Robust Fault Detection for Networked Systems with Communication Delay and Data Missing ⋆

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Abstract

In this paper, the robust fault detection problem is investigated for a class of discrete-time networked systems with unknown input and multiple state delays. A novel measurement model is utilized to represent both the random measurement delays and the stochastic data missing phenomenon, which are typically resulted from the limited capacity of the communication networks. Network status is assumed to vary in a Markovian fashion and its transition probability matrix is uncertain but reside in a known convex set of a polytopic type. The main purpose of this paper is to design a robust fault detection filter such that, for all unknown inputs, possible parameter uncertainties as well as incomplete measurements, the error between residual and fault is made as small as possible. By casting the addressed robust fault detection problem into an auxiliary robust H_∞ filtering problem of a certain Markovian jumping system, a sufficient condition for the existence of the desired robust fault detection filter is established in terms of linear matrix inequalities. A numerical example is provided to illustrate the effectiveness and applicability of the proposed technique.

Key words: Fault detection; networked systems; parameter uncertainty; random measurement delay; data missing; Markovian jumping system.

1 Introduction

Control systems where sensors, controllers, actuators and other system components communicate over a communication network are a type of distributed control systems referred to as networked control systems (NCSs) [1, 2]. The use of a communication network offers advantages in terms of reliability, enhanced resource utilization, reduced wiring and reconfigurability. As such, network-based analysis and designs have many industrial applications in, for example, automobiles, manufacturing plants, aircrafts, and HVAC systems. However, implementing a control network over a communication network induces stochastic delays and packet dropouts that inevitably degrade performance and could be a source of instability. The problem of designing NCSs against network-induced communication delays and packet dropouts has recently attracted considerable research attention, see [3, 4] for some representative works.

Fault detection and isolation (FDI), on the other hand, has been an active field of research over the past decades because of the ever increasing demand for higher performance, higher safety and reliability standards [5,6]. Generally speaking, a fault detection process consists of constructing a residual signal which can then be compared with a predefined threshold. When the residual exceeds the threshold, the fault is detected and an alarm is generated [7]. In view of the wide usage of the network cables in today's world, a seemingly natural research problem is to study the FDI problems for networked systems in the presence of network-induced delays or data missing, see e.g. [8,9]. Since network-induced delays and data missing phenomenon are inherently random and time-varying [10], they have been modeled in various probabilistic ways [11–14]. Very recently, in [15], the network-induced delay and data dropout problems have been investigated within a unified framework and the robust filtering problem with polytopic uncertainties has been thoroughly studied.

It should be pointed out that, in all the aforementioned results, it has been implicitly assumed that the delay or missing characteristics are statistically mutually *independent* from transfer to transfer. Obviously, such an assumption is quite restrictive since network-induced characteristics are highly related to each other over the time. One possible way to remove such an assumption is to describe the residual dynamics by a discrete-time Markovian jumping system (MJS) [16]. Unfortunately, exact transition probability matrix can not be obtained in practice and to the best of the authors' knowledge, the *robust* fault detection problem for networked MJSs with uncertain transition probability matrices has not been fully investigated, which constitutes the main focus of this paper.

In this paper, the robust fault detection problem is studied for a class of networked systems with unknown input, multiple state delays and data missing. A sequence varying in a Markovian fashion is employed in the measurement model, and both the measurement delays and data missing are simultaneously considered. Polytopic-type uncertainty in the transition probability matrix of the Markov process is taken into account. The addressed robust fault detection problem is converted into an aux-

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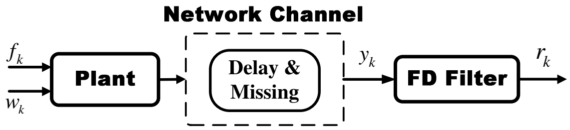

Fig. 1. Fault detection for networked systems

iliary robust H_∞ filtering problem for a certain MJS, and a sufficient condition for the existence of the desired robust fault detection filters is established in terms of linear matrix inequalities (LMIs).

Notations used in the present paper are fairly standard except where otherwise stated. $Pr\{e\}$ and $\mathbf{E}\{x\}$ represent, respectively, the occurrence probability of the event "e" and the mathematical expectation of the stochastic variable x. $l_2[0,\infty)$ is the space of all square-summable vector functions over $[0,\infty)$, and $\|x\|$ is the standard l_2 norm of x, i.e., $\|x\| = (x^T x)^{1/2}$. In symmetric block matrices, "$*$" denotes the term that is induced by symmetry. diag$\{\cdots\}$ stands for a block-diagonal matrix.

