THE PRICING RELATIONSHIP BETWEEN THE FTSE 100 STOCK INDEX AND FTSE 100 STOCK INDEX FUTURES CONTRACT

by

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For Karen, who puts up with me so well

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ABSTRACT

This thesis investigates the pricing relationship between the FTSE 100 Stock Index and the FTSE 100 Stock Index futures market. We develop and apply a framework in which it is possible to evaluate whether or not markets can be said to function effectively and efficiently. The framework is applied to both the daily and intra-daily pricing relationship between the aforementioned markets. In order to analyse the pricing relationship within days, we develop a new method to remove the effects of nonsynchronous trading from the FTSE 100 Index. We find that on a daily basis the markets generally function effectively, although this does not carryover to the intra-daily pricing relationship. This is especially true during the October 1987 stock market crash, where it is argued that a possible cause of the breakdown lies with the stock market. If this is the case, then any regulation should be aimed at the stock market, not the stock index futures market.

INTRODUCTION

> 'Of course, the real reason the market reacts one way or the other is because many traders are irrational and emotional.' (Quoted in Antoniou (1986, p.1)).

This somewhat traditional notion that prices react in the way they do because of the emotional and irrational reaction of market participants to new information has led to the charge that the introduction of futures markets helps only to worsen the situation. Emotional and irrational reaction in one market is bad enough. Emotional and irrational reaction in another market closely related to the underlying spot market can only make spot market traders' reactions worse. This charge seems particularly true of stock index futures markets.

Moreover, to support this argument, proponents need look no further than the growing body of evidence emerging from the US that the stock index futures market leads the stock market. Of course, such reactions could just as easily be indicative of effectively functioning and efficient spot and futures markets. The problem is that there is no framework in the existing literature in which these charges and countercharges can be objectively appraised. This thesis provides such a framework. In addition, although we discuss this framework in the context of the pricing relationship between the stock market and stock index futures market in the UK, it is sufficiently general that it can be applied to an analysis of pricing relationships between any markets with little or no modification.

In chapter two, we start with a critique of the existing methods available for appraising and analysing pricing relationships between stock and stock index futures markets. We then move on to consider how the problems noted with, for want of a better word, 'traditional' approaches can be overcome.

Within the 'traditional' approach to the analysis of pricing relationships there are two distinct and quite separate areas of interest : lead-lag relationships between spot and futures prices and deviations of the futures price from its theoretically correct price to determine the presence of arbitrage opportunities. It is the fact that these two areas are studied independently that gives the anti-futures markets lobby its ammunition : because they are studied independently they can be used independently to justify the claim that futures markets provide no benefits.

Using the framework provided by a general to specific modelling methodology (see Hendry and Richard (1982, 1983) for an overview of general to specific modelling and Hendry and Ericsson (1991a,b) and Garrett and Priestley (1991) for applications), we unify these two independent areas into a coherent whole. By treating these two areas as a whole, we derive a framework within which it is possible to appraise whether or not equity markets are effectively functioning and efficient in their processing of information. The advantage of this framework is that if they are not functioning effectively or efficiently, it is possible to pinpoint reasons why this might be the case. Using this framework, chapter three empirically investigates the functioning of the UK stock market, proxied by the FTSE 100 Share Index, and the UK stock index futures market, where a contract written on the FTSE 100 Index is traded.

In chapter four, we move on to consider the nature of the pricing relationship between the FTSE 100 Index and futures contract on an intra-daily time scale. In recent studies, primarily in the US, where intra-daily data has been used, concern has been expressed about the use of an Index that is calculated in real-time (often minute by minute) without taking the fact that not all of its constituent shares will have traded within that minute into account. This has come to be known as the nonsynchronous trading problem. We propose a method for removing its effects which overcomes the at times substantial problems, whether they be conceptual or data-related, with other methods that have been proposed. Having calculated indices which are devoid of nonsynchronicity, we analyse the minute by minute relationship between the FTSE 100 Index and futures contract, noting the implications of the results obtained by applying the framework developed in chapter two for those studies, particularly in the US, which identify what is potentially false volatility in the pricing relationship. The empirical findings in this chapter only serve to reinforce the advantage of using the framework developed in chapter two.

In chapter five, we utilise the models derived in chapters two and four to analyse the still controversial October 1987 stock market crash. Following the crash, many commentators have argued that stock index futures markets played a substantial role in the decline and as such they should be subject to greater regulatory control. The anti-futures lobby have even argued that the crash is just one more piece of evidence against the justification for the existence of futures markets. We assess the validity of these claims using minute by minute data on the FTSE 100 Index and futures contract for the 19th and

20th October 1987. Once again, the advantages of utilising the framework developed in chapter two to evaluate the validity of the 'regulate futures to prevent it happening again' argument are emphasised and the findings of this chapter merely confirm the advantages such a framework generates.

Chapter six, the final chapter of the thesis, briefly restates the conclusions of the analysis undertaken in this thesis and points out some extensions of the results herein that should prove fruitful and interesting.

CHAPTER ONE : THE ECONOMICS OF STOCK INDEX FUTURES MARKETS

1.1. INTRODUCTION

The past two decades or so has witnessed an explosion in the growth and availability of derivatives on financial instruments. The growth in financial products and markets introduces constant change which causes problems for regulatory regimes and may introduce new sources of systematic risk, especially if the innovation is an innovation for innovation's sake. This growth has created a need for a more systematic investigation of the functioning of markets, especially the interrelationships between spot and derivative markets. Knowledge of the interrelationships between these markets is vital from a regulatory and policy-making perspective, for if the nature of these relationships are not well understood, incorrect regulatory and policy decisions may result.

One aspect of this growth has been the emergence of stock index futures markets, which have been phenomenally successful in the US and have an ever-increasing role to play in the UK as well. The extraordinary growth in the market for stock index futures in the US has prompted a great deal of research, both theoretical and empirical, into various aspects of their functioning. One of the purposes of this chapter is to evaluate this literature, which is almost exclusively concerned with the US markets. This allows us to focus on the weaknesses of the existing approaches to analysing pricing relationships between the stock and stock index futures markets. These weaknesses provide the rationale for the analysis that follows in the subsequent chapters of this thesis.

We start by discussing the general nature and features of futures contracts and discussing how they provide an alternative to forward contracts in order to set the scene for what follows. In section three, we focus on the issue of market completeness and how futures markets can aid in completing markets, providing a rationale for the existence of futures markets in general, and stock index futures markets in particular. We demonstrate that, theoretically, the introduction of a futures contract on an index (portfolio) completes an otherwise incomplete market. Of course, in practice markets are not complete but stock index futures do contribute by providing a means for hedging otherwise unhedgable stock market risk.

We show that this is indeed the case in section four, where we demonstrate that the introduction of a futures contract on an index, such as the Financial Times Stock Exchange (FTSE) 100 Index enables stock market participants to introduce negative correlations into their portfolio of stocks and as such reduce systematic risk. Having discussed the economic justification(s) for the existence of a futures contract on a stock portfolio, we move on to provide an appraisal of the literature in section five. The literature on futures markets, and its spillover into stock index futures, is now so vast that a full review is impossible to undertake. Therefore, the discussion of the literature will be quite selective, focusing on the predominant and most important papers directly relevant to the task at hand, that is, the analysis of pricing relationships between the stock market and the market for stock index futures. Section six concludes.

1.2. The Nature Of Futures Contracts

Futures contracts in financial instruments are a relatively new innovation (for example, the London International Financial Futures Exchange (LIFFE) was only established in 1982), futures contracts on commodities have been in existence for much longer. Essentially, one can argue that futures contracts emerged from forward contracts which have been established in one form or another for centuries. As such, futures markets bear the same feature of forwards, that is, they trade deferred claims on assets. Forward markets evolved as parallel markets to commodity spot markets (the place where the commodity is physically sold) not only to facilitate greater trading in the spot market but also to create some form of certainty for traders, whether they be buyers or sellers. Futures contracts can in some respects be viewed as a refinement of, although not necessarily a substitute for,[1] forward contracts and have special features which readily distinguish them from forward contracts.

The first apparent difference between the two concerns the terms of the respective contracts. Forward contracts are tailor-made to the individual's requirements. Given that individual's requirements will invariably differ, forward contracts are in this respect heterogeneous goods. For example, with a forward contract the delivery date is agreed between the two parties to the contract rather than being fixed. Futures contracts, on the other hand, are standardised in just about every aspect : they are homogeneous with well-specified commitments for a

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[1] This is readily shown by the failure of foreign currency futures contracts traded at LIFFE. Trading ceased because the forward market overwhelmed the futures market : the futures market could not compete with the demand for forward foreign exchange contracts.

carefully described commodity (whether it be financial or physical) which will be delivered at a certain time on a certain date. Unlike the forward contract, this standardisation means that futures contracts have fixed expiration dates, although these do differ across contracts on different assets.[2] In addition, the size of the futures contract, unlike a forward contract, is fixed. These features may appear to make the futures contract unattractive at first sight. However, in conjunction with the other features of futures contracts they actually serve to enhance their appeal to a wider range of economic agents.

The second difference that emerges between the two stems form the fact that futures contracts are standardised. They are standardised because, unlike forward contracts, they trade on organised exchanges, which are non-profit-making organisations, according to a prespecified set of trading rules. Thus futures contracts are relatively easier to trade. In addition to trading on organised exchanges, futures markets are characterised by the presence of a clearing house

[2] To illustrate the point, all futures contracts traded on LIFFE (which are contracts for financial instruments) expire in March, June, September and December whereas futures contracts traded on the London Grain Futures Market expire in January, March, May, June, September and November. Further, the expiry day within the expiration month will differ for different commodities but will not differ for contracts on the same commodity.

which again is a non-profit-making institution. The purpose of the clearing house is to ensure that the futures market functions effectively and can substantially aid reduction in transaction costs. It does this by interposing itself between the buyer and seller of the futures contract without actually taking an active position in the market on its own behalf. Therefore, traders are undertaking transactions with an impersonal, non-profit-seeking body.[3]

The final difference between the two is the margin system that operates in futures markets. The margin is a deposit the trader must make with the broker in order to trade in a futures contract.[4] The margin may be posted in the form of interest-bearing securities such that the opportunity cost of investing in a futures contract is effectively zero. This margin is then adjusted every day in the so-called marking-to-market process where any losses that reduce the margin posted with the broker to below the minimum level are made good. The principle idea behind the margin is,

[3] The argument is analogous to that used in justifying the development of a financial system to overcome the problems of an economy trading through a barter system, namely one does not have to seek out another party to the contract : it is already there in the form of the clearing house and there is no need for a double coincidence of wants.

[4] According to Franklin and Ma (1992), the margin is calculated as $\mu + 3\sigma$ where μ is the average of the absolute daily price change of the futures contract and σ is the standard deviation of this change.

as mentioned earlier, to act as a deposit and thus to reduce as far as possible the risk of default. Since the margin system works as a deposit system, an investor who has promised to buy gold, say, at a price of $400 per ounce, is less likely to default if the price of gold is $350 at the time the trade is due to take place because the trader can retrieve the 'deposit' simply by reversing the futures trade. In this situation, then, the margin only serves to enhance the attractiveness of futures markets by virtually eliminating default risk, something that is not so readily done with forward contracts.

To summarise thus far, then, futures differ from forwards through the standardisation of contracts which are traded on an organised exchange through a clearing house, with a deposit equal to only a fraction of the price being necessary to undertake the futures trade.[5] It is precisely these differences that provide futures contracts with several advantages over forward contracts.

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[5] One must be careful not to treat futures and forward prices as synonymous. In a theoretical framework, Cox, Ingersoll and Ross (1981) show that forward and futures prices will only be equal if interest rates are nonstochastic, a rather restrictive assumption. From an empirical standpoint the evidence is rather mixed, pointing to no difference between futures and forward prices for financial instruments but significant differences for commodities (see Cornell and Reinganum (1981), French (1983) and Park and Chen (1985)).

The standardisation of futures contracts in the first instance reduces the (indirect) transactions costs faced by the futures market participant since there is no need to incur costs in ensuring and ascertaining not only the quality of the contract but also the integrity of the other party to the contract. This is achieved because the other party to the transaction is the clearing house. Having the clearing house as the other party to the transaction, in conjunction with the margin system, virtually ensures the removal of default risk. This represents another reduction in transactions costs to the futures trader.

Second, given that a futures contract is a deferred claim on another asset, it should provide information about likely future movements in the spot asset's price. Of obvious importance is whether or not the futures market can do this efficiently for it seems a natural place for new information about the asset to appear first. The centralisation of the futures market, coupled with the speed with which a futures transaction can be effected (there is no need to spend time finding out about the quality of the contract since it is standardised) and the very low transactions costs (the margin can be posted in the form of interest-bearing securities so there is no opportunity cost and there is only one transaction to be undertaken so brokers fees are minimal), means that the futures market is likely to serve its price discovery role well which can only improve the information reflected in the underlying asset's price such that market participants, whether they be hedgers, speculators or arbitrageurs, receive accurate price signals.

One of the criticisms that has been levelled at futures markets, however, is the supposed destabilising effects they have on the underlying spot market, especially through increased volatility in the spot asset. The argument here is that futures markets, given the advantages outlined above, will attract speculators who have no interest other than making a quick profit. Therefore, so the argument goes, speculative trade will be destabilising and this will manifest itself in increased price volatility in the spot market, distorting signals sent by prices to other types of trader. Speculators, however, are necessary in any futures market to provide liquidity for hedgers.[6] Moreover, if speculative traders are well informed, then any increase in spot market volatility could be due to the more efficient processing of information.

[6] The argument here is that hedgers are more risk averse than speculators and therefore are looking to transfer price risk. Speculators will take on this price risk and hedgers are prepared to pay them a premium (in the form of a lower futures price), the so-called risk premium.

The counter argument to the charge of destabilisation is that speculative trade decreases price volatility since speculators buy low and sell high, thus avoiding large price swings. The evidence, at least for the UK, seems to suggest that futures markets decrease spot price volatility (see Antoniou and Foster (1992) for evidence for Brent Crude Oil and Antoniou and Holmes (1992) for evidence for the UK stock market). To conclude this section, then, futures markets have proved to be a very useful innovation in terms of what they offer to investors. The rest of this chapter will now focus on various aspects of stock index futures. In the next section, we shall consider a powerful economic justification for their existence : that of completing markets.

1.3. STOCK INDEX FUTURES AND MARKET COMPLETENESS

Securities exist so that economic agents, whether they be firms or individuals, can postpone current consumption in order to invest in productive opportunities to increase future consumption. Given that an investment involves the postponement of consumption now for consumption in the future, the consumption-investment decision is necessarily an intertemporal one. However, the intertemporal nature of the consumption-investment decision introduces risk and this can be problematic. The problem arises through the very nature of securities. A security can be viewed as a collection of possible future payoffs out of which only one will occur and that one which does occur is dependent upon the state of the world in the future. The risk occurs rather obviously because the state of the world that will occur is, *ex ante*, unknown and as such risk averse investors will wish to minimise this risk.

However, in an risky world without markets for all assets, economic agents cannot create payoffs that reflect their own preferences and hence they cannot create payoffs that cover every possible future state of the world. They cannot translate their *preferred* bundle of 'goods' into an equivalent *actual* bundle of 'goods'. To demonstrate, consider the following example discussed in Copeland and Weston (1988, p.112). Suppose there are three states of nature and three assets available. The first asset, a risk-free one, generates the following payoffs : (1,1,1). The second asset, a risky security, generates a payoff of (1,0,0) and the third asset, which is risky debt, has a payoff pattern given by (0,1,1). We have three assets and three states of nature but we do not have complete markets. The reason for this is that the payoff pattern for the risk-free asset can be constructed as the sum of the payoffs for the other two assets. Thus, because the number of linearly independent assets/securities is not equal to the number of possible outcomes, a portfolio that covers all possible outcomes cannot be constructed. For example, the payoff pattern (0,1,0) cannot be constructed from the three securities and as such there is, in some sense, an absence of a market.

The implication of this is that with the absence of markets for some goods and services economic agents cannot have a unanimous ranking of alternative opportunities. This is illustrated by the example above whereby because the payoff (0,1,0) cannot be constructed, and hence a value cannot be attributed to it, one agent may think it is worth more than another agent : prices assigned to this hypothetical security need not be the same. Thus, the absence of a unanimous ranking of assets through market incompleteness implies that risk-sharing arrangements in an uncertain world are sub-optimal and this sub-optimality arises through incomplete markets.

1.3.1. How Can Futures Complete Markets?

If incomplete markets generate sub-optimal risk-sharing arrangements, the interesting question is how can markets be completed. This is where derivative instruments written on the underlying asset enter the picture. The issue of how derivative markets can complete otherwise incomplete markets was discussed by Ross (1976) in relation to options. Ross's work built upon the work of Arrow (1964), who analysed the role of securities in the allocation of risk.

Arrow (1964) formulated the argument about complete markets in terms of spanning a space whereby the space represents each potential state of nature thta can occur. The argument is then one of incomplete markets through incomplete spanning of the space. In this framework, Arrow (1964) demonstrates that an inadequate number of state contingent claims will lead to inefficiency[7] because the feasible set of pure contingent claims[8] fails to span the space. Ross builds on this by noting that the possibility of writing options on securities (or primitive assets in Ross's terminology) opens up new opportunities for spanning the space of the natural states of the world that can occur. The argument for using derivative markets to provide the opportunities for spanning the state space are intuitively appealing. As Ross (1976, p.76) argues,

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[7] Efficiency in the context of market completeness is in relation to Pareto optimality as opposed to definitions of efficiency that arise from the Efficient Markets Hypothesis.

[8] Pure contingent claims, or pure securities, are ones which offer a claim to wealth if a particular single state occurs and zero otherwise. We will return to this point later.

> 'If the introduction of a contingent claims market will use more resources than it will save, in an opportunity cost sense, by moving closer to efficiency, then within the context of the institutional structure of the economy the absence of a market is required for efficiency...[However], in general, it is less costly to market a derived asset generated by a primitive than to issue a new primitive, and there is at least some reason to believe that options will be created until the gains are outweighed by the set-up costs.'

Although Ross illustrates his argument with respect to options, the same argument applies to futures markets given their low transaction costs, as discussed earlier. Thus, we can use Ross's framework to demonstrate that stock index futures can be used to complete the market.

Following Ross, assume for simplicity that the only assets are securities which yield a return in each state of the world and there are n such assets, denoted by x_i, $i=1,\ldots,n$. Further assume that the number of future states is finite and that there are m states of the world denoted by ω_j, $j=1,\ldots,m$. Denote by $\mathbf{X}$ the mxn matrix with typical element x_{ij} which represents the return on asset x_i in state ω_j and let $\Omega = \{\omega_1,\ldots,\omega_m\}$. If rank $(\mathbf{X}) = m$, then the market is complete and there will exist a matrix of portfolios, $\mathbf{A}$, formed by combining the n primitive assets, such that we have

$$XA = I \tag{1.1}$$

where $\mathbf{I}$ is an mxm identity matrix. If this is possible, then combining the primitive assets allows the formation of a complete set of pure securities, where a pure security is defined as yielding a return of 1 if that state of the world occurs and zero if that state does not occur. If it is possible to form such a portfolio of securities, then because the identity matrix is of full rank, a pure security payoff can be created for each state and markets will be complete. The problem that is faced is that typically there are more ω_j than there are x_i and therefore the space cannot be spanned and markets are incomplete. However, if it is possible to derive new assets from the primitives, it is possible to provide more spanning opportunities. This is where futures enter the picture. However, as we shall see, creating a futures contract for each primitive asset does not necessarily provide a solution and this is where stock index futures become useful. To demonstrate, consider examples one and two from Ross (1976, p.80).

Suppose $\mathbf{X}$ contains a single asset, x, and there are three states of the world. Further, suppose x offers the following payoff pattern for the three states :

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \tag{1.2}$$

Since $\mathbf{x}$ is a vector, it has a rank of 1 and cannot therefore span the space of future states of nature. Therefore, markets are incomplete. Suppose now that two futures contracts are available on x with settlement prices 1 and 2 and these have the following payoff patterns[9] :

$$f_x(1) = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad \textit{and} \quad f_x(2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{1.3}$$

$\mathbf{X}$ now contains three assets and the payoffs are given as

$$\begin{bmatrix} x & f_x(1) & f_x(2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \tag{1.4}$$

Clearly, rank ($\mathbf{X}$) = 3 and there are three states so the market is complete. Thus, there will exist some

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[9] The two futures contracts could have different expiration months, for example.

A such that **XA** = **I** and from this, **X** = $\mathbf{A}^{-1}$ which tells an investor which securities to buy (if the elements of $\mathbf{A}^{-1}$ are positive) and sell (if the elements are negative) to obtain the particular pure security payoff they prefer (in this case either (1,0,0), (0,1,0) or (0,0,1)). In actual fact, if markets are complete economic agents can internalise the risk by constructing their portfolios such that they will receive a payoff of 1 regardless of which state of the world occurs. To see this, note that $\mathbf{A}^{-1}$ is given by

$$\boldsymbol{A}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \tag{1.5}$$

Now to create a pure security payoff of (1,0,0) buy one unit of the security that yields (1,2,3), one unit of that which yields (0,0,1) and sell two units of (0,1,2). Similarly for (0,1,0) buy one unit of (0,1,2) and sell two units of (0,0,1) and to obtain (0,0,1) simply buy one unit of (0,0,1). If the economic agent engages in these strategies the payoff will be 1 regardless of which state of the world occurs, such that there is no uncertainty about future payoffs.

However, this situation will not necessarily always occur, for if there are some states in which assets have the same returns it is impossible to derive futures contracts that will distinguish them. This possibility leads Ross to propose the following theorem which provides a sufficient condition for completeness (Theorem 1 of Ross (1976), p.81) :

THEOREM : The dimension of $F_{\mathbf{x}}(\mathbf{X})$ is full if and only if no two rows of $\mathbf{X}$ are identical.[10]

The reason why completeness requires $F_{\mathbf{x}}(\mathbf{X})$ as opposed to $\mathbf{X}$ to be of full rank is that, as the example above showed, $\mathbf{X}$ does not have to be full rank if it can be augmented by futures, giving $F_{\mathbf{x}}(\mathbf{X})$ which must be of full rank if markets are to be complete. In other words, if the theorem is true $F_{\mathbf{x}}(\mathbf{X})$ will span Ω.

The requirement from Theorem 1 of Ross (1976) that markets can only be made complete through the use of derivatives if no two assets offer identical payoff patterns is somewhat problematic. In addition, when the number of primitive securities is large, there will be a need for multiple, complex futures contracts which are very rare, if they exist at all. To overcome these objections, Ross derives an interesting

[10] $F_{\mathbf{x}}(\mathbf{X})$ is simply $\mathbf{X}$ augmented by all futures written on $\mathbf{X}$.

and powerful result relating to the role a simple futures contract written on a portfolio can play in completing the market. Suppose that investors can now form portfolios of the primitive assets and denote these by $\mathbf{P_x}$. In addition, simple futures contracts can be written on $\mathbf{P_x}$. Denoting $F(\mathbf{P_x})$ as the space spanned by the futures written on the portfolio, then Ross demonstrates the following theorem (Theorem 4 of Ross (1976, p.84)) :

THEOREM : A necessary and sufficient condition for rank $F(\mathbf{P_x}) = m$ is that there exists a single portfolio such that $\mathbf{XA} = \mathbf{b}$ with $b_i \neq b_j$ for all i,j.

The importance of this result should not be understated. As Ross (1976, pp.85-86) points out,[11]

> 'When we are permitted to write options on portfolios, a necessary as well as sufficient condition for efficiency is that there exists a single portfolio *a* with the property that options written on it can span Ω.'

Herein lies the importance of the stock index futures contract and, for that matter, the stock index options contract. The stock index futures contract represents the simple futures contract written on the underlying

[11] Since Ross talks about completeness with regard to options, and since it is a direct quote, we have left in the word options. The quote is still equally applicable.

portfolio constructed from primitive assets. Whilst in practice markets are not complete, a stock index futures contract on a portfolio can reduce market incompleteness and provide a step nearer to complete markets. This is a powerful economic justification for their existence.

The stock index futures contract that exists in the UK is the FTSE 100 index futures contract and the underlying portfolio which it is written on is the FTSE 100 Share Index. We will discuss some of the features of these instruments in the next subsection and this will clarify how stock index futures can help to complete the market.

1.3.2. THE FTSE 100 INDEX AND INDEX FUTURES CONTRACT

The FTSE 100 Share Index was introduced in January 1984 in order to support a futures contract based on the UK equity market. The FTSE 100 Share Index is a representation of the market value of the outstanding shares of the 100 companies that comprise it. The Index is calculated as a real-time weighted arithmetic average of the market value of the outstanding shares of the component companies, the weight each company's share price receives being dependent upon its market capitalisation. The design of the FTSE 100 Index is such that it mirrors the movements of a 'typical' well-diversified institutional portfolio. However, whilst the FTSE 100 Index serves this important function, investors cannot buy the Index *per se*.[12] However, the Index does represent a good underlying instrument for a futures contract based on the stock market precisely because it closely approximates the size and composition of institutional portfolios. Indeed, such is its success in this role that it provides not only a benchmark against which portfolio performance can be compared but also it is widely recognised as an indicator of the performance of the stock market as a whole.

The FTSE 100 stock index futures contract, on the other hand, represents the purchase or sale (depending upon whether the futures contract is bought or sold) of the 'basket' of shares that comprise the Index in a proportion consistent with their allocated weights within the portfolio. The trade date for the transaction is specified as the expiry date of the futures contract. Of course, since the Index does not exist as a tradeable instrument in itself, actual purchase or delivery of the Index cannot take place.[13]

[12] Although some unit trusts may well provide a close approximation to it.

[13] Even if it did exist as a tradeable instrument, it is unlikely that investors would wish to purchase or deliver the Index in any case.

Consequently, the stock index futures market is a cash settlement market rather than a delivery market, with all contracts outstanding on the expiration date deemed to be settled by either the purchase or sale of the Index (depending upon whether the position taken in the futures market was short or long) at its closing price on the expiration date. In all cases, investors stand to gain or lose the difference between the price at which they bought (sold) the futures contract when taking their initial position and the price at which they sell (buy) the futures contract upon expiration in order to close their position. Given that in the space of one transaction the purchase of the stock index futures contract represents a purchase of a future claim on all of the shares of the Index and given the low transaction costs involved in the purchase of a stock index futures contract,[14] stock index futures provide increased opportunities for investors to create payoffs in accordance with their preferences. Moreover, whilst they are designed with the FTSE 100 Index as the underlying portfolio, they can still be utilised by investors who hold portfolios that differ from the FTSE 100 because their purpose is to provide

[14] Recall that the purchase of a futures contract only requires the posting of a margin which is a fraction of the price, and this margin can be posted in the form of interest-bearing securities so that there is effectively no opportunity cost to investing in futures.

a vehicle for, say, hedging against general movements in the stock market.

1.3.3. THE USES OF STOCK INDEX FUTURES

In general, the use of stock index futures contracts falls into three categories : hedging, speculative trading and arbitrage trading. Whilst all three of these categories are important, perhaps the most important from the point of view of the existence of futures markets is the hedging category,[15] which we will return to in more detail shortly. Briefly, hedging involves the purchase or sale of a stock index futures contract in anticipation of an intended transaction in the spot market. Thus, futures allow investors to reduce risk since they provide the investor with some level of compensation should there be adverse movements in the spot market prior to the intended transaction taking place.[16]

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[15] Futures are, after all, primarily a risk-reducing instrument, allowing hedgers to transfer their own price risk to speculators.

[16] It should be noted, however, that a cross hedge is nearly always involved when hedging with stock index futures. The reason for this is that the portfolio being hedged invariably differs from the portfolio underlying the stock index futures contract. This does not mean they cannot reduce risk. It means they cannot reduce all risk since there is a risk that the portfolio and stock index futures contract will not track each other exactly. See Anderson and Danthine (1981).

Speculative traders can use stock index futures simply to speculate on the direction of future price movements in the stock market as a whole. Given the low transaction costs and the speed with which a futures trade can be executed, it is much easier for a speculator to take a position in the futures market. Thus, given that speculators will only take a position in one market, those that use futures to speculate will provide much needed liquidity to the market. It is perhaps no coincidence that those futures markets that survive tend to have a bigger speculative trader component. This is especially true if the futures market is in an asset for which prices can be quite volatile. If a market is more volatile, 'abnormal' price movements will tend to be more frequent and this is what speculators try to take advantage of. By the very nature of a speculative trade, it is more risky than a hedge trade. In addition, speculative traders do not hold the futures contract for great lengths of time since this makes their position even riskier.[17]

The third category of traders that stock index futures support are arbitrageurs. Arbitrageurs simultaneously purchase and sell stocks and futures in order to capitalise on perceived mispricing. Technically, such

[17] For example, 'scalpers' are speculative traders who try to take advantage of very short term intra-daily price movements.

a trade is risk free, although this may not necessarily be true since the portfolio of stocks they sell may not be of exactly the same makeup as the index on which the futures contract is written. To take advantage of the mispricing they will take opposite positions in the markets (buy stocks and sell futures or vice versa). The effect this has is to drive prices back to their true levels. As such, arbitrage is very important if futures and the underlying spot prices are to be kept in line.

To see this, suppose an arbitrageur believes the futures price is undervalued because speculative trade, say, has driven the futures price below what can be termed its fair value.[18] The natural trade to undertake is to buy the futures. In addition to this, the arbitrageur will also sell the stocks. Therefore, the initial selling pressure in the futures market is transmitted as selling pressure to the stock market.

Now, suppose that trading activity after the arbitrage trade is such the futures contract becomes overvalued. The arbitrageur will then sell the futures contract and buy the stock. Thus, the buying pressure initiated by arbitrage trades is transmitted to the spot market. In addition, the arbitrageur has made a

[18] The fair value is given by equation (1.17) (see section 1.5.3.).

risk free profit.[19] Arbitrageurs will thus ensure that prices in both the spot and the futures market will not drift to far away from their equilibrium values and as a consequence hedgers, for example, will receive accurate signals upon which they can base their hedging strategy. The whole idea of a hedge is that the two prices move in the same direction such that risk reduction is possible (gains in one offset by losses in the other). If arbitrage does not function effectively, or arbitrage opportunities are left unexploited, this will not happen and the futures market becomes redundant.

1.4. STOCK INDEX FUTURES AND DIVERSIFICATION

In the previous section we saw how, at the theoretical level, stock index futures contracts can complete otherwise incomplete markets. Of course, in reality markets are not complete but if stock index futures are serving their purpose correctly, they can greatly enhance the range of risk-sharing arrangements

[19] An assumption implicit within this example is that the portfolio of stocks sold by the arbitrageur is exactly the same as the underlying stock index. This assumption is not necessary, but it does indicate the risk-free nature of the trade. Suppose the arbitrageur anticipated the movement of the markets incorrectly. A profit would still have been made even if the futures market and the stock market continued to fall, for in closing out the position, the loss on the futures would be offset by the gains from the spot. This is what is meant by a risk-free profit : the arbitrageur could not lose.

available to the investor through the provision of hedging opportunities. Indeed, not only do they enhance risk-sharing arrangements, they also allow investors to reduce systematic risk through diversification, something they could not do prior to the introduction of derivative assets.

The notion of risk reduction through diversification stems from the work of Markowitz (1952). The argument is that rational, risk averse investors should concentrate on combining assets into a portfolio that gives the maximum expected return for a given level of risk. Equivalently, they will minimise the level of risk for a given level of expected return. Whichever way the issue is approached, the argument is one of the optimal management of risk. By assuming that investors are rational, risk averse utility maximisers and that returns are normally distributed, Markowitz shows that combining different spot assets into a portfolio can substantially reduce risk. Moreover, by assuming normality, the expected return is given by the mean return and the level of risk is given by the variance of returns. Thus, investors make their investment decision solely on the basis of the mean and variance of the distribution of returns. For the sake of exposition, suppose there are two risky assets A and B, the returns of which are random variables. The variance of the portfolio combining these two assets is given by the well-known formula[20]

$$\sigma_p^2 = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B cov(R_A,R_B) \tag{1.6}$$

which, noting that $cov(R_A, R_B) = \sigma_A\sigma_B\rho_{AB}$, can be rewritten as

$$\sigma_p^2 = x_A^2\sigma_A^2 + x_B^2\sigma_B^2 + 2x_Ax_B\sigma_A\sigma_B\rho_{AB} \tag{1.7}$$

where x_i is the proportion of funds invested in asset i, σ_i is the standard deviation of the returns of asset i and ρ_{ij} is the correlation between returns on assets i and j. The extent to which diversification is possible is determined by the correlation between returns on the assets. As far as risk reduction is concerned, the important term in (1.7) is the last one on the right hand side. From this term, the lower the correlation, the greater is the reduction in risk that can be achieved through diversification. If the asset returns are perfectly positively correlated there is no gain in reducing risk from diversification.

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[20] For the more general case of n assets, the variance of the portfolio is given by

$$\sigma_p^2 = X'\Sigma X$$

where **X** is an nx1 vector of weights and Σ is the nxn variance-covariance matrix of returns.

However, the implication of perfect negative correlation is quite different : with perfect negative correlation it is possible to construct a portfolio which is risk-free. Noting that $x_B=(1-x_A)$, then differentiating (1.7) with respect to x_A and solving the derivative for x_A yields

$$x_A^* = \frac{\sigma_B^2 - \rho_{AB}\sigma_A\sigma_B}{\sigma_A^2 + \sigma_B^2 - 2\rho_{AB}\sigma_A\sigma_B} \qquad (1.8)$$

which is the proportion of funds that should be invested in asset A if the minimum variance portfolio is desired. Now, if the correlation between A and B is perfectly negative then investing x_A^* in A will set σ_p^2 equal to zero.

Thus, diversification and risk reduction are most fruitful when negative correlation can be introduced into the portfolio. The problem with this, however, is that most spot assets such as stocks and bonds tend to be positively correlated[21] such that there will always be some risk that cannot be diversified away. This risk is known as systematic risk.

One method by which this problem could be overcome is

[21] Even international asset markets are positively correlated.

through the short sale of stocks such that gains in one are offset by losses in the other. In this way, short sales would expand the opportunity set available to investors, which in turn helps to complete markets. However, there are problems with short trading. The first is a rather insurmountable one : the only people who can short sell in the UK are market makers and they can only short sell during the account period; it is illegal otherwise. Such short selling restrictions are not present in the stock index futures market. The second is that even if short selling were allowed, although short selling would expand the trading opportunity set, it would not expand it as much as stock index futures can. With stock index futures, there are four expiry months and typically three different expiration contracts trading at any one time. Thus, there are three/four index futures contracts that can be used in the investment opportunity set at any one time, offering more trading opportunities through the potential of at times quite complex spreads across different maturity contracts. In addition, transactions costs in the stock index futures market, and futures markets in general, are less.

A more viable possibility for introducing negative correlation into the portfolio, then, is to allow trade in stock index futures contracts and allow investors to introduce negative correlation into their portfolios this way. It is straightforward to demonstrate how stock index futures can do this, and indeed how it is possible for stock index futures to facilitate construction of a risk free portfolio (the so-called hedged portfolio).[22]

Suppose a rational, utility maximising, risk averse investor holds a stock portfolio that mimics the FTSE 100 Index and wishes to use stock index futures to reduce the risk associated with the portfolio. To reduce the risk of the portfolio, the investor will short stock index futures. Denoting the number of stock index futures contracts the investor will short by h, then the investor's expected return comprises the expected return from the spot portfolio and the expected return from the futures position :

$$E(R_{hp}) = E(R_s) + hE(R_f) \qquad (1.9)$$

where $E(R_{hp})$ is the expected return on the new hedged portfolio, $E(R_s)$ is the expected return from the spot portfolio and $E(R_f)$ is the expected return from the futures position. The variance of the hedged

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[22] The arguments that follow are also used by Tucker, Madura and Chiang (1991) to demonstrate the use of forward foreign exchange contracts in constructing a risk-free hedged portfolio.

portfolio is given by

$$\sigma_{hp}^2 = \sigma_s^2 + h^2\sigma_f^2 + 2h\sigma_{sf} \tag{1.10}$$

where σ_{sf} is the covariance between spot and futures returns. To determine h, minimise (1.10) with respect to h :

$$\frac{\partial \sigma_{hp}^2}{\partial h} = 2h\sigma_f^2 + 2\sigma_{sf} = 0 \tag{1.11}$$

Solving (1.11) for h, the futures position is given by

$$h = \frac{-\sigma_{sf}}{\sigma_f^2} \tag{1.12}$$

The optimal futures position has a negative sign because the investor is participating on the short side of the futures market. If h=0 such that the investor does not use the stock index futures market, then $\sigma_{hp}^2 = \sigma_s^2$. Therefore, the percentage of risk eliminated by participating on the short side of the futures market is

$$x = \frac{\sigma_s^2 - \sigma_{hp}^2}{\sigma_s^2} \tag{1.13}$$

Substituting (1.12) into (1.10) and the result into (1.13) yields

$$x = \frac{h^2 \sigma_f^2}{\sigma_s^2} \tag{1.14}$$

Now, suppose that the spot and futures prices are perfectly positively correlated (x=1), such that the futures position is -1, that is, sell one contract. For this to be true, then from (1.12), σ_{hp}^2 must be equal to zero. Thus by participating on the short side of the stock index futures market it is possible to create a risk free portfolio by hedging. This must be so because if the spot and futures prices move together exactly, gains from the stock portfolio are exactly offset by losses in the stock index futures market : short sales of the derivative instrument introduce the negative correlation necessary for diversification to be most effective.[23] The full range of possibilities open to the investor with the introduction of stock index futures are plotted in figure 1.1. Notice that the introduction of futures caters not only to the conservative investor (h=-1) but also to the aggressive investor (h=1) who can use futures to increase expected return (with a corresponding increase in risk). Obviously, rational investors will only be interested in portfolios that lie on the curve. Those beneath it are sub-optimal and those above it are unobtainable.

