Robust H_∞ Filtering for Time-Delay Systems With Probabilistic Sensor Faults

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***Abstract*—In this paper, a new robust H_∞ filtering problem is investigated for a class of time-varying nonlinear system with norm-bounded parameter uncertainties, bounded state delay, sector-bounded nonlinearity and probabilistic sensor gain faults. The probabilistic sensor reductions are modeled by using a random variable that obeys a specific distribution in a known interval $[\alpha, \beta]$, which accounts for the following two phenomenon: 1) signal stochastic attenuation in unreliable analog channel and 2) random sensor gain reduction in severe environment. The main task is to design a robust H_∞ filter such that, for all possible uncertain measurements, system parameter uncertainties, nonlinearity as well as time-varying delays, the filtering error dynamics is asymptotically mean-square stable with a prescribed H_∞ performance level. A sufficient condition for the existence of such a filter is presented in terms of the feasibility of a certain linear matrix inequality (LMI). A numerical example is introduced to illustrate the effectiveness and applicability of the proposed methodology.**

***Index Terms*—Linear matrix inequality (LMI), parameter uncertainties, robust H_∞ filtering, sensor gain reduction.**

I. Introduction

THE state estimation problem of dynamic systems has attracted persistent research attention and has found many practical applications during the last decades. Fundamentally, two classes of performance indices have been considered in the literature based on the assumptions on the input noise [5]. In the classical H_2 filtering approach, the noise characteristics are assumed to be known, leading to the minimization of the H_2 norm of the transfer function from the process noise to the estimation error. The alternative H_∞ filtering, which was first introduced in 1989 [2], has relaxed the boundedness assumption of the noise variance [12]. Over the past decades, much work has been done on the robust H_∞ filtering problem in the presence of parameter uncertainties in various settings [3], [4], [18].

In most literature concerning with the H_∞ filtering problems, the assumption of consecutive measurements has been made, which means that the true measurement signal can always be obtained by the filtering node. Unfortunately, this is not always the case in practice. Taking the networked control system (NCS) [11] for instance, the limited capacity communication networks that are generally shared by a group of systems have brought us new challenges in the analysis and design of H_∞ filters with missing and/or delayed measurements, which can be collectively called "incomplete measurements" [8].

Recently, the binary switching sequence approach has been introduced to model the missing measurements for its simplicity and practicality [15]–[17]. However, in many cases such as the signal transmission process in unreliable analog communication channel [10] and sensor gain variation under abnormal work conditions [14], the measurement may be stochastically distorted. Such kind of "stochastic sensor faults" cannot be simply described by 0 (completely missing) or 1 (completely normal). Therefore, there is an urgent need to look into a more general description for the measurement with probabilistic sensor faults, and this constitutes the main motivation of the present study.

In this paper, we are concerned with a new filtering problem for a class of nonlinear time-varying systems with parameter uncertainties and probabilistic sensor faults. It is assumed that the "range" of the possible sensor faults can be estimated statistically and therefore the faulty sensor gain obeys a specific distribution law, which is a natural reflection of the signal stochastic attenuation in unreliable analog channel as well as the random sensor gain reduction in severe environment. The main task is to design a robust H_∞ filter such that for all norm-bounded parameter uncertainties, bounded state delay, sector-bounded nonlinearity and probabilistic sensor gain faults, the filtering error system is asymptotically mean-square stable and a prescribed H_∞ noise attenuation level is achieved. A linear matrix inequality (LMI) approach is developed to solve the addressed problem.

II. Problem Formulation

The plant we are interested in is supposed to be modeled by the following system:

$$\begin{cases} x_{k+1} = A_k x_k + A_{dk} x_{k-d_k} + E_k f(x_k) + B_k w_k, \\ z_k = C_0 x_k + D_0 w_k, \\ x_k = \varphi_k, \quad k = -\overline{d}, -\overline{d}+1, \ldots, 0, \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state vector; $z_k \in \mathbb{R}^r$ is the signal to be estimated; $w_k \in l_2[0,\infty) \subset \mathbb{R}^q$ is disturbance signal; A_k, A_{dk}, E_k, B_k are time-varying matrices with appropriate dimensions, which are assumed to be of the form $A_k = A + \Delta A_k$, $A_{dk} = A_d + \Delta A_{dk}$, $E_k = E + \Delta E_k$, and $B_k = B + \Delta B_k$. Here, A, A_d, E, and B are known constant matrices; ΔA_k, ΔA_{dk},

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ΔE_k, and ΔB_k are unknown matrices satisfying the following norm-bounded condition

$$[\Delta A_k\ \Delta A_{dk}\ \Delta E_k\ \Delta B_k] = NF_k[H_1\ H_2\ H_3\ H_4] \quad (2)$$

with N, H_1, H_2, H_3, H_4 being known matrices and F_k satisfying $F_kF_k^T \le I$.