2 Problem Formulation and Preliminaries

Consider a class of discrete-time networked systems with the structure shown in Fig. 1. The plant and the robust fault detection filter are located at different places and connected with communication channel, which can be characterized by the following discrete-time linear system model:

$$\begin{cases} x_{k+1} = \sum_{i=0}^{q} A_i x_{k-i} + B_w w_k + B_f f_k, \\ y_k = \sum_{i=0}^{q} \delta(\tau_k, i) C_i x_{k-i} + \bar{\delta}(\tau_k, -1) D w_k, \\ x_k = \varphi_k, \quad k = -q, -q+1, \ldots, 0, \end{cases} \tag{1}$$

where $x_k \in \mathbf{R}^n$ is the state vector; $w_k \in \mathbf{R}^p$ is the unknown input belonging to $l_2[0,\ \infty)$; $f_k \in \mathbf{R}^l$ is the fault signal to be detected; $y_k \in \mathbf{R}^m$ is the measurement, which may contain *both* the random communication delays ($1 \le i \le q$) *and* stochastic data missing ($q = -1$) induced by the limited capacity of the communication networks. All system matrices in (1) are assumed to have appropriate dimensions and φ_k is a given real initial sequence on $[-q, 0]$. $\bar{\delta}(j,l) = 1 - \delta(j,l)$, and $\delta(\cdot,\cdot)$ is the Kronecker delta function, i.e.,

$$\delta(a,b) = \begin{cases} 0, & \text{if } a \neq b \\ 1, & \text{if } a = b \end{cases}$$

τ_k is a random variable introduced to describe the possibility of data missing as well as the size of the occurred delay at time instant k [15]. In the present paper, we consider the sequence $\{\tau_k\}$ obeying a discrete-time homogeneous Markov chain taking values in the following finite state space

$$\Xi = \{-1, 0, \ldots, q\} \tag{2}$$

and $\Lambda = [\lambda_{ij}]$ is the stationary transition probability matrix with its entities defined as

$$\lambda_{ij} = Pr\{\tau_{k+1} = j | \tau_k = i\}. \tag{3}$$

In this paper, we deal with the robust fault detection problem for system (1) with polytopic uncertainties in the transition probability matrix Λ of the Markov chain $\{\tau_k\}$ [17], namely $\Lambda \in \Pi$, where Π is a polytope with u vertices

$$\Pi = \left\{ \Lambda \mid \Lambda = \sum_{s=1}^{u} \beta_s \Lambda_s; \quad \sum_{s=1}^{u} \beta_s = 1, \quad \beta_s \ge 0 \right\}, \tag{4}$$

where $\Lambda_s = [\lambda_{ij}^{(s)}]$ ($i, j \in \Xi$, $s = 1, \ldots, u$) are given transition probability matrices. It can be directly confirmed that the convex combination of these transition probability matrices is also a possible transition probability matrix.

Consider the following discrete-time full-order fault detection filter

$$\begin{cases} \tilde{x}_{k+1} = G(\tau_k)\tilde{x}_k + K(\tau_k) y_k, \\ r_k = L(\tau_k)\tilde{x}_k + M(\tau_k) y_k, \end{cases} \tag{5}$$

here, $\tilde{x}_k \in \mathbf{R}^n$ is the state of the fault detection filter; $r_k \in \mathbf{R}^l$ is its output (also called "residual") that is compatible with f_k. For each $\tau_k = i \in \Xi$, we notate matrices $G_i = G(\tau_k = i)$, $K_i = K(\tau_k = i)$, $L_i = L(\tau_k = i)$ and $M_i = M(\tau_k = i)$ as $G(\tau_k)$, $K(\tau_k)$, $L(\tau_k)$ and $M(\tau_k)$, respectively. In our present work, it is intended to make the error between the residual r_k and the fault signal f_k as small as possible in H_∞ framework.

Introduce the following new vectors

$$\begin{aligned} &\zeta_k = [w_k^T\ f_k^T]^T, \quad \tilde{r}_k = r_k - \hat{f}_k, \\ &\bar{x}_k = [x_{k-1}^T\ \cdots\ x_{k-q}^T]^T, \quad \eta_k = [x_k^T\ \bar{x}_k^T\ \tilde{x}_k^T]^T, \end{aligned} \tag{6}$$

and let matrices $\tilde{A}(\tau_k)$, $\tilde{B}(\tau_k)$, $\tilde{C}(\tau_k)$ and $\tilde{D}(\tau_k)$ represent $\tilde{A}_i = \tilde{A}(\tau_k = i)$, $\tilde{B}_i = \tilde{B}(\tau_k = i)$, $\tilde{C}_i = \tilde{C}(\tau_k = i)$ and $\tilde{D}_i = \tilde{D}(\tau_k = i)$. The overall fault detection dynamics governed by the following system can be obtained

$$\begin{cases} \eta_{k+1} = \tilde{A}_i \eta_k + \tilde{B}_i \zeta_k, \\ \tilde{r}_k = \tilde{C}_i \eta_k + \tilde{D}_i \zeta_k, \end{cases} \tag{7}$$

