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FRACTIONAL COINTEGRATION IN US TERM SPREADS

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Abstract

This note examines the stochastic properties of US term spreads with parametric and semi-parametric fractional integration techniques. Since the observed data (rather than the estimated residuals from a cointegrating regression) are used for the analysis, standard methods can be applied. The results indicate that US Treasury maturity rates are I(1) in most cases, although the order of integration decreases with maturity. Further, mean reversion occurs for the 5, 7 and 10 year rates as well as for several term spreads, suggesting that the expectation hypothesis of the term structure is satisfied empirically.

JEL Classification: *C22, E43*

Keywords: *Term structure, Long memory, Fractional integration, Fractional cointegration.*

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1. Introduction

The term structure of interest rates has been extensively investigated given the information it can provide about agents' expectations on future rates and inflation, as well as its role in the transmission mechanism of monetary policy (see Mankiw and Summers, 1984). In an influential paper Campbell and Shiller (1987) showed that in the presence of rational expectations long-term interest rates should be equal to the present discounted value of expected future short-term rates. The implication is that the term spread should be stationary, which the same authors found to be the case in the US using standard Dickey-Fuller tests. Alternatively, cointegration between short- and long-term rates should hold, a result which was reported by Hall et al. (1992) amongst others.

However, the discrete options I(0) and I(1) offered by classical unit root and cointegration analysis might be too restrictive to model the term structure, as the adjustment to equilibrium might in fact be a rather slow process. In a recent paper, therefore, Barassi and Zhang (2009) carry out fractional cointegration tests following a residual-based approach based on the exact local Whittle estimator, which is shown to outperform rival methods by means of Monte Carlo experiments, and report that the term structure in both the US and the UK is a mean-reverting process with long memory. [1]

In this present note we also consider a general model allowing for long memory in order to analyse the term structure of US interest rates; however, unlike Barassi and Zhang (2009), we do not regress longer maturity rates against shorter ones with the aim of testing the stationarity or long-memory properties of the estimated residuals, but instead construct term spreads and analyse their fractional integration properties. In other words, we test for

[1] Caporale and Gil-Alana (2004) use fractional cointegration techniques to test present value models of stock prices.

fractional cointegration under the assumption of a given (1, -1) cointegrating vector. This has the advantage that the tests are based on the raw data instead of the estimated residuals, and therefore standard fractional methods can be applied directly to the observed data without any prior estimation.

2. Methodology

The analysis carried out in this note is based on fractional integration, which is a more general framework than the standard unit root tests based on the I(0)/I(1) dichotomy. Specifically, we obtain estimates of d based on the Whittle function in the frequency domain (Dahlhaus, 1989) using parametric (Robinson, 1994) and semi-parametric techniques (Robinson, 1995). The model considered is the following:

$$y_t \;=\; \mu \;+\; x_t; \qquad (1 - L)^d x_t \;=\; u_t\,, \qquad t \;=\; 1, 2, \ldots, \qquad (1)$$

where y_t stands for the observed raw data, d is a real value, and u_t is assumed to be I(0). In the parametric model we assume first that u_t is uncorrelated, and then an AR(1) structure is imposed. Other methods of estimating d parametrically (Sowell, 1992) or semiparametrically (Phillips and Shimotsu, 2005) lead to essentially the same results.

3. Data and empirical results

The dataset includes the US Treasury constant maturity rates, monthly, for the time period 1982M1 – 2009M12, for different maturities, namely 3 and 6 months (M3 and M6), and 1, 2, 3, 5, 7 and 10 years (Y1, Y2, Y3, Y5, Y7 and Y10). These series were obtained from the US Federal Reserve Board.

[Insert Figures 1 – 4 about here]

Figure 1 shows plots of the raw time series, which decline steadily with some oscillations over the sample period under investigation. Figures 2, 3 and 4 display the term spreads for the 10, 7 and 5 year maturity rates respectively; there appears to be a relatively small degree of dependence in the data.

