Robust Finite-Horizon Filtering for Stochastic Systems With Missing Measurements

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***Abstract*—In this letter, we consider the robust finite-horizon filtering problem for a class of discrete time-varying systems with missing measurements and norm-bounded parameter uncertainties. The missing measurements are described by a binary switching sequence satisfying a conditional probability distribution. An upper bound for the state estimation error variance is first derived for all possible missing observations and all admissible parameter uncertainties. Then, a robust filter is designed, guaranteeing that the variance of the state estimation error is not more than the prescribed upper bound. It is shown that the desired filter can be obtained in terms of the solutions to two discrete Riccati difference equations, which are of a form suitable for recursive computation in online applications. A simulation example is presented to show the effectiveness of the proposed approach by comparing to the traditional Kalman filtering method.**

***Index Terms*—Kalman filtering, missing measurements, parameter uncertainty, robust filtering, time-varying systems.**

I. Introduction

IT IS well known that filtering problems play an important role in signal processing [1]. Various filtering schemes have been proposed recently, such as robust filtering [8], [13], and H_∞ filtering [4]. In most existing literature, however, it is implicitly assumed that the measurements always contain a true signal, which may be corrupted by the noise. In fact, in practical applications, due to various reasons such as sensor temporal failures and network data transmission delay or loss, the measurement sequence may contain noise only. In other words, the system may have missing measurements with probability less than one [2], [3], [6].

The filtering problem with missing measurements was first studied in [3] and [6], where the missing data were modeled by a binary switching sequence specified by a conditional probability distribution. Such a problem has recently stirred renewed research interests since missing data phenomena become more and more popular in today's data-intensive engineering (e.g., networked control systems; see [5], [7], and [11]). For example, in [9], [10], and [12], the filtering problem with missing data has been investigated by using a jump Riccati equation approach. The variance-constrained filtering problem has been considered in [14] for discrete-time stochastic systems with probabilistic missing measurements subject to norm-bounded parameter uncertainties. The statistical convergence properties of Kalman filter with missing measurements have been addressed in [11], where a critical value has been shown to exist for the arrival rate of the observations.

In this letter, along the lines of [3], [6], [11], and [14], we model the missing measurements by a Bernoulli distributed white sequence with a known conditional probability distribution. In [14], the robust *stationary* (infinite-horizon) filtering problem has been discussed subject to missing measurements, but the finite-horizon filter design case has not been considered. Note that finite-horizon filters could provide a better transient filtering performance if the noise inputs are nonstationary. In [13], the robust finite-horizon filtering problem has been studied thoroughly, but the missing measurement phenomenon has not been taken into account. It is, therefore, our intention in this letter to deal with the challenging problem: the finite-horizon filtering problem for uncertain systems with missing measurements.

II. Problem Formulation and Preliminaries

Consider the following class of uncertain linear discrete time-varying systems

$$x_{k+1} = (A_k + \Delta A_k)x_k + B_k w_k \tag{1}$$

where $x_k \in \mathbb{R}^n$ is a state vector, and $w_k \in \mathbb{R}^n$ is a zero mean Gaussian white noise sequence with covariance $Q_k > 0$. The initial state x_0 has the mean $\bar{x}_0$ and covariance P_0. A_k and B_k are known real time-varying matrices with appropriate dimensions. ΔA_k is a real-valued uncertain matrix satisfying

$$\Delta A_k = H_k F_k E_k, \quad F_k F_k^T \leq I. \tag{2}$$

Here, H_k and E_k are known time-varying matrices of appropriate dimensions, and F_k represents the time-varying uncertainties. The measurements, which may contain missing data, are described by

$$y_k = \gamma_k C_k x_k + v_k \tag{3}$$

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where the stochastic variable $\gamma_k \in \mathbb{R}$ is a Bernoulli distributed white sequence taking the values of 0 and 1 with

