Distributed H_∞-Consensus Filtering in Sensor Networks with Multiple Missing Measurements: The Finite-Horizon Case ⋆

Bo Shen [a], Zidong Wang [a,b], Y. S. Hung [c]

[a] *School of Information Science and Technology, Donghua University, Shanghai 200051, China.*

[b] *Department of Information Systems and Computing, Brunel University, Uxbridge, Middlesex, UB8 3PH, U.K.*

[c] *Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong.*

Abstract

This paper is concerned with a new distributed H_∞-consensus filtering problem over a finite-horizon for sensor networks with multiple missing measurements. The so-called H_∞-consensus performance requirement is defined to quantify bounded consensus regarding the filtering errors (agreements) over a finite-horizon. A set of random variables are utilized to model the probabilistic information missing phenomena occurring in the channels from the system to the sensors. A sufficient condition is first established in terms of a set of difference linear matrix inequalities (DLMIs) under which the expected H_∞-consensus performance constraint is guaranteed. Given the measurements and estimates of the system state and its neighbors, the filter parameters are then explicitly parameterized by means of the solutions to a certain set of DLMIs that can be computed recursively. Subsequently, two kinds of robust distributed H_∞-consensus filters are designed for the system with norm-bounded uncertainties and polytopic uncertainties. Finally, two numerical simulation examples are used to demonstrate the effectiveness of the proposed distributed filters design scheme.

Key words: Sensor networks; distributed H_∞-consensus filtering; discrete time-varying systems; difference linear matrix inequalities; finite-horizon; data missing.

1 Introduction

The past few decades have witnessed constant research interests on various aspects of sensor networks due primarily to the fact that sensor networks have been extensively applied in many fields such as information collection, environmental monitoring, industrial automation and intelligent buildings. In particular, the distributed filtering or estimation for sensor networks has been an ongoing research issue that attracts increasing attention from researchers in the area. Compared to the single sensor, filter i in a sensor network estimates the system state based not only on the sensor i's measurement, but also on its neighboring sensors' measurements according to the topology of the given sensor network. Such a problem is usually referred to as the distributed filtering or estimation problem. The main difficulty in designing distributed filters lies in how to deal with the complicated coupling between one sensor and its neighboring sensors.

So far, considerable research efforts have been made with respect to distributed filtering and some novel distributed filters are proposed, see e.g [2, 11, 19]. Also, the consensus problems of multi-agent networks have stirred a great deal of research interests, and a rich body of research results has been reported in the literature, see e.g. [1, 5, 6, 8, 10, 13, 15, 16]. Recently, the consensus problem has also been studied for designing distributed Kalman filters (DKFs) [7, 9, 12]. For example, a distributed filter has been introduced in [9] that allows the nodes of a sensor network to track the average of n sensor measurements using an average consensus based distributed filter called consensus filter. The DKF algorithm presented in [9] has been modified in [7], where another two novel DKF algorithms have been proposed and the communication complexity as well as packet-loss issues have been discussed. As is well known, a variety of robust and/or H_∞ filtering approaches have been proposed in the literature to improve the robustness of the filters against parameter uncertainties and exogenous disturbances. In this sense, it seems natural to include the robust and/or H_∞ performance requirements for the distributed consensus filtering problems, and this constitutes one of the two motivations for our current investigation.

Virtually all practical engineering systems are time-varying. A finite-horizon filter could provide better transient performance for filtering process especially when the noise inputs are non-stationary. Therefore, it is of vital importance to consider the filtering problems for time-varying system over a finite horizon. Some efforts have been made on this issue. For example, in [18], a robust finite-horizon Kalman filter has been designed for uncertain systems with multiplicative noises by means of two discrete Riccati difference equations. Using the same approach, the robust finite-horizon filtering problem has been investigated for uncertain system with randomly varying sensor delay in [17]. In addition to the recursive Riccati equation approach, the difference linear matrix inequality (DLMI) method serves as an-

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Email address: `Zidong.Wang@brunel.ac.uk` (Zidong Wang).

other effective tool for handling finite-horizon control and filtering problems for time-varying systems. DLMI approach has been originally proposed in [3, 4], which has proven to be computationally appealing due mainly to the numerical efficiency of LMI algorithms. Up to now, the robust and/or H_∞ distributed consensus filtering problem has not been adequately addressed for time-varying systems over a finite horizon, which gives rise to the second motivation of our research.

In response to the above discussion, in this paper, we aim to deal with the distributed H_∞-consensus filtering problem for sensor networks with multiple missing measurements. The main contributions can be summarized as follows: 1) the concept of H_∞-consensus is introduced to quantify the consensus degree over a finite horizon; 2) the distributed filtering problem is addressed for a class of time-varying systems in the sensor network represented by a directed graph; and 3) a set of random variables is introduced to model the probabilistic data missing occurring in the process of information transmission from the system to each sensor. By resorting to the DLMI technique, the filter parameters can be designed in a recursive way, subject to the H_∞-consensus performance constraint, via the measurements and estimates from the system state as well as its neighbors. Based on the analysis and synthesis results established, we further discuss the robust distributed H_∞-consensus filtering problem for system with norm-bounded uncertainties and polytopic uncertainties, respectively, in terms of a set of DLMIs which can be solved by using available software. Finally, two numerical simulation examples are exploited to show the effectiveness of the distributed filtering techniques proposed in this paper.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. $\|A\|$ refers to the norm of a matrix A defined by $\|A\| = \sqrt{\text{trace}(A^TA)}$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semidefinite (respectively, positive definite). M^T represents the transpose of the matrix M. I denotes the identity matrix of compatible dimension. $\text{diag}_n\{A_i\}$ stands for a block-diagonal matrix with the ith diagonal element being A_i and the notation $\text{diag}_n^i\{A\}$ is employed to stand for the block-diagonal matrix with the ith diagonal element being A and others being zero matrices. $\text{vec}_n\{x_i\}$ denotes $\begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}$. $\mathbb{E}\{x\}$ stands for the expectation of the stochastic variable x. $\text{Prob}\{\cdot\}$ means the occurrence probability of the event "$\cdot$". $l_2[0 \ \ N-1]$ is the space of square summable vector-value functions $f(k)$ in interval $[0 \ \ N-1]$ with the norm $\|f\|_2 = (\sum_{k=0}^{N-1} \|f(k)\|^2)^{\frac{1}{2}}$. In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

2 Problem Formulation and Preliminaries

Consider a sensor network whose topology is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ of order n with the set of nodes (sensors) $\mathcal{V} = \{1, 2, \cdots, n\}$, set of edges $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]$ with nonnegative adjacency elements a_{ij}. An edge of $\mathcal{G}$ is denoted by (i, j). The adjacency elements associated with the edges of the graph are positive, i.e., $a_{ij} > 0 \Longleftrightarrow (i, j) \in \mathcal{E}$. Moreover, we assume $a_{ii} = 1$ for all $i \in \mathcal{V}$, and therefore (i, i) can be regarded as an additional edge. The set of neighbors of node $i \in \mathcal{V}$ plus the node itself are denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$.