Let d_k denote the time-varying state delay with lower and upper bounds $\underline{d} \le d_k \le \overline{d}$. φ_k is a given real initial sequence on $[-\overline{d}, 0]$. $f(x_k)$ is a vector-valued nonlinear function satisfying the sector-bounded condition [6]

$$[f(x_k) - T_1x_k]^T [f(x_k) - T_2x_k] \le 0, \quad \forall x_k \in \mathbb{R}^n \quad (3)$$

where T_1 and T_2 are known real constant matrices and $T = T_1 - T_2$ is symmetric positive definite matrices.

Consider the following measurement model with probabilistic gain reduction faults

$$y_k = \lambda_k C x_k + D w_k \quad (4)$$

where C and D are real constant matrices and the stochastic variable λ_k distributes in the interval $[\alpha, \beta]$ $(0 \le \alpha \le \beta \le 1)$, with its mathematical expectation $\mathbb{E}\{\lambda_k\} = \tau$ and variance $Cov\{\lambda_k\} = \sigma^2$. α, β, τ and σ are known scalars.

Remark 1: One can use (4) to describe the measurements affected by stochastic signal attenuation or sensor gain faults. Note that the gain degradation parameter λ_k can be obtained by statistic inference.

Remark 2: In our assumption, only the mathematical expectation and the variance of the stochastic variable λ_k are required. Note that if we take the distribution law as

$$Pr\{\lambda_k\} = \begin{cases} p, & \lambda_k = 0, \\ 1-p, & \lambda_k = 1 \end{cases}$$

where $Pr\{\lambda_k\}$ represents the probability of λ_k, and p is a known value satisfying $0 \le q \le 1$, the our measurement model can be specialized to those studied in [15], [16].

Consider a full-order filter of the form

$$\begin{cases} \hat{x}_{k+1} = G\hat{x}_k + K(y_k - \tau C\hat{x}_k), \\ \hat{z}_k = L\hat{x}_k \end{cases} \quad (5)$$

where G, K and L are the parameters to be determined. By defining $\eta_k = [x_k^T\ \hat{x}_k^T]^T$, we have the following augmented filtering dynamics:

$$\begin{cases} \eta_{k+1} = \mathcal{A}_k\eta_k + (\lambda_k - \tau)\mathcal{A}_0\eta_k + \mathcal{A}_{dk}Z\eta_{k-d_k} \\ \quad +\mathcal{E}_k f(Z\eta_k) + \mathcal{B}_k w_k, \\ \tilde{z}_k = z_k - \hat{z}_k = \mathcal{C}\eta_k + D_0 w_k \end{cases} \quad (6)$$

where

$$\mathcal{A}_k = \begin{bmatrix} A & 0 \\ \tau KC & G - \tau KC \end{bmatrix} + \begin{bmatrix} N \\ 0 \end{bmatrix} F_k [H_1 \quad 0] = \mathcal{A} + \mathcal{N}F_k\mathcal{H}_1,$$

$$\mathcal{A}_{dk} = \begin{bmatrix} A_d \\ 0 \end{bmatrix} + \begin{bmatrix} N \\ 0 \end{bmatrix} F_k H_2 = \mathcal{A}_d + \mathcal{N}F_kH_2,$$

$$\mathcal{E}_k = \begin{bmatrix} E \\ 0 \end{bmatrix} + \begin{bmatrix} N \\ 0 \end{bmatrix} F_k H_3 = \mathcal{E} + \mathcal{N}F_kH_3,$$

$$\mathcal{B}_k = \begin{bmatrix} B \\ KD \end{bmatrix} + \begin{bmatrix} N \\ 0 \end{bmatrix} F_k H_4 = \mathcal{B} + \mathcal{N}F_kH_4,$$

$$\mathcal{A}_0 = \begin{bmatrix} 0 & 0 \\ KC & 0 \end{bmatrix}, \ \mathcal{C} = [C_0 \quad -L], \ Z = [I \quad 0].$$

Consider the existence of the stochastic variable λ_k, in the rest of this paper, we aim to design a filter such that the filtering error system satisfies both the requirements (R1) and (R2):

(**R1**) The filtering error system (6) is asymptotically mean-square stable [9].