where

$$\begin{aligned} \tilde{A}_i &= \begin{bmatrix} A_0 & A_d & 0 \\ \hat{A}_{21} & \hat{A}_{22} & 0 \\ \delta(i,0)K_0C_0 & K_iC_ie_i & G_i \end{bmatrix}, \\ \tilde{B}_i &= \begin{bmatrix} B_w & B_f \\ 0_{qn\times p} & 0_{qn\times l} \\ \bar{\delta}(i,-1)K_iD & 0 \end{bmatrix}, \\ \hat{A}_{21} &= \begin{bmatrix} I_n \\ 0_{(q-1)n\times n} \end{bmatrix}, \ \hat{A}_{22} = \begin{bmatrix} 0 & 0 \\ I_{(q-1)n} & 0 \end{bmatrix}, \\ A_d &= \begin{bmatrix} A_1 & \cdots & A_q \end{bmatrix}, \\ e_i &= \begin{bmatrix} \delta(i,1)I_n & \cdots & \delta(i,q)I_n \end{bmatrix}, \\ \tilde{C}_i &= \begin{bmatrix} \delta(i,0)M_0C_0 & M_iC_ie_i & L_i \end{bmatrix}, \\ \tilde{D}_i &= \begin{bmatrix} \bar{\delta}(i,-1)M_iD & -I \end{bmatrix}. \end{aligned} \tag{8}$$

After the above treatments, the possible communication delays and data missing introduced by network cable can be converted as the jumping parameters of the Markovian jumping system (7) with the same transition probability matrix Λ in original system (1).

Recall the following definition of mean square stability for MJSs.

Definition 1 *[18, 19] System (7) with $\zeta_k = 0$ is said to be mean square stable if*

$$\mathbf{E}\{\|\eta_k\|^2\} \to 0, \quad \text{as } k \to \infty$$

for any initial condition η_0 and initial distribution $\tau_0 \in \Xi$.

Considering the existence of uncertainty in the transition probability matrix of MJSs, we further introduce the following definition.

Definition 2 *Markovian system (7) with uncertain transition probability matrix $\Lambda \in \Pi$ is robustly mean square stable if (7) is mean square stable for each $\Lambda \in \Pi$.*

With Definition 2, the original robust fault detection filter design problem for system (1) can be further converted to a robust H_∞ filtering problem [20] for an MJS (7): Finding a series of filter parameters G_i, K_i, L_i and M_i ($i \in \Xi$) such that the augmented fault detection dynamics (7) is robustly mean square stable and the infimum of γ is made as small as possible in the feasibility of

$$\sup_{\zeta_k \neq 0} \mathbf{E}\left\{\|\tilde{r}_k\|^2/\|\zeta_k\|^2\right\} < \gamma^2, \qquad \gamma > 0. \tag{9}$$

Introduce a residual evaluation function $J(k)$ with a form of quadratic sum

$$J(k) = \left\{\sum_{h=0}^{k} r_h^T r_h\right\}^{1/2}, \tag{10}$$

and the occurrence of faults can then be alarmed by comparing the incremental version of $J(k)$ with a prescribed threshold J_{th}, according to the following logic

$$\begin{cases} J(k) - J(k-\mathcal{L}) > J_{th} \Longrightarrow \text{fault detected}, \\ J(k) - J(k-\mathcal{L}) \leq J_{th} \Longrightarrow \text{no faults}, \end{cases}$$

where

$$J_{th} = \sup_{k \in \mathbf{N}+, w_k \in l_2, f_k = 0} \mathbf{E}\{J(k+\mathcal{L}) - J(k)\},$$

and $\mathcal{L}$ is the length of a finite evaluating time horizon.

3 Main Results

In this section we shall discuss the robust fault detection filter design problem of system (1). The following Bounded Real Lemma (BRL) will help us in deriving the main result.

Lemma 3 *[19] (Discrete BRL for MJSs) Consider the MJS (7) with fixed and known transition probability matrix (3). Let $\gamma > 0$ be a given scalar. Then the system (7) is mean square stable with $\zeta_k = 0$ and, under zero initial conditions, satisfies (9), if there exist matrices $\tilde{P}_i \in \mathbf{R}^{(q+2)n}$ such that the following LMIs*

$$\begin{bmatrix} -P_i & \tilde{A}_i^T \mathcal{P}^T \mathcal{S}_i & 0 & \tilde{C}_i^T \\ * & -\mathcal{P}^T \mathcal{S}_i & \mathcal{P}^T \mathcal{S}_i \tilde{B}_i & 0 \\ * & * & -\gamma^2 I & \tilde{D}_i^T \\ * & * & * & -I \end{bmatrix} < 0 \tag{11}$$

hold for any $i \in \Xi$, where $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{C}_i$, $\tilde{D}_i$ are defined in (8) and

$$\mathcal{P} = \begin{bmatrix} P_{-1} & \cdots & P_q \end{bmatrix}^T,$$
$$\mathcal{S}_i = \begin{bmatrix} \lambda_{i(-1)} I_{(q+2)n} & \cdots & \lambda_{iq} I_{(q+2)n} \end{bmatrix}^T. \tag{12}$$

Now, we establish an alternative sufficient condition for MJS (7) with fixed and known transition probability matrix from Lemma 3. The proof is similar with that of Corollary 1 in [15] and is omitted here.