$$\text{Prob}\{\gamma_k = 1\} = \mathbb{E}\{\gamma_k\} := \beta_k \quad (4)$$
$$\text{Prob}\{\gamma_k = 0\} = 1 - \mathbb{E}\{\gamma_k\} := 1 - \beta_k \quad (5)$$

and $\beta_k \in \mathbb{R}$ is a known time-varying positive scalar. $y_k \in \mathbb{R}^p$ is a measured output vector, $v_k \in \mathbb{R}^p$ is a zero mean Gaussian white noise sequence with covariance $R_k > 0$, and C_k is a known time-varying matrix of appropriate dimension. For simplicity, we assume that γ_k, v_k, w_k, and x_0 are mutually uncorrelated.

Remark 1: The system measurement mode (3), which can be used to represent missing measurements or uncertain observations, was first introduced in [6] and has been subsequently studied in many papers [11], [14]. On the other hand, Markovian jumping systems have been employed to model the missing phenomenon in [9], [10], and [12], and the filtering problem has been investigated by solving several sets of *coupled* Riccati equations. In this letter, we let the stochastic variable $\gamma(k)$ be a Bernoulli sequence, and therefore, we are able to obtain more practical solutions. For example, the algorithm developed in this letter will not involve solving *coupled* matrix equations/inequalities.

We consider the following filter for system (1)–(3):

$$\hat{x}_{k+1} = \hat{A}_k \hat{x}_k + \hat{K}_k (y_k - \beta_k C_k \hat{x}_k) \quad (6)$$

where $\hat{x}_k$ is the state estimate, and $\hat{A}_k$ and $\hat{K}_k$ are filter parameters to be determined.

The purpose of this letter is first to design finite-horizon filters such that an upper bound for the estimation error variance is guaranteed, that is, there exists a sequence of positive-definite matrices Θ_k $(0 < k \le N)$ satisfying

$$\mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \le \Theta_k, \quad \forall k \quad (7)$$

and then minimize such a bound Θ_k in the sense of the matrix norm, therefore obtaining the expected filter. This problem will be referred to as a finite-horizon robust filtering problem.

III. Upper Bound for the Error Covariance

It is noted that the system (1)–(3) involves uncertain and stochastic terms, and the accurate error covariance is impossible to determine. Therefore, in this section, we aim to derive an upper bound for the time-varying estimation error covariance. Define a new state vector by

$$\tilde{x}_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix} \quad (8)$$

and then an augmented state-space model combining the system (1)–(3) and the filter (6) can be expressed by

$$\tilde{x}_{k+1} = (\tilde{A}_k + \tilde{H}_k F_k \tilde{E}_k)\tilde{x}_k + \tilde{A}_{ek}\tilde{x}_k + \tilde{B}_{1k} w_k + \tilde{B}_{2k} v_k \quad (9)$$

where

$$\tilde{A}_k = \begin{bmatrix} A_k & 0 \\ \beta_k \hat{K}_k C_k & \hat{A}_k - \beta_k \hat{K}_k C_k \end{bmatrix}, \quad \tilde{H}_k = \begin{bmatrix} H_{1,k} \\ 0 \end{bmatrix} \quad (10)$$

$$\tilde{E}_k = [E_k \quad 0], \quad \tilde{A}_{ek} = \begin{bmatrix} 0 & 0 \\ (\gamma_k - \beta_k)\hat{K}_k C_k & 0 \end{bmatrix} \quad (11)$$

$$\tilde{B}_{1k} = \begin{bmatrix} B_k \\ 0 \end{bmatrix}, \quad \tilde{B}_{2k} = \begin{bmatrix} 0 \\ \hat{K}_k \end{bmatrix}. \quad (12)$$

It is easy to see that the augmented system (9) is actually a stochastic parameter system. The state covariance matrix of the augmented system (9) is denoted by

$$\tilde{\Sigma}_k := \mathbb{E}\left[\tilde{x}_k \tilde{x}_k^T\right]. \quad (13)$$