The plant is described by the following class of discrete time-varying systems defined on $k \in \{0, 1, \cdots, N-1\}$:

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)v(k) \\ \quad\ \ z(k) = M(k)x(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the immeasurable state, $z(k) \in \mathbb{R}^{n_z}$ is the output to be estimated, and $v(k) \in \mathbb{R}^{n_v}$ is the disturbance input belonging to $l_2[0 \ \ N-1]$. The initial state $x(0)$ is an unknown vector.

For every i $(1 \leq i \leq n)$, the model of sensor node i is given as follows:

$$y_i(k) = \gamma_i(k)C_i(k)x(k) + D_i(k)v(k) \tag{2}$$

where $y_i(k) \in \mathbb{R}^{n_y}$ is the measured output received by the sensor i from the plant, and the stochastic variable $\gamma_i(k) \in \mathbb{R}$ is a Bernoulli distributed white sequence taking values of 1 and 0 with $\text{Prob}\{\gamma_i(k) = 1\} = \beta_i$ and $\text{Prob}\{\gamma_i(k) = 0\} = 1 - \beta_i$. Here, $\beta_i \in [0 \ \ 1]$ is a known constant. The matrices concerned above, i.e., $A(k)$, $B(k)$, $M(k)$, $C_i(k)$, and $D_i(k)$ are known matrices with appropriate dimensions. Moreover, throughout this paper, all stochastic variables $\gamma_i(k)$ $(1 \leq i \leq n, 0 \leq k \leq N-1)$ are assumed to be independent in k and i.

In the sensor network, the information available for the filter on the sensor node i comes from not only the sensor i but also its neighbors. Motivated by this fact, the filter is of the following structure on sensor node i:

$$\begin{cases} \hat{x}_i(k+1) = W_{ii}(k)\hat{x}_i(k) + H_{ii}(k)[y_i(k) \\ \qquad\qquad\qquad - \beta_iC_i(k)\hat{x}_i(k)] + u_i(k) \\ \quad\ \ \hat{z}_i(k) = M(k)\hat{x}_i(k) \end{cases} \tag{3}$$

where $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ is the state estimate of sensor node i and $\hat{z}_i(k) \in \mathbb{R}^{n_z}$ is the estimate for $z(k)$ from the filter on sensor node i. $u_i(k) \in \mathbb{R}^{n_x}$, which represents how the sensor i communicates the information with its neighboring sensors j $(j \in \mathcal{N}_i/\{i\})$, is expressed as follows:

$$\begin{aligned} u_i(k) = &\sum_{j\in\mathcal{N}_i/\{i\}} W_{ij}(k)a_{ij}\hat{x}_j(k) \\ &+ \sum_{j\in\mathcal{N}_i/\{i\}} H_{ij}(k)a_{ij}[y_j(k) - \beta_jC_j(k)\hat{x}_j(k)]. \end{aligned} \tag{4}$$

Here, matrices $W_{ij}(k)$, $H_{ij}(k)$ $(j \in \mathcal{N}_i)$ in (3) and (4) are parameters of the filter for sensor node i which are to be determined. Moreover, the initial values of filters are assumed to be $\hat{x}_i(0) = 0$ for all $1 \leq i \leq n$.

Remark 1 *In the framework of sensor networks, it is important to establish a filter structure to suitably represent how each node communicates information with its* *neighboring nodes. For this purpose, some types of filters (estimators) have been proposed, see [7, 9, 11, 19] for more details. Filter (3) with (4) proposed here consists of two parts: one is used to describe the contribution to the estimate from the node itself; and the other is employed to represent the communications between the underlying node and its neighboring nodes. On the other hand, it is well known that Kalman filtering is an effective approach and the structure of Kalman filter is widely adopted due to its simplicity and practicality. In fact, the structure of filter (3) with (4) stems from that of the Kalman filters by taking into account the communications between the sensor nodes. In order to show the generality of such a filter structure, let us consider the case where there is no communication between the node i and its neighboring nodes, the filter (3) with (4) will be reduced to*

$$\hat{x}_i(k+1) = W_{ii}(k)\hat{x}_i(k) + H_{ii}(k)[y_i(k) - \beta_i C_i(k)\hat{x}_i(k)],$$

which covers the existing ones in available literature.

Letting $e_i(k) = x(k) - \hat{x}_i(k)$ and $\tilde{z}_i(k) = z(k) - \hat{z}_i(k)$, we can obtain the following system that governs the filtering error dynamics for the sensor network:

$$\begin{cases} e_i(k+1) = \sum_{j\in\mathcal{N}_i} W_{ij}(k)a_{ij}e_j(k) \\ \quad - \sum_{j\in\mathcal{N}_i} \beta_j H_{ij}(k)a_{ij}C_j(k)e_j(k) \\ \quad + \big(B(k) - \sum_{j\in\mathcal{N}_i} H_{ij}(k)a_{ij}D_j(k)\big)v(k) \\ \quad + \big(A(k) - \sum_{j\in\mathcal{N}_i} W_{ij}(k)a_{ij} \\ \quad - \sum_{j\in\mathcal{N}_i} (\gamma_j(k) - \beta_j)H_{ij}(k)a_{ij}C_j(k)\big)x(k) \\ \tilde{z}_i(k) = M(k)e_i(k) \end{cases} \tag{5}$$

for $i = 1, 2, \cdots, n$.

Definition 1 *The filtering errors $\tilde{z}_i(k)$ $(i = 1, 2, \cdots, n)$ are said to satisfy the H_∞-consensus performance constraints if the following inequalities hold*

$$\frac{1}{n}\sum_{i=1}^{n} \|\tilde{z}_i\|^2_{\mathbb{E}_2} \le \bar{\gamma}^2 \left\{ \|v\|^2_2 + \frac{1}{n}\sum_{i=1}^{n} e_i^T(0)S_i e_i(0) \right\} \tag{6}$$

where $\|\tilde{z}_i\|_{\mathbb{E}_2} = \left(\mathbb{E}\sum_{k=0}^{N-1} \|\tilde{z}_i(k)\|^2\right)^{\frac{1}{2}}$, for some given disturbance attenuation level $\bar{\gamma} > 0$ and for some given positive definite matrices $S_i = S_i^T > 0$ $(1 \le i \le n)$.

