Effect of Connecting Shunt Capacitor on Nonlinear Load Terminals

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***Abstract*—The use of terminal shunt capacitance has different effects on the displacement factor and distortion factor components of the power factor. These effects are considered for nonlinear loads with ideal supply, and also where the supply impedance exists but is small compared with the load impedance. Optimization of the displacement factor is found to result in reduction of the distortion factor to a minimum value.**

***Index Terms*—Distortion, harmonics, power factor.**

NOMENCLATURE

C	Capacitance, F.
DIPF	Displacement factor.
DISF_1	Distortion factor of load.
DISF_2	Distortion factor after compensation.
$\mathrm{DISF}_{2\,\min}$	Minimum value of DISF_2.
E	rms supply voltage, V.
I	rms current, A.
I_C	rms value of capacitor current, A.
I_k	rms value of k-th component of load current, A.
I_S	rms value of supply current, A.
I_1	rms value of fundamental component of load current, A.
$\overline{I}_1$	rms value of fundamental component of supply current, after compensation, A.
K_I	Current ratio $I_1/\sqrt{\sum_{k=2} I_k^2}$.
K_V	Voltage ratio $V_1/\sqrt{\sum_{k=2} V_k^2}$.
P	Average power, W.
PF	Power factor after compensation.
ϕ_{1I}	Fundamental phase-angle (displacement angle) of load current, rad.
$\overline{\phi}_{1I}$	Fundamental phase-angle of supply current after compensation, rad.
ϕ_{1V}	Fundamental phase-angle of load voltage, rad.
ω	Angular supply frequency, rad/s.

I. INTRODUCTION

THE power factor of a load is universally defined as the ratio of the real power input to the volt-ampere product at the terminals [1].

For systems with sinusoidal supply voltage (i.e., negligible supply impedance) and nonlinear load impedance, the load current waveform is periodic but nonsinusoidal. It is sometimes convenient to consider the power factor of such systems as the product of a displacement factor and a supply current distortion factor. It is only valid for loads with sinusoidal voltage. The displacement factor is partly, but not entirely, related to reactive properties of the load impedance [1]. The distortion factor is largely related to the degree and kind of nonlinearity of the load impedance. In realistic power supply and distribution systems, the supply has finite and measurable impedance. This can usually be interpreted as series inductance in the supply lines between the generator and the load terminals. If the load impedance is nonlinear, the effect of the nonsinusoidal current drawn through the supply reactance is to create distorted voltage at the load terminals. When both the voltage and the current at the supply point are of different nonsinusoidal waveforms, it is generally not possible to express the power factor in terms of displacement factor and a supply current distortion factor [2].

A straightforward way of obtaining power factor correction for loads with a lagging phase-angle is to use shunt capacitance at the supply point. For many nonlinear loads comes power factor compensation that can be thereby realized even if the load impedance is purely resistive.

This way of power factor correction installation can be used when it is not necessary to take measures to avoid resonance problems or to reduce harmonics. This is generally the case when the resonant frequency given by the network inductance and the capacitance of the power factor correction installation is relatively high and the harmonic content of the network (i.e., bus voltage and harmonic currents generated by the load) are very low [3].

Harmonic currents tend to flow from the nonlinear loads (harmonic sources) toward the lowest impedance, usually the utility source. The impedance of the utility source is usually much lower than parallel paths offered by loads. However, the harmonic currents will split depending on the impedance ratios. Higher harmonics will flow to capacitors that are low impedance to high frequencies [2].

The main purpose of connecting shunt capacitance is to reduce the displacement angle between the supply voltage vector and the fundamental supply current vector, thereby increasing the displacement factor of the overall burden. The most important feature, however, is the effect on the overall power factor [4]–[7].

This paper considers this effect, with ideal supply, and also where the supply impedance exists but is small compared with the load impedance.

In this analysis, the following assumptions are made.

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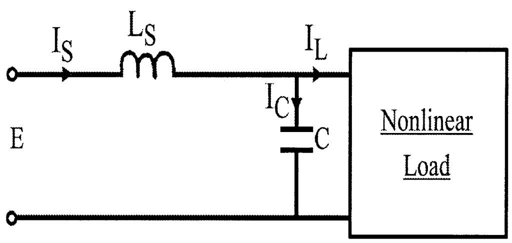

Fig. 1. Nonlinear load with nonideal single-phase supply.

1) The waveform of the nonlinear load input voltage is undisturbed by connecting the compensating capacitor across the load input terminals. This assumption will give reasonable results if the capacitor current is small relative to the load current.
2) Harmonics of the capacitor current are small relative to the fundamental [8].
3) The harmonic current components drawn from the supply are not affected by connecting the capacitor across the supply terminals [9].

