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Determination of the gravitational constant at an effective mass separation of 22 m

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A vacuum balance that compares the weights of 10-kg stainless-steel masses suspended in evacuated tubes at different levels in a hydroelectric reservoir is being used to measure the gravitational attractions of layers of lake water up to 10 m in depth. The mean effective distance between interacting masses in this experiment is 22 m, making it the largest-scale measurement of G using precisely controlled moving masses. The experiment extends laboratory-type measurements into the range previously explored only by geophysical methods. Assuming purely Newtonian physics the value of the gravitational constant determined from data obtained so far is $G = 6.689(57)\times 10^{-11}$ m^3 kg^{-1}s^{-2}, which agrees with laboratory estimates. The data admit at a 0.6 standard deviation level the parameters of non-Newtonian gravity inferred from geophysical measurements in mines and a tower. These measurements push the estimated ranges of non-Newtonian forces down to a scale accessible to our reservoir experiment, so that experimental improvements now at hand may provide a critical test of non-Newtonian effects.

INTRODUCTION

Speculations about the existence of forces of macroscopic but finite ranges arise principally from ideas about the unification of gravity with the electroweak and strong forces.[1] These ideas were stimulated by measurements of gravity in mines that give estimates of the gravitational constant G that are higher, typically by 0.6%, than the laboratory value.[2,3] Experiments that seek evidence of gravitational nonequivalence of chemically different masses[4–8] have attracted particular attention to this problem, but we are here concerned with inverse-square-law tests and not with composition dependence effects.

We present here details of an experiment that uses a hydroelectric reservoir to measure the gravitational constant on a scale of a few tens of meters, which, it now appears, could be particularly relevant to the search for an inverse-square-law breakdown. The results of a preliminary analysis of the first data from this experiment were referred to by Stacey *et al.*,[3] who presented, as the curves labeled 4 of their Fig. 2, the restriction imposed by these results on the amplitude and range parameters of a single Yukawa term added to the Newtonian potential. Subsequently all of the geophysical data available in early 1987 were reanalyzed[9] in terms of a gravitational potential with a pair of Yukawa terms, as favored for theoretical reasons Goldman *et al.*,[10] whose notation we follow:

$$V = -\frac{G_\infty m}{r}(1 - ae^{-r/v} + be^{-r/s}) \ . \qquad (1)$$

Now we have also an observational reason for considering Eq. (1) rather than a single Yukawa term. Gravity data from a 600-m TV mast, obtained by the U.S. Air Force Geophysics Laboratory[11,12] (USAFGL), require dominance of the attractive (b) term in Eq. (1), whereas measurements in mines[2] indicate a dominant repulsion (the a term) and it is impossible, even qualitatively, to satisfy both data sets simultaneously with a single Yukawa term. We therefore consider the present results in terms of Eq. (1) and find that, with the restrictions on the parameters v and s imposed by the USAFGL tower data, the lake experiment is more sensitive to the effects postulated than we had previously supposed. However, if either the mine data or the tower data are discounted then there is no observational requirement for more than one Yukawa term and if we return to consideration of a single term, then we cannot yet significantly improve the parameter limits in Fig. 2 of Ref. 3.

The lake that is used for the experiment is the Splityard Creek reservoir, which has an area of about 1 km^2 at a surface level about 90 m above a much larger lake impounded by the recently completed Wivenhoe Dam on the Brisbane River. The smaller lake is a storage reservoir for water pumped up to it through a hydroelectric station during periods of surplus electricity generation elsewhere in the supply system and used for generation during peak demand. Level changes up to 10 m occur daily. The Wivenhoe hydroelectric station was under development when discussion of non-Newtonian gravity was becoming serious, allowing an experimental tower to be constructed in the lake before filling commenced. The tower itself is an electricity pylon with an added observing platform, sited 150 m from the nearest shoreline, so that successive layers of lake water pumped into and removed from the lake, are perfectly horizontal, plane layers of almost uniform density and, in the zeroth approximation, infinite in extent. Thus the metrology required to measure G by comparing the weights of masses suspended (in evacuated tubes) below low water and above high water is very simple. Apart from the weighing, the only precise measurements required are of water depth and density. Corrections for the water excluded by the material of the tower are very small and the perimeter correction is straightforward.