Remark 2 *In a sensor network, each sensor node can only receive the information from its neighboring nodes. Therefore, it turns out to be conservative to require every filtering error from a sensor node to satisfy the central H_∞ performance constraints. Actually, only an average consensus needs to be reached by all nodes of the network regarding the value of filtering error $\tilde{z}_i$ over a finite-time interval, i.e. H_∞-consensus performance constraint guarantees that each filter estimates the system state well. Such an average consensus can be understood as an approximate agreement within a bounded set quantified by the H_∞-norm.*

Definition 2 *Filters of the form (3)-(4) $(i = 1, 2, \cdots, n)$ are said to be distributed H_∞-consensus filters if their filtering errors $\tilde{z}_i(k)$ $(i = 1, 2, \cdots, n)$ satisfy the H_∞-consensus performance constraints (6).*

In this paper, we are interested in finding the filter gain matrices $W_{ij}(k)$, $H_{ij}(k)$ $(i = 1, 2, \cdots, n,\ j \in \mathcal{N}_i)$ such that the filtering errors $\tilde{z}_i(k)$ $(i = 1, 2, \cdots, n)$ from (5) satisfy the H_∞-consensus performance constraints (6).

3 Finite-Horizon Distributed H_∞-Consensus Filtering

In this section, we investigate the distributed H_∞-consensus filtering problem for system (1) with n sensors whose topology is determined by the given graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$. For convenience of later analysis, we denote

$$\begin{aligned} &e(k) = \mathrm{vec}_n^T\{e_i^T(k)\}, && \bar{x}(k) = \mathrm{vec}_n^T\{x^T(k)\}, \\ &\tilde{z}(k) = \mathrm{vec}_n^T\{\tilde{z}_i^T(k)\}, && \bar{A}(k) = \mathrm{diag}_n\{A(k)\}, \\ &\bar{B}(k) = \mathrm{vec}_n^T\{B^T(k)\}, && \bar{D}(k) = \mathrm{vec}_n^T\{D_i^T(k)\}, \\ &\bar{M}(k) = \mathrm{diag}_n\{M(k)\}, && E_n^i(k) = \mathrm{diag}_n^i\{C_i(k)\}, \\ &G_\beta(k) = \mathrm{diag}_n\{\beta_i C_i(k)\}, && \alpha_i = \beta_i(1-\beta_i). \end{aligned} \tag{7}$$

Then, the error dynamics governed by (5) can be rewritten in the following compact form

$$\begin{cases} e(k+1) = \Big(\bar{A}(k) - \bar{W}(k) - \sum_{i=1}^{n}(\gamma_i(k) - \beta_i) \\ \quad \times \bar{H}(k)E_n^i(k)\Big)\bar{x}(k) + \big(\bar{W}(k) - \bar{H}(k)G_\beta(k)\big) \\ \quad \times e(k) + \big(\bar{B}(k) - \bar{H}(k)\bar{D}(k)\big)\, v(k) \\ \tilde{z}(k) = \bar{M}(k)e(k) \end{cases} \tag{8}$$

where

$$\begin{aligned} \bar{W}(k) &= [O_{ij}(k)]_{n\times n}, \text{ with } O_{ij}(k) = W_{ij}(k)a_{ij}, \\ \bar{H}(k) &= [\bar{O}_{ij}(k)]_{n\times n}, \text{ with } \bar{O}_{ij}(k) = H_{ij}(k)a_{ij}. \end{aligned} \tag{9}$$

Obviously, since $a_{ij} = 0$ when $j \notin \mathcal{N}_i$, $\bar{W}(k)$ and $\bar{H}(k)$ are two sparse matrices which can be expressed as

$$\bar{W}(k) \in \mathscr{W}_{n_x\times n_x}, \quad \bar{H}(k) \in \mathscr{W}_{n_x\times n_y} \tag{10}$$

where $\mathscr{W}_{p\times q} = \{\bar{U} = [U_{ij}] \in \mathbb{R}^{np\times nq} | U_{ij} \in \mathbb{R}^{p\times q},\ U_{ij} = 0 \text{ if } j \notin \mathcal{N}_i\}$.

It is not difficult to see that the set $\mathscr{W}_{p\times q}$ has a nice property in the following lemma that will be used in analyzing the filter performance. The proof of the lemma is straightforward and is therefore omitted.

Lemma 1 *Let $Q = diag\{Q_1, Q_2, \cdots, Q_n\}$ with $Q_i \in \mathbb{R}^{p\times p}$ $(1 \le i \le n)$ being invertible matrices. For $W \in \mathbb{R}^{np\times nq}$, if $X = QW$, then we have $W \in \mathscr{W}_{p\times q} \Longleftrightarrow X \in \mathscr{W}_{p\times q}$.*

Setting $\eta(k) = \left[\bar{x}^T(k)\ e^T(k)\right]^T$, the combination of (8) and (1) yields the following augmented system

$$\begin{cases} \eta(k+1) = \Big(\mathcal{A}(k) + \sum_{i=1}^{n}(\gamma_i(k)-\beta_i)\mathcal{F}_i(k)\Big) \\ \qquad\qquad \times\, \eta(k) + \mathcal{B}(k)v(k) \\ \tilde{z}(k) = \mathcal{M}(k)\eta(k) \end{cases} \tag{11}$$

where

$$\begin{aligned} \mathcal{A}(k) &= \begin{bmatrix} \bar{A}(k) & 0 \\ \bar{A}(k)-\bar{W}(k) & \bar{W}(k)-\bar{H}(k)G_\beta(k) \end{bmatrix}, \\ \mathcal{M}(k) &= \begin{bmatrix} 0 & \bar{M}(k) \end{bmatrix}, \mathcal{F}_i(k) = \begin{bmatrix} 0 & 0 \\ -\bar{H}(k)E_n^i(k) & 0 \end{bmatrix}, \\ \mathcal{B}(k) &= \begin{bmatrix} \bar{B}(k) \\ \bar{B}(k)-\bar{H}(k)\bar{D}(k) \end{bmatrix}. \end{aligned} \tag{12}$$

For presentation convenience, the H_∞-consensus performance constraints (6) can be rewritten as

$$\|\tilde{z}\|_{\mathbb{E}_2}^2 \le \bar{\gamma}^2 \left\{ n\|v\|_2^2 + e^T(0)Re(0) \right\} \tag{13}$$

where $R = \mathrm{diag}\{S_1, S_2, \cdots, S_n\}$.

