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Cubature Kalman Fusion Filtering Under Amplify-and-Forward Relays with Randomly Varying Channel Parameters

Jiaxing Li, Zidong Wang, *Fellow, IEEE*, Jun Hu, *Member, IEEE*, Hongli Dong, *Senior Member, IEEE*, and Hongjian Liu

***Abstract*—In this paper, the problem of cubature Kalman fusion filtering (CKFF) is addressed for multi-sensor systems under amplify-and-forward (AaF) relays. For the purpose of facilitating data transmission, AaF relays are utilized to regulate signal communication between sensors and filters. Here, the randomly varying channel parameters are represented by a set of stochastic variables whose occurring probabilities are permitted to exhibit bounded uncertainty. Employing the spherical-radial cubature principle, a local filter under AaF relays is initially constructed. This construction ensures and minimizes an upper bound of the filtering error covariance by designing an appropriate filter gain. Subsequently, the local filters are fused through the application of the covariance intersection fusion rule. Furthermore, the uniform boundedness of the filtering error covariance's upper bound is investigated through establishing certain sufficient conditions. The effectiveness of the proposed CKFF scheme is ultimately validated via a simulation experiment concentrating on a three-phase induction machine.**

***Index Terms*—Cubature Kalman filtering, amplify-and-forward relays, covariance intersection fusion, multi-sensor systems, uniform boundedness.**

I. INTRODUCTION

Accompanying the increasing popularity of multi-sensor networks, research interest in multi-sensor fusion filtering has surged due primarily to its extensive applications in areas such as inertial navigation, environmental monitoring, and fault diagnosis [2], [15], [27], [38]. Multi-sensor fusion filtering is generally categorized into centralized fusion filtering and distributed fusion filtering. The centralized approach is known for achieving optimal state estimation, albeit at the cost of significant resource consumption [40]. In contrast, the distributed fusion filtering scheme, which involves first computing a local estimate before sending it to a fusion center for further processing, offers notable advantages in reducing network load and enhancing reliability [19], [43]. Considerable efforts have been dedicated to exploring distributed fusion filtering, particularly for linear systems [3], [4], [44]. However, it is important to recognize that the practical systems often exhibit nonlinearity due to complex external environments, and the scarcity of research on distributed fusion filtering algorithms for nonlinear systems serves as the impetus for the current study.

Various filtering schemes have been proposed for estimating the state of dynamic systems in nonlinear contexts, see e.g. extended Kalman filtering (EKF) [5], [12], [26], unscented Kalman filtering (UKF) [11], [24], [46], and cubature Kalman filtering (CKF) methods [25], [29], [30]. The CKF, in particular, seeks to approximate the posterior probability distribution of the real state using a set of cubature points with equal weights, which operates under the common assumption that the predictive probability density functions of the system variables conform to a Gaussian distribution. Compared to other nonlinear filtering methods, CKF is seen as an effective approach for addressing nonlinear state estimation problems. Notably, CKF offers advantages over the popular EKF by avoiding the computation of the Jacobian matrix, thereby improving computational efficiency. Furthermore, CKF requires fewer sampling points than the UKF scheme, effectively reducing the computational load. Recent efforts in CKF research for nonlinear systems have seen notable developments. For instance, the CKF problem has been investigated in [20] within the context of nonlinear systems under stochastic communication scheduling. Moreover, the event-based CKF issue has been addressed in [22] for nonlinear dynamical systems.

The widespread deployment of wireless communication networks in practical engineering can be attributed to their notable advantages such as high scalability, low cost, and straightforward installation [8], [13], [14], [16], [41], [47]. Nevertheless, it is crucial to acknowledge that network com-

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J. Li is with the Department of Applied Mathematics, Harbin University of Science and Technology, Harbin 150080, China. (Email: 1820900028@stu.hrbust.edu.cn).

Z. Wang is with the Department of Computer Science, Brunel University London, Uxbridge UB8 3PH, United Kingdom. (Email: Zidong.Wang@brunel.ac.uk).

J. Hu is with the Department of Applied Mathematics, Harbin University of Science and Technology, Harbin 150080, China; and is also with the School of Automation, Harbin University of Science and Technology, Harbin 150080, China. (Email: jhu@hrbust.edu.cn).

H. Dong is with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing 163318, China; is also with the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Northeast Petroleum University, Daqing 163318, China; and is also with the National Key Laboratory of Continental Shale Oil of China, Northeast Petroleum University, Daqing 163318, China. (Email: shiningdhl@vip.126.com).

H. Liu is with the Key Laboratory of Advanced Perception and Intelligent Control of High-end Equipment, Ministry of Education, Anhui Polytechnic University, Wuhu 241000, China; and is also with the School of Mathematics and Physics, Anhui Polytechnic University, Wuhu 241000, China. (Email: hjliu2006@ahpu.edu.cn).

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munication quality is significantly affected by the distance between sensors and receivers [6], [9], [34], [42]. Due to the limited transmission capabilities of sensors, directly delivering data signals to receivers often poses challenges. To extend signal propagation distance and ensure transmission quality, relays are employed to receive and process data from source nodes before transmitting it to destination nodes [1], [21], [32], [48]. Relays are generally classified into several types, including amplify-and-forward (AaF) relays [17], [37], compute-and-forward relays [35], filtering-and-forward relays [36], and decode-and-forward relays [18], [33]. Notably, AaF relays have garnered significant research interest due to their clear engineering significance and ease of implementation.

It is noteworthy that filtering approaches involving AaF relays have been developed based mainly on deterministic channel parameters [39]. However, in practical engineering scenarios, channel parameters are often stochastic due to various external factors such as multi-path propagation, hardware imperfections, and environmental influences [10], [23], [28]. Therefore, it is both valuable and urgent to devise a novel filtering algorithm that incorporates AaF relays with stochastic channel parameters so as to reflect real-world conditions more accurately. Furthermore, it is crucial to acknowledge that much of the existing literature on AaF relays assumes precise probabilities for stochastic channel parameters. This assumption can be overly restrictive since variations in statistical experiments could influence the determination of these probabilities. A potential solution to this challenge is to incorporate *uncertain probabilities* into the characterization of stochastic channel parameters. Despite its importance, the problem of filtering in multi-sensor systems under relay scheduling, particularly when dealing with stochastic channel parameters with uncertain probabilities, remains underexplored, and this gives rise to another key motivation for the current paper.

Inspired by the discussions outlined above, this paper addresses the challenging task of cubature Kalman fusion filtering (CKFF) under AaF relays by focusing on minimizing the filtering error covariance (FEC). The key difficulties we face include: 1) how to accurately represent the stochastic nature of channel parameters? 2) how to develop a CKFF algorithm for multi-sensor systems operating under AaF relays with stochastic channel parameters? 3) how to theoretically assess the performance of the developed CKFF scheme in conjunction with AaF relays? In tackling these difficulties, the paper makes significant contributions in the following areas.

1) AaF relays are introduced into the design of the cubature Kalman fusion filter for multi-sensor systems, thereby offering a representation closer to practical scenarios.
2) An innovative CKFF strategy is developed that ensures enhanced filtering precision, which is achieved by deriving a minimum upper bound for the FEC, facilitated through the construction of an appropriate filter gain.
3) Sufficient conditions are established for the uniform boundedness of the proposed CKFF algorithm, ensuring robustness and reliability in its application.

Overall, these contributions aim to advance the field of CKFF under conditions involving AaF relays, addressing the unique complexities presented by stochastic channel parameters in multi-sensor systems.

Notations. $\mathbb{R}^m$ signifies the m-dimensional Euclidean space. The symbol I represents the identity matrix of a suitable dimension. I_ζ is used to denote the ζ-th column of the identity matrix. $\mathrm{Prob}\{\mathcal{E}\}$ stands for the probability of an event $\mathcal{E}$. $\mathbb{E}\{c\}$ is the mathematical expectation of a stochastic variable c. $\mathrm{diag}\{\cdot\}$ refers to the block-diagonal matrix. For a real-valued matrix $\mathcal{U}$, $\mathcal{U} > 0$ ($\mathcal{U} \geq 0$) indicates that $\mathcal{U}$ is positive definite (positive semi-definite) and $\mathrm{tr}\{\mathcal{U}\}$ is the trace of $\mathcal{U}$. Superscripts T and -1 signify the transpose and inverse of matrices, respectively.

II. Problem Formulation and Preliminaries

Consider the following class of nonlinear systems:

$$x_{s+1} = f(x_s) + \omega_s, \tag{1}$$

$$y_{i,s} = g_i(x_s) + \nu_{i,s}, \tag{2}$$

where s is the time instant, $x_s \in \mathbb{R}^{n_x}$ is the system state, the initial value x_0 obeys the Gaussian distribution with mean $\bar{x}_0$ and covariance P_0, and $y_{i,s} \in \mathbb{R}^{n_y}$ is the measurement of the i-th node for $i \in S \triangleq \{1, 2, \cdots, M\}$ with M being the number of sensors. ω_s is the zero-mean Gaussian white noise (ZMGWN) in the process with covariance R_s and $\nu_{i,s}$ is the ZMGWN in the measurement with covariance $Q_{i,s}$. $f(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ and $g_i(\cdot) : \mathbb{R}^{n_x} \to \mathbb{R}^{n_y}$ stand for known and continuously differentiable nonlinear functions.

In this paper, the AaF relays are deployed in the communication channels between sensors and filters to enlarge the signal propagation distance. Specifically, there exist two transmission channels between sensors and filters, i.e., the sensor-relay-channel and the relay-filter-channel.

Let $z_{i,s}$ be the measurement obtained by the i-th relay through the sensor-relay-channel, which satisfies

$$z_{i,s} = \sqrt{E_i^l}\gamma_{i,s} y_{i,s} + m_{i,s}, \tag{3}$$

where $E_i^l \in \mathbb{R}$ is the average signal energy of the i-th sensor, $m_{i,s}$ is the ZMGWN with covariance $Q_{i,s}^m > 0$ in the sensor-relay-channel, and $\gamma_{i,s}$ is the channel parameter satisfying the following probability distribution

$$\mathrm{Prob}\{\gamma_{i,s} = \gamma_{i,s}^{(l)}\} = \bar{\gamma}_{i,s}^{(l)} + \Delta\bar{\gamma}_{i,s}^{(l)},\ l = 1, 2, \cdots, \phi \tag{4}$$

where $\gamma_{i,s}^{(l)}$ is a scalar, ϕ is a positive integer, $0 \leq \bar{\gamma}_{i,s}^{(l)} + \Delta\bar{\gamma}_{i,s}^{(l)} \leq 1$, $\sum_{l=1}^{\phi}(\bar{\gamma}_{i,s}^{(l)} + \Delta\bar{\gamma}_{i,s}^{(l)}) = 1$, and $\Delta\bar{\gamma}_{i,s}^{(l)}$ is used to characterize the uncertainty satisfying $|\Delta\bar{\gamma}_{i,s}^{(l)}| \leq \rho_{i,s}^{(l)}$ with $\rho_{i,s}^{(l)}$ being a non-negative scalar.

After receiving the signal $z_{i,s}$, the AaF relay retransmits it to the filter side. Denote measurement received by the i-th filter via the relay-filter-channel as $\vec{y}_{i,s}$ modeled by

$$\vec{y}_{i,s} = \alpha_{i,s}\sqrt{E_i^r}\xi_{i,s} z_{i,s} + n_{i,s}, \tag{5}$$

where $E_i^r \in \mathbb{R}$ is the average signal energy of the i-th relay, $\alpha_{i,s} \in \mathbb{R}$ is the amplification factor, $n_{i,s}$ is the ZMGWN with covariance $Q_{i,s}^n > 0$ from the relay to the filter channel, and $\xi_{i,s}$ is the channel parameter satisfying

$$\mathrm{Prob}\{\xi_{i,s} = \xi_{i,s}^{(r)}\} = \bar{\xi}_{i,s}^{(r)} + \Delta\bar{\xi}_{i,s}^{(r)},\ r = 1, 2, \cdots, \varphi \tag{6}$$

where $\xi_{i,s}^{(r)}$ is a scalar, φ is a positive integer, $0 \leq \bar{\xi}_{i,s}^{(r)} + \Delta\bar{\xi}_{i,s}^{(r)} \leq 1$, $\sum_{r=1}^{\varphi}(\bar{\xi}_{i,s}^{(r)} + \Delta\bar{\xi}_{i,s}^{(r)}) = 1$, and $\Delta\bar{\xi}_{i,s}^{(r)}$ is utilized to describe the uncertainty satisfying $|\Delta\bar{\xi}_{i,s}^{(r)}| \leq \kappa_{i,s}^{(r)}$ with $\kappa_{i,s}^{(r)}$ being a non-negative scalar. Furthermore, it is assumed that stochastic variables x_0, ω_s, $\nu_{i,s}$, $\gamma_{i,s}$, $\xi_{i,s}$, $m_{i,s}$ and $n_{i,s}$ are mutually independent.