As a final point to consider in this section, note that perfect positive correlation between the two markets actually completes the market since risk can be eliminated. This is confirmed by the example in the previous section (the matrix given by (1.5)). To construct a portfolio that offers a payoff of one in every state that can occur, the investor shorts futures contracts. In the example, the overall net position is sold one futures contract. If there is perfect positive correlation between spot and futures prices, then the model in this section shows that the investor should be short on one futures contract. If

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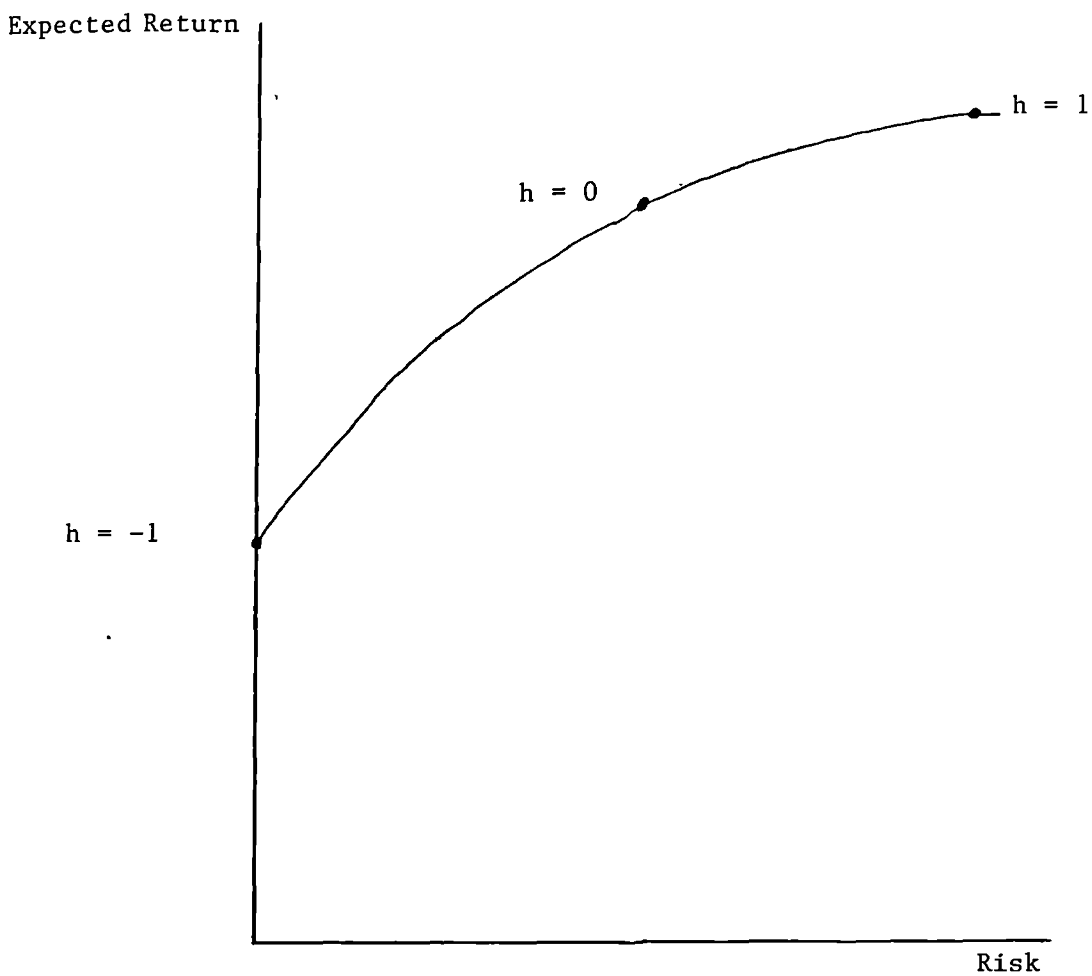
[23] Note that, if prices are perfectly positively correlated the hedge portfolio constructed from a combination of a stock portfolio and short sales of stock index futures provides an alternative measure of the zero beta portfolio in Black's (1972) Capital Asset Pricing Model (CAPM).



there is perfect positive correlation, then, futures must complete the market. Moreover, it is the allowance of short sales of the futures contract that actually enables it to complete the market. Without this short sale facility, their usefulness would be questionable.

1.5. Literature Review

In this section, we will review the literature relevant to our study. The literature review will be quite selective, focusing on those papers considered to be of primary importance in relation to the subject under study here. The reason for this is that the literature that on futures markets in general, and stock index futures markets in particular, is vast and the number of areas of interest is quite wide-ranging. Thus we keep the review selective to avoid detracting from the main issue(s) under consideration in this thesis. Before doing this, there are two areas that are worthy of minor comment. These are hedging and volatility. In the literature on hedging, emphasis has been on the effectiveness of futures in reducing/removing spot market risk. Figlewski (1984,1985) analyses the effectiveness of hedging with stock index futures, explicitly accounting for the risk introduced into the hedge by the basis. The risk arises because the portfolio being hedged may not be the same as the portfolio the stock index futures is written on. Therefore, they may not track each other exactly. Junkus and Lee (1985) also examine the effectiveness of hedging with stock index futures by comparing three hedging strategies : a classic hedge (positions of the same size), one based on Johnson's (1960) mean-variance model of hedger behaviour and one based on the basis risk approach to hedging (Working (1953)). Rolfo (1980) analyses the effect of differing degrees of risk aversion on hedging with cocoa futures.

With volatility, the focus has been on the appropriate measure of volatility to use (see Garman and Klass (1980), Parkinson (1980) and Kunitomo (1992)) and the effect of volume on volatility (Grammatikos and Saunders (1986)). For a review, see Karpoff (1987). An interesting critique of these tests comes from Lamoureux and Lastrapes (1990), who show that if volatility is modelled in the more preferable ARCH framework, volatility in stock prices is completely explained by volume, that is, once volume is included in the conditional variance equation, ARCH effects disappear. For a different application of ARCH to the volatility issue, see Chan, Chan and Karolyi (1991).

The reason why these areas are of minor interest is that hedging requires correct specification of the pricing relationship and volatility, especially if modelled as an ARCH process, could be due to misspecification. Thus, correct specification of the pricing relationship can affect them both.

1.5.1. LEAD-LAG RELATIONSHIPS AND NONSYNCHRONOUS TRADING

Whilst stock index futures were originally proposed to aid investors in spreading risk (achieved through aiding in the completion of markets and allowing investors to introduce artificial negative correlations into their portfolios, as discussed earlier), another important function has emerged which stock index futures should fulfil. This function relates to the role of information and how information is transmitted and incorporated into prices, an issue intimately related to market efficiency. Market efficiency is explicitly tied to the notion of how prices reflect information, as embodied in the three forms of efficiency discussed in Fama (1970) : weak efficiency, where prices fully reflect all information implied by past price movements; semi-strong efficiency, where prices fully reflect all publicly available information; and strong efficiency, where prices reflect all relevant information, whether publicly available or not. Clearly, since the stock market is a major allocator of resources, an informationally efficient stock market is vital.

This notion of efficiency has prompted a good deal of research testing the various forms of efficiency (see Fama (1970) for a review of early work and LeRoy

(1989) for more recent tests). The empirical evidence on this issue is somewhat mixed, although the more recent evidence has tended to point to inefficiency. These results, coupled with the theoretical results in Grossman and Stiglitz (1976, 1980) which show the impossibility of an informationally efficient market, have prompted concern over the functioning of equity markets and attention has focused on the role stock index futures can play in improving the informational efficiency of the underlying stock market.

The argument concerning the informational role of stock index futures stems from the nature of the stock index futures contract. As mentioned earlier, a trade in a stock index futures contract is a trade in a future claim on the shares that comprise the underlying index in a proportion consistent with their allocated weights. Therefore, they reach a new equilibrium price with each trade.

For the underlying stock index, and hence the stock market, to reach a new equilibrium price, however, all of the constituent shares must trade. As a result, the stock index futures price will lead the stock index because it reacts instantaneously to the new information causing the price movement. This is indeed one of the implications of the model of stock index futures trading derived by Subrahmanyam (1991).

This conclusion also arises from models of restrictions on trading in the stock market, see for example Diamond and Verrechia (1987). In their model, short selling constraints means that stock prices are slower to adjust to private information, especially if it is bad. Such short sale constraints are not present in the stock index futures market and hence, if traders are willing to use futures, the futures will convey predictive information to the stock market.

These theoretical arguments showing that the stock index futures market will lead the stock market have generated a good deal of empirical work in the US[24] that tests precisely this proposition. These tests are typically carried out using Granger-Sims causality tests (see Granger (1969) and Sims (1972)), with variable deletion tests being used to determine what leads what, if at all. The hypotheses to be tested are formulated in different ways, but essentially they are testing the same thing. Whilst the theory models suggest that the futures will lead the spot, the empirical models are modified slightly to allow for the possibility of feedback, that is, not only does the futures lead the spot but the spot also leads the

[24] Unfortunately there are no papers that test lead-lag relationships between the stock and stock index futures market in the UK.

futures.

The justification for formulating the econometric model in which the hypotheses are nested in this fashion are that although the stock index futures price will lead the underlying index price, the futures price will tend to react to information that is economy-wide.[25] The underlying stock index, on the other hand, will not only react to economy-wide information but also to information that may only affect a subset of securities that comprise its make up. In this situation, the spot may lead the futures.

One of the first papers testing lead-lag relationships between the stock market and stock index futures market was written by Kawaller, Koch and Koch (1987, 1988). Using minute by minute price data for Standard and Poor's 500 Index (the S&P 500) and index futures contract[26] they sought to investigate first whether the index futures price lead the stock market and second whether the observed lead-lag behaviour differed according to the time to expiration of the

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[25] By economy-wide information, we mean such things as inflation announcements, trade figures and so on.

[26] Using data from 1984 and 1985, they examine lead-lag relationships eighty-eight days, sixty days, thirty days, fourteen days and one day before expiration and the expiration day itself for the March, June, September and December 1984 and 1985 S&P 500 Index futures contract.

futures contract. They formulated their tests in the context of the following model :

$$\begin{aligned} \Delta i_t &= \gamma_{10} + \sum_{k=1}^{60} \alpha_{1k} \Delta i_{t-k} + \sum_{k=0}^{45} \beta_{1k} \Delta f_{t-k} + e_{1t} \\ \Delta f_t &= \gamma_{20} + \sum_{k=0}^{45} \alpha_{2k} \Delta i_{t-k} + \sum_{k=1}^{60} \beta_{2k} \Delta f_{t-k} + e_{2t} \end{aligned} \tag{1.15}$$

where Δi_t is the change in the log of the index price, Δf_t is the change in the log of the futures price and e_{it} are white noise error terms (i=1,2). Kawaller, Koch and Koch (1987) quite correctly treat (1.15) as a system because first contemporaneous values enter the equations and it is well recognised that spot and futures prices are jointly determined; second, if spot and futures prices are jointly determined, and determine each other, then the e_{it} will be correlated across equations since the markets will tend to react to the same information. Kawaller, Koch and Koch (1987) then estimate (1.15) by 3SLS[27] and test the hypotheses $H_o^1 : \beta_{1k} = 0$ and $H_o^2 : \alpha_{2k} = 0$. Rejection of H_o^1 and acceptance of H_o^2 means that the stock index futures price leads the stock market and vice versa. Rejection of both means that there is feedback.

Their general conclusion is that whilst there is quite

[27] Even if spot and futures prices were not simultaneously determined, estimation of (1.1) by OLS would be inefficient. To ensure efficiency (1.1) would have to be estimated by SURE.

weak evidence of a feedback relationship, the futures leads the spot, the lead from the futures to the cash price being some twenty to forty-five minutes whilst the lead from the spot to the futures price rarely moves beyond one minute. In addition, they find that this relationship is stable across both futures contracts and days to expiration of the futures contract.

The results in Kawaller, Koch and Koch (1987, 1988) have, however, been criticised for being potentially misleading. Indeed, although they do not recognise it as such, they actually present the reason why in their conclusion :

> 'The length of the lead from futures to the index reflects, in part, inertia in the stock market. Stocks are not traded as readily as futures contracts.' (Kawaller, Koch and Koch (1987), p.1327).

The criticism of their results is that this so-called inertia in the stock index in actual fact leads to potentially spurious conclusions. This is because the measure of the index used may not actually reflect its true value due to nonsynchronous, or thin, trading. Intuitively, the nonsynchronous trading problem arises because not all shares within the index (or portfolio) will trade in any one given minute.[28] Therefore, if they react to information with a lag, they will generate serial correlation in index returns, serial correlation which may not be genuine but rather may be due simply to the way the index is constructed.

The issue of nonsynchronous trading is not new, it was recognised by Fisher (1966) with regard to the construction of stock indices. However, much of the attention until recently focused on the effect it had on the estimates of beta used in tests of the CAPM.[29] The more recent literature has concentrated on estimating the probability of nontrading (Lo and MacKinlay (1990)) and removing its effects (Harris (1989), Stoll and Whaley (1990) and Garrett (1991)). Nonsynchronous trading cannot be ignored, as the following ten-point 'checklist' from Lo and MacKinlay

[28] Essentially, the nonsynchronous trading problem, certainly for stock returns, is founded on the notion that share *i* will not trade with probability p_i and will trade with probability $(1-p_i)$. This forms the basis of Lo and MacKinlay's (1990) model of nonsynchronous trading.

[29] This is perhaps not surprising given the controversy surrounding the validity of asset pricing models and the mixed results from tests of such models, particularly the CAPM, in the late 1970's and early 1980's. See the attack on the CAPM by Roll (1977) and the alternative Arbitrage Pricing Theory of Ross (1976). For the effects of, and how to correct for, nonsynchronous trading in estimates of beta see Scholes and Williams (1977) and Dimsom (1979).

(1990, pp.198-199) demonstrates[30] :

Nonsynchronous trading

a) Does not affect the mean of returns, whether they be an individual security's or a portfolio's.

b) For a non-zero mean return, the variance of individual security returns increases.

c) If a well diversified portfolio consists of securities with the same probability of not trading then the variance of that portfolios returns will decrease.

d) Induces *negative* serial correlation which declines geometrically in securities where the mean return is non-zero. (Their emphasis).

e) Induces *positive* serial correlation which declines geometrically in observed portfolio returns when the portfolios are well diversified with common nontrading probability of the constituent shares, yielding an AR(1) process for observed returns. (Their emphasis).

f) Induces *asymmetric* cross correlation between the returns of two different securities which declines geometrically. This is due to the assumption that different securities have different nontrading probabilities. (Their emphasis).

g) Induces *positive* cross correlation between two well diversified portfolios with common nontrading probabilities. Again, this correlation declines geometrically. (Their emphasis).

[30] Although Lo and MacKinlay (1990) derive a model of nonsynchronous trading, they note (p.198) that the implications that follow are 'consistent with earlier models of nonsynchronous trading.'

h) Induces *positive* serial correlation in an equally weighted index if the betas of the securities generally have the same sign. (Their emphasis).

i) Coupled with aggregation over time, the maximal negative serial correlation in observed security returns induced by nontrading increases. However, this is attained as time aggregation increases, with the nontrading probability nearing unity.

j) Coupled with time aggregation, the autocorrelation in portfolio returns induced by nontrading decreases. This is true for all nontrading probabilities.

Unfortunately, Kawaller, Koch and Koch do not report the α_{1k}'s so it is difficult to assess whether nonsynchronous trading is a problem in their study. However, the above concerns would suggest that it may well be.

Harris (1989) examines lead-lag relationships between the S&P 500 Index and the S&P 500 index futures contract around and on the stock market crash of October 1987. Harris uses observations five minutes apart over the period 12th October 1987 to 23rd October 1987. However, unlike Kawaller, Koch and Koch (1987, 1988) Harris incorporates nonsynchronous trading effects into his calculation of the S&P 500 Index. To do this, Harris constructs what is essentially a latent factor model to extract the common factor, which is the true underlying value of the Index, from transactions and price data for each of the stocks that comprise the S&P 500 Index. Harris then analyses the lead-lag relationship between the adjusted index and the stock index futures price using cross correlations.

Harris's main findings are that first, nonsynchronous trading effects are present in the data, although removal of this effect still leaves autocorrelation in the Index, suggesting that the autocorrelation is genuine and price changes are not independent of each other. Second, Harris finds that the futures strongly leads the Index, with apparently no feedback although again this is difficult to assess since Harris does not report the standard errors for the cross correlation coefficients. Analysis of the cross correlation coefficients between the change in the adjusted indices and the first lead of the change in the futures price (panel c of table II, Harris (1989, p.89)) does seem to suggest that again there is weak feedback from the spot to the futures on most of the days in Harris's sample.[31] However, given that Harris's case stands and falls on the analysis of the correlation properties of the data alone some caution must be used in interpreting his results.

[31] For example, for the 12th October the cross correlation coefficient between the perfect foresight index and the first lead of the futures price is 0.390. Likewise, for the 13th, 14th, 15th and 16th they are 0.027, -0.201, 0.218 and -0.026 respectively.

Stoll and Whaley (1990) analyse lead-lag relationships between the S&P 500 Index, the S&P 500 Index futures contract, the Major Market Index (MMI) and IBM share prices over a stable time period. Again, they analyse 5 minute movements using data over the period April 21st 1982 to March 31st 1987. As with the results of Harris (1989), Stoll and Whaley find that nonsynchronous trading is somewhat of a problem with the effects of infrequent trading more prevalent in the indices, especially the S&P 500. To remove the effects of infrequent trading, Stoll and Whaley derive a model whereby observed returns follow an ARMA(p,q) process when there is infrequent trading.[32] They then interpret the residuals from the ARMA model as the adjusted index and use these residuals in their tests of lead-lag relationships. They find that an ARMA (2,3) is required to remove the effects of nonsynchronous trading (and the bid-ask spread) from the S&P 500 and MMI indices whilst for IBM an ARMA (0,3) is the appropriate model.

Using the residuals from these models as proxies for the true returns on the indices they then test the lead-lag relationship in the context of the following model :

[32] In fact, Stoll and Whaley's model takes account not only of nonsynchronous trading but also the bid-ask price effect.

$$\Delta S_t^* = \alpha + \sum_{i=-3}^{+3} \beta_i \Delta F_{t+i} + u_t \tag{1.16}$$

where ΔS_t^* is the change in the true value of the relevant index and ΔF_{t+i} is the lead/lagged change in the futures price. If the coefficients with negative subscripts are significant, the futures leads the spot and if the coefficients with positive subscripts are significant the spot leads the futures. Stoll and Whaley estimate (1.16) by OLS and they conclude that the futures leads the spot, although again there is weak evidence of feedback between the two.

There are, however, several points that must be borne in mind when interpreting their results. First, use of an ARMA(p,q) model to extract true returns is problematic given the well-known identification problems that accompany such models, not only in terms of identifying the orders of the autoregressive and moving average terms but also in the identification of the coefficients if the model is over-parameterised. Second, use of the residuals as the proxy for true returns *forces* true returns to be white noise since the residuals from all of their estimated models are indeed white noise. This imposes weak efficiency, which may or may not be true. In addition, use of the residuals is incorrect since they represent not the adjusted index but the adjustment that should be *made* to the index. Third, futures and true spot returns will still be contemporaneously correlated and therefore estimation by OLS is inappropriate. We will return to some of these points later in the thesis.

A final study we can consider with respect to lead-lag relationships is that of Chan (1992). Chan uses data on the MMI, the MMI futures contract and the S&P 500 futures contract over two periods : from August 1984 to June 1985 and from January 1987 to September 1987. The reason for doing this is to try and assess the robustness of the results to improved trading practices.[33] To overcome the nonsynchronous trading problem, Chan uses transaction and price data from the 20 component stocks that make up the MMI and recomputes the Index over a five minute time interval. This reduces the infrequent trading problem to negligible levels. In a preliminary analysis of the data, Chan finds that the nontrading probability, whilst small for the 1984/1985 sample, is virtually nonexistent for the 1987 sample.[34]

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[33] This is based on an observation by Froot and Perold (1990) that the autocorrelation in short-term returns on several market indices decline over time. One interpretation of this is that improved and changed trading practices increase the speed with which stock prices react to new information.

[34] Chan defines the probability of nontrading as the proportion of five minute intervals in which a stock comprising the MMI Index does not trade.

Chan uses (1.16) to test the lead-lag relationship but, unlike Stoll and Whaley (1990), the model is estimated using Hansen's (1982) Generalised Method of Moments (GMM). Thus, Chan does in some respects recognise the simultaneity issue. As with the other studies discussed earlier, Chan finds that the futures leads the spot but again there is some weak evidence of feedback. Moreover, the lead-lag relationship is an asymmetric one, a point not drawn out in previous studies. An interesting finding that does emerge from Chan's results, however, is the lead-lag relationship changes over time. In particular the lead from the futures to the spot price is shorter in the 1987 sample, a result consistent with the arguments of Froot and Perold (1990).

To further assess the robustness of the results, Chan also analyses the lead-lag relationship under good and bad news, the intensity of trading and in relation to market-wide movement in prices. To test the good news-bad news relationship, the sample is sorted into quintiles by size and sign, with trading time being split into thirty minute intervals. The bad news group is then represented by the first quintile and the good news group by the fifth quintile. Chan interprets the bad news group as the one most likely to be subject to short sale constraints and in this respect he provides an indirect test of Diamond and

Verrechia's (1987) proposition. The conclusions from this test are illuminating :

> 'Summarising the results, it does not seem that the futures leads the cash index only under bad news. Neither is there a stronger tendency for the futures to lead the cash index under bad news than under good news. This may suggest that short-sale restrictions are not a constraint to marginal arbitrageurs, who are able to exploit their information by selling stocks under bad news.' (Chan (1992) p.137).

This is an important finding given that arbitrageurs are vital in ensuring futures and spot prices remain close together. Following a similar procedure for trading volume, Chan finds that the lead-lag relationship remains essentially unaltered under different intensities of trading. However, market-wide movement does affect the lead-lag relationship. Indeed, Chan finds that when there is substantial market-wide movement the feedback from the futures to the cash market is stronger, implying that indeed cash and futures markets have different access to information. Chan also suggests that this finding might explain the apparently strong lead-lag relationships that exist between the S&P 500 Index and Index futures contract : it is market-wide movement that drives it because information appears first in the futures market.

1.5.2. MISPRICING

Another issue that has received prominent attention is that of mispricing. The mispricing literature is geared towards the identification of arbitrage opportunities, and thus how good the stock index futures market is at pricing these contracts, and how the mispricing that may arise behaves over time. The starting point for analysing mispricing is based on the notion of the theoretically correct stock index futures price and how this differs from the actual futures price. Typically, the fair (theoretically correct) futures price is calculated from the formula derived by Cornell and French (1983a,b)[35] and is given as[36]

[35] Black (1976) derives a different formula based on the Black-Scholes (1973) option pricing model. Whilst Black uses his model for forward prices, Cox, Ingersoll and Ross (1981) demonstrate that in the presence of nonstochastic interest rates forward and futures prices will be equivalent.

[36] This is rather a simplistic statement of the fair futures price for a stock index futures contract since taxes and the timing option associated with tax payment are ignored. However, Figlewski (1984, p.666) suggests that the magnitude of the tax-timing option 'is probably not large'. In addition, Yadav and Pope (1990) point out that it is probably not a problem in the UK since the tax liability only arises when the position is closed. The evidence in Cornell (1985) seems to confirm the point that tax-timing is negligible. For the pricing formula with taxes see Cornell and French (1983a,b).

$$F^*_{t,T} = S_t e^{r(T-t)} - \sum_{\tau=t+1}^{T} D_\tau e^{r(T-\tau)} \qquad (1.17)$$

where $F^*_{t,T}$ is the theoretically correct stock index futures price quoted at time t for delivery at time T, S_t is the current index value, r is a risk-free interest rate of approximately the same duration as the futures contract has to expiration, (T-t) is the time to expiration and the second term on the left hand side represents the present value of the flow of dividends derived from holding the index portfolio until expiration. Analysis of mispricing then consists of comparing the price calculated from (1.17) with the actual futures prices to see if arbitrage opportunities are available. Defining $F_{t,T}$ as the actual futures price, then if $F^*_{t,T} < F_{t,T}$, the futures contract is undervalued and the arbitrageur would buy futures and sell stock. If the futures contract is overvalued, the reverse trade would be initiated.

In practice, however, these trading strategies will not be so straightforward because of the presence of transactions costs. In one of the first studies of mispricing and the behaviour of futures prices, Modest and Sundaresan (1983) show that the differential will fall between an upper and lower bound determined by transactions costs and as long as the futures price is within these bounds arbitrage will not be profitable and therefore will not take place. The bounds they derive assuming dividends are paid are[37]

$$C_{pl} + C_{fs} \leq F_{t,T} - F^{*}_{t,T} \leq - C_{ps} - C_{fl} \qquad (1.18)$$

where C_{pl} is the cost of being long in the spot index, C_{ps} is the cost of being short in the spot index, C_{fl} is the cost of being long in the futures contract and C_{fs} is the cost of being short in the futures contract. The evidence in Modest and Sundaresan (1983) is that these bounds are infrequently violated such that arbitrage opportunities are few.

MacKinlay and Ramaswamy (1988) analyse mispricing between the S&P 500 Index futures and its fair value using intra-daily data from April 1982 (the start of trading in the S&P 500 contract) to June 1987. MacKinlay and Ramaswamy construct their mispricing series' based on index and futures prices quoted at fifteen minute intervals for each of the March, June September and December contracts in their sample. To assess the significance or otherwise of nonsynchronous trading in their data, MacKinlay and Ramaswamy

[37] Modest and Sundaresan (1983) compare the futures price with the spot price rather than the fair price. However, the argument is still the same.

calculate autocorrelation coefficients for fifteen, thirty, sixty and one hundred and twenty minute intervals. They find that lengthening the time interval reduces the size of the first order autocorrelation coefficient and they interpret this as evidence of a nonsynchronous trading effect. However, they also note that nonsynchronous trading is not the only source of autocorrelation in the index, suggesting that perhaps prices are predictable and the market is inefficient on an intra-daily basis.

The question of interest that MacKinlay and Ramaswamy examine is whether any degree of mispricing that falls outside of the no arbitrage bands determined by transaction costs persists or whether arbitrage activity is sufficient to drive the mispricing back within the no arbitrage bands. This is an important hypothesis, for if arbitrage opportunities persist the implication is that markets are inefficient. If markets are inefficient then the assumption that prices are correct is not warranted and therefore any hedging decisions, for example, may be incorrect and sub-optimal.

MacKinlay and Ramaswamy set their total transaction cost bands at ± 0.6% based on a round-trip stock commission of 0.7%, a round-trip futures commission of

0.08%, a market impact cost[38] of 0.05% in the futures market and a market impact cost of 0.35% in the stock market, with the S&P 500 Index level set at 200. To assess the robustness of the results they also used transaction cost bands of ± 0.4% and ± 0.8% (although they do not report results for these latter bands since the results are apparently unaltered).

The hypotheses of interest in this study are the behaviour of mispricing with time to maturity and the path dependence of mispricing. With regard to the behaviour of mispricing and time to expiration the question they investigate is does the average magnitude of the observed mispricing increase with time to maturity. The reason for analysing this hypothesis is that if average mispricing is dependent upon time to maturity, then the boundaries within which mispricing can fall without triggering arbitrage are not constant, indicating factors other than transactions costs determine arbitrage opportunities the further away the contract is from maturity.

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[38] The market impact cost reflects the impact of a trade on the bid-ask spread. Given that the spread represents a transaction cost in buying and selling, it is important in determining the transaction cost bands.

To test this hypothesis, MacKinlay and Ramaswamy estimate

$$z_{t,T} = \beta_0 + \beta_1 (T-t) + \epsilon_{t,T} \text{ , } z_{t,T} = ABS\left[\sum_{j=1}^{N_t} X_{t,T}(j) / N_t\right] \qquad \textbf{(1.19)}$$

where $X_{t,T}(j)$ is mispricing for the jth quarter-hour observation, N_t is the number of observations in day t and (T-t) is time to maturity of the futures contract. For the sixteen contracts used in their study the estimate of β_1 was positive for fourteen of them and statistically significant for eleven. For the two contracts with negative coefficients, they were small, with the coefficients being insignificantly different from zero. Thus, they find strong evidence to support the hypothesis that the absolute value of mispricing is positively related to time to maturity.

The path dependence argument is one of the behaviour of arbitrage. The path dependence or otherwise of mispricing arises because arbitrageurs have what is termed an unwinding option. This is the situation where arbitrageurs will close out a position taken when mispricing was outside of one of the bounds *before* it reaches the other bound. There are two possibilities (MacKinlay and Ramaswamy (1988, pp.155-156):

i) Mispricing is path independent, following a stochastic process that is pinned on zero at expiration.

ii) Mispricing is path dependent such that the probability of mispricing hitting the upper (lower) bound, conditional on it having hit the lower (upper) bound is smaller.

With i), the probability of mispricing hitting the upper or lower bound having returned to zero after hitting either of the bounds is equal. With ii), it is lower ($p < 0.5$). From their sample, they identify one hundred and forty two cases where mispricing crosses the upper or lower bound, returns to zero and then crosses the upper or lower bound again. Of these, eighty two were situations where mispricing hit the upper bound and 60 were cases where mispricing hit the lower bound. For the upper bound cases, they find that

$$p\ (x \text{ hitting upper} \mid x \text{ hit lower and crossed } 0) = 0.36$$

$$p\ (x \text{ hitting upper} \mid x \text{ hit upper and crossed } 0) = 0.73$$

For the lower bound cases,

$$p\ (x \text{ hitting lower} \mid x \text{ hit upper and crossed } 0) = 0.27$$

$$p\ (x \text{ hitting lower} \mid x \text{ hit lower and crossed } 0) = 0.64$$

MacKinlay and Ramaswamy's results, then, provide clear evidence that for the S&P 500 index futures market, mispricing is positively related to time to maturity, indicating that factors other than transactions costs affect the arbitrage bands, and it is also path dependent, implying that arbitrageurs do tend to exercise the unwinding option available to them.

Using MacKinlay and Ramaswamy's (1988) data, Brennan and Schwartz (1990) also investigate mispricing and the presence of arbitrage opportunities. The innovation of this paper, however, is that they explicitly model the stochastic behaviour of the mispricing series and use the parameter estimates of the stochastic process to examine the behaviour of the profits obtained from a simulated arbitrage strategy. Brennan and Schwartz begin by defining two sets of transactions costs.

The first, which they denote by C_1, is the transaction cost involved in executing the 'simple' arbitrage trade, where the simple long (short) arbitrage position involves a long (short) position in the underlying portfolio and a short (long) position in the futures, held to maturity of the futures contract. If this strategy is followed, then transactions costs consist of two stock commissions, two futures commissions and one market impact cost. The second, denoted by C_2, is associated with the early unwinding of the arbitrage position and the transactions costs

Brennan and Schwartz use for this strategy is simply one market impact cost.[39] They assume that mispricing evolves according to the following Brownian Bridge process (Brennan and Schwartz (1990), p.S12, equation 8)

$$d\varepsilon(\tau) = -\frac{\mu\varepsilon}{\tau}dt + \gamma dz \qquad (1.20)$$

where ε is mispricing, τ is the time to maturity of the futures contract, dz is the increment to a Gauss-Wienèr process (Brownian motion), γ is the instantaneous standard deviation of the increment to the Gauss-Wiener process and μ represents the speed of mean reversion.

The reason why Brennan and Schwartz use this stochastic process is that arbitrage profit has a tendency towards zero (it is mean reverting) and will equal zero upon maturity with probability equal to one, which must happen anyway through institutional arrangements (see section 1.3.2.). By deriving an expression to value the right to unwind the arbitrage position early, and calculating it using the estimated parameters from the Brownian Bridge process, Brennan

[39] Using the transactions costs reported in Stoll and Whaley (1987), Brennan and Schwartz calculate C_1 to be 1.2 index points and C_2 to be 0.5 index points.

and Schwartz compare the actual profit with the theoretical profit predicted by their model, both under transaction cost scenario 1 and 2. They find that their model underpredicts, that is, the actual profit figures are greater than the theoretical value of the arbitrage opportunity. They attribute this to sampling error, the discreteness of the observations on the spot and futures prices and the possible misspecification of the stochastic process.

The latter is the most likely source of the problem with their model for the simple reason that, using the same data set, MacKinlay and Ramaswamy (1988) established the path dependence of mispricing. The Brownian Bridge process is, however, path independent. That misspecification of the stochastic process is the source of the problem with Brennan and Schwartz's model seems to be confirmed by the instability of the estimate of γ, the speed at which mispricing mean reverts, across futures contracts.

Yadav and Pope (1990) analyse mispricing and test for the presence of arbitrage opportunities for the UK markets using daily data on the FTSE 100 Index and the FTSE 100 Index futures contract. They replicate MacKinlay and Ramaswamy's (1988) models to determine whether or not findings from the UK markets are consistent with findings from the US markets. One of the major differences between the US and UK markets that Yadav and Pope (1990) identify is the size of transactions costs, which are typically higher in the UK.[40] In addition, given that different market participants have different transactions costs, Yadav and Pope use transaction cost bands of 0.5%, 1.0%, 1.5% and 2% in determining the presence or otherwise of arbitrage opportunities.

Yadav and Pope find that before the Big Bang of October 1986, mispricing is consistently negative whilst post Big Bang there is a greater tendency for mispricing to reverse back to zero. In addition, mispricing is best described as a stationary AR(1) process, indicating that it may be a mean-reverting, path independent process. They point out that the behaviour of mispricing pre and post Big Bang is consistent with the growth and improvement of the arbitrage sector, though they also note that the mispricing seems too great to be accounted for solely by transactions costs.

In order to investigate the potential for arbitrage profits, they use four trading rules (Yadav and Pope

[40] For example, according to Yadav and Pope (1990, p.579) one extra source of transaction cost is stamp duty, payable by all investors, with the exception of market makers and broker/dealers, at a rate of 0.5% on every purchase transaction.

(1990, pp.590-593) from a simple hold to expiration strategy through to hold with an early unwind option or a roll forward into the next contract option, whichever is the more profitable.

An interesting result that emerges from these *ex post* trading rules is that roll forward and early unwinding strategies generate additional profit which are not eradicated by transaction costs. Yadav and Pope argue (Yadav and Pope (1990, p.593) that this implies substantial transaction cost discount and as such, it should generate arbitrage activity *even if* mispricing falls within the no arbitrage boundaries. It is interesting to note, however, that executing the trading rules *ex ante* substantially *reduces* the arbitrage profit available relative to the *ex post* strategies.

A final issue Yadav and Pope (1990) examine is whether mispricing is path independent or path dependent. To test this hypothesis they estimate two models :

$$X_{t,T} = \alpha + \beta(T-t) + \epsilon_t \qquad (1.21)$$

and

$$|X_{t,T}| = \alpha + \beta(T-t) + \epsilon_t \qquad (1.22)$$

where $X_{t,T}$ and $|X_{t,T}|$ are mispricing and absolute mispricing respectively, (T-t) is the time to expiration of the futures contract and ε_t is a white noise error term. They estimate (1.21) using Beach and MacKinnon's (1978) maximum likelihood method with AR(1) errors, and they estimate (1.22) as a Tobit model, censored at zero. They find evidence from both of these models that supports path dependence, consistent with MacKinlay and Ramaswamy's (1988) results for the US.

However, one must be a little cautious in interpreting these particular results, for both models are misspecified. This is especially evident from (1.21). The residual autocorrelation in this model is most unlikely to be genuine. Rather, given that earlier in their paper they found that mispricing follows a stationary AR(1) process, it probably results from the omission of $(X_{t,T})_{t-1}$ from the model.

In addition, it is well known that static models with AR(1) errors impose non-linear restrictions (the so-called COMFAC restrictions) on more general linear dynamic models. Given that these restrictions have been imposed rather than tested, and given the evidence that mispricing follows an AR(1) process, the conclusion that mispricing is path dependent appears unwarranted.[41]

Yadav and Pope (1992) further investigate the issue of mispricing, in particular the existence of seasonalities, both intra-weekly and intra-daily, using hourly data on the FTSE 100 Index and the FTSE 100 Index futures contract over the period April 26th 1986 to March 23rd 1990.

The aim of this study is to see whether the institutional features and settlement procedures in the UK stock and stock index futures markets contribute to a seasonal pattern in mispricing returns. The reason for analysing this is to determine whether the markedly different trading systems and settlement procedures in the stock and stock index futures market contribute to any observed seasonalities.

There are three major differences between the stock market and stock index futures market in the UK. First, trading in the stock index futures market is open outcry, as opposed to the predominantly screen-

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[41] Note that, although MacKinlay and Ramaswamy (1988) do not report any test statistics for serial correlation, these same criticisms may apply.

based trading system that operates in the stock market (the so-called pure dealership system). Second, settlement in the futures market is undertaken daily with marking to market (see section 1.2.) whereas settlement in the stock market is based on a two (sometimes three) week account period. Third, transactions costs are lower in the futures market and liquidity is higher. The question is do these differences present themselves in the form of seasonalities?