However, in cases where the assumptions are not valid, the conclusion from this paper will be an approximate estimate rather than an exact solution. To obtain the exact effect in such cases, one has to study each case individually considering simultaneously the effects of supply current and distorted supply voltage harmonics.

II. Power Factor Properties With Nonideal Supply

Consider a nonlinear load impedance connected to nonideal, sinusoidal supply (i.e., generator), Fig. 1.

The supply current is periodic of rms value I_S, and has a fundamental component of rms value I_1, lagging E by angle ϕ_{11} (Fig. 2). The supply current distortion factor DISF_1 is given by

$$\mathrm{DISF}_1 = \frac{I_1}{I_S} = \frac{I_1}{\sqrt{I_1^2 + \sum\limits_{k=2}^{\infty} I_k^2}}. \tag{1}$$

Now let a capacitor be connected across the supply terminals to correct the power factor of the total load. The capacitor will draw from the supply a sinusoidal current of supply frequency and rms value I_C. This current adds vectorially to the fundamental supply current I_1 drawn by the load branch to give the fundamental supply current $\overline{I}_1$Fig. 2, lagging E by $\overline{\phi}_{1I}$. The load voltage has rms value V_L and a fundamental component of rms value V_1, lagging E by angle ϕ_{1V}, Fig. 2. The input voltage to the load is not sinusoidal and has a distortion factor

$$\begin{aligned} K_V =& \frac{V_1}{V_L} \\ =& \frac{V_1}{\sqrt{\sum\limits_{k=1} V_k^2}}. \end{aligned}$$

According to the assumptions, the capacitor will draw from the supply a supply frequency current of rms value $I_C = V_1 \omega \mathrm{C}$ and phase-angle $\phi_{1V} + 90^\circ$. This current adds vectorially to the fundamental supply current I_1 drawn by the load branch to give the fundamental supply current $\overline{I}_1$, lagging E by $\overline{\phi}_{1I}$, Fig. 2.

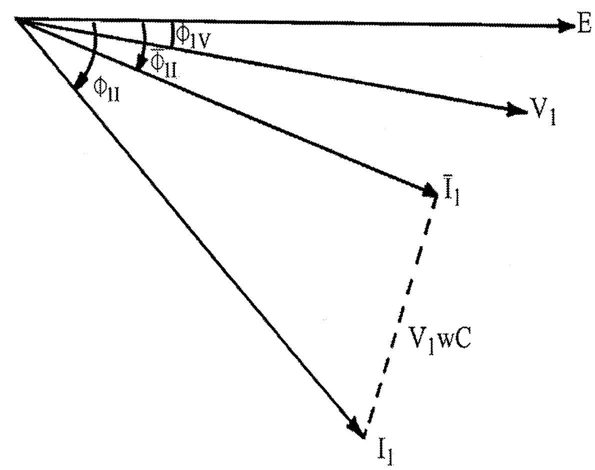

Fig. 2. Phasor diagram representing fundamental supply current and load voltage components.

The harmonic current components drawn from the supply are not affected by connecting the capacitor across the supply terminals and hence the terms $\sum_{k=2}^{\infty} I_k^2$ of (1) remain constant. The distortion factor DISF_2 of the supply current after correction is now given by

$$\mathrm{DISF}_2 = \frac{\overline{I}_1}{\sqrt{\overline{I}_1^2 + \sum\limits_{k=2}^{\infty} I_k^2}}. \tag{2}$$

For the distortion factor to improve after compensation, it is necessary that

$$\mathrm{DISF}_2 > \mathrm{DISF}_1.$$

Hence

$$(\mathrm{DISF}_2)^2 > (\mathrm{DISF}_1)^2. \tag{3}$$

Substituting from (1) and (2) into (3) gives the requirement

$$\overline{I}_1^2 > I_1^2 \tag{4}$$

or

$$\overline{I}_1 > I_1.$$

The inequality of (4) is obviously not true for the system. In Fig. 2, for example, the displacement factor is increased due to the capacitor. But in reducing the displacement angle, it is seen that the value of fundamental component of supply current, after compensation, is reduced. It therefore follows from (4) that the distortion factor is reduced. It also follows that the distortion factor can only be improved by reducing the displacement factor [10], [11]. Referring to Fig. 2, one can write

$$\overline{I}_1 = \frac{I_1 \cos(\phi_I - \phi_{1V})}{\cos\left(\overline{\phi}_{1I} - \phi_{1V}\right)} \tag{5}$$

$$I_1 \sin(\phi_1 - \phi_{1V}) - \overline{I}_1 \sin\left(\overline{\phi}_{1I} - \phi_{1V}\right) = V_1 \omega \mathrm{C}. \tag{6}$$