Remark 1: It is noted that when the distance between the sensor and filter is exceedingly long, the difficulties in data communication might be caused. In order to cope with such an issue, the AaF relays are adopted in the transmission process. To be specific, the deployment of AaF relays in transmission channels serves two primary purposes: to extend the signal propagation distance and to improve communication quality. In this context, the channel parameters are characterized by a collection of stochastic variables, each associated with uncertain probabilities. Unfortunately, the introduction of AaF relays into the system also introduces additional complexities to the CKFF problem. These challenges stem mainly from the stochastic nature of the channel parameters and the presence of channel noises, which complicate the process of accurately estimating and filtering the state of the system. Addressing these challenges is crucial for the effective application of CKFF in environments where AaF relays are utilized.

The local filter is designed as

$$\hat{x}_{i,s+1|s+1} = \hat{x}_{i,s+1|s} + K_{i,s+1}r_{i,s+1}, \tag{7}$$

where $r_{i,s+1} \triangleq \vec{y}_{i,s+1} - \alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}\hat{y}_{i,s+1|s}$, $\bar{\gamma}_{i,s} \triangleq \sum_{l=1}^{\phi}\gamma_{i,s}^{(l)}\bar{\gamma}_{i,s}^{(l)}$, $\bar{\xi}_{i,s} \triangleq \sum_{r=1}^{\varphi}\xi_{i,s}^{(r)}\bar{\xi}_{i,s}^{(r)}$, $E_i \triangleq E_i^l E_i^r$, $\hat{x}_{i,s+1|s}$ denotes the one-step prediction, $\hat{x}_{i,s|s}$ represents the state estimate, $\hat{y}_{i,s+1|s}$ is the measurement prediction, and $K_{i,s+1}$ denotes the filter gain to be designed.

To improve the estimation precision of the filtering algorithm, the covariance intersection (CI) fusion criterion is utilized in this paper to fuse the obtained local filters. Let $\hat{x}_{s|s}^{c}$ denote the fusion filter and $\vec{\Xi}_{s|s}$ stand for the fusion covariance. Then, the CI fusion rule can be described as

$$\begin{aligned}\hat{x}_{s|s}^{c} =& \vec{\Xi}_{s|s}\sum_{i=1}^{M}\varsigma_{i,s}\Xi_{i,s|s}^{-1}\hat{x}_{i,s|s},\\ \vec{\Xi}_{s|s} \triangleq& \left(\sum_{i=1}^{M}\varsigma_{i,s}\Xi_{i,s|s}^{-1}\right)^{-1},\end{aligned} \tag{8}$$

where $\Xi_{i,s|s}$ is an upper bound on the FEC, and $\varsigma_{i,s}$ is the weight parameter with $0 \leq \varsigma_{i,s} \leq 1$ and $\sum_{i=1}^{M}\varsigma_{i,s} = 1$.

The objectives of this paper are threefold as described as follows.

1) The first objective is to identify an upper bound of the FEC for each local filter, which involves constructing an appropriate filter gain with the aim of minimizing the identified upper bound. This step is crucial for ensuring that the local filters operate within an optimal error range, enhancing the overall accuracy and reliability of the system.
2) The second objective is to fuse all the local filters using the CI fusion rule. This fusion process is essential for combining the information from multiple filters, thereby creating a more comprehensive and accurate estimation of the system state.
3) The third objective is to investigate the uniform boundedness of the upper bound of the FEC. This analysis is key to understanding and evaluating the performance of the developed CKFF scheme. By ensuring that the upper bound is uniformly bounded, the paper aims to demonstrate the stability and effectiveness of the CKFF scheme under the challenging conditions posed by the presence of AaF relays and stochastic channel parameters.

III. Main Results

In this section, the focus is on proposing a CKFF scheme incorporating AaF relays and the development of this scheme involves several key steps: 1) computation of the upper bound of the FEC; 2) derivation of local filter gain; 3) design of the fusion filter; and 4) proof of the uniform boundedness of the upper bound.

Before proceeding, we denote

$$\begin{aligned}
&\Delta\bar{\gamma}_{i,s} \triangleq \sum_{l=1}^{\phi}\gamma_{i,s}^{(l)}\Delta\bar{\gamma}_{i,s}^{(l)},\ \Delta\bar{\xi}_{i,s} \triangleq \sum_{r=1}^{\varphi}\xi_{i,s}^{(r)}\Delta\bar{\xi}_{i,s}^{(r)},\\
&\vec{\gamma}_{i,s} \triangleq \sum_{l=1}^{\phi}(\gamma_{i,s}^{(l)})^2\bar{\gamma}_{i,s}^{(l)},\ \vec{\xi}_{i,s} \triangleq \sum_{r=1}^{\varphi}(\xi_{i,s}^{(r)})^2\bar{\xi}_{i,s}^{(r)},\\
&\Delta\vec{\gamma}_{i,s} \triangleq \sum_{l=1}^{\phi}(\gamma_{i,s}^{(l)})^2\Delta\bar{\gamma}_{i,s}^{(l)},\ \Delta\vec{\xi}_{i,s} \triangleq \sum_{r=1}^{\varphi}(\xi_{i,s}^{(r)})^2\Delta\bar{\xi}_{i,s}^{(r)},\\
&e_{i,s|s} \triangleq x_s - \hat{x}_{i,s|s},\ P_{i,s|s} \triangleq \mathbb{E}\{e_{i,s|s}e_{i,s|s}^T\},\\
&e_{i,s+1|s} \triangleq x_{s+1} - \hat{x}_{i,s+1|s},\ P_{i,s+1|s} \triangleq \mathbb{E}\{e_{i,s+1|s}e_{i,s+1|s}^T\},\\
&e_{i,s+1|s}^{y} \triangleq y_{i,s+1} - \hat{y}_{i,s+1|s},\ P_{i,s+1|s}^{xy} \triangleq \mathbb{E}\{e_{i,s+1|s}(e_{i,s+1|s}^{y})^T\},\\
&P_{i,s}^{yy} \triangleq \mathbb{E}\{y_{i,s}y_{i,s}^T\},\ P_{i,s+1|s}^{yy} \triangleq \mathbb{E}\{e_{i,s+1|s}^{y}(e_{i,s+1|s}^{y})^T\}.
\end{aligned} \tag{9}$$

The following lemmas are necessary in obtaining the main results of this paper.

Lemma 1: [24] For any positive scalar β and vectors $a, b \in \mathbb{R}^m$, we have

$$ab^T + ba^T \leq \beta aa^T + \beta^{-1}bb^T.$$

Lemma 2: [7] For matrices $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ and $\mathcal{O}$ of suitable dimensions, one has

$$\begin{aligned}\frac{\partial \mathrm{tr}\{\mathcal{X}\mathcal{Y}\mathcal{Z}\}}{\partial \mathcal{Y}} =& \mathcal{X}^T\mathcal{Z}^T,\ \frac{\partial \mathrm{tr}\{\mathcal{X}\mathcal{Y}^T\mathcal{Z}\}}{\partial \mathcal{Y}} = \mathcal{Z}\mathcal{X},\\ \frac{\partial \mathrm{tr}\{\mathcal{X}\mathcal{Y}\mathcal{Z}\mathcal{Y}^T\mathcal{O}\}}{\partial \mathcal{Y}} =& \mathcal{X}^T\mathcal{O}^T\mathcal{Y}\mathcal{Z}^T + \mathcal{O}\mathcal{X}\mathcal{Y}\mathcal{Z}.\end{aligned}$$

A. Local Filter Design

An upper bound of the local FEC is computed in the following theorem.

Theorem 1: For given scalars $\beta_1 > 0$ and $\beta_2 > 0$, an upper bound of the FEC $\Xi_{i,s+1|s+1}$ is computed as

$$\begin{aligned}&\Xi_{i,s+1|s+1}\\ =&(1+\beta_1)P_{i,s+1|s} + \alpha_{i,s+1}^2 E_i\check{\xi}_{i,s+1}(\vec{\gamma}_{i,s+1} + \vec{\rho}_{i,s+1})\end{aligned}$$

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$$\begin{aligned}&\times K_{i,s+1}P^{yy}_{i,s+1}K^T_{i,s+1}+\alpha^2_{i,s+1}E_i(\bar{\xi}_{i,s+1}+\bar{\kappa}_{i,s+1})^2\\&\times\breve{\gamma}_{i,s+1}K_{i,s+1}P^{yy}_{i,s+1}K^T_{i,s+1}+(1+\beta_2)\alpha^2_{i,s+1}E_i\bar{\xi}^2_{i,s+1}\\&\times\bar{\gamma}^2_{i,s+1}K_{i,s+1}P^{yy}_{i,s+1|s}K^T_{i,s+1}+(1+\beta_1^{-1}+\beta_2^{-1})\\&\times\alpha^2_{i,s+1}E_i\check{\lambda}^2_{i,s+1}K_{i,s+1}P^{yy}_{i,s+1}K^T_{i,s+1}+\alpha^2_{i,s+1}E^r_i\\&\times(\vec{\xi}_{i,s+1}+\vec{\kappa}_{i,s+1})K_{i,s+1}Q^m_{i,s+1}K^T_{i,s+1}+K_{i,s+1}\\&\times Q^n_{i,s+1}K^T_{i,s+1}-\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}\\&\times\big(P^{xy}_{i,s+1|s}K^T_{i,s+1}+K_{i,s+1}(P^{xy}_{i,s+1|s})^T\big),\end{aligned}\tag{10}$$

under the initial condition $P_{i,0|0}\leq\Xi_{i,0|0}$, where

$$\begin{aligned}\bar{\rho}_{i,s}&\triangleq\sum_{l=1}^{\phi}|\gamma^{(l)}_{i,s}|\rho^{(l)}_{i,s},\ \bar{\kappa}_{i,s}\triangleq\sum_{r=1}^{\varphi}|\xi^{(r)}_{i,s}|\kappa^{(r)}_{i,s},\\\vec{\rho}_{i,s}&\triangleq\sum_{l=1}^{\phi}(\gamma^{(l)}_{i,s})^2\rho^{(l)}_{i,s},\ \vec{\kappa}_{i,s}\triangleq\sum_{r=1}^{\varphi}(\xi^{(r)}_{i,s})^2\kappa^{(r)}_{i,s},\\\check{\lambda}_{i,s}&\triangleq\bar{\xi}_{i,s}\bar{\rho}_{i,s}+\bar{\kappa}_{i,s}\bar{\gamma}_{i,s}+\bar{\kappa}_{i,s}\bar{\rho}_{i,s},\\\breve{\xi}_{i,s}&\triangleq(\vec{\xi}_{i,s}+\vec{\kappa}_{i,s})+(\bar{\xi}_{i,s}-\bar{\kappa}_{i,s})^2,\\\breve{\gamma}_{i,s}&\triangleq(\vec{\gamma}_{i,s}+\vec{\rho}_{i,s})+(\bar{\gamma}_{i,s}-\bar{\rho}_{i,s})^2.\end{aligned}\tag{11}$$

Proof: According to (7) and (9), the filtering error is computed as

$$\begin{aligned}&e_{i,s+1|s+1}\\=&e_{i,s+1|s}-K_{i,s+1}\big(\alpha_{i,s+1}\sqrt{E_i}\xi_{i,s+1}\gamma_{i,s+1}y_{i,s+1}\\&+\alpha_{i,s+1}\sqrt{E^r_i}\xi_{i,s+1}m_{i,s+1}+n_{i,s+1}\\&-\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}\hat{y}_{i,s+1|s}\big).\end{aligned}\tag{12}$$

Adding the following term

$$\begin{aligned}&\alpha_{i,s+1}\sqrt{E_i}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\gamma_{i,s+1}K_{i,s+1}y_{i,s+1}\\&-\alpha_{i,s+1}\sqrt{E_i}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\gamma_{i,s+1}K_{i,s+1}y_{i,s+1}\\&+\alpha_{i,s+1}\sqrt{E_i}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})(\bar{\gamma}_{i,s+1}+\Delta\bar{\gamma}_{i,s+1})\\&\times K_{i,s+1}y_{i,s+1}-\alpha_{i,s+1}\sqrt{E_i}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\\&\times(\bar{\gamma}_{i,s+1}+\Delta\bar{\gamma}_{i,s+1})K_{i,s+1}y_{i,s+1}\end{aligned}\tag{13}$$

into the right-hand side of (12), it follows from (9) that

$$\begin{aligned}&e_{i,s+1|s+1}\\=&e_{i,s+1|s}-\alpha_{i,s+1}\sqrt{E_i}\tilde{\xi}_{i,s+1}\gamma_{i,s+1}K_{i,s+1}y_{i,s+1}\\&-\alpha_{i,s+1}\sqrt{E_i}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\tilde{\gamma}_{i,s+1}K_{i,s+1}y_{i,s+1}\\&-\alpha_{i,s+1}\sqrt{E_i}\lambda_{i,s+1}K_{i,s+1}y_{i,s+1}-K_{i,s+1}n_{i,s+1}\\&-\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}K_{i,s+1}e^y_{i,s+1|s}\\&-\alpha_{i,s+1}\sqrt{E^r_i}\xi_{i,s+1}K_{i,s+1}m_{i,s+1},\end{aligned}\tag{14}$$

where

$$\begin{aligned}\tilde{\xi}_{i,s}&\triangleq\xi_{i,s}-(\bar{\xi}_{i,s}+\Delta\bar{\xi}_{i,s}),\\\tilde{\gamma}_{i,s}&\triangleq\gamma_{i,s}-(\bar{\gamma}_{i,s}+\Delta\bar{\gamma}_{i,s}),\\\lambda_{i,s}&\triangleq\bar{\xi}_{i,s}\Delta\bar{\gamma}_{i,s}+\Delta\bar{\xi}_{i,s}\bar{\gamma}_{i,s}+\Delta\bar{\xi}_{i,s}\Delta\bar{\gamma}_{i,s}.\end{aligned}\tag{15}$$