[Insert Table 1 about here]

Table 1 reports the estimates of d for the original time series. Specifically, the second and third columns show the estimates of d (Dahlhaus, 1989) along with the 95% confidence interval using Robinson's (1994) parametric approach; in the former column, the errors are assumed to be white noise, whilst in the latter an AR(1) structure is imposed. It can be seen that the estimates are all significantly above 1 in the case of uncorrelated errors. However, when an AR(1) specification is adopted, the I(1) hypothesis cannot be rejected for the M3, M6, Y1, Y2 and Y3 rates, but it is rejected in favour of mean reversion (i.e. $d < 1$) for the remaining three rates (Y5, Y7 and Y10). Owing to the disparity of the results depending on the specification of the error term, we employ two additional methods. The first assumes that the errors follow the Bloomfield (1973) model. This is a non-parametric method that produces autocorrelations for the error term that decay exponentially as in the AR(MA) case. The second approach is the Whittle semi-parametric method of Robinson (1995). The estimates based on these methods are displayed in the fourth and fifth columns of Table 1. The results based on the Bloomfield model imply that the I(1) hypothesis cannot be rejected in any case. By contrast, when using the semi-parametric Whittle method, the parameter d is estimated to be significantly smaller than 1 in the cases of Y5, Y7 and Y10. It is also noteworthy that the estimated values of d decrease monotonically with the maturity rate.

[Insert Table 2 about here]

Table 2 has the same structure as Table 1 but concerns the term spreads. In particular, we focus on the following spreads: Y10-M3; Y10-M6; Y10-Y1; Y7-M3; Y7-M6; Y7-Y1; and Y5-M3; Y5-M6 and Y5-Y1. The first noticeable feature here is that the estimated values of d are in all cases smaller, suggesting that there is a decrease in the degree of dependence compared with the individual series. As before, differences emerge depending on the specification of the error term. Using the Bloomfield's (1973) approach and the Whittle semi-parametric method (Robinson, 1995), mean reversion is found for the following three series: Y10-M3, Y7-M3 and Y5-M3. In the remaining cases, although the estimates are smaller than in Table 1, the unit root hypothesis cannot be rejected.

4. Conclusions

This note has analysed the stochastic properties of US term spreads with parametric and semi-parametric fractional integration methods. The approach employed here is preferable to other methods that test the Expectations Hypothesis by means of standard stationary/unit root tests, or by testing the order of integration of the estimated residuals from cointegrating regressions as in Barassi and Zhang (2009). The reason is that, in addition to allowing for any real values of the fractional parameter d, the chosen method uses the raw data, without requiring any preliminary estimation step. The main finding is that the term spreads are non-stationary but mean-reverting with orders of integration strictly smaller than 1, implying that the Expectations Hypothesis is satisfied in the long run, which is consistent with the results reported in Barassi and Zhang (2009).

References

Barassi, M.R. and D. Zhang (2009), “Fractional integration and cointegration: testing the term structure of interest rates”, DP no. 09-17, Department of Economics. University of Birmingham.

Bloomfield, P. (1973) An exponential model in the spectrum of a scalar time series. Biometrika 60, 217-226.

Campbell, J.Y. and R.J. Shiller (1987), “Cointegration and tests of present value models”, Journal of Political Economy, 95, 1062-1088.

Caporale, G.M. and L.A. Gil-Alana (2004), “Fractional cointegration and tests of present value models”, Review of Financial Economics, 13, 3, 245-258.

Dahlhaus, R. (1989) Efficient parameter estimation for self-similar process. Annals of Statistics 17, 1749-1766.

Hall, A.D., Anderson, H.M. and C.W.J. Granger (1992), “A cointegration analysis of Treasury bill yields”, Review of Economics and Statistics, 74, 116-126.

Mankiw, G. and L. Summers (1984), “Do long-term interest rates overreact to short-term interest rates?”, Brooking Papers on Economic Activity, 223-242.

Phillips, P.C.B. and K. Shimotsu (2005), “Exact local Whittle estimation of fractional integration”, Annals of Statistics, 33, 4, 1890-1933.

Robinson, P.M. (1994), “Efficient tests of nonstationary hypotheses”, Journal of the American Statistical Association, 89, 1420-1437.

Robinson, P.M. (1995) Gaussian semiparametric estimation of long range dependence. Annals of Statistics 23, 1630-1661.

Sowell, F. (1992) “Maximum likelihood estimation of stationary univariate fractionally integrated time series models”, Journal of Econometrics, 53, 165-188.