Substituting from (5) into (6) gives (See the equation at the bottom of the page) for negative values of $\overline{\phi}_{1I}-\phi_{1V}$ (See the equation at the bottom of the page) for positive value of

$$\overline{\phi}_{1I}-\phi_{1V}. \tag{7}$$

If the value $\bar{I}_1$ is substituted from (5) into (2) one obtains

$$\mathrm{DISF}_2 = \frac{\frac{I_1\cos(\phi_{1I}-\phi_{1V})}{\cos(\overline{\phi}_{1I}-\phi_{1V})}}{\sqrt{\frac{I_1^2\cos^2(\phi_{1I}-\phi_{1V})}{\cos^2(\overline{\phi}_{1I}-\phi_{1V})+\sum_{k=2}^{\infty} I_k^2}}}. \tag{8}$$

The overall displacement factor after capacitor correction is now given Fig. 2 by

$$\mathrm{DIPF} = \cos(\overline{\phi}_{1I}-\phi_{1V}). \tag{9}$$

It has been shown [12] that if the supply voltage distortion factor K_V is high and the magnitude of the fundamental supply current I_1 is greater than that of any higher harmonic, the power factor PF after capacitance compensation of the circuit of Fig. 1 may be approximated to

$$PF = \frac{K_V I_1 \cos(\phi_{1I}-\phi_{1V})}{\sqrt{\frac{I_1^2\cos^2(\phi_{1I}-\phi_{1V})}{\cos^2(\overline{\phi}_{1I}-\phi_{1V})+\sum_{k=2}^{\infty} I_k^2}}}. \tag{10}$$

If ϕ_{1V} is equal to zero, ideal supply, (9) reduces to

$$PF = \frac{I_1\cos(\phi_{1I})}{\sqrt{\frac{I_1^2\cos^2(\phi_{1I})}{\cos^2(\overline{\phi}_{1I})+\sum_{k=2}^{\infty} I_k^2}}}.$$

Consider the following nonlinear load with $\phi_{1I}=-30^\circ$, $\phi_{1V}=-15^\circ$, $K_I=3.316$, $K_V=0.9962$, $I_1=922.550$.

Fig. 3 shows consistent variations of modified supply current distortion factor (DISF_2), corrected displacement factor (DISF), and modified power factor (PF) with the modified fundamental supply current phase angle.

Variations of modified fundamental supply current phase angle can be easily converted to corresponding variations in the value of compensating capacitor C by using (7).

Comparing for the case of ideal supply, one can see that the general effect of supply impedance is to shift the fundamental phase-angle ϕ_{1V} of the distorted supply voltage.

Fig. 3 shows also that unlike the case with ideal supply, the minimum supply current distortion factor is not equal to the