In light of (9) and (14), it is clear that

$$\begin{aligned}&P_{i,s+1|s+1}\\=&\mathbb{E}\Big\{e_{i,s+1|s}e^T_{i,s+1|s}\Big\}+\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\tilde{\xi}^2_{i,s+1}\gamma^2_{i,s+1}\\&\times K_{i,s+1}y_{i,s+1}y^T_{i,s+1}K^T_{i,s+1}\Big\}+\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\\&\times(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})^2\tilde{\gamma}^2_{i,s+1}K_{i,s+1}y_{i,s+1}\\&\times y^T_{i,s+1}K^T_{i,s+1}\Big\}+\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\bar{\xi}^2_{i,s+1}\bar{\gamma}^2_{i,s+1}\\&\times K_{i,s+1}e^y_{i,s+1|s}(e^y_{i,s+1|s})^TK^T_{i,s+1}\Big\}\\&+\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\lambda^2_{i,s+1}K_{i,s+1}y_{i,s+1}y^T_{i,s+1}K^T_{i,s+1}\Big\}\\&+\mathbb{E}\Big\{\alpha^2_{i,s+1}E^r_i\xi^2_{i,s+1}K_{i,s+1}m_{i,s+1}m^T_{i,s+1}K^T_{i,s+1}\Big\}\\&+\mathbb{E}\Big\{K_{i,s+1}n_{i,s+1}n^T_{i,s+1}K^T_{i,s+1}\Big\}\\&+\sum_{\ell=1}^{21}\big(\Im_{i\ell,s+1}+\Im^T_{i\ell,s+1}\big),\end{aligned}\tag{16}$$

where

$$\begin{aligned}\Im_{i1,s+1}\triangleq&-\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}e_{i,s+1|s}\\&\times(e^y_{i,s+1|s})^TK^T_{i,s+1}\Big\},\\\Im_{i2,s+1}\triangleq&-\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}\lambda_{i,s+1}e_{i,s+1|s}y^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i3,s+1}\triangleq&\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\lambda_{i,s+1}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}K_{i,s+1}e^y_{i,s+1|s}\\&\times y^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i4,s+1}\triangleq&-\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}\tilde{\xi}_{i,s+1}\gamma_{i,s+1}e_{i,s+1|s}\\&\times y^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i5,s+1}\triangleq&-\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\tilde{\gamma}_{i,s+1}\\&\times e_{i,s+1|s}y^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i6,s+1}\triangleq&-\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E^r_i}\xi_{i,s+1}e_{i,s+1|s}m^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i7,s+1}\triangleq&-\mathbb{E}\Big\{e_{i,s+1|s}n^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i8,s+1}\triangleq&\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\tilde{\xi}_{i,s+1}\gamma_{i,s+1}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\\&\times\tilde{\gamma}_{i,s+1}K_{i,s+1}y_{i,s+1}y^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i9,s+1}\triangleq&\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\tilde{\xi}_{i,s+1}\gamma_{i,s+1}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}K_{i,s+1}\\&\times y_{i,s+1}(e^y_{i,s+1|s})^TK^T_{i,s+1}\Big\},\\\Im_{i10,s+1}\triangleq&\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i\tilde{\xi}_{i,s+1}\gamma_{i,s+1}\lambda_{i,s+1}K_{i,s+1}y_{i,s+1}\\&\times y^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i11,s+1}\triangleq&\mathbb{E}\Big\{\alpha^2_{i,s+1}\sqrt{E}_i\sqrt{E^r_i}\tilde{\xi}_{i,s+1}\gamma_{i,s+1}\xi_{i,s+1}K_{i,s+1}\\&\times y_{i,s+1}m^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i12,s+1}\triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}\tilde{\xi}_{i,s+1}\gamma_{i,s+1}K_{i,s+1}y_{i,s+1}\\&\times n^T_{i,s+1}K^T_{i,s+1}\Big\},\\\Im_{i13,s+1}\triangleq&\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\tilde{\gamma}_{i,s+1}\bar{\xi}_{i,s+1}\\&\times\bar{\gamma}_{i,s+1}K_{i,s+1}y_{i,s+1}(e^y_{i,s+1|s})^TK^T_{i,s+1}\Big\},\\\Im_{i14,s+1}\triangleq&\mathbb{E}\Big\{\alpha^2_{i,s+1}E_i(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\tilde{\gamma}_{i,s+1}\lambda_{i,s+1}\end{aligned}$$

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$$
\begin{aligned}
&\quad \times K_{i,s+1}y_{i,s+1}y_{i,s+1}^TK_{i,s+1}^T\Big\},\\
\Im_{i15,s+1} \triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}^2\sqrt{E_i}\sqrt{E_i^r}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\tilde{\gamma}_{i,s+1}\\
&\quad \times \xi_{i,s+1}K_{i,s+1}y_{i,s+1}m_{i,s+1}^TK_{i,s+1}^T\Big\},\\
\Im_{i16,s+1} \triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})\tilde{\gamma}_{i,s+1}\\
&\quad \times K_{i,s+1}y_{i,s+1}n_{i,s+1}^TK_{i,s+1}^T\Big\},\\
\Im_{i17,s+1} \triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}^2\sqrt{E_i}\sqrt{E_i^r}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}\xi_{i,s+1}K_{i,s+1}\\
&\quad \times e_{i,s+1|s}^y m_{i,s+1}^TK_{i,s+1}^T\Big\},\\
\Im_{i18,s+1} \triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}K_{i,s+1}e_{i,s+1|s}^y\\
&\quad \times n_{i,s+1}^TK_{i,s+1}^T\Big\},\\
\Im_{i19,s+1} \triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}^2\sqrt{E_i}\sqrt{E_i^r}\lambda_{i,s+1}\xi_{i,s+1}K_{i,s+1}y_{i,s+1}\\
&\quad \times m_{i,s+1}^TK_{i,s+1}^T\Big\},\\
\Im_{i20,s+1} \triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i}\lambda_{i,s+1}K_{i,s+1}y_{i,s+1}n_{i,s+1}^T\\
&\quad \times K_{i,s+1}^T\Big\},\\
\Im_{i21,s+1} \triangleq&\mathbb{E}\Big\{\alpha_{i,s+1}\sqrt{E_i^r}\xi_{i,s+1}K_{i,s+1}m_{i,s+1}n_{i,s+1}^T\\
&\quad \times K_{i,s+1}^T\Big\}.
\end{aligned}
$$

Considering the statistical properties of stochastic variables $\xi_{i,s}$, $\gamma_{i,s}$, $m_{i,s}$ and $n_{i,s}$, we can see easily that $\Im_{i\ell,s}$ and $\Im_{i\ell,s}^T$ are zero matrices for $\ell=4,5,\cdots,21$. As such, according to (9), the FEC (16) is rewritten as

$$
\begin{aligned}
&P_{i,s+1|s+1}\\
=&P_{i,s+1|s}+\alpha_{i,s+1}^2E_i\mathbb{E}\{\tilde{\xi}_{i,s+1}^2\gamma_{i,s+1}^2\}K_{i,s+1}P_{i,s+1}^{yy}\\
&\times K_{i,s+1}^T+\alpha_{i,s+1}^2E_i(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})^2\mathbb{E}\{\tilde{\gamma}_{i,s+1}^2\}\\
&\times K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T+\alpha_{i,s+1}^2E_i\bar{\xi}_{i,s+1}^2\bar{\gamma}_{i,s+1}^2\\
&\times K_{i,s+1}P_{i,s+1|s}^{yy}K_{i,s+1}^T+\alpha_{i,s+1}^2E_i\lambda_{i,s+1}^2K_{i,s+1}\\
&\times P_{i,s+1}^{yy}K_{i,s+1}^T+\alpha_{i,s+1}^2E_i^r\mathbb{E}\{\xi_{i,s+1}^2K_{i,s+1}m_{i,s+1}\\
&\times m_{i,s+1}^TK_{i,s+1}^T\}+\mathbb{E}\{K_{i,s+1}n_{i,s+1}n_{i,s+1}^TK_{i,s+1}^T\}\\
&+\sum_{\ell=1}^{3}\big(\Im_{i\ell,s+1}+\Im_{i\ell,s+1}^T\big).
\end{aligned}\tag{17}
$$

It is worth pointing out that the *accurate* FEC is unattainable. Consequently, we are dedicated to finding an upper bound of the FEC as a suboptimal solution to facilitate the design of the CKFF scheme. Firstly, it follows from (9), $|\Delta\bar{\gamma}_{i,s}^{(l)}|\le\rho_{i,s}^{(l)}$, and $|\Delta\bar{\xi}_{i,s}^{(r)}|\le\kappa_{i,s}^{(r)}$ that

$$
\begin{aligned}
|\Delta\bar{\gamma}_{i,s}|&\le\sum_{l=1}^{\phi}|\gamma_{i,s}^{(l)}|\rho_{i,s}^{(l)}=\bar{\rho}_{i,s},\\
|\Delta\bar{\xi}_{i,s}|&\le\sum_{r=1}^{\varphi}|\xi_{i,s}^{(r)}|\kappa_{i,s}^{(r)}=\bar{\kappa}_{i,s},\\
|\Delta\vec{\gamma}_{i,s}|&\le\sum_{l=1}^{\phi}(\gamma_{i,s}^{(l)})^2\rho_{i,s}^{(l)}=\vec{\rho}_{i,s},
\end{aligned}
$$

$$
|\Delta\vec{\xi}_{i,s}|\le\sum_{r=1}^{\varphi}(\xi_{i,s}^{(r)})^2\kappa_{i,s}^{(r)}=\vec{\kappa}_{i,s},\tag{18}
$$

where $\bar{\rho}_{i,s}$, $\bar{\kappa}_{i,s}$, $\vec{\rho}_{i,s}$ and $\vec{\kappa}_{i,s}$ are denoted in (11). With the help of (9), (15) and (18), we have

$$
\begin{aligned}
\mathbb{E}\{\tilde{\xi}_{i,s+1}^2\}=&\mathbb{E}\{\xi_{i,s+1}^2-(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})^2\}\\
\le&(\vec{\xi}_{i,s+1}+\vec{\kappa}_{i,s+1})+(\bar{\xi}_{i,s+1}-\bar{\kappa}_{i,s+1})^2\\
=&\breve{\xi}_{i,s+1},
\end{aligned}\tag{19}
$$

where $\breve{\xi}_{i,s+1}$ is defined in (11). Similarly, one has

$$
\begin{aligned}
\mathbb{E}\{\tilde{\gamma}_{i,s+1}^2\}\le&(\vec{\gamma}_{i,s+1}+\vec{\rho}_{i,s+1})+(\bar{\gamma}_{i,s+1}-\bar{\rho}_{i,s+1})^2\\
=&\breve{\gamma}_{i,s+1},
\end{aligned}\tag{20}
$$

where $\breve{\gamma}_{i,s+1}$ is defined in (11).