Yadav and Pope (1992) find that this is indeed the case. To summarise their results, the stock market does exhibit seasonality within the settlement period, particularly on the first Monday of the period, a feature which carries over to the futures market. In addition, they find evidence in the stock market that refutes the 'bad news arrives over the weekend' hypothesis, with any abnormal returns on Monday accruing during the trading day rather than over the weekend non-trading period. Several other anomalies suggest that trades based on mispricing can yield positive abnormal returns. In particular, two observations stand out : systematic falls in the UK markets when the US markets open, and a tendency for prices in the stock market to rise whilst the market is open whilst prices in the futures market rise while the market is closed. This latter result seems somewhat surprising. However, in this situation Yadav and Pope show that a long futures-short cash hedge yields significant positive returns during trading periods and a short cash-long futures yields significant positive returns during non-trading periods.

A final paper we consider is that of Chung (1991). Chung's paper is of interest because he casts serious doubts on the conclusions reached in other studies. The reasons for such potentially false conclusions are twofold. First, there is no allowance for an execution lag in the arbitrage trade. The second reason relates to Chan's (1992) critique of tests of lead-lag relationships, that is, failure to take proper account of the fact that the quoted value of the index is not necessarily synonymous with its true value.

The reason why these issues are ignored, Chung argues, is because the issue under consideration is not correct. The issue to be investigated is not the size and frequency of violations of the no arbitrage boundaries. It is the size and frequency of profitable arbitrage opportunities.[42] As Chung (1990,

[42] Note that this particular point is not applicable to Yadav and Pope (1990) for they expressly test for profitable arbitrage opportunities.

p.1792) observes,

> 'A market efficiency test should be carried out as an ex ante test to see the extent to which arbitrageurs can make positive ex ante profits after observing ex post mispricings. What appears ex post as a riskless profit opportunity is not necessarily a real ex ante exploitable profit opportunity because there is no guarantee that the prices at the next available transaction will still be favourable to the arbitrageur.'

This is an important point, for as long as there are no *ex ante* profitable arbitrage opportunities, the presence of *ex post* ones seems to be of less importance. In addition, the presence and persistence of apparent ex post opportunities may not be indicative of inefficiency at all but rather indicative of model inadequacy in terms of the misspecification of the no arbitrage boundaries.

To overcome the shortcomings of other studies, Chung uses minute by minute prices of individual stocks that comprise the MMI and of the MMI Index futures contract in the construction of the mispricing series. Chung conducts all of his tests using ex ante arbitrage trading strategies, allowing for various execution lags (twenty seconds, two minutes and five minutes) and various transaction cost scenarios (0.5%, 0.75% and 1.0%).

By approaching the issue in this way, Chung finds that previous studies have overestimated the size, persistence and frequency of arbitrage opportunities. Moreover, the persistence of profitable ex ante opportunities have declined as the stock index futures market has matured. The overall conclusion Chung reaches is that there are some profitable arbitrage opportunities available, but not nearly as many as previous studies tend to suggest. Moreover, those profits that are available cannot be solely attributed to inefficiency because profits realised from the arbitrage strategies are not riskless.

1.6. SUMMARY AND CONCLUSIONS

In this chapter, we set out to analyse the nature of futures contracts in general, and stock index futures contracts in particular, looking at economic justifications for their existence. Futures contracts represent a deferred claim on an asset and in this sense they are the same as forward contracts. However, futures contracts do offer several advantages over forward contracts, such as a substantial reduction in transaction costs through such factors as the virtually complete removal of default risk.

From an economic viewpoint, we demonstrated in the context of Ross' (1976) framework for completing markets using options that stock index futures markets are a powerful addition to any economy by the fact that, theoretically at least, they complete markets and thus substantially reduce risk. Even in this case where markets are not complete, stock index futures can complete them if stock index futures prices are perfectly positively correlated with the stock index price. This is because in their role as instruments for hedging and diversification, the ability of investors to short stock index futures allows them to construct risk-free portfolios.

The fact, then, that stock index futures can complete markets, not just theoretically but potentially in practice as well, is a powerful economic justification for their existence. Moreover, it makes them worthy of systematic investigation, both at the theoretical level and the empirical level.

In terms of the literature analysing the behaviour of stock index futures, two growth areas, indeed areas that are beginning to predominate the stock index futures literature, are the analysis of lead-lag relationships between the stock and stock index futures market and the analysis of mispricing of the stock index futures contract. The former is concerned with whether prices in one market lead prices in the other. This issue is founded on the notion that because trade in stock index futures is less costly and there are less frictions in the stock index futures market, for example short sales are allowed, information should manifest itself first in the stock index futures price. Therefore, given that, in principle, stock and stock index futures prices track each other very closely (they must do otherwise futures would not fulfil their functions discussed earlier), the information will be transmitted from the futures to the stock market and thus the futures price leads.

The issue is, however, not as clear cut as it may seem. First, it is entirely feasible that, at least empirically, a feedback relationship exists such that the stock index price also leads the stock index futures price.

Second, and more important, is how are the results influenced by nonsynchronous trading in the stock index? Nonsynchronous trading is the situation where not all stocks within an index trade in any given trading interval. As a result, the lag in their reaction to information generates serial correlation in observed returns on the index. The problem is that it is not genuine serial correlation one is observing. It is serial correlation that arises because of the way the index is constructed. Therefore, failure to account for it leads to invalid inference. As such, before analysing lead-lag relationships the index must be adjusted for nonsynchronous trading.

The mispricing literature is concerned with analysing whether or not deviations of the futures price from its fair price are of sufficient size and persistence to allow arbitrageurs to trade profitably. The reason for analysing this is that it allows an analysis of the efficiency of the market, with sustained, profitable arbitrage opportunities being indicative of systematic mispricing and hence inefficiency.

There are several objections that can be raised with regard to the analysis of lead-lag relationships and mispricing and we address these in the following chapters. First and foremost is the way these two issues are analysed, both conceptually and empirically. If one were to read a paper on lead-lag relationships, then it would be difficult to conceive that there is another equally important area of the pricing relationship. The same applies in reading papers on mispricing : the two subject areas tend to be treated entirely independently of each other. This is problematic to say the least, for they are far from independent. Indeed, the literature on mispricing has a great deal to contribute to the analysis of lead-lag relationships.

Accordingly, in the next chapter we unify these two apparently diverse strands of the literature in a coherent, error correction framework. This is where mispricing contributes, for the literature on theoretically correct stock index futures prices actually provides us with the error correction mechanism. As it happens, the natural error correction mechanism is the futures to cash price differential, known in the parlance of futures markets as the basis, which we show to be equivalent to mispricing of the stock index futures contract. The natural result of analysing the pricing relationship in this framework is that the issue of lead-lag relationships and mispricing become the single issue of effectively functioning equity markets, an issue that has become so much more important since the October 1987 stock market crash. We show how effective price functioning can be tested objectively within this framework and we distinguish between two categories of effective functioning.

Having demonstrated how it is possible to test for effectively functioning equity markets, we show how this framework can be used to provide an objective test of the efficiency of both the stock and stock index futures markets. In fact, we show that efficiency and effective functioning are virtually synonymous. We also discuss the contribution the analysis of effectively functioning equity markets can make to the vexed issue of the behaviour of mispricing discussed above. In chapter three we utilise this framework to test whether the stock and stock index futures markets in the UK are effectively functioning using daily data on the FTSE 100 Index and FTSE 100 Index futures contract.

In chapter four we move on to consider the issue of nonsynchronous trading and how it may be estimated. We propose a new method for estimating the nonsynchronous trading effect which is more intuitively appealing and easier to implement than extant methods. We use this method to construct a new Index and, using the new Index, we analyse the intra-daily pricing relationship over a stable period to investigate whether markets can be said to be effectively functioning on an intra-daily basis.

In chapter five, we use the framework developed in chapter two and the method for estimating the nonsynchronous trading adjustment in chapter four to examine the minute by minute pricing relationship on October 19th and 20th 1987, the time of the stock market crash. The value of analysing the effective functioning of equity markets as opposed to lead-lag relationships and mispricing is demonstrated once more in this chapter, for using this framework we are able to determine the cause of the massive downward spiral in prices that occurred on the 19th and the reason why they stabilised somewhat on the 20th. Chapter six summarises and concludes the thesis.

Chapter Two : A Framework For Modelling The Pricing Relationship Between Stock Index Futures And The Underlying Stock Index

2.1. Introduction

The establishment of the FTSE 100 stock index futures contract in May 1984, coupled with the FTSE 100 option contract, offered investors a much greater degree of flexibility in the construction of their investment portfolios and in the timing of transactions associated with such portfolios. With the emergence of such markets worldwide, there is a growing body of literature, primarily concerned with the stock index futures contracts in the United States (especially the S&P 500 index futures contract), examining the pricing relationship between the stock and stock index futures markets (see *inter alia* Kawaller, Koch and Koch (1987), MacKinlay and Ramaswamy (1988), Stoll and Whaley (1990) and Chan (1992)).[1] Much of this examination of the pricing relationship has been

[1] Given the much wider availability of finer data bases in the US, many of the studies that examine the pricing relationship use intra-day data over long periods of time. For example, Stoll and Whaley (1990) use prices quoted at five minute intervals from April 1982 to March 1987. Unfortunately, such data over reasonable periods of time is not widely available in the UK and thus we are restricted to using daily data. Nevertheless, this does not diminish the arguments that will follow.

concerned with identifying the lead-lag relationship between prices in the two markets to try and determine which market, if either, reacts to new information first. In this chapter we focus on the nature of the pricing relationship between the two markets, arguing that the focus of those studies of the US stock index futures markets are inappropriate and flawed, both in the way they approach the issue conceptually and in the econometric methods they employ to test their proposed models.

The rest of this chapter is organised as follows. The second section discusses the nature of lead-lag relationships and how they might arise. Section three focuses on the 'traditional' method of estimating and testing for lead-lag relationships and points out the deficiencies with such an approach. A new and alternative framework and method for addressing the question of lead-lag relationships is proposed in section four. This framework demonstrates that the issue to be examined is one of whether equity markets function effectively. In section five, we link this framework to the issue of market efficiency, suggesting that tests of efficiency should be conducted in the framework of effectively functioning equity markets. In section six, we focus our attention on the behaviour of mispricing, using the framework proposed in this chapter to argue that it is a path independent, stationary, mean reverting stochastic process. Section seven concludes.

2.2. THE NATURE OF LEAD-LAG RELATIONSHIPS

The argument that underlies the analysis of lead-lag relationships between indices and index futures is predicated on the observation that this relationship is indicative first of how well integrated the markets are and second of how quickly the markets reflect the arrival of new (and relevant) information relative to each other. If markets were perfect and investors fully rational with costless and equal access to the same information set then as Zeckhauser and Niederhoffer (1983) point out, it is not unreasonable to assume that stock index futures prices would carry no predictive information and would therefore have no role to play.[2] However, the existence of transactions costs and other imperfections ensure that stock index futures do have a role to play because in this situation, they will convey relevant information about future movements in the stock index.

There are several reasons as to why this may be the

[2] The argument here is that if markets were perfect and investors fully rational then because markets would be frictionless, the spot price would contain all the relevant information and investors could buy and sell whole portfolios given the absence of transactions costs.

case. One intuitive reason is similar to Black's (1975) analogy concerning the role of option contracts in the provision of relevant information for the underlying asset. Futures markets are very liquid with relatively low transactions costs. Moreover, investing in a futures contract requires no capital outlay since the margin can be posted in the form of interest-bearing securities and as such there is no opportunity cost. Thus, suppose an investor acquires new information on the health of the economy, say, that is worth acting upon. The investor has to decide whether to purchase stocks or a stock index futures contract. Purchase of the stocks requires a substantial amount of capital, a substantial amount of time and relatively substantial transactions costs. Purchase of the index futures contract, on the other hand, can be affected immediately with little up-front cash. Therefore, if the investor is willing to trade in futures, the futures transaction is the one to choose. The information will be incorporated in the futures price, driving it upwards. This will widen the differential between the futures and spot price which in turn will attract arbitrageurs. Since arbitrageurs trade simultaneously in cash and futures markets the information will be transmitted from the futures to the cash market. Thus, the futures price will lead the cash price.

Other reasons as to why the futures will lead the cash stem from institutional arrangements such as short-sale restrictions that are present in the cash market but not in the futures market. In this setting, Diamond and Verrechia (1987) demonstrate that prices will be slower to adjust especially to bad news if traders who have private information are not allowed to short the security/securities. Such constraints are not present in the futures market, hence traders can short the futures contract. This will drive the futures price down, narrowing the differential between spot and futures prices and again attracting arbitrageurs. The futures price will thus lead the cash price. The relationship will, of course, not be as one-sided as it appears from the above discussion. A stock index futures price will tend to react to economy-wide information[3] as opposed to security-specific information. Thus, information concerning a specific security or group of securities may cause the cash market to lead the futures market, such that a (potentially complex) feedback relationship exists. This recognition that the futures price should lead

[3] This actually serves to enhance the justification for the existence of stock index futures, especially given the increasing body of evidence that macroeconomic variables can help predict stock market returns, violating market efficiency. If the stock index futures price reacts first to new information contained in these variables then this can only help improve the efficiency of the underlying stock market.

the stock price has formed the basis of a great deal of empirical work geared to testing this very proposition (see for example Kawaller, Koch and Koch (1987), Stoll and Whaley (1990) and Chan (1992)). However, as we shall see, these studies are flawed and the results that they generate are potentially misleading.

2.3. Testing Lead-Lag Relationships : The 'Traditional' Approach.

Typically, tests of lead-lag relationships in the extant literature are similar in spirit to Granger-Sims-type causality tests (see Granger (1969) and Sims (1972)). The model that is usually estimated is of the following form :

$$\Delta S_t = \alpha_0 + \alpha_i \sum_{i=-k}^{i=+k} \Delta F_{t-i} + u_t \tag{2.1}$$

where ΔS_t is the change in the spot price, ΔF_t is the change in the futures price and u_t is the usual white noise error term. Tests of the lead-lag relationship then consist of testing the significance of the lag and lead coefficients on the futures prices. If the lags are significant and the leads are zero, the futures leads the spot. If the opposite is true then the spot leads the futures. If some of both the lead and lag coefficients are statistically non-zero, then a feedback relationship exists.

There are, however, two important and inter-related criticisms that can be addressed to the 'traditional' method of testing lead-lag relationships. One is concerned with the estimation of, and inference about, models such as (2.1) and the second is concerned with the specification of such models. To formalise matters, first note that whilst theory models suggest that an asymmetric feedback relationship is likely to exist, they give little guidance about the nature and form this asymmetry takes. Thus, models such as (2.1) are inevitably *statistical* models within which what effectively amounts to Granger-Sims causality tests are undertaken. The method of estimation in this context becomes vitally important if valid inference is to be sustained. This is one of the criticisms that can be levelled at Stoll and Whaley (1990) who estimate (2.1) by Ordinary Least Squares, immediately casting doubt on their results. To see why this is the case, consider the following, very general, data generation process (DGP) describing the joint density of *all* variables[4] (see, *inter alia*, Hendry and Richard

[4] This approach to econometric modelling has come to be known as the LSE approach following the work of Sargan and its formalisation and extension primarily by Hendry (see, for example, Hendry and Richard (1983)). For a very readable exposition of this approach, see Gilbert (1986).

(1982, 1983) and Hendry, Pagan and Sargan (1984)) :

$$\prod_{t=1}^{T} D(\boldsymbol{x}_t | \boldsymbol{X}_{t-1}; \boldsymbol{\Theta}) \tag{2.2}$$

where $\mathbf{x}_t$ is a vector of observations on all variables in period t, $\mathbf{X}_{t-1} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{t-1})'$ and Θ is a vector of unknown parameters. This density function is far too general to be useful. However, the vector $\mathbf{x}$ can be partitioned into those variables of interest and those of no interest. We then have (Ericsson (1992))

$$D(\boldsymbol{x}_t | \boldsymbol{X}_{t-1}; \boldsymbol{\Theta}) = D_1(\boldsymbol{y}_t | \boldsymbol{Y}_{t-1}, \boldsymbol{Z}_t; \boldsymbol{\lambda}_1) D_2(\boldsymbol{z}_t | \boldsymbol{Z}_{t-1}, \boldsymbol{Y}_{t-1}; \boldsymbol{\lambda}_2) \tag{2.3}$$

where D_1 is the density function of those variables of interest that are selected as endogenous, $\mathbf{y}_t$, which are conditioned on lagged $\mathbf{y}$ and current and lagged values of the exogenous variables, $\mathbf{Z}$. D_1, then, represents the conditional model. D_2 is the density function of those variables of interest that are deemed to be (at least weakly) exogenous, $\mathbf{z}_t$ (the marginal model).

The crucial issue now with regard to estimation is the partitioning of density function of the DGP with respect to the conditional density for y_t and a marginal density for the exogenous variables. Define $\lambda = (\lambda_1', \lambda_2')'$ in (2.3) as the parameters of the conditional and marginal models and Λ_1 and Λ_2 as the parameter spaces for λ_1 and λ_2. For the conditioning assumptions (that is, conditioning $\mathbf{y}_t$ on current and lagged values of $\mathbf{z}_t$) to be valid, and for efficient estimation and inference using OLS, we require that the variables $\mathbf{z}_t$ be weakly exogenous for the endogenous variables of interest. For weak exogeneity, we require a sequential cut such that parameter space Λ is the product of Λ_1 and Λ_2 and $\lambda \in \Lambda_1 \times \Lambda_2$. In addition, we require that the parameters of interest are a function of λ_1 alone. For the purpose of inference, if $\mathbf{z}_t$ is weakly exogenous for the parameters of interest then the marginal model for $\mathbf{z}_t$ does not need to be estimated since it contains no information relevant to the estimation of the conditional model for $\mathbf{y}_t$. This is what is required for Stoll and Whaley's (1990) results to be valid, for by implication they have ΔS_t being equivalent to y_t and ΔF_t being equivalent to $\mathbf{z}_t$. However, it is well recognised in the literature that spot and futures prices are simultaneously determined[5] and therefore, in principle, weak exogeneity *does not hold*, though it is testable.

It is also illuminating to consider the role of

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[5] This is especially recognised in the literature on hedging (see, for example, Stein (1961) and Anderson and Danthine (1981)). Garbade and Silber (1983) also consider the simultaneity of spot and futures prices in their model of price dynamics. We will return to this point later.

Granger non-causality in this context. Granger non-causality is related to *marginalisation* as opposed to conditioning in so far as, in the context of marginalising the DGP, Granger non-causality relates to strong exogeneity, whereby strong exogeneity is weak exogeneity plus Granger non-causality of $\mathbf{Y}_{t-1}$ for $\mathbf{z}_t$. Strong exogeneity is necessary if we wish to make valid forecasts of $\mathbf{y}$ using the conditional model, given forecasts of $\mathbf{Z}$ from the marginal model. Moreover, in this context Granger causality tests are only meaningful if we have weak exogeneity. Thus, as far as the results of Stoll and Whaley (1990) are concerned, they must be viewed as suspect.

It is possible to argue that the approach adopted by Chan (1992) overcomes this problem since he estimates (2.1) using Hansen's (1982) Generalised Method of Moments (GMM), thereby allowing for simultaneity. However, it is illuminating to consider Chan's (1992) reasons for using GMM : it is to correct for serial correlation and heteroscedasticity in order to provide consistent standard errors for the purposes of inference. This suggests that whilst, indirectly, Chan (1992) goes some way to recognising the simultaneity problem, the need for valid conditioning is completely ignored, hence the need to 'correct' serial correlation, which in itself is suspect given that it most likely arises not because it is genuine but because the dynamics of the model are misspecified. The model estimated also imposes common unit restrictions (which are not tested) and which may therefore be invalid (see Hendry and Mizon (1978)). This leads to the second concern regarding the specification of models such as (2.1).

As far as the specification argument is concerned, leaving the issue of exogeneity to one side for a moment, (2.1) is incorrectly specified. This arises through invalid conditioning, primarily through the exclusion of, when the system is written out in full, ΔS_{t-i} and ΔF_{t-i}, i=1...k, without these zero restrictions being tested (see equation (2.16) and the discussion that follows it). This essentially *forces* the spot and futures prices individually to be well approximated by martingale sequences which, *a priori*,[6] may or may not be true. The fact that one may draw on arguments of weak form efficiency do not in themselves stand up unless weak form efficiency, which is a testable proposition in itself, is verified. There is, however, no *a priori* empirical reason why markets should be weakly efficient *all* of the time.

[6] Such an exclusion would probably be defended on the grounds of market efficiency. However, such a defence is dubious when intra-day data is employed, especially when the autocorrelation properties of the spot series reported in some of the above-mentioned studies tend to support the data following an AR(p) process rather than being white noise.

Perhaps more important here, however, is the nature of the interaction between spot and futures markets and the effect this has on the specification of models such as (2.1). The reason for such specification problems stems from the fact that in considering the pricing relationship between stock index futures markets and the underlying stock market, two quite distinct and seemingly independent strands have emerged in the literature : those studies that analyse mispricing by comparing the actual futures price with its fair, or theoretically correct, value to determine whether profitable arbitrage opportunities are available (see *inter alia* MacKinlay and Ramaswamy (1988), Yadav and Pope (1990) and Chung (1991)) and those that analyse the lead-lag relationship between the two markets (Kawaller, Koch and Koch (1987), Harris (1989) and Stoll and Whaley (1990)). Most studies tend to focus on either the former or the latter issue, but not both.

This is where the specification problems arise for rather than being apparently independent areas of investigation, the former, that is, mispricing, provides some valuable insights into the likely behaviour of lead-lag relationships and indicates that, in addition to those points mentioned above, results from studies of the lead-lag relationship must be viewed with some caution. To demonstrate, consider two commonly used and well known theoretical models showing the relationship between the stock index futures price and the underlying stock index portfolio. First, we have (see Cornell and French (1983a,b))

$$F^*_{t,T} = S_t e^{r(T-t)} - \sum_{k=t+1}^{T} D_k e^{r(T-k)} \qquad (2.4)$$

where $F^*_{t,T}$ is the fair or, equivalently, the theoretically correct stock index futures price quoted at time t for delivery at time T, S_t is the value of the underlying stock index (spot portfolio), r is a riskless interest rate of approximately the same duration as the time to expiration of the futures contract and D is the daily dividend inflow from the portfolio until maturity of the stock index futures contract. Alternatively, we can consider the following model (MacKinlay and Ramaswamy (1988))

$$F^*_{t,T} = S_t e^{(r-d)(T-t)} \qquad (2.5)$$

where $F^*_{t,T}$ and S_t are defined as above, r is the risk-free rate of interest, d is the yield on dividends from the underlying portfolio and (T-t) is the time to maturity of the futures contract. The expression (r-d)(T-t) is generally referred to as the cost of carrying the spot portfolio until maturity.

Now, studies that analyse mispricing and the existence of arbitrage opportunities typically compare the differential between the actual futures price quoted at time t for delivery at time T, $F_{t,T}$, with the fair futures price, $F^*_{t,T}$.

However, it is straightforward to demonstrate the role of the simple basis[7] in this analysis. For ease of exposition, we will work with (2.5), though similar arguments follow for (2.4). The theoretical basis, $F_{t,T} - F^*_{t,T}$, is compared with transactions costs to determine if arbitrage opportunities are present.[8] If the theoretical basis falls outside of the no arbitrage window determined by transactions costs then dependent on whether the futures contract is undervalued (overvalued) due to, say, bearish (bullish) speculation in the stock index futures market, arbitrageurs will buy (sell) futures and sell (buy) stocks. It is clear that the theoretical basis is very important in the pricing relationship given that index arbitrage links the two markets and the theoretical basis determines whether arbitrage

[7] One must be careful in talking about the basis for there are several definitions. Where there may be confusion, we will refer to the futures to cash price differential as the simple basis. When there is no risk of confusion, we will refer to it as the basis. The futures to fair price differential will be referred to as the theoretical basis.

[8] Note that since the yield on dividends is typically less than the yield on the riskless asset, the basis should theoretically be positive.

opportunities are available.

To see the importance of the basis itself in the pricing relationship, take natural logs of (2.5) (lower case letters denote variables in natural logarithms) :

$$f^*_{t,T} = s_t + (r-d)(T-t) \tag{2.6}$$

Clearly, if the futures market is pricing the stock index futures contract correctly then we have that

$$f_{t,T} - f^*_{t,T} = 0 \tag{2.7}$$

Now, to see the importance of the basis in the pricing relationship, substitute (2.6) into (2.7) and rearrange to obtain

$$f_{t,T} - s_t = (r-d)(T-t) \tag{2.8}$$

It is clear from (2.8) that the simple basis also has an important role to play in the arbitrage process. It is also apparent that upon expiration (t=T), the basis will equal zero whilst with non—expiration it will, theoretically, equal the cost of carry, though if the contract is near to maturity carrying costs become trivial. From a theoretical viewpoint, the basis is crucial given that arbitrage provides an important link between the two markets. From an econometric point of view, the basis also has the rather appealing interpretation as the error correction mechanism which prevents prices in the two markets drifting apart without bound. The importance of the basis cannot be understated, for as Harris (1989, p.77) points out,

> 'The (simple) basis is studied because it is a key determinant of whether arbitrage opportunities exist, because variance in the basis is a measure of how well integrated the markets are, and because the basis is related to tests for causality among the prices in the two markets.'

The problems with the specification of (2.1) can now be made clear and can be redressed.

2.4. FORMULATING TESTS OF THE LEAD-LAG RELATIONSHIP AND THE ISSUE OF EFFECTIVELY FUNCTIONING MARKETS

To recap the problems mentioned earlier, the results from extant tests must be viewed with extreme caution given the problems of invalid conditioning and in some cases (implicit) invalid (and untested) exogeneity assertions. To combat these problems, it is important to note that, essentially, (2.1) is a VAR (Vector

AutoRegression). If (2.1) is treated as a VAR, then any empirical model of lead-lag relationships must be viewed as a *system*. Treating (2.1) as a VAR has important implications for the marginalisation of the DGP, for now we require any non-modelled variables to be *strongly* exogenous. However, in analysing lead-lag relationships, the variables under consideration are the stock index futures price and the value of the stock index, which are both jointly determined. Thus, there are no exogenous variables and as such the DGP becomes

$$D(\boldsymbol{x}_t|\boldsymbol{X}_{t-1};\boldsymbol{\Theta}) = D(\boldsymbol{y}_t|\boldsymbol{Y}_{t-1};\boldsymbol{\theta}) \tag{2.9}$$

where $\mathbf{y}_t' = [f_t':s_t']$. Clearly, if there are no exogenous variables then, in conjunction with the conditioning assumptions, the system will be closed and it will have a VAR representation. Any tests of lead-lag relationships must then be undertaken within the VAR framework.

Within the VAR framework, tests of the lead-lag relationship then become ones of testing zero restrictions on the VAR in order to arrive at a parsimonious representation of the VAR. As proposed by Hendry and Mizon (1990),[9] only once a parsimonious parameterisation of the VAR has been achieved can structural models be considered and any *valid* structural model should encompass the VAR, the encompassing test being a test for the validity of the overidentifying restrictions (Hendry and Mizon (1990)).

Consider the following closed system where $\mathbf{y}_t$ is an Nx1 vector of endogenous time series variables :

$$\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\Pi}_1 \boldsymbol{y}_{t-1} + \boldsymbol{\Pi}_2 \boldsymbol{y}_{t-2} + \ldots + \boldsymbol{\Pi}_p \boldsymbol{y}_{t-p} + \boldsymbol{u}_t \qquad (2.10)$$

where the Π_i are NxN coefficient matrices, μ is a vector of deterministic components (e.g. deterministic constants) with $\mathbf{u}_t$ being random disturbances with mean zero and variance-covariance matrix Σ. Using lag operator notation, we can rewrite (2.10) as

$$\boldsymbol{\Pi}(L)\boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{u}_t \qquad (2.11)$$

where (L) is the lag operator and Π is a p-th order matrix polynomial with $\Pi_0 = \mathbf{I}_N$, an NxN identity matrix.

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[9] See also Mizon and Clements (1991) and Clements (1991). Monfort and Rabemananjara (1990) discuss similar proposals for VAR models in stationary variables.

Consider traditional tests of lead-lag relationships. These are conducted in the context of variables in first differences and thus we have

$$A(L)\Delta y_t = \mu + u_t^* \qquad (2.12)$$

where Δ is the first difference operator. It can be clearly seen that (2.12) is equivalent to the 'non-structural' formulation of lead-lag tests. First differences are often used because the series are stationary and, conveniently, log first differences gives the variables in returns form. However, consider the following definitions provided by Engle and Granger (1987, pp.83-84) :

i) 'A series with no deterministic component which has a stationary, invertible ARMA representation after differencing d times is said to be integrated of order d, denoted $y_t \sim I(d)$.[10]

ii) The components of the vector $\mathbf{y}_t$ are said to be cointegrated of order d,b, denoted $\mathbf{y}_t \sim CI(d,b)$, if (i) all components of $\mathbf{y}_t$ are I(d); (ii) there exists a vector α ($\neq 0$) so that $\mathbf{z}_t = \alpha'\mathbf{y}_t \sim I(d-b)$, $b>0$. The vector α is called the cointegrating vector.'

[10] That is, for some p and q y_t will belong to the ARIMA (p,d,q) class of models proposed by Box and Jenkins (1970).

Clearly, definition i) provides us with (2.12), the 'quasi-unrestricted'[11] model used in the 'traditional' tests of lead-lag relationships discussed above. However, 'traditional' tests ignore the second definition and it is this, in conjunction with invalid conditioning, that leads to specification problems.

An important case with respect to cointegration is when d=b=1. When d=b=1, then from the Granger Representation Theorem (Engle and Granger (1987)) if both definitions i) and ii) hold there exists an error correction representation of the VAR, given the natural isomorphism between cointegration and error correction models. Now, if both the spot and the futures prices are nonstationary, or I(1) in the terminology of Engle and Granger (1987), then their first differences will be I(0), that is, stationary. However, from definition ii), it is possible that spot and futures prices cointegrate, that is, a linear combination of the two I(1) series will be I(0). In fact, from the mispricing literature we know that the cointegrating vector α should equal 1, that is, the basis should be the error correction mechanism.

[11] We refer to the model in first differences as the quasi-unrestricted model for such a model still imposes common unit restrictions. We use the term quasi-unrestricted to denote the fact that zero restrictions on the coefficients of lagged dependent variables are not imposed.

The implication of this is clear : models such as (2.1) or (2.12) will be misspecified unless there is no cointegration or the error correction term is insignificant in *all* equations. Estimation of the VAR in first differences without taking into account the information provided by theory about the long-run equilibrium between the two markets will be misspecified and hence any test of lead-lag relationships through the testing of zero restrictions will be invalid, as will any use of the VMA (Vector Moving Average) representation of (2.12) to analyse impulse response functions. Thus, rather than using (2.12), (2.11) should be reparameterised in error correction form as (Johansen (1988) and Johansen and Juselius (1990)) :

$$\Delta \boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\Gamma}_1 \Delta \boldsymbol{y}_{t-1} + \ldots\ldots + \boldsymbol{\Gamma}_{k-1} \Delta \boldsymbol{y}_{t-k+1} + \boldsymbol{\Pi} \boldsymbol{y}_{t-k} + \boldsymbol{u}_t \qquad (2.13)$$

where the lagged levels term $\mathbf{y}_{t-k}$ represents the short-run equilibrium error, or deviations from the long-run equilibrium, and of which at least one element must be non-zero. The information about the impact of the equilibrium error is provided by Π. Johansen (1988) and Johansen and Juselius (1990) factor the long-run response matrix into $\Pi = \alpha\beta'$ where β' is the matrix of cointegrating coefficients such that $\beta'\mathbf{y}_{t-k} \sim I(0)$ and α is the matrix of adjustment coefficients. Note that the matrix Π will have reduced rank if any of the variables in the vector $\mathbf{y}$ are cointegrated and this forms the basis of the test for cointegration proposed by Johansen (1988). If we denote rank(Π) by r, there are three possibilities. First, r=0 in which case all of the variables are I(1) and there are no cointegrating vectors. Second, r=N in which case all of the variables are I(0) and there are N cointegrating vectors since any linear combination of stationary variables is in itself stationary. Finally, 0<r<N in which case there will be r linear combinations of the nonstationary variables that are stationary. Equivalently, there will be N-r common stochastic trends (Stock and Watson (1986)).

That the issue of cointegration is ignored might seem surprising. However, as already mentioned the issues of mispricing and lead-lag relationships are incorrectly analysed independently of each other. Nowhere is this more clear than the frequently-used model of simultaneous cash-futures price dynamics proposed by Garbade and Silber (1983). This model typically provides the foundation for analysing which market dominates the other and how mispricing behaves (see for example Yadav and Pope (1990)). Garbade and Silber's (1983) final estimable model is given by

$$\begin{bmatrix} f_t \\ s_t \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \end{bmatrix} + \begin{bmatrix} (1-\alpha_{11}) & \alpha_{11} \\ \alpha_{12} & (1-\alpha_{12}) \end{bmatrix} \begin{bmatrix} f_{t-1} \\ s_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \qquad \textbf{(2.14)}$$

where f_t is the log of the futures price, s_t is the log of the spot price and the u_{it} are random error terms.[12] Rewriting this model with the restrictions imposed yields

$$\begin{bmatrix} \Delta f_t \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \end{bmatrix} + \begin{bmatrix} -\alpha_{11} \\ \alpha_{21} \end{bmatrix} \left[f_{t-1} - s_{t-1} \right] + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \qquad \textbf{(2.15)}$$

(2.15) is nothing more than a restricted VAR(1), the restriction being long-run homogeneity in both equations of the system. Clearly, this (modified) model shows explicitly the intimate ties between mispricing and the pricing relationship between the two markets, although this connection has been ignored in the extant literature. Making use of the VAR, we can also see a further problem with the traditional approach to testing lead-lag relationships. Consider the following typical 'structural' models that are estimated, viz. (2.1), with k set equal to 1 for ease

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[12] Garbade and Silber (1983) actually derive a model for the relationship between the *fair* futures price and the spot price. However, the discussion in section 2.1.1 shows that the interesting relationship is the one between the *actual* futures price and the spot price.

of exposition :

$$\begin{aligned} \Delta f_t &= \alpha_{10} + \alpha_{11}\Delta s_t + \alpha_{12}\Delta f_{t-1} + \alpha_{13}\Delta s_{t-1} + u_{1t} \\ \Delta s_t &= \alpha_{20} + \alpha_{21}\Delta f_t + \alpha_{22}\Delta f_{t-1} + \alpha_{23}\Delta s_{t-1} + u_{2t} \end{aligned} \qquad (2.16)$$

When equation (2.1) is written out in full as a system, it becomes clear that in addition to the specification problems already discussed, particularly the omission of any cointegrating vectors, the models used *are not identified* since the equations differ only by normalisation and as such cannot be distinguished. Seen in this light, the results in Chan (1992), who indirectly recognises simultaneity, must be viewed as suspect. If we solve the equations in (2.16) for their respective reduced forms then we obtain the following estimable just-identified representation of the system

$$\begin{aligned} \Delta f_t &= \gamma_{10} + \gamma_{11}\Delta f_{t-1} + \gamma_{12}\Delta s_{t-1} + v_{1t} \\ \Delta s_t &= \gamma_{20} + \gamma_{21}\Delta f_{t-1} + \gamma_{22}\Delta s_{t-1} + v_{2t} \end{aligned} \qquad (2.17)$$

where

$$\gamma_{10} = \frac{\alpha_{10} + \alpha_{11}\alpha_{20}}{1 - \alpha_{11}\alpha_{21}} \qquad \gamma_{11} = \frac{\alpha_{12} + \alpha_{11}\alpha_{22}}{1 - \alpha_{11}\alpha_{21}} \qquad \gamma_{12} = \frac{\alpha_{13} + \alpha_{11}\alpha_{23}}{1 - \alpha_{11}\alpha_{21}}$$

$$\gamma_{20} = \frac{\alpha_{20} + \alpha_{21}\alpha_{10}}{1 - \alpha_{21}\alpha_{11}} \qquad \gamma_{21} = \frac{\alpha_{22} + \alpha_{21}\alpha_{12}}{1 - \alpha_{21}\alpha_{11}} \qquad \gamma_{22} = \frac{\alpha_{23} + \alpha_{21}\alpha_{13}}{1 - \alpha_{21}\alpha_{11}}$$

Estimation of models of the lead-lag relationship by OLS, as in Stoll and Whaley (1990) is only justifiable in the context of (2.17), unless there is weak exogeneity. However, (2.17) is equivalent to a VAR in first differences and as such is subject to exactly the same critique discussed earlier, that is, the misspecification problems generated by the omission of any cointegrating vectors.