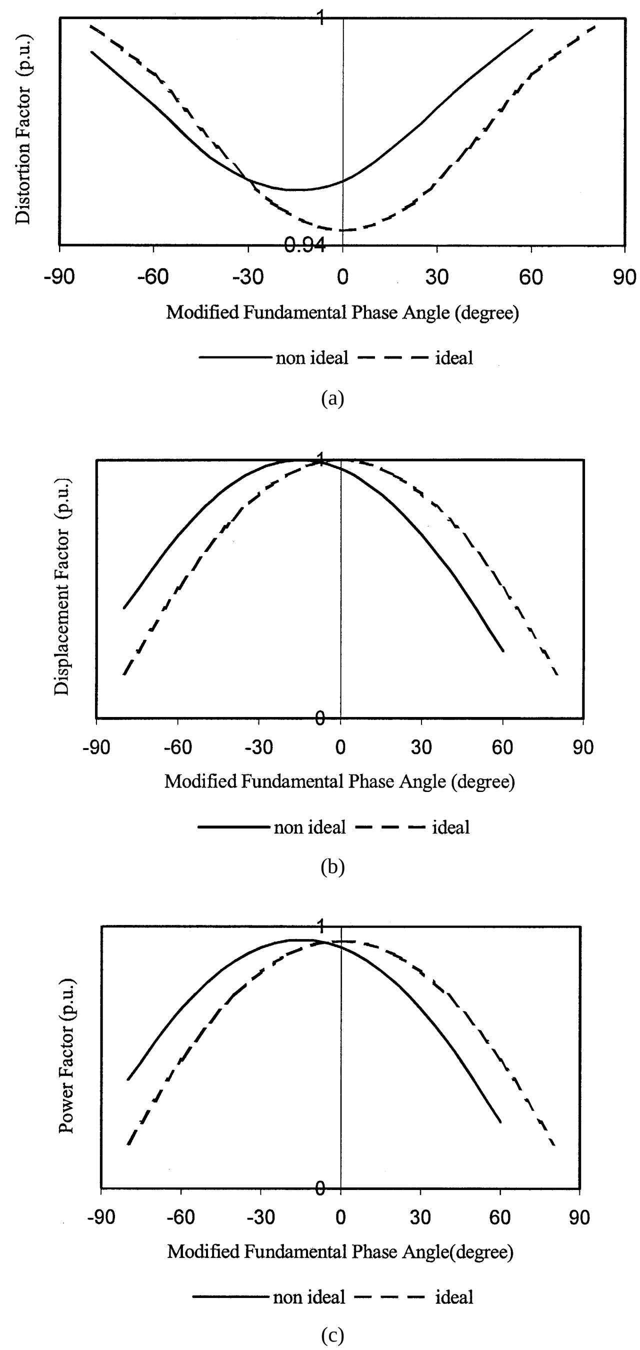

Fig. 3. Variation of distortion factor (DISF_2), displacement factor (DISF), and power factor (PF) with compensated phase-angle $\overline{\phi}_{1I}$.

$$C = \frac{I_1\left[\sin\left|(\phi_{1I}-\phi_{1V})\right| - \cos(\phi_{1I}-\phi_{1V})\tan\left|(\overline{\phi}_{1I}-\phi_{1V})\right|\right]}{V_1\omega}$$

$$C = \frac{I_1\left[\sin\left|(\phi_{1I}-\phi_{1V})\right| + \cos(\phi_{1I}-\phi_{1V})\tan\left|(\overline{\phi}_{1I}-\phi_{1V})\right|\right]}{V_1\omega}$$

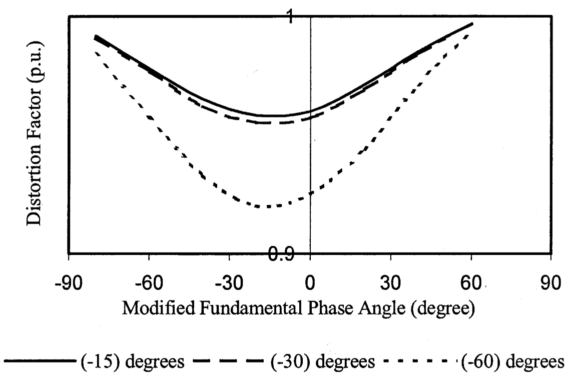

Fig. 4. Distortion factor (DISF_2) versus modified fundamental phase-angle $\overline{\phi}_{1I}$ for different values of uncorrected fundamental supply current phase angle ϕ_{1I}.

maximum power factor. This is due to the effect of the supply voltage distortion factor K_V as shown by (10).

A set of curves showing supply current distortion factor DISF_2 versus corrected fundamental supply current phase-angle $\overline{\phi}_{1I}$ for different value of $\overline{\phi}_{1I}$ (Fig. 4). Comparing for the case of ideal supply, one can see that the general effect of supply inductance is to shift the point of minimum distortion factor by an angle equal to the fundamental phase-angle of the distorted supply voltage.

One may note the following points for the case of ideal supply

1) Maximum power factor is obtained at $\overline{\phi}_{1I} = 0$ where the modified displacement factor is unity.
2) At this value, the distortion factor is a minimum and the maximum realizable power factor is therefore equal to the minimum realizable distortion factor.
3) For the range $-\phi_{1I} < \overline{\phi}_{1I} < \phi_{1I}$, the three curves of Fig. 3 are symmetrical about the compensated fundamental phase-angle $(\phi_{1I} = 0)$ axis.
4) Although the distortion factor is increased for values of $\phi_{1I} > 0$, the total power factor then decreases due to decrease in the displacement factor. This condition is accompanied by increased fundamental supply current and increased transmission loss.
5) To increase the compensated distortion factor above its uncompensated value, angle ϕ_{1I} must be increased beyond $+\phi_{1I}$. The effect of such increase is to reduce the power factor, due to reduction of the displacement factor.

As the fundamental phase-angle ϕ_{1I} before compensation is increased negatively, the minimum distortion factor decreases and consequently the maximum realizable compensated power factor will also decrease.