Figure 1: Treasury Constant Maturity Rates: M3, M6, Y1, Y2, Y3, Y5, Y7 and Y10

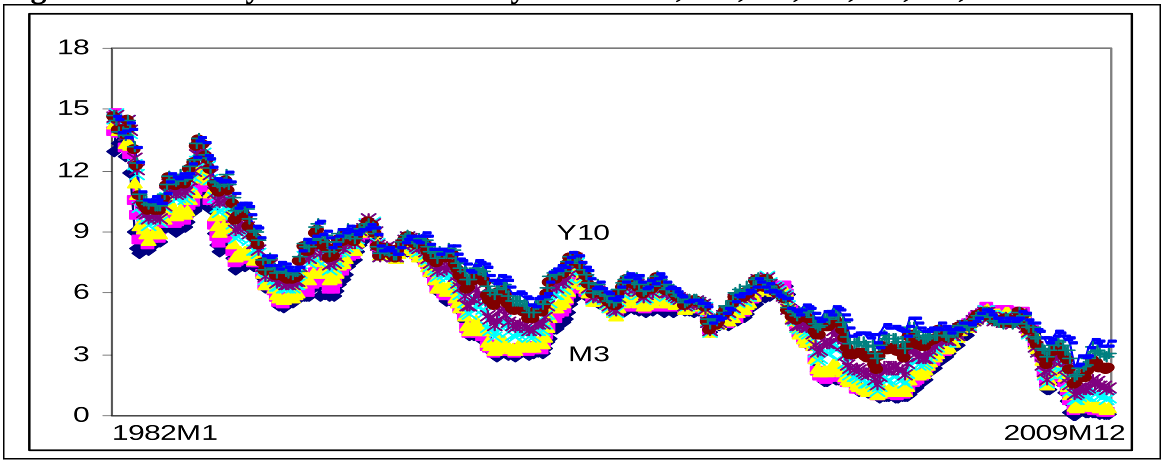

Figure 2: Term spreads: Y10-M3; Y10-M6; Y10-Y1

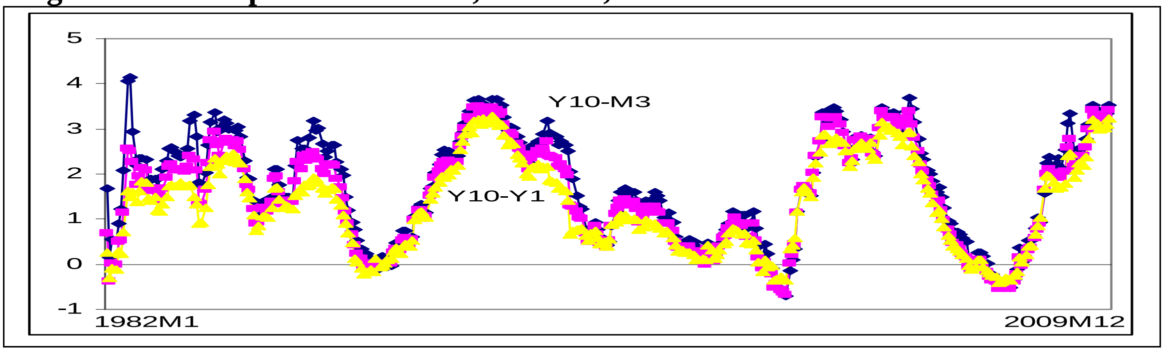

Figure 3: Term spreads: Y7-M3; Y7-M6; Y7-Y1

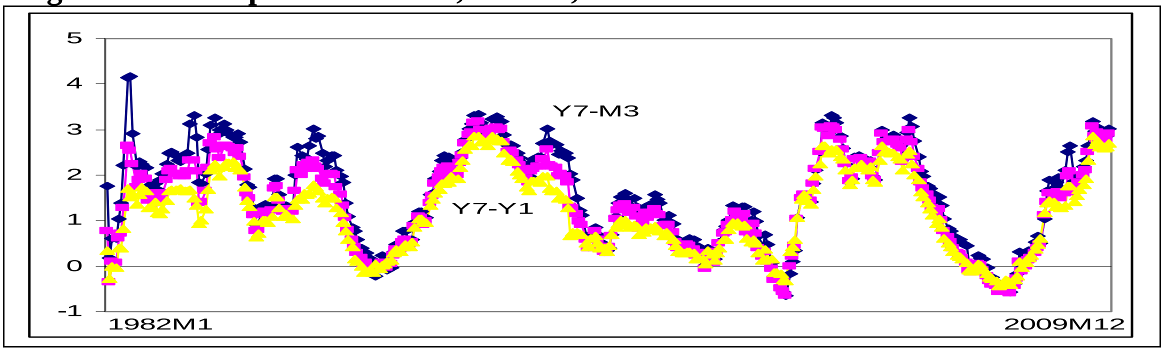

Figure 4: Term spreads: Y5-M3; Y5-M6; Y5-Y1

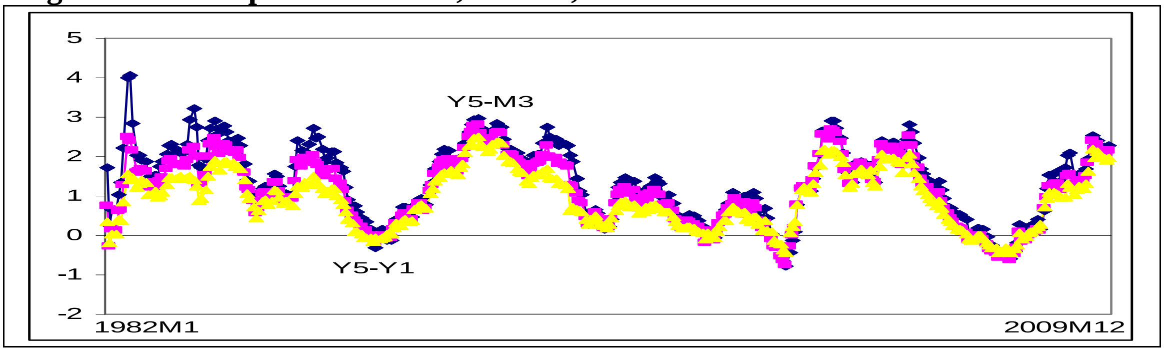

Table 1: Estimates of d using the original time series

	Robinson's (1994) parametric approach			Rob. (1995)
	White noise	AR (1)	Bloomfield	$M = (T)^{0.5}$
M3	1.310 (1.222, 1.424)	1.044 (0.900, 1.233)	1.101 (0.983, 1.239)	1.159
M6	1.311 (1.231, 1.413)	1.128 (0.959, 1.331)	1.179 (1.051, 1.336)	1.148
Y1	1.334 (1.247, 1.445)	1.007 (0.924, 1.262)	1.123 (0.984, 1.270)	1.104
Y2	1.324 (1.227, 1.445)	0.939 (0.832, 1.132)	1.021 (0.874, 1.182)	0.999
Y3	1.315 (1.211, 1.442)	0.957 (0.801, 1.054)	0.943 (0.782, 1.111)	0.926
Y5	1.300 (1.191, 1.437)	**0.861*** **(0.776, 0.985)**	0.870 (0.653, 1.049)	**0.815***
Y7	1.271 (1.164, 1.406)	**0.850*** **(0.777, 0.969)**	0.842 (0.622, 1.030)	**0.753***
