Reliable H_∞ Filtering for Discrete Time-Delay Systems with Randomly Occurred Nonlinearities via Delay-Partitioning Method

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Abstract

In this paper, the reliable H_∞ filtering problem is investigated for a class of uncertain discrete time-delay systems with randomly occurred nonlinearities (RONs) and sensor failures. RONs are introduced to model a class of sector-like nonlinearities that occur in a probabilistic way according to a Bernoulli distributed white sequence with a known conditional probability. The failures of sensors are quantified by a variable varying in a given interval. The time-varying delay is unknown with given lower and upper bounds. The aim of the addressed reliable H_∞ filtering problem is to design a filter such that, for all possible sensor failures, RONs, time-delays as well as admissible parameter uncertainties, the filtering error dynamics is asymptotically mean-square stable and also achieves a prescribed H_∞ performance level. Sufficient conditions for the existence of such a filter are obtained by using a new Lyapunov-Krasovskii functional and delay-partitioning technique. The filter gains are characterized in terms of the solution to a set of linear matrix inequalities (LMIs). A numerical example is given to demonstrate the effectiveness of the proposed design approach.

Keywords

H_∞ filtering; Reliable filtering; Sensor failure; Time-delay; Randomly occurred nonlinearities; Delay partitioning.

I. Introduction

Filtering problem has been playing an important role in control engineering and signal processing that has attracted constant research attention [2, 14]. The well-known Kalman filtering is the most representative one among various filtering approaches. For Kalman filtering, the variance of the estimation error is minimized under the assumption that the noise processes have exactly known statistical properties. However, it has been recognized that the standard Kalman filtering algorithm might not guarantee satisfactory performance when the statistical information of the noise is unknown [1]. To handle this problem, the H_∞ filtering scheme has been well developed whose main idea is to design an estimator for a given system to estimate a combination of unknown states such that the L_2 gain from the exogenous disturbance to the estimation error is less than some prescribed level $\gamma > 0$. In the past years, various approaches, which include the linear matrix inequality (LMI) approach [3, 5, 10, 23, 26, 28, 35] and Riccati equation approach [7, 33], have been developed to deal with the H_∞ filtering problem.

In practice, nonlinearity is a main resource that contributes significantly to the system complexity [13, 27]. As such, the H_∞ filtering problem for nonlinear systems has been gaining increasing research attention and a great deal of results have been available in the literature. For example, the H_∞ filtering problem has been investigated in [21, 23] where the nonlinearities are assumed to satisfy the sector-bounded conditions.

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With respect to general stochastic systems, the nonlinear H_∞ filtering problem has also been paid great efforts in [15, 16, 34]. It is worth mentioning that, however, a number of practical systems are influenced by additive randomly occurred nonlinear disturbances that are caused by environmental circumstances. For example, in a networked environment, such nonlinear disturbances may be subject to random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, modification of the operating point of a linearized model of nonlinear systems, etc. As explained in [20], such nonlinear disturbances may occur in a probabilistic way and are randomly changeable in terms of their types and/or intensity, which are then named as randomly occurred nonlinearities (RONs). It should be pointed out that, up to now, the control and filtering problems for discrete-time systems with RONs have not received adequate research attention yet despite their engineering importance in networked control systems.

Due to the finite switching speed of the amplifiers, time-delays are frequently encountered in dynamical systems. The existence of time-delays may deteriorate the system performance and even result in the instability of the systems. As such, in the past few years, a great number of results have been reported for the systems with various types of delays, such as constant time-delay [17,18,25,31], time-varying delay [6,24,35], distributed delay [12,22,29], etc. Recently, the co-called delay partitioning technique has been widely used to address the stability analysis problem of time-delay systems, which has proven to be very effective in reducing the possible conservatism of the stability criteria. For example, in [9], a delay decomposition approach was proposed to deal with the stability issue for linear neutral systems with time delays. In [36], the stability and stabilization problem was investigated for delayed T-S fuzzy systems by using delay partitioning approach. On the other hand, it is quite common in practice that the measurement output of a stochastic dynamic system contains incomplete observations because of temporal sensor failures. Therefore, it is not surprising that the reliable filtering problem in the presence of possible sensor failures has recently attracted much attention. In the past few years, a number of results have been reported for linear or nonlinear systems, see, e.g., [8, 11, 19, 32, 37]. However, up to now, the H_∞ reliable filtering problem for uncertain discrete-time systems with *randomly occurred nonlinearities and time-varying delays* has not been fully investigated, which gives the motivation of our present investigation.

In this paper, we consider the reliable H_∞ filtering problem against sensor failures for a class of uncertain discrete-time systems with norm-bounded uncertainties, time-varying delay and RONs. The main contributions are as follows. 1) First of all, the RONs model and the sensor failure model are introduced. RONs are introduced to model a class of sector-like nonlinearities whose occurrence is governed by a Bernoulli distributed white sequence with a known conditional probability. 2) The sensor failures are described by a variable taking values in some interval, and such a description is more practical than the conventional outage case. 3) Next, asymptotically mean-square stability conditions of the filtering error dynamics with a prescribed H_∞ performance level are obtained by using a novel Lyapunov-Krasovskii functional and delay-partitioning technique. 4) Then, we introduce a variable to realize the decoupling between the Lyapunov matrices and the filtering error system matrices, which can reduce the conservativeness. Based on the decoupling idea, we design a reliable H_∞ filter whose gains can be obtained by solving a set of LMIs. Finally, a simulation example is utilized to illustrate the effectiveness of the developed approach.

Notation The following notation will be used in this paper. $\mathbb{R}^n$ denotes the n dimensional Euclidean space. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. I and 0 represent the identity matrix and a zero matrix with appropriate dimension, respectively. For a matrix R, R^T represents its transpose and $\mathrm{diag}\{R_1, R_2, ...\}$ denotes a block diagonal matrix whose diagonal blocks are given by $R_1, R_2, ...$ In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions.

II. Problem Formulation

Consider the following discrete-time uncertain stochastic nonlinear system:

$$\begin{aligned} x_{k+1} &= (A+\Delta A)\, x_k + (A_d + \Delta A_d)\, x_{k-d_k} + Dw_k + \xi_k E f(x_k), \\ y_k &= C_1 x_k + D_1 w_k, \\ z_k &= C x_k, \\ x_k &= \varphi_k, \quad k = -d_M, -d_M+1, \ldots, 0, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state; $y_k \in \mathbb{R}^m$ is the measured output vector; $z_k \in \mathbb{R}^r$ is the signal to be estimated; $w_k \in \mathbb{R}^q$ is the exogenous disturbance signal belonging to $l_2[0,\infty)$; d_k denotes the time-varying delay with lower and upper bounds $d_m \le d_k \le d_M$. Note that the lower bound of delay d_m can be always described by $d_m = \tau m$ where τ and m are integers. φ_k is the initial state of the system. A, A_d, C, C_1, D, D_1 and E are known real matrices with appropriate dimensions. ΔA and ΔA_d are unknown matrices representing parameter uncertainties that are assumed to satisfy the following admissible condition:

$$[\Delta A \quad \Delta A_d] = MF[N \quad N_d], \qquad FF^T \le I \tag{2}$$

where M, N and N_d are known constant matrices with appropriate dimensions.

The nonlinear function $f(x)$ satisfies the following sector-bounded condition:

$$[f(x) - T_1 x]^T [f(x) - T_2 x] \le 0, \qquad \forall x \in \mathbb{R}^n \tag{3}$$

where T_1 and T_2 are known real matrices of appropriate dimensions and $T = T_1 - T_2$ is a symmetric positive definite matrix.

Remark 1: It is customary that the nonlinear function $f(x)$ is said to belong to sectors $[T_2,\ T_1]$. The description in (3) is quite general that includes the usual Lipschitz conditions as a special case, see [20] for the discussion on the sector-like nonlinearities. Note that both the control analysis and model reduction problems for systems with sector-like nonlinearities have been intensively studied, see e.g. [12, 21].