Since $\tilde{A}_{ek}$ is a zero-mean stochastic matrix sequence in (9), we have the following Lyapunov equation that governs the evolution of the covariance matrix $\tilde{\Sigma}_k$ from (9)

$$\tilde{\Sigma}_{k+1} = (\tilde{A}_k + \tilde{H}_k F_k \tilde{E}_k)\tilde{\Sigma}_k(\tilde{A}_k + \tilde{H}_k F_k \tilde{E}_k)^T + \Psi_k + \tilde{B}_{1k} Q_k \tilde{B}_{1k}^T + \tilde{B}_{2k} R_k \tilde{B}_{2k}^T \quad (14)$$

where

$$\begin{aligned}\Psi_k &:= \mathbb{E}\left[\tilde{A}_{ek}\tilde{\Sigma}_k \tilde{A}_{ek}^T\right] \\ &= \mathbb{E}[(\gamma_k - \beta_k)^2] \begin{bmatrix} 0 & 0 \\ \hat{K}_k C_k & 0 \end{bmatrix} \tilde{\Sigma}_k \begin{bmatrix} 0 & 0 \\ \hat{K}_k C_k & 0 \end{bmatrix}^T \\ &= (1 - \beta_k)\beta_k \begin{bmatrix} 0 & 0 \\ \hat{K}_k C_k & 0 \end{bmatrix} \tilde{\Sigma}_k \begin{bmatrix} 0 & 0 \\ \hat{K}_k C_k & 0 \end{bmatrix}^T. \end{aligned} \quad (15)$$

Before giving the upper bound for $\tilde{\Sigma}_k$, we present two lemmas.

Lemma 1: (Lemma 2 of [15]) Given matrices A, H, E, and F with compatible dimensions such that $FF^T \le I$. Let X be a positive definite matrix and $\alpha > 0$ be an arbitrary constant such that $\alpha^{-1}I - EXE^T > 0$; then, we have

$$(A + HFE)X(A + HFE)^T \le A(X^{-1} - \alpha E^T E)^{-1} A^T + \alpha^{-1} HH^T. \quad (16)$$

Lemma 2: (Lemma 3.2 of [13]) For $0 \le k \le N$, suppose $X = X^T > 0$, and $s_k(X) = s_k^T(X) \in \mathbb{R}^{n \times n}, h_k(X) = h_k^T(X) \in \mathbb{R}^{n \times n}$. If there exists $Y = Y^T > X$ such that $s_k(Y) \ge s_k(X)$ and $h_k(Y) \ge s_k(Y)$, then the solutions M_k and N_k to the following difference equations

$$M_{k+1} = s_k(M_k), \quad N_{k+1} = h_k(N_k), \quad M_0 = N_0 > 0 \quad (17)$$

satisfy $M_k \le N_k$.

The following corollary can be obtained immediately from Lemma 1 and (14), which provides a matrix recursive inequality for computing the actual covariance.

Corollary 1: If there exists an α_k such that $\alpha_k^{-1} I - \tilde{E}_k \tilde{\Sigma}_k \tilde{E}_k^T > 0$, then the following inequality holds:

$$\tilde{\Sigma}_{k+1} \le \tilde{A}_k \left(\tilde{\Sigma}_k^{-1} - \alpha_k \tilde{E}_k^T \tilde{E}_k\right)^{-1} \tilde{A}_k^T + \alpha_k^{-1} \tilde{H}_k \tilde{H}_k^T + \tilde{B}_{1k} Q_k \tilde{B}_{1k}^T + \tilde{B}_{2k} R_k \tilde{B}_{2k}^T + \Psi_k. \quad (18)$$