(**R2**) Under the zero-initial condition, the filtering error $\tilde{z}_k$ satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|\tilde{z}_k\|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|w_k\|^2\right\}, \quad (7)$$

for all nonzero w_k, where $\gamma > 0$ is a prescribed scalar.

III. Main Results

In this section, we give the main results of our paper. Firstly we consider the H_∞ filtering performance analysis for system (6).

Theorem 1: Given a scalar $\gamma > 0$ and the filter parameters G, K and L. If there exist a scalar δ, positive definite matrices $P = P^T > 0$ and $Q = Q^T > 0$ satisfying

$$\Phi = [\Phi_{i,j}]_{4\times 4} < 0 \quad (8)$$

where $\Phi_{1,1} = \mathcal{A}_k^T P\mathcal{A}_k + \theta Z^TQZ - P + \sigma^2\mathcal{A}_0^TP\mathcal{A}_0 + \mathcal{C}^T\mathcal{C} - \delta\bar{\mathcal{T}}_1$, $\Phi_{1,2} = \mathcal{A}_k^TP\mathcal{A}_{dk}$, $\Phi_{1,3} = \mathcal{A}_k^TP\mathcal{E}_k - \delta\bar{\mathcal{T}}_2$, $\Phi_{1,4} = \mathcal{A}_k^TP\mathcal{B}_k$, $\Phi_{2,2} = \mathcal{A}_{dk}^TP\mathcal{A}_{dk} - Q$, $\Phi_{2,3} = \mathcal{A}_{dk}^TP\mathcal{E}_k$, $\Phi_{2,4} = \mathcal{A}_{dk}^TP\mathcal{B}_k$, $\Phi_{3,3} = \mathcal{E}_k^TP\mathcal{E}_k - \delta I$, $\Phi_{3,4} = \mathcal{E}_k^TP\mathcal{B}_k$, $\Phi_{4,4} = \mathcal{B}_k^TP\mathcal{B}_k - \gamma^2 I$, $\mathcal{T}_1 = (T_1^TT_2 + T_2^TT_1)/2$, $\mathcal{T}_2 = -(T_1^T + T_2^T)/2$, $\bar{\mathcal{T}}_1 = \text{diag}\{\mathcal{T}_1, 0\}$, $\bar{\mathcal{T}}_2 = [\mathcal{T}_2^T\ 0]^T$, and $\theta = \overline{d} - \underline{d} + 1$, then the filtering error system (6) satisfies (R1) and (R2).

Proof: Consider the Lyapunov–Krasovskii functional $V_k = V_{1k} + V_{2k} + V_{3k}$ with

$$V_{1k} = \eta_k^TP\eta_k, \quad V_{2k} = \sum_{i=k-d_k}^{k-1} \eta_i^TZ^TQZ\eta_i,$$

$$V_{3k} = \sum_{j=k-\overline{d}+1}^{k-\underline{d}} \sum_{i=j}^{k-1} \eta_i^TZ^TQZ\eta_i.$$

Noticing that $\mathbb{E}\{(\lambda_k - \tau)^2\} = \sigma^2$, we calculate the difference of V_k with $w_k = 0$, take the mathematical expectation and obtain

$$\mathbb{E}\{\Delta V_{1k}\} = \bar{\eta}_{k+1}P\bar{\eta}_{k+1} - \eta_k^TP\eta_k,$$
$$\mathbb{E}\{\Delta V_{2k} + \Delta V_{3k}\} \le \theta Z^TQZ - \eta_{k-d_k}^TZ^TQZ\eta_{k-d_k}$$

where $\bar{\eta}_{k+1} = \mathcal{A}_k\eta_k + \sigma^2\mathcal{A}_0\eta_k + \mathcal{A}_{dk}Z\eta_{k-d_k} + \mathcal{E}_kf(Z\eta_k)$. Also, noting that (3) implies

$$\begin{bmatrix} x_k \\ f(x_k) \end{bmatrix}^T \begin{bmatrix} \mathcal{T}_1 & \mathcal{T}_2 \\ \mathcal{T}_2^T & I \end{bmatrix}^T \begin{bmatrix} x_k \\ f(x_k) \end{bmatrix} \le 0$$

and by defining $\xi_k = [\eta_k^T, \eta_{k-d_k}^TZ^T, f^T(Z\eta_k)]^T$, we can further obtain