To this end, the addressed distributed H_∞-consensus filtering problem for system (1) over the finite time-horizon amounts to analyzing the L_2-gain of the augmented system (11), which is a state-multiplicative stochastic system due to the existence of the stochastic variables γ_i $(1 \le i \le n)$. The problem of L_2-gain analysis for such a stochastic system has been fully studied in [4] and the corresponding bounded real lemma (BRL) has been derived. Corresponding to the problem addressed in this paper, we modify the BRL in [4] in the following lemma.

Lemma 2 *Consider the stochastic system (11). Given a positive scalar $\bar{\gamma} > 0$ and a positive definite matrix $R^T = R > 0$, if there exists a positive definite matrix $Q^T(k) = Q(k) > 0$ satisfying*

$$\begin{aligned} &\mathcal{A}^T(k)Q(k+1)\mathcal{A}(k) + \sum_{i=1}^{n}\alpha_i\mathcal{F}_i^T(k)Q(k+1)\mathcal{F}_i(k) \\ &+\mathcal{A}^T(k)Q(k+1)\mathcal{B}(k)\Theta^{-1}(k)\mathcal{B}^T(k)Q(k+1)\mathcal{A}(k) \\ &+\mathcal{M}^T(k)\mathcal{M}(k) - Q(k) \le 0 \end{aligned} \tag{14}$$

with the initial condition $\eta^T(0)Q(0)\eta(0) \le \bar{\gamma}^2 e^T(0)Re(0)$ and such that

$$\Theta(k) := n\bar{\gamma}^2 I - \mathcal{B}^T(k)Q(k+1)\mathcal{B}(k) > 0, \tag{15}$$

for all $k = 0, 1, \cdots, N-1$, then the filtering error $\tilde{z}(k)$ satisfies the H_∞ performance constraints (13).

Proof: By considering the facts of $\mathbb{E}\{(\gamma_i(k)-\beta_i)\} = 0$, $\mathbb{E}\{(\gamma_i(k)-\beta_i)^2\} = \alpha_i$ and $\mathbb{E}\{(\gamma_i(k)-\beta_i)(\gamma_j(k)-\beta_j)\} = 0$ $(i \ne j)$, and using the "completing the square" technique to $v(k)$, it can be shown from (14) that

$$\begin{aligned} &\mathbb{E}\Big\{\|\tilde{z}(k)\|^2\Big\} - n\bar{\gamma}^2\|v(k)\|^2 \\ &+\mathbb{E}\Big\{\eta^T(k+1)Q(k+1)\eta(k+1) - \eta^T(k)Q(k)\eta(k)\Big\} \\ =& \,\mathbb{E}\Big\{-\big(v(k)-v^*(k)\big)^T\Theta(k)\big(v(k)-v^*(k)\big) + \eta^T(k) \\ &\times\Big(\mathcal{A}^T(k)Q(k+1)\mathcal{B}(k)\Theta^{-1}(k)\mathcal{B}^T(k)Q(k+1)\mathcal{A}(k) \\ &+\mathcal{A}^T(k)Q(k+1)\mathcal{A}(k) + \sum_{i=1}^{n}\alpha_i\mathcal{F}_i^T(k)Q(k+1)\mathcal{F}_i(k) \\ &+\mathcal{M}^T(k)\mathcal{M}(k) - Q(k)\Big)\eta(k)\Big\} \\ \le& \,\mathbb{E}\Big\{-\big(v(k)-v^*(k)\big)^T\Theta(k)\big(v(k)-v^*(k)\big)\Big\} \end{aligned} \tag{16}$$

where $v^*(k) = \Theta^{-1}(k)\mathcal{B}^T(k)Q(k+1)\mathcal{A}(k)\eta(k)$ and $\Theta(k)$ is defined in (15). Noting that $\Theta(k) > 0$, it follows from (16) that

$$\begin{aligned} \mathbb{E}\Big\{\sum_{k=0}^{N-1}\|\tilde{z}(k)\|^2\Big\} &\le n\bar{\gamma}^2\sum_{k=0}^{N-1}\|v(k)\|^2 \\ &- \mathbb{E}\Big\{\eta^T(N)Q(N)\eta(N)\Big\} + \eta^T(0)Q(0)\eta(0) \end{aligned} \tag{17}$$

which concludes that the H_∞ performance constraints (13) is satisfied under the initial condition $\eta^T(0)Q(0)\eta(0) \le \bar{\gamma}^2 e^T(0)Re(0)$ as long as $Q(N) > 0$. The proof is complete.

Next, let us focus our attention on the design problem of the finite-horizon distributed H_∞-consensus filters for system (1).

The following theorem provides a design method for the distributed H_∞-consensus filtering problem.

Theorem 1 *Given a positive scalar $\bar{\gamma} > 0$ and positive definite matrices $S_i = S_i^T > 0$ $(1 \le i \le n)$. The finite-horizon distributed H_∞-consensus filtering problem is solvable if there exist some families of positive definite matrices $\{Q_1(k)\}_{0\le k\le N}$, $\{Q_2(k) = diag_n\{P_i(k)\}\}_{0\le k\le N}$ and two families of matrices $\{X(k)\}_{0\le k\le N-1}$, $\{Y(k)\}_{0\le k\le N-1}$ satisfying the constraints*

$$X(k) \in \mathscr{W}_{n_x\times n_x}, \quad Y(k) \in \mathscr{W}_{n_x\times n_y}, \tag{18}$$

the initial condition

$$\begin{aligned} &\bar{x}^T(0)Q_1(0)\bar{x}(0) + \sum_{i=1}^{n}e_i^T(0)P_i(0)e_i(0) \\ &\qquad\qquad \le \bar{\gamma}^2\sum_{i=1}^{n}e_i^T(0)S_ie_i(0), \end{aligned} \tag{19}$$

and the following set of DLMIs

$$\Gamma(k) = \Gamma^T(k) \le 0 \tag{20}$$

for all $0 \le k \le N-1$, where $\Gamma(k) = [\Gamma_{ij}(k)]_{6\times6}$ and $\Gamma_{11}(k) = -Q_1(k)$, $\Gamma_{13}(k) = \bar{A}^T(k)Q_1(k+1)$, $\Gamma_{14}(k) = \bar{A}^T(k)Q_2(k+1) - X^T(k)$, $\Gamma_{16}(k) =$