From (9), (18) and (19), we obtain

$$
\begin{aligned}
&\alpha_{i,s+1}^2E_i\mathbb{E}\{\tilde{\xi}_{i,s+1}^2\gamma_{i,s+1}^2\}K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T\\
\le&\alpha_{i,s+1}^2E_i\breve{\xi}_{i,s+1}(\vec{\gamma}_{i,s+1}+\vec{\rho}_{i,s+1})\\
&\times K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T.
\end{aligned}\tag{21}
$$

In view of (18) and (20), it is clear that

$$
\begin{aligned}
&\alpha_{i,s+1}^2E_i(\bar{\xi}_{i,s+1}+\Delta\bar{\xi}_{i,s+1})^2\mathbb{E}\{\tilde{\gamma}_{i,s+1}^2\}\\
&\times K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T\\
\le&\alpha_{i,s+1}^2E_i(\bar{\xi}_{i,s+1}+\bar{\kappa}_{i,s+1})^2\breve{\gamma}_{i,s+1}\\
&\times K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T.
\end{aligned}\tag{22}
$$

Noting (15) and (18), we have

$$
\begin{aligned}
\lambda_{i,s+1}^2\le&(\bar{\xi}_{i,s+1}\bar{\rho}_{i,s+1}+\bar{\kappa}_{i,s+1}\bar{\gamma}_{i,s+1}+\bar{\kappa}_{i,s+1}\bar{\rho}_{i,s+1})^2\\
=&\check{\lambda}_{i,s+1}^2,
\end{aligned}\tag{23}
$$

where $\check{\lambda}_{i,s+1}$ is defined in (11). According to (23), one has

$$
\begin{aligned}
&\alpha_{i,s+1}^2E_i\lambda_{i,s+1}^2K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T\\
\le&\alpha_{i,s+1}^2E_i\check{\lambda}_{i,s+1}^2K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T.
\end{aligned}\tag{24}
$$

Considering (9), (18) and the statistical properties of the channel noise sequences, one obtains

$$
\begin{aligned}
&\alpha_{i,s+1}^2E_i^r\mathbb{E}\Big\{\xi_{i,s+1}^2K_{i,s+1}m_{i,s+1}m_{i,s+1}^TK_{i,s+1}^T\Big\}\\
&+\mathbb{E}\Big\{K_{i,s+1}n_{i,s+1}n_{i,s+1}^TK_{i,s+1}^T\Big\}\\
\le&\alpha_{i,s+1}^2E_i^r(\vec{\xi}_{i,s+1}+\vec{\kappa}_{i,s+1})K_{i,s+1}Q_{i,s+1}^mK_{i,s+1}^T\\
&+K_{i,s+1}Q_{i,s+1}^nK_{i,s+1}^T.
\end{aligned}\tag{25}
$$

By virtue of (9), we have

$$
\begin{aligned}
&\Im_{i1,s+1}+\Im_{i1,s+1}^T\\
=&-\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}\big(P_{i,s+1|s}^{xy}K_{i,s+1}^T\\
&+K_{i,s+1}(P_{i,s+1|s}^{xy})^T\big).
\end{aligned}\tag{26}
$$

Using Lemma 1, other cross terms in the right-hand side of (17) can be handled as

$$
\begin{aligned}
&\Im_{i2,s+1}+\Im_{i2,s+1}^T\\
\le&\beta_1P_{i,s+1|s}+\beta_1^{-1}\alpha_{i,s+1}^2E_i\check{\lambda}_{i,s+1}^2K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T,\\
&\Im_{i3,s+1}+\Im_{i3,s+1}^T
\end{aligned}
$$

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$$\begin{aligned}&\leq\beta_2\alpha_{i,s+1}^2E_i\bar{\xi}_{i,s+1}^2\bar{\gamma}_{i,s+1}^2K_{i,s+1}P_{i,s+1|s}^{yy}K_{i,s+1}^T\\&\quad+\beta_2^{-1}\alpha_{i,s+1}^2E_i\check{\lambda}_{i,s+1}^2K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T,\end{aligned}\tag{27}$$

where β_1 and β_2 are positive scalars.

Substituting (21), (22) and (24)–(27) into (17) yields

$$\begin{aligned}&P_{i,s+1|s+1}\\&\leq(1+\beta_1)P_{i,s+1|s}+\alpha_{i,s+1}^2E_i\breve{\xi}_{i,s+1}(\vec{\gamma}_{i,s+1}+\vec{\rho}_{i,s+1})\\&\quad\times K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T+\alpha_{i,s+1}^2E_i(\bar{\xi}_{i,s+1}+\bar{\kappa}_{i,s+1})^2\\&\quad\times\breve{\gamma}_{i,s+1}K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T+(1+\beta_2)\alpha_{i,s+1}^2E_i\bar{\xi}_{i,s+1}^2\\&\quad\times\bar{\gamma}_{i,s+1}^2K_{i,s+1}P_{i,s+1|s}^{yy}K_{i,s+1}^T+(1+\beta_1^{-1}+\beta_2^{-1})\\&\quad\times\alpha_{i,s+1}^2E_i\check{\lambda}_{i,s+1}^2K_{i,s+1}P_{i,s+1}^{yy}K_{i,s+1}^T+\alpha_{i,s+1}^2E_i^r\\&\quad\times(\vec{\xi}_{i,s+1}+\vec{\kappa}_{i,s+1})K_{i,s+1}Q_{i,s+1}^mK_{i,s+1}^T+K_{i,s+1}\\&\quad\times Q_{i,s+1}^nK_{i,s+1}^T-\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}\\&\quad\times\big(P_{i,s+1|s}^{xy}K_{i,s+1}^T+K_{i,s+1}(P_{i,s+1|s}^{xy})^T\big).\end{aligned}\tag{28}$$

Consequently, it follows from (10) and (28) that $\Xi_{i,s+1|s+1}$ is an upper bound of $P_{i,s+1|s+1}$, and the proof of this theorem is complete. ■

In Theorem 1, the upper bound of the FEC is computed via solving matrix equations. Next, we aim to derive a proper filter gain, which is achieved through minimizing the trace of the obtained upper bound matrix.

Theorem 2: Let the filter gain $K_{i,s+1}$ be calculated as

$$K_{i,s+1}=\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}P_{i,s+1|s}^{xy}\Omega_{i,s+1}^{-1},\tag{29}$$

where

$$\begin{aligned}&\Omega_{i,s+1}\\&\triangleq\alpha_{i,s+1}^2E_i\breve{\xi}_{i,s+1}(\vec{\gamma}_{i,s+1}+\vec{\rho}_{i,s+1})P_{i,s+1}^{yy}\\&\quad+\alpha_{i,s+1}^2E_i(\bar{\xi}_{i,s+1}+\bar{\kappa}_{i,s+1})^2\breve{\gamma}_{i,s+1}P_{i,s+1}^{yy}\\&\quad+(1+\beta_2)\alpha_{i,s+1}^2E_i\bar{\xi}_{i,s+1}^2\bar{\gamma}_{i,s+1}^2P_{i,s+1|s}^{yy}\\&\quad+\alpha_{i,s+1}^2E_i^r(\vec{\xi}_{i,s+1}+\vec{\kappa}_{i,s+1})Q_{i,s+1}^m+Q_{i,s+1}^n\\&\quad+(1+\beta_1^{-1}+\beta_2^{-1})\alpha_{i,s+1}^2E_i\check{\lambda}_{i,s+1}^2P_{i,s+1}^{yy}.\end{aligned}\tag{30}$$

Then, $\mathrm{tr}\{\Xi_{i,s+1|s+1}\}$ is minimized, and the minimum upper bound $\Xi_{i,s+1|s+1}$ is computed as

$$\begin{aligned}\Xi_{i,s+1|s+1}=&(1+\beta_1)P_{i,s+1|s}-\alpha_{i,s+1}^2E_i\bar{\xi}_{i,s+1}^2\bar{\gamma}_{i,s+1}^2\\&\times P_{i,s+1|s}^{xy}\Omega_{i,s+1}^{-1}(P_{i,s+1|s}^{xy})^T.\end{aligned}\tag{31}$$

Proof: Taking the partial derivative of $\mathrm{tr}\{\Xi_{i,s+1|s+1}\}$ (as shown in (10)) regarding $K_{i,s+1}$ and using Lemma 2, one obtains

$$\begin{aligned}&\frac{\partial\mathrm{tr}\{\Xi_{i,s+1|s+1}\}}{\partial K_{i,s+1}}\\&=2\alpha_{i,s+1}^2E_i\breve{\xi}_{i,s+1}(\vec{\gamma}_{i,s+1}+\vec{\rho}_{i,s+1})K_{i,s+1}P_{i,s+1}^{yy}\\&\quad+2\alpha_{i,s+1}^2E_i(\bar{\xi}_{i,s+1}+\bar{\kappa}_{i,s+1})^2\breve{\gamma}_{i,s+1}K_{i,s+1}P_{i,s+1}^{yy}\\&\quad+2(1+\beta_2)\alpha_{i,s+1}^2E_i\bar{\xi}_{i,s+1}^2\bar{\gamma}_{i,s+1}^2K_{i,s+1}P_{i,s+1|s}^{yy}\\&\quad+2(1+\beta_1^{-1}+\beta_2^{-1})\alpha_{i,s+1}^2E_i\check{\lambda}_{i,s+1}^2K_{i,s+1}P_{i,s+1}^{yy}\\&\quad+2\alpha_{i,s+1}^2E_i^r(\vec{\xi}_{i,s+1}+\vec{\kappa}_{i,s+1})K_{i,s+1}Q_{i,s+1}^m\\&\quad+2K_{i,s+1}Q_{i,s+1}^n-2\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\\&\quad\times\bar{\gamma}_{i,s+1}P_{i,s+1|s}^{xy}.\end{aligned}\tag{32}$$

Letting

$$\frac{\partial\mathrm{tr}\{\Xi_{i,s+1|s+1}\}}{\partial K_{i,s+1}}=0,$$

the filter gain $K_{i,s+1}$ can be determined as in (29).

Next, substituting (29) into (10), one has

$$\begin{aligned}&\Xi_{i,s+1|s+1}\\&=(1+\beta_1)P_{i,s+1|s}+K_{i,s+1}\Omega_{i,s+1}K_{i,s+1}^T-\alpha_{i,s+1}\sqrt{E_i}\\&\quad\times\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}P_{i,s+1|s}^{xy}K_{i,s+1}^T-\alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\\&\quad\times\bar{\gamma}_{i,s+1}K_{i,s+1}(P_{i,s+1|s}^{xy})^T\\&=(1+\beta_1)P_{i,s+1|s}-\alpha_{i,s+1}^2E_i\bar{\xi}_{i,s+1}^2\bar{\gamma}_{i,s+1}^2P_{i,s+1|s}^{xy}\\&\quad\times\Omega_{i,s+1}^{-1}(P_{i,s+1|s}^{xy})^T,\end{aligned}\tag{33}$$

and the proof of this theorem is complete. ■

B. Fusion Filter Design

For the obtained local filters $\hat{x}_{i,s|s}$ $(i\in S)$ and the minimum upper bound matrices $\Xi_{i,s|s}$ $(i\in S)$, we employ the CI fusion principle of the form (8) to realize the fusion estimation, in which the weight parameters are determined by the solutions to the following optimization issue

$$\begin{aligned}&\min_{\varsigma_{1,s},\varsigma_{2,s},\cdots,\varsigma_{M,s}}\mathrm{tr}\{\vec{\Xi}_{s|s}\},\\&\text{s.t.}\sum_{i=1}^{M}\varsigma_{i,s}=1,\ 0\leq\varsigma_{i,s}\leq1.\end{aligned}\tag{34}$$

It is significant to observe that the nonlinearity leads to certain difficulties in calculating the error covariances. With the purpose of coping with such a problem, the CKF approach is utilized to approximate the one-step prediction and the related error covariances. Specifically, the steps of the developed CKFF algorithm are outlined in Algorithm 1. Next, we shall analyze the performance of the developed CKFF algorithm from a theoretical viewpoint.

C. Boundedness Analysis

The primary purpose of this section is to analyze the boundedness of the upper bound of the FEC to evaluate the performance of the developed CKFF algorithm.

Employing the Taylor expansion method, by resorting to the result in [45] and (9), we expand x_{s+1} and $\hat{x}_{i,s+1|s}$ around $\hat{x}_{i,s|s}$ to obtain

$$e_{i,s+1|s}=\Phi_{i,s}e_{i,s|s}+o(|e_{i,s|s}|)+\omega_s,\tag{35}$$

where

$$\begin{aligned}\Phi_{i,s}&\triangleq\frac{\partial f(x_s)}{\partial x_s}\Big|_{x_s=\hat{x}_{i,s|s}},\\o(|e_{i,s|s}|)&\triangleq M_{i,s}\Pi_{i,s}N_{i,s}e_{i,s|s},\end{aligned}$$

$M_{i,s}$ and $N_{i,s}$ are given matrices of suitable dimensions, and $\Pi_{i,s}$ is an unknown matrix describing the linearization error with $\Pi_{i,s}\Pi_{i,s}^T\leq I$. To this end, (35) can be rewritten as

$$e_{i,s+1|s}=(\Phi_{i,s}+M_{i,s}\Pi_{i,s}N_{i,s})e_{i,s|s}+\omega_s.\tag{36}$$

Algorithm 1: The CKFF Algorithm Under AaF Relays

Step 1. Initialize the parameters $\hat{x}_{i,0|0}$ and $\hat{\Xi}_{i,0|0}$, and set $s=0$.