Lemma 4 *LMIs (11) are feasible if there exist matrices $P_i \in \mathbf{R}^{(q+2)n}$ and $\mathcal{Q}_i \in \mathbf{R}^{(q+2)n}$ satisfying*

$$\begin{bmatrix} -P_i & \tilde{A}_i^T \mathcal{Q}_i^T & 0 & \tilde{C}_i^T \\ * & \bar{\Gamma}_{22} & \mathcal{Q}_i \tilde{B}_i & 0 \\ * & * & -\gamma^2 I & \tilde{D}_i^T \\ * & * & * & -I \end{bmatrix} < 0, \tag{13}$$

where $\bar{\Gamma}_{22} = \mathcal{P}^T \mathcal{S}_i - \mathcal{Q}_i - \mathcal{Q}_i^T$.

Next, we give the following sufficient H_∞ filter analysis condition for MJS (7) with uncertain transition probability matrix $\Lambda \in \Pi$.

Lemma 5 *Consider system (1) with uncertain transition probability matrix $\Lambda \in \Pi$. For a given fault detection filter of the form (5), the augmented dynamic (7) is robustly mean square stable and satisfies the constraint (9) if there exist matrices $P_{is} \in \mathbf{R}^{(q+2)n}$, $H_i \in \mathbf{R}^{(q+2)n \times (q+2)^2 n}$, $E_i \in \mathbf{R}^{(q+2)^2 n \times (q+2)^2 n}$, $\mathcal{Q}_i \in \mathbf{R}^{(q+2)n \times (q+2)n}$ such that the following LMIs*

$$\begin{bmatrix} -P_{is} & \tilde{A}_i^T \mathcal{Q}_i^T & 0 & 0 & \tilde{C}_i^T \\ * & \Upsilon_{22} & \Upsilon_{23} & \mathcal{Q}_i \tilde{B}_i & 0 \\ * & * & \Upsilon_{33} & 0 & 0 \\ * & * & * & -\gamma^2 I & \tilde{D}_i^T \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{14}$$

hold for all $i \in \Xi$ and $s = 1, \ldots, u$, where $\tilde{A}_i$, $\tilde{B}_i$, $\tilde{C}_i$, $\tilde{D}_i$ are defined in (8) and

$$\Upsilon_{22} = -\mathcal{Q}_i - \mathcal{Q}_i^T + H_i \mathcal{S}_{is} + \mathcal{S}_{is}^T H_i^T,$$
$$\Upsilon_{23} = -0.5 \mathcal{P}_s^T + H_i + \mathcal{S}_{is}^T E_i, \quad \Upsilon_{33} = E_i + E_i^T,$$
$$\mathcal{P}_s = \begin{bmatrix} P_{(-1)s} & \cdots & P_{qs} \end{bmatrix}^T,$$
$$\mathcal{S}_{is} = \begin{bmatrix} \lambda_{i(-1)}^{(s)} I_{(q+2)n} & \cdots & \lambda_{iq}^{(s)} I_{(q+2)n} \end{bmatrix}^T. \tag{15}$$

Proof We firstly consider MJS (7) with an arbitrary fixed transition probability matrix. By taking into account (13) in Lemma 4, we further give the following sufficient condition ensuring that system (7) is mean square stable and satisfies the constraint (9): there exist matrices P_i, $\mathcal{Q}_i$, H_i, E_i such that the following LMIs hold for all $i \in \Xi$.

$$\begin{bmatrix} -P_i & \tilde{A}_i^T \mathcal{Q}_i^T & 0 & 0 & \tilde{C}_i^T \\ * & \bar{\Upsilon}_{22} & \bar{\Upsilon}_{23} & \mathcal{Q}_i \tilde{B}_i & 0 \\ * & * & \Upsilon_{33} & 0 & 0 \\ * & * & * & -\gamma^2 I & \tilde{D}_i^T \\ * & * & * & * & -I \end{bmatrix} < 0 \qquad (16)$$

where $\bar{\Upsilon}_{22} = -\mathcal{Q}_i - \mathcal{Q}_i^T + H_i \mathcal{S}_i + \mathcal{S}_i^T H_i^T$, $\bar{\Upsilon}_{23} = -0.5\mathcal{P}^T + H_i + \mathcal{S}_i^T E_i$; Υ_{33} and $\mathcal{P}$ are the same as defined in (15) and (12), respectively. Pre- and post-multiply (16) by $\mathrm{diag}\{I, I, -\mathcal{S}_i^T, I, I\}$ and its transpose, and after some proper elementary transformation, we can easily get (13).