Definition 1: The filter (6) is said to be a quadratic filter associated with a sequence of matrices $\Sigma_k = \Sigma_k^T \ge 0$ $(0 \le k \le N)$ if, for some positive scalars α_k $(0 \le k \le N))$, the sequence Σ_k satisfies

$$\Sigma_{k+1} = \tilde{A}_k \left(\Sigma_k^{-1} - \alpha_k \tilde{E}_k^T \tilde{E}_k\right)^{-1} \tilde{A}_k^T + \alpha_k^{-1} \tilde{H}_k \tilde{H}_k^T + \tilde{B}_{1k} Q_k \tilde{B}_{1k}^T + \tilde{B}_{2k} R_k \tilde{B}_{2k}^T + \Psi_k \quad (19)$$

$$\alpha_k^{-1} I - \tilde{E}_k \Sigma_k \tilde{E}_k^T > 0. \quad (20)$$

Remark 2: It will be shown in the sequel that, if we could design a quadratic filter of the form (6), i.e., there exist positive definite solutions Σ_k to (19) and (20), then Σ_k is an expected upper bound, and a solution to the optimization problem can be found. Hence, it is important to investigate the existence as well as the solving algorithm for the solution to the recursive matrix equation (19).

From Definition 1 and Lemma 2, it is not difficult to obtain the following theorem, which confirms that the solution Σ_k to (19) and (20) indeed provides an upper bound for the error covariance matrix $\tilde{\Sigma}_k$ in (14).

Theorem 1: Given $\tilde{\Sigma}_k$ and Σ_k satisfying (14) and (19)–(20), respectively. If $\Sigma_0 = \tilde{\Sigma}_0$, then we have

$$\tilde{\Sigma}_k \leq \Sigma_k. \tag{21}$$

Furthermore, the following corollary is accessible from Definition 1 and Theorem 1 readily.

Corollary 2: $\forall k$, the following inequality holds:

$$\mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \leq [I \quad -I]\Sigma_k[I \quad -I]^T. \tag{22}$$

IV. Finite-Horizon Filter Design

Theorem 2: Let $\alpha_k > 0$ be a sequence of positive scalars. If the following two discrete-time Riccati-like difference equations

$$\begin{aligned}\Theta_{k+1} = &-\beta_k^2 A_k \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} C_k^T R_{1,k}^{-1} C_k \\ &\cdot \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} A_k^T \\ &+ A_k \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} A_k^T \\ &+ \alpha_k^{-1} H_{1,k} H_{1,k}^T + B_k Q_k B_k^T, \quad \Theta_0 = S_1\end{aligned} \tag{23}$$

and

$$\begin{aligned}P_{k+1} = &A_k \left(P_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} A_k^T + \alpha_k^{-1} H_{1,k} H_{1,k}^T \\ &+ B_k Q_k B_k^T, \quad P_0 = S_0 \geq S_1\end{aligned} \tag{24}$$

have positive-definite solutions Θ_k and P_k such that

$$\alpha_k^{-1} I - E_k P_k E_k^T > 0 \tag{25}$$

then there exists a quadratic filter (6) with parameters

$$\begin{aligned}\hat{A}_k = &A_k + (A_k - \beta_k \hat{K}_k C_k)\Theta_k E_k^T \\ &\cdot \left(\alpha_k^{-1} I - E_k \Theta_k E_k^T\right)^{-1} E_k\end{aligned} \tag{26}$$

and

$$\hat{K}_k = \beta_k A_k \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} C_k^T R_{1,k}^{-1} \tag{27}$$

where

$$\begin{aligned}R_{1,k} = &R_k + (1 - \beta_k)\beta_k C_k P_k C_k^T \\ &+ \beta_k^2 C_k \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} C_k^T\end{aligned} \tag{28}$$

such that the state estimation error variance satisfies boundedness condition

$$\mathbb{E}[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \leq \Theta_k, \quad \forall k. \tag{29}$$