The site of the tower was previously a quarry, about 800 m from the nearest point of the dam wall, which, apart from clay and sand cores, is made of loose rock allowing water penetration, making it a poorly defined section of the perimeter. The hard-rock quarry wall, which is the section of the lake perimeter closest to the observing platform, is an unambiguous and particularly well measured boundary.

The weighing is by means of an automated balance of novel design[13] that incorporates a mercury-level tiltmeter to servocontrol a supporting leg, maintaining a precisely horizontal reference axis and dramatically reducing sensitivity to tower instability.

PRINCIPLE OF THE EXPERIMENT— THE NEWTONIAN ANALYSIS

The contribution to gravity by an infinite horizontal sheet of density ρ and thickness t, $2\pi G\rho t$, is independent of distance from it, so that the contribution of the sheet to the gravity difference between points above and below it is

$$\Delta g = 4\pi G\rho t \ . \tag{2}$$

It follows that in measuring such a difference by comparing the weights of masses suspended above and below the layer, the positions and shapes of the masses have no influence on the observations. This remains true as an approximation in the real case of a layer of finite horizontal extent provided the distance to the nearest point of the perimeter is much greater than the dimensions of the masses or the separation of their centers.

Consider a layer of density ρ and infinitesimal thickness dx at a distance x below a point where we wish to estimate its gravity. The distance to the edge of the layer $R(\theta)$ is an arbitrary function of azimuthal angle θ in the horizontal plane and may be multivalued (as in the case of reentrant arms of the lake). Expanding the gravity due to a sector $d\theta$ in powers of the small quantity x/R and integrating over all azimuths, the gravity due to the layer is

$$\begin{aligned} dg &= G\rho\, dx \int_0^{2\pi} \left[1 - \frac{x}{(x^2+R^2)^{1/2}} \right] d\theta \\ &= G\rho\, dx \int_0^{2\pi} \left[1 - \frac{x}{R} + \frac{1}{2}\frac{x^3}{R^3} + \cdots \right] d\theta \\ &= 2\pi G\rho \left[1 - x\langle R^{-1}\rangle + x^3 \frac{\langle R^{-3}\rangle}{2} + \cdots \right] dx \ , \end{aligned} \tag{3}$$

where $\langle R^{-1}\rangle$ and $\langle R^{-3}\rangle$ are azimuthal averages. For values of θ at which R is multivalued, i.e., $R=R_1, R_2, R_3, \ldots, R_n$ (n odd), we take $R^{-1}=(R_1^{-1}-R_2^{-1}+R_3^{-1}-\ldots)$, etc., in calculating the averages. It is a matter of convenience that there is no second-order term in Eq. (3). The third-order term is of no significance at the present accuracy of our experiments and is neglected in the following discussion. Now consider a point at distance y below the layer (and directly below the first point). An expression exactly similar to Eq. (3) is obtained with opposite sign and y replacing x. Taking the difference between the gravity contributions at the two points, we have

$$d(\Delta g) = 4\pi G\rho \left[1 - \frac{x+y}{2}\langle R^{-1}\rangle \right] dx \ . \tag{4}$$

Now $(x+y)$ is simply the vertical separation H of the two observation points and is the same for all layers, independently of x,y, so that integration of Eq. (4) over a finite layer is trivial. The gravity difference due to a layer of thickness $(Z-Z_0)$ extending from $x=Z_0$ to Z is

$$\Delta g = 4\pi G\rho \left\{ (Z-Z_0) \left[1 - \left[\frac{H}{2} \right] \langle R^{-1}\rangle \right] \right\} , \tag{5}$$

where $\langle R^{-1}\rangle$ is now an average with respect to both azimuth and depth over the range $(Z-Z_0)$. If Newtonian gravity is assumed, as in the derivation of Eq. (5), an experiment with two pairs of masses, having different vertical separations H_1 and H_2, and using the same water depth $(Z-Z_0)$, avoids the need to know $\langle R^{-1}\rangle$. Provision for this is incorporated in the balance design,[13] which uses the single knife-edge or mass-substitution method of weighing, so that pairs of masses with different separations may be compared on opposite ends of the balance. The balance was oriented to give the same boundary correction at each end. However, as we show, elimination of $\langle R^{-1}\rangle$ in this way could defeat the object of the experiment, because we also eliminate to first order any non-Newtonian terms that should be added to the inverse square law. It was, therefore, necessary to conduct a detailed lake perimeter survey to obtain the perimeter correction term in Eq. (5).