For an arbitrary uncertain system with transition probability matrix $\Lambda \in \Pi$, one can always find a set of coefficients $\beta_s \geq 0$ $(s = 1, \ldots, u)$ such that (4) holds. Noticing that LMIs (14) are affine in the matrices P_{is} and $\mathcal{S}_{is}$, we multiply suitable inequalities of (14) by appropriate scalars β_s and then summing up, it can be readily shown that (16) hold with matrices $P_i(\beta) = \sum_{s=1}^{u} \beta_s P_{is}$. Thus, from (14), we can confirm that (13) holds for any choice of Λ. Form Lemma 4 and Lemma 3, it follows that system (1) is robustly mean square stable and (9) is satisfied. This concludes the proof.

Next, we consider the robust fault detection filter design problem for system (1) with uncertain transition probability matrix $\Lambda \in \Pi$.

Theorem 6 *Consider system (1) with uncertain transition probability matrix $\Lambda \in \Pi$ and let $\gamma > 0$ be a given scalar. Then there exists a full-order robust fault detection filter of the form (5) ensuring that the overall augmented dynamics (7) is robustly mean square stable and the constraint (9) is satisfied if, there exist matrices $0 < X_{is}^T = X_{is} \in \mathbf{R}^{(q+2)n \times (q+2)n}$, $S_i \in \mathbf{R}^{n \times n}$, $Z_i \in \mathbf{R}^{n \times n}$, $Y_i \in \mathbf{R}^{n \times n}$, $\bar{G}_i \in \mathbf{R}^{n \times n}$, $\bar{K}_i \in \mathbf{R}^{n \times m}$, $\bar{L}_i \in \mathbf{R}^{l \times n}$, $\bar{M}_i \in \mathbf{R}^{l \times m}$, $N_i \in \mathbf{R}^{qn \times qn}$, $0 < \mathcal{H}_{is}^T = \mathcal{H}_{is} \in \mathbf{R}^{(q+2)n \times (q+2)n}$, $\mathcal{E}_{is} \in \mathbf{R}^{(q+2)n \times (q+2)n}$ such that the following LMIs*

$$\begin{bmatrix} -X_{is} & \Psi_{12} & 0 & 0 & \Psi_{15} \\ * & \Psi_{22} & \Psi_{23} & \Psi_{24} & 0 \\ * & * & \Psi_{33} & 0 & 0 \\ * & * & * & -\gamma^2 I & \tilde{D}_i^T \\ * & * & * & * & -I \end{bmatrix} < 0 \qquad (17)$$

hold for $i \in \Xi$ and $s = 1, \ldots, u$, where

$$\Psi_{12} = \begin{bmatrix} A_0^T Z_i^T & \Psi_{12}^{(12)} & \Psi_{12}^{(13)} \\ A_d^T Z_i^T & \Psi_{12}^{(22)} & \Psi_{12}^{(23)} \\ A_0^T Z_i^T & \Psi_{12}^{(32)} & \Psi_{12}^{(33)} \end{bmatrix},$$

$$\Psi_{15} = \begin{bmatrix} \delta(i,0)\bar{M}_0 C_0 + \bar{L}_i & \bar{M}_i C_i e_i & \delta(i,0)\bar{M}_0 C_0 \end{bmatrix}^T,$$

$$\Psi_{22} = \mathcal{H}_{is} + \mathcal{H}_{is}^T - \begin{bmatrix} \Psi_{22}^{(11)} & 0 & \Psi_{22}^{(13)} \\ * & \Psi_{22}^{(22)} & 0 \\ * & * & \Psi_{22}^{(33)} \end{bmatrix},$$

$$\Psi_{23} = -0.5\mathcal{X}_s \mathcal{S}_{is} + \mathcal{H}_{is} + \mathcal{E}_{is},$$

$$\Psi_{24} = \begin{bmatrix} Z_i B_w & Z_i B_f \\ 0 & 0 \\ Y_i B_w + \bar{\delta}(i,-1)\bar{K}_i D & Y_i B_f \end{bmatrix},$$

$$\Psi_{33} = \mathcal{E}_{is} + \mathcal{E}_{is}^T, \quad \mathcal{X}_s = \begin{bmatrix} X_{(-1)s} & \cdots & X_{qs} \end{bmatrix}^T,$$

$$\Psi_{12}^{(12)} = \Psi_{12}^{(32)} = \begin{bmatrix} I_n & 0_{n \times (q-1)n} \end{bmatrix} N_i^T,$$

$$\Psi_{12}^{(13)} = A_0^T Y_i^T + \delta(i,0) C_0^T \bar{K}_0^T + \bar{G}_i^T,$$

$$\Psi_{12}^{(22)} = \begin{bmatrix} 0_{(q-1)n \times n} & I_{(q-1)n} \\ 0_n & 0_{n \times (q-1)n} \end{bmatrix} N_i^T,$$