Y10	1.240 (1.137, 1.372)	**0.826*** **(0.770, 0.964)**	0.814 (0.597, 1.010)	**0.708***

In bold, statistical evidence of mean reversion at the 5% level. M is the bandwidth number.

Table 2: Estimates of d for the term spreads

	Robinson's (1994) parametric approach			Rob. (1995)
	White noise	AR (1)	Bloomfield	$M = (T)^{0.5}$
Y10 - M3	1.201 (1.097, 1.343)	**0.834*** **(0.630, 0.986)**	**0.824*** **(0.709, 0.950)**	0.995
Y10 – M6	1.224 (1.139, 1.335)	0.993 (0.813, 1.152)	1.012 (0.892, 1.142)	1.120
Y10 – Y1	1.219 (1.144, 1.314)	1.089 (0.940, 1.225)	1.082 (0.960, 1.208)	1.1236
Y7 - M3	1.192 (1.082, 1.340)	**0.764*** **(0.349, 0.938)**	**0.790*** **(0.671, 0.929)**	**0.900***
Y7 – M6	1.217 (1.128, 1.333)	0.928 (0.791, 1.120)	0.988 (0.865, 1.124)	1.040
Y7 – Y1	1.213 (1.136, 1.312)	1.063 (0.849, 1.214)	1.072 (0.952, 1.209)	1.165
Y5 - M3	1.198 (1.077, 1.359)	**0.691*** **(0.495, 0.866)**	**0.731*** **(0.603, 0.881)**	**0.774***
Y5 – M6	1.215 (1.118, 1.340)	0.922 (0.733, 1.054)	0.931 (0.812, 1.086)	0.943
Y5 – Y1	1.193 (1.113, 1.295)	1.014 (0.827, 1.189)	1.039 (0.923, 1.181)	1.110