The separation between the test masses, H in these equations, is the distance between centers of mass. In our experiment all of the masses were stainless-steel cylinders of the same size, so that it is easy to see that the equations apply to all corresponding pairs of elements of the masses individually and therefore collectively. The precision with which H must be known is the same as the precision needed in $\langle R^{-1}\rangle$, being smaller than the error in determining G by the factor $H/2\langle R^{-1}\rangle \approx 0.024$, with $H=11.972$ m and $1/\langle R^{-1}\rangle = 250$ m. If we measure H to better than ± 45 mm (which is obviously easy) and $1/R^{-1}\rangle$ to better than 1 m (which we claim to have done) the corresponding error in G does not exceed 1 part in 10^4, which is the target accuracy of our experiment.

THE NON-NEWTONIAN ANALYSIS

The foregoing analysis can be generalized to consider a gravitational interaction described by Eq. (1). Using this equation and integrating over the layer as before we can write the gravity difference between the test masses in the form of Eq. (5) plus additional terms. Rewriting in terms of $G_{\rm lab}=G_\infty(1-a+b)$, which assumes that $v,s \gg 0.07$ m, the mass separation in the best laboratory determination of G, (Ref. 14) we have Eq. (25) of Stacey *et al.*:[9]

$$\Delta g=4\pi G_{\rm lab}\rho\left[(Z-Z_0)\left[1-\frac{H}{2}\langle R^{-1}\rangle\right]+C+D\right], \tag{6}$$

where

$$C=-\frac{a}{1-a+b}\left[\frac{v}{2}(-e^{-Z/v}+e^{-(H-Z)/v}+e^{-Z_0/v}-e^{-(H-Z_0)/v})-(Z-Z_0)\right]$$
$$+\frac{b}{1-a+b}\left[\frac{s}{2}(-e^{-Z/s}+e^{-(H-Z)/s}+e^{-Z_0/s}-e^{-(H-Z_0)/s})-(Z-Z_0)\right]$$

and

$$D=\frac{(Z-Z_0)H}{1-a+b}\left[\left\langle\frac{a}{R}(1-e^{-R/v})\right\rangle+\left\langle\frac{b}{R}(1-e^{-R/s})\right\rangle\right].$$

This adds to Eq. (5) the contributions of the Yukawa terms in Eq. (1) due to an infinite layer (C) and the boundary correction (D)

In the approximation that v,s are both larger than H and recognizing that $H>Z$ or Z_0, we can expand the term C in powers of $H/v,H/s$:

$$C=\frac{H(Z-Z_0)}{2(1-a+b)}\left[\frac{a}{v}-\frac{b}{s}\right]+O\left[\left[\frac{H}{v}\right]^2,\left[\frac{H}{s}\right]^2\right]. \tag{7}$$

Thus, to first order, both C and D are proportional to H, as is the Newtonian boundary correction in Eq. (5), and the effect of using measurements with different values of H to eliminate the boundary correction also would remove the non-Newtonian term to first order in $H/v,H/s$. This would therefore appear to confirm the value of $G_{\rm lab}$ even if the measurements were sensitive enough to observe a discrepancy. A careful determination of the boundary correction is necessary.

This analysis is required to specify parameter limits on non-Newtonian gravity, but it is convenient to refer to the effective scale of the experiment in a simpler way. By weighting all of the elements of water according to their gravity contributions at the suspended masses we obtain an average interaction distance of 22 m.

THE LAKE PERIMETER SURVEY AND ANALYSIS

The outlines of the boundary of the lake at level zero Z_0 (maximum water level) and -10 m, shown in Fig. 1, were obtained before filling of the lake, by surveying from a control network of reference stations marked on the figure. Those with prefixes SY were survey pillars established by the Queensland Water Resources Commission and those prefixed UQ were established for this experiment. A fixed mark on the instrument platform was included, so that all points on the boundary could ultimately be referred to it. Relative positions of the reference stations to 30-mm accuracy were determined horizontally using a laser rangefinder and a 1-s theodolite and vertically by differential leveling. The perimeter was then surveyed by ranging from the reference stations using a corner cube on a monopod placed at intervals around the lake. Survey points near to the tower were at approximately 2-m spacing, but the spacing was expanded up to 10 m for the most remote, and least critical, survey points. Positions of more than 5600 individual survey points were accurate to 0.1 m, which is much finer than the irregularity of the boundary. These raw data were first translated to coordinates based on the instrument platform and then used to calculate the boundary distance from the tower at 720 azimuths ($\frac{1}{2}$-deg intervals) and 1-m depth intervals from the highest water level to -10 m. Each of the points of this equal azimuthal angle tabulation was obtained by interpolation from the three raw data points forming the closest triangle about it. The systematic error in boundary distance arising from this process of approximating the boundary by a network of triangles is 2.2 parts in 10^5 and the corresponding error in G determination is 5 parts in 10^7.