$$\mathbb{E}\{\Delta V_k\} \le \xi_k^T \begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} \\ * & * & \tilde{\Phi}_{33} \end{bmatrix} \xi_k \quad (9)$$

where $\tilde{\Phi}_{11} = \mathcal{A}_k^TP\mathcal{A}_k + \sigma^2\mathcal{A}_0^TP\mathcal{A}_0 + \theta Z^TQZ - P - \delta\bar{\mathcal{T}}_1$, $\tilde{\Phi}_{12} = \mathcal{A}_k^TP\mathcal{A}_{dk}$, $\tilde{\Phi}_{13} = \mathcal{A}_k^TP\mathcal{E}_k - \delta\bar{\mathcal{T}}_2$, $\tilde{\Phi}_{22} = \mathcal{A}_{dk}^TP\mathcal{A}_{dk} - Q$, $\tilde{\Phi}_{23} = \mathcal{A}_{dk}^TP\mathcal{E}_k$, $\tilde{\Phi}_{33} = \mathcal{E}_k^TP\mathcal{E}_k - \delta I$.

From (8), we can verify that

$$\begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} \\ * & \tilde{\Phi}_{22} & \tilde{\Phi}_{23} \\ * & * & \tilde{\Phi}_{33} \end{bmatrix} + \begin{bmatrix} \mathcal{C}^T\mathcal{C} & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} < 0$$

and then it follows $\mathbb{E}\{\Delta V_k\} < 0$. We can now confirm that the filtering error system (6) is asymptotically mean-square stable [9].

Next, for any nonzero w_k, it follows from (8) and (9) that $\mathbb{E}\{\Delta V_k\} + \mathbb{E}\{\tilde{z}_k^T \tilde{z}_k\} - \gamma^2 \mathbb{E}\{w_k^T w_k\} = \mathbb{E}\{\tilde{\xi}_k^T \Phi \tilde{\xi}_k\} < 0$, where $\tilde{\xi}_k = [\eta_k^T, \eta_{k-d_k}^T Z^T, f^T(Z\eta_k), w_k^T]^T$. Summing up this relationship from 0 to ∞ with respect to k yields

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\tilde{z}_k^T \tilde{z}_k\right\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{w_k^T w_k\right\} - \mathbb{E}\{V_\infty\} + \mathbb{E}\{V_0\}.$$

Since the system (6) is asymptotically mean-square stable, it is straightforward to see that (7) holds under the zero initial condition. The proof is completed. ■

Next, we will provide a solution to the H_∞ filtering problem for time-varying nonlinear system (1)–(4) with probabilistic sensor faults.

Theorem 2: For the time-varying nonlinear system (1)–(4), an H_∞ filter of the form (5) can be designed such that the filtering error system (6) is asymptotically mean-square stable with norm constraints (7) fulfilled for all nonzero w_k if, for a given scalar $\gamma > 0$, there exist positive definite matrices $R = R^T > 0$, $S = S^T > 0$, $Q = Q^T > 0$, real matrices U_1, U_2, U_3 and real scalars ρ, $\varepsilon > 0$, such that the following LMI holds

$$\Psi = [\Psi_{i,j}]_{12\times 12} < 0 \tag{10}$$

where Ψ is a symmetric block-matrix with its entities being $\Psi_{1,1} = \Psi_{1,2} = \theta Q - \rho \mathcal{T}_1 - S$, $\Psi_{1,4} = \Psi_{2,4} = -\rho \mathcal{T}_2$, $\Psi_{1,6} = \Psi_{2,6} = A^T S$, $\Psi_{1,7} = A^T R + \tau C^T U_2^T + U_1^T$, $\Psi_{1,9} = \sigma C^T U_2^T$, $\Psi_{1,10} = C_0^T - U_3^T$, $\Psi_{1,12} = \Psi_{2,12} = \varepsilon H_1^T$, $\Psi_{2,2} = \theta Q - \rho \mathcal{T}_1 - R$, $\Psi_{2,7} = A^T R + \tau C^T U_2^T$, $\Psi_{2,9} = \sigma C^T U_2^T$, $\Psi_{2,10} = C_0^T$, $\Psi_{3,3} = Q$, $\Psi_{3,6} = A_d^T S$, $\Psi_{3,7} = A_d^T R$, $\Psi_{3,12} = \varepsilon H_2^T$, $\Psi_{4,4} = -\rho I$, $\Psi_{4,6} = E^T S$, $\Psi_{4,7} = E^T R$, $\Psi_{4,12} = \varepsilon H_3^T$, $\Psi_{5,5} = -\gamma^2 I$, $\Psi_{5,6} = B^T S$, $\Psi_{5,7} = B^T R + D^T U_2^T$, $\Psi_{5,10} = D_0^T$, $\Psi_{5,12} = \varepsilon H_4^T$, $\Psi_{6,6} = \Psi_{6,7} = \Psi_{8,8} = \Psi_{8,9} = -S$, $\Psi_{6,11} = SN$, $\Psi_{7,7} = \Psi_{9,9} = -R$, $\Psi_{7,11} = RN$, $\Psi_{10,10} = -I$, $\Psi_{11,11} = \Psi_{12,12} = -\varepsilon I$, and all other entities being zeros. Moreover, if (10) is true, the desired filter parameters are given by