The stochastic variable $\xi_k \in \mathbb{R}$, which accounts for the phenomena of randomly occurred nonlinearities (RONs), is a Bernoulli distributed white sequence taking values of 1 and 0 with

$$\begin{aligned} \mathrm{Prob}\{\xi_k = 1\} &= \bar{\xi}, \\ \mathrm{Prob}\{\xi_k = 0\} &= 1 - \bar{\xi}, \end{aligned} \tag{4}$$

where $\bar{\xi} \in [0 \ \ 1]$ is a known constant.

When the sensors experience failures, we consider the following sensor failure model to describe the measured signal sent from sensors:

$$y_k^F = G y_k \tag{5}$$

where the sensor fault matrix G is defined as follows:

$$0 \leq \underline{G} = \mathrm{diag}\{\underline{g}_1, \ldots, \underline{g}_p\} \leq G = \mathrm{diag}\left\{g_1, \ldots, g_p\right\} \leq \bar{G} = \mathrm{diag}\left\{\bar{g}_1, \ldots, \bar{g}_p\right\} \leq I \tag{6}$$

in which the variables $g_i\,(i = 1, \ldots, p)$ quantify the failures of the sensors.

Let

$$G_0 = \mathrm{diag}\left\{g_{01}, \ldots, g_{0p}\right\} := \frac{\underline{G} + \bar{G}}{2} = \mathrm{diag}\left\{\frac{\underline{g}_1 + \bar{g}_1}{2}, \ldots, \frac{\underline{g}_p + \bar{g}_p}{2}\right\}, \tag{7}$$

$$\tilde{G} = \mathrm{diag}\left\{\tilde{g}_1, \ldots, \tilde{g}_p\right\} := \frac{\bar{G} - \underline{G}}{2} = \mathrm{diag}\left\{\frac{\bar{g}_1 - \underline{g}_1}{2}, \ldots, \frac{\bar{g}_p - \underline{g}_p}{2}\right\}. \tag{8}$$

We can rewrite G as follows:

$$G = G_0 + \Delta = G_0 + \mathrm{diag}\left\{\phi_1, \ldots, \phi_p\right\}, \qquad |\phi_i| \leq \tilde{g}_i, \qquad (i = 1, \ldots, p)\,. \tag{9}$$

In this paper, we consider the following reliable filter:

$$\begin{aligned} \hat{x}_{k+1} &= A_f \hat{x}_k + B_f y_k^F, \\ \hat{z}_k &= C_f \hat{x}_k \end{aligned} \tag{10}$$

where A_f, B_f and C_f are parameters to be determined. By defining $\eta_k = \begin{bmatrix} x_k^T & \hat{x}_k^T \end{bmatrix}^T$, we have the following filtering error system:

$$\begin{aligned} \eta_{k+1} &= \tilde{A}\eta_k + \tilde{A}_d \eta_{k-d_k} + \tilde{B} w_k + \xi_k \tilde{E} f\left(Z\eta_k\right), \\ e_k &= \tilde{C}\eta_k \end{aligned} \tag{11}$$

where $e_k = z_k - \hat{z}_k$ is the estimated error, and

$$\begin{aligned} \tilde{A} &= \begin{bmatrix} A + \Delta A & 0 \\ B_f G C_1 & A_f \end{bmatrix} = \begin{bmatrix} A & 0 \\ B_f G C_1 & A_f \end{bmatrix} + \begin{bmatrix} M \\ 0 \end{bmatrix} F \begin{bmatrix} N & 0 \end{bmatrix} = \bar{A} + \bar{M} F \bar{N}, \\ \tilde{A}_d &= \begin{bmatrix} A_d + \Delta A_d & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} M \\ 0 \end{bmatrix} F \begin{bmatrix} N_d & 0 \end{bmatrix} = \bar{A}_d + \bar{M} F \bar{N}_d, \\ \tilde{B} &= \begin{bmatrix} D \\ B_f G D_1 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C & -C_f \end{bmatrix}, \quad \tilde{E} = \begin{bmatrix} E \\ 0 \end{bmatrix}, \quad Z = \begin{bmatrix} I & 0 \end{bmatrix}. \end{aligned}$$

In this paper, we aim to determine the parameters A_f, B_f and C_f for the reliable filter (10) such that, for all admissible sensor failures, randomly occurred nonlinearities (RONs), time-varying delay, parameter uncertainties and exogenous disturbance, the filtering error system (11) satisfies the following requirements:

(a) The filtering error system (11) is asymptotically mean-square stable.

(b) Under the zero-initial condition, the estimated error e_k satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\left\{\|e_k\|^2\right\} < \gamma^2 \sum_{k=0}^{\infty} \mathbb{E}\left\{\|w_k\|^2\right\} \tag{12}$$

for all nonzero w_k, where $\gamma > 0$ is a prescribed scalar.

III. Main Results

The following lemmas are needed in the proofs of our main results in this paper.

Lemma 1: Let $J = J^T$, M and N be real matrices of appropriate dimensions with F satisfying $FF^T \leq I$, then $J + MFN + N^T F^T M^T < 0$ if and only if there exists a positive scalar ε such that

$$J + \varepsilon^{-1} M M^T + \varepsilon N^T N < 0. \tag{13}$$

Lemma 2: [30] Let $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$. Then, for any scalar $\mu > 0$, we have

$$x^T y + y^T x \leq \mu x^T x + \mu^{-1} y^T y. \tag{14}$$

First of all, let us deal with both the stability analysis issue and the H_∞ performance analysis issue in the case that the parameter matrix describing the sensor failures is known. Sufficient conditions for the addressed problem are represented via an LMI approach.

Theorem 1: Consider the filtering error system (11) with known sensor failure parameter matrix G and a prescribed H_∞ performance index $\gamma > 0$. If there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $S_1 > 0$, $S_2 > 0$, R_1, R_2, R_3 and a scalar $\mu > 0$ such that the following linear matrix inequalities hold,

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * \\ \lambda_2 R_2^T & 0 & -S_2 & * & * & * & * & * & * & * \\ PW_{P_1} & 0 & 0 & -P & * & * & * & * & * & * \\ \lambda_1 S_1 \Xi & 0 & 0 & 0 & -S_1 & * & * & * & * & * \\ \lambda_2 S_2 \Xi & 0 & 0 & 0 & 0 & -S_2 & * & * & * & * \\ \lambda_3 PW_{P_2} & 0 & 0 & 0 & 0 & 0 & -P & * & * & * \\ \lambda_4 S_1 W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & -S_1 & * & * \\ \lambda_5 S_2 W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_2 & * \\ W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * \\ \lambda_2 R_3^T & 0 & -S_2 & * & * & * & * & * & * & * \\ PW_{P_1} & 0 & 0 & -P & * & * & * & * & * & * \\ \lambda_1 S_1 \Xi & 0 & 0 & 0 & -S_1 & * & * & * & * & * \\ \lambda_2 S_2 \Xi & 0 & 0 & 0 & 0 & -S_2 & * & * & * & * \\ \lambda_3 PW_{P_2} & 0 & 0 & 0 & 0 & 0 & -P & * & * & * \\ \lambda_4 S_1 W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & -S_1 & * & * \\ \lambda_5 S_2 W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -S_2 & * \\ W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{16}$$