Step 2. Select $2n_x$ cubature points

$$\hat{\chi}^{\zeta}_{i,s|s} = \hat{x}_{i,s|s} + \sqrt{n_x \hat{\Xi}_{i,s|s}} I_{\zeta},\ \zeta = 1,2,\cdots,n_x,$$
$$\hat{\chi}^{\zeta+n_x}_{i,s|s} = \hat{x}_{i,s|s} - \sqrt{n_x \hat{\Xi}_{i,s|s}} I_{\zeta},\ \zeta = 1,2,\cdots,n_x.$$

Step 3. Prediction

$$\hat{\chi}^{\zeta}_{i,s+1|s} = f\left(\hat{\chi}^{\zeta}_{i,s|s}\right),\ \zeta = 1,2,\cdots,2n_x,$$
$$\hat{x}_{i,s+1|s} = \frac{1}{2n_x}\sum_{\zeta=1}^{2n_x} \hat{\chi}^{\zeta}_{i,s+1|s},$$
$$\hat{P}_{i,s+1|s} = \frac{1}{2n_x}\sum_{\zeta=1}^{2n_x} \hat{\chi}^{\zeta}_{i,s+1|s}\left(\hat{\chi}^{\zeta}_{i,s+1|s}\right)^T - \hat{x}_{i,s+1|s}\hat{x}^T_{i,s+1|s} + R_s,$$
$$(\hat{\chi}^{\zeta}_{i,s+1|s})^* = \hat{x}_{i,s+1|s} + \sqrt{n_x \hat{P}_{i,s+1|s}} I_{\zeta},\ \zeta = 1,2,\cdots,n_x,$$
$$(\hat{\chi}^{\zeta+n_x}_{i,s+1|s})^* = \hat{x}_{i,s+1|s} - \sqrt{n_x \hat{P}_{i,s+1|s}} I_{\zeta},\ \zeta = 1,2,\cdots,n_x,$$
$$\hat{y}^{\zeta}_{i,s+1|s} = g_i\left((\hat{\chi}^{\zeta}_{i,s+1|s})^*\right),\ \zeta = 1,2,\cdots,2n_x,$$
$$\hat{y}_{i,s+1|s} = \frac{1}{2n_x}\sum_{\zeta=1}^{2n_x} \hat{y}^{\zeta}_{i,s+1|s},$$
$$\hat{P}^{yy}_{i,s+1|s} = \frac{1}{2n_x}\sum_{\zeta=1}^{2n_x} \hat{y}^{\zeta}_{i,s+1|s}\left(\hat{y}^{\zeta}_{i,s+1|s}\right)^T - \hat{y}_{i,s+1|s}\hat{y}^T_{i,s+1|s} + Q_{i,s+1},$$
$$\hat{P}^{xy}_{i,s+1|s} = \frac{1}{2n_x}\sum_{\zeta=1}^{2n_x} \hat{\chi}^{\zeta}_{i,s+1|s}\left(\hat{y}^{\zeta}_{i,s+1|s}\right)^T - \hat{x}_{i,s+1|s}\hat{y}^T_{i,s+1|s},$$
$$\hat{P}^{yy}_{i,s+1} = \frac{1}{2n_x}\sum_{\zeta=1}^{2n_x} \hat{y}^{\zeta}_{i,s+1|s}\left(\hat{y}^{\zeta}_{i,s+1|s}\right)^T + Q_{i,s+1}.$$

Step 4. Calculate the filter gain

$$\hat{K}_{i,s+1} = \alpha_{i,s+1}\sqrt{E_i}\bar{\xi}_{i,s+1}\bar{\gamma}_{i,s+1}\hat{P}^{xy}_{i,s+1|s}\hat{\Omega}^{-1}_{i,s+1},$$

where

$$\begin{aligned}\hat{\Omega}_{i,s+1} \triangleq& \alpha^2_{i,s+1}E_i\breve{\xi}_{i,s+1}(\vec{\gamma}_{i,s+1}+\vec{\rho}_{i,s+1})\hat{P}^{yy}_{i,s+1} + \alpha^2_{i,s+1}E_i \\ &\times(\bar{\xi}_{i,s+1}+\bar{\kappa}_{i,s+1})^2\breve{\gamma}_{i,s+1}\hat{P}^{yy}_{i,s+1} + (1+\beta_2)\alpha^2_{i,s+1} \\ &\times E_i\bar{\xi}^2_{i,s+1}\bar{\gamma}^2_{i,s+1}\hat{P}^{yy}_{i,s+1|s} + \alpha^2_{i,s+1}E^r_i \\ &\times(\vec{\xi}_{i,s+1}+\vec{\kappa}_{i,s+1})Q^m_{i,s+1} + Q^n_{i,s+1} \\ &+(1+\beta_1^{-1}+\beta_2^{-1})\alpha^2_{i,s+1}E_i\check{\lambda}^2_{i,s+1}\hat{P}^{yy}_{i,s+1}.\end{aligned}$$

Step 5. Update the local filter $\hat{x}_{i,s+1|s+1}$ using (7) and compute the minimum upper bound

$$\hat{\Xi}_{i,s+1|s+1} = (1+\beta_1)\hat{P}_{i,s+1|s} - \alpha^2_{i,s+1}E_i\bar{\xi}^2_{i,s+1}\bar{\gamma}^2_{i,s+1} \times \hat{P}^{xy}_{i,s+1|s}\hat{\Omega}^{-1}_{i,s+1}(\hat{P}^{xy}_{i,s+1|s})^T.$$

Step 6. Conduct the CI fusion operation

$$\hat{x}^c_{s+1|s+1} = \hat{\bar{\Xi}}_{s+1|s+1}\sum_{i=1}^{M}\varsigma_{i,s+1}\hat{\Xi}^{-1}_{i,s+1|s+1}\hat{x}_{i,s+1|s+1},$$
$$\hat{\bar{\Xi}}_{s+1|s+1} \triangleq \left(\sum_{i=1}^{M}\varsigma_{i,s+1}\hat{\Xi}^{-1}_{i,s+1|s+1}\right)^{-1},$$

where the weight parameter is optimized as

$$\min_{\varsigma_{1,s+1},\varsigma_{2,s+1},\cdots,\varsigma_{M,s+1}} \mathrm{tr}\{\hat{\bar{\Xi}}_{s+1|s+1}\},$$
$$\text{s.t.} \sum_{i=1}^{M}\varsigma_{i,s+1} = 1,\ 0 \le \varsigma_{i,s+1} \le 1.$$

Step 7. Repeat the related steps if not stopped.

Noting the statistical property of process noise ω_s, (9) and (36), we have

$$\begin{aligned}&P_{i,s+1|s} \\ =&\mathbb{E}\{(\Phi_{i,s}+M_{i,s}\Pi_{i,s}N_{i,s})e_{i,s|s}e^T_{i,s|s}(\Phi_{i,s}+M_{i,s}\Pi_{i,s} \\ &\times N_{i,s})^T\} + \mathbb{E}\{\omega_s\omega_s^T\} + \mathbb{E}\{(\Phi_{i,s}+M_{i,s}\Pi_{i,s}N_{i,s}) \\ &\times e_{i,s|s}\omega_s^T\} + \mathbb{E}\{\omega_s e^T_{i,s|s}(\Phi_{i,s}+M_{i,s}\Pi_{i,s}N_{i,s})^T\} \\ =&(\Phi_{i,s}+M_{i,s}\Pi_{i,s}N_{i,s})P_{i,s|s}(\Phi_{i,s}+M_{i,s}\Pi_{i,s}N_{i,s})^T \\ &+R_s. \end{aligned} \tag{37}$$

Next, it follows from Lemma 1 that

$$\begin{aligned}P_{i,s+1|s} \le& (1+\epsilon^{-1})\mathrm{tr}\{N_{i,s}P_{i,s|s}N^T_{i,s}\}M_{i,s}M^T_{i,s} \\ &+(1+\epsilon)\Phi_{i,s}P_{i,s|s}\Phi^T_{i,s} + R_s,\end{aligned} \tag{38}$$

where ϵ is a positive scalar.

For simplicity, we denote

$$\begin{aligned}\tilde{P}_{i,s+1|s} \triangleq& \hat{P}_{i,s+1|s} - P_{i,s+1|s}, \\ \aleph_{i,s+1} \triangleq& (1+\epsilon^{-1})\mathrm{tr}\{N_{i,s}(\Xi_{i,s|s}-\hat{\Xi}_{i,s|s})N^T_{i,s}\} \\ &\times M_{i,s}M^T_{i,s} + (1+\epsilon)\Phi_{i,s}(\Xi_{i,s|s}-\hat{\Xi}_{i,s|s}) \\ &\times\Phi^T_{i,s} + \tilde{P}_{i,s+1|s} + R_s.\end{aligned} \tag{39}$$

To proceed, the following assumption is useful in obtaining the uniform bound.

Assumption 1: There are positive constants $\bar{a}_i, \bar{n}_i, \bar{m}_i$ and $\bar{\chi}_i$ such that

$$\Phi_{i,s}\Phi^T_{i,s} \le \bar{a}_i I,\ N_{i,s}N^T_{i,s} \le \bar{n}_i I,$$
$$M_{i,s}M^T_{i,s} \le \bar{m}_i I,\ \aleph_{i,s} \le \bar{\chi}_i I.$$

Theorem 3: Under Assumption 1 and the initial value $\hat{\Xi}_{i,0|0} \le \vartheta_i I$, let $\theta_i \le \vartheta_i$ be satisfied with

$$\theta_i \triangleq (1+\beta_1)\big((1+\epsilon)\bar{a}_i\vartheta_i + (1+\epsilon^{-1})n_x\bar{n}_i\vartheta_i\bar{m}_i + \bar{\chi}_i\big). \tag{40}$$

Then, one has $\hat{\Xi}_{i,s|s} \le \vartheta_i I$ for any time instant $s \ge 1$.

Proof: The proof is performed by using the mathematical inductive method. Supposing $\hat{\Xi}_{i,s|s} \le \vartheta_i I$, we aim to prove that $\hat{\Xi}_{i,s+1|s+1} \le \vartheta_i I$ holds.

In view of Assumption 1, (38), (39) and $P_{i,s|s} \le \Xi_{i,s|s}$, we obtain

$$\begin{aligned}&\hat{P}_{i,s+1|s} \\ \le& (1+\epsilon^{-1})\mathrm{tr}\{N_{i,s}\hat{\Xi}_{i,s|s}N^T_{i,s}\}M_{i,s}M^T_{i,s} \\ &+(1+\epsilon)\Phi_{i,s}\hat{\Xi}_{i,s|s}\Phi^T_{i,s} + \aleph_{i,s+1} \\ \le& \big((1+\epsilon)\bar{a}_i\vartheta_i + (1+\epsilon^{-1})\mathrm{tr}\{\bar{n}_i\vartheta_i I\}\bar{m}_i + \bar{\chi}_i\big)I \\ =& \big((1+\epsilon)\bar{a}_i\vartheta_i + (1+\epsilon^{-1})n_x\bar{n}_i\vartheta_i\bar{m}_i + \bar{\chi}_i\big)I.\end{aligned} \tag{41}$$

It is not difficult to observe that $\hat{\Omega}_{i,s+1}$ (shown in Step 4 in Algorithm 1) is positive definite. As a result, combining the expression of $\hat{\Xi}_{i,s+1|s+1}$ given in Step 5 in Algorithm 1 with (41), one obtains

$$\begin{aligned}&\hat{\Xi}_{i,s+1|s+1} \\ =& -\alpha^2_{i,s+1}E_i\bar{\xi}^2_{i,s+1}\bar{\gamma}^2_{i,s+1}\hat{P}^{xy}_{i,s+1|s}\hat{\Omega}^{-1}_{i,s+1}(\hat{P}^{xy}_{i,s+1|s})^T \\ &+(1+\beta_1)\hat{P}_{i,s+1|s} \\ \le& (1+\beta_1)\hat{P}_{i,s+1|s}\end{aligned}$$

$$\leq \theta_i I, \tag{42}$$

where θ_i is defined in (40). It follows from (42) and $\theta_i \leq \vartheta_i$ that $\hat{\Xi}_{i,s+1|s+1} \leq \vartheta_i I$ holds, which ends the proof. ■

By virtue of the CI fusion rule and Theorem 3, there is an upper bound such that the fused upper bound of the FEC satisfies

$$\hat{\bar{\Xi}}_{s|s} \leq \left(\sum_{i=1}^{M} \varsigma_{i,s} \vartheta_i^{-1} \right)^{-1} I.$$

Remark 2: In this paper, a fusion filter for multi-sensor systems with AaF relays is designed within the framework of the CKF based on the CI fusion principle. Key theoretical developments in this paper are presented through several theorems. Theorem 1 deals with the computation of the upper bound of the FEC using matrix theory. Establishing this upper bound is crucial for assessing the potential error range of the filter's estimations. In Theorem 2, the optimal filter gain is determined by minimizing the upper bound matrix identified in Theorem 1. The derivation of an appropriate filter gain is essential for enhancing the accuracy and efficiency of the filter. Theorem 3 extensively discusses the boundedness of the upper bound of the FEC. Proving the boundedness is important for validating the theoretical robustness and reliability of the CKFF scheme. The paper also includes Algorithm 1, which outlines the specific implementation strategy of the proposed method. This algorithm is a step-by-step guide to applying the developed CKFF scheme in practical scenarios. In particular, due to the presence of the nonlinearity, a sequence of cubature points with equal weights is firstly selected in Step 2, and then in light of the spherical-radial cubature rule, the one-step prediction and its related error covariances are calculated in Step 3. An important aspect of the designed filter gain is its incorporation of various significant factors, such as time-varying system parameters, channel parameters, and channel noise covariances. The consideration of these factors ensures that the filter gain is dynamically adapted to the specific conditions of the multi-sensor systems with AaF relays, thereby improving the overall performance and applicability of the CKFF scheme in real-world scenarios.

Remark 3: As is well recognized, the traditional Kalman filtering can be viewed as an optimal estimation strategy within the context of linear systems subjected to Gaussian noise. Nevertheless, when the nonlinear systems are concerned, the traditional Kalman filtering method will be no longer effective. Compared with the traditional Kalman filtering scheme, the advantages of the designed filtering scheme can be highlighted as follows: 1) the CKF is adopted in this paper to provide an effective estimation algorithm for dealing with the nonlinear state estimation problems; 2) the local filters are fused based on the CI fusion rule to enhance the estimation accuracy; and 3) the AaF relays are deployed in the transmission channels to enlarge the transmission distance of data signals and ensure the communication quality.