$$\begin{aligned} G &= X_{12}^{-1} U_1 (S-R)^{-1} X_{12} + \sigma K C, \\ K &= X_{12}^{-1} U_2, \quad L = U_3 (S-R)^{-1} X_{12} \end{aligned} \tag{11}$$

where X_{12} comes from the factorization of $I - RS^{-1} = X_{12} Y_{12}^T < 0$.

Proof: The proof is similar with the treatment in [7], and is therefore omitted here for the limitation of space. ■

IV. An Illustrative Example

Consider the system (1)–(4) with parameters as follows:

$$A = \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.3 \end{bmatrix},\ A_d = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0.3 \end{bmatrix},\ B = \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix},$$
$$E = \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},\ N = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},\ H_1 = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix}^T,$$
$$H_2 = [0.1\quad 0],\ H_3 = [0.3\quad 0.1],\ H_4 = [0.2],$$
$$C_0 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}^T,\ D_0 = 0.4, C = \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix}^T,\ D = 0.3$$

$\underline{d} = 1$, $\overline{d} = 2$, and λ_k in (4) is supposed to obey the truncated standardized normal distribution in [0.4, 1], with $\tau = 0.7$ and $\sigma = 0.1719$. The nonlinear term $f(x_k) = [f_1(x_{1k})^T f_2(x_{2k})^T]^T$ is defined as

$$f_1(x_{1k}) = \begin{cases} 0.2x_{1k}, & |x_{1k}| \le 0.25, \\ 0.2\mathrm{sgn}(x_{1k}), & \text{otherwise}, \end{cases}$$
$$f_2(x_{2k}) = \begin{cases} -0.1x_{2k}, & |x_{2k}| \le 0.15, \\ -0.1\mathrm{sgn}(x_{2k}), & \text{otherwise} \end{cases}$$

which can be bounded by

$$T_1 = \begin{bmatrix} 0.25 & 0 \\ 0 & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} 0 & 0 \\ 0 & -0.15 \end{bmatrix}.$$

We are interested in finding an H_∞ filter with the minimal H_∞ attenuation level. For this purpose, we can minimize γ^2 when solving the feasibility problem (10). With help from LMI ToolBox [1], we obtain the minimum disturbance attenuation level as $\gamma^* = \sqrt{\gamma_{opt}^2} = 0.6134$, where γ_{opt}^2 is the sub-optimal solution of the corresponding convex optimization problem. A sub-optimal H_∞ filter can then be obtained as

$$G = \begin{bmatrix} -0.0877 & 0.3286 \\ -0.0813 & -0.6207 \end{bmatrix},\ K = \begin{bmatrix} 0.5529 \\ -0.1601 \end{bmatrix},$$
$$L = [0.6544 \quad -0.9936].$$

Let $w_k = \exp(-k/10) \times n_k$ and $F_k = \sin(k/2)$. Using the above designed H_∞ filter, we depict time-domain simulation within 50 time steps in Fig. 1. Fig. 1(a) illustrates the signal to be estimated z_k and the output of the above robust H_∞ filter. Fig. 1(b) shows the estimation error $\tilde{z}_k$ which tends to 0 as time tends to ∞. In Fig. 1(c), the real time proportion between the error energy and the noise energy γ_k versus time k is provided, from which we can see that γ_k is always less than the worst case disturbance attenuation level γ_{opt}^2.