It is also useful to consider the role of Sims' (1972) interpretation of Granger causality tests in this context as tests for exogeneity, required if interpretations of 'structural' models such as (2.1) or (2.16) are too have any meaning (cf. the results of Stoll and Whaley (1990)). Cooley and LeRoy (1985) point out that Granger causality tests *cannot* be interpreted as tests of exogeneity or predeterminedness in the usual sense of systems of equations and as such any defence of models such as (2.1) or (2.16) on these grounds are invalid. The argument is as follows. *For the futures price to be* strictly exogenous, say, in (2.16) we require that $\alpha_{11}=\alpha_{13}=0$ whereas if $\alpha_{13}\neq0$, the futures price is predetermined. In the context of the reduced form (2.17) Granger non-causality requires $\gamma_{12}=0$. From the definition of γ_{12} above, $\gamma_{12}=0$ implies that $\alpha_{13}+\alpha_{11}\alpha_{23}$ is zero. However, it is obvious that it does not automatically follow just because γ_{12} is zero that

$\alpha_{11}=\alpha_{13}=0$, or even $\alpha_{11}=0$. Thus, assertions about exogeneity in the traditional sense cannot be sustained from tests of Granger non-causality. Again, in this context it would appear that Granger non-causality is only relevant in testing strong exogeneity which, as mentioned earlier, is relevant not so much to conditioning but to marginalising the DGP.

Consider again the VAR model proposed by Garbade and Silber (1983).[13] Suppose that, for the sake of argument, we accept Granger causality tests as valid tests of lead-lag relationships. How are the conclusions of these tests affected by Garbade and Silber's (1983) model? The answer is the conclusion that the stock index futures market leads the spot market is unwarranted and invalid. The reason for this stems from the fact that the restricted version of Garbade and Silber's (1983) model requires cointegration and requires the cointegrating vector to be the basis. *Now in this model, the basis is* important in both the futures and spot equation. As such, *even if* the lagged changes in spot and futures prices are zero in both equations (as in the 'standard' test for lead-lag relationships), there

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[13] Note that Garbade and Silber (1983) do not interpret their model as a VAR and do not estimate it as one in their empirical study.

must be Granger causality from the futures to spot and spot to futures, that is, feedback must be present within the system. In this context, both markets would appear to lead each other. In fact, a more reasonable interpretation of this apparent feedback is Campbell and Shiller's (1988) argument that cointegration arises not so much because there is true causality between two variables but because one is a good forecast of the other. In this case, then, we have the fact that the futures and spot prices are good contemporaneous forecasts of each other, a not unreasonable proposition since on the one hand one is a derivative of the other yet on the other one reacts to information quicker than the other.

One final point that can be considered is the notion that once we analyse lead-lag relationships and mispricing as interdependent issues, then conceptually issues of lead-lag relationships and mispricing effectively become the single issue of whether or not equity markets can be said to function effectively. To demonstrate the argument, let us write the reduced form VAR system (2.15) in more conventional notation as

$$\boldsymbol{Y} = \boldsymbol{XB} + \boldsymbol{e} \tag{2.18}$$

Since (2.18) is a VAR, it can be estimated by OLS and the system estimator is given by

$$\hat{\boldsymbol{B}} = [\boldsymbol{I} \otimes (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}']\boldsymbol{Y} \tag{2.19}$$

From the discussion earlier we know that upon expiration of the futures contract, the futures price will equal the spot price and thus, by implication and through the nature of short-run deviations from the long-run equilibrium, the spot and futures prices will converge upon each other as expiration approaches. The implication of this is that as (T-t) $\rightarrow$ 0, so **X** $\rightarrow$ **Y** and the estimator for **B** becomes

$$\hat{\boldsymbol{B}} = [\boldsymbol{I} \otimes (\boldsymbol{Y}'\boldsymbol{Y})^{-1}\boldsymbol{Y}']\boldsymbol{Y} \quad \rightarrow \quad \hat{\boldsymbol{B}} = \boldsymbol{I} \tag{2.20}$$

Substituting into (2.18) we have that

$$\boldsymbol{Y} = \boldsymbol{X} + \boldsymbol{e} \;\rightarrow\; \boldsymbol{Y} = \boldsymbol{Y} \;\; \textit{since} \;\; \lim_{t \rightarrow T} \boldsymbol{X} = \boldsymbol{Y} \;\; \textit{and} \;\; \boldsymbol{e} = \boldsymbol{0} \tag{2.21}$$

such that the two series become indistinguishable and we have an identification problem, which follows somewhat trivially from the long-run equilibrium. However, what is not trivial is the fact that if (2.15) is the correct model regardless of the time to expiration then the system cannot be identified. Thus, once we treat lead-lag relationships and mispricing as interdependent, non-identification of the system becomes indicative of effectively functioning equity markets since with non-identification the two series are indistinguishable.

To summarise thus far, ignoring the insights that can be gained from the mispricing literature when conducting tests of lead-lag relationships can be disastrous given the misspecification problems that arise. In addition, failure to treat the models used as a system can generate invalid results since the models typically used are not identified. Estimating such models by GMM or instrumental variables does not overcome the problem and estimation of the 'structural' models by OLS yields biased and inconsistent estimates through invalid (and untested) exogeneity assumptions.

Considering the two strands of the literature in unison provides a whole new conceptual outlook on the problem. From the mispricing literature, we have that the long-run equilibrium between stock index futures prices and the value of the underlying stock index is a homogeneous one such that short-run deviations from this equilibrium are given by the basis. This result is reinforced by the reduced form VAR model of Garbade and Silber (1983).

Taken together, these models imply that the system relating the spot and futures prices is not identified and as such, the two markets are indistinguishable. If this is the case then equity markets can be said to function effectively. Moreover, analysing the issue of effectively functioning equity markets in this framework allows us to be more specific about what we mean by effectively functioning. It is possible to categorise two forms of effectively functioning financial markets : those that are strongly effectively functioning and those that are weakly effectively functioning. Obviously, if markets are neither of these then they are said to be ineffectively functioning. We will deal with the conditions for these classifications in turn below.[14]

a) Strongly Effectively Functioning

For the functioning of spot and futures markets to be classified as being strongly effective we require three conditions to hold :

i) the price series cointegrate, with the cointegrating vector being the basis. In other words, the homogeneity restriction must hold.

[14] We discuss these conditions in the context of the model of the pricing relationship between stock index futures prices and the underlying stock index price. These conditions can obviously be readily generalised for a system comprising more than two equations.

ii) the system cannot be identified from the reduced form.

iii) the reduced form is stable such that the pricing relationship does not change over time.

If these conditions hold then prices in the two markets will be indistinguishable and they will both depend only upon the same common factor which is the basis, or equivalently the degree of mispricing, so important in the arbitrage process.

Taken individually, these conditions are necessary but not sufficient to ensure that markets are strongly effectively functioning. However, if we take all of the conditions together, then they are both necessary and sufficient to ensure that markets are strongly effective in their functioning.

b) Weakly Effectively Functioning

For the functioning of spot and futures markets to be classified as being weakly effective, then we require conditions i) and iii) from the definition of strongly effective functioning to hold. However, condition ii) is relaxed somewhat in the case of weakly effectively functioning markets. Specifically, if some, but not all, of the equations in the system can be identified markets are said to be weakly effectively functioning. Again, taken individually these conditions are necessary but not sufficient to ensure weak effectiveness. Taken together, they are both necessary and sufficient for markets to be weakly effectively functioning. In addition, if markets are only weakly effectively functioning, we can obtain some indication as to which market regulation (in whatever guise) should be aimed at.

If markets are neither strongly nor weakly effectively functioning then they must be ineffectively functioning. If they are ineffectively functioning then it is important that the source of the ineffectiveness be pinpointed or, if it cannot be pinpointed exactly, it must be possible at least to obtain an indication of what is causing the ineffectiveness. This is possible within this framework.

Specifically, if conditions ii) and iii) for strong effectiveness hold but there is either no cointegration or the cointegrating vector is not the basis then this is suggestive of the fact that markets are ineffectively functioning because arbitrage is not functioning effectively. Thus, regulation (reform) of trading systems may be in order.

If the first and third conditions for strong effectiveness hold but the system can be identified, then markets are ineffectively functioning because prices do not incorporate all the available information. Again, one reason may be frictions in the trading system.

If the first and second conditions for strong effectiveness hold but there is instability in the system the pricing relationship could change over time, which would be indicative of ineffectively functioning financial markets. Again, this information can be used for pinpointing the nature of the ineffectiveness. For example, if the instability occurs because of a specific incident and persists after this incident, the source of the instability can be pinpointed.

Being able to classify markets in this way, then, allows any necessary regulation to be targeted at the correct market. For example, it would be possible in principle to appraise Kleidon and Whaley's (1992) argument that the source of the October 1987 market breakdown in the US was the stock market.

2.5. THE LINK BETWEEN EFFECTIVE FUNCTIONING AND MARKET EFFICIENCY

A further advantage of analysing effective functioning of financial markets, as opposed to analysing lead-lag relationships and mispricing separately, is that, with a slight modification of the interpretation of the results, it is possible to use exactly the same framework to test the efficiency of both markets jointly. Thus, effective functioning in some sense becomes synonymous with market efficiency. In this section, we will reinterpret the approach in the light of market efficiency and provide conditions for which the system exhibits market efficiency similar, but not quite as restrictive, to those provided for determining whether financial markets are effectively functioning. Before doing this, however, there is a point to note about the definition of efficiency we use here. The definition of efficiency we use is different to those definitions in Fama (1970). We use the definition discussed by Dwyer and Wallace (1990), which is the definition that underlies the discussion of exchange rate market efficiency in Levich (1985) and from an economic viewpoint is more interesting. A market is said to be efficient if there are no profit opportunities available which will increase agent's expected utility. As Dwyer and Wallace (1990, p.2) note,

> '...it is hard to see how a market with no expected-utility increasing profit opportunities available to agents based on expected-utility maximising acquisition of information could be characterised as inefficient in any interesting sense of the word.'

In the context of tests of the efficiency of spot markets that have derivative instruments traded upon them, the emphasis has predominantly been on the relationship between spot and forward prices. In particular, the methodology usually employed is to test whether the forward price is an optimal predictor of the future spot price, or whether it is biased and there is a (possibly time-varying) risk premium. This hypothesis has been tested in the context of exchange rates by, *inter alia*, Geweke and Feige (1979), Hansen and Hodrick (1980), Baillie, Lippens and McMahon (1983), Fama (1984), Domowitz and Hakkio (1985) and Hakkio and Rush (1989), for the London Metal Exchange by MacDonald and Taylor (1988, 1989) and for commodity spot and futures markets by Antoniou and Foster (1991).

The general approach underlying these tests is based on the notion that under rational expectations, with agent risk neutrality and the absence of profitable arbitrage opportunities, we must have that

$$E(S_{t+n} \mid \Omega_t) = F_{t,t+n} \tag{2.22}$$

where S_{t+n} is the spot price at time t+n, $F_{t,t+n}$ is the forward/futures price quoted at time at time t for delivery at time t+n, Ω_t is the information set as of time t and E(.) is the mathematical expectations operator. Rational expectations further implies that

$$S_{t+n} = E(S_{t+n} \mid \Omega_t) + \varepsilon_{t+n} \quad where \quad E(\varepsilon_{t+n} \mid \Omega_t) = 0 \tag{2.23}$$

where ε_{t+n} is a zero mean, MA(n-1) error which is independent of the information set. Substituting (2.22) into (2.23) we have that

$$S_{t+n} = F_{t,t+n} + \varepsilon_{t+n} \tag{2.24}$$

Equation (2.24) states that the forward/futures price quoted at time t for delivery at t+n is an unbiased predictor of the spot price at time t+n. Further, ε_{t+n} can now be interpreted as the forecast error. In testable form, (2.24) is

$$S_{t+n} = \beta F_{t,t+n} + \varepsilon_{t+n} \tag{2.25}$$

where for efficiency we require that β=1.

Tests of efficiency based on the forward/futures price being an optimal predictor of the future spot price have yielded mixed results. Tests based on (2.25) tend to accept the restriction and conclude that the forward/futures price is indeed an unbiased predictor of the future spot price. Other studies have formulated the model to be tested as (as before, lower case letters denote variables in natural logarithms)

$$\Delta s_{t+n} = \alpha + \beta(f-s)_t + \varepsilon_{t+n} \tag{2.26}$$

and have rejected the restrictions α=0, β=1 such that the forward rate is a biased predictor and there is a risk premium ($\alpha \neq 0$).[15] The problem with this

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[15] More recent studies (Domowitz and Hakkio (1985) for example) have tested for a time varying risk premium using the ARCH-M(q) formulation discussed in Engle, Lilien and Robins (1987), its generalisation to a GARCH-M(p,q) model (Bollerslev, Engle and Wooldridge (1988)) or the multivariate GARCH-M model (Baillie and Bollerslev (1990)). In this formulation, the intercept term in (2.26) is modified to allow the conditional variance of the residuals to directly affect the mean, that is,

$$\alpha = \theta_0 + \theta_1 h_{t+1} \quad \textit{where} \quad h_{t+1}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i+1}^2 + \sum_{i=1}^{p} \beta_i h_{t-i+1}^2$$

For more details on ARCH models, see chapter 5 and the review in either Engle and Bollerslev (1986) or Bollerslev, Chou and Kroner (1992).

conflicting evidence is the approach adopted. Tests based on (2.25) will find it hard to reject $\beta=1$ first because the two prices will track each other closely and second, the model is capturing not the true relationship between the two prices but the fact that they both follow very similar trends. This second point is very damaging to tests based on (2.25) because the data are nonstationary and it is well known that in this situation (nonstationarity), standard inference procedures are invalid. Rejections based on (2.26) have been attributed by some authors (for example, Hakkio (1981)) to misspecification of the model. Hakkio and Rush (1989) propose a framework within which this dilemma can be resolved.

First, given the well-documented evidence that spot and forward/futures prices are stochastic nonstationary (see *inter alia* Baillie and Bollerslev (1989) and Antoniou and Foster (1991)) (2.25) can be viewed as a cointegrating regression. Thus, a first step is to test for cointegration, or equivalently test for a unit root in $(s_{t+n} - f_{t,t+n})$. In this instance, cointegration (with $\beta=1$) is *necessary* for efficiency. As Dwyer and Wallace (1990) point out, studies that find that $(s_{t+n} - f_{t,t+n})$ is the cointegrating vector (for example, Baillie and Bollerslev (1989)) are wrong to conclude that cointegration is evidence of inefficiency.[16] Non-cointegration is evidence of inefficiency.

Cointegration, with the forecast error being the cointegrating vector, however, is not sufficient to ensure efficiency. A second step must be used and this is to test for efficiency in the reduced form error correction model that follows given cointegration. Specifically, setting n equal to one for ease of exposition, estimate

$$\Delta s_{t+1} = \alpha \Delta f_{t,t+1} + \rho(s_t - \beta f_{t-1,t+1}) + \varepsilon_{t+1} \qquad (2.27)$$

and test the joint hypothesis that $H_0\!:-\rho=\alpha=\beta=1$. If

[16] The temptation to conclude that cointegration implies inefficiency, as Baillie and Bollerslev (1989) do, stems from the apparently contradictory observation by Granger (1986) that first, there should be no cointegration between two speculative markets if they are efficient and second, markets in which prices move closely together, such as spot and futures markets, should cointegrate. Baillie and Bollerslev (1989) appear to be treating spot and forward foreign exchange markets as separate speculative markets since they argue that prices can be predicted from the error correction term. This is incorrect. For example, we have already seen that if spot and futures markets are functioning effectively, they are indistinguishable, that is, they function as one market. If they are functioning as one market, then cointegration *must* imply efficiency. The way to think of this is that if the forward or futures price is an optimal predictor of the spot price, then the forward market now reflects the spot market in n periods time. Therefore, they are essentially the same market and in essence one is testing the efficiency of one market at two different points in time. Thus, Granger's (1986) observation is not contradictory.

this is valid, then (2.27) collapses to (2.25) and we have market efficiency. Moreover, test statistics from (2.27) are powerful because standard inference applies.

This principle is equally applicable to the stock and stock index futures markets, although it does need modification. In fact, with the modification the conditions for efficiency are in some senses less stringent. Recall from theory that the stock index futures market will be efficiently pricing contracts if the actual stock index futures price is equal to

$$f^{*}_{t,T} = s_t + (r-d)(T-t) \tag{2.28}$$

The long-run equilibrium from this is simply f=s, implying the restriction $\beta=1$ in $f_t = \beta s_t + \varepsilon_t$. This can be tested by checking for the presence of a unit root in $(f_t - s_t)$, the null hypothesis of a unit root being rejected if *the restriction is valid. The* system reduced form error correction model that follows on naturally from this is already familiar to us :

$$\begin{bmatrix} \Delta f_t \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} -\alpha_{11} \\ \alpha_{21} \end{bmatrix} \left[f_{t-1} - s_{t-1} \right] + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \tag{2.29}$$

In terms of the system (2.29) reducing to f=s (s=f), the very formulation of the error correction term ensures that this will happen.[17] The advantage of addressing the efficiency question in this systems framework lies in the fact that, unlike the 'traditional' framework discussed above, we are concerned with efficiency in both markets.

In the context of the systems approach we require slightly different conditions than those required in using the approach discussed above. As it turns out these conditions are not at all dissimilar to those required to ensure strongly effectively functioning equity markets.[18]

a) Both Markets are Efficient

For both markets to be classified as efficient, we

[17] To see this, note that in a steady-state, static equilibrium, $f_t=f_{t-1}=f$ and $s_t=s_{t-1}=s$. f=s and s=f then follows automatically.

[18] As with the conditions for effectively functioning markets, we discuss the conditions for efficiency in the context of the stock and stock index futures markets. They can obviously be generalised for systems containing more than two markets.

require the basis to be the cointegrating vector and that the system as a whole cannot be identified. These two conditions are exactly the same as the first two necessary conditions for markets to be strongly effectively functioning. If these conditions hold, then together they are necessary and sufficient to ensure market efficiency.

The difference between efficiency and strongly effectively functioning financial markets is that there is no *a priori* reason to require stability as a necessary condition. If the model is stable, so much the better since this implies that the market will be efficient all of the time. However, given that there is no *a priori* reason to suppose that markets are efficient all of the time[19] (and the increasing body of literature on stock market inefficiency suggests that this is the case), stability is not necessary for efficiency. If the model is unstable, it simply means that efficiency will be specific to the sample period under consideration.

b) One Market is Efficient

The conditions for one of the markets to be efficient

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[19] The argument here is that efficiency is not necessarily a time invariant, intrinsic property of markets.

correspond primarily to those for weak effective functioning of financial markets. We require that the cointegrating vector be the basis and that one of the equations comprising the system to be under identified. The equation which can be identified then corresponds to the market which is inefficient. Moreover, the inefficiency should be an exploitable one since identification will require the presence of either lagged futures returns, lagged Index returns or both in the equation, which in turn implies forecastability of future returns in whichever market is inefficient. The stability condition is not required for the same reasons given in the discussion on efficiency in both markets.

Obviously, as with the discussion of the conditions for markets to function effectively, we can also identify situations when both markets will be inefficient. If there is no cointegration, or the cointegrating vector is not the basis, then inefficiency in both markets is implied. Given that, by its designated role, the stock index futures price and the stock index price should track each other almost exactly, the absence of cointegration between the two implies that they will drift apart without bound. If this is the case, it should be possible to develop a trading strategy that exploits with the express purpose of earning abnormal returns.

In the case where there is cointegration but the basis is not the cointegrating vector, predictability of prices is implied and again investors should be able to take advantage of this to earn abnormal returns. Finally, if both equations of the system can be identified, the implication is that future returns in both markets can be predicted and investors again should be able to formulate a trading strategy that exploits these inefficiencies.

As a final point in this section, the paradox here, as it is with the Efficient Markets Hypothesis, is that investors who take advantage of the inefficiencies will ensure that the market is efficient. If investors believe that both markets are efficient and operate buy-and-hold-type hedging strategies, the markets will remain inefficient since there will be nothing to correct prices so that they fully reflect information available.

2.6. Is Mispricing Path Dependent ?

A final issue that we can consider in this chapter is the implication of the above exposition for the behaviour of mispricing. In particular, does the above model have anything to offer on the issue of path independence versus path dependence? The answer to this question is yes, not so much as a direct implication of the model but as a direct implication of the arguments used in its construction.

Recall from chapter one that in analysing the behaviour of mispricing, MacKinlay and Ramaswamy (1988) found evidence of path dependence in the measure of mispricing for the S&P 500 Index futures contract in the US and Yadav and Pope (1990) found similar evidence for the FTSE 100 Index futures contract in the UK. However, in attempting to model the theoretical properties of mispricing, Brennan and Schwartz (1990) allow mispricing to evolve according to the following continuous time stochastic process, known as a Brownian Bridge process :

$$d\varepsilon(\tau) = -\frac{\mu\varepsilon}{\tau}dt + \gamma dz \qquad (2.30)$$

where ε is mispricing, τ is the time to maturity of the futures contract and μ is the speed of mean reversion. The distinguishing feature of this stochastic process is that it is path independent, with mispricing reverting back to zero and equalling zero with a probability of one when τ=T, i.e., expiration of the futures contract.

Consider now the nature of mispricing (the basis) and the nature of tests for path dependence. It was argued earlier that 'traditional' tests of lead-lag relationships are misspecified because they ignore any cointegrating vectors. From cointegration, a cointegrating vector will exist if a linear combination of two I(1) variables is I(0). An I(1) variable is a variable in which the nonstationarity is stochastic rather than deterministic such that shocks to the process will be permanent. Therefore, if a cointegrating vector is a linear combination of two stochastic nonstationary variables, it is not unreasonable to hypothesise that the linear combination will be a stationary *stochastic* variable, following an ARMA(p,q) process. We know that in effectively functioning equity markets, the cointegrating vector is the basis, which is the measure of mispricing and which will be a stationary stochastic process.

Tests for path dependence in mispricing on the other hand typically involve *regressing mispricing on time* to expiration and testing whether time to expiration is significant. The sign of the coefficient then gives the nature of the path dependence. Time to maturity is measured by (T-t),[20] where T is the

[20] Some studies express time to maturity as a fraction of a year, that is, (T-t)/365. The argument is equally applicable to this measure of time to maturity.

maturity date, which is fixed. Define t as the number of trading days the contract has to run. (T-t) then represents the number of trading days to maturity. Since T is fixed, (T-t) decreases by one unit each day until expiration is reached. Viewed in this light, (T-t) is nothing more than a deterministic time trend. Therefore, if it is significant, mispricing is stationary around a deterministic trend. However, this is ruled out by the definition of cointegration. The implication of this is that if stock and stock index futures prices are stochastic nonstationary and the basis (mispricing) is the cointegrating vector, then mispricing should follow a stationary stochastic process.

These arguments, then, are suggestive of the fact that the finding of path dependence in MacKinlay and Ramaswamy (1988) and Yadav and Pope (1990) is in actual fact a finding of misspecification in the model used to test for path dependence. Indeed, we know this to be the case with Yadav and Pope (1990), for before they test for path dependence they model mispricing and find that a stationary AR(1) process adequately describes its behaviour. This finding is consistent with the theoretical model in Brennan and

Schwartz (1990) which is a path independent model.[21]

It would appear, then, that a path independent process that has mean reverting features, as discussed in Brennan and Schwartz (1990), is the way forward in terms of modelling the theoretical behaviour of mispricing.

2.6. CONCLUSIONS

In this chapter, we have set out to provide a framework which allows us to test whether or not the stock market and the stock index futures market are effectively functioning, a framework which is also easy to generalise to the case of more markets.[22]

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[21] If mispricing is a stationary stochastic process, it will be consistent with a mean reverting process. To see this, suppose mispricing, denoted by X_t, follows the following, stationary AR(1) process

$$X_t = \rho X_{t-1} + u_t \quad , \quad |\rho| < 1, \quad u_t \sim N(0,\sigma_u^2)$$

The s-period-ahead forecast of this is given by

$$E(y_{t+s} \mid y_t) = \rho^s y_t$$

It is clear that $\lim_{s\to\infty} \rho^s = 0$. Clearly, in this case, the limit of s is the expiration date, upon which time mispricing must be zero. Thus, mispricing following an AR(1) process that is stationary will be mean reverting, the mean being zero. This is consistent with a Brownian Bridge type stochastic process.

[22] One could think of models designed to test the term structure relationship between short and long term interest rates, for example, where if the short

That this framework allows us to investigate whether or not markets are effectively functioning comes about by unifying two apparently diverse strands of the literature on stock index futures into a single, coherent framework.

Two issues in the analysis of stock index futures that have been the subject of quite indepth inquiries have been the presence of lead-lag relationships between the two markets and the behaviour of mispricing and the profitability of arbitrage strategies based on mispricing. However, the interrelationship between the two is typically ignored such that results from tests of lead-lag relationships, for example, are unreliable and must be interpreted with some caution. We rectify this state of affairs by explicitly considering the interrelationship, making use of the valuable information provided by theory about the form mispricing takes. The argument is that the models typically used to test lead-lag relationships are misspecified through inappropriate conditioning, that is, the omission of lagged variables without testing the zero coefficients these restrictions implies, and the omission of any cointegrating vectors.

The implication of the omission of lagged variables is

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and long gilt futures contracts are included, a four-equation system will result.

that, when the system is written out in full, both spot and futures returns are *forced* to be martingale difference processes, which may or may not be true but is testable. Moreover, if the zero *restrictions that* force returns to be martingale difference processes are invalid, the resultant system is under identified such that any interpretation of the coefficients and their meaning is inappropriate. The implication of the omission of any cointegrating vectors is that even if the system is identified, there is an omitted variables problem such that, again, interpretation of, and inference about, the coefficients is hazardous. Mispricing enters the picture in terms of providing information about the form of the cointegrating vector. Indeed, it is shown by manipulation of the model giving the theoretically correct futures price that mispricing in itself is given by the basis, which is defined as the futures to cash price differential. The implication of this is that if both the spot and futures prices are stochastic nonstationary, then from the results in Engle and Granger (1987), it is possible that a linear combination of the two prices is stationary.

In fact, this linear combination is the basis, such that the long-run equilibrium relationship between the stock and stock index futures prices is homogeneous. Therefore, the cointegrating vector is the basis and, as is shown using Garbade and Silber's (1983) model, it explains movements in both markets. Therefore, given that stock and stock index futures prices are jointly determined, the relationship must be treated in a systems context.

When we treat the relationship between the two markets as a system, a further interesting implication emerges with regard to the identification of the equations of the system. In particular, we have that upon expiration the two prices become indistinguishable such that the system cannot be identified. This result follows rather trivially from the long run equilibrium condition. However, what is not trivial about this result is the fact that if the cointegrating vector is the basis and the system cannot be identified further away from expiration, the stock and stock index futures prices are indistinguishable, both being dependent solely upon the same common factor, the basis, which is the degree of mispricing.

This framework, then, shows that the natural question to ask is not what is the nature of the lead-lag relationship between the two markets, nor whether arbitrage opportunities are profitable based on simulated (and possibly unrealistic) trading strategies. Rather, it is whether equity markets are effectively functioning. If the two markets cannot be distinguished, they must be.

Whilst being able to determine whether or not markets are effectively functioning is of immense importance, of equal importance is being able to target the correct market for regulation if they are not functioning effectively. This framework allows us to do precisely this, splitting effectively functioning as a whole into two sub-components : strongly effectively functioning markets and weakly effectively functioning markets.

We provide an objective and testable necessary and sufficient condition, which in itself is the amalgamation of three necessary conditions, for these two types of effective functioning to be determined empirically. For strong effectiveness we have that the cointegrating vector must be the basis, the system must be under identified and the reduced form must be stable such that the pricing relationship does not change. For weak effectiveness, we require that the basis be the cointegrating vector, that some, but not all, of the equations in the system be under identified and that the reduced form be stable, again such that the pricing relationship does not change over time. If some of these conditions are violated, markets function ineffectively but within this framework it is possible to determine, for the purpose of regulation, likely sources of ineffectiveness.

This framework for testing effectively functioning equity markets also allows us to test efficiency of both the stock and stock index futures markets. Indeed, the two are virtually synonymous, the difference between them being the requirement of stability for effective functioning, a condition which is not necessary for efficiency.

The final point which we addressed in this chapter concerns the behaviour of mispricing and in particular whether mispricing is path independent, as proposed by Brennan and Schwartz (1990), or path independent as evidence in MacKinlay and Ramaswamy (1988) and Yadav and Pope (1990) suggests. Using the assumptions under which the model in this chapter was derived, we showed that mispricing should be path independent stochastic process and as such any theoretical models of the process driving mispricing should proceed along the lines of path independence.

To summarise, then, the issue is not one of which market leads each other. It is one of whether markets are effectively functioning. This then ties into the question of whether markets are efficient. The implication of this framework is then that mispricing should only be analysed after an investigation of the effective functioning or otherwise of the markets. If markets are strongly effective and hence efficient, the purpose of analysing the stochastic properties of the basis becomes relevant only in so far as it provides information as to the likely stochastic process that underlies any theoretical model of mispricing (cf. Brennan and Schwartz (1990)).

If markets function ineffectively (in which case both will be inefficient) or one of them is inefficient then analysis of the stochastic properties of mispricing might suggest potentially profitable arbitrage trading strategies. Note, however, that it does not automatically follow that ineffectiveness holds the key to profitable arbitrage opportunities. For example, if we have ineffectiveness through instability, the market can still be efficient. Hence just because the market is ineffectively functioning, it does not necessarily follow that it is inefficient and therefore analysis of mispricing in this situation will not necessarily yield profitable arbitrage opportunities.

In the next chapter, we turn our attention to analysing these issues through analysing the functioning of the stock and stock index futures markets in the UK on a daily basis.

CHAPTER THREE : MODELLING THE DAILY PRICING RELATIONSHIP BETWEEN THE FTSE 100 INDEX AND FUTURES CONTRACT.

3.1. INTRODUCTION

In this chapter, we use the framework developed in chapter two to analyse the price functioning of the stock market, represented by the FTSE 100 Index, and the stock index futures market, where the contract on the FTSE 100 Index is traded. The previous chapter raised several issues that are worthy of empirical investigation. First, from the point of view of regulators and from a policy perspective the ideal situation is where the markets are strongly effective in their functioning. Less appealing is that they be weakly effective but this is preferable to both being ineffective. If they are strongly effective, then by implication the stock index futures market is carrying out its prescribed role effectively, a most desirable state of affairs.

Second, if markets can be characterised as strongly effectively functioning then by implication they are efficient (in the sense that no profitable arbitrage opportunities that increase expected utility are available). However, if they are not strongly effective in their functioning, they can still be efficient, again an obviously desirable state of affairs.

The final issue raised is the behaviour of mispricing. There are several reasons for wishing to analyse this, even in the presence of strongly effective and efficient equity markets. If the cointegrating vector is the basis, which is a measure of mispricing, then it should be mean reverting, that is, a stationary, autoregressive stochastic process. If this is the case, then any theoretical model geared to helping us understand the behaviour of mispricing must proceed along the lines of that suggested by Brennan and Schwartz (1990).

Analysis of these issues is either conducted within, or is derived from, the following model

$$\begin{bmatrix} \Delta f_t \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \end{bmatrix} + \begin{bmatrix} -\alpha_{11} \\ \alpha_{21} \end{bmatrix} \begin{bmatrix} f_{t-1} - s_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

which is a first order VAR reparameterised in error correction form. It is this model that provides us with the conditions not only for effective functioning but also for efficiency. First, the error correction term should be the basis, that is, long-run homogeneity should hold. This is necessary for both effectiveness and efficiency. Second, as written both equations in th system are under identified. This is a necessary condition for strong effectiveness and efficiency in both markets. A final condition necessary for strong effectiveness, although not for efficiency, is that the model be stable. If all of these conditions hold, then both markets depend upon the same common factor, the basis. We investigate the validity of these conditions in this chapter.

The rest of the chapter is organised as follows. In the next section, we test to see whether the stock market and stock index futures market in the UK are effectively functioning and efficient. In section three, we analyse the stochastic properties of the basis. Section four concludes.

3.2. Modelling The Daily Pricing Relationship

The discussion in the previous chapter, and the definitions provided by Engle and Granger (1987), suggest that an appropriate starting point for analysing the pricing relationship is determining the properties of the series in question, that is, are the two series individually nonstationary (possibly random walks) and do they cointegrate with the basis being the cointegrating vector? To investigate these issues and the nature of the pricing relationship we utilise daily closing prices[1] of the FTSE 100 Stock Index and the FTSE 100 Stock Index futures contract from January 1985 to December 1990.[2] The two series are plotted in figures 1 and 2 overleaf.

The futures price is constructed as a rollover, that is, we take the three months of prices quoted for each nearest maturity contract, so that the two series are comparable. The graphs are remarkable in that it is obvious that the two prices track each other virtually exactly. There are also two noteworthy events that are apparent from the graphs : the Big Bang of October 1986 and the stock market crash of October 1987. However, even in these periods of 'disruption' the two markets seem to track each other exactly.[3]

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[1] Obviously, the Index does not have a price as such because it is an index rather than a tradeable instrument. Technically, the Index has a quoted value rather than a price. Whilst we recognise this, we will refer to the value of the Index as its price for the sake of convenience.

[2] The data was obtained from the International Stock Exchange, LIFFE and Datastream. I am grateful to Stephen Wells at the ISE and Maggy Keefe at LIFFE for providing some of the data.

[3] As a little aside, it would appear from visual inspection of the graphs that the crash was nothing more than a long-overdue price correction, returning prices to their pre-Big Bang trend. This would seem to confirm that lowering interest rates for fear of an ensuing recession was unnecessary given that the crash would appear to have been nothing more than a price correction.

FIGURE 3.1 : FTSE 100 INDEX FUTURES CLOSING PRICE
2 JANUARY 1985 TO 31 DECEMBER 1990

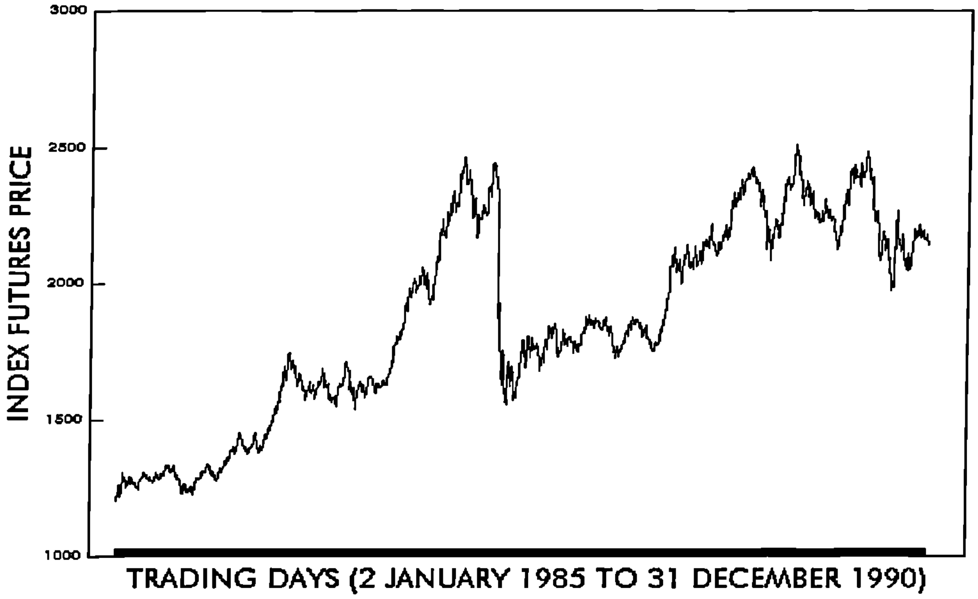

FIGURE 3.2 : FTSE 100 INDEX CLOSING PRICE
2 JANUARY 1985 TO 31 DECEMBER 1990

Another intriguing feature of the data is that, with the exception of the Big Bang-crash period, the two prices seem to follow a stable upward trend, suggesting that the prices are indeed nonstationary (although whether they are, in the terminology of Nelson and Plosser (1982), difference stationary as opposed to trend stationary remains to be seen.)

For the purposes of empirical analysis, an interesting question is whether or not the pricing relationship differs according to time to expiration. Thus, we analyse the behaviour of the two markets according to the times to expiration of the contracts. For example, we analyse the pricing relationship between the Index and the March contract three months from expiration, two months from expiration and in the expiration month (full details of the construction of the data are provided in the appendix). The reasons for analysing the pricing relationship in this way are straightforward and appealing : we know that in the expiration month and upon expiration the two prices are indistinguishable and hence the system is not identified. The question is does this hold further away from expiration : are the futures and the spot price still indistinguishable further away from expiration, for if they are not this would indicate the presence of perhaps profitable arbitrage opportunities amongst other things, raising questions over the efficiency of the markets.

We begin our analysis by examining the univariate time series properties of the Index and Index futures price for the various contracts and the various months to expiration. In recent years it has emerged that many economic and financial time series are stochastic nonstationary, i.e. they have a unit root in their auutoregressive time series representation (see Nelson and Plosser (1982), Perron (1988) and Baillie and Bollerslev (1989) for example). Nelson and Plosser (1982) have referred to such series as difference stationary since such series have a constant mean when they are first differenced. The other scenario is that the nonstationarity is deterministic in nature such that deviations about the trend are stationary.