Remark 4: In this paper, a comprehensive and systematic investigation is conducted on the CKFF problem for nonlinear systems equipped with AaF relays. The study stands out for its novel contributions in several key areas given as follows.

1) *Novelty of the CKFF Problem*: The CKFF problem as proposed in this study is a new exploration for the specified multi-sensor systems that are influenced by AaF relays. This introduces a unique perspective to the field, considering the specific challenges and dynamics introduced by the inclusion of AaF relays in multi-sensor systems.
2) *Integration of AaF Relays with Stochastic Channel Parameters*: A significant novelty of this study is the arrangement of AaF relays with stochastic channel parameters within the transmission channels. This arrangement is designed to enhance signal communication quality. The consideration of stochastic channel parameters in the design and operation of AaF relays represents a forward-thinking approach, acknowledging and addressing the inherent uncertainties in real-world communication environments.
3) *Examination of Boundedness in the CKFF Scheme*: Another notable contribution of this research is the thorough examination of the boundedness of the CKFF scheme. By providing sufficient conditions for boundedness, the study ensures that the CKFF algorithm remains robust and reliable under various operating conditions. This aspect is crucial for practical applications, where stability and predictability of the filtering process are essential.

IV. Simulation Results

In this section, a simulation example is adopted to validate the efficacy and superiority of the proposed CKFF algorithm. The objective is to remotely monitor/estimate the flux and the angular velocity of an induction machine through a communication network subject to AaF relay mechanism.

As given in [31], the state nonlinear function is described as

$$f(x_s) = \begin{bmatrix} x_s^{[1]} + h\left(b_1 x_s^{[1]} + k_1 x_s^{[2]} + b_2 x_s^{[3]} + k_2\right) \\ x_s^{[2]} + h\left(-k_1 x_s^{[1]} + b_1 x_s^{[2]} + b_2 x_s^{[4]}\right) \\ x_s^{[3]} + h\left(b_3 x_s^{[1]} + b_4 x_s^{[3]} + (k_1 - x_s^{[5]}) x_s^{[4]}\right) \\ x_s^{[4]} + h\left(b_3 x_s^{[2]} - (k_1 - x_s^{[5]}) x_s^{[3]} + b_4 x_s^{[4]}\right) \\ x_s^{[5]} + h\left(b_5 (x_s^{[1]} x_s^{[4]} - x_s^{[2]} x_s^{[3]}) + b_6 k_3\right) \end{bmatrix},$$

where $x_s = \begin{bmatrix} x_s^{[1]} & x_s^{[2]} & x_s^{[3]} & x_s^{[4]} & x_s^{[5]} \end{bmatrix}^T$ with $x_s^{[1]}$, $x_s^{[2]}$, $x_s^{[3]}$ and $x_s^{[4]}$ representing the components of the stator and the rotor flux in order, and $x_s^{[5]}$ being the angular velocity. k_1 denotes the frequency, k_2 refers to the amplitude of the stator voltage, and k_3 stands for the load torque.

The output equations are described as

$$g_1(x_s) = \begin{bmatrix} b_7 x_s^{[1]} + b_8 x_s^{[3]} \\ b_7 x_s^{[2]} + b_8 x_s^{[4]} \end{bmatrix},$$

$$g_2(x_s) = \begin{bmatrix} b_9 x_s^{[1]} + b_{10} x_s^{[3]} \\ b_9 x_s^{[2]} + b_{10} x_s^{[4]} \end{bmatrix},$$

$$g_3(x_s)=\begin{bmatrix} b_{11}x_s^{[1]}+b_{12}x_s^{[3]} \\ b_{11}x_s^{[2]}+b_{12}x_s^{[4]} \end{bmatrix},$$
$$g_4(x_s)=\begin{bmatrix} b_{13}x_s^{[1]}+b_{14}x_s^{[3]} \\ b_{13}x_s^{[2]}+b_{14}x_s^{[4]} \end{bmatrix}.$$

The initial values are selected as $(i=1,2,3,4)$

$$\begin{aligned} P_0 =&0.0001I,\ \hat{\Xi}_{i,0|0}=1.25I, \\ \bar{x}_0 =&\begin{bmatrix}0.25 & -0.55 & -0.4 & 0.2 & 0.15\end{bmatrix}^T, \\ \hat{x}_{1,0|0} =&\begin{bmatrix}0.22 & -0.45 & -0.54 & 0.23 & 0.22\end{bmatrix}^T, \\ \hat{x}_{2,0|0} =&\begin{bmatrix}0.21 & -0.38 & -0.54 & 0.32 & 0.19\end{bmatrix}^T, \\ \hat{x}_{3,0|0} =&\begin{bmatrix}0.42 & -0.55 & -0.51 & 0.31 & 0.21\end{bmatrix}^T, \\ \hat{x}_{4,0|0} =&\begin{bmatrix}0.09 & -0.62 & -0.52 & 0.33 & 0.21\end{bmatrix}^T. \end{aligned}$$

The related parameters with respect to the induction machine are chosen as $b_1=-0.186$, $b_2=-1.778$, $b_3=6.125$, $b_4=-6.234$, $b_5=0.081$, $b_6=4.643$, $b_7=-4.448$, $b_8=1.11$, $b_9=-4.428$, $b_{10}=1.21$, $b_{11}=-4.435$, $b_{12}=1.05$, $b_{13}=-4.441$, $b_{14}=1.14$, $h=0.1$, $k_1=2$, $k_2=0.1$ and $k_3=0.55$. The noise covariance matrices are set as $R_s=1\times10^{-6}I$, $Q_{i,s}=0.25I$, $Q_{i,s}^m=0.2I$ and $Q_{i,s}^n=0.1I$. The amplification factor is selected as $\alpha_{i,s}=0.25$. The average signal energy is set as $E_i^l=E_i^r=0.05$. The uncertainties $\Delta\bar{\gamma}_{i,s}^{(1)}$, $\Delta\bar{\gamma}_{i,s}^{(2)}$, $\Delta\bar{\gamma}_{i,s}^{(3)}$, $\Delta\bar{\xi}_{i,s}^{(1)}$, $\Delta\bar{\xi}_{i,s}^{(2)}$ and $\Delta\bar{\xi}_{i,s}^{(3)}$ are uniformly distributed over $[-0.01,0.01]$. Other parameters are selected as $\xi_{i,s}^{(1)}=\gamma_{i,s}^{(1)}=0.1$, $\xi_{i,s}^{(2)}=\gamma_{i,s}^{(2)}=0.5$, $\xi_{i,s}^{(3)}=\gamma_{i,s}^{(3)}=1$, $\bar{\xi}_{i,s}^{(1)}=0.2$, $\bar{\xi}_{i,s}^{(2)}=0.3$, $\bar{\xi}_{i,s}^{(3)}=0.5$, $\bar{\gamma}_{i,s}^{(1)}=0.4$, $\bar{\gamma}_{i,s}^{(2)}=0.3$, $\bar{\gamma}_{i,s}^{(3)}=0.3$, $\kappa_{i,s}^{(1)}=\kappa_{i,s}^{(2)}=\kappa_{i,s}^{(3)}=\rho_{i,s}^{(1)}=\rho_{i,s}^{(2)}=\rho_{i,s}^{(3)}=0.01$, $\beta_1=1\times10^{-6}$ and $\beta_2=1\times10^{-4}$.

For the sake of evaluating the performance of the developed CKFF algorithm, the root mean square error (RMSE) is adopted in this paper, which is defined as follows:

$$\mathrm{RMSE}_{i,s}\triangleq\sqrt{\frac{1}{\Upsilon}\sum_{j=1}^{\Upsilon}\sum_{\iota=1}^{5}(x_s^{[\iota],j}-\hat{x}_{i,s|s}^{[\iota],j})^2},$$

where Υ denotes the number of the Monte Carlo experiment runs with $\Upsilon=50$ in this paper, $x_s^{[\iota],j}$ and $\hat{x}_{i,s|s}^{[\iota],j}$ denote the ι-th element of the real state and its estimate at the j-th Monte Carlo experiment, respectively.

Building on the theoretical foundations, the CKFF scheme can be implemented to deal with the proposed issue, and the related experiment results are depicted in Figs. 1–12. Specifically, the trajectories of the real states and their estimates are displayed in Figs. 1–5. These figures obviously demonstrate that the developed CKFF algorithm can reliably estimate the system states, even taking into consideration the effects of AaF relays. The trajectories of the trace of the minimum upper bound under local filters and the fusion filter are portrayed in Fig. 6. It is clear from this figure that the minimum upper bound under the fusion filter significantly stays below the local filters. The curves of the RMSE and its upper bound are exhibited in Fig. 7, in which the RMSE definitely remains below the upper bound, thus demonstrating the effectiveness of the CKFF scheme in practical scenarios.

It is well known that the EKF is based on the linearization technique and therefore works well for systems with relatively weak nonlinearities. In this example, the nonlinearities addressed are relatively strong (described by 2nd-order polynomials) and therefore the CKF is expected to outperform the corresponding EKF, which is indeed the case as confirmed by the simulation. It is seen from Figs. 8–11 that the RMSE calculated by the local CKF always keeps below the local EKF. Moreover, with the purpose of exhibiting the superiority of the developed CKFF algorithm, the curves of RMSE regarding the CKFF and the local CKF are given in Fig. 12, from which we observe that the RMSE of the CKFF stays below the CKF. All these experiment results confirm that the effectiveness of the designed CKFF algorithm can be well ensured in handling the state estimation problem in relation to the induction machine.

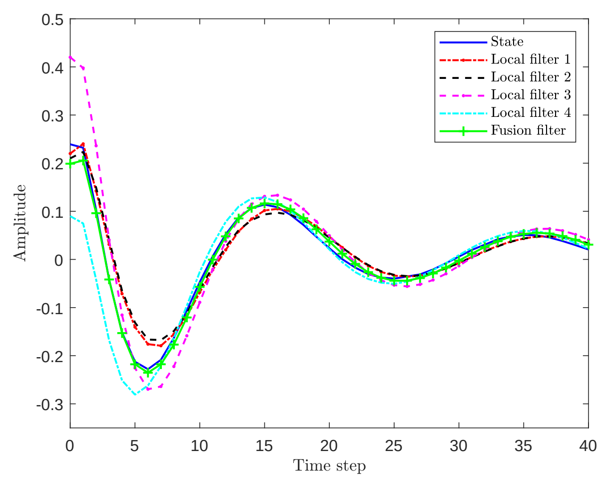

Fig. 1. Trajectories of $x_s^{[1]}$ and its estimates.

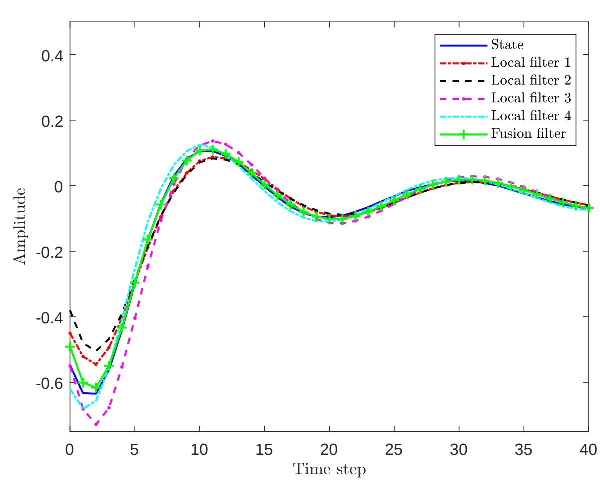

Fig. 2. Trajectories of $x_s^{[2]}$ and its estimates.

V. Conclusion

In this paper, the CKFF issue has been handled for multi-sensor systems with AaF relays. To enhance communication

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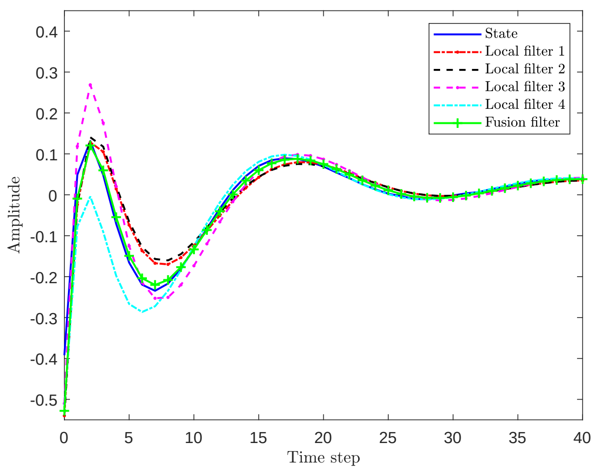

Fig. 3. Trajectories of $x_s^{[3]}$ and its estimates.