Proof: Let us start with finding a solution to (19). Suppose that Σ_k is of the following form:

$$\Sigma_k = \begin{bmatrix} P_k & P_k - \Theta_k \\ P_k - \Theta_k & P_k - \Theta_k \end{bmatrix} \tag{30}$$

where Θ_k and P_k are defined in (23) and (24), respectively. Define

$$\begin{aligned}h(P_k) = &A_k \left(P_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} A_k^T \\ &+ \alpha_k^{-1} H_{1,k} H_{1,k}^T + B_k Q_k B_k^T\end{aligned} \tag{31}$$

and

$$\begin{aligned}s(\Theta_k) = &-\beta_k^2 A_k \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} C_k^T R_{1,k}^{-1} C_k \\ &\cdot \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} A_k^T \\ &+ A_k \left(\Theta_k^{-1} - \alpha_k E_k^T E_k\right)^{-1} A_k^T \\ &+ \alpha_k^{-1} H_{1,k} H_{1,k}^T + B_k Q_k B_k^T.\end{aligned} \tag{32}$$

The functional h and s satisfy the conditions in Lemma 2, hence, $P_k - \Theta_k \geq 0$ and $\Sigma_k \geq 0$. From (25), it can also be verified that (20) holds. Substituting (26), (27), and (30) into (19) and considering conditions (23) and (24), we can further show that (30) is a solution to (19). Finally, from Corollary 2, we can conclude that (29) holds. This ends the proof. ■

The following theorem shows that filter (6) with parameters (26) and (27) is an optimal filter.

Theorem 3: If (23) and (24) have positive-definite solutions Θ_k and P_k such that $\alpha_k^{-1} I - E_k P_k E_k^T > 0$, then, the quadratic filter (6) with parameters (26) and (27) minimizes the bound Θ_k.

Proof: We give the sketch of the proof here. From (10), (12), (19), and (30), we have

$$\begin{aligned}\Theta_{k+1} &= [I \quad -I]\Sigma_{k+1}[I \quad -I]^T \\ &= \alpha_k^{-1} H_{1,k} H_{1,k}^T + B_k Q_k B_k^T + \hat{K}_k R_k \hat{K}_k^T \\ &\quad + (1 - \beta_k)\beta_k \hat{K}_k C_k P_k C_k^T \hat{K}_k^T \\ &\quad + [A_k - \beta_k \hat{K}_k C_k \quad \beta_k \hat{K}_k C_k - \hat{A}_k] \\ &\quad \cdot \left(\Sigma_k^{-1} - \alpha_k \tilde{E}_k^T \tilde{E}_k\right)^{-1} \\ &\quad \cdot [A_k - \beta_k \hat{K}_k C_k \quad \beta_k \hat{K}_k C_k - \hat{A}_k]^T.\end{aligned} \tag{33}$$

In order to find the optimal filter parameters $\hat{A}_k$ and $\hat{K}_k$ that minimize Θ_{k+1}, we take the first variation to (33) with respect to $\hat{A}_k$ and $\hat{K}_k$ and obtain

$$\begin{aligned}&[A_k - \beta_k \hat{K}_k C_k \quad \beta_k \hat{K}_k C_k - \hat{A}_k] \\ &\quad \cdot \left(\Sigma_k^{-1} - \alpha_k \tilde{E}_k^T \tilde{E}_k\right)^{-1} [0 \quad -I]^T = 0\end{aligned} \tag{34}$$

and

$$\begin{aligned}&\hat{K}_k R_k + (1 - \beta_k)\beta_k \hat{K}_k C_k P_k C_k^T \\ &\quad + [A_k - \beta_k \hat{K}_k C_k \quad \beta_k \hat{K}_k C_k - \hat{A}_k] \\ &\quad \cdot \left(\Sigma_k^{-1} - \alpha_k \tilde{E}_k^T \tilde{E}_k\right)^{-1} [-\beta_k C_k \quad \beta_k C_k]^T = 0.\end{aligned} \tag{35}$$