Tables of $\langle R^{-1}\rangle$, $\langle R^{-3}\rangle$, and $\langle(1-e^{-R/\lambda})/R\rangle$ for a range of values of λ ($=v$ or s) were prepared for 1-m levels and found to fit simple polynomials in water level Z. The polynomial coefficients were used in data processing.

Although nominally of far greater accuracy than necessary, the boundary correction is subject to a possible

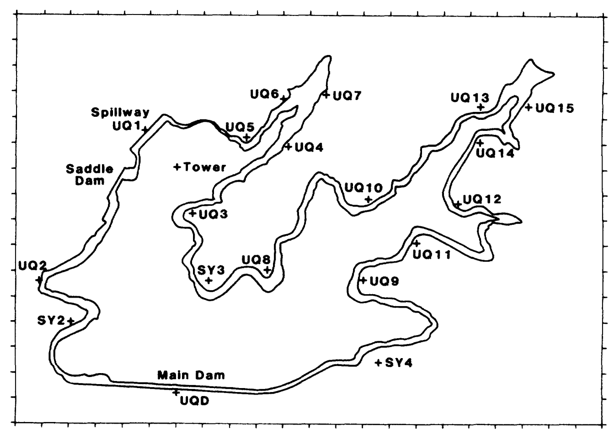

FIG. 1. Outlines of lake perimeter at maximum level and 10 m lower. The scale interval is 100 m.

systematic error, due to seepage of water into the banks of the lake. In the case of the dam wall, we used the advice of the construction engineers that its porosity was about 30% between the surface and the core, adding to the effective perimeter distance 0.3 times the thickness of the porous zone. Elsewhere the seepage could only be estimated roughly, but, thanks to early clearing of the water line in the process of preparing the dam site there appeared no more than a few centimeters of highly porous material in most areas and none at all in the area of the quarry wall, close to the tower. Since very little of the perimeter appears to have more than a few percent porosity, even very close to the surface, it is evident that reversible seepage to a depth of order 30 m would be required to influence the determination of G at the level of 1 part in 10^4 and the seepage problem is of no concern at the presently claimed accuracy (0.8%).

WATER LEVEL AND DENSITY

The determination of G can be no more accurate than the measurements of water level changes and water density. Water level is measured by lowering a weighted steel tape with a magnetically encoded layer, of the type used in numerical control of lathes, etc., past a recording head on the base plate of the balance until a sharp tip makes electrical contact with the water in a still well. The tape is then withdrawn 5 cm and the process repeated, so that the water level is measured to the nearest 0.1 mm, relative to the balance, once every 20 s while the experiment is running. The tape reading is accurate at 20 °C and is corrected for thermal expansion at other temperatures. At the maximum water level change (10 m), the possible error is 1 part in 10^5.

The greatest variability in water density is due to temperature changes. We initially supposed that a simple temperature measurement would suffice because vigorous pumping would stir the water sufficiently to make it uniform in temperature, but this was not the case. The water is often thermally layered, with a warm surface layer rising and falling as cooler water is pumped in and out underneath it. For the measurements reported here profiles of water temperature were obtained by hand, although we now have a line of thermistors to monitor the water temperature profile to better than 0.05 K, so that the mean density can be calculated to about 1 part in 10^5 for the layers of interest.