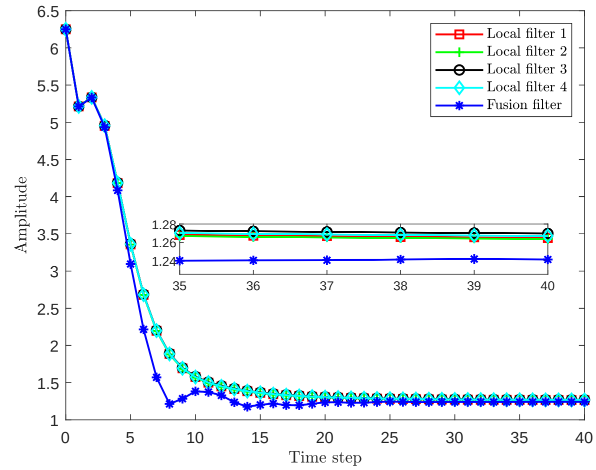

Fig. 6. Trace of the upper bounds.

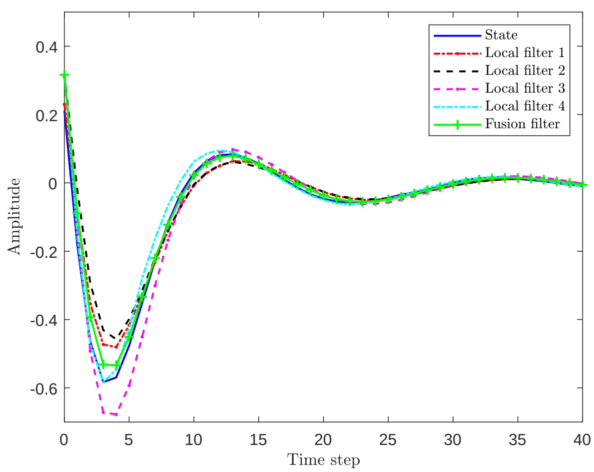

Fig. 4. Trajectories of $x_s^{[4]}$ and its estimates.

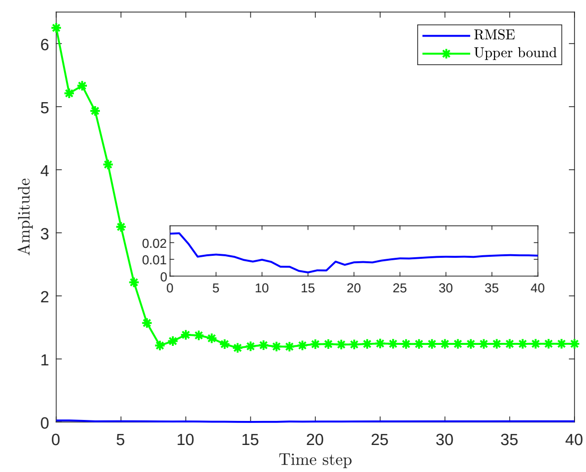

Fig. 7. The RMSE and the trace of upper bound.

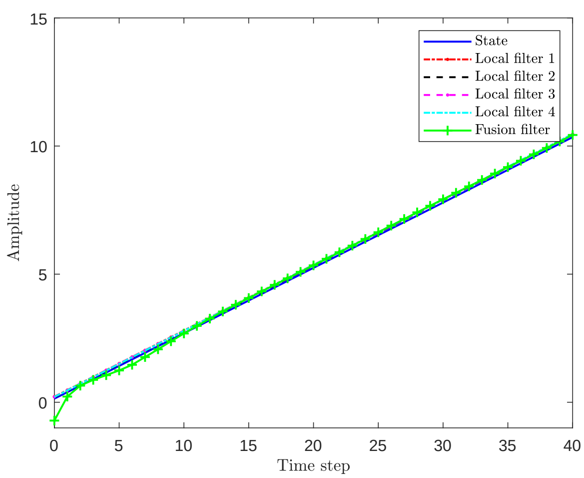

Fig. 5. Trajectories of $x_s^{[5]}$ and its estimates.

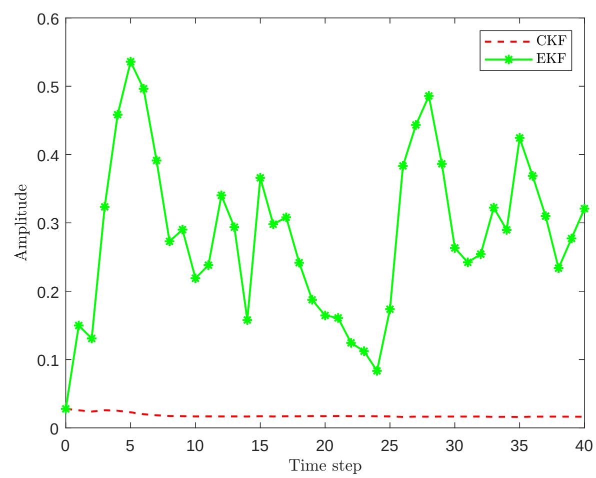

Fig. 8. Comparisons of RMSE with CKF and EKF for node 1.

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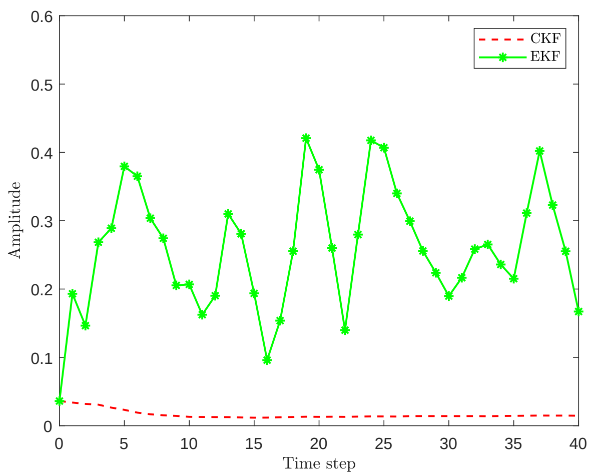

Fig. 9. Comparisons of RMSE with CKF and EKF for node 2.

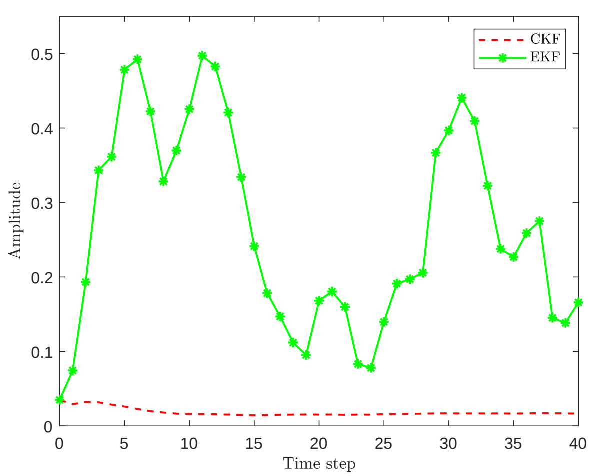

Fig. 10. Comparisons of RMSE with CKF and EKF for node 3.

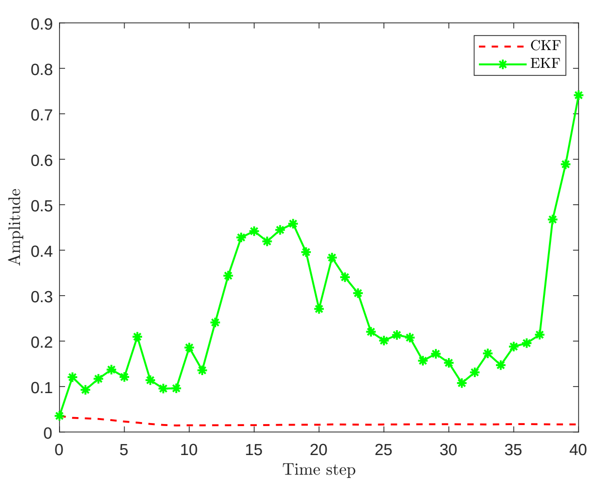

Fig. 11. Comparisons of RMSE with CKF and EKF for node 4.

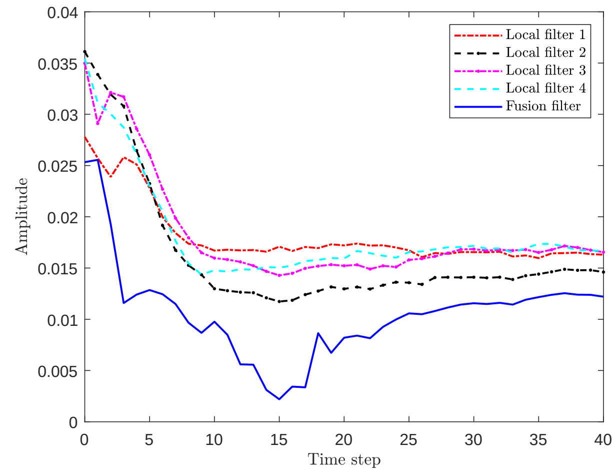

Fig. 12. Comparisons of RMSE with fusion filter and local filters.

quality, AaF relays have been deployed in transmission channels to prolong the signal communication distance. Specifically, the local filters have been firstly constructed such that, in the presence of AaF relays, an upper bound of the FEC has been ensured and minimized by selecting an expected filter gain. After that, the fusion filter has been designed according to the obtained local filters and CI fusion rule. Moreover, the boundedness of the upper bound of the FEC has been examined in light of some sufficient conditions. Finally, a simulation experiment concerning a three-phase induction machine system has been utilized to validate the effectiveness of the developed CKFF algorithm.

REFERENCES

[1] E. Antonio-Rodriguez, S. Werner, R. López-Valcarce, and R. Wichman, MMSE filter design for full-duplex filter-and-forward MIMO relays under limited dynamic range, *Signal Processing*, vol. 156, pp. 208–219, 2019.

[2] R. Caballero-Águila, A. Hermoso-Carazo, and J. Linares-Pérez, Networked fusion estimation with multiple uncertainties and time-correlated channel noise, *Information Fusion*, vol. 54, pp. 161–171, 2020.

[3] R. Caballero-Águila and J. Linares-Pérez, Distributed fusion filtering for uncertain systems with coupled noises, random delays and packet loss prediction compensation, *International Journal of Systems Science*, vol. 54, no. 2, pp. 371–390, 2023.

[4] B. Chen, G. Hu, D. W. C. Ho, W.-A. Zhang, and L. Yu, Distributed robust fusion estimation with application to state monitoring systems, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 11, pp. 2994–3005, 2017.

[5] S. Chen and D. W. C. Ho, Information-based distributed extended Kalman filter with dynamic quantization via communication channels, *Neurocomputing*, vol. 469, pp. 251–260, 2022.

[6] W. Chen, D. Ding, H. Dong, and G. Wei, Distributed resilient filtering for power systems subject to denial-of-service attacks, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 8, pp. 1688–1697, 2019.

[7] Y. Chen, X. Meng, Z. Wang, and H. Dong, Event-triggered recursive state estimation for stochastic complex dynamical networks under hybrid attacks, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 34, no. 3, pp. 1465–1477, 2023.

[8] D. Ciuonzo, A. Aubry, and V. Carotenuto, Rician MIMO channel- and jamming-aware decision fusion, *IEEE Transactions on Signal Processing*, vol. 65, no. 15, pp. 3866–3880, 2017.

[9] D. Ding, Q.-L. Han, X. Ge, and J. Wang, Secure state estimation and control of cyber-physical systems: A survey, *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 51, no. 1, pp. 176–190, 2021.