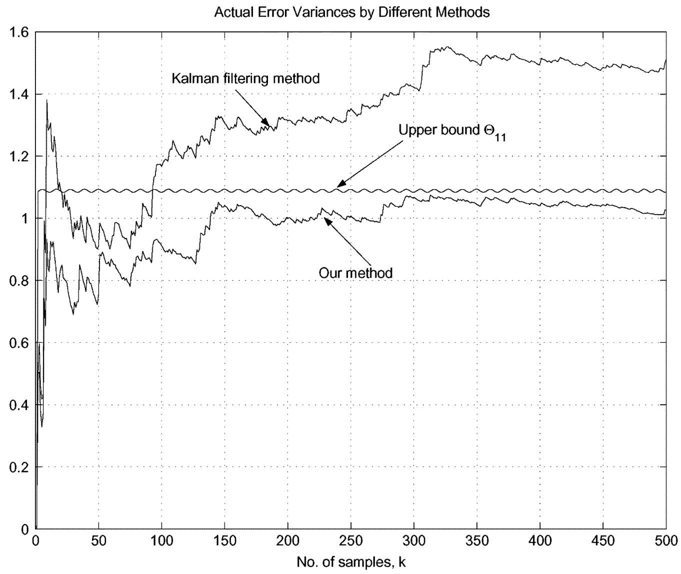

Fig. 1. Actual error variances for the first state.

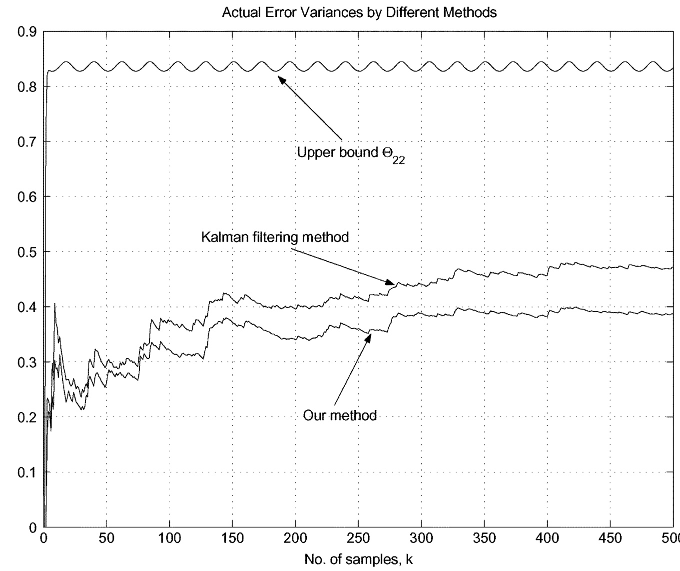

Fig. 2. Actual error variances for the second state.

Then, the filter parameters $\hat{A}_k$ and $\hat{K}_k$ in (26) and (27) can be derived, respectively, by tedious manipulations on (34) and (35). This completes the proof. ■

Remark 3: Theorems 2 and 3 provide the optimal filter design by optimizing the upper bound for state estimation error variance. One-step ahead variance bound is optimized by selecting the filter parameters $\hat{A}_k$ and $\hat{K}_k$, as in (26) and (27), under given scaling parameter α_k. The optimization is step by step by solving the Riccati-like difference equations (23) and (24).

V. Illustrative Example

In this section, we present an illustrative example to demonstrate the effectiveness of the proposed algorithms.

Consider the following discrete time-varying uncertain system with missing measurement

$$
\begin{aligned}
x_{k+1} &= \left(\begin{bmatrix} 0 & 0.1\sin(6k) \\ 0.2 & 0.3 \end{bmatrix} \right. \\
&\quad \left. + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} F_k [0.2 \quad 0.1] \right) x_k + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} w_k \\
y_k &= \gamma_k [0.5 + 0.3\sin(6k) \quad 1] x_k + v_k \\
x_0 &= [1 \quad 0]^T
\end{aligned}
$$

where $F_k = \sin(0.6k)$ is a deterministic perturbation matrix satisfying $F_k F_k^T \le I$, and both w_k and v_k are zero-mean Gaussian white noise sequences with unity covariances. The stochastic variable $\gamma_k \in \mathbb{R}$ is a Bernoulli distributed white sequence taking values on 0 and 1 with $\text{Prob}\{\gamma_k = 1\} = \mathbb{E}\{\gamma_k\} = 0.9$.