III. Minimum Realizable Value of Compensated Distortion Factor

From (8)

$$(\mathrm{DISF}_2)^2 = \frac{1}{1 + \frac{\cos^2(\overline{\phi}_{1I} - \phi_{1V})}{K_1^2 \cos^2(\phi_{1I} - \phi_{1V})}}. \tag{11}$$

Let the load branch be undisturbed while capacitor C is varied. Parameter K_I, ϕ_{1I}, and ϕ_{1V} of (10) then remain

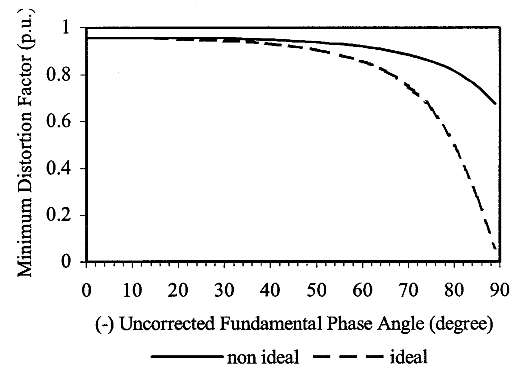

Fig. 5. Variation of minimum compensated distortion factor with uncorrected fundamental phase angle ϕ_{1I}.

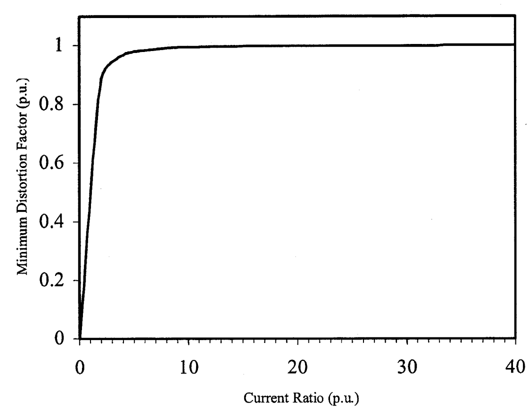

Fig. 6. Variation of minimum compensated distortion factor with K_I.

constant while $\overline{\phi}_{1I}$ is varied. For a maximum or minimum value of distortion factor, one then has

$$\frac{\partial\,(\mathrm{DISF}_2)}{\partial \overline{\phi}_{1I}} = 0$$

from which

$$\sin 2(\overline{\phi}_{1I} - \phi_{1v}) = 0$$

so that

$$\overline{\phi}_{1I} - \phi_{1v} = 0 \text{ or } \overline{\phi}_{1I} - \phi_{1v} = 90^\circ.$$

It is clear from Figs. 3 and 4 that $\overline{\phi}_{1I} - \phi_{1V} = 0$ represents a maximum. Substituting this value into (10), gives the following expression for the minimum value $\mathrm{DISF}_{2\,\min}$ of the modified distortion factor

$$\mathrm{DISF}_{2\,\min} = \frac{K_I \cos(\phi_{1I} - \phi_{1V})}{\sqrt{K_I^2 \cos^2(\phi_{1I} - \phi_{1V}) + 1}}. \tag{12}$$

If $\phi_{1V} = 0$, for ideal supply, (11) reduces to

$$\mathrm{DISF}_{2\,\min} = \frac{K_I \cos(\phi_{1I})}{\sqrt{K_I^2 \cos^2(\phi_{1I}) + 1}}. \tag{13}$$

Fig. 5 shows variation of $\mathrm{DISF}_{2\min}$ with ϕ_{11}.

Fig. 6 shows variation of $\mathrm{DISF}_{2\,\min}$ with K_I. Variation of function (11) for changes of K_I shows that $\mathrm{DISF}_{2\,\min} = 0$, with $K_I = 0$, when the fundamental supply current is then zero.

One can conclude that for values of $K_I > 10$, the minimum distortion factor becomes constant at a value near unity, because $K_I > 10$ represents only a small distortion effect.

IV. Conclusions

The function of terminal shunt capacitance compensation is to reduce the phase (displacement) angle between the fundamental components of the terminal voltage and current. When the displacement angle is zero, the displacement factor is unity and this can be realized even with nonlinear loads. The power factor of the compensated load is then a maximum.

The presence of supply reactance causes a nonzero value ϕ_{1V} of the fundamental load (and supply) voltage phase-angle. With supply reactance, an approximate analysis shows that the curves of displacement factor, distortion factor, and power factor are symmetrical about $\overline{\phi}_{1I} = \phi_{1V}$ axis. The presence of supply impedance also reduces the maximum value of power factor realizable by shunt capacitance compensation.

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