Next, let us provide a comparison with the case of binary variable perturbation on the measurements [16], where λ_k is of the form as stated in Remark 2 with $p = 0.7$. Considering the H_∞ filter with binary missing measurements in the design process

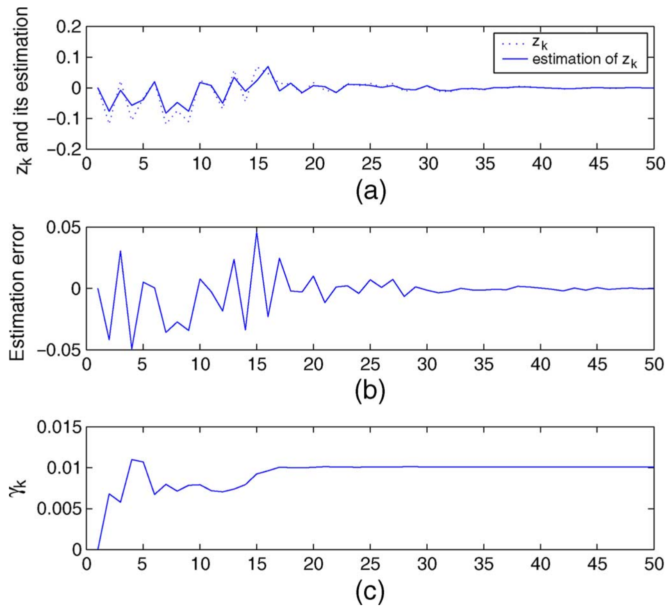

Fig. 1. Filter performance using our proposed method.

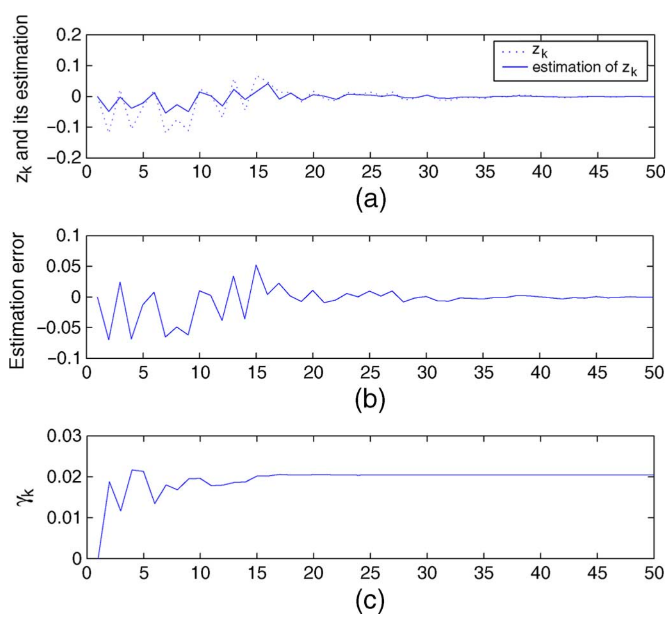

Fig. 2. Filter performance with H_∞ filter considering binary missing measurements.

and without taking the parameter uncertainties into consideration, we can obtain a less sub-optimal value $\gamma^* = \sqrt{\gamma_{opt}^2} = 0.5911$ and the following filter parameters:

$$G = \begin{bmatrix} -0.0469 & 0.3513 \\ -0.2175 & -0.7482 \end{bmatrix}, \; K = \begin{bmatrix} 0.2557 \\ -0.1662 \end{bmatrix},$$
$$L = [1.1520 \quad -0.9763].$$

It should be noticed that although we get a less disturbance attenuation level in designing the second filter, it cannot be inferred that one can use the second filter to get a better filter performance. To show this point, we use the second H_∞ filter for the state estimation of system (1)–(4) with the same parameters as aforementioned, and the filtering performance can be seen in Fig. 2. Fig. 2(a)–(c) shows the signal to be estimated and the output of the second H_∞ filter, the estimation error, and the real time disturbance attenuation level, respectively.

It can be seen from the comparison between Fig. 1(c) and Fig. 2(c) that the proposed robust H_∞ filter provides a less estimation error and a better H_∞ attenuation performance than the filter only considering binary missing phenomenon of system measurements, which demonstrates the effectiveness of the result in this paper.

V. Conclusion

In this paper, the robust H_∞ filtering problem for a class of nonlinear systems has been studied in the presence of probabilistic sensor faults, where the system is subject to time-varying norm-bounded parameters, sector-bounded nonlinearities, time-varying bounded state delays, and energy bounded disturbance input. The sensor fault is described using a sequence of stochastic variables that are of any distribution in an interval $[\alpha, \beta]$, and only the mathematical expectation and the covariance of the stochastic variables are required. A numerical example has been given to show the usefulness of the results derived.

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