We choose $\alpha_k = 3$ in this example. Using Theorem 2 under the initial conditions of $S_0 = 2I_2$ and $S_1 = I_2$, the filtering performance is obtained by solving (23) and (24). In Figs. 1 and 2, we show the actual estimation error variances for the first and second state, respectively, which are obtained by the proposed method in this letter, and by the traditional Kalman filtering method (without taking into account the parameter uncertainties and measurement missing problem).

It can be seen that the performance of our designed filter is better than the typical Kalman filter in the presence of modeling error and measurement missing, and the actual estimation error variances for the states stay below their upper bounds.

References

[1] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Englewood Cliffs, NJ: Prentice-Hall, 1979.

[2] G. Chen, "A simple treatment for suboptimal Kalman filtering in case of measurement data missing," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 26, no. 2, pp. 413–415, Mar. 1990.

[3] M. Hadidi and S. Schwartz, "Linear recursive state estimators under uncertain observations," *IEEE Trans. Autom. Control*, vol. 24, no. 6, pp. 944–948, Jun. 1979.

[4] Y. S. Hung and F. Yang, "Robust H_∞ filtering with error variance constraints for uncertain discrete time-varying systems with uncertainty," *Automatica*, vol. 39, no. 7, pp. 1185–1194, 2003.

[5] Q. Ling and M. D. Lemmon, "Optimal dropout compensation in networked control systems," in *Proc. IEEE Conf. Decision Control*, Hawaii, Dec. 2003, pp. 670–675.

[6] N. Nahi, "Optimal recursive estimation with uncertain observation," *IEEE Trans. Inf. Theory*, vol. IT-15, no. 4, pp. 457–462, Apr. 1969.

[7] J. Nilsson, "Real-time control systems with delays," Ph.D. dissertation, Lund Institute of Technology, Lund, Sweden, 1998.

[8] I. Petersen and A. V. Savkin, *Robust Kalman Filtering for Signals and Systems with Large Uncertainties*, ser. Control Engineering. Boston, MA: Birkhaüser, 1999.

[9] A. V. Savkin and I. R. Petersen, "Robust filtering with missing data and a deterministic description of noise and uncertainty," *Int. Syst. Sci.*, vol. 28, no. 4, pp. 373–378, 1997.

[10] A. V. Savkin, I. R. Petersen, and S. O. R. Moheimani, "Model validation and state estimation for uncertain continuous-time systems with missing discrete-continuous data," *Comput. Elect. Eng.*, vol. 25, no. 1, pp. 29–43, 1999.

[11] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordan, and S. S. Sastry, "Kalman filtering with intermittent observations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1453–1464, Sep. 2004.

[12] S. Smith and P. Seiler, "Estimation with lossy measurements: Jump estimators for jump systems," *IEEE Trans. Autom. Control*, vol. 48, no. 12, pp. 2163–2171, Dec. 2003.

[13] Y. Theodor and U. Shaked, "Robust discrete-time minimum-variance filtering," *IEEE Trans. Signal Process.*, vol. 44, no. 2, pp. 181–189, Feb. 1996.

[14] Z. Wang, D. Ho, and X. Liu, "Variance-constrained filtering for uncertain stochastic systems with missing measurements," *IEEE Trans. Autom. Control*, vol. 48, no. 7, pp. 1254–1258, Jul. 2003.

[15] Z. Wang, J. Zhu, and H. Unbehauen, "Robust filter design with time-varying parameter uncertainty and error variance constraints," *Int. J. Control*, vol. 72, no. 1, pp. 30–38, 1999.