Determining whether the nonstationarity is stochastic or deterministic is important for whilst the two series may look the same graphically, they behave very differently. To see this, consider the following two models, one of which has a deterministic trend and one of which has a stochastic trend. First, we have

$$y_t = \alpha + \beta T + u_t \ , \ u_t = \rho u_{t-1} + e_t \ , \ e_t \sim N(0, \sigma_e^2) \ , \ |\rho| < 1 \qquad (3.1)$$

where T is a deterministic time trend (T=1,2,.....T) and the errors follow a first order autoregressive process. Alternatively, we have

$$y_t = \mu + y_{t-1} + v_t \ , \ v_t \sim N(0, \sigma_v^2) \tag{3.2}$$

which is a random walk (with drift if $\mu \neq 0$). Suppose that $y_0 = \mu$. Then by repeated substitution (3.2) can be written as

$$y_t = t\mu + \sum_{i=1}^{t} v_i \tag{3.3}$$

Now to see the differences between the two series take the s-period ahead forecast of (3.1) and (3.2) respectively :

$$E(y_{t+s} \mid y_t) = \alpha + \beta(T+s) + u_{t+s|t} \ , \ u_{t+s|t} = \rho^s u_t \tag{3.4}$$

$$E(y_{t+s} \mid y_t) = s\mu + y_t \tag{3.5}$$

Substituting (3.3) into (3.5) we obtain

$$E(y_{t+s} \mid y_t) = \mu(t+s) + \sum_{i=1}^{t} v_i \tag{3.6}$$

Now, as $\lim_{s\to\infty}$ it is clear that in (3.4) $\rho^s u_t \to 0$ and the series becomes independent of the errors, that is, shocks are transitory. However, for the series with the stochastic trend, it is clear that the shocks persist, that is they are permanent. Thus, the implications for the persistence of shocks are very different for the two series. Another difference between the two series can be shown by taking the first difference of (3.1) (assuming no serial correlation for ease of exposition) and (3.2).

An alternative way in which to view taking the first difference of (3.2) is to see the unit root as a coefficient restriction. Thus, for (3.2) we obtain

$$\Delta y_t = \mu + v_t \tag{3.7}$$

which, by the definition of white noise, is stationary. However, for (3.1) the result is potentially quite different :

$$\Delta y_t = \beta + u_t - u_{t-1} \tag{3.8}$$

For the AR(1) error in (3.1), we have

$$u_t = \frac{e_t}{1-\rho L}$$

$$\Rightarrow \quad \Delta u_t = \frac{1-L}{1-\rho L} e_t \tag{3.9}$$

If $\rho = 0$, then differencing the trend stationary series will induce a moving average error with a unit root which is not invertible. This would lead to nonstationarity in the error term (viz. spurious regressions, see Granger and Newbold (1974)).

In order to determine which type of nonstationarity is present, a great deal of research has been aimed at deriving statistics for testing the unit root hypothesis, that is, testing for the presence of a stochastic trend (see *inter alia* Dickey and Fuller (1981), Phillips (1987), Perron (1988) and Phillips and Perron (1988)). Most empirical applications of unit root tests follow the by now very well known procedure set out in Dickey and Fuller (1981).

An alternative, and arguably more preferable, approach is the nonparametric procedure proposed by Phillips (1987) and Perron (1988) (see also Phillips and Perron (1988)) which allows for various kinds of heterogeneity in the residuals and allows for serial correlation not by adding extra lags as in the Dickey-Fuller procedure but by allowing for it in the calculation of a consistent estimator of the variance in the unit root regressions. Testing the unit root hypothesis using the Phillips-Perron procedure involves estimating the following three models

$$y_t = \mu + \beta\left(T - \frac{n}{2}\right) + \alpha y_{t-1} + u_t \qquad \textbf{(3.10a)}$$

$$y_t = \mu^* + \alpha^* y_{t-1} + u_t^* \qquad \textbf{(3.10b)}$$

$$y_t = \hat{\alpha} y_{t-1} + \hat{u}_t \qquad \textbf{(3.10c)}$$

and using the modified F or t statistics (referred to as Z statistics by Phillips-Perron) to test the appropriate null hypothesis.

Taking model (a), there are three hypotheses that can be tested.[4] First, we have $H_0 : (\mu, \beta, \alpha) = (\mu, 0, 1)$, which tests for a stochastic trend with drift. This is tested using the modified F statistic $Z(\Phi_3)$ against the alternative that the process is deterministic nonstationary. The second hypothesis is given by $H_0 : (\mu, \beta, \alpha) = (0, 0, 1)$ which tests for a stochastic trend without drift using the statistic $Z(\Phi_2)$. Finally we can simply test $H_0 : \alpha = 1$ using the $Z(\alpha)$ or $Z(t_\alpha)$ statistics. With model (b), the null hypothesis is H_0: $(\mu^*, \alpha^*) = (0, 1)$ or H_0: $\alpha^* = 1$. These hypotheses are tested using the $Z(\Phi_1)$ and $Z(\alpha^*)$ or $Z(t_{\alpha*})$ statistics respectively. These hypotheses are tested against the alternative that $\alpha^* < 1$, that is, the series is a

[4] Note that with the vast majority of unit root tests, the null hypothesis is that the series has a stochastic trend.

stationary stochastic process. For model (c) the null is $H_0 : \hat{\alpha} = 1$. The alternative is the same as model (b). The appropriate statistic in this case is $Z(\hat{\alpha})$ or $Z(t_{\hat{\alpha}})$.

The reason for specifying three models to test the unit root hypothesis becomes clear when we consider the testing strategy to be adopted.[5] Dickey, Bell and Miller (1986) argue that the appropriate model to use is (b). Their reasons for suggesting this are that if the series has a unit root with drift then the statistics from model (c) would have low power. If on the other hand the series has a unit root without drift the statistics from model (a) would have low power through the inclusion of both drift and a deterministic trend.

However, Theorem 1 of Perron (1988) proves that as the sample size increases it becomes impossible to distinguish between a unit root and a deterministic time trend. Moreover, the discussion above would suggest that failing to allow for a deterministic trend could prove disastrous at the modelling stage. Therefore, Perron (1988) suggests the following strategy : estimate model (a) and test the hypotheses associated with it. If they cannot be rejected it may

[5] For a detailed account of this strategy, see Perron (1988). We only summarise it here.

be due to low power of the tests. Therefore, estimate model (b). If the hypotheses cannot be rejected for model (b) then the appropriate model for testing is model (c).

The problem with the Phillips-Perron approach is in the choice of the truncation lag l since the results may be sensitive to this choice. In order to check the robustness of the results the unit root tests were calculated using truncation lags of 1, 4, 7 and 10. The results were qualitatively the same and quantitatively very similar.[6]

Results of the unit root tests for l=1 are presented in table 3.1 overleaf.[7] Starting with model (a), it is clear that the null of a stochastic trend with drift and without drift cannot be rejected in favour of the alternative of a deterministic trend in any of the series. Moving on to model (b), again the null hypothesis of a unit root without drift cannot be rejected. Thus, the appropriate model would appear to

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[6] The unit root test statistics were calculated using Peter Burridge's ROOTINE program.

[7] 5% critical values for the test statistics are (see Perron (1988))

$Z(\Phi_3)$	= 6.25	$Z(\Phi_2)$	= 4.68	$Z(\Phi_1)$	= 4.59
$Z(\alpha)$	= -21.8	$Z(\alpha^*)$	= -14.1		
$Z(t_\alpha)$	= -3.41	$Z(t_{\alpha^*})$	= -2.86		

TABLE 3.1 : PHILLIPS-PERRON UNIT ROOT TEST RESULTS (l=1)

	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(\alpha)$	$Z(t_\alpha)$	$Z(\Phi_1)$	$Z(\alpha^*)$	$Z(t_{\alpha*})$
MARCH 1							
s_t	2.380	2.056	-8.592	-2.074	2.824	-3.028	-1.663
f_t	2.424	1.888	-8.183	-2.090	2.693	-3.040	-1.647
Δs_t					57.54	-119.1	-10.79
Δf_t					63.94	-124.4	-11.32
MARCH 2							
s_t	2.574	5.032	-10.35	-2.263	1.725	-2.071	-1.225
f_t	2.777	2.094	-10.89	-2.352	2.287	-2.283	-1.273
Δs_t					56.27	-115.3	-10.78
Δf_t					54.40	-113.2	-10.58
MARCH 3							
s_t	2.831	1.900	-11.55	-2.402	3.887	-1.150	-0.849
f_t	2.933	7.600	-11.84	-2.441	2.400	-1.279	-0.921
Δs_t					48.57	-108.2	-9.905
Δf_t					52.26	-112.9	-10.30

TABLE 3.1 CONT...

		$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(\alpha)$	$Z(t_\alpha)$	$Z(\Phi_1)$	$Z(\alpha^*)$	$Z(t_{\alpha^*})$
JUNE 1								
	s_t	2.200	1.483	-8.487	-2.078	1.680	-3.456	-1.505
	f_t	2.214	2.090	-8.498	-2.082	1.817	-3.568	-1.534
	Δs_t					43.73	-104.7	-9.547
	Δf_t					47.13	-110.3	-9.970
JUNE 2								
	s_t	2.603	2.167	-9.762	-2.277	2.310	-3.359	-1.492
	f_t	2.681	1.797	-10.17	-2.316	2.085	-3.456	-1.505
	Δs_t					54.96	-113.1	-10.59
	Δf_t					63.14	-119.9	-11.28
JUNE 3								
	s_t	3.078	3.667	-11.66	-2.403	1.857	-4.083	-1.796
	f_t	3.329	2.489	-12.34	-2.507	2.124	-4.297	-1.813
	Δs_t					40.16	-91.37	-8.892
	Δf_t					56.83	-114.5	-10.83

TABLE 3.1 CONT...

	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(\alpha)$	$Z(t_\alpha)$	$Z(\Phi_1)$	$Z(\alpha^*)$	$Z(t_{\alpha*})$
SEPTEMBER 1							
s_t	2.275	1.706	-8.678	-2.017	1.884	-5.637	-1.924
f_t	2.371	1.583	-9.027	-2.067	1.937	-5.820	-1.953
Δs_t					66.89	-129.9	-11.79
Δf_t					73.50	-135.8	-12.40
SEPTEMBER 2							
s_t	2.123	1.985	-7.516	-1.903	1.844	-4.247	-1.811
f_t	2.182	2.096	-7.725	-1.943	2.123	-4.333	-1.832
Δs_t					39.26	-91.58	-8.709
Δf_t					46.89	-102.9	-9.588
SEPTEMBER 3							
s_t	2.170	2.241	-8.163	-2.012	1.964	-3.881	-1.662
f_t	2.236	1.843	-8.453	-2.061	1.808	-3.902	-1.650
Δs_t					50.12	-114.6	-10.26
Δf_t					59.29	-124.5	-11.07

TABLE 3.1 CONT...

	$Z(\Phi_3)$	$Z(\Phi_2)$	$Z(\alpha)$	$Z(t_\alpha)$	$Z(\Phi_1)$	$Z(\alpha^*)$	$Z(t_{\alpha*})$
DECEMBER 1							
s_t	2.476	1.707	-10.40	-2.233	2.387	-2.076	-1.140
f_t	2.376	2.655	-10.22	-2.182	1.306	-2.107	-1.158
Δs_t					72.11	-126.4	-12.11
Δf_t					74.28	-127.0	-12.13
DECEMBER 2							
s_t	4.090	7.801	-16.98	-2.865	2.825	-2.680	-1.353
f_t	4.598	3.453	-17.75	-3.051	3.669	-2.576	-1.306
Δs_t					42.11	-104.0	-9.483
Δf_t					48.92	-109.4	-10.01
DECEMBER 3							
s_t	2.706	2.686	-10.23	-2.239	1.952	-5.288	-1.918
f_t	3.020	2.064	-11.38	-2.390	2.001	-5.665	-1.963
Δs_t					56.10	-119.5	-10.66
Δf_t					56.01	-120.2	-10.70

be model (c).[8] In fact it appears that the index futures and Index prices are best described as martingale sequences. Table 3.1 also reports unit root tests on the first differences of the series and the null hypothesis that the first differences of the series contain a unit root is rejected in all cases.

Given that the series are I(1) in levels, I(0) in first differences, it is possible that they cointegrate (in fact they should cointegrate given the arguments presented earlier). In order to test for cointegration, we utilise the systems approach presented in Johansen (1988) and Johansen and Juselius (1990). Recall from chapter 2 the VAR parameterised in error correction form :

$$\Delta \boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\Gamma}_1 \Delta \boldsymbol{y}_{t-1} + \ldots\ldots + \boldsymbol{\Gamma}_{k-1} \Delta \boldsymbol{y}_{t-k+1} + \boldsymbol{\Pi} \boldsymbol{y}_{t-k} + \boldsymbol{u}_t \qquad (3.11)$$

The tests for cointegration proposed by Johansen (1988) and Johansen and Juselius (1990) then consist of testing the rank of the matrix Π which will have reduced rank if there is cointegration. However, an issue that does arise in testing the number of cointegrating vectors is the lag length used in the

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[8] Although, as an anonymous referee of The Economic Journal has pointed out, model (c) is rarely used in practice since the mean is rarely known *a priori* and this causes problems with regard to invariance in finite samples.

VAR, for as Hall (1991) demonstrates, the test statistics can be sensitive to the choice of lag length. Consequently, we start with a VAR(5) to allow for any possible trading day anomalies that may be present and test the restrictions imposed by reducing the order of the VAR sequentially by one lag. Once we obtain a rejection we have the order of the VAR that should be used in testing for cointegration.

Likelihood ratio statistics testing for the appropriate lag length in the VAR are presented in table 3.2 overleaf.[9] With the exception of the September contract three months from expiration and the December contract three months from expiration, which contains the effects of both the Big Bang and the crash, the appropriate lag length for the VAR is one. September 3 and December 3 both reject the restrictions imposed in moving from the VAR(2) to the VAR(1).

The tests for the number of cointegrating vectors are presented in table 3.3, based on a VAR with a

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[9] In calculating the likelihood ratio test statistic we make an adjustment for the relatively small sample size. Thus, we have

$$-2\log\lambda = (T-k)(\log|\Sigma_R| - \log|\Sigma_U|)$$

where k is the average number of regressors in the VAR, $|\Sigma_R|$ is the determinant of the covariance matrix from the restricted model and $|\Sigma_U|$ is the determinant of the covariance matrix from the unrestricted model.

TABLE 3.2 : LIKELIHOOD RATIO TESTS FOR VAR LENGTH (DISTRIBUTED $\chi^2(4)$)

	MARCH 1	MARCH 2	MARCH 3
5 to 4	2.813	3.575	3.231
4 to 3	0.949	4.254	5.600
3 to 2	2.214	4.617	6.042
2 to 1	5.361	4.237	9.381
	JUNE 1	**JUNE 2**	**JUNE 3**
5 to 4	1.274	0.416	4.722
4 to 3	3.714	4.190	2.249
3 to 2	3.512	5.149	0.269
2 to 1	5.467	3.078	3.959

* denotes significant at 5%
** denotes significant at 1%

TABLE 3.2 CONT...

	SEPTEMBER 1	SEPTEMBER 2	SEPTEMBER 3
5 to 4	3.216	0.774	2.382
4 to 3	1.070	1.728	0.229
3 to 2	4.474	1.346	2.247
2 to 1	1.904	6.718	17.66**
	DECEMBER 1	**DECEMBER 2**	**DECEMBER 3**
5 to 4	3.730	3.200	2.155
4 to 3	1.833	1.839	9.390
3 to 2	2.836	3.969	7.062
2 to 1	1.996	8.997	9.641*

* denotes significant at 5%
** denotes significant at 1%

Table 3.3 : Tests For The Number Of Cointegrating Vectors

(H_o^a:r=0, H_1^a:r=1; H_o^b:r≤1, H_1^b:r=2)

	MARCH 1		MARCH 2		MARCH 3	
	H_o^a	H_o^b	H_o^a	H_o^b	H_o^a	H_o^b
λ_{max}	51.362	2.7372	22.063	1.1870	32.472	0.6431
λ_{trace}	54.099	2.7372	23.251	1.1870	33.115	0.6431

	JUNE 1		JUNE 2		JUNE 3	
	H_o^a	H_o^b	H_o^a	H_o^b	H_o^a	H_o^b
λ_{max}	22.774	2.0861	15.135	2.2129	24.494	3.0414
λ_{trace}	24.860	2.0861	17.348	2.2129	27.535	3.0414

Critical Values (Johansen and Juselius (1990) Table A.2.)

		90%	95%
λ_{max}	r=0	6.691	8.083
	r=1	12.78	14.60
λ_{trace}	r=0	6.691	8.083
	r=1	15.58	17.84

TABLE 3.3 CONT...

(H_o^a: r=0, H_1^a: r=1; H_o^b: r≤1, H_1^b: r=2)

	SEPTEMBER 1		SEPTEMBER 2		SEPTEMBER 3	
	H_o^a	H_o^b	H_o^a	H_o^b	H_o^a	H_o^b
λ_{max}	38.107	3.4075	13.344	3.2202	4.1211	2.3407
λ_{trace}	41.514	3.4075	16.564	3.2202	6.4618	2.3407
	DECEMBER 1		**DECEMBER 2**		**DECEMBER 3**	
	H_o^a	H_o^b	H_o^a	H_o^b	H_o^a	H_o^b
λ_{max}	30.310	1.2532	26.126	1.8362	13.665	3.6783
λ_{trace}	31.563	1.2532	27.962	1.8362	17.344	3.6783

Critical Values (Johansen and Juselius (1990) Table A.2.)

		90%	95%
λ_{max}	r=0	6.691	8.083
	r=1	12.78	14.60
λ_{trace}	r=0	6.691	8.083
	r=1	15.58	17.84

constant. With the exception of the September contract three months from expiration (and the possible exception of the September and December contracts two and three months from expiration respectively, where the null hypothesis of zero cointegrating vectors is rejected at 90% but not 95%) the null hypothesis of zero cointegrating vectors is rejected whilst the null of one cointegrating vector cannot be rejected, confirming the results of the unit root tests. The implications of failing to account for cointegration discussed earlier, then, appear to be well justified.

The interesting question now is whether or not the basis is the cointegrating vector. The cointegrating vectors and unit root tests on the basis are presented in tables 3.4 and 3.5. The cointegrating vectors are plotted in figures 3 to 14. In all cases (even when there is apparently no cointegration) the cointegrating vector is the basis with the estimates of the coefficient on the Index price being remarkably close to one. The first necessary condition for effectively functioning markets, and by implication efficiency, then, would appear to hold for the UK stock and stock index futures markets.

Let us now focus on the identification issue mentioned earlier. The arguments presented in the previous

TABLE 3.4 : ESTIMATED COINTEGRATING VECTORS

MARCH 1	f = 1.014s
MARCH 2	f = 1.104s
MARCH 3	f = 1.016s
JUNE 1	f = 1.008s
JUNE 2	f = 1.028s
JUNE 3	f = 1.020s
SEPTEMBER 1	f = 0.998s
SEPTEMBER 2	f = 0.999s
SEPTEMBER 3	f = 1.026s
DECEMBER 1	f = 0.999s
DECEMBER 2	f = 1.023s
DECEMBER 3	f = 1.018s

TABLE 3.5 : BASIS UNIT ROOT TEST RESULTS (l=1)

	$Z(\Phi_1)$	$Z(\alpha^*)$	$Z(t_{\alpha^*})$
MARCH 1	27.98	-58.55	-6.306
MARCH 2	11.96	-28.92	-4.185
MARCH 3	18.48	-32.14	-4.523
JUNE 1	16.26	-32.03	-4.452
JUNE 2	4.629	-14.93	-2.845
JUNE 3	14.64	-30.79	-4.329
SEPTEMBER 1	21.60	-59.23	-6.323
SEPTEMBER 2	9.771	-19.20	-3.281
SEPTEMBER 3	5.025	-8.257	-1.952
DECEMBER 1	18.57	-48.24	-5.564
DECEMBER 2	17.16	-29.90	-4.221
DECEMBER 3	11.85	-28.71	-4.072