$$\Psi_{12}^{(23)} = A_d^T Y_i^T + e_i^T C_i^T \bar{K}_i^T, \quad \Psi_{12}^{(33)} = A_0^T Y_i^T + \delta(i,0) C_0^T \bar{K}_0^T,$$

$$\Psi_{22}^{(11)} = Z_i + Z_i^T, \quad \Psi_{22}^{(13)} = Z_i + Y_i^T + S_i^T,$$

$$\Psi_{22}^{(22)} = N_i + N_i^T, \quad \Psi_{22}^{(33)} = Y_i + Y_i^T.$$

A_d, $\tilde{D}_i$ are shown in (8); $\mathcal{S}_{is}$ is the same in (15). Moreover, if (17) is feasible, the parameters of the desired robust fault detection filter can be given by

$$G_i = V_i^{-1} \bar{G}_i S_i^{-1} V_i, \quad K_i = V_i^{-1} \bar{K}_i,$$
$$L_i = \bar{L}_i S_i^{-1} V_i, \quad M_i = \bar{M}_i, \qquad (18)$$

where $V_i \in \mathbf{R}^{n \times n}$ is any invertible matrix (for example, V_i could be set as I).

Proof Consider the augmented parameters in (8), we take a special structure of $\mathcal{Q}_i$ into account. Let

$$\mathcal{Q}_i^T = \left[Q_i^{ab} \right]_{3 \times 3}, \qquad (19)$$

and introduce new matrices

$$\bar{\mathcal{Q}}_i^T = \left[\bar{Q}_i^{ab} \right]_{3 \times 3} \qquad (20)$$

with their entities $Q_i^{11} = Y_i$, $Q_i^{13} = V_i$, $Q_i^{22} = X_i$, $Q_i^{12} = Q_i^{21} = Q_i^{23} = Q_i^{32} = 0$, $\bar{Q}_i^{11} = Z_i^{-1}$, $\bar{Q}_i^{13} = U_i$, $\bar{Q}_i^{22} = I$, $\bar{Q}_i^{12} = \bar{Q}_i^{21} = \bar{Q}_i^{23} = \bar{Q}_i^{32} = 0$ and Q_i^{31}, Q_i^{33}, $\bar{Q}_i^{31}$, $\bar{Q}_i^{33}$ are uniquely determined from the following equalities

$$\begin{bmatrix} Y_i & V_i \\ Q_i^{31} & Q_i^{33} \end{bmatrix} \begin{bmatrix} Z_i^{-1} & U_i \\ \bar{Q}_i^{31} & \bar{Q}_i^{33} \end{bmatrix} = I$$
$$\begin{bmatrix} Z_i^{-1} & U_i \\ \bar{Q}_i^{31} & \bar{Q}_i^{33} \end{bmatrix} \begin{bmatrix} Y_i & V_i \\ Q_i^{31} & Q_i^{33} \end{bmatrix} = I \qquad (21)$$

Furthermore, we have the following relationship

$$\mathcal{Q}_i \bar{\mathcal{Q}}_i = \bar{\mathcal{Q}}_i \mathcal{Q}_i = \mathrm{diag}\{I, N_i, I\}. \qquad (22)$$

Define

$$\mathcal{T}_i = \left[T_i^{ab} \right]_{3 \times 3},$$

with $T_i^{11} = Z_i^T$, $T_i^{13} = Y_i^T$, $T_i^{22} = I$, $T_i^{33} = V_i^T$ and other entities all zeros, performing congruence transformations to (14) by $\mathrm{diag}\{\bar{\mathcal{Q}}_i^T \mathcal{T}_i, \bar{\mathcal{Q}}_i^T \mathcal{T}_i, \mathcal{S}_{is} \bar{\mathcal{Q}}_i^T \mathcal{T}_i, I, I\}$ and define

$$X_{is} = \mathcal{T}_i^T \bar{\mathcal{Q}}_i P_{is} \bar{\mathcal{Q}}_i^T \mathcal{T}_i, \quad \mathcal{H}_{is} = \mathcal{T}_i^T \bar{\mathcal{Q}}_i H_i \mathcal{S}_{is} \bar{\mathcal{Q}}_i^T \mathcal{T}_i,$$
$$\mathcal{E}_{is} = \mathcal{T}_i^T \bar{\mathcal{Q}}_i \mathcal{S}_{is}^T E_i \mathcal{S}_{is} \bar{\mathcal{Q}}_i^T \mathcal{T}_i, \quad \bar{G}_i = V_i G_i U_i^T Z_i^T,$$
$$\bar{K}_i = V_i K_i, \ \bar{L}_i = L_i U_i^T Z_i^T, \ \bar{M}_i = M_i, \ S_i = V_i U_i^T Z_i^T,$$

we can easily obtain that LMIs (17) with constraints (19) and (20) are sufficient condition for LMIs (14). Hence, if there exist matrices $X_{is} > 0$, S_i, Z_i, Y_i, $\bar{G}_i$, $\bar{K}_i$, $\bar{L}_i$, $\bar{M}_i$, N_i, $\mathcal{H}_{is} > 0$ and $\mathcal{E}_{is}$ such that LMIs (17) are feasible, then the overall fault detection dynamics (7) is robustly mean square stable and the constraint (9) is satisfied.