where

$$
\begin{aligned}
\Omega &= -W_{P_3}^T P W_{P_3} + W_{Q_1}^T \bar{Q}_1 W_{Q_1} + W_{Q_2}^T \bar{Q}_2 W_{Q_2} + W_{Q_3}^T \bar{Q}_3 W_{Q_3} - \mu \breve{T} - \gamma^2 W_w^T W_w \\
&\quad + R_1 W_{R_1} + (R_1 W_{R_1})^T + R_2 W_{R_2} + (R_2 W_{R_2})^T + R_3 W_{R_3} + (R_3 W_{R_3})^T, \\
\lambda_1 &= \sqrt{\tau}, \quad \lambda_2 = \sqrt{d_M - \tau m}, \quad \lambda_3 = \sqrt{\bar{\xi}\left(1-\bar{\xi}\right)}, \quad \lambda_4 = \sqrt{\tau\bar{\xi}\left(1-\bar{\xi}\right)}, \\
\lambda_5 &= \sqrt{\bar{\xi}\left(1-\bar{\xi}\right)(d_M - \tau m)}, \quad \lambda_6 = \sqrt{d_M - \tau m + 1}, \\
\bar{Q}_1 &= \begin{bmatrix} Q_1 & 0 \\ 0 & -Q_1 \end{bmatrix}, \quad \bar{Q}_2 = \begin{bmatrix} Q_2 & 0 \\ 0 & -Q_2 \end{bmatrix}, \quad \bar{Q}_3 = \begin{bmatrix} Q_3 & 0 \\ 0 & -Q_3 \end{bmatrix}, \\
W_{P_1} &= \begin{bmatrix} \tilde{A} & 0_{2n,2mn} & \tilde{A}_d & 0_{2n} & \bar{\xi}\tilde{E} & \tilde{B} \end{bmatrix}, \quad W_{P_2} = \begin{bmatrix} 0_{2n,2mn+6n} & \tilde{E} & 0_{2n,q} \end{bmatrix}, \\
W_{P_3} &= \begin{bmatrix} I_{2n} & 0_{2n,2mn+5n+q} \end{bmatrix}, \quad W_{P_4} = \begin{bmatrix} \tilde{C} & 0_{n_C,2mn+5n+q} \end{bmatrix}, \quad \Xi = W_{P_1} - W_{P_3}, \\
W_{Q_1} &= \begin{bmatrix} I_{2mn} & 0_{2mn,7n+q} & \\ 0_{2mn,2n} & I_{2mn} & 0_{2mn,5n+q} \end{bmatrix}, \quad W_{Q_2} = \begin{bmatrix} I_{2n} & 0_{2n,2mn+5n+q} & \\ 0_{2n,2mn+4n} & I_{2n} & 0_{2n,n+q} \end{bmatrix}, \\
W_{Q_3} &= \begin{bmatrix} \lambda_6 I_{2n} & 0_{2n,2mn+5n+q} & \\ 0_{2n,2mn+2n} & I_{2n} & 0_{2n,3n+q} \end{bmatrix}, \\
W_{R_1} &= \begin{bmatrix} I_{2n} & -I_{2n} & 0_{2n,2mn+3n+q} \end{bmatrix}, \quad W_{R_2} = \begin{bmatrix} 0_{2n,2mn} & I_{2n} & -I_{2n} & 0_{2n,3n+q} \end{bmatrix}, \\
W_{R_3} &= \begin{bmatrix} 0_{2n,2mn+2n} & I_{2n} & -I_{2n} & 0_{2n,n+q} \end{bmatrix}, \quad W_w = \begin{bmatrix} 0_{q,2mn+7n} & I_q \end{bmatrix}, \\
\breve{T} &= \begin{bmatrix} Z^T \tilde{T}_1 Z & * & * & * \\ 0_{2mn+4n,2n} & 0_{2mn+4n} & * & * \\ \tilde{T}_2^T Z & 0_{n,2mn+4n} & I_n & * \\ 0_{q,2n} & 0_{q,2mn+4n} & 0_{q,n} & 0_q \end{bmatrix}, \quad \tilde{T}_1 = \left(T_1^T T_2 + T_2^T T_1\right)/2, \quad \tilde{T}_2 = -\left(T_1^T + T_2^T\right)/2,
\end{aligned}
$$

in which n_C is the number of row in matrix C, then the filtering error system (11) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ.

Proof: Let us first show that, under the zero-initial condition, the estimated error e_k satisfies (12) for all nonzero w_k. Choose a new Lyapunov-Krasovskii functional candidate:

$$V_k = V_{1k} + V_{2k} + V_{3k} + V_{4k} \tag{17}$$

where

$$
\begin{aligned}
V_{1k} &= \eta_k^T P \eta_k, \\
V_{2k} &= \sum_{i=k-\tau}^{k-1} \Gamma_i^T Q_1 \Gamma_i + \sum_{i=k-d_M}^{k-1} \eta_i^T Q_2 \eta_i, \\
V_{3k} &= \sum_{j=-d_M+1}^{-\tau m+1} \sum_{i=k-1+j}^{k-1} \eta_i^T Q_3 \eta_i, \\
V_{4k} &= \sum_{j=-\tau}^{-1} \sum_{i=k+j}^{k-1} \delta_i^T S_1 \delta_i + \sum_{j=-d_M}^{-\tau m-1} \sum_{i=k+j}^{k-1} \delta_i^T S_2 \delta_i
\end{aligned}
$$

with

$$\delta_i = \eta_{i+1} - \eta_i, \qquad \Gamma_i = \begin{bmatrix} \eta_i \\ \eta_{i-\tau} \\ \vdots \\ \eta_{i-(m-1)\tau} \end{bmatrix}.$$

Calculating the difference of V_k along the system (11) under the zero-initial condition, we have

$$\mathbb{E}\{\Delta V_k\} = \mathbb{E}\{\Delta V_{1k}\} + \mathbb{E}\{\Delta V_{2k}\} + \mathbb{E}\{\Delta V_{3k}\} + \mathbb{E}\{\Delta V_{4k}\} \tag{18}$$

where

$$\begin{aligned}
\mathbb{E}\{\Delta V_{1k}\} &= \mathbb{E}\Big\{\Big[\tilde{A}\eta_k + \tilde{A}_d\eta_{k-d_k} + \tilde{B}w_k + \bar{\xi}\tilde{E}f(Z\eta_k) + \big(\xi_k - \bar{\xi}\big)\tilde{E}f(Z\eta_k)\Big]^T P \\
&\quad \times \Big[\tilde{A}\eta_k + \tilde{A}_d\eta_{k-d_k} + \tilde{B}w_k + \bar{\xi}\tilde{E}f(Z\eta_k) + \big(\xi_k - \bar{\xi}\big)\tilde{E}f(Z\eta_k)\Big] - \eta_k^T P\eta_k\Big\} \\
&= \mathbb{E}\Big\{\Big[\tilde{A}\eta_k + \tilde{A}_d\eta_{k-d_k} + \tilde{B}w_k + \bar{\xi}\tilde{E}f(Z\eta_k)\Big]^T P\Big[\tilde{A}\eta_k + \tilde{A}_d\eta_{k-d_k} + \tilde{B}w_k + \bar{\xi}\tilde{E}f(Z\eta_k)\Big] \\
&\quad + \bar{\xi}\big(1-\bar{\xi}\big) f^T(Z\eta_k)\tilde{E}^T P\tilde{E}f(Z\eta_k) - \eta_k^T P\eta_k\Big\} \\
&= \mathbb{E}\left\{\alpha_k^T\left(W_{P_1}^T P W_{P_1} + \bar{\xi}\big(1-\bar{\xi}\big)W_{P_2}^T P W_{P_2} - W_{P_3}^T P W_{P_3}\right)\alpha_k\right\},
\end{aligned} \tag{19}$$

$$\begin{aligned}
\mathbb{E}\{\Delta V_{2k}\} &= \mathbb{E}\left\{\Gamma_k^T Q_1\Gamma_k - \Gamma_{k-\tau}^T Q_1\Gamma_{k-\tau} + \eta_k^T Q_2\eta_k - \eta_{k-d_M}^T Q_2\eta_{k-d_M}\right\} \\
&= \mathbb{E}\left\{\alpha_k^T\left(W_{Q_1}^T\bar{Q}_1 W_{Q_1} + W_{Q_2}^T\bar{Q}_2 W_{Q_2}\right)\alpha_k\right\},
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathbb{E}\{\Delta V_{3k}\} &= \mathbb{E}\left\{(d_M - \tau m + 1)\eta_k^T Q_3\eta_k - \sum_{i=k-d_M}^{k-\tau m}\eta_i^T Q_3\eta_i\right\} \\
&\le \mathbb{E}\left\{(d_M - \tau m + 1)\eta_k^T Q_3\eta_k - \eta_{k-d_k}^T Q_3\eta_{k-d_k}\right\} \\
&= \mathbb{E}\left\{\alpha_k^T\left(W_{Q_3}^T\bar{Q}_3 W_{Q_3}\right)\alpha_k\right\},
\end{aligned} \tag{21}$$