The effect of impurities on the water density is measured by comparing the density of the lake water with distilled water, using a hydrometer sensitive to differences of 1 part in 10^5 and applying a density-temperature tabulation for pure water. Observations have indicated rather little variation with time. The lake water is systematically denser than distilled water at the same temperature by about 2.4 parts in 10^4. The difference is attributed to dissolved ingredients as we have not observed any change after suspended matter settled out. The effective water density for the purpose of this experiment is the difference between the densities of water and air, because air replaces absent water. The air density at pressure P and temperature T is taken as $1.293(P/p_0)(T_0/T)$ $\mathrm{kg\,m^{-3}}$, where $P_0=760$ mmHg, $T_0=273$ K, which gives value close to 1.20 $\mathrm{kg\,m^{-3}}$ most of the time.

The effect of compression of the water is not significant because at the maximum water depth of interest, 10 m, the density is increased by less than 5 parts in 10^6, which is well below our target accuracy.

OTHER CORRECTIONS AND PRECAUTIONS

Structural members of the tower, vacuum tubes enclosing suspension wires for the lower masses and the walls of the still well used for water-level measurement all exclude water from the layers causing the measured gravitational attractions. Small corrections are added to the observed gravity differences to obtain the values that would be observed with uninterrupted layers of water. These corrections involve only the geometry of about 100 solid components, to calculate the effect of the excluded water as a function of water level Z. The results of detailed numerical calculations were represented by a series of polynomials in Z, one for each of seven "layers" of the tower. The maximum correction amounts to 2.8×10^{-4}, which is certainly obtained to better than the 30% accuracy required to meet the target precision in determining G, 1 part in 10^4.

There are two small corrections involving thermal expansions of the suspension wires for the lower test masses. Separation H of the upper and lower masses varies slightly with temperature due to thermal expansions of the wires. This affects the boundary correction terms in Eqs. (5) and (6) and so is only a correction to a 2.5% correction. The 11.972-m separation of the upper and lower masses increases by 0.14 mm per degree, so that a one degree error in temperature introduces an error in G smaller than 1 part in 10^5, but to accommodate seasonal temperature changes a correction may be necessary at the target accuracy of the experiment. We also consider the effect of the displacement of a mass in the gradient of Earth's gravity as its supporting wire extends. At the positions of the lower masses, allowing for the effect of the water and the contours of the lake floor and shores, the gravity gradient is calculated to be 2×10^{-6} $\mathrm{s^{-2}}$ (0.2 mGal/m), so that a vertical displacement of 0.14 mm per degree gives an increase in gravity on the deeper of the lower test masses by 2.8×10^{-10} $\mathrm{ms^{-2}}$ per degree or $2.8_6\times10^{-11}$ of g per degree. For temperature changes of several degrees this becomes significant at the target accuracy of the experiment, but not for the results reported here.

The suspension wires have mass per unit length $(m/l)=2.1_2$ g/m and a correction is routinely applied for the gravitational forces on these distributed masses. The correction is equivalent to allowing for a variation in the lower mass with water level, because although the lower mass is always below all of the water layers of interest, part of the wire that is weighed with it appears above the water and so experiences the same attraction as the upper mass. Since the correction is small we can neglect the variability of the boundary correction, as it affects the wire. Then the mass that is pulled upward by a layer of water between Z_0 and some arbitrary lower level Z is $[M+m/l(Z_l-Z)]$ where M is the lower mass and Z_l is the level of the top of it, that is the end of the supporting wire. There is no net attraction of the length of wire below water level, so that we correct for the gravitational attraction of the wire by using the "corrected" value of the lower mass. The correction is a significant one, amounting to a maximum of 2 parts in 10^3 over 10 m of water level change.

The following effects have been examined but found to be too small to lead to corrections.

(i) The lake floor and the instrument tower are slightly depressed by addition of water, moving the whole apparatus vertically. Observations showed that the movements relative to the nearest SY stations in Fig. 1 did not exceed 3 mm. This affects the weighing by moving the pairs of separated masses that are compared with respect to the second derivative of Earth's gravity, which is estimated to be $d^2g/dZ^2 = 5.8\times10^{-11}\ \mathrm{m^{-1}s^{-2}}$. By moving a pair of masses that are separated by a vertical distance $H = 11.972$ m through a small distance $\delta Z < 3$ mm, the change in gravity difference between them is

$$|\delta(\Delta g)| = \frac{d^2g}{dZ^2} H\delta Z < 2\times10^{-12}\ \mathrm{m\,s^{-2}}$$
$$= 2\times10^{-13}\ g \qquad (8)$$

which is clearly insignificant.