FIGURE 3.3 : MARCH ONE MONTH (EXPIRATION MONTH) COINTEGRATING VECTOR

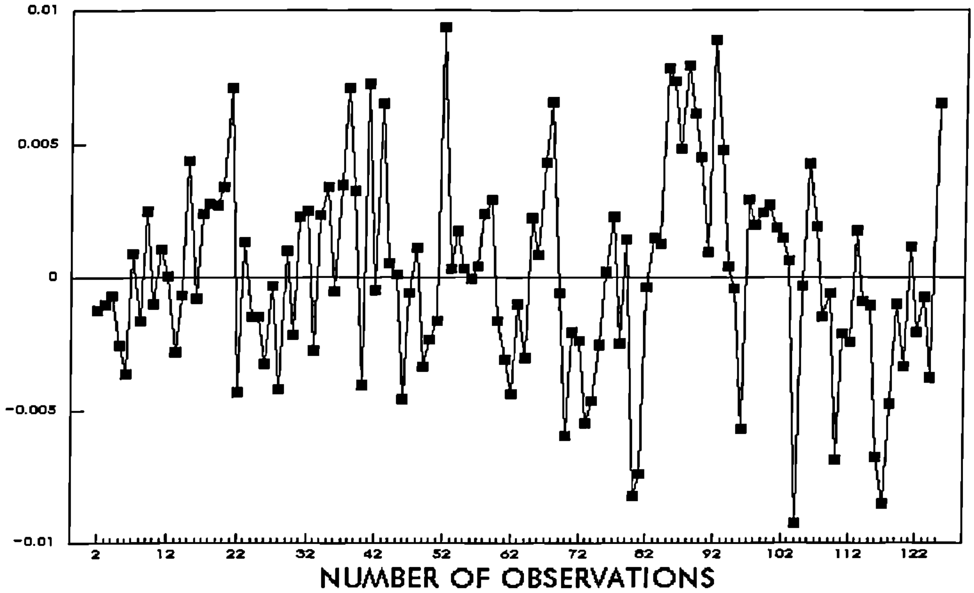


FIGURE 3.4 : MARCH TWO MONTHS TO EXPIRATION COINTEGRATING VECTOR

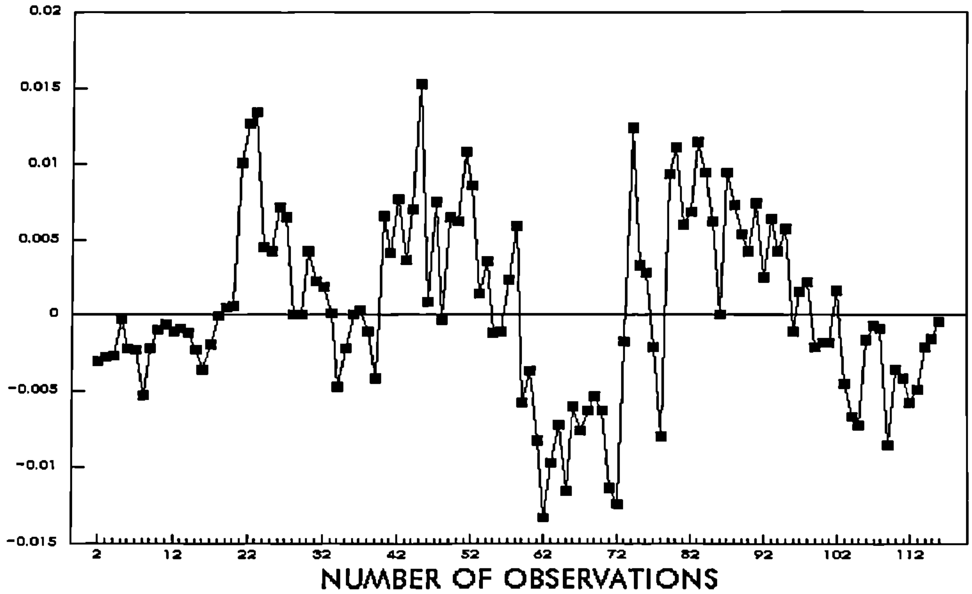


FIGURE 3.5 : MARCH THREE MONTHS TO EXPIRATION
COINTEGRATING VECTOR

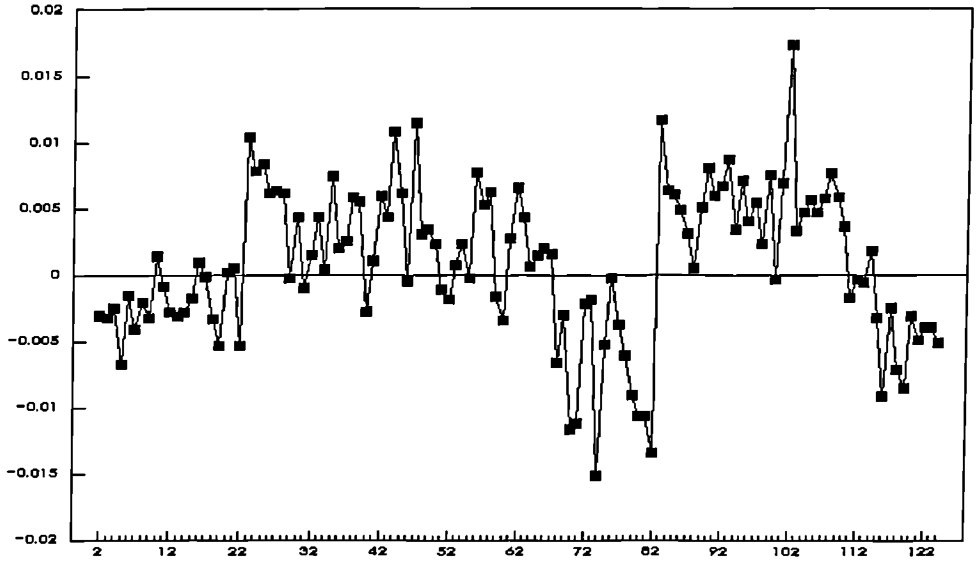


FIGURE 3.6 : JUNE ONE MONTH (EXPIRATION MONTH)
COINTEGRATING VECTOR

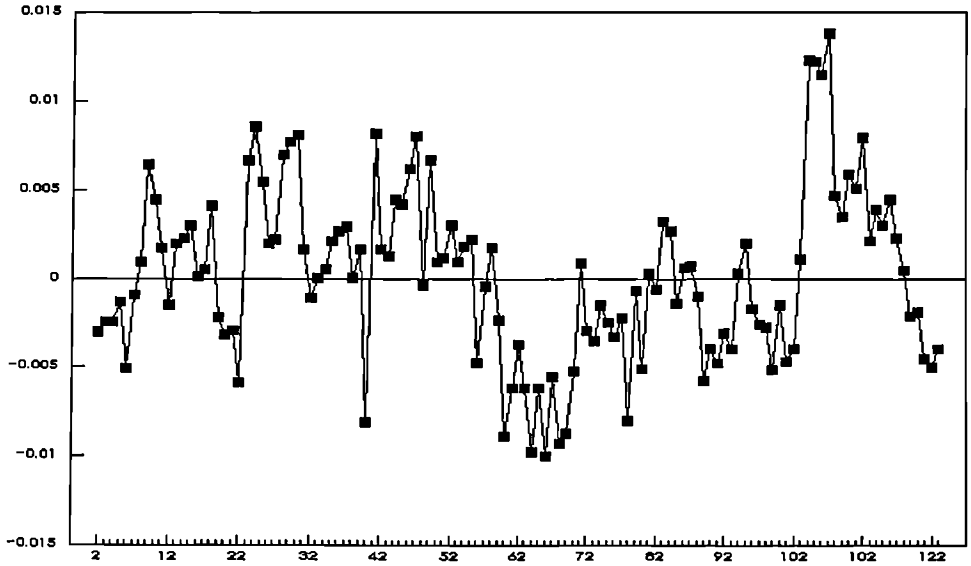


FIGURE 3.7 : JUNE TWO MONTHS TO EXPIRATION
COINTEGRATING VECTOR

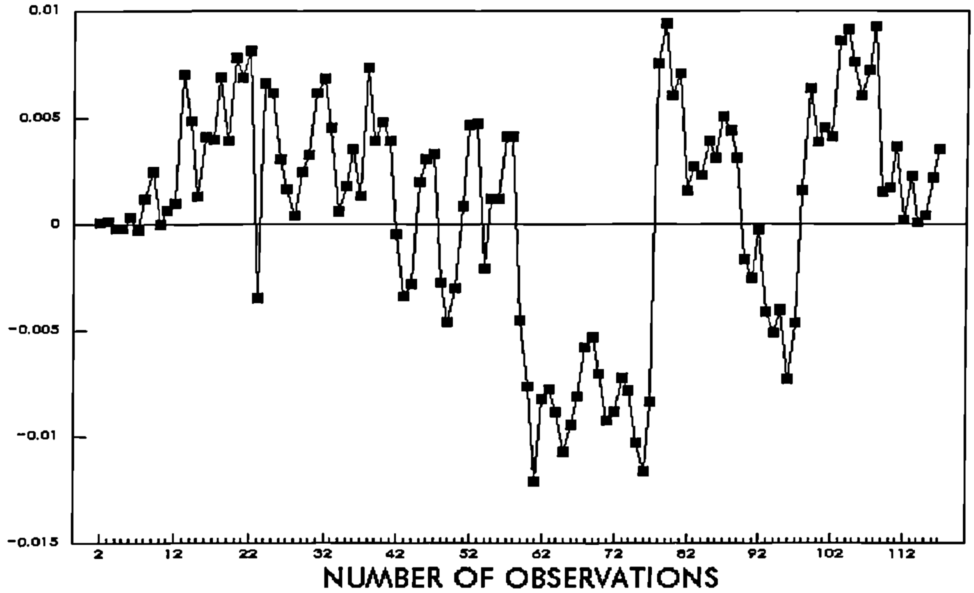


FIGURE 3.8 : JUNE THREE MONTHS TO EXPIRATION
COINTEGRATING VECTOR

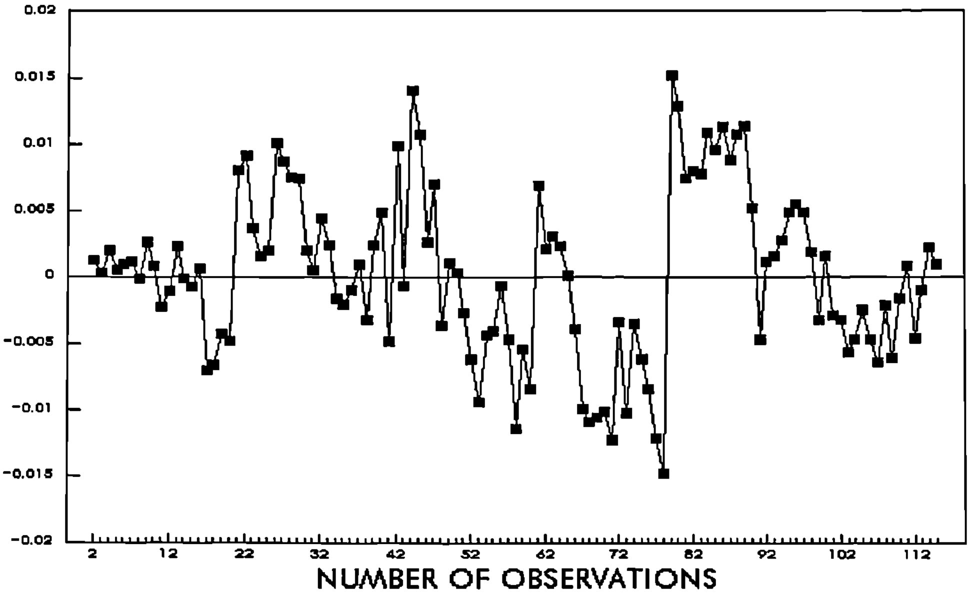


FIGURE 3.9 : SEPTEMBER ONE MONTH (EXPIRATION MONTH) COINTEGRATING VECTOR

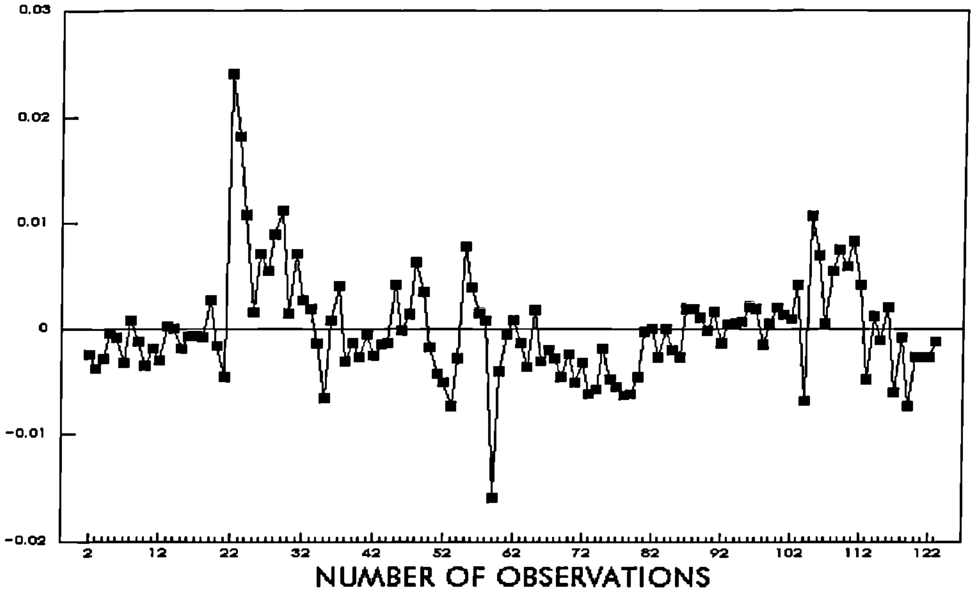


FIGURE 3.10 : SEPTEMBER TWO MONTHS TO EXPIRATION COINTEGRATING VECTOR

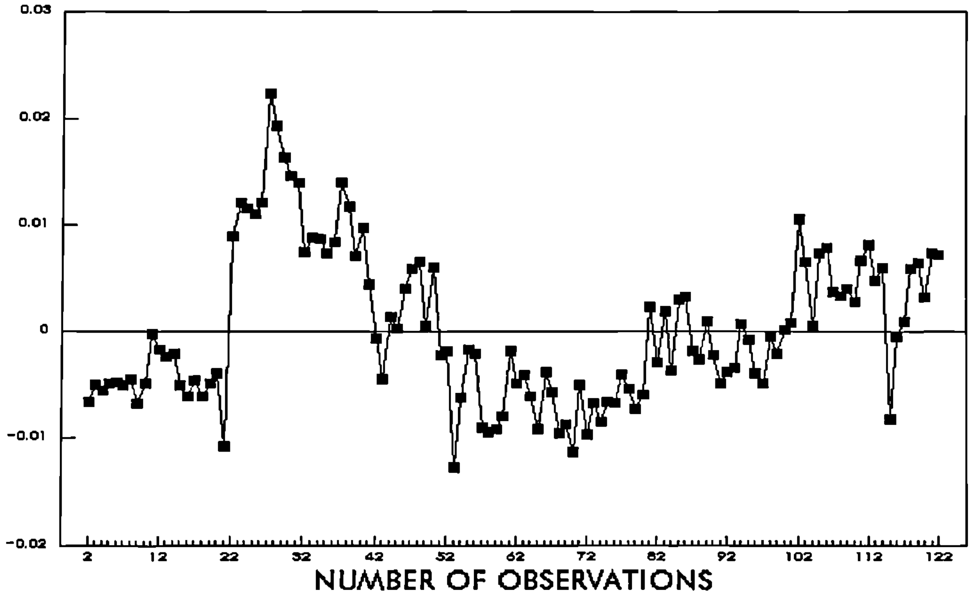


FIGURE 3.11 : SEPTEMBER THREE MONTHS TO EXPIRATION COINTEGRATING VECTOR

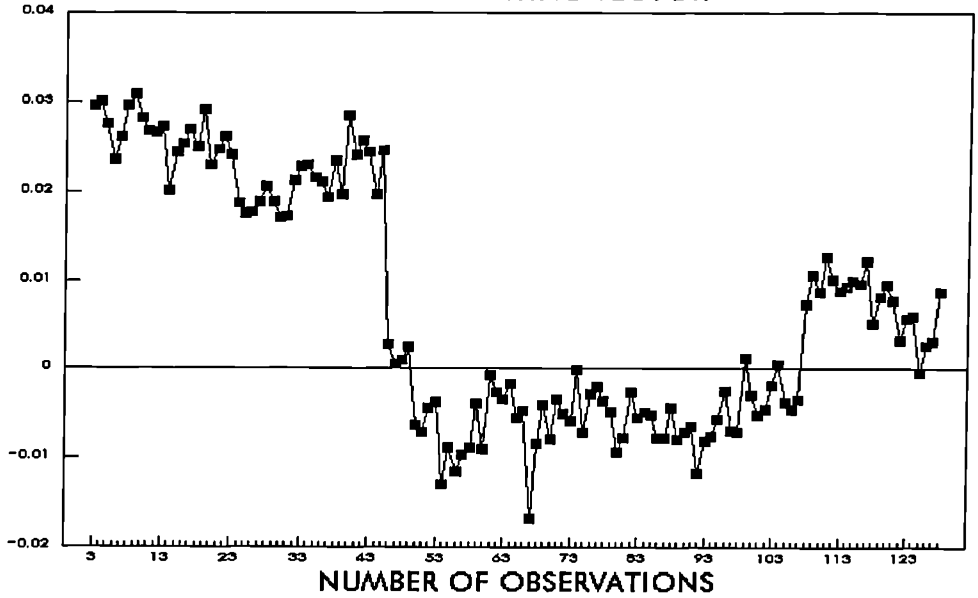


FIGURE 3.12 : DECEMBER ONE MONTH (EXPIRATION MONTH) COINTEGRATING VECTOR

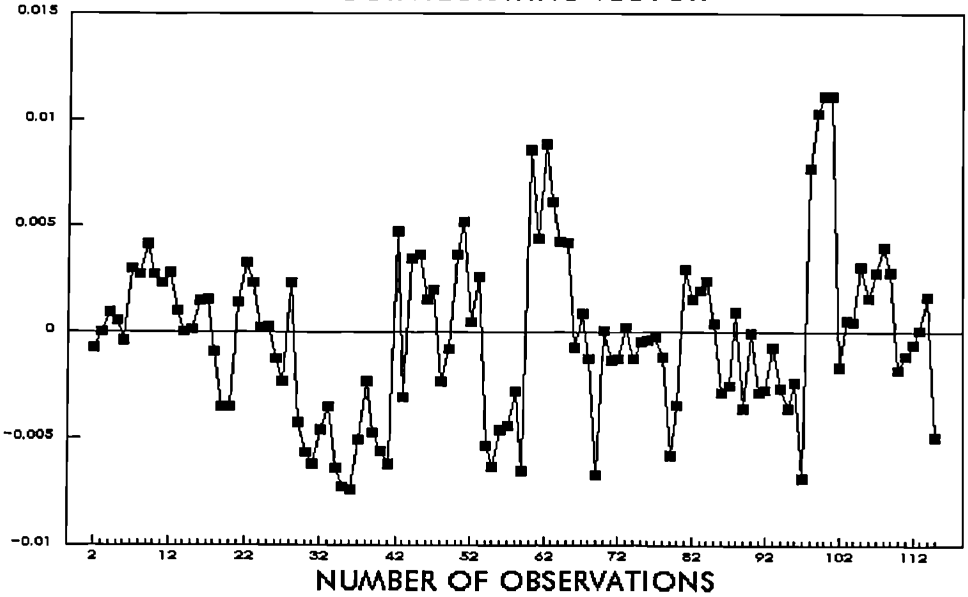


FIGURE 3.13 : DECEMBER TWO MONTHS TO EXPIRATION COINTEGRATING VECTOR

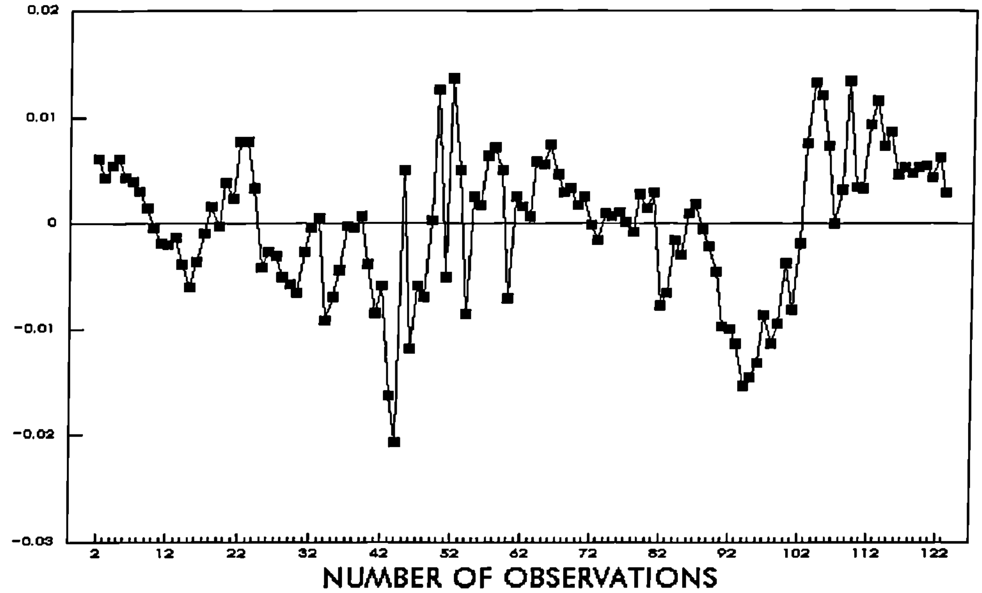
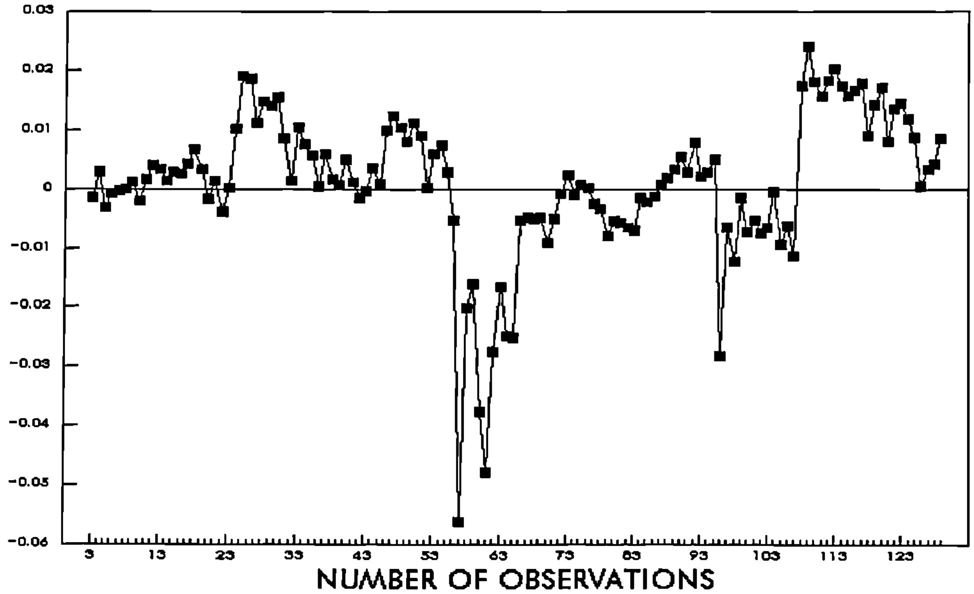


FIGURE 3.14 : DECEMBER THREE MONTHS TO EXPIRATION COINTEGRATING VECTOR



chapter are that the equity market can be said to be functioning effectively if the system describing the pricing relationship between the FTSE 100 Index and index futures contract cannot be identified. By implication, of course, the futures market must be fulfilling its prescribed role if this lack of identification is the case. An alternate way to phrase this is that both markets depend upon a common factor only and that common factor is the short-run disequilibrium between prices in the two markets which, as we know, theoretically and empirically is the basis.

We have already seen that with the exception of the September and December futures contract three months from expiration the most appropriate model is a VAR(1) reparameterised in error correction form. That the VAR(1) is the appropriate model for all bar two of the contracts confirms that the identification condition is satisfied, that is, from the VAR(1), the system cannot be identified.[10]

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[10] This can be confirmed by writing out the structural model for the reduced form system :

$$\beta_{11}\Delta f_t + \beta_{12}\Delta i_t + \beta_{13}(f-s)_{t-1} + \beta_{14} = e_{1t}$$
$$\beta_{21}\Delta i_t + \beta_{22}\Delta f_t + \beta_{23}(f-s)_{t-1} + \beta_{24} = e_{2t}$$

Clearly, both equations cannot be distinguished and will differ only by normalisation.

TABLE 3.6 : MISSPECIFICATION AND SPECIFICATION OF THE PREFERRED MODEL

$$\begin{bmatrix} \Delta f_t \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \end{bmatrix} + \begin{bmatrix} -\alpha_{11} \\ \alpha_{21} \end{bmatrix} \begin{bmatrix} f_{t-1} - s_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

	$\chi^2(2)$	$\chi^2(12)$	$F(,..,)$	$F_1(,.,)$	$F_2(,.,)$
MARCH 1					
FUTURES	3.508	6.100	16.96**		
INDEX	4.594	5.144	(2,122)		
MARCH 2					
FUTURES	1.998	23.83*	7.20**		
INDEX	3.482	10.76	(2,113)		
MARCH 3					
FUTURES	3.478	7.345	7.28**		
INDEX	0.670	14.21	(2.121)		

* denotes significant at 5%
** denotes significant at 1%

$\chi^2(2)$ is a test for normality, $\chi^2(12)$ is the Box-Pierce (1970) test for serial correlation $F(,.,)$ tests the joint significance of the basis and where appropriate $F_1(,.,)$ and $F_2(,.,)$ test the joint significance of Δf_{t-1} and Δs_{t-1} respectively (degrees of freedom in parentheses).

TABLE 3.6 CONT...

$$\begin{bmatrix}\Delta f_t \\ \Delta s_t\end{bmatrix} = \begin{bmatrix}\alpha_{10} \\ \alpha_{20}\end{bmatrix} + \begin{bmatrix}-\alpha_{11} \\ \alpha_{21}\end{bmatrix}\left[f_{t-1} - s_{t-1}\right] + \begin{bmatrix}u_{1t} \\ u_{2t}\end{bmatrix}$$

	$\chi^2(2)$	$\chi^2(12)$	F(,.,)	$F_1(,.,)$	$F_2(,.,)$
JUNE 1					
FUTURES	2.576	13.85	9.32**		
INDEX	2.703	11.23	(2,120)		
JUNE 2					
FUTURES	1.019	6.079	3.62*		
INDEX	2.703	8.467	(2,114)		
JUNE 3					
FUTURES	0.103	11.04	4.86**		
INDEX	0.879	14.85	(2,111)		

* denotes significant at 5%
** denotes significant at 1%

$\chi^2(2)$ is a test for normality, $\chi^2(12)$ is the Box-Pierce (1970) test for serial correlation F(,.,) tests the joint significance of the basis and where appropriate $F_1(,.,)$ and $F_2(,.,)$ test the joint significance of Δf_{t-1} and Δs_{t-1} respectively (degrees of freedom in parentheses).

TABLE 3.6 CONT...

$$\begin{bmatrix} \Delta f_t \\ \Delta s_t \end{bmatrix} = \begin{bmatrix} \alpha_{10} \\ \alpha_{20} \end{bmatrix} + \begin{bmatrix} -\alpha_{11} \\ \alpha_{21} \end{bmatrix} \begin{bmatrix} f_{t-1} - s_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix}$$

	$\chi^2(2)$	$\chi^2(12)$	F(,.,)	F_1(,.,)	F_2(,.,)
SEPTEMBER 1					
FUTURES	5.687	10.49	21.94**		
INDEX	5.186	18.38	(2,120)		
SEPTEMBER 2					
FUTURES	0.618	15.97	5.01**		
INDEX	0.469	21.32*	(2,118)		
SEPTEMBER 3					
FUTURES	0.998	5.965	1.02	10.65**	6.87**
INDEX	0.469	9.866	(2,122)	(2,122)	(2,122)

* denotes significant at 5%
** denotes significant at 1%

$\chi^2(2)$ is a test for normality, $\chi^2(12)$ is the Box-Pierce (1970) test for serial correlation F(,.,) tests the joint significance of the basis and where appropriate F_1(,.,) and F_2(,.,) test the joint significance of Δf_{t-1} and Δs_{t-1} respectively (degrees of freedom in parentheses).

TABLE 3.6 CONT...

$$\begin{bmatrix}\Delta f_t \\ \Delta s_t\end{bmatrix} = \begin{bmatrix}\alpha_{10} \\ \alpha_{20}\end{bmatrix} + \begin{bmatrix}-\alpha_{11} \\ \alpha_{21}\end{bmatrix}[f_{t-1} - s_{t-1}] + \begin{bmatrix}u_{1t} \\ u_{2t}\end{bmatrix}$$

	$\chi^2(2)$	$\chi^2(12)$	F(,..,)	F_1(,..,)	F_2(,..,)
DECEMBER 1					
FUTURES	1.822	9.594	6.54**		
INDEX	2.086	12.88	(2,111)		
DECEMBER 2					
FUTURES	2.573	8.290	9.36**		
INDEX	1.675	10.12	(2,119)		
DECEMBER 3					
FUTURES	1.793	14.91	6.29**	25.44**	15.38**
INDEX	3.972	19.78	(2,122)	(2,122)	(2,122)

* denotes significant at 5%
** denotes significant at 1%

$\chi^2(2)$ is a test for normality, $\chi^2(12)$ is the Box-Pierce (1970) test for serial correlation F(,.,) tests the joint significance of the basis and where appropriate F_1(,.,) and F_2(,.,) test the joint significance of Δf_{t-1} and Δs_{t-1} respectively (degrees of freedom in parentheses).

Tests of the specification (that is, the joint significance of the basis in the VAR and where appropriate the significance of Δf_{t-1} and Δs_{t-1}) and misspecification tests for normality and serial correlation are reported in table 3.6. It is clear from this that the VAR, as given by (2.15) in chapter two, which is reproduced at the beginning of this chapter, is well specified in every case, with only marginal evidence of serial correlation in two cases.

With the exception of September three months from expiration the basis is jointly significant at least at the 5% level in every other case, confirming the fact that both markets depend on one common factor, the basis, and that essentially they are indistinguishable. Moreover, examination of figures 15 through 38, which plot break point Chow tests constructed from recursive estimation of the VAR model for each of the data sets together with scaled 1% critical values, reveals that the error correction parameterisation of the VAR, as well as capturing the salient features of the pricing relationship between the two markets, provides in most cases a remarkably stable model.

The cases where the model seems to be quite unstable, with varying degrees of instability, relate almost exclusively to the December stock index futures

FIGURE 3.15 : FUTURES EQUATION, MARCH ONE MONTH
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

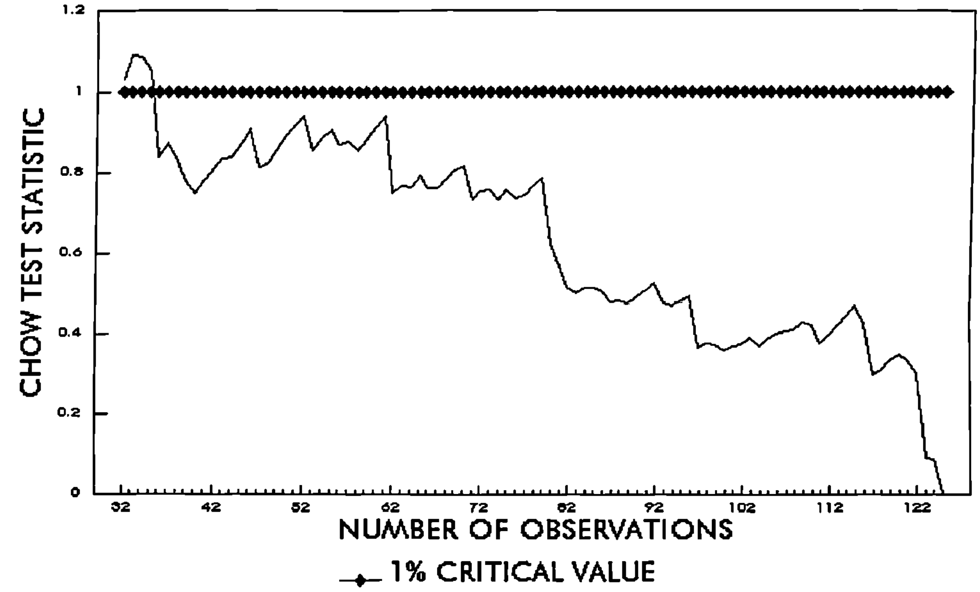


FIGURE 3.16 : INDEX EQUATION, MARCH ONE MONTH
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

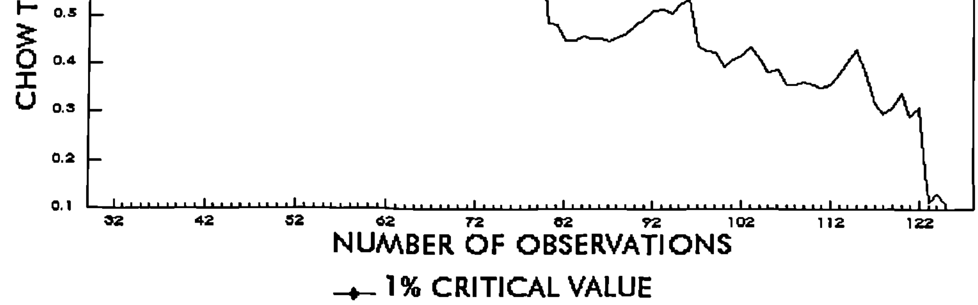


FIGURE 3.17 : FUTURES EQUATION, MARCH TWO MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

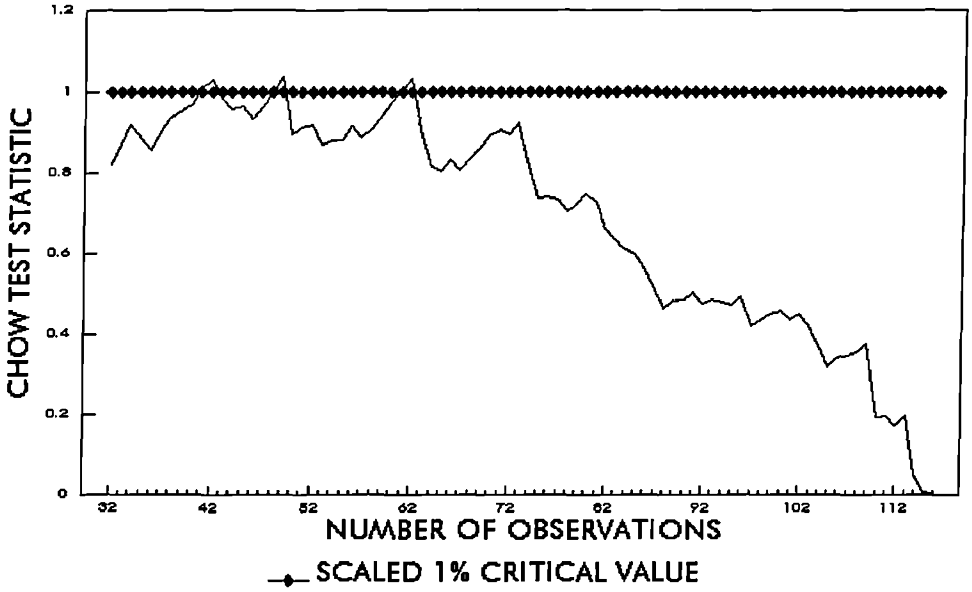


FIGURE 3.18 : INDEX EQUATION, MARCH TWO MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

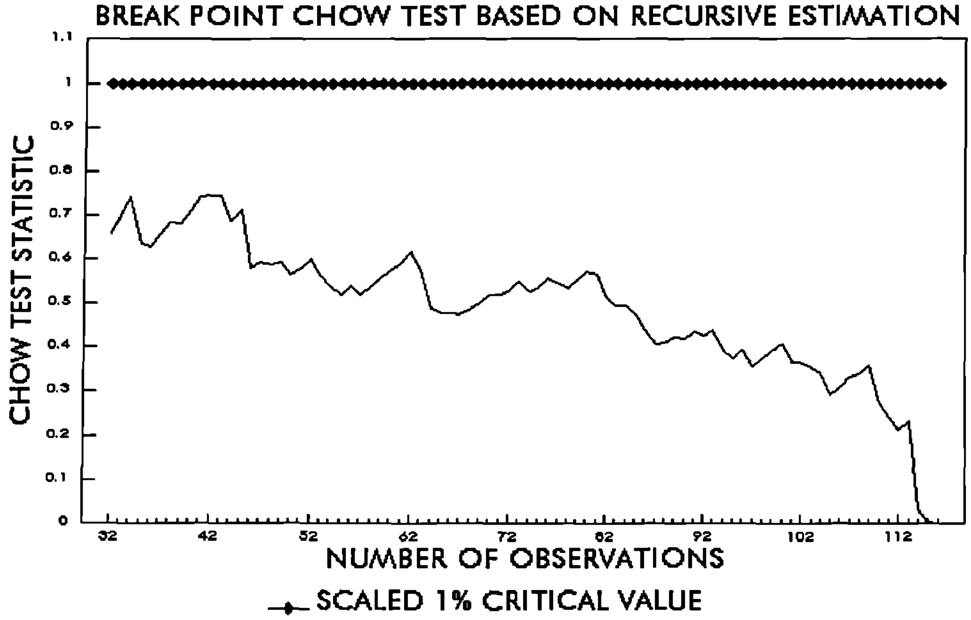


FIGURE 3.19 : FUTURES EQUATION, MARCH THREE MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

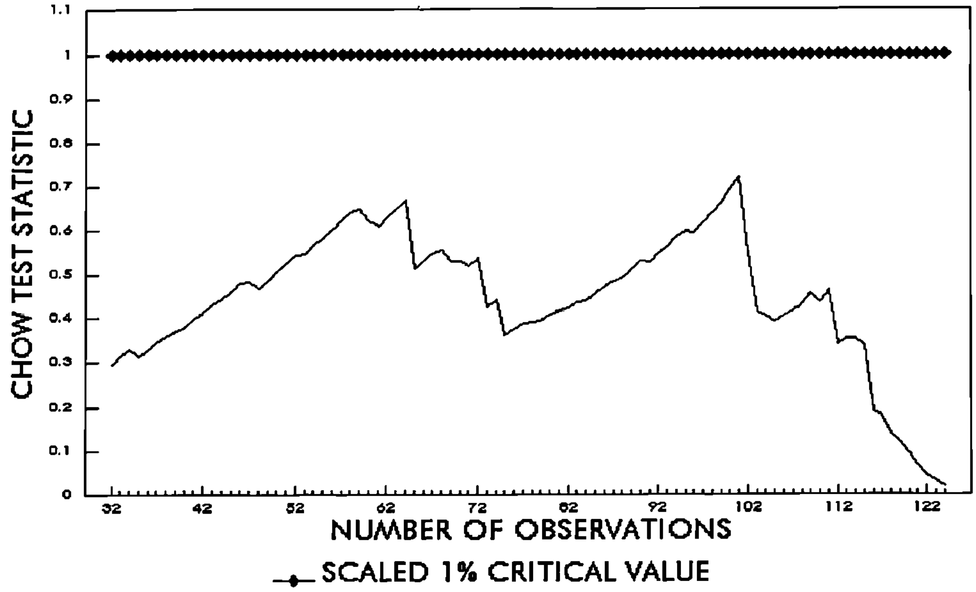


FIGURE 3.20 : INDEX EQUATION, MARCH THREE MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

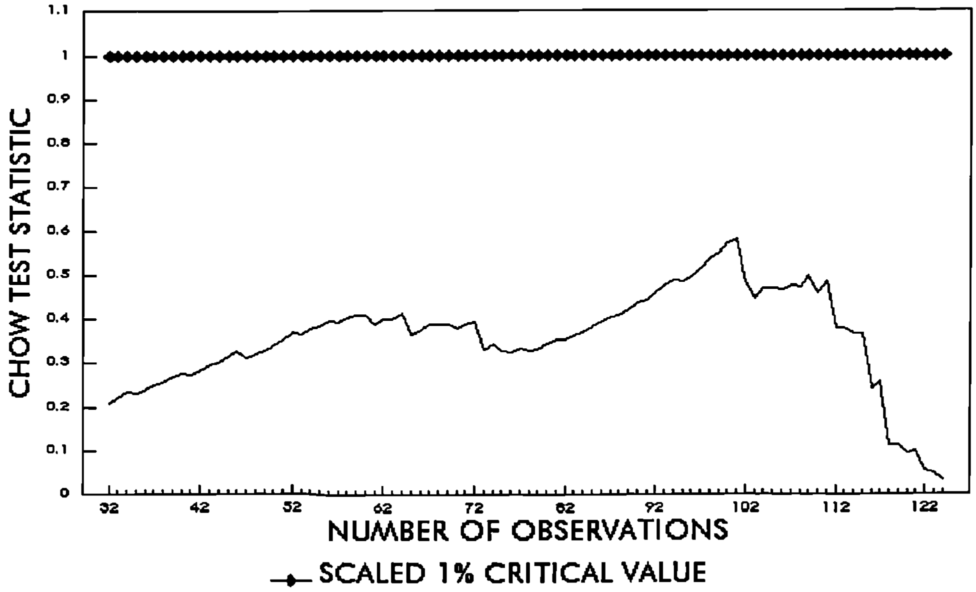


FIGURE 3.21 : FUTURES EQUATION, JUNE ONE MONTH
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

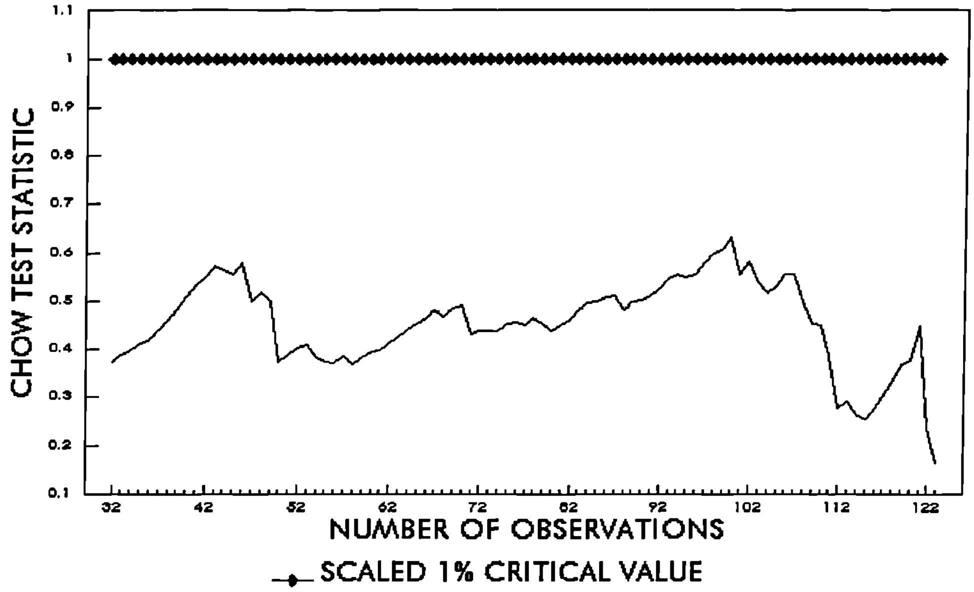


FIGURE 3.22 : INDEX EQUATION, JUNE ONE MONTH
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

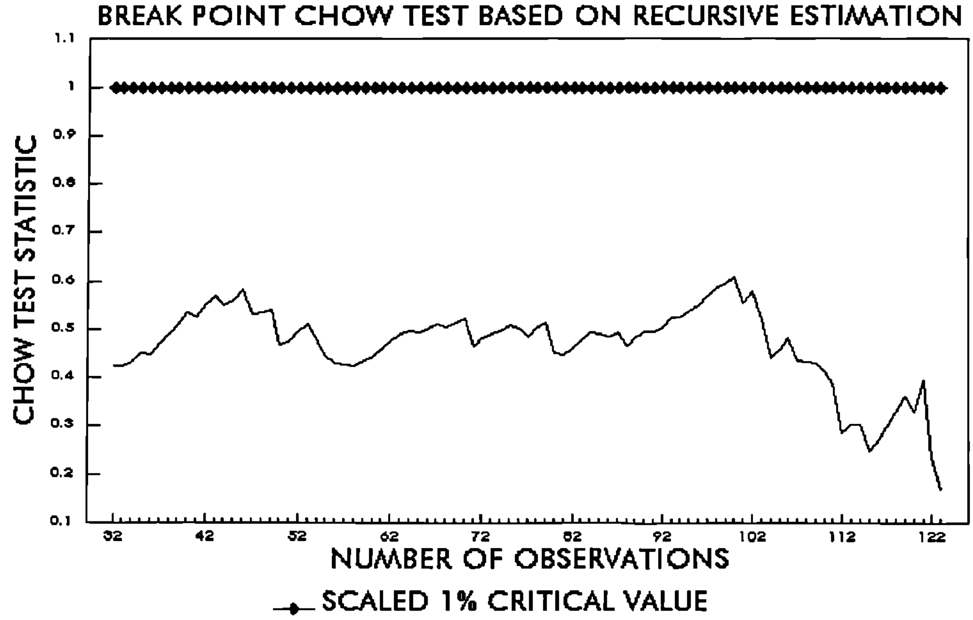


FIGURE 3.23 : FUTURES EQUATION, JUNE TWO MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

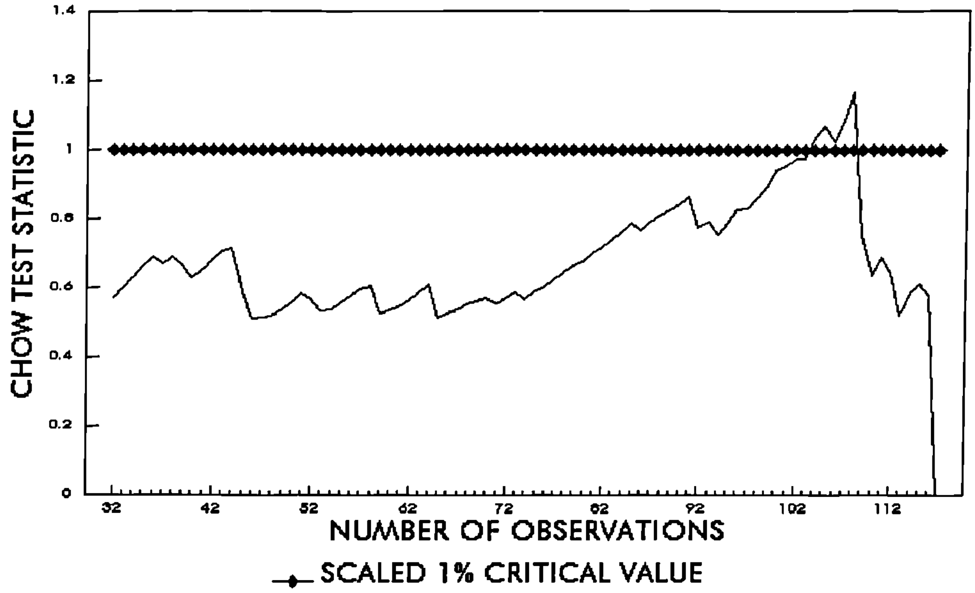


FIGURE 3.24 : INDEX EQUATION, JUNE TWO MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

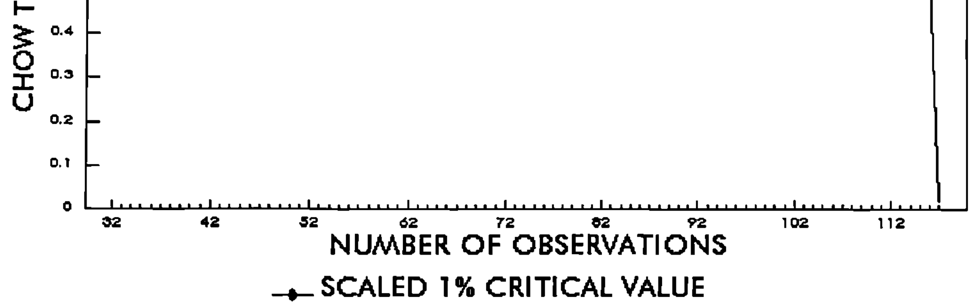


FIGURE 3.25 : FUTURES EQUATION, JUNE THREE MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

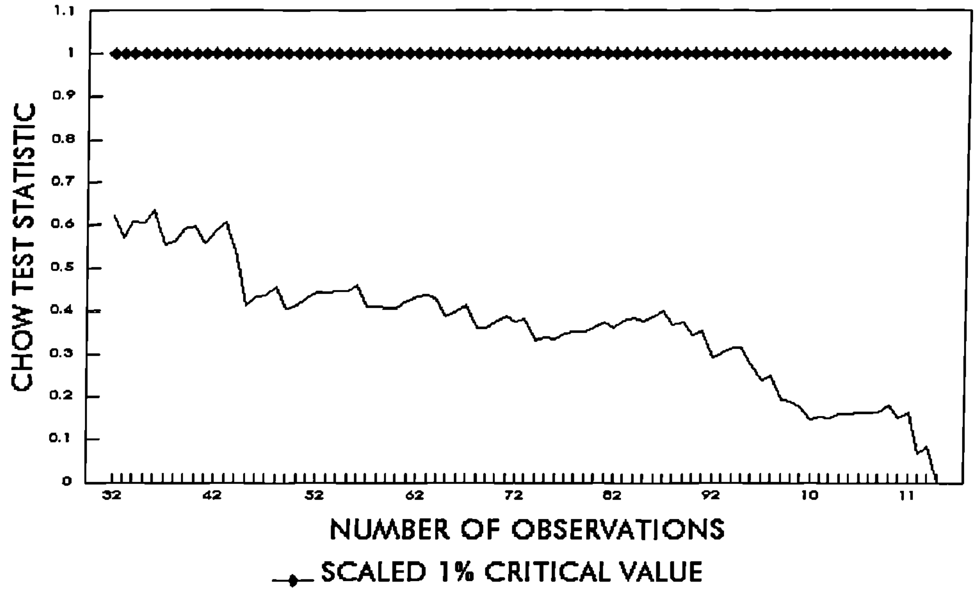


FIGURE 3.26 : INDEX EQUATION, JUNE THREE MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

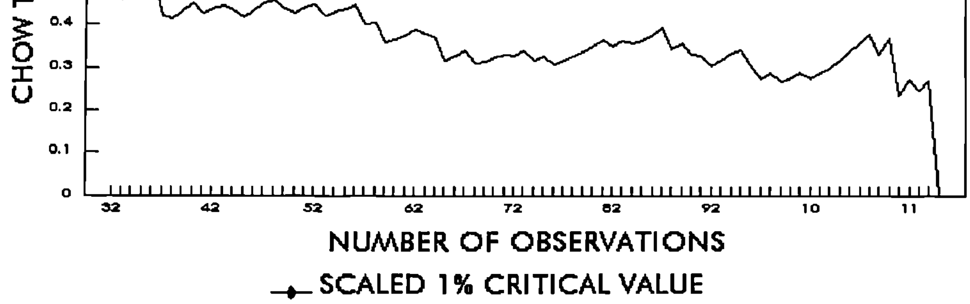


FIGURE 3.27 : FUTURES EQUATION, SEPTEMBER ONE MONTH BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

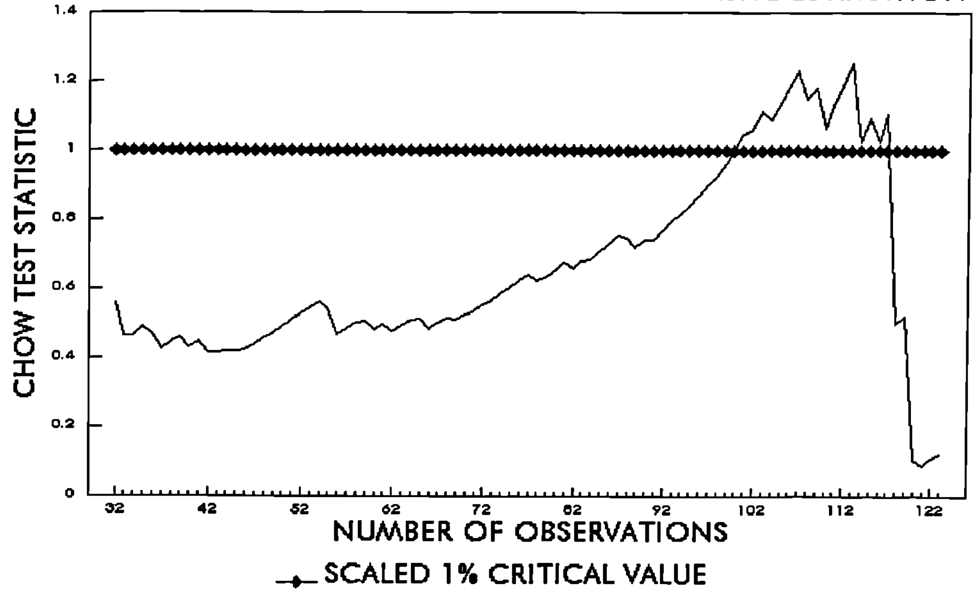


FIGURE 3.28 : INDEX EQUATION, SEPTEMBER ONE MONTH BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

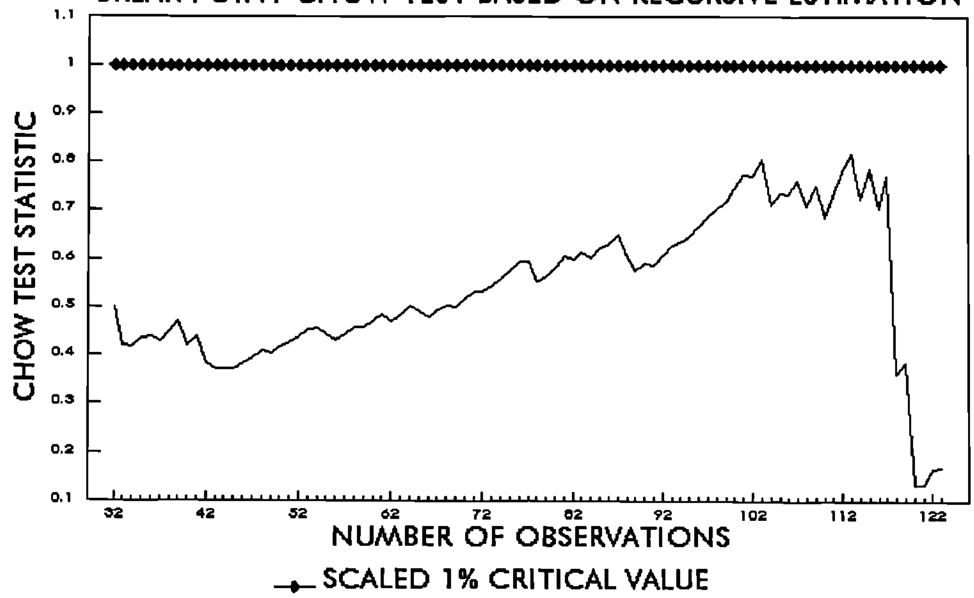


FIGURE 3.29 : FUTURES EQUATION, SEPTEMBER 2 MONTHS BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

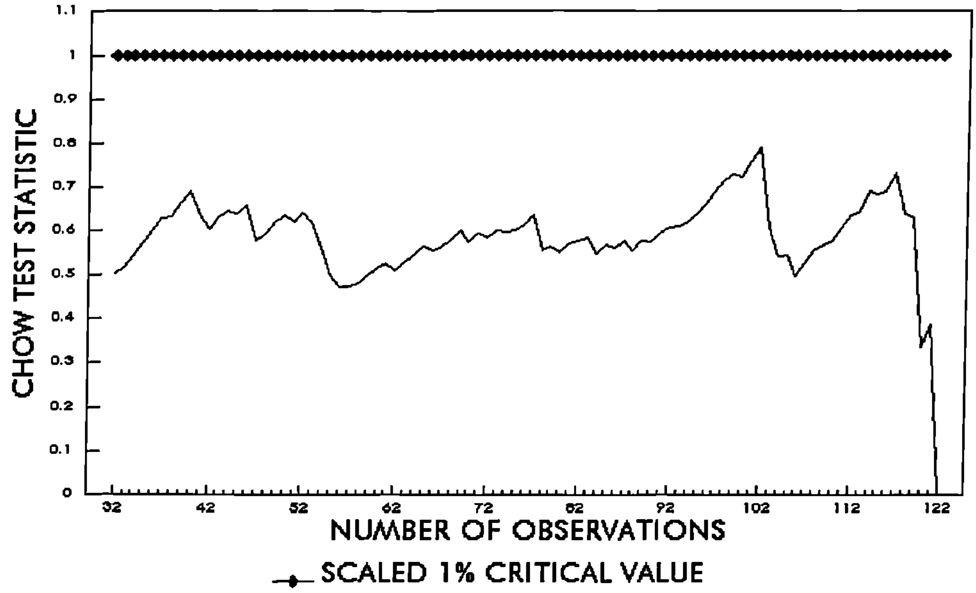


FIGURE 3.30 : INDEX EQUATION, SEPTEMBER 2 MONTHS BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