$$\begin{aligned}
\mathbb{E}\{\Delta V_{4k}\} &= \mathbb{E}\left\{\delta_k^T(\tau S_1 + (d_M - \tau m)S_2)\delta_k - \sum_{i=k-\tau}^{k-1}\delta_i^T S_1\delta_i - \sum_{i=k-d_k}^{k-\tau m-1}\delta_i^T S_2\delta_i - \sum_{i=k-d_M}^{k-d_k-1}\delta_i^T S_2\delta_i\right\} \\
&= \mathbb{E}\Big\{\Big[\tilde{A}\eta_k + \tilde{A}_d\eta_{k-d_k} + \tilde{B}w_k + \bar{\xi}\tilde{E}f(Z\eta_k) - \eta_k\Big]^T(\tau S_1 + (d_M - \tau m)S_2) \\
&\quad \times\Big[\tilde{A}\eta_k + \tilde{A}_d\eta_{k-d_k} + \tilde{B}w_k + \bar{\xi}\tilde{E}f(Z\eta_k) - \eta_k\Big] \\
&\quad + \bar{\xi}\big(1-\bar{\xi}\big)f^T(Z\eta_k)\tilde{E}^T(\tau S_1 + (d_M - \tau m)S_2)\tilde{E}f(Z\eta_k) \\
&\quad - \sum_{i=k-\tau}^{k-1}\delta_i^T S_1\delta_i - \sum_{i=k-d_k}^{k-\tau m-1}\delta_i^T S_2\delta_i - \sum_{i=k-d_M}^{k-d_k-1}\delta_i^T S_2\delta_i\Bigg\} \\
&= \mathbb{E}\Big\{\alpha_k^T\Big(\tau\Xi^T S_1\Xi + (d_M - \tau m)\Xi^T S_2\Xi + \tau\bar{\xi}\big(1-\bar{\xi}\big)W_{P_2}^T S_1 W_{P_2} \\
&\quad + \bar{\xi}\big(1-\bar{\xi}\big)(d_M - \tau m)W_{P_2}^T S_2 W_{P_2}\Big)\alpha_k \\
&\quad - \sum_{i=k-\tau}^{k-1}\delta_i^T S_1\delta_i - \sum_{i=k-d_k}^{k-\tau m-1}\delta_i^T S_2\delta_i - \sum_{i=k-d_M}^{k-d_k-1}\delta_i^T S_2\delta_i\Bigg\},
\end{aligned} \tag{22}$$

with

$$\alpha_k = \begin{bmatrix} \Gamma_k^T & \eta_{k-\tau m}^T & \eta_{k-d_k}^T & \eta_{k-d_M}^T & f^T\left(Z\eta_k\right) & w_k^T \end{bmatrix}^T.$$

According to the definition of δ_i, for any matrices R_1, R_2 and R_3, the following equations always hold

$$2\alpha_k^T R_1 \left[\eta_k - \eta_{k-\tau} - \sum_{i=k-\tau}^{k-1} \delta_i\right] = 0, \tag{23}$$

$$2\alpha_k^T R_2 \left[\eta_{k-\tau m} - \eta_{k-d_k} - \sum_{i=k-d_k}^{k-\tau m-1} \delta_i\right] = 0, \tag{24}$$

$$2\alpha_k^T R_3 \left[\eta_{k-d_k} - \eta_{k-d_M} - \sum_{i=k-d_M}^{k-d_k-1} \delta_i\right] = 0. \tag{25}$$

On the other hand, note that (3) is equivalent to

$$\begin{bmatrix} x \\ f\left(x\right) \end{bmatrix}^T \begin{bmatrix} \tilde{T}_1 & \tilde{T}_2 \\ \tilde{T}_2^T & I \end{bmatrix} \begin{bmatrix} x \\ f\left(x\right) \end{bmatrix} \leq 0. \tag{26}$$

which implies

$$-\mu\alpha_k^T \breve{T} \alpha_k \geq 0, \tag{27}$$

where $\mu > 0$.

To analyze the H_∞ performance of the filtering error system (11), we introduce the following index:

$$\begin{aligned} J\left(e, w\right) &= \sum_{k=0}^{\infty} \mathbb{E}\left\{e_k^T e_k - \gamma^2 w_k^T w_k\right\} \\ &= \sum_{k=0}^{\infty} \mathbb{E}\left\{e_k^T e_k - \gamma^2 w_k^T w_k + \Delta V_k\right\} + \mathbb{E}\left\{V_0\right\} - \mathbb{E}\left\{V_\infty\right\} \\ &\leq \sum_{k=0}^{\infty} \mathbb{E}\left\{e_k^T e_k - \gamma^2 w_k^T w_k + \Delta V_k\right\}. \end{aligned} \tag{28}$$

From (19)-(25) and (27), we have

$$
\begin{aligned}
&\mathbb{E}\left\{e_k^T e_k-\gamma^2 w_k^T w_k+\Delta V_k\right\}\\
\leq\ &\mathbb{E}\left\{\alpha_k^T\left\{\Omega+W_{P_1}^T P W_{P_1}+\tau\Xi^T S_1\Xi+\left(d_M-\tau m\right)\Xi^T S_2\Xi\right.\right.\\
&+\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T P W_{P_2}+\tau\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T S_1 W_{P_2}+\bar{\xi}\left(1-\bar{\xi}\right)\left(d_M-\tau m\right)W_{P_2}^T S_2 W_{P_2}\\
&\left.\left.+W_{P_4}^T W_{P_4}+\tau R_1 S_1^{-1}R_1^T+\left(d_k-\tau m\right)R_2 S_2^{-1}R_2^T+\left(d_M-d_k\right)R_3 S_2^{-1}R_3^T\right\}\alpha_k\right\}\\
&-\sum_{i=k-\tau}^{k-1}\left(S_1\delta_i+R_1^T\alpha_k\right)^T S_1^{-1}\left(S_1\delta_i+R_1^T\alpha_k\right)\\
&-\sum_{i=k-d_k}^{k-\tau m-1}\left(S_2\delta_i+R_2^T\alpha_k\right)^T S_2^{-1}\left(S_2\delta_i+R_2^T\alpha_k\right)\\
&-\sum_{i=k-d_M}^{k-d_k-1}\left(S_2\delta_i+R_3^T\alpha_k\right)^T S_2^{-1}\left(S_2\delta_i+R_3^T\alpha_k\right)\\
\leq\ &\mathbb{E}\left\{\alpha_k^T\left\{\Omega+W_{P_1}^T P W_{P_1}+\tau\Xi^T S_1\Xi+\left(d_M-\tau m\right)\Xi^T S_2\Xi\right.\right.\\
&+\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T P W_{P_2}+\tau\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T S_1 W_{P_2}+\bar{\xi}\left(1-\bar{\xi}\right)\left(d_M-\tau m\right)W_{P_2}^T S_2 W_{P_2}\\
&\left.\left.+W_{P_4}^T W_{P_4}+\tau R_1 S_1^{-1}R_1^T+\left(d_k-\tau m\right)R_2 S_2^{-1}R_2^T+\left(d_M-d_k\right)R_3 S_2^{-1}R_3^T\right\}\alpha_k\right\}\\
=\ &\mathbb{E}\left\{\alpha_k^T\left\{\Omega+W_{P_1}^T P W_{P_1}+\tau\Xi^T S_1\Xi+\left(d_M-\tau m\right)\Xi^T S_2\Xi+\tau R_1 S_1^{-1}R_1^T\right.\right.\\
&+\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T P W_{P_2}+\tau\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T S_1 W_{P_2}+\bar{\xi}\left(1-\bar{\xi}\right)\left(d_M-\tau m\right)W_{P_2}^T S_2 W_{P_2}\\
&\left.\left.+W_{P_4}^T W_{P_4}+\left(\frac{d_k-\tau m}{d_M-\tau m}\right)\left(d_M-\tau m\right)R_2 S_2^{-1}R_2^T+\left(\frac{d_M-d_k}{d_M-\tau m}\right)\left(d_M-\tau m\right)R_3 S_2^{-1}R_3^T\right\}\alpha_k\right\}\\
=\ &\mathbb{E}\left\{\alpha_k^T\left\{\left(\frac{d_k-\tau m}{d_M-\tau m}\right)\left(\Omega+W_{P_1}^T P W_{P_1}+\tau\Xi^T S_1\Xi+\left(d_M-\tau m\right)\Xi^T S_2\Xi\right.\right.\right.\\
&+\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T P W_{P_2}+\tau\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T S_1 W_{P_2}+\bar{\xi}\left(1-\bar{\xi}\right)\left(d_M-\tau m\right)W_{P_2}^T S_2 W_{P_2}+W_{P_4}^T W_{P_4}\\
&\left.+\tau R_1 S_1^{-1}R_1^T+\left(d_M-\tau m\right)R_2 S_2^{-1}R_2^T\right)+\left(\frac{d_M-d_k}{d_M-\tau m}\right)\left(\Omega+W_{P_1}^T P W_{P_1}+\tau\Xi^T S_1\Xi+\left(d_M-\tau m\right)\Xi^T S_2\Xi\right.\\
&+\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T P W_{P_2}+\tau\bar{\xi}\left(1-\bar{\xi}\right)W_{P_2}^T S_1 W_{P_2}+\bar{\xi}\left(1-\bar{\xi}\right)\left(d_M-\tau m\right)W_{P_2}^T S_2 W_{P_2}+W_{P_4}^T W_{P_4}\\
&\left.\left.\left.+\tau R_1 S_1^{-1}R_1^T+\left(d_M-\tau m\right)R_3 S_2^{-1}R_3^T\right)\right\}\alpha_k\right\}. \qquad (29)
\end{aligned}
$$