Furthermore, from LMIs in (17), we can confirm that $\Psi_{22} < 0$, which fuhrer indicates $\mathcal{H}_{is} + \mathcal{H}_{is}^T - \Psi_{22} > 0$ and Z_i and Y_i are nonsingular. Define $\mathcal{W} = [I \ \ 0 \ -I]$, we have

$$\mathcal{W}\left[\mathcal{H}_{is} + \mathcal{H}_{is}^T - \Psi_{22}\right]\mathcal{W}^T = -S_i - S_i^T > 0, \qquad (23)$$

which implies that S_i is nonsingular and also ensures the existence of parameter matrices G_i, K_i, L_i and M_i in (18). The proof is completed.

Remark 7 *In Theorem 6, an uncertainty-dependent robust fault detection filter design result is obtained, which is less conservative than the uncertainty-independent ones. In fact, if we impose*

$$\begin{aligned}&X_{is} = X_i, \ \ \mathcal{H}_{is} = \mathcal{H}_i, \ \ \mathcal{E}_{is} = \mathcal{E}_i,\\ &i \in \Xi, \ \ s = 1, \dots, u,\end{aligned} \qquad (24)$$

to (17), the uncertainty-independent result is recovered.

Remark 8 *In most practical cases in NCSs, we are able to know the size of the measurement delay or whether the data is missing at a certain time by using the time-stamp technique, and therefore the jumping parameters of the transformed MJS are accessible. In this sense, Theorem 6 provides us a network-status-dependent fault detection filter design result. On the other hand, if the network status is not accessible, i.e., the jumping parameters of the transformed MJS are unavailable, a network-status-independent result can be easily obtained by imposing*

$$\begin{aligned}&S_i = S, \ \ V_i = V, \ \ \bar{G}_i = \bar{G},\\ &\bar{K}_i = \bar{K}, \ \ \bar{L}_i = \bar{L}, \ \ \bar{M}_i = \bar{M}, \ \ i \in \Xi,\end{aligned} \qquad (25)$$

to Theorem 6.

Note that (17) is a set of LMIs over both the matrix variables and the prescribed scalar γ^2, which gives rise to the following two conclusions: (1) the robust full-order fault detection filter can be obtained from the solution of convex optimization problems in terms of LMIs that can be solved via efficient interior-point algorithms [21] and, (2) the scalar γ^2 can be also included as one of the optimization variables for LMIs (17), which makes it possible to obtain the minimal noise attenuation level bound for the fault detection dynamics (7). Thus, a sub-optimal robust fault detection filter can be readily found by solving the following convex optimization problem.

Problem 1: Consider networked system (1) with multiple state-delays, unknown inputs and uncertain transition probability matrix $\Lambda \in \Pi$, a uncertainty-dependent sub-optimal robust fault detection filter can be obtained by solving the following problem:

$$\min_{\substack{X_{is} > 0, S_i, Z_i, Y_i, N_i,\\ \bar{G}_i, \bar{K}_i, \bar{L}_i, \bar{M}_i, \mathcal{H}_{is} > 0, \mathcal{E}_{is},\\ i \in \Xi, \ s = 1, \dots, u}} \gamma^2, \ \ \text{s.t. (17) holds.}$$

The parameters of the sub-optimal robust fault detection filter can be determined by (18), and the sub-optimal robust H_∞ attenuation level for fault detection dynamics is given by $\gamma^* = \sqrt{\gamma_{opt}^2}$, where γ_{opt}^2 are the sub-optimal solution of the corresponding convex optimization problem.

4 A Numerical Example

To illustrate the effectiveness of the proposed method, we provide a numerical example in this section. The parameters of the discrete-time networked system (1) are given as the follows:

$$A_0 = \begin{bmatrix} 0 & 0.5 \\ 0.2 & 0.2 \end{bmatrix}, \ A_1 = \begin{bmatrix} 0.2 & 0 \\ 0.7 & 0.1 \end{bmatrix}, \ B_w = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix},$$

$$B_f = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \ C_0 = C_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \ D = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}.$$

The initial state values φ_k are set to be $\varphi_{-1} = \varphi_0 = 0$. Letting $q = 1$, the state-space of the Markov chain $\{\tau_k\}$ can be obtained as $\Xi = \{-1, \ 0, \ 1\}$. The transition probability matrix of the Markov process is unknown but resides in a polytope with the following two vertices:

$$\Lambda_1 = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix}, \ \Lambda_2 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix},$$