(ii) The gravity tide is not correlated in time with the water levels and so could only appear as noise on the record, but we are only concerned with the vertical gradient of the gravity tide, which is about $2\times10^{-13}\ \mathrm{s^{-2}}$ and so causes a semidiurnal variation in the gravity difference between masses separated vertically by 11.972 m that is only slightly greater than 2 parts in 10^{12}, well below the level of detectability.

(iii) The layers of lake water are assumed to be perfectly plane and horizontal, but in fact follow the curvature of Earth. This can be allowed for by adding to the bracketed factor in Eq. (3) a term of order $x/R_E \approx 10^{-6}$ where R_E is Earth's radius. This is also insignificant.

(iv) In the process of interchanging the masses that are weighed, they are raised and lowered by approximately $\delta h = 1.5$ mm, changing slightly the cross attractions between them. The closest pair are separated horizontally by the length of the balance arm, $l = 320$ mm, so that the cross attraction causes a change in gravity.

$$\delta g = \frac{GM}{l^3}\delta h = 3.05\times10^{-11}\mathrm{m\,s^{-2}}$$
$$\approx 3.1\times10^{-12} g \ . \qquad (9)$$

Variability of this quantity between mass interchanges is not greater than about 10% so that the error in weighing is only a few parts in 10^{13} and is well below the level of significance.

RESULTS

Values of the weight differences as functions of water level are accumulated during undisturbed periods. After the various corrections are applied, they are identified with gravity differences due to the variations in the product of depth and density of the water appearing above the level of the lower mass. It is convenient to correct for variations in density by calculating a corrected water depth Z' corresponding to the same mass of water, but at the reference density $\rho' = 1000\ \mathrm{kg\,m^{-3}}$, i.e.,

$$Z' = H - (H - Z)\rho/\rho' \ . \qquad (10)$$

Then the value of G is inferred from a least-squares fit of Δg and $(Z' - Z_0)$ in Eq. (5). Using the results plotted in Fig. 2 we obtain

$$G = 6.689(57)\times10^{-11}\ \mathrm{m^3\,kg^{-1}s^{-2}} \ . \qquad (11)$$

This result coincides with the best laboratory estimate,[14]

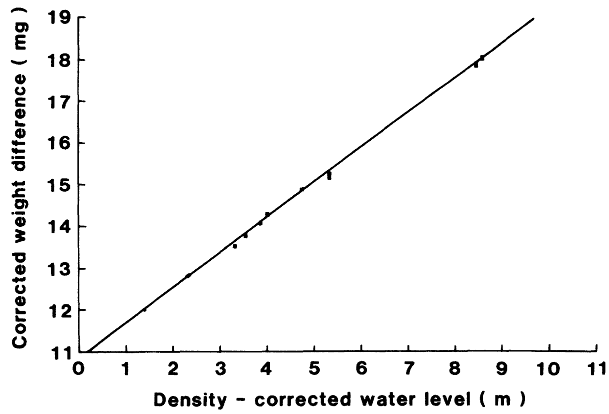

FIG. 2. A plot of the variation in weight difference between 10-kg masses, separated vertically by 11.97 m, as a function of the level of the lake surface below the upper mass. The weights are corrected for the variable length of suspension wire above the water level and the "missing" water due to components of the tower. Water height is adjusted to a reference density of 1000 $\mathrm{kg\,m^{-3}}$ and "corrected" for the lake boundary to correspond to an infinite lake. Error brackets are comparable to the sizes of the marked data points, some of which appear as close superpositions of multiple observations.

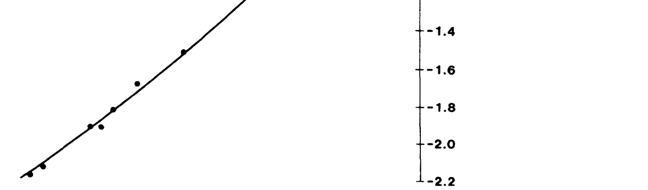

FIG. 3. A combined fit of the Hilton mine data and the USAF tower data (Refs. 11 and 12) to Eqs. (12) and (13). The curve uses the parameters for $a = 0.05$ in Table I. For the purpose of this fit (and the others in Table I) all of the data points are weighted equally except that at $z = 183$ m which is ignored.