By Schur complement, it follows from (15) and (16) that $\mathbb{E}\left\{e_k^T e_k-\gamma^2 w_k^T w_k+\Delta V_k\right\}<0$, which implies that $J\left(e,w\right)<0$. Therefore, the inequality (12) holds for all nonzero w_k. Similar to the above deduction, we can show that the forward difference of V_k with $w=0$ satisfies $\Delta V_k<0$, which indicates the filtering error system (11) is asymptotically mean-square stable. This completes the proof. ■

Remark 2: The delay partitioning technique has been widely used to deal with time-delay systems that has shown the potential of reducing conservatism, see [36]. In Theorem 1, asymptotically mean-square stability conditions of the filtering error system with a prescribed H_∞ performance level have been obtained based on the delay partitioning technique. The conditions can be checked by solving a set of LMIs. Note that the dimensions of the LMIs depend on the partitioning number m.

Theorem 2: Consider the filtering error system (11) with known sensor failure parameter matrix G and a prescribed H_∞ performance index $\gamma>0$. If there exist matrices $P>0$, $Q_1>0$, $Q_2>0$, $Q_3>0$, $S_1>0$,

$S_2 > 0$, R_1, R_2, R_3, H and a scalar $\mu > 0$ such that the following linear matrix inequalities hold,

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * \\ \lambda_2 R_2^T & 0 & -S_2 & * & * & * & * & * & * & * \\ H^T W_{P_1} & 0 & 0 & H_P & * & * & * & * & * & * \\ \lambda_1 H^T \Xi & 0 & 0 & 0 & H_{S_1} & * & * & * & * & * \\ \lambda_2 H^T \Xi & 0 & 0 & 0 & 0 & H_{S_2} & * & * & * & * \\ \lambda_3 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & H_P & * & * & * \\ \lambda_4 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_1} & * & * \\ \lambda_5 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_2} & * \\ W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * \\ \lambda_2 R_3^T & 0 & -S_2 & * & * & * & * & * & * & * \\ H^T W_{P_1} & 0 & 0 & H_P & * & * & * & * & * & * \\ \lambda_1 H^T \Xi & 0 & 0 & 0 & H_{S_1} & * & * & * & * & * \\ \lambda_2 H^T \Xi & 0 & 0 & 0 & 0 & H_{S_2} & * & * & * & * \\ \lambda_3 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & H_P & * & * & * \\ \lambda_4 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_1} & * & * \\ \lambda_5 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_2} & * \\ W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{31}$$

where

$$H_P = P - H - H^T, \qquad H_{S_1} = S_1 - H - H^T, \qquad H_{S_2} = S_2 - H - H^T,$$

Ω, W_{P_1}, W_{P_2}, W_{P_4}, Ξ, λ_1, λ_2, λ_3, λ_4 and λ_5 are defined as in Theorem 1, then the filtering error system (11) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ.

Proof: Using the fact $P - H - H^T \geq -H^T P^{-1} H = \tilde{H}_P$, $S_1 - H - H^T \geq -H^T S_1^{-1} H = \tilde{H}_{S_1}$ and $S_2 - H - H^T \geq -H^T S_2^{-1} H = \tilde{H}_{S_2}$, we can obtain that

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * \\ \lambda_2 R_2^T & 0 & -S_2 & * & * & * & * & * & * & * \\ H^T W_{P_1} & 0 & 0 & \tilde{H}_P & * & * & * & * & * & * \\ \lambda_1 H^T \Xi & 0 & 0 & 0 & \tilde{H}_{S_1} & * & * & * & * & * \\ \lambda_2 H^T \Xi & 0 & 0 & 0 & 0 & \tilde{H}_{S_2} & * & * & * & * \\ \lambda_3 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & \tilde{H}_P & * & * & * \\ \lambda_4 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{H}_{S_1} & * & * \\ \lambda_5 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{H}_{S_2} & * \\ W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{32}$$

Then, pre- and post-multiplying (32) by diag$\{I, I, I, PH^{-T}, S_1 H^{-T}, S_2 H^{-T}, PH^{-T}, S_1 H^{-T}, S_2 H^{-T}, I\}$ and its transpose lead to (15). Similar to the above deduction, from (31), we can obtain that (16) holds. Thus, the proof is completed. ∎

Remark 3: In Theorem 2, by introducing a variable H, the coupling between the Lyapunov matrices and the filtering error system matrices will be eliminated. Such a newly introduced variable H does not present any structural constraint such as symmetry, which is supposed to lead to potentially less conservative results.

In the following theorem, the uncertainties satisfying (2) will be eliminated in order to facilitate the actual filter design.

Theorem 3: Consider the filtering error system (11) with known sensor failure parameter matrix G and a prescribed H_∞ performance index $\gamma > 0$. If there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $S_1 > 0$, $S_2 > 0$, R_1, R_2, R_3, H and scalars $\mu > 0$, $\varepsilon > 0$ such that the following linear matrix inequalities hold,

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * & * & * \\ \lambda_2 R_2^T & 0 & -S_2 & * & * & * & * & * & * & * & * & * \\ H^T \tilde{W}_{P_1} & 0 & 0 & H_P & * & * & * & * & * & * & * & * \\ \lambda_1 H^T \tilde{\Xi} & 0 & 0 & 0 & H_{S_1} & * & * & * & * & * & * & * \\ \lambda_2 H^T \tilde{\Xi} & 0 & 0 & 0 & 0 & H_{S_2} & * & * & * & * & * & * \\ \lambda_3 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & H_P & * & * & * & * & * \\ \lambda_4 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_1} & * & * & * & * \\ \lambda_5 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_2} & * & * & * \\ W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & \bar{M}^T H & \lambda_1 \bar{M}^T H & \lambda_2 \bar{M}^T H & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ \varepsilon \Pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \qquad (33)$$

$$\begin{bmatrix} \Omega & * & * & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * & * & * \\ \lambda_2 R_3^T & 0 & -S_2 & * & * & * & * & * & * & * & * & * \\ H^T \tilde{W}_{P_1} & 0 & 0 & H_P & * & * & * & * & * & * & * & * \\ \lambda_1 H^T \tilde{\Xi} & 0 & 0 & 0 & H_{S_1} & * & * & * & * & * & * & * \\ \lambda_2 H^T \tilde{\Xi} & 0 & 0 & 0 & 0 & H_{S_2} & * & * & * & * & * & * \\ \lambda_3 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & H_P & * & * & * & * & * \\ \lambda_4 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_1} & * & * & * & * \\ \lambda_5 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_2} & * & * & * \\ W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & \bar{M}^T H & \lambda_1 \bar{M}^T H & \lambda_2 \bar{M}^T H & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ \varepsilon \Pi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \qquad (34)$$

where

$$\begin{aligned} \tilde{W}_{P_1} &= \begin{bmatrix} \bar{A} & 0_{2n,2mn} & \bar{A}_d & 0_{2n} & \bar{\xi}\bar{E} & \tilde{B} \end{bmatrix}, \quad W_{P_3} = \begin{bmatrix} I_{2n} & 0_{2n,2mn+5n+q} \end{bmatrix}, \\ \Pi &= \begin{bmatrix} \bar{N} & 0_{n_N,2mn} & \bar{N}_d & 0_{n_N,3n+q} \end{bmatrix}, \quad \tilde{\Xi} = \tilde{W}_{P_1} - W_{P_3}, \end{aligned}$$

n_N is the number of row in matrix N, Ω, W_{P_2}, W_{P_4}, λ_1, λ_2, λ_3, λ_4 and λ_5 are defined as in Theorem 1, and H_P, H_{S_1} and H_{S_2} are defined as in Theorem 2, then the filtering error system (11) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ.