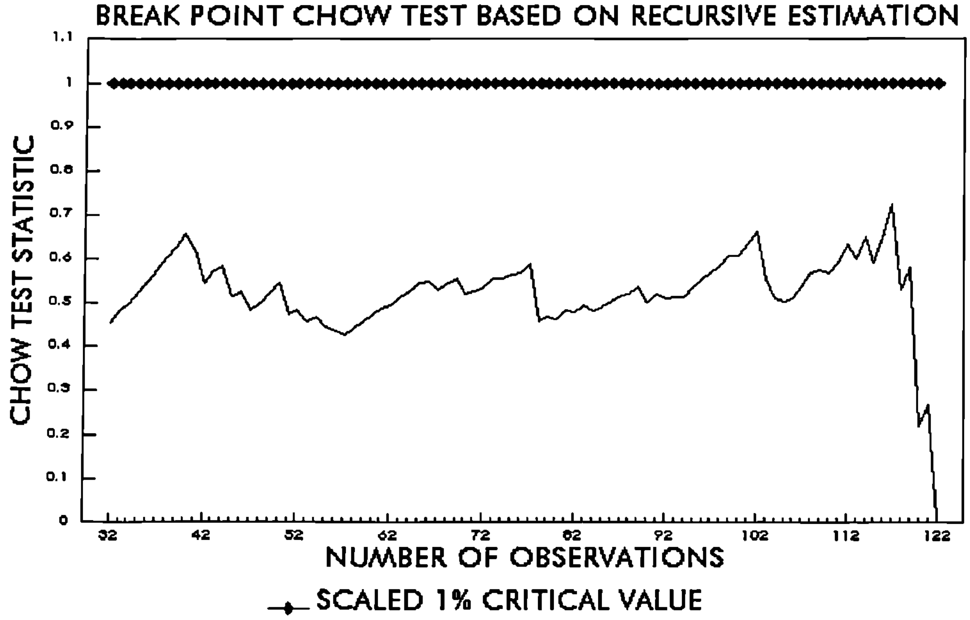


FIGURE 3.31 : FUTURES EQUATION, SEPTEMBER THREE MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

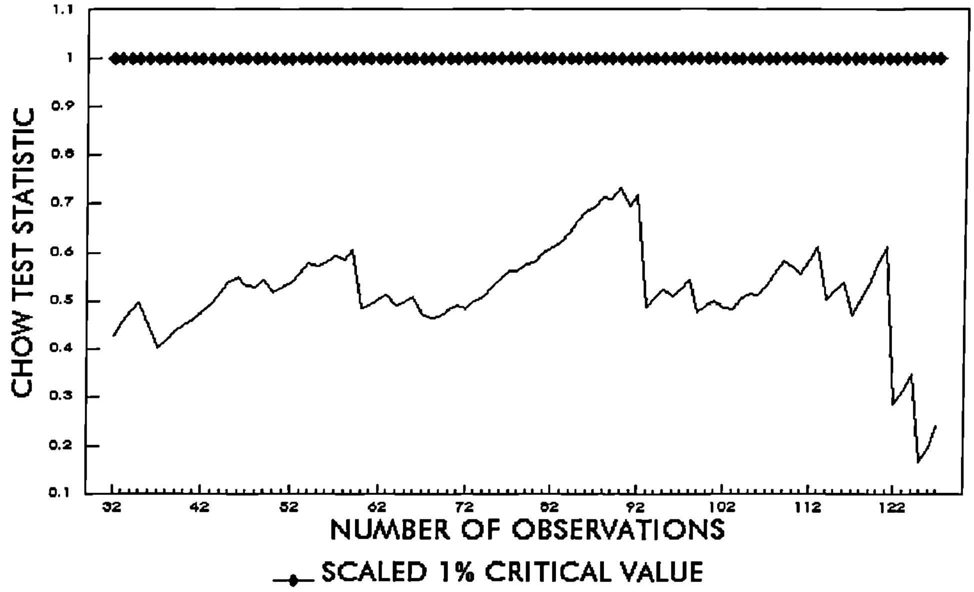


FIGURE 3.32 : INDEX EQUATION, SEPTEMBER THREE MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

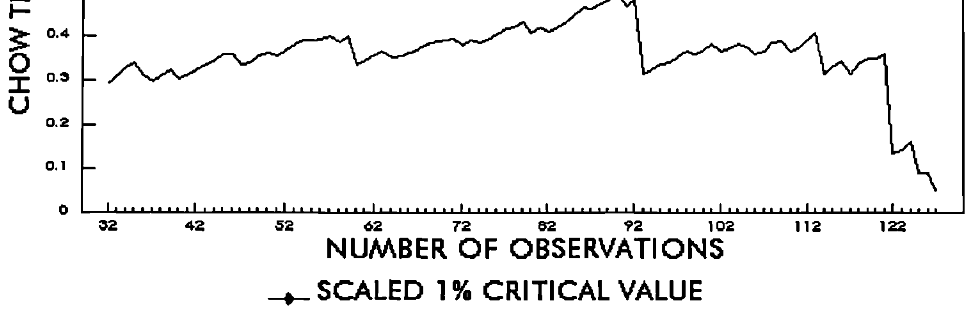


FIGURE 3.33 : FUTURES EQUATION, DECEMBER ONE MONTH BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

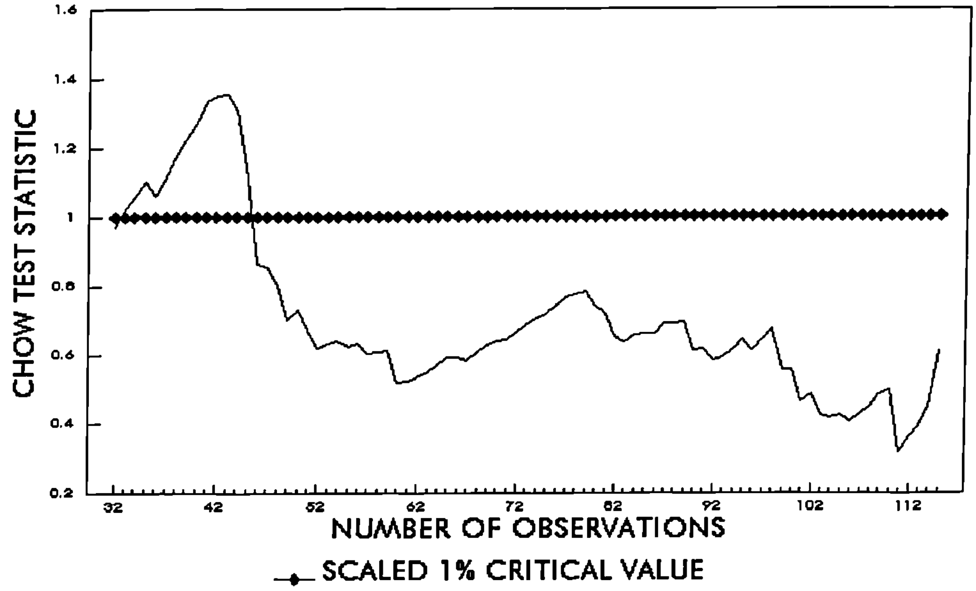


FIGURE 3.34 : INDEX EQUATION, DECEMBER ONE MONTH BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

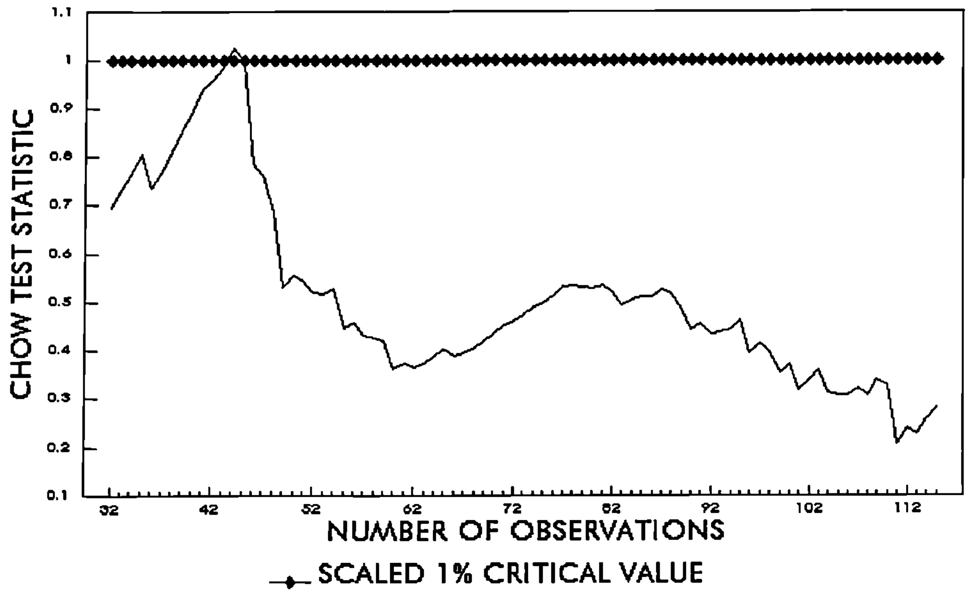


FIGURE 3.35 : FUTURES EQUATION, DECEMBER TWO MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

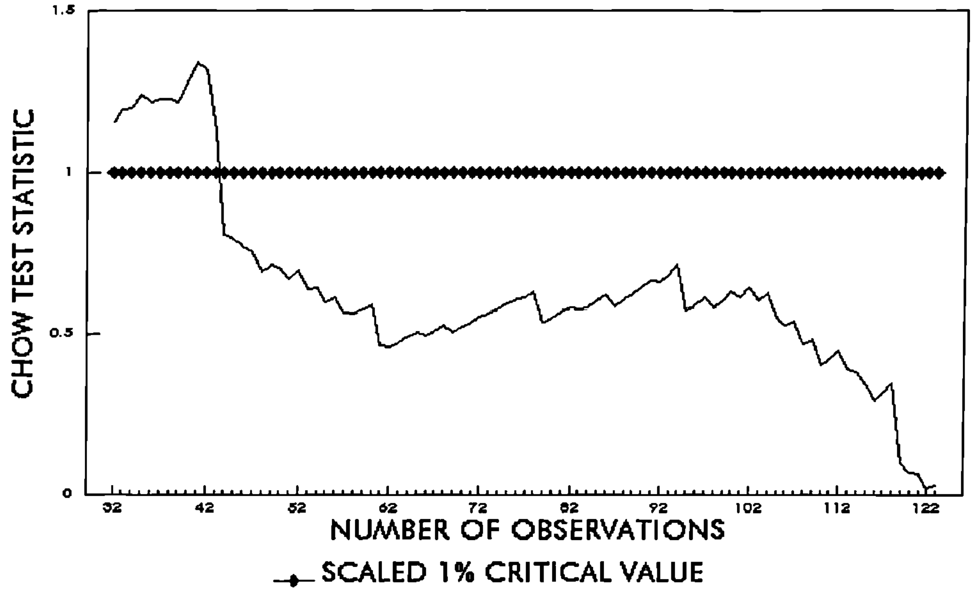


FIGURE 3.36 : INDEX EQUATION, DECEMBER TWO MONTHS
BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

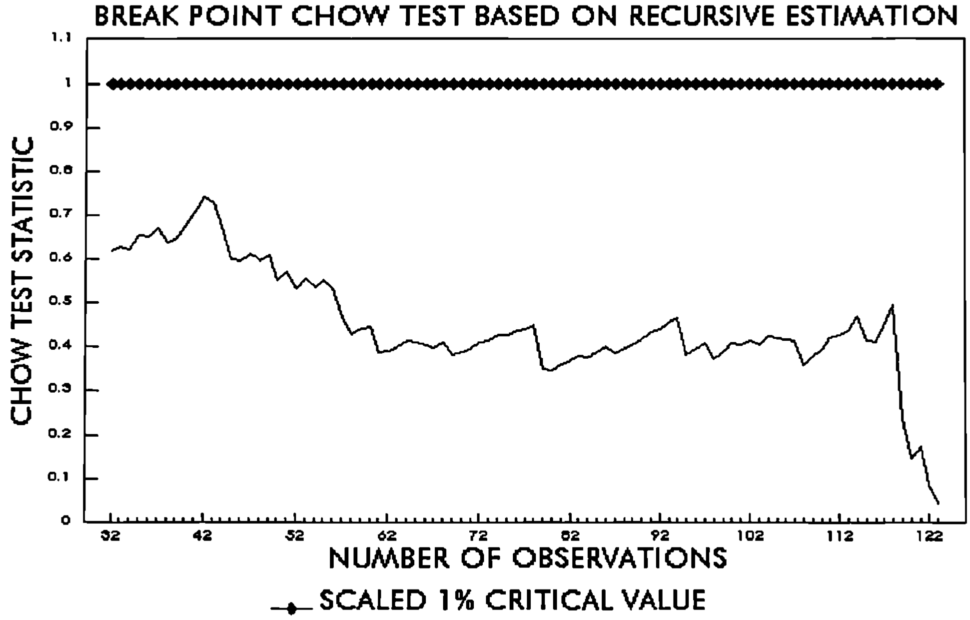


FIGURE 3.37 : FUTURES EQUATION, DECEMBER THREE MONTHS BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION

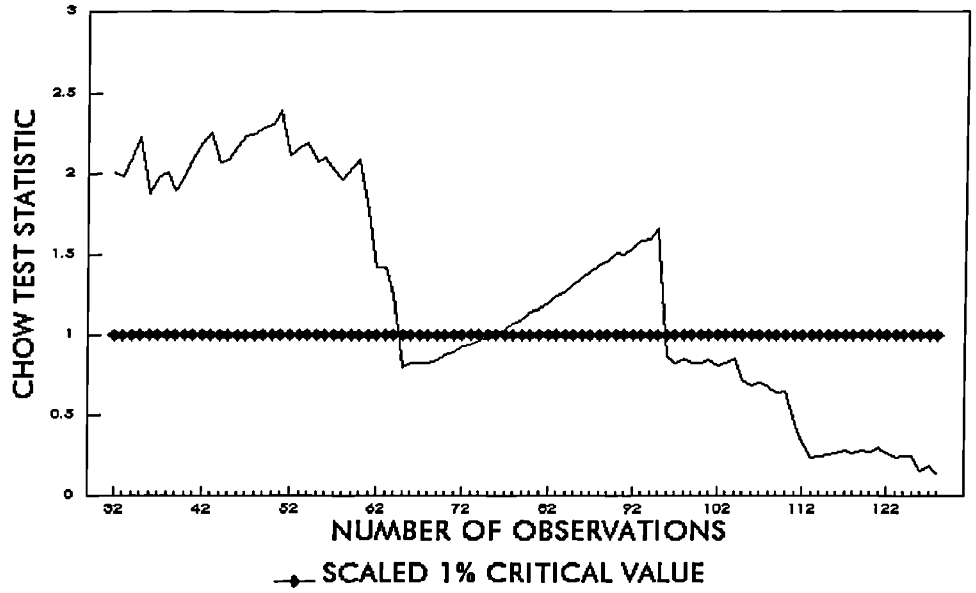
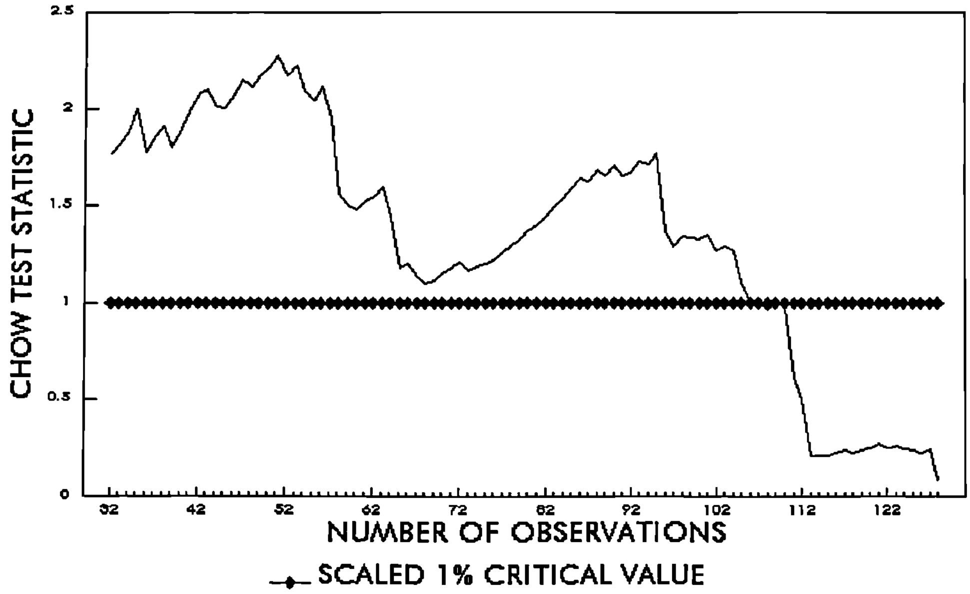


FIGURE 3.38 : INDEX EQUATION, DECEMBER THREE MONTH BREAK POINT CHOW TEST BASED ON RECURSIVE ESTIMATION



contract. When one considers what the December stock index futures contract has been through this is perhaps not so surprising, especially the December contract three months from expiration. Specifically, we have from October 1986 the Big Bang at the Stock Exchange and from October 1987 the stock market crash, both of which had a profound effect on the stock market. Indeed, the effects of these two events spilled over into November and December of their respective years and hence had an effect on the December contract two months from expiration and in the expiration month, though the effect relative to the December contract three months from expiration is much less pronounced.

The important point to note here is that these two events had a very profound impact on the stability of the model which is otherwise an adequate description of the pricing relationship. To anticipate conclusions that will emerge later on, one possible reason for such an effect is the fact that whilst the two markets should effectively function as one, and the evidence presented above certainly seems to confirm this view during stable time periods, they are regulated as two entirely separate entities.[11] The

[11] For example, one difference that immediately springs to mind is the different trading systems operational in both markets, specifically open outcry in the futures market versus the pure dealership,

implication that follows from this is if the markets are treated as two separate entities then during times of pressure they will behave as separate entities.

One final issue is with regard to those contracts and months to expiration for which the equity and futures market appear not to be effectively functioning. Specifically, for the September and December contracts three months from expiration a VAR(2) seems to be the best representation of the system and from this it is possible that a structural model can be developed. If this is the case then the two series are distinguishable from each other.

However, in attempting to move from the reduced form to a structural model the September contract is best described as both series (Δf_t and Δs_t) being independent white noise processes such that the levels of the futures and index series are independent random walks. For the December contract, the structural model is in fact *not identified once contemporaneous* values are included. Indeed, inclusion of the contemporaneous values renders the lagged variables insignificant such that the system, at the structural level, becomes equivalent to the model in footnote 9. Moreover, examination of the graph of the break point

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screen-based trading system operational in the stock market.

chow test statistic reveals in recent years the relationship has stabilised. We will return to this point in the conclusion.

To conclude this section, then, it appears that on a daily basis the necessary conditions, and hence necessary and sufficient condition, for strongly effectively functioning equity markets are satisfied for the FTSE 100 Index and the FTSE 100 Index futures contract. There appear to be only two aberrations : the September contract three months from expiration, where the stability and identification conditions are satisfied but the cointegration condition is not, and the December contract three months from expiration, where the cointegration and identification conditions are satisfied but the stability condition is not, certainly in the pre stock market crash period.

A final point to note here is the efficiency of the markets. With the exception of the September futures contract three months from expiration, the necessary conditions, and hence the necessary and sufficient condition, are satisfied for the system and hence both markets can be said to be efficient. Given that they are efficient, we should observe the basis behaving in a stochastic fashion. Moreover, it should be mean reverting, that is, stationary. We analyse the stochastic properties of the basis in the next section.

3.3. THE STOCHASTIC PROPERTIES OF THE BASIS

The discussion in the previous chapters suggested that by the very definition of a cointegrating vector, the basis will be a stochastic process rather than a path dependent one. The argument is that if the basis is path dependent, it will be a deterministic process. However, given that a cointegrating vector is a stationary linear combination of two stochastic nonstationary variables, one might anticipate that the basis will be path independent. Results from modelling the basis are presented in table 3.7 overleaf. In each case, ten observations at the end of the sample have been retained to evaluate the stability of the coefficients. All of the equations show no signs of misspecification, with diagnostic tests for serial correlation, nonlinearity, non normal errors, heteroscedasticity and ARCH all being insignificant at conventional significance levels. In addition, the model appears to be stable, as shown by the insignificance of the Chow test for predictive failure.

With the exception of the September contract three months from expiration, the basis is adequately described by an autoregressive process of at most

TABLE 3.7 : MODELLING THE BASIS

MARCH 1 $b_t = 0.000 + 0.371b_{t-1} + 0.239b_{t-2}$

(1.107) (4.031) (2.599)

$\eta_1(1,109)=0.940$	$\eta_2(1,109)=0.061$	$\eta_3(2)=2.031$
$\eta_4(1,111)=1.315$	$\eta_5(1,109)=0.004$	$\eta_6(10,110)=1.214$
$\eta_7(1,119)=4.883$		

MARCH 2 $b_t = 0.000 + 0.627b_{t-1} + 0.184b_{t-2}$

(0.937) (7.149) (2.104)

$\eta_1(1,99)=1.250$	$\eta_2(1,99)=1.443$	$\eta_3(2)=2.374$
$\eta_4(1,102)=0.677$	$\eta_5(1,99)=0.284$	$\eta_6(10,100)=0.449$
$\eta_7(1,99)=0.082$		

MARCH 3 $b_t = 0.001 + 0.545b_{t-1} + 0.275b_{t-2}$

(2.419) (7.513) (3.831)

$\eta_1(1,107)=0.194$	$\eta_2(1,107)=0.293$	$\eta_3(2)=0.307$
$\eta_4(1,110)=0.447$	$\eta_5(1,107)=1.143$	$\eta_6(10,108)=0.786$
$\eta_7(1,107)=4.775$		

Notes :

Figures in parentheses are t ratios.

η_1 is a test for 1st order serial correlation, distributed F(,.,) under the null of no serial correlation.
η_2 is a test for nonlinearity, distributed F(,.,) under the null of linearity (correct functional form).
η_3 is a test for nonnormality, distributed $\chi^2(2)$ under the null of normality.
η_4 is a test for heteroscedasticity, distributed F(,.,) under the null of homoscedasticity.
η_5 is a test for ARCH, distributed F(,.,) under the null of no ARCH.
η_6 is a test for predictive failure, distributed F(,.,) under the null of no predictive failure.
η_7 tests the significance of the addition of a deterministic time trend, distributed F(,.,) under the null of a zero coefficient.

TABLE 3.7 CONT...

JUNE 1 $b_t = 0.000 + 0.624b_{t-1} + 0.177b_{t-2}$ + dummy

(1.690) (8.165) (2.305)

$\eta_1(1,106)=2.453$ $\eta_2(1,106)=0.993$ $\eta_3(2)=2.192$

$\eta_4(1,109)=2.729$ $\eta_5(1,106)=0.215$ $\eta_6(10,107)=0.250$

$\eta_7(1,106)=0.492$

JUNE 2 $b_t = 0.000 + 0.929b_{t-1}$ + dummy

(1.836) (23.69)

$\eta_1(1,102)=0.149$ $\eta_2(1,102)=0.478$ $\eta_3(2)=3.241$

$\eta_4(1,104)=0.775$ $\eta_5(1,102)=0.075$ $\eta_6(10,103)=1.139$

$\eta_7(1,102)=0.223$

JUNE 3 $b_t = 0.000 + 0.800b_{t-1}$ + dummy

(1.528) (15.37)

$\eta_1(1,100)=2.410$ $\eta_2(1,100)=0.762$ $\eta_3(2)=0.866$

$\eta_4(1,102)=0.080$ $\eta_5(1,100)=0.041$ $\eta_6(10,101)=0.669$

$\eta_7(1,100)=3.620$

Notes :

Figures in parentheses are t ratios.

η_1 is a test for 1st order serial correlation, distributed F(,.,) under the null of no serial correlation.
η_2 is a test for nonlinearity, distributed F(,.,) under the null of linearity (correct functional form).
η_3 is a test for nonnormality, distributed $\chi^2(2)$ under the null of normality.
η_4 is a test for heteroscedasticity, distributed F(,.,) under the null of homoscedasticity.
η_5 is a test for ARCH, distributed F(,.,) under the null of no ARCH.
η_6 is a test for predictive failure, distributed F(,.,) under the null of no predictive failure.
η_7 tests the significance of the addition of a deterministic time trend, distributed F(,.,) under the null of a zero coefficient.

TABLE 3.7 CONT...

SEPTEMBER 1 $b_t = 0.000 + 0.572b_{t-1}$ + dummy
$\quad\quad (0.956) \quad (10.09)$

$\eta_1(1,108)=0.414$	$\eta_2(1,108)=0.934$	$\eta_3(2)=3.080$
$\eta_4(1,110)=0.000$	$\eta_5(1,108)=0.310$	$\eta_6(10,109)=1.449$
$\eta_7(1,108)=0.005$		

SEPTEMBER 2 $b_t = 0.000 + 0.854b_{t-1}$ + dummy
$\quad\quad (1.913) \quad (18.16)$

$\eta_1(1,107)=3.846$	$\eta_2(1,107)=2.619$	$\eta_3(2)=4.903$
$\eta_4(1,109)=0.109$	$\eta_5(1,107)=0.247$	$\eta_6(10,108)=0.794$
$\eta_7(1,107)=0.043$		

SEPTEMBER 3 $\Delta b_t = \varepsilon_t - 0.615\varepsilon_{t-1} - 0.000$ + dummy
$\quad\quad (-7.887) \quad (-1.069)$

Notes :

Figures in parentheses are t ratios.

η_1 is a test for 1st order serial correlation, distributed F(,.,) under the null of no serial correlation.
η_2 is a test for nonlinearity, distributed F(,.,) under the null of linearity (correct functional form).
η_3 is a test for nonnormality, distributed $\chi^2(2)$ under the null of normality.
η_4 is a test for heteroscedasticity, distributed F(,.,) under the null of homoscedasticity.
η_5 is a test for ARCH, distributed F(,.,) under the null of no ARCH.
η_6 is a test for predictive failure, distributed F(,.,) under the null of no predictive failure.
η_7 tests the significance of the addition of a deterministic time trend, distributed F(,.,) under the null of a zero coefficient.

TABLE 3.7 CONT...

DECEMBER 1 $b_t = 0.001 + 0.681b_{t-1}$ + dummy

(2.821) (9.560)

$\eta_1(1,99)=2.247$ $\eta_2(1,99)=5.032$ $\eta_3(2)=0.021$

$\eta_4(1,102)=1.468$ $\eta_5(1,99)=0.670$ $\eta_6(10,100)=0.755$

$\eta_7(1,99)=0.020$

DECEMBER 2 $b_t = 0.001 + 0.738b_{t-1}$ + dummy

(2.958) (13.97)

$\eta_1(1,108)=1.630$ $\eta_2(1,108)=1.757$ $\eta_3(2)=4.108$

$\eta_4(1,110)=0.005$ $\eta_5(1,108)=0.461$ $\eta_6(10,109)=0.378$

$\eta_7(1,108)=7.004$

DECEMBER 3 $b_t = 0.003 + 0.762b_{t-1}$ + dummy

(5.151) (21.47)

$\eta_1(1,114)=1.558$ $\eta_2(1,114)=4.808$ $\eta_3(2)=5.257$

$\eta_4(1,116)=0.081$ $\eta_5(1,114)=1.392$ $\eta_6(10,115)=0.858$

$\eta_7(1,114)=1.036$

Notes :

Figures in parentheses are t ratios.

η_1 is a test for 1st order serial correlation, distributed F(,.,) under the null of no serial correlation.
η_2 is a test for nonlinearity, distributed F(,.,) under the null of linearity (correct functional form).
η_3 is a test for nonnormality, distributed $\chi^2(2)$ under the null of normality.
η_4 is a test for heteroscedasticity, distributed F(,.,) under the null of homoscedasticity.
η_5 is a test for ARCH, distributed F(,.,) under the null of no ARCH.
η_6 is a test for predictive failure, distributed F(,.,) under the null of no predictive failure.
η_7 tests the significance of the addition of a deterministic time trend, distributed F(,.,) under the null of a zero coefficient.

order 2. The September contract three months from expiration is best described by an ARIMA(0,1,1) process. That this contract should behave differently from the others is no surprise given that there was no cointegration. However, it is interesting to note that even in this case the basis has a nonstationary stochastic trend.

Turning to the other contracts, they are all stationary, with the some of the autoregressive coefficients for each contract being less than one. In addition, with the exception of the December contract two months from expiration, the statistic testing the null hypothesis that the coefficient on the deterministic time trend cannot be rejected at the 1% level.[12] Even with the December contract two months from expiration, the rejection is marginal (a p-value of 0.009). These results confirm our earlier intuition that the basis is indeed stochastic and that evidence of path dependence is in fact evidence of misspecification.

Let us now consider the behaviour of the basis in more detail, focusing particularly on any mean reverting behaviour it may exhibit. Recall from chapter two

[12] With the exception of the March contract one and three months from expiration, the time trend is insignificant at the 5%level. The rejection for March one and three is marginal, though.

that with a stationary autoregressive process, it will always converge to zero (or to its mean if this is non-zero) such that it is, by definition mean reverting. It is clear from table 3.7 that this is indeed the case for all contracts, with the exception of the September contract three months from expiration.

In addition, it is clear that (with the exception of the June contract) the size of the autoregressive coefficient increases monotonically with time to expiration.[13] Thus, the further away the contract is from expiration, the longer the basis takes to revert to zero.[14] Note also that in most of the equations there is a need for a dummy variable to capture outliers, suggesting that mispricing is subject to discrete jumps. All of these individual pieces of evidence would suggest that a stochastic process with solely mean reverting properties is not sufficient to capture the behaviour of mispricing. Future research on this matter might do well to concentrate on a mean

[13] The coefficients are March 1 : 0.61; March 2 : 0.811; March 3 : 0.820; June 1 : 0.801; June 2 : 0.929; June 3 : 0.800; September 1 : 0.572; September 2 : 0.854; December 1 : 0.681; December 2 : 0.738; December 3 : 0.762. Where the appropriate model is an AR(2), we have added the coefficients together to give some idea of the 'persistence' of the mean reversion.

[14] The mean of the basis is given by the constant in the regression. With the exception of the March contract three months from expiration and the December contract, the mean is zero.

reverting process subject to discrete jumps.

3.4 CONCLUSIONS

In this chapter, we have analysed the pricing relationship between the stock market and stock index futures market using daily data on the FTSE 100 Index and the FTSE 100 Index futures contract. The question of interest is whether or not these markets can be said to function effectively, particularly further from expiration. In order to test this proposition we began by testing for a unit root in each individual time series. The null hypothesis of a unit root could not be rejected in any of the series, making them candidates for cointegration. Indeed, with the exception of the September contract three months from expiration, the spot and futures prices do cointegrate, with the cointegrating vector being the basis. Thus, the first condition for effective functioning and efficiency was found to be valid.

In terms of the identification of the system from the reduced form, the appropriate model for all contracts and expiration months, with the exception of the September and December contracts three months from expiration, is a VAR(1). This, coupled with the evidence that the cointegrating vector is the basis, suggests that the VAR(1) reparameterised in error correction form is the most appropriate model, in which case the identification condition is satisfied. This is reassuring for, from the conditions necessary for efficiency, both markets are efficient regardless of time to expiration.

The third condition for strong effectiveness is that the model be stable. With the exception of the December contract three months from expiration, which shows signs of substantial instability, the models are remarkably stable. This provides reassuring evidence that the stock and stock index futures markets in the UK are strongly effectively functioning. Even the December contract three months from expiration begins to show signs of stability after the stock market crash of 1987 and this reveals the power of analysing the issue of effectively functioning equity markets in this framework.

Analysis of figures 37 and 38 reveal several interesting points about the markets' ability to cope with what may be termed radical events. In particular, the December futures contract three months from expiration has seen some extraordinary changes : the Big Bang of October 1986 and the stock market crash of October 1987. Figures 37 and 38 show the effect these events have had on the stability of the relationship very clearly indeed. Whilst the pricing relationship between the markets is stable, both markets depending only upon the same common factor, the basis, this only applies to stable time periods. In times of intense pressure, the markets appear to have difficulty in maintaining their links. This is suggestive of the fact that if the two are regulated as separate entities, as indeed they are, they will behave as separate entities in times of radical change, a conclusion which is borne out in chapter five.

The final issue that we examined was the behaviour of the basis. The purpose of this analysis was to try and shed some light on the path dependence versus path independence debate. The results clearly support path independence, with the basis, with one exception, being adequately described by an AR process of at most order 2. However, whilst the results support path independence, they also show that the Brownian Bridge process used by Brennan and Schwartz (1990) to model the behaviour of mispricing is inadequate. Whilst it retains the important mean reverting feature, is does not capture the apparent presence of discrete jumps. The implication of this is that a modification of the Brownian Bridge process to incorporate discrete jumps (and possibly the monotonic decrease in the mean reversion coefficient as maturity approaches) should fare better, although this is left to future research.

To summarise the findings in this chapter, then, we find reassuring evidence that the stock and stock index futures markets in the UK are, on the whole, strongly effectively functioning and efficient, a reassuring conclusion because the implication of this is that first the futures market generally serves its prescribed role well, and second, that the stock market generally functions reliably in its role as resource allocator.

Chapter Four : Nonsynchronous Trading And The Nature Of The Intra–Daily Pricing Relationship

4.1. Introduction

In recent years, there has been a growth in the availability of higher frequency price data in relation to stock indices and related stock index futures contracts, especially in the US and to a much lesser degree in the UK. The availability of such data has prompted renewed interest in the issue of nonsynchronous trading. Nonsynchronous trading is concerned with the possibility that some shares within an index or portfolio, say, do not trade in every time interval. The effect of this is that the *observed* price of the index or portfolio is not necessarily a reflection of its true price since it contains 'stale' prices. Moreover, if these so-called thinly traded stocks react to relevant new information with a time lag they generate autocorrelation in the observed behaviour of returns on the portfolio, potentially generating false inferences with regard to the predictability of returns.

It is these latter concerns that have prompted the renewed interest in nonsynchronous trading, with the emphasis shifting from its effects on estimates of beta in the CAPM (Scholes and Williams (1977) and Dimson (1979)) and factors in the APT (Shanken (1987)) to the effect it has on security returns. More recent models of nonsynchronous trading have concentrated on estimating the probability of nontrading (Lo and MacKinlay (1990)) and removing its effects by filtering the data according to some model of the relationship between the observed returns generating process and the true returns generating process (Harris (1989) and Stoll and Whaley (1990)).