TABLE I. Values of the parameters in Eq. (1) least-square fitted by Eqs. (12) and (13) to the two data sets plotted in Fig. 3.

a	b	v(m)	s(m)	δg(mGal)	σ(mGal)	$(\Delta G/G)_{\rm lake}$(%)	$(bs-av)$(m)
0.02	0.015 23	$\to 0^{a}$	618	0.3746	0.044 55	1.95	9.4
0.03	0.023 87	16.8	270.8	0.4942	0.041 73	0.82	6.0
0.05	0.043 27	39.3	165.0	0.5523	0.040 45	0.54	5.2
0.1	0.093 00	59.0	116.0	0.5773	0.040 38	0.49	4.9
0.3	0.2929	73.0	91.1	0.5858	0.040 72	0.49	4.7
1	0.9928	70.1	75.3	0.5932	0.043 79	0.59	4.6
10	9.9927	68.0	68.5	0.5934	0.046 31	0.64	4.5

[a]This solution drives v to zero. A value of 1 m is assumed in fitting $b,s,\delta g$.

$6.6726(5)\times 10^{-11}$ m^3 kg^{-1} s^{-2}, although with much lower accuracy, and so has been used to impose a lower bound on the ranges of the forces that are postulated to explain mine gravity data.[3,9] Now, however the new data set obtained in a 600-m TV transmission mast,[11,12] favors ranges close to this bound, so that the lake experiment assumes a new significance.

Although we are not addressing the composition-dependence problem, it may be noted for reference that our measurements refer to the gravitational attraction of 18/8 stainless steel to fresh water.

A COMPARISON WITH OTHER DATA

We have examined the result of fitting simultaneously the best mine data set (from the Hilton mine in Northwest Queensland[2]) and the USAF tower data[11,12] to alternative sets of values for the parameters in Eq. (1). One such fit is plotted as Fig. 3. For this purpose there are two adjustments to the "raw" data. The tower gravity residuals are multiplied by the factor 2750/2350 with the supposition that if they had been observed in an area where the local rock density was 2750 kg m^{-3}, as in the Hilton mine, instead of about 2350 kg m^{-3}, the values would have been correspondingly larger. We also allow an arbitrary zero shift δg (downward) of all of the mine data, to acknowledge that we do not have satisfactory densities for the top 20 m of rock and therefore cannot properly connect the surface data point to the others. The procedure is then to assume a value of a and for this value, least-squares fit the parameters b, v, s, and δg to the two data sets, using equations for the gravity residuals as functions of depth z and height h:

$$\Delta g(z)=4\pi G_{\rm lab}\rho\left[-\frac{a}{1-a+b}\left[z-\frac{v}{2}(1-e^{-z/v})\right]+\frac{b}{1-a+b}\left[z-\frac{s}{2}(1-e^{z/s})\right]\right], \tag{12}$$

$$\Delta g(h)=2\pi G_{\rm lab}\rho\left[\frac{av}{1-a+b}(1-e^{-h/v})\frac{bs}{1-a+b}(1-e^{-h/s})\right], \tag{13}$$

and repeat the process for different values of a.

The interesting results are listed in Table I with the standard deviations of the misfits σ which are found to be virtually independent of the choice of a. We then used Eq. (6) to determine the discrepancies that these parameter sets predict between $G_{\rm lab}$ and the lake value. These results are listed as percentages $(\Delta G/G)_{\rm lake}$ in the table. The misfit approaches 0.6 standard deviation for all values of a greater than 0.03. We also list in the table the values of $(bs-av)$, which are seen to be well below the limit of 14 m imposed by the agreement between surface and satellite gravity data.[3] The listed parameters are accepted by the present lake data at about 0.6 standard deviation level, so we now recognize more clearly than hitherto the urgency to obtain a much improved lake result.

The uncertainty in the present result arises entirely from the problem of weighing on a rather unstable platform at a very exposed site. All systematic errors are very much smaller. The requirements for an improved result are therefore dependent upon the instrumental refinements which we have in hand, in particular a better tilt servo and improved thermal compensation in the knife-edge support system.

CONCLUSION

At this stage the parameter fits to geophysical data listed in Table I are indicative rather than definitive, but it is clear that gravitational forces with ranges of many kilometers should no longer be considered and that ranges 100 m or less are generally favored. The Splityard Creek lake experiment is therefore sensitive to the effects considered.

ACKNOWLEDGMENTS

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