Proof: According to Schur complement, it can be seen that (33) is equivalent to

$$\Psi_1 + \varepsilon^{-1}\tilde{M}\tilde{M}^T + \varepsilon\tilde{N}^T\tilde{N} < 0, \tag{35}$$

where

$$\begin{aligned}
\Psi_1 &= \begin{bmatrix}
\Omega & * & * & * & * & * & * & * & * & * \\
\lambda_1 R_1^T & -S_1 & * & * & * & * & * & * & * & * \\
\lambda_2 R_2^T & 0 & -S_2 & * & * & * & * & * & * & * \\
H^T\tilde{W}_{P_1} & 0 & 0 & H_P & * & * & * & * & * & * \\
\lambda_1 H^T\tilde{\Xi} & 0 & 0 & 0 & H_{S_1} & * & * & * & * & * \\
\lambda_2 H^T\tilde{\Xi} & 0 & 0 & 0 & 0 & H_{S_2} & * & * & * & * \\
\lambda_3 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & H_P & * & * & * \\
\lambda_4 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_1} & * & * \\
\lambda_5 H^T W_{P_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{S_2} & * \\
W_{P_4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I
\end{bmatrix}, \\
\tilde{M} &= \begin{bmatrix} 0_{n_M,2mn+11n+q} & \bar{M}^T H & \lambda_1\bar{M}^T H & \lambda_2\bar{M}^T H & 0_{n_M,7n} \end{bmatrix}^T, \\
\tilde{N} &= \begin{bmatrix} \Pi & 0_{n_N,17n} \end{bmatrix},
\end{aligned}$$

in which n_M is the number of column in matrix M.

By Lemma (1), we can obtain that

$$\Psi_1 + \tilde{M}F\tilde{N} + \left(\tilde{M}F\tilde{N}\right)^T < 0, \tag{36}$$

which is equivalent to (30). Similar to the above deduction, from (34), we can obtain that (31) holds. Therefore, the filtering error system (11) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ. This completes the proof. ■

Based on Theorem 3, we will solve the problem of reliable H_∞ filter design.

Theorem 4: Consider the filtering error system (11) with known sensor failure parameter matrix G and a prescribed H_∞ performance index $\gamma > 0$. If there exist matrices $P_1 > 0$, P_2, $P_3 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $S_{11} > 0$, S_{12}, $S_{13} > 0$, $S_{21} > 0$, S_{22}, $S_{23} > 0$, R_1, R_2, R_3, H_1, H_2, H_3, $\hat{A}$, $\hat{B}$, $\hat{C}$ and scalars $\mu > 0$, $\varepsilon > 0$ such that the following linear matrix inequalities hold,

$$\Phi = \begin{bmatrix}
\Phi_1 & * & * & * & * & * & * & * & * & * & * & * \\
\lambda_1 R_1^T & -\Phi_8 & * & * & * & * & * & * & * & * & * & * \\
\lambda_2 R_2^T & 0 & -\Phi_9 & * & * & * & * & * & * & * & * & * \\
\Phi_2 & 0 & 0 & \Phi_{10} & * & * & * & * & * & * & * & * \\
\lambda_1\Phi_3 & 0 & 0 & 0 & \Phi_{11} & * & * & * & * & * & * & * \\
\lambda_2\Phi_3 & 0 & 0 & 0 & 0 & \Phi_{12} & * & * & * & * & * & * \\
\lambda_3\Phi_4 & 0 & 0 & 0 & 0 & 0 & \Phi_{10} & * & * & * & * & * \\
\lambda_4\Phi_4 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{11} & * & * & * & * \\
\lambda_5\Phi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{12} & * & * & * \\
\Phi_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\
0 & 0 & 0 & \Phi_7 & \lambda_1\Phi_7 & \lambda_2\Phi_7 & 0 & 0 & 0 & 0 & -\varepsilon I & * \\
\varepsilon\Phi_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix} < 0, \tag{37}$$

$$\tilde{\Phi}=\begin{bmatrix} \Phi_1 & * & * & * & * & * & * & * & * & * & * & * \\ \lambda_1 R_1^T & -\Phi_8 & * & * & * & * & * & * & * & * & * & * \\ \lambda_2 R_3^T & 0 & -\Phi_9 & * & * & * & * & * & * & * & * & * \\ \Phi_2 & 0 & 0 & \Phi_{10} & * & * & * & * & * & * & * & * \\ \lambda_1\Phi_3 & 0 & 0 & 0 & \Phi_{11} & * & * & * & * & * & * & * \\ \lambda_2\Phi_3 & 0 & 0 & 0 & 0 & \Phi_{12} & * & * & * & * & * & * \\ \lambda_3\Phi_4 & 0 & 0 & 0 & 0 & 0 & \Phi_{10} & * & * & * & * & * \\ \lambda_4\Phi_4 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{11} & * & * & * & * \\ \lambda_5\Phi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{12} & * & * & * \\ \Phi_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\ 0 & 0 & 0 & \Phi_7 & \lambda_1\Phi_7 & \lambda_2\Phi_7 & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ \varepsilon\Phi_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix}<0, \quad (38)$$

where

$$\begin{aligned}
\Phi_1 &= -W_{P_3}^T P W_{P_3} + W_{Q_1}^T \bar{Q}_1 W_{Q_1} + W_{Q_2}^T \bar{Q}_2 W_{Q_2} + W_{Q_3}^T \bar{Q}_3 W_{Q_3} - \mu\breve{T} - \gamma^2 W_w^T W_w \\
&\quad + R_1 W_{R_1} + (R_1 W_{R_1})^T + R_2 W_{R_2} + (R_2 W_{R_2})^T + R_3 W_{R_3} + (R_3 W_{R_3})^T, \\
P &= \begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix}, \quad \bar{Q}_1 = \begin{bmatrix} Q_1 & 0 \\ 0 & -Q_1 \end{bmatrix}, \quad \bar{Q}_2 = \begin{bmatrix} Q_2 & 0 \\ 0 & -Q_2 \end{bmatrix}, \quad \bar{Q}_3 = \begin{bmatrix} Q_3 & 0 \\ 0 & -Q_3 \end{bmatrix}, \\
\Phi_2 &= \begin{bmatrix} H_1^T A + \hat{B}GC_1 & \hat{A} & 0_{n,2mn} & H_1^T A_d & 0_{n,3n} & \bar{\xi} H_1^T E & H_1^T D + \hat{B}GD_1 \\ H_3^T A + \hat{B}GC_1 & \hat{A} & 0_{n,2mn} & H_3^T A_d & 0_{n,3n} & \bar{\xi} H_3^T E & H_3^T D + \hat{B}GD_1 \end{bmatrix}, \\
\Phi_3 &= \begin{bmatrix} H_1^T A - H_1^T + \hat{B}GC_1 & \hat{A} - H_2^T & 0_{n,2mn} & H_1^T A_d & 0_{n,3n} & \bar{\xi} H_1^T E & H_1^T D + \hat{B}GD_1 \\ H_3^T A - H_3^T + \hat{B}GC_1 & \hat{A} - H_2^T & 0_{n,2mn} & H_3^T A_d & 0_{n,3n} & \bar{\xi} H_3^T E & H_3^T D + \hat{B}GD_1 \end{bmatrix}, \\
\Phi_4 &= \begin{bmatrix} 0_{n,2mn+6n} & H_1^T E & 0_{n,q} \\ 0_{n,2mn+6n} & H_3^T E & 0_{n,q} \end{bmatrix}, \quad \Phi_5 = \begin{bmatrix} C & -\hat{C} & 0_{n_C,2mn+5n+q} \end{bmatrix}, \\
\Phi_6 &= \begin{bmatrix} N & 0_{n_N,2mn+n} & N_d & 0_{n_N,4n+q} \end{bmatrix}, \quad \Phi_7 = \begin{bmatrix} M^T H_1 & M^T H_3 \end{bmatrix}, \\
\Phi_8 &= \begin{bmatrix} S_{11} & * \\ S_{12} & S_{13} \end{bmatrix}, \quad \Phi_9 = \begin{bmatrix} S_{21} & * \\ S_{22} & S_{23} \end{bmatrix}, \quad \Phi_{10} = \begin{bmatrix} P_1 - H_1 - H_1^T & * \\ P_2 - H_2 - H_3^T & P_3 - H_2 - H_2^T \end{bmatrix}, \\
\Phi_{11} &= \begin{bmatrix} S_{11} - H_1 - H_1^T & * \\ S_{12} - H_2 - H_3^T & S_{13} - H_2 - H_2^T \end{bmatrix}, \quad \Phi_{12} = \begin{bmatrix} S_{21} - H_1 - H_1^T & * \\ S_{22} - H_2 - H_3^T & S_{23} - H_2 - H_2^T \end{bmatrix},
\end{aligned}$$