The initial mode is set to be $\tau_0 = 0$. For $k = 0, 1, \dots, 300$, the unknown input w_k is supposed to be a random noise uniformly distributed over $[-0.5, 0.5]$, and the fault signal f_k is given as:

$$f_k = \begin{cases} 0.5, & \text{for } \ k = 100, 101, \dots, 200, \\ 0, & \text{others.} \end{cases}$$

With the above parameters, after solving Problem 1 by using the Matlab LMI toolbox [21], we can obtain the minimal noise attenuation level of the fault detection dynamic $\gamma_{opt} = 1.0000$. Furthermore, the parameters of the sub-optimal fault detection filter in different modes are given by

$$G_{-1} = \begin{bmatrix} 0.0025 & 1.2515 \\ 0.0857 & 0.6038 \end{bmatrix}, \ K_{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$L_{-1} = \begin{bmatrix} -0.6317 & 0.3461 \end{bmatrix}, \ M_{-1} = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

$$G_0 = \begin{bmatrix} -0.1872 & 0.8228 \\ -0.0576 & 0.2541 \end{bmatrix}, \ K_0 = 10^{-6} \times \begin{bmatrix} -1.656 & -4.389 \\ -0.772 & -1.206 \end{bmatrix},$$

$$L_0 = \begin{bmatrix} 0.0734 & -0.3254 \end{bmatrix}, \ M_0 = 10^{-6} \times \begin{bmatrix} 1.592 & -0.097 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} -0.3622 & 1.0477 \\ -0.2268 & 0.6846 \end{bmatrix}, \ K_1 = 10^{-6} \times \begin{bmatrix} -3.552 & -0.159 \\ -3.246 & -0.199 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.3731 & 0.4892 \end{bmatrix}, \ M_1 = 10^{-6} \times \begin{bmatrix} 1.534 & 0.159 \end{bmatrix}.$$

We consider the real transition probability matrix as

$$\Lambda(\beta) = \begin{bmatrix} 0.62 & 0.28 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{bmatrix},$$

which means $\beta_1 = 0.4$, $\beta_2 = 0.6$ in (4). For the sake of reducing false alarm as well as easy implementation,

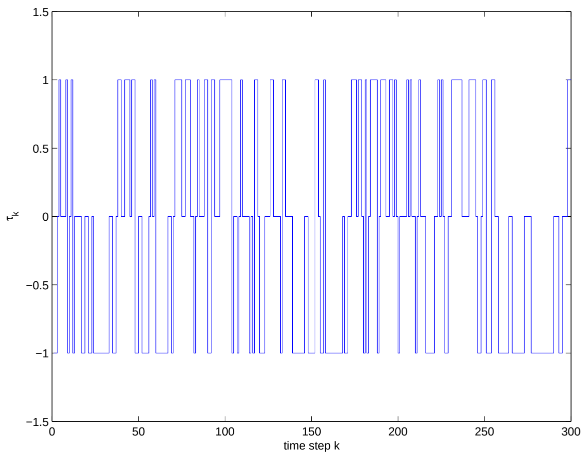

Fig. 2. Measurement mode over network

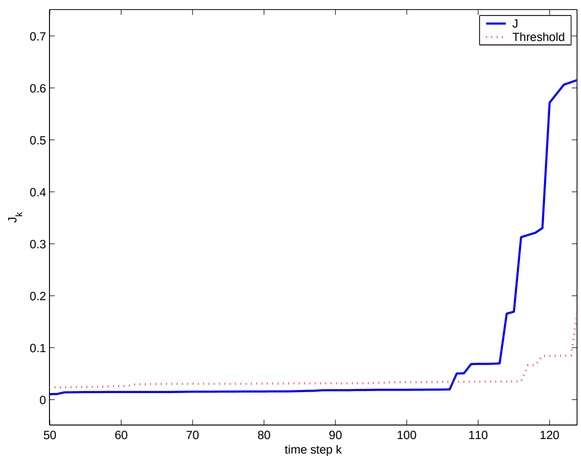

Fig. 3. Evolution of $J(k)$

we properly adjust the gain of the transfer function, enlarging the threshold be acceptable in practice. In this simulation, we multiple K_0, M_0, K_1 and M_1 with 10^7. Using the resulted fault detection filter, we provide a time-domain simulation result.

Fig. 2 shows the measurement mode with random delay and missing phenomenon, where $\tau_k = -1$ (respectively, 0, 1) means the measurement is missing (transmitted over the network perfectly, delayed for one-step, respectively).

The evolution function $J(k)$ defined in (10) is presented in Fig. 3. We set the length of the finite evaluating time horizon as $\mathcal{L} = 10$ and determine a threshold by using 400 times Monte Carlo simulation as $J_{th} = 0.0156$. From Fig. 3, we can observe that when $k = 107$, $J(k) > J(k - \mathcal{L}) + J_{th}$ for the first time, and the designed fault detection filter can alarm the fault within 7 time steps after the fault occurred at $k = 100$.

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