$vec_n\{-\alpha_i E_n^{iT}(k)Y^T(k)\}$, $\Gamma_{22}(k) = -Q_2(k)+\bar{M}^T(k)\bar{M}(k)$, $\Gamma_{24}(k) = X^T(k) - G_\beta^T(k)Y^T(k)$, $\Gamma_{33}(k) = -Q_1(k+1)$, $\Gamma_{35}(k) = Q_1(k+1)\bar{B}(k)$, $\Gamma_{44}(k) = -Q_2(k+1)$, $\Gamma_{45}(k) = Q_2(k+1)\bar{B}(k) - Y(k)\bar{D}(k)$, $\Gamma_{55}(k) = -n\bar{\gamma}^2 I$, $\Gamma_{66}(k) = diag_n\{-\alpha_i Q_2(k+1)\}$, and others are zero matrices. Furthermore, if the set of DLMIs (20) with (18)-(19) are feasible, the desired filter parameters are given by

$$\begin{aligned} \bar{W}(k) &= Q_2^{-1}(k+1)X(k) \\ \bar{H}(k) &= Q_2^{-1}(k+1)Y(k) \end{aligned} \tag{21}$$

for all $0 \le k \le N-1$.

Proof: In terms of Lemma 2, the filter parameters $\bar{W}(k)$ and $\bar{H}(k)$ should satisfy the condition (14) which is guaranteed by

$$\begin{bmatrix} -Q_1(k) & 0 & \bar{A}^T(k)Q_1(k+1) & \Omega_1(k) & 0 & \Omega_2(k) \\ * & \Gamma_{22}(k) & 0 & \Omega_4(k) & 0 & 0 \\ * & * & -Q_1(k+1) & 0 & Q_1(k+1)\bar{B}(k) & 0 \\ * & * & * & -Q_2(k+1) & \Omega_5(k) & 0 \\ * & * & * & * & -n\bar{\gamma}^2 I & 0 \\ * & * & * & * & * & \Gamma_{66}(k) \end{bmatrix} \le 0 \tag{22}$$

and $Q(k) = \text{diag}_2\{Q_i(k)\}$, where

$$\begin{aligned} \Omega_1(k) &= \bar{A}^T(k)Q_2(k+1) - \bar{W}^T(k)Q_2(k+1), \\ \Omega_2(k) &= \text{vec}_n\{-\alpha_i E_n^{iT}(k)\bar{H}^T(k)Q_2(k+1)\}, \\ \Omega_4(k) &= \bar{W}^T(k)Q_2(k+1) - G_\beta^T(k)\bar{H}^T(k)Q_2(k+1), \\ \Omega_5(k) &= Q_2(k+1)\bar{B}(k) - Q_2(k+1)\bar{H}(k)\bar{D}(k). \end{aligned}$$

By noting (21), (22) follows from (20) directly. Moreover, we know that $\bar{W}(k)$ and $\bar{H}(k)$ satisfy the constraints (10) by Lemma 1, and the initial condition $\eta^T(0)Q(0)\eta(0) \le \bar{\gamma}^2 e^T(0)Re(0)$ can be guaranteed by (19). The rest of the proof follows from Lemma 2.

4 Robust Distributed H_∞-Consensus Filtering for Uncertain Systems

In this section, the problem of robust finite-horizon distributed H_∞-consensus filtering is considered for two classes of uncertain systems, i.e., systems with norm-bounded uncertainties and systems with polytopic uncertainties.

4.1 Norm-bounded uncertainties

In this case, the matrix $A(k)$ in plant (1) and the matrices $C_i(k)$ $(1 \le i \le n)$ in sensor model (2) are supposed to be in the form of $A(k) = \vec{A}(k)+\Delta A(k)$ and $C_i(k) = \vec{C}_i(k)+\Delta C_i(k)$ $(1 \le i \le n)$. Here, matrices $\vec{A}(k)$, $\vec{C}_i(k)$ $(1 \le i \le n)$ are known while $\Delta A(k)$, $\Delta C_i(k)$ $(1 \le i \le n)$ are unknown matrices representing parameter uncertainties that satisfy the following admissible condition:

$$\begin{bmatrix} \Delta A(k) \\ \Delta C_i(k) \end{bmatrix} = \begin{bmatrix} S(k) \\ S_i(k) \end{bmatrix} F(k)T(k) \tag{23}$$

where $S(k)$, $T(k)$, $S_i(k)$ $(1 \le i \le n)$ are known real matrices and $F(k)$ is the unknown matrix-valued function subject to $F^T(k)F(k) \le I$, for all $k = 0, 1, \cdots, N-1$. Denote

$$\begin{aligned} &\tilde{A}(k) = \text{diag}_n\{\vec{A}(k)\}, \ \tilde{G}_\beta(k) = \text{diag}_n\{\beta_i \vec{C}_i(k)\}, \\ &\tilde{S}_1(k) = \text{diag}_n\{S(k)\}, \ \tilde{S}_2(k) = \text{diag}_n\{\beta_i S_i(k)\}, \\ &\tilde{T}(k) = \text{diag}_n\{T(k)\}, \quad \tilde{E}_n^i(k) = \text{diag}_n^i\{\vec{C}_i(k)\}, \\ &\tilde{F}(k) = \text{diag}_n\{F(k)\}, \quad \tilde{S}_{3i}(k) = \text{diag}_n^i\{S_i(k)\}. \end{aligned} \tag{24}$$

Then, the matrices $\bar{A}(k)$, $G_\beta(k)$ and $E_n^i(k)$ can be rewritten as

$$\begin{aligned} \bar{A}(k) &= \tilde{A}(k) + \tilde{S}_1(k)\tilde{F}(k)\tilde{T}(k), \\ G_\beta(k) &= \tilde{G}_\beta(k) + \tilde{S}_2(k)\tilde{F}(k)\tilde{T}(k), \\ E_n^i(k) &= \tilde{E}_n^i(k) + \tilde{S}_{3i}(k)\tilde{F}(k)\tilde{T}(k). \end{aligned} \tag{25}$$

Based on Theorem 1, the problem of robust finite-horizon distributed H_∞-consensus filtering is solved in the following theorem for time-varying system (1) with norm-bounded uncertainties.