and W_{P_3}, W_{Q_1}, W_{Q_2}, W_{Q_3}, W_{R_1}, W_{R_2}, W_{R_3}, W_w, $\breve{T}$, λ_1, λ_2, λ_3, λ_4 and λ_5 are defined as in Theorem 1, then the filtering error system (11) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ. Moreover, the parameters of the desired filter are given as follows:

$$A_f = H_2^{-T}\hat{A}, \qquad B_f = H_2^{-T}\hat{B}, \qquad C_f = \hat{C}. \quad (39)$$

Proof: First, let us partition H as

$$H = \begin{bmatrix} H_1 & H_3 \\ H_2 & H_2 \end{bmatrix} \quad (40)$$

where H_2 is nonsingular without loss of generality. Furthermore, partition P, S_1 and S_2 as

$$P = \begin{bmatrix} P_1 & * \\ P_2 & P_3 \end{bmatrix}, \quad S_1 = \begin{bmatrix} S_{11} & * \\ S_{12} & S_{13} \end{bmatrix}, \quad S_2 = \begin{bmatrix} S_{21} & * \\ S_{22} & S_{23} \end{bmatrix}. \tag{41}$$

Then substituting (39)-(41) into (33) and (34), we can get (37) and (38) immediately. This completes the proof. ■

In Theorems 1-4, with known sensor failure parameter and disturbance attenuation lever γ, we obtain the asymptotic stability conditions of the filtering error system (11) and design an H_∞ filter based on a linear matrix inequality approach. In the following theorem, a design procedure for the desired filter parameters will be provided in the case that the failure parameter matrix is unknown but satisfies the constraints (6)-(9).

Theorem 5: Consider the filtering error system (11) with a prescribed H_∞ performance index $\gamma > 0$. If there exist matrices $P_1 > 0$, P_2, $P_3 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $S_{11} > 0$, S_{12}, $S_{13} > 0$, $S_{21} > 0$, S_{22}, $S_{23} > 0$, R_1, R_2, R_3, H_1, H_2, H_3, $\hat{A}$, $\hat{B}$, $\hat{C}$ and scalars $\mu > 0$, $\varepsilon > 0$, $\sigma > 0$ such that the following linear matrix inequalities hold,

$$\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_{11} & * \\ \hat{\Phi}_{21} & \hat{\Phi}_{22} \end{bmatrix} < 0, \tag{42}$$

$$\breve{\Phi} = \begin{bmatrix} \breve{\Phi}_{11} & * \\ \breve{\Phi}_{21} & \breve{\Phi}_{22} \end{bmatrix} < 0, \tag{43}$$

where

$$\hat{\Phi}_{11} = \begin{bmatrix} \Phi_1 & * & * & * & * & * \\ \lambda_1 R_1^T & -\Phi_8 & * & * & * & * \\ \lambda_2 R_2^T & 0 & -\Phi_9 & * & * & * \\ \bar{\Phi}_2 & 0 & 0 & \Phi_{10} & * & * \\ \lambda_1 \bar{\Phi}_3 & 0 & 0 & 0 & \Phi_{11} & * \\ \lambda_2 \bar{\Phi}_3 & 0 & 0 & 0 & 0 & \Phi_{12} \end{bmatrix},$$

$$\breve{\Phi}_{11} = \begin{bmatrix} \Phi_1 & * & * & * & * & * \\ \lambda_1 R_1^T & -\Phi_8 & * & * & * & * \\ \lambda_2 R_3^T & 0 & -\Phi_9 & * & * & * \\ \bar{\Phi}_2 & 0 & 0 & \Phi_{10} & * & * \\ \lambda_1 \bar{\Phi}_3 & 0 & 0 & 0 & \Phi_{11} & * \\ \lambda_2 \bar{\Phi}_3 & 0 & 0 & 0 & 0 & \Phi_{12} \end{bmatrix},$$

$$\hat{\Phi}_{21} = \breve{\Phi}_{21} = \begin{bmatrix} \lambda_3 \Phi_4 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 \Phi_4 & 0 & 0 & 0 & 0 & 0 \\ \lambda_5 \Phi_4 & 0 & 0 & 0 & 0 & 0 \\ \Phi_5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Phi_7 & \lambda_1 \Phi_7 & \lambda_2 \Phi_7 \\ \varepsilon \Phi_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Phi_{13} & \lambda_1 \Phi_{13} & \lambda_2 \Phi_{13} \\ \sigma \Phi_{14} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
\hat{\Phi}_{22} &= \breve{\Phi}_{22} = \mathrm{diag}\{\Phi_{10}, \Phi_{11}, \Phi_{12}, -I, -\varepsilon I, -\varepsilon I, -\sigma\tilde{G}^{-2}, -\sigma I\}, \\
\bar{\Phi}_2 &= \begin{bmatrix} H_1^T A + \hat{B}G_0C_1 & \hat{A} & 0_{n,2mn} & H_1^T A_d & 0_{n,3n} & \bar{\xi}H_1^T E & H_1^T D + \hat{B}G_0D_1 \\ H_3^T A + \hat{B}G_0C_1 & \hat{A} & 0_{n,2mn} & H_3^T A_d & 0_{n,3n} & \bar{\xi}H_3^T E & H_3^T D + \hat{B}G_0D_1 \end{bmatrix}, \\
\bar{\Phi}_3 &= \begin{bmatrix} H_1^T A - H_1^T + \hat{B}G_0C_1 & \hat{A} - H_2^T & 0_{n,2mn} & H_1^T A_d & 0_{n,3n} & \bar{\xi}H_1^T E & H_1^T D + \hat{B}G_0D_1 \\ H_3^T A - H_3^T + \hat{B}G_0C_1 & \hat{A} - H_2^T & 0_{n,2mn} & H_3^T A_d & 0_{n,3n} & \bar{\xi}H_3^T E & H_3^T D + \hat{B}G_0D_1 \end{bmatrix}, \\
\Phi_{13} &= \begin{bmatrix} \hat{B}^T & \hat{B}^T \end{bmatrix}, \qquad \Phi_{14} = \begin{bmatrix} C_1 & 0_{n_{C_1},2mn+6n} & D_1 \end{bmatrix},
\end{aligned}$$

n_{C_1} is the number of row in matrix C_1, Φ_1, Φ_4, Φ_5, Φ_6, Φ_7, Φ_8, Φ_9, Φ_{10}, Φ_{11} and Φ_{12} are defined as in Theorem 4, and λ_1, λ_2, λ_3, λ_4 and λ_5 are defined as in Theorem 1, then the filtering error system (11) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ. Moreover, the parameters of the desired filter are given as follows:

$$A_f = H_2^{-T}\hat{A}, \qquad B_f = H_2^{-T}\hat{B}, \qquad C_f = \hat{C}. \tag{44}$$

