Journal of Econometrics 14, 227–238.
doi:10.1016/0304-4076(80)90092-5

Granger, C.W.J., (1981), Some properties of time series data and their use in econometric model specification. Journal of Econometrics 16, 121–130.
doi:10.1016/0304-4076(81)90079-8

Granger, C.W.J. and R. Joyeux (1980), An introduction to long memory time series and fractional differencing, Journal of Time Series Analysis 1, 1 ,15-29
doi:10.1111/j.1467-9892.1980.tb00297.x

Hashemzadeh, N., and P. Taylor, (1988). Stock prices, money supply, and interest rates: The question of causality. Applied Economics, 20(12), 1603-1611.
doi:10.1080/00036848800000091

Hassler, U., and Wolters, J., (1994). On the power of unit root tests against fractional alternatives. Economic Letters 45, 1–5.
doi:10.1016/0165-1765(94)90049-3

Hosking, J.R.M. (1981) Modelling persistence in hydrological time series using fractional differencing. Water Resources Research 20, 1898–1908
doi:10.1029/wr020i012p01898

Hu, J., Jiang, G.J. and Pan, G. (2020) 'Market reactions to central bank interest rate changes: Evidence from the Chinese stock market*', *Asia-Pacific Journal of Financial Studies*, 49(5), pp. 803–831. doi:10.1111/ajfs.12316.

Huang, W., Mollick, A.V. and Nguyen, K.H. (2016) 'U.S. stock markets and the role of real interest rates', The Quarterly Review of Economics and Finance, 59, 231–242.
doi:10.1016/j.qref.2015.07.006.

Kwiatkowski, D., Phillips, P.C.D., Schmidt, P., and Y. Shin (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?, Journal of Econometrics 54, 159–178.
doi:10.1016/0304-4076(92)90104-y

Lee, D., and Schmidt, P. (1996). On the power of the KPSS test of stationary against fractionally integrated alternatives, Journal of Econometrics 73, 285-302.
doi:10.1016/0304-4076(95)01741-0

Nelson C.R. and C.I. Plosser (1982), Trends and random walks in macroeconomic time series, Journal of Monetary Economics 10, 139-162.
doi:10.1016/0304-3932(82)90012-5

Phillips, P.C.B., and Perron, P. (1988). ‘Testing for a unit root in time series regression’, *Biometrika*, Vol. 75, No. 2, pp.335–346. doi:10.1093/biomet/75.2.335

Rigobon, R. and B. Sack, (2003), Measuring the reaction of monetary policy to the stock market. Quarterly Journal of Economics 118, 639–669.
DOI:1162/003355303321675473

Robinson, P. M. (1994), ‘Semiparametric analysis of long-memory time series’, Annals of Statistics 22, 515—539.
doi:10.1214/aos/1176325382

Robinson, P. M. and F.J. Hidalgo, (1997), ‘Time series regression with long-range dependence’, Annals of Statistics 25, 77—104.
doi:10.1214/aos/1034276622

Shimotsu, K. and P.C.B. Phillips (2005), Exact Local Whittle Estimation of Fractional Integration, The Annals of Statistics 33, 4, 1890-1933.

Sowell, F. (1992), Maximum likelihood estimation of stationary univariate fractionally integrated time series models, Journal of Econometrics 53, 1-3, 165-188.
doi.org/10.1016/0304-4076(92)90084-5