Theorem 2 *Given a positive scalar $\bar{\gamma} > 0$ and positive definite matrices $S_i = S_i^T > 0$ $(1 \le i \le n)$. The robust finite-horizon distributed H_∞-consensus filtering problem for the system (1) with norm-bounded uncertainties is solvable if there exist some families of positive definite matrices $\{Q_1(k)\}_{0 \le k \le N}$, $\{Q_2(k) = diag_n\{P_i(k)\}\}_{0 \le k \le N}$, two families of matrices $\{X(k)\}_{0 \le k \le N-1}$, $\{Y(k)\}_{0 \le k \le N-1}$, and two families positive scalars $\{\varepsilon_1(k)\}_{0 \le k \le N-1}$, $\{\varepsilon_2(k)\}_{0 \le k \le N-1}$ satisfying the constraint (18), initial condition (19) and the following set of DLMIs*

$$\Xi(k) = \Xi^T(k) \le 0 \tag{26}$$

for all $0 \le k \le N-1$, where $\Xi(k) = [\Xi_{ij}(k)]_{8\times 8}$ and $\Xi_{11}(k) = -Q_1(k) + \varepsilon_1(k)\tilde{T}^T(k)\tilde{T}(k)$, $\Xi_{13}(k) = \tilde{A}^T(k)Q_1(k+1)$, $\Xi_{14}(k) = \tilde{A}^T(k)Q_2(k+1) - X^T(k)$, $\Xi_{16}(k) = vec_n\{-\alpha_i \tilde{E}_n^{iT}(k)Y^T(k)\}$, $\Xi_{22}(k) = -Q_2(k) + \bar{M}^T(k)\bar{M}(k) + \varepsilon_2(k)\tilde{T}^T(k)\tilde{T}(k)$, $\Xi_{24}(k) = X^T(k) - \tilde{G}_\beta^T(k)Y^T(k)$, $\Xi_{33}(k) = -Q_1(k+1)$, $\Xi_{35}(k) = Q_1(k+1)\bar{B}(k)$, $\Xi_{37}(k) = Q_1(k+1)\tilde{S}_1(k)$, $\Xi_{44}(k) = -Q_2(k+1)$, $\Xi_{45}(k) = \Gamma_{45}(k)$, $\Xi_{47}(k) = Q_2(k+1)\tilde{S}_1(k)$, $\Xi_{48}(k) = -Y(k)\tilde{S}_2(k)$, $\Xi_{55}(k) = -n\bar{\gamma}^2 I$, $\Xi_{66}(k) = \Gamma_{66}(k)$, $\Xi_{67}(k) = vec_n^T\{-\alpha_i \tilde{S}_{3i}^T(k)Y^T(k)\}$, $\Xi_{77}(k) = -\varepsilon_1(k)I$, $\Xi_{88}(k) = -\varepsilon_2(k)I$, and others are zero matrices. Furthermore, if the set of DLMIs (26) subject to (18)-(19) are feasible, the desired filter parameters are given by (21) for all $0 \le k \le N-1$.

Proof: By noting (25), the matrix $\Gamma(k)$ in (20) can be expressed as the summation of two parts, i.e., $\Gamma(k) = \tilde{\Gamma}(k) + \Delta\Gamma(k)$. Here, $\tilde{\Gamma}(k)$ has the same form of $\Gamma(k)$ with all $\bar{A}(k)$, $G_\beta(k)$, and $E_n^i(k)$ being replaced by $\tilde{A}(k)$, $\tilde{G}_\beta(k)$, and $\tilde{E}_n^i(k)$, respectively. By denoting

$$\begin{aligned}
\tilde{M}_1^T(k) &= \Big[0\ 0\ \tilde{S}_1^T(k)Q_1(k+1) \\
&\qquad\qquad \tilde{S}_1^T(k)Q_2(k+1)\ 0\ \Xi_{67}^T(k)\Big]^T, \\
\tilde{M}_2^T(k) &= \Big[0\ 0\ 0\ -\tilde{S}_2^T(k)Y^T(k)\ 0\ 0\Big]^T, \qquad (27)\\
\tilde{N}_1(k) &= \Big[\tilde{T}(k)\ 0\ 0\ 0\ 0\ 0\Big], \\
\tilde{N}_2(k) &= \Big[0\ \tilde{T}(k)\ 0\ 0\ 0\ 0\Big],
\end{aligned}$$

the other part $\Delta\Gamma(k)$ can be written as follows

$$\begin{aligned}
\Delta\Gamma(k) \le{}& \varepsilon_1^{-1}(k)\tilde{M}_1(k)\tilde{M}_1^T(k) + \varepsilon_1(k)\tilde{N}_1^T(k)\tilde{N}_1(k) \\
&+\varepsilon_2^{-1}(k)\tilde{M}_2(k)\tilde{M}_2^T(k) + \varepsilon_2(k)\tilde{N}_2^T(k)\tilde{N}_2(k).
\end{aligned}$$

This inequality can be obtained by noting $\tilde{F}^T(k)\tilde{F}(k) \le I$ and employing a well-known elementary inequality (see, e.g., Lemma 1 in [14]). Subsequently, by using the Schur complement, we know that $\Gamma(k) = \tilde{\Gamma}(k)+\Delta\Gamma(k) \le 0$ is implied by (26), and the rest of the proof follows directly from Theorem 1.

4.2 Polytopic uncertainties

To consider the problem of robust finite-horizon distributed H_∞-consensus filtering for system with ploytopic uncertainties, the matrix $A(k)$ in plant (1) and the matrices $C_i(k)$ $(1 \le i \le n)$ in sensor model (2) are rewritten as $A^{(\xi)}(k)$, and $C_i^{(\xi)}(k)$ $(1 \le i \le n)$, respectively. Here, we assume that $A^{(\xi)}(k)$ and $C_i^{(\xi)}(k)$ $(1 \le i \le n)$ are unknown time-varying matrices which contain polytopic uncertainties as follows:

$$\Phi^{(\xi)} := (A^{(\xi)}(k), C_i^{(\xi)}(k), i = 1, 2, \cdots, n) \in \mathfrak{R} \qquad (28)$$

where $\mathfrak{R}$ is a convex polyhedral set described by v vertices

$$\begin{aligned}
\mathfrak{R} := \Big\{\Phi^{(\xi)} | \Phi^{(\xi)} = \sum_{m=1}^{v} \xi_m \Phi^{(m)}, \\
\sum_{m=1}^{v} \xi_m = 1, \xi_m \ge 0, m = 1, 2, \cdots, v\Big\} \qquad (29)
\end{aligned}$$

and $\Phi^{(m)} := (A^{(m)}(k), C_i^{(m)}(k), i = 1, 2, \cdots, n)$ are known matrices for all $m = 1, 2, \cdots, v$.

The following theorem provides a DLMI approach to the design problem of robust finite-horizon distributed H_∞-consensus filters for time-varying systems with polytopic uncertainties.