Proof: From (9), we know that Φ in (37) can be rewritten as

$$\Phi = \Phi' + W^T \Delta V + V^T \Delta W, \tag{45}$$

where

$$\begin{aligned}
\Phi' &= \begin{bmatrix}
\Phi_1 & * & * & * & * & * & * & * & * & * & * & * \\
\lambda_1 R_1^T & -\Phi_8 & * & * & * & * & * & * & * & * & * & * \\
\lambda_2 R_2^T & 0 & -\Phi_9 & * & * & * & * & * & * & * & * & * \\
\bar{\Phi}_2 & 0 & 0 & \Phi_{10} & * & * & * & * & * & * & * & * \\
\lambda_1\bar{\Phi}_3 & 0 & 0 & 0 & \Phi_{11} & * & * & * & * & * & * & * \\
\lambda_2\bar{\Phi}_3 & 0 & 0 & 0 & 0 & \Phi_{12} & * & * & * & * & * & * \\
\lambda_3\Phi_4 & 0 & 0 & 0 & 0 & 0 & \Phi_{10} & * & * & * & * & * \\
\lambda_4\Phi_4 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{11} & * & * & * & * \\
\lambda_5\Phi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_{12} & * & * & * \\
\Phi_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I & * & * \\
0 & 0 & 0 & \Phi_7 & \lambda_1\Phi_7 & \lambda_2\Phi_7 & 0 & 0 & 0 & 0 & -\varepsilon I & * \\
\varepsilon\Phi_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon I
\end{bmatrix}, \\
W &= \begin{bmatrix} 0 & 0 & 0 & \Phi_{13} & \lambda_1\Phi_{13} & \lambda_2\Phi_{13} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
V &= \begin{bmatrix} \Phi_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.
\end{aligned}$$

From Lemma 2 and (9), we have

$$\Phi \le \Phi' + \sigma^{-1} W^T \tilde{G}^2 W + \sigma V^T V = \Theta. \tag{46}$$

By Schur complement, (42) implies that $\Phi \le \Theta < 0$. Similarly, from (43), we can get that (38) holds. Therefore, the filtering error system (11) is asymptotically mean-square stable with an H_∞ disturbance attenuation level γ. This completes the proof. ■

Remark 4: The robust H_∞ filter design problem is solved in Theorems 5 for the addressed uncertain nonlinear stochastic time-delay systems. We derive an LMI-based sufficient condition for the existence of desired filters that ensure the mean-square asymptotic stability of the filtering error dynamics and reduce the effect of the disturbance input on the estimated output to a prescribed level for all admissible uncertainties. The delay-partitioning approach has been exploited to deal with time-delay systems that has shown the advantages of reducing conservatism when thinning the delay fractions.

Remark 5: Our main results are based on the LMI conditions. The LMI Control Toolbox implements state-of-the-art interior-point LMI solvers. While these solvers are significantly faster than classical convex optimization algorithms, it should be kept in mind that the complexity of LMI computations remains higher than that of solving, say, a Riccati equation. For instance, problems with a thousand design variables typically take over an hour on today's workstations. However, research on LMI optimization is a very active area in the applied math, optimization and the operations research community, and substantial speed-ups can be expected in the future.

IV. An Illustrative Example

In this section, we present an illustrative example to demonstrate the effectiveness of the proposed method. Consider the system (1)-(4) with parameters as follows:

$$\begin{aligned} A &= \begin{bmatrix} 0.3 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}, \quad E = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\ C_1 &= \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \quad D_1 = 0.5, \quad C = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}, \\ M &= \begin{bmatrix} 1 & 1 \end{bmatrix}^T, \quad N = \begin{bmatrix} 0.001 & 0.001 \end{bmatrix}, \quad N_d = \begin{bmatrix} 0.001 & 0.001 \end{bmatrix}, \quad \bar{\xi} = 0.8. \end{aligned}$$

The nonlinear function $f(x_k)$ is chosen as

$$f(x_k) = \frac{1}{2} \begin{bmatrix} \frac{0.3(x_{1k}+x_{2k})}{1+x_{1k}^2+x_{2k}^2} + 0.1x_{1k} + 0.1x_{2k} & 0.3x_{1k} + 0.3x_{2k} \end{bmatrix}^T$$

which can be bounded by

$$T_1 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} -0.1 & 0 \\ -0.1 & 0.1 \end{bmatrix}.$$

The time-varying delay d_k satisfies $2 \leq d_k \leq 4$ and let $m = 1$. The sensor fault matrix G is assumed to satisfy $0.5 \leq G \leq 0.9$. Then, we can obtain that $G_0 = 0.7$ and $\tilde{G} = 0.2$. Let $F = \sin(k)$ and $w_k = \exp(-k/20) \times n_k$ with n_k being uniformly distributed over $[-0.05, 0.05]$. The H_∞ performance level is taken as $\gamma = 0.5$.

Solving the LMIs (42) and (43) by using the Matlab LMI Toolbox, we can obtain the parameters of the desired filter as follows:

$$A_f = \begin{bmatrix} 0.2558 & 0.0619 \\ -0.1240 & 0.5129 \end{bmatrix}, \qquad B_f = \begin{bmatrix} -0.4257 \\ -1.7332 \end{bmatrix}, \qquad C_f = \begin{bmatrix} -0.0767 & 0.0883 \end{bmatrix}.$$

The simulation results are shown in Figs. 1-2, where the trajectory and estimation of z_k are given in Fig. 1 and the estimated error e_k is given in Fig. 2, which confirm that all the expected system performance requirements are well achieved.

V. Conclusions

In this paper, the reliable H_∞ filtering problem has been studied for a class of discrete-time systems with sensor failures, randomly occurred nonlinearities, bounded state delay and norm-bounded parameter

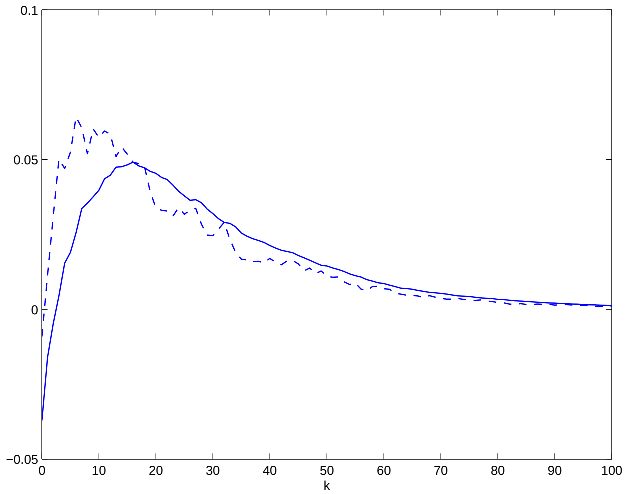

Fig. 1. z_k (solid) and $\hat{z}_k$ (dashed)

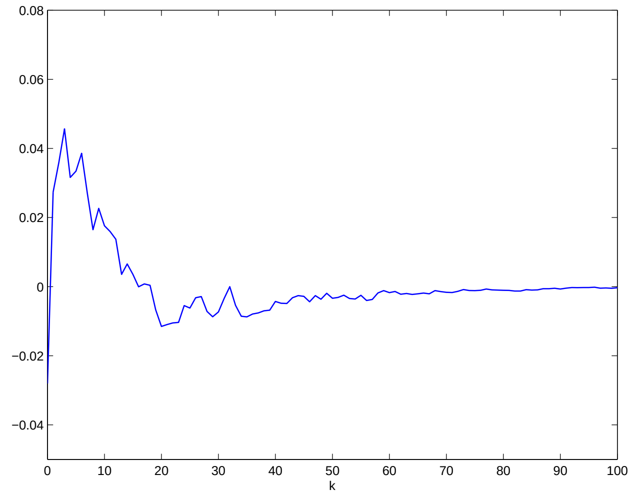

Fig. 2. the estimated error e_k

uncertainties. A new Lyapunov-Krasovskii functional and delay-partitioning technique have been used to design a filter for all admissible uncertainties such that the filtering error system is asymptotically mean-square stable and achieves a prescribed H_∞ performance level. The filter gains have been characterized by the solution of a set of LMIs. An illustrative example has been exploited to show the usefulness of the results obtained. The future research topics would include the extension of the main results developed in this paper to more general complex systems such as networked systems with random packet losses, general stochastic systems, polynomial nonlinear systems and functional differential equations of the neutral type.

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