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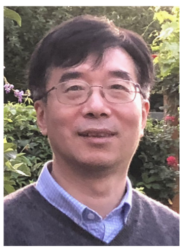
[10] Y. Ding, L. Gong, M. Zhang, C. Li, and Z. Qin, A multi-path adaptive fusion network for multimodal brain tumor segmentation, *Neurocomputing*, vol. 412, pp. 19–30, 2020.
[11] S. Feng, X. Li, S. Zhang, Z. Jian, H. Duan, and Z. Wang, A review: State estimation based on hybrid models of Kalman filter and neural network, *Systems Science & Control Engineering*, vol. 11, no. 1, art. no. 2173682, 2023.
[12] H. Gao, Y. Li, L. Yu, and H. Yu, Collaborative-prediction-based recursive filtering for nonlinear systems with sensor saturation under duty cycle scheduling, *Systems Science & Control Engineering*, vol. 11, no. 1, art. no. 2247007, 2023.
[13] X. Ge, Q.-L. Han, Q. Wu, and X.-M. Zhang, Resilient and safe platooning control of connected automated vehicles against intermittent denial-of-service attacks, *IEEE/CAA Journal of Automatica Sinica*, vol. 10, no. 5, pp. 1234–1251, 2023.
[14] X. Ge, Q.-L. Han, X. Zhang, L. Ding, and F. Yang, Distributed event-triggered estimation over sensor networks: A survey, *IEEE Transactions on Cybernetics*, vol. 50, no. 3, pp. 1306–1320, 2020.
[15] H. Geng, Z. Wang, Y. Chen, F. E. Alsaadi, and Y. Cheng, Multi-sensor filtering fusion with parametric uncertainties and measurement censoring: Monotonicity and boundedness, *IEEE Transactions on Signal Processing*, vol. 69, pp. 5875–5890, 2021.
[16] F. Han, J. Liu, J. Li, J. Song, M. Wang, and Y. Zhang, Consensus control for multi-rate multi-agent systems with fading measurements: The dynamic event-triggered case, *Systems Science & Control Engineering*, vol. 11, no. 1, art. no. 2158959, 2023.
[17] S. Huang, H. Chen, J. Cai, and F. Zhao, Energy efficiency and spectral-efficiency tradeoff in amplify-and-forward relay networks, *IEEE Transactions on Vehicular Technology*, vol. 62, no. 9, pp. 4366–4378, 2013.
[18] A. Jee, K. Agrawal, and S. Prakriya, A coordinated direct AF/DF relay-aided NOMA framework for low outage, *IEEE Transactions on Communications*, vol. 70, no. 3, pp. 1559–1579, 2022.
[19] Y. Jin, X. Ma, X. Meng, and Y. Chen, Distributed fusion filtering for cyber-physical systems under round-robin protocol: A mixed H_2/H_∞ framework, *International Journal of Systems Science*, vol. 54, no. 8, pp. 1661–1675, 2023.
[20] Y. Ju, D. Liu, D. Ding, and G. Wei, Distributed cubature Kalman filtering for nonlinear systems with stochastic communication protocol, *Asian Journal of Control*, vol. 24, no. 6, pp. 3566–3579, 2022.
[21] S. Kiani, S. ShahbazPanahi, M. Dong, and G. Boudreau, Power-optimal distributed beamforming for multi-carrier asynchronous bidirectional relay networks, *IEEE Transactions on Signal and Information Processing over Networks*, vol. 7, pp. 114–129, 2021.
[22] M. Kooshkbaghi and H. J. Marquez, Event-triggered discrete-time cubature Kalman filter for nonlinear dynamical systems with packet dropout, *IEEE Transactions on Automatic Control*, vol. 65, no. 5, pp. 2278–2285, 2020.
[23] A. S. Leong and D. E. Quevedo, Kalman filtering with relays over wireless fading channels, *IEEE Transactions on Automatic Control*, vol. 61, no. 6, pp. 1643–1648, 2016.
[24] C. Li, Z. Wang, W. Song, S. Zhao, J. Wang, and J. Shan, Resilient unscented Kalman filtering fusion with dynamic event-triggered scheme: Applications to multiple unmanned aerial vehicles, *IEEE Transactions on Control Systems Technology*, vol. 31, no. 1, pp. 370–381, 2023.
[25] S. Li, Y. Hu, L. Zheng, Z. Li, X. Chen, T. Fernando, H. H. C. Iu, Q. Wang, and X. Liu, Stochastic event-triggered cubature Kalman filter for power system dynamic state estimation, *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 66, no. 9, pp. 1552–1556, 2019.
[26] W. Li, Y. Jia, and J. Du, Variance-constrained state estimation for nonlinearly coupled complex networks, *IEEE Transactions on Cybernetics*, vol. 48, no. 2, pp. 818–824, 2018.
[27] W. Li and F. Yang, Information fusion over network dynamics with unknown correlations: An overview, *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 2, art. no. 100003, 2023.
[28] Y. Liu, Z. Wang, C. Liu, M. Coombes, and W.-H. Chen, Auxiliary particle filtering over sensor networks under protocols of amplify-andforward and decode-and-forward relays, *IEEE Transactions on Signal and Information Processing over Networks*, vol. 8, pp. 883–893, 2022.
[29] Y.-W. Lv and G.-H. Yang, An adaptive cubature Kalman filter for nonlinear systems against randomly occurring injection attacks, *Applied Mathematics and Computation*, vol. 418, art. no. 126834, 2022.
[30] Y.-W. Lv and G.-H. Yang, Centralized and distributed adaptive cubature information filters for multi-sensor systems with unknown probability of measurement loss, *Information Sciences*, vol. 630, pp. 173–189, 2023.
[31] K. Reif, F. Sonnemann, and R. Unbehauen, An EKF-based nonlinear observer with a prescribed degree of stability, *Automatica*, vol. 34, no. 9, pp. 1119–1123, 1998.
[32] M. A. Sadr and B. Champagne, Joint channel estimation and robust beamforming design for AF relaying using IMM Kalman filters, *IEEE Transactions on Vehicular Technology*, vol. 72, no. 2, pp. 1775–1790, 2023.
[33] R. Senanayake, S. Atapattu, J. S. Evans, and P. J. Smith, Decentralized relay selection in multi-user multihop decode-and-forward relay networks, *IEEE Transactions on Wireless Communications*, vol. 17, no. 5, pp. 3313–3326, 2018.
[34] Y. Shen and S. Sun, Distributed recursive filtering for multi-rate uniform sampling systems with packet losses in sensor networks, *International Journal of Systems Science*, vol. 54, no. 8, pp. 1729–1745, 2023.
[35] O. Shmuel, A. Cohen, and O. Gurewitz, Compute-and-forward in large relaying systems: Limitations and asymptotically optimal scheduling, *IEEE Transactions on Information Theory*, vol. 67, no. 9, pp. 6243–6265, 2021.
[36] S. M. J. A. Tabatabaee, M. Rajabzadeh, and M. Towliat, A novel low-complexity GFDM relay communication system: Relay selection and filter-and-forward, *IEEE Transactions on Signal Processing*, vol. 69, pp. 5147–5158, 2021.
[37] H. Tan, B. Shen, Q. Li, and W. Qian, Recursive filtering for nonlinear systems with self-interferences over full-duplex relay networks, *IEEE/CAA Journal of Automatica Sinica*, vol. 9, no. 11, pp. 2037–2040, 2022.
[38] T. Tian, S. Sun, and N. Li, Multi-sensor information fusion estimators for stochastic uncertain systems with correlated noises, *Information Fusion*, vol. 27, pp. 126–137, 2016.
[39] F.-S. Tseng, C.-T. Lin, J.-Y. Chen, and K.-Y. Chen, Finite-order source and relay filtering design for wideband full-duplex amplify-and-forward relaying networks, *IEEE Transactions on Communications*, vol. 68, no. 12, pp. 7395–7409, 2020.
[40] X. Wang, A. K. Gostar, T. Rathnayake, B. Xu, A. Bab-Hadiashar, and R. Hoseinnezhad, Centralized multiple-view sensor fusion using labeled multi-Bernoulli filters, *Signal Processing*, vol. 150, pp. 75–84, 2018.
[41] Y. Wang, H.-J. Liu, and H.-L. Tan, An overview of filtering for sampled-data systems under communication constraints, *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 3, art. no. 100011, 2023.
[42] Y.-A. Wang, B. Shen, L. Zou, and Q.-L. Han, A survey on recent advances in distributed filtering over sensor networks subject to communication constraints, *International Journal of Network Dynamics and Intelligence*, vol. 2, no. 2, art. no. 100007, 2023.
[43] M. Xie, D. Ding, H. Dong, Q.-L. Han, and G. Wei, Reliable fusion estimation over sensor networks with outliers and energy constraints, *International Journal of Robust and Nonlinear Control*, vol. 29, no. 17, pp. 5913–5929, 2019.
[44] Z. Xing and Y. Xia, Distributed federated Kalman filter fusion over multi-sensor unreliable networked systems, *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 63, no. 10, pp. 1714–1725, 2016.
[45] K. Xiong, C. Wei, and L. Liu, Robust extended Kalman filtering for nonlinear systems with stochastic uncertainties, *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, vol. 40, no. 2, pp. 399–405, 2010.
[46] K. Xiong, H. Zhang, and C. Chan, Performance evaluation of UKF-based nonlinear filtering, *Automatica*, vol. 42, no. 2, pp. 261–270, 2006.
[47] Y. Xu, R. Lu, H. Peng, K. Peng, and A. Xue, Asynchronous dissipative state estimation for stochastic complex networks with quantized jumping coupling and uncertain measurements, *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 2, pp. 268–277, 2015.
[48] T. Zhang, G. Liu, H. Zhang, W. Kang, G. K. Karagiannidis, and A. Nallanathan, Energy-efficient resource allocation and trajectory design for UAV relaying systems, *IEEE Transactions on Communications*, vol. 68, no. 10, pp. 6483–6498, 2020.

Jiaxing Li received the B.Sc. degree in Mathematics and Applied Mathematics from Harbin University of Commerce, Harbin, China, in 2017 and M.Sc. degree in Mathematics from Harbin University of Science and Technology, Harbin, China, in 2020, respectively. She is currently working toward the Ph.D. degree in Mathematics from Harbin University of Science and Technology, Harbin, China. From March 2023 to March 2024, she was a Visiting Ph.D. Student with the Department of Computer Science, Brunel University London, Uxbridge, U.K. Her current research interests include optimal distributed filtering for time-varying state-saturated systems, optimal distributed filtering for sensor networks, and cubature Kalman filtering. She is an active reviewer for many international journals.



Zidong Wang (Fellow, IEEE) received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, the M.Sc. degree in applied mathematics and the Ph.D. degree in electrical engineering both from Nanjing University of Science and Technology, Nanjing, China, in 1990 and 1994, respectively.

He is currently a Professor of Dynamical Systems and Computing in the Department of Computer Science, Brunel University London, Uxbridge, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany, and the U.K. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published a number of papers in international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, and the William Mong Visiting Research Fellowship of Hong Kong.

Prof. Wang serves (or has served) as the Editor-in-Chief for *International Journal of Systems Science*, the Editor-in-Chief for *Neurocomputing*, the Editor-in-Chief for *Systems Science & Control Engineering*, and an Associate Editor for 12 international journals, including IEEE TRANSACTIONS ON AUTOMATIC CONTROL, IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY, IEEE TRANSACTIONS ON NEURAL NETWORKS, IEEE TRANSACTIONS ON SIGNAL PROCESSING, and IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS–PART C. He is a Member of the Academia Europaea, a Member of the European Academy of Sciences and Arts, an Academician of the International Academy for Systems and Cybernetic Sciences, a Fellow of the IEEE, a Fellow of the Royal Statistical Society, and a member of program committee for many international conferences.

Jun Hu (Member, IEEE) received the B.Sc. degree in Information and Computing Science and M.Sc. degree in Applied Mathematics from Harbin University of Science and Technology, Harbin, China, in 2006 and 2009, respectively, and the Ph.D. degree in Control Science and Engineering from Harbin Institute of Technology, Harbin, China, in 2013.

From September 2010 to September 2012, he was a Visiting Ph.D. student in the Department of Information Systems and Computing, Brunel University London, Uxbridge, U.K. From May 2014 to April 2016, he was an Alexander von Humboldt research fellow at the University of Kaiserslautern, Kaiserslautern, Germany. From January 2018 to January 2021, he was a research fellow at University of South Wales, Pontypridd, U.K. He is with the Department of Applied Mathematics, Harbin University of Science and Technology, Harbin, China, and is also with the School of Automation, Harbin University of Science and Technology, Harbin, China. His research interests include nonlinear control, filtering and fault estimation, time-varying systems and complex networks. He has published more than 90 papers in refereed international journals.

Dr. Hu serves as a reviewer for Mathematical Reviews, as an editor for *Neurocomputing*, *Journal of Intelligent & Fuzzy Systems*, *Neural Processing Letters*, *Systems Science & Control Engineering*, and as a guest editor for *International Journal of General Systems* and *Information Fusion*.

Hongli Dong (Senior Member, IEEE) received the Ph.D. degree in control science and engineering from the Harbin Institute of Technology, Harbin, China, in 2012.

From 2009 to 2010, she was a Research Assistant with the Department of Applied Mathematics, City University of Hong Kong, Hong Kong. From 2010 to 2011, she was a Research Assistant with the Department of Mechanical Engineering, The University of Hong Kong, Hong Kong. From 2011 to 2012, she was a Visiting Scholar with the Department of Information Systems and Computing, Brunel University London, Uxbridge, U.K. From 2012 to 2014, she was an Alexander von Humboldt Research Fellow with the University of Duisburg–Essen, Duisburg, Germany. She is currently a Professor with the Artificial Intelligence Energy Research Institute, Northeast Petroleum University, Daqing, China. She is also the Director of the Heilongjiang Provincial Key Laboratory of Networking and Intelligent Control, Daqing, China. Her current research interests include robust control and networked control systems.

Dr. Dong is a very active reviewer for many international journals.

Hongjian Liu received his B.Sc. degree in applied mathematics in 2003 from Anhui University, Hefei, China, and the M.Sc. degree in detection technology and automation equipment in 2009 from Anhui Polytechnic University, Wuhu, China, and the Ph.D. degree in control science and engineering in 2018 from Donghua University, Shanghai, China. In 2016, he was a Research Assistant with the Department of Mathematics, Texas A&M University at Qatar, Doha, Qatar, for two months. From March 2017 to March 2018, he was a Visiting Scholar in the Department of Information Systems and Computing, Brunel University London, Uxbridge, U.K. He is currently a Professor in the School of Mathematics and Physics, Anhui Polytechnic University, Wuhu, China.

Dr. Liu's current research interests include filtering theory, memristive neural networks, and network communication systems. He is a very active reviewer for many international journals.