Theorem 3 *Given a positive scalar $\bar{\gamma} > 0$ and positive definite matrices $S_i = S_i^T > 0$ $(1 \le i \le n)$. The robust finite-horizon distributed H_∞-consensus filters can be designed for the time-varying systems (1) with polytopic uncertainties if there exist some families of positive definite matrices $\{Q_1(k)\}_{0\le k\le N}$, $\{Q_2(k) = diag_n\{P_i(k)\}\}_{0\le k\le N}$ and two families of matrices $\{X(k)\}_{0\le k\le N-1}$, $\{Y(k)\}_{0\le k\le N-1}$ satisfying the constraint (18), initial condition (19) and the following set of DLMIs*

$$\Pi(k) = \Pi^T(k) \le 0 \qquad (30)$$

for all $m = 1, 2, \cdots, v$ and all $0 \le k \le N-1$, where $\Pi(k) = [\Pi_{ij}(k)]_{6\times 6}$ and $\Pi_{11}(k) = -Q_1(k)$, $\Pi_{13}(k) = \bar{A}^{(m)T}(k)Q_1(k+1)$, $\Pi_{14}(k) = \bar{A}^{(m)T}(k)Q_2(k+1) - X^T(k)$, $\Pi_{16}(k) = vec_n\{-\alpha_i E_n^{i(m)T}(k)Y^T(k)\}$, $\Pi_{22}(k) = \Gamma_{22}(k)$, $\Pi_{24}(k) = X^T(k) - G_\beta^{(m)T}(k)Y^T(k)$, $\Pi_{33}(k) = -Q_1(k+1)$, $\Pi_{35}(k) = Q_1(k+1)\bar{B}(k)$, $\Pi_{44}(k) = -Q_2(k+1)$, $\Pi_{45}(k) = \Gamma_{45}(k)$, $\Pi_{55}(k) = -n\bar{\gamma}^2 I$, $\Pi_{66}(k) = \Gamma_{66}(k)$, $\bar{A}^{(m)}(k) = diag_n\{A^{(m)}(k)\}$, $E_n^{i(m)}(k) = diag_n^i\{C_i^{(m)}(k)\}$, $G_\beta^{(m)}(k) = diag_n\{\beta_i C_i^{(m)}(k)\}$, and others are zero matrices. Furthermore, if the set of DLMIs (30) with (18)-(19) are solvable, the desired filter parameters are given by (21) for all $0 \le k \le N-1$. Here, $(A^{(m)}(k), C_i^{(m)}(k), i = 1, 2, \cdots, n)$ are the matrices in (29) at the mth vertex of the polytope.

Proof: The proof is straightforward and is therefore omitted.

5 Illustrative Examples

Consider the sensor network (with 6 nodes) whose topology is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with the set of nodes $\mathcal{V} = \{1, 2, 3, 4, 5, 6\}$, set of edges $\mathcal{E} = \{(1,1), (1,3), (1,5), (2,1), (2,2), (2,4), (3,3), (3,6), (4,2), (4,4), (4,6), (5,3), (5,5), (6,1), (6,4), (6,6)\}$ and the adjacency matrix $\mathcal{A} = [a_{ij}]_{6\times 6}$ where adjacency elements $a_{ij} = 1$ when $(i,j) \in \mathcal{E}$; otherwise, $a_{ij} = 0$.

The nominal time-varying system considered here is given by

$$\begin{cases}
x(k+1) = \begin{bmatrix} 0 & -0.4 \\ 0.6 & 0.7\sin(6k) \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} v(k) \\
z(k) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} x(k)
\end{cases} \qquad (31)$$

with the initial value $x(0) = \begin{bmatrix} 0.2 & -0.1 \end{bmatrix}^T$. The exogenous disturbance input $v(k)$ is selected as $v(k) = 0.3\cos(5t)$. For each i, $(i = 1, 2, 3, 4, 5, 6)$, the model of sensor i is described as follows:

$$y_i(k) = \gamma_i(k)\begin{bmatrix} 0.3 & 0.2\sin(6k) \end{bmatrix} x(k) + v(k). \qquad (32)$$

The probabilities are taken as $\beta_1 = 0.9$, $\beta_2 = 0.95$, $\beta_3 = 0.85$, $\beta_4 = 0.9$, $\beta_5 = 0.8$, and $\beta_6 = 0.85$. The disturbance attenuation level and the positive definite matrix are given as $\bar{\gamma} = 1$ and $S_1 = S_2 = S_3 = S_4 = S_5 = S_6 = \mathrm{diag}\{2, 2\}$, respectively.

Example 1: In this example, the system (31) is assumed to be subject to the norm-bounded uncertainties which satisfy the admissible condition (23) with the following parameters

$$\begin{aligned} S(k) = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}, \; T(k) = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \; S_1(k) = S_2(k) \\ = S_3(k) = S_4(k) = S_5(k) = S_6(k) = 0.1. \end{aligned} \tag{33}$$

We first choose the initial positive definite matrices $Q_1(0) = I_{12}$ and $P_1(0) = P_2(0) = P_3(0) = P_4(0) = P_5(0) = P_6(0) = I_2$ to satisfy the initial condition (19). Then the set of DLMIs in Theorem 2 can be solved recursively by using Matlab (with the YALMIP 3.0 and SeDuMi 1.1). Accordingly, all filter parameters can be obtained in terms of (21).

In the simulation, we set $F(k) = \cos(0.1k)$, and repeat the experiment 100 times. Simulation results are presented in Figs. 1-2. The output $z(k)$ and its average estimates are depicted in Fig. 1. All average filtering errors are given in Fig. 2. The simulation has confirmed that the designed distributed filters perform very well.

Example 2: In this example, we assume that the system (31) and the sensor models are subject to the polytopic uncertainties as follows:

$$\begin{cases} x(k+1) = \begin{bmatrix} 0 & -0.4+\xi \\ 0.6 & 0.7\sin(6k) \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} v(k) \\ z(k) = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix} x(k) \end{cases} \tag{34}$$

$$y_i(k) = \gamma_i(k) \begin{bmatrix} 0.3+\xi & 0.2\sin(6k) \end{bmatrix} x(k) + v(k) \tag{35}$$

where the uncertain parameter ξ is unknown but assumed to belong to the known range $[-0.05 \;\; 0.05]$.

We first choose the same initial positive definite matrices as those in *Example 1*. Then, by employing Matlab (with the YALMIP 3.0 and SeDuMi 1.1), we can solve the set of DLMIs (30) in Theorem 3 recursively and derive all desired filters parameters. When the uncertain parameter in the system (34) is taken as $\xi = 0.02$, we can obtain the corresponding simulation results as shown in Figs. 3-4, which also have demonstrated the effectiveness of the distributed filtering technology presented in this paper.

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Bo Shen received his B.S. degree in Mathematics from Northwestern Polytechnical University, Xi'an, China, in 2003. He is currently pursuing the Ph.D. degree in the School of Information Science and Technology, Donghua University, Shanghai, China. From August 2009 to February 2010, he was a Research Assistant in the Department of Electrical and Electronic Engineering, the University of Hong Kong, Hong Kong. He is now a Visiting PhD Student in the Department of Information Systems and Computing, Brunel University, U.K. His research interest is primarily in nonlinear stochastic control and filtering. He is a very active reviewer for many international journals.

Zidong Wang was born in Jiangsu, China, in 1966. He received the B.Sc. degree in mathematics in 1986 from Suzhou University, Suzhou, China, and the M.Sc. degree in applied mathematics in 1990 and the Ph.D. degree in electrical engineering in 1994, both from Nanjing University of Science and Technology, Nanjing, China.
He is currently Professor of Dynamical Systems and Computing in the Department of Information Systems

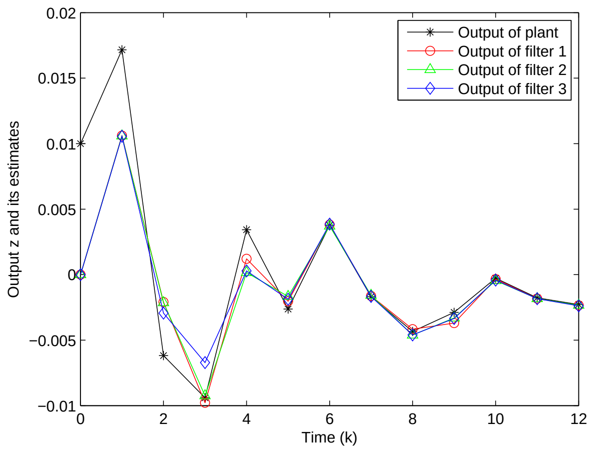

Fig. 1. Output $z(k)$ and its average estimates.

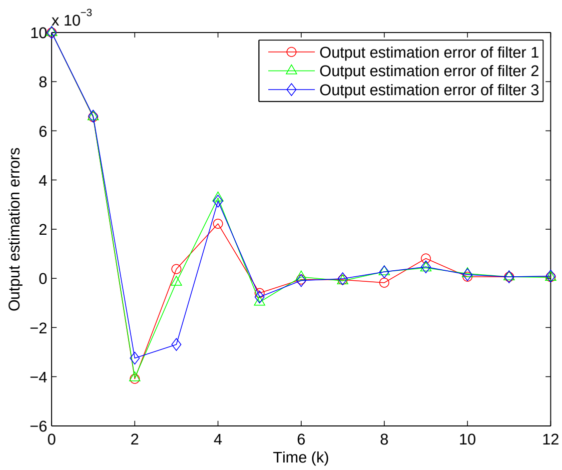

Fig. 2. Average filtering errors.

and Computing, Brunel University, U.K. From 1990 to 2002, he held teaching and research appointments in universities in China, Germany and the UK. Prof. Wang's research interests include dynamical systems, signal processing, bioinformatics, control theory and applications. He has published more than 100 papers in refereed international journals. He is a holder of the Alexander von Humboldt Research Fellowship of Germany, the JSPS Research Fellowship of Japan, William Mong Visiting Research Fellowship of Hong Kong.
Prof. Wang serves as an Associate Editor for 11 international journals, including IEEE Transactions on Automatic Control, IEEE Transactions on Control Systems Technology, IEEE Transactions on Neural Networks, IEEE Transactions on Signal Processing, and IEEE Transactions on Systems, Man, and Cybernetics - Part C. He is a Senior Member of the IEEE, a Fellow of the Royal Statistical Society and a member of program committee for many international conferences.

Y. S. Hung received his B.Sc. (Eng) in Electrical Engineering and B.Sc. in Mathematics, both from the University of Hong Kong, and his M.Phil. and Ph.D. degrees from the University of Cambridge. He has worked

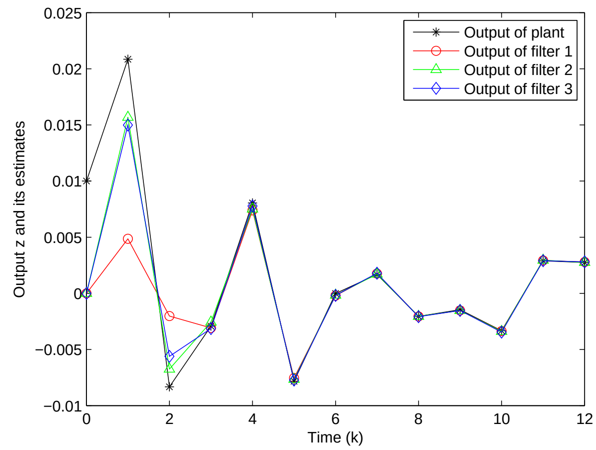

Fig. 3. Output $z(k)$ and its average estimates.

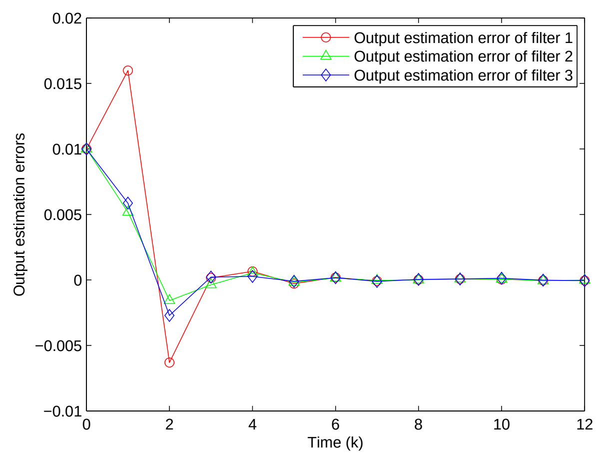

Fig. 4. Average filtering errors.

at the University of Cambridge and the University of Surrey before he joined the University of Hong Kong, where he is now a professor in the Department of Electrical and Electronic Engineering. He has authored and co-authored over 150 publications in books, journals and conferences. His research interests include control systems, robotics, computer vision and bioinformatics.