The first section of this chapter is devoted to an analysis of the nonsynchronous trading problem. If one wishes to analyse the pricing relationship between a stock index and a stock index futures contract on an intra-daily time scale, then this problem cannot be overlooked for, as already mentioned, its effects on portfolio (index) returns can be potentially so serious that no reliable inferences can be made with regard to the pricing relationship.

In order to provide some perspective on the approaches adopted to removing its effects, the next section focuses on a discussion of two very recent models proposed in the literature : those of Harris (1989) and Stoll and Whaley (1990). Shortcomings with these models, and what we see as a problem with one of the implications of Lo and MacKinlay's (1990) model, are addressed. To overcome the problems with extant approaches to the estimation of nonsynchronous trading effects, we propose a new model which conceptually has a similar starting point to the models discussed in Harris (1989) and Stoll and Whaley (1990). The advantage of our model, however, is that it overcomes the problems associated with these other models without sacrificing intuitive appeal in the face of (perhaps unnecessary) complexity.

We utilise this new model to estimate the nonsynchronous trading adjustment and armed with this we construct a new measure of the FTSE 100 Index, adjusted for nonsynchronous trading, for use in analysing the intra-daily pricing relationship between the two markets.

The rest of the chapter is organised as follows. In section two we consider in some detail the models proposed by Harris (1989) and Stoll and Whaley (1990), pointing out their shortcomings. We also consider a problem with Lo and MacKinlay's (1990) model with regard to the implications of nonsynchronous trading. Having done this, we move on to consider a new method for estimating the nonsynchronous trading adjustment. We use this new model in section three to generate a measure of the FTSE 100 Index, adjusted for nonsynchronous trading. Armed with this adjusted Index, we utilise the framework developed in chapter two and applied in chapter three to analyse the functioning of the FTSE 100 Index and FTSE 100 Index futures markets during a week in June 1991. Section four summarises and concludes.

4.2. MODELS OF NONSYNCHRONOUS TRADING

Let us consider first of all Harris's (1989) model of nonsynchronous trading. Harris developed his model in order to analyse high frequency data on the S&P 500 Index and Index futures contract around and during the October 1987 stock market crash. Harris starts by considering two definitions of the value of a portfolio at time t. First, we have that, in Harris's notation

$$S_t = \sum_{i=1}^{N} q_i P_{it} \tag{4.1}$$

where S_t is the observed value of the portfolio, N is the number of securities in the portfolio, q_i is the number of shares held in the ith security and P_{it} is the price of the ith security at time t. The observed price of the portfolio at time t is then given by the product of the number of shares outstanding for the ith security and the ith security's most recently observed price, summed across the N securities comprising the portfolio.

Second, we have the true value of the portfolio as of time t which is given by

$$S_t^* = \sum_{i=1}^{N} q_i V_{it} \tag{4.2}$$

where S_t^* is the true value of the portfolio, q_i is as defined earlier and V_{it} is the value (to be defined shortly) of a share in the ith firm as of time t. Subtracting (4.1) from (4.2) then gives us the difference between the true value of the portfolio and its observed value :

$$S_t^* - S_t = \sum_{i=1}^{N} q_i (V_{it} - P_{it}) \tag{4.3}$$

To arrive at an expression for the nonsynchronous trading adjustment, Harris assumes that value is equal to price when price is observed. If this is the case, then any difference between the true portfolio value and the observed portfolio value will arise if the last observed price is an old one, that is, it was observed at t-k. In this case, (4.3) can be written as

$$S_t^* - S_t = \sum_{i=1}^{N} q_i \Delta_{k_{it}} V_{it} \tag{4.4}$$

where Δ_k is the k period difference operator, that is, $\Delta_k = (1-L^k)V_{it}$ where L is the lag operator and $L^k = t-k$ and k_{it} is the number of periods since the last share price for security i was observed. Clearly, if all prices are observed at time t, such that k=0, then the observed portfolio value will equal the true portfolio value. If $k \neq 0$, then ΔV_{it} is a measure of the nonsynchronous trading adjustment. Therefore, if ΔV_{it} can be estimated so can the true value of the portfolio, S_t^*.

To estimate ΔV_{it}, Harris uses a factor model, with the factor to be estimated being the nonsynchronous trading adjustment. Thus, values evolve according to (lower case letters denote variables in natural logarithms)

$$\Delta v_{it} = \lambda_t + e_{it} \tag{4.5}$$

where λ is a factor common to all the securities in the portfolio and e_{it} is a zero-mean, firm-specific component. Estimated changes in the unobserved value of the portfolio are then given by

$$\Delta V_{k_{it}} = P_{it} e^{\left(\sum_{i=1}^{k_{it}} \hat{f}_{t-i+1}\right)} \tag{4.6}$$

To estimate these changes in value, Harris reformulates the problem of the extraction of the common factor into a minimisation problem. Specifically, Harris calculates the percentage change in *observed* portfolio value ($\%\Delta S_t = (S_t - S_{t-1})/S_{t-1}$) by minimising the following function

$$\min_{f_t} \sum_{i=1}^{N} w_i (\%\Delta P_{it} - f_t)^2 \tag{4.7}$$

where $w_i = q_i P_{it-1}/S_{t-1}$, which is the value weight given to security i in the portfolio. The reason for specifying the problem in this way is that it yields an equation similar to (4.5), viz.

$$\%\Delta P_{it} = f_t + e_{it}, \qquad i = 1,\ldots\ldots,N \tag{4.8}$$

where e_{it} is as above, but with variance proportional to $1/w_i$. In a multiperiod framework the model is

$$\Delta_{k_{it}} p_{it} = \sum_{j=1}^{k_{it}} f_{t-j+1} + e_{it} \tag{4.9}$$

for all the observed P_{it} for the i=1,....,N securities in the portfolio. (4.8) and (4.9) impose a unit coefficient on the common factor and therefore the estimated factor reflects estimated true percentage returns on the portfolio.

Formulating the nonsynchronous trading problem as one of extracting a common factor has intuitive appeal. However, the major problem with this approach lies in the data requirements : essentially, the common factor has to be estimated for each individual security and then aggregated across all securities in the portfolio to generate true portfolio returns. This requires immense amounts of very specific data. To quote Harris (1989, p.82),

> 'The stock sample consists of all primary market trades of *each* S&P 500 stock from the open of trading on Monday October 12 1987 to the close of trading on Friday, October 23. The data...include the date, time, price and shares traded for *each transaction* on *each exchange* in the United States.' (emphasis added).

The data requirements for this method, then, effectively preclude its use unless one has access to trade by trade data on each individual stock in the index (to estimate Δv_{it}) and information on the number of shares traded and the number of shares outstanding (to calculate q_i). In addition, it could be computationally quite expensive.[1] This effectively precludes it from use.

As an alternative to Harris (1989), Stoll and Whaley (1990) consider a model that removes both nonsynchronous trading and bid-ask price effects. They initially derive separate models for each of these effects and then combine them to produce a single model that corrects the data for both. Beginning with bid-ask effects, consider first of all a single stock and assume that it trades at least once in every time interval (so there is no nonsynchronous trading effect). The observed return, which equals the true return, is given by

$$R_{it}^{*} = \mu_i + \epsilon_{it} + \nu_{it} - \theta_i \nu_{it-1} \tag{4.10}$$

where $\mu_i = E(R_i^*)$ and ε_{it} and ν_{it} are mean zero disturbances. (4.10) says that the observed return on the ith security in period t is equal to the expected return, plus any random deviations around the expected

[1] For example, Harris (1989) calculates the value of the S&P 500 Index at 5 minute intervals over ten trading days.

return, and a disturbance which follows an MA(1) process. The MA(1) process represents the effects of the bid-ask spread, for there will be an error introduced by the bid-ask spread at the beginning of the period and at the end of the period. Maintaining the assumption of no nonsynchronous trading effects, then extending this to a portfolio is straightforward:

$$R_{pt}^{*} = \sum_{i=1}^{N} q_i R_{it}^{*} \tag{4.11}$$

where q_i and N are as defined earlier. Substituting (4.10) in to (4.11) yields, after some manipulation,

$$R_{pt}^{*} = \mu_p + \epsilon_p + \sum_{i=1}^{N} q_i (\nu_{it} - \theta_i \nu_{it-1}) \tag{4.12}$$

Consider now the nonsynchronous trading problem. Assuming that a stock trades at least once every n time periods, Stoll and *Whaley (1990) represent this* by

$$R_{pt}^{o} = \sum_{k=0}^{n-1} \omega_{pk} R_{pt-k}^{*} + u_t \tag{4.13}$$

where R_{pt}^{o} represents the observed return on the portfolio, which is no longer equal to the true return, and $\omega_{p,k}$ represents that fraction of true returns reflected in observed returns.

The interpretation of (4.11) is straightforward. At time t, only a fraction of the true portfolio return is reflected in the observed portfolio return. Obviously, this fraction will depend upon the number of stocks within the portfolio that trade in every time interval (and the weight these stocks take within the portfolio). If $\omega_{p0}=1$ then the observed portfolio return is equal to the true portfolio return and there are no nonsynchronous trading effects. If $\omega_{p0}\neq 1$ then ω_{pk} represents that fraction of true returns that is observed t+k periods later.

To obtain an estimable model, Stoll and Whaley rewrite (4.13) expressing the ω_{pk}'s ($k\neq 0$) as a proportion of ω_{p0}, rewrite the model using lag operator notation and solve to give

$$R_{pt}^{o} = \omega_{p0} R_{pt}^{*} + \sum_{k=1}^{\infty} \gamma_{pk} R_{pt-k}^{o} + \nu_{t} - \sum_{k=1}^{\infty} \gamma_{pk} \nu_{pt-k} \qquad (4.14)$$

Substituting (4.12) into (4.14) and gathering terms yields

$$R_{pt}^{o} = \omega_{p0}\mu_p + \sum_{k=1}^{\infty}\phi_k R_{pt-k}^{o} + \epsilon_{pt} - \sum_{k=1}^{\infty}\theta_k\epsilon_{pt-k} \tag{4.15}$$

which shows that in the presence of nonsynchronous trading and bid-ask price effects, observed returns will follow an ARMA(p,q) process with p,q = ∞. Stoll and Whaley then argue that true returns are given by observed returns minus the fitted values from the model, although this is incorrect : true returns are given by the fitted values, as we shall see.

In practice, p and q cannot obviously equal infinity. Stoll and Whaley find that an ARMA(3,2) process for returns adequately removes these effects. However, selection of the orders for p and q in ARMA models is notoriously problematic and very subjective. Moreover, overparameterisation of the model can distort the results. If the true process is an ARMA(1,1), an ARMA(2,2) will fit equally as well. The notion of correct selection of the orders of p and q is especially important given Stoll and Whaley's interpretation of what constitutes true portfolio returns. Overparameterisation in this case will yield misleading inferences later on.

Another cause for concern with Stoll and Whaley's model is the use of autoregressive terms to model nonsynchronous trading. This cause for concern also applies to one of the implications of Lo and MacKinlay's (1990) model of nonsynchronous trading, that is, nonsynchronous trading induces positive serial correlation in observed portfolio returns, yielding an AR(1) process for observed returns.

To demonstrate, suppose we have a time series of returns on a portfolio, shown by figure 4.1. Armed with this returns series, we proceed to calculate the autocorrelation coefficients, obtaining the results shown in table 4.1, and plot a graph of the autocorrelation function which is as shown in figure 4.2.

Observing first of all the size of the autocorrelation coefficients and their significance at all lags, as witnessed by the Box-Pierce (1970) statistic, we also note that the serial correlation is generally positive and declines geometrically. We then recall the fact that one of the implications of *Lo and MacKinlay's* (1990) model is nonsynchronicity generates an AR(1) process in returns, and Stoll and Whaley's (1990) model shows that observed returns also evolve according to a more general ARMA process, of which an AR(1) model is a special case. Estimating an AR(1) model for returns, we find that $R_{pt} = 0.2 + 0.4R_{pt-1}$, confirming that indeed returns do follow an AR(1)

FIGURE 4.1 : TIME SERIES OF SIMULATED RETURNS
R(t) = 0.2 + 0.4 R(t−1) + U(t)

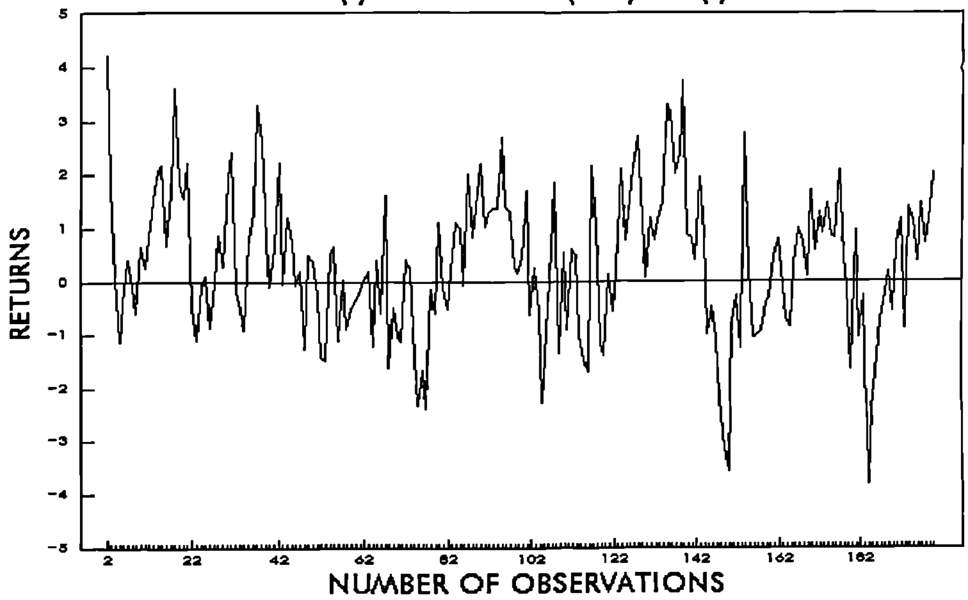

FIGURE 4.2 : AUTOCORRELATION FUNCTION
R(t) = 0.2 + 0.4 R(t−1) + U(t)

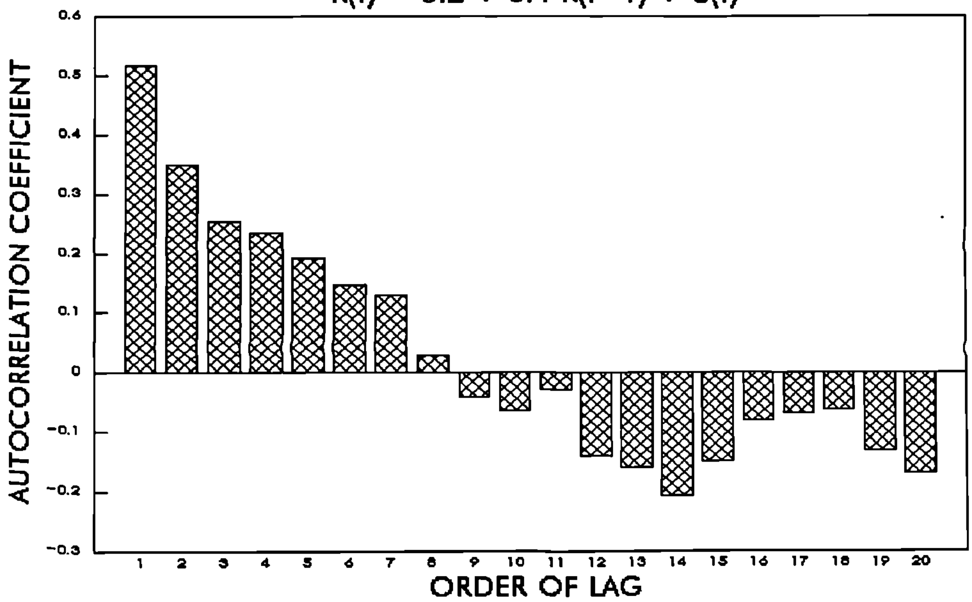

TABLE 4.1 : AUTOCORRELATION FUNCTION FOR SIMULATED RETURNS ($R_t = 0.2 + 0.4R_{t-1} + u_t$, $u_t \sim N(0,1)$)

Order	Autocorrelation Coefficient	Standard Error	Box-Pierce Statistic
1	.51842	.070888	53.4839[.000]
2	.34989	.087899	77.8465[.000]
3	.25304	.094640	90.5879[.000]
4	.23574	.097980	101.6470[.000]
5	.19081	.10079	108.8921[.000]
6	.14531	.10259	113.0941[.000]
7	.13028	.10362	116.4718[.000]
8.	.029248	.10444	116.6421[.000]
9	-.039901	.10448	116.9589[.000]
10	-.064712	.10456	117.7922[.000]
11	-.028572	.10476	117.9547[.000]
12	-.14130	.10480	121.9281[.000]
13	-.15932	.10575	126.9790[.000]
14	-.20868	.10695	135.6448[.000]
15	-.14997	.10898	140.1204[.000]
16	-.079951	.11001	141.3925[.000]
17	-.067833	.11030	142.3082[.000]
18	-.062274	.11051	143.0799[.000]
19	-.13103	.11069	146.4964[.000]
20	-.16873	.11146	152.1621[.000]

Notes :

Figures in square parentheses are p-values.

process. On the basis of this evidence, we conclude that we have a nonsynchronous trading problem and recall Stoll and Whaley's (1990) argument that true returns are given by the residuals from this model. We therefore use the residuals, which we find to be white noise, to reflect true portfolio returns and we have a time series of returns with the effects of nonsynchronous trading expunged.

The problem here is that we do not. The reason for this is that the true returns generating process is AR(1) : what we have is a genuine inefficiency. We have interpreted it otherwise. This example is a little contrived but it illustrates the point well : just because returns follow an AR process, it does not mean that this is evidence of nonsynchronous trading. It could be that returns are genuinely predictable.[2] Clearly, this is a cause for concern and needs to be overcome. We suggest a method that does this in the

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[2] As a little aside, table 4.1 also illustrates the danger of making inferences solely on the basis of calculated autocorrelation coefficients. It would be tempting to conclude that there is strong autocorrelation which we can make use of in forecasting future returns. This is wrong : the autocorrelations merely illustrate the effect of an autoregressive process on the autocorrelation function. Examination of the partial autocorrelation function would demonstrate that indeed the correlation persists for one lag only, that is, we have an AR(1) process. Estimation yields a coefficient of 0.4. Using this to forecast five periods ahead, say, we find that $E(R_{t+5}|R_t) = 0.01R_t$. The autocorrelation is not as strong as the autocorrelation function indicates.

next subsection.

4.2.1. A Model Of Nonsynchronous Trading

The last part of the previous section highlights some of the dangers that can occur if we argue that nonsynchronicity generates an autoregressive structure in returns. The problem here is that genuine autocorrelation may be treated as evidence of nonsynchronous trading and, although failure to account for nonsynchronous trading can generate misleading inferences, incorrectly accounting for it can generate equally misleading inferences.[3] The solution to this problem is found in rethinking the effects of nonsynchronous trading on prices and hence returns.

An alternative way to analyse the nonsynchronous trading problem is to think of nontrading as a lag in the reaction of security prices to new information. If new *information arrives in a random fashion, such* that shocks generated by new information are exogenous and unpredictable, failure to react immediately to

[3] For example, by using the residuals from their ARMA models for observed returns as true returns, Stoll and Whaley (1990) force true returns to be white noise, that is, they impose market efficiency. However, there is no *a priori* reason why market efficiency should hold. Indeed, one could even argue that with high frequency intra-daily it will not hold, even in the futures market.

this new information will not generate autoregressive behaviour in prices and hence returns. It will generate *moving average* behaviour. Therefore, in the presence of nonsynchronous trading, returns will have a moving average component which reflects this delayed reaction to new information. Whilst any moving average process that is invertible can be written as a stationary autoregressive process, this is not advisable in this particular instance because confusion such as that identified above could ensue. What is required, then, is a model which is interpretable as a moving average model.

We know from the arguments outlined in Harris (1989) that the observed value of the portfolio consists of the true underlying value plus an adjustment for nonsynchronous trading. Therefore, we have

$$S_t = S_t^* + u_t \tag{4.16}$$

where, as noted earlier, S_t is the observed value of the portfolio and S_t^* is the true underlying value. u_t is a zero mean process which can be interpreted as the nonsynchronous trading adjustment. This model is very similar to Harris's (1989) starting point. Our model, however, differs from Harris's (1989) by the way we treat the estimation of both the unobserved true portfolio value and the unobserved nonsynchronous trading adjustment. Following Garrett (1991), (4.16) can be viewed as an unobserved components model in which the observed series consists of a signal (the true underlying portfolio price) and noise (the nonsynchronous trading adjustment). Treated in this way, the nonsynchronous trading problem becomes a signal extraction problem, with the signal to be extracted being the true value of the portfolio. Therefore, if we can extract this signal, we also have a measure of the nonsynchronous trading adjustment.

A method that can be used to extract the signal is the Kalman Filter. Using the notation in Harvey (1987),[4] we can set up the model in state space form as follows. We have the measurement equation, which is given by

$$y_t = z_t^{\prime}\alpha_t + u_t \tag{4.17}$$

and the transition equation, given by

[4] For a very detailed exposition of the econometrics of the Kalman Filter, see Cuthbertson, Hall and Taylor (1992) and for an application see Haldane and Hall (1991).

$$\alpha_t = \theta \alpha_{t-1} + v_t \tag{4.18}$$

In the above equations, y_t *is the observed variable,* $\mathbf{z}_t$ is a vector of parameters and α_t is known as the state vector. (4.18) describes the evolution of the state vector through time, with θ being a matrix of parameters. u_t and v_t are zero mean random variables with variances $\sigma_u^2 h_t$ and $\sigma_v^2 Q_t$ respectively. Define now $\hat{\alpha}_{t-1}$ as the best estimate α_{t-1} and the covariance matrix of this estimate as $\mathbf{P}_{t-1}$. We then have the following prediction equations

$$\hat{\alpha}_{t|t-1} = \theta \hat{\alpha}_{t-1} \tag{4.19}$$

and

$$P_{t|t-1} = \theta P_{t-1} \theta' + Q_t \tag{4.20}$$

The idea behind the Kalman Filter is that as observations on the observed variable y_t become available, so we can use this information to update the prediction equations. The updating equations, which define the Kalman Filter, are given by

$$\hat{\alpha}_t = \hat{\alpha}_{t|t-1} + P_{t|t-1} z_t (y_t - z_t' \hat{\alpha}_{t|t-1}) / z_t' P_{t|t-1} z_t + h_t \tag{4.21}$$

and

$$P_t = P_{t|t-1} - P_{t|t-1}z_t z_t' P_{t|t-1} / z_t' P_{t|t-1} z_t + h_t \tag{4.22}$$

This model is estimated by maximum likelihood, with the likelihood function being expressed as a function of the one-step prediction errors. The final requirement before estimation is the specification of the various vectors involved in the state space formulation. The specification we use is the local linear trend model (see Harvey (1984, 1987) and Chatfield (1989)). This is given by

$$S_t = S_t^* + u_t \tag{4.23a}$$

$$S_t^* = S_{t-1}^* + \beta_{t-1} + v_t \tag{4.23b}$$

$$\beta_t = \beta_{t-1} + \zeta_t \tag{4.23c}$$

where β_t represents a stochastic trend such that true returns can evolve in a stochastic fashion if need be. Formulating the problem in this manner provides several advantages. First, it represents an intuitive and simple way to analyse the problem. Second, as is pointed out by Harvey (1984) and Chatfield (1989), the system (4.23a-c) has the same properties as an

ARIMA(0,2,2) process and as such, it captures the moving average behaviour we require. Third, it is quite general since β can be restricted to zero.[5] If it is, then the model can be written as an ARIMA(0,1,1). The ARIMA(0,1,1) has been adopted in some studies as an *ad hoc* method of accounting for nonsynchronous trading effects (see, for example, Baillie and Bollerslev (1990)). This practice now has a sound justification. Given that the model, either with or without $\beta=0$, can always be written as an ARIMA process, estimation is straightforward. Thus, this model avoids the complexities of Harris's (1989) model. We use this model in the next section as a first step in modelling the intra-daily pricing relationship between the FTSE 100 Index and the FTSE 100 Index futures contract.

4.3. MODELLING THE INTRA-DAILY PRICING RELATIONSHIP BETWEEN THE FTSE 100 INDEX AND FTSE 100 INDEX FUTURES CONTRACT

The data used in this chapter are minute by minute values of the FTSE 100 Index and trade by trade data for the FTSE 100 Index futures contract over the period Monday 10th June 1991 to Friday 14th June 1991.

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[5] In fact, in the applications that follow in this chapter and the next chapter, β is constrained to zero.

The transactions prices for the June 1991 contract, the nearest to maturity at the time, were used for the futures price. However, a problem with the futures data is that there was not always a transaction in any one given minute. Indeed, there are some quite long periods without any transactions in the futures market, some lasting fifteen minutes or more. Where there are no transactions in any one given minute but bid and ask prices are quoted, we have used an average of the bid and ask prices as the futures price. Those minutes where there are no prices quoted at all, whether they be bid, ask or transaction prices, for the June contract have been deleted.

The nonsynchronous trading adjustment was estimated using the log of the Index price for each day (with any observations corresponding to those were no futures transactions took place deleted) and is plotted for each trading day is plotted in figures 3 through 7 overleaf. Figures have been multiplied by 100 for readability of the scales on the graphs. It is *evident from the graphs that the nonsynchronous* trading adjustment is very small, implying that nonsynchronous trading is not a major problem with the FTSE 100 Index. This finding is consistent with the fact that first, the FTSE 100 Index generally comprises big (so-called blue chip) companies whose shares tend to trade more frequently and second, the

FIGURE 4.3 : NONSYNCHRONOUS TRADING ADJUSTMENT
10TH JUNE 1991

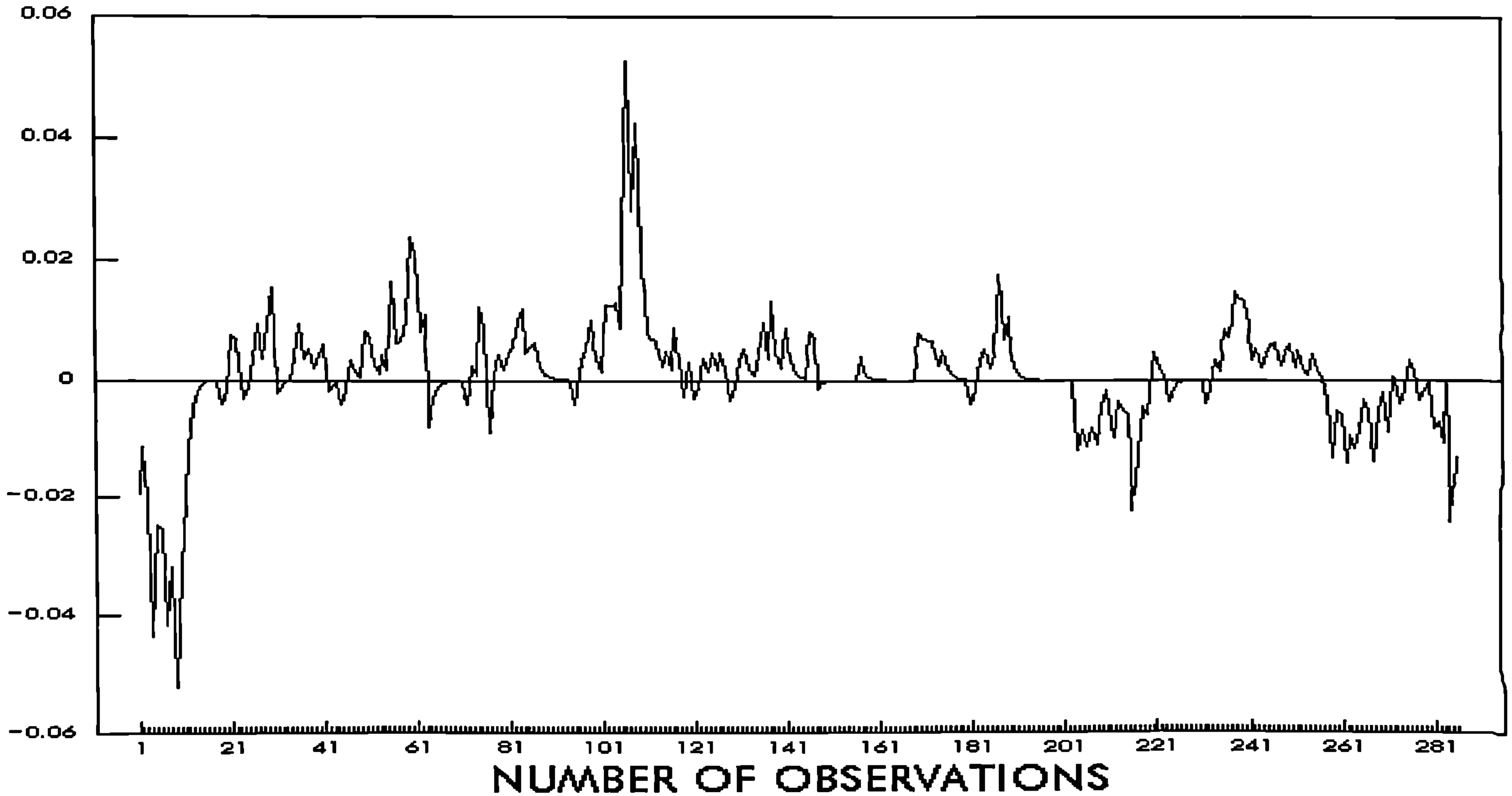


FIGURE 4.4 : NONSYNCHRONOUS TRADING ADJUSTMENT
11TH JUNE 1991

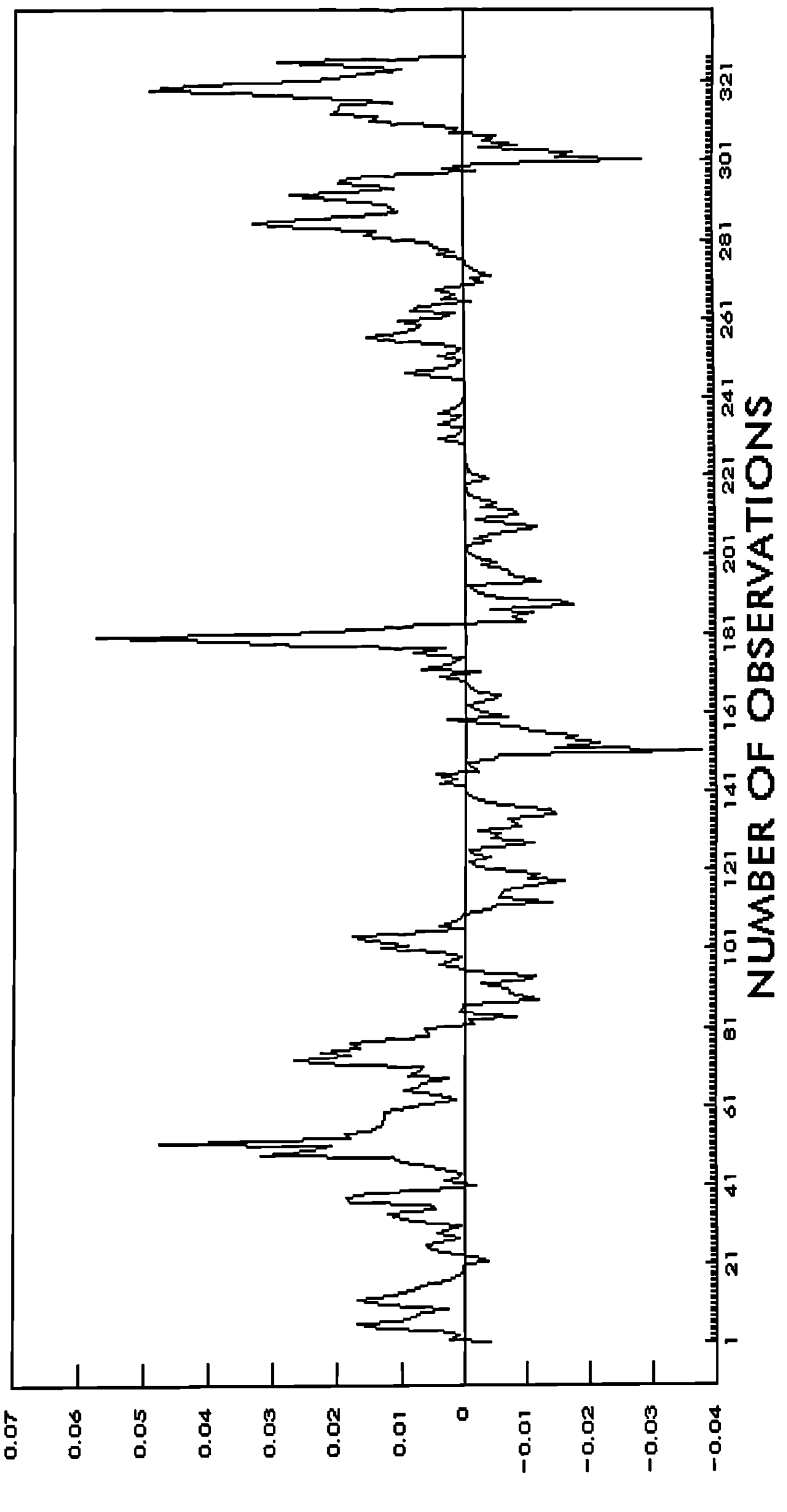


FIGURE 4.5 : NONSYNCHRONOUS TRADING ADJUSTMENT
12TH JUNE 1991

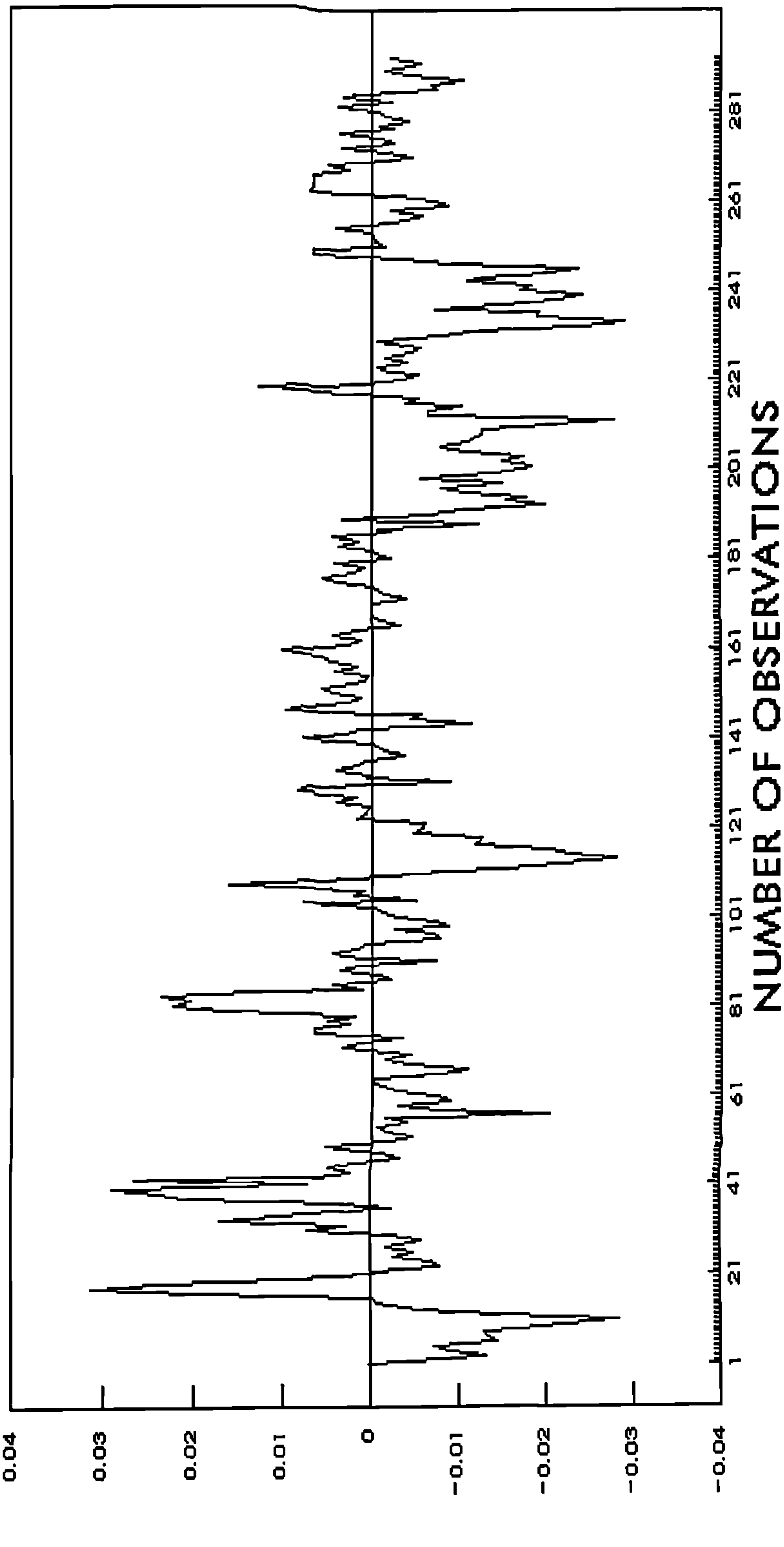


FIGURE 4.6 : NONSYNCHRONOUS TRADING ADJUSTMENT
13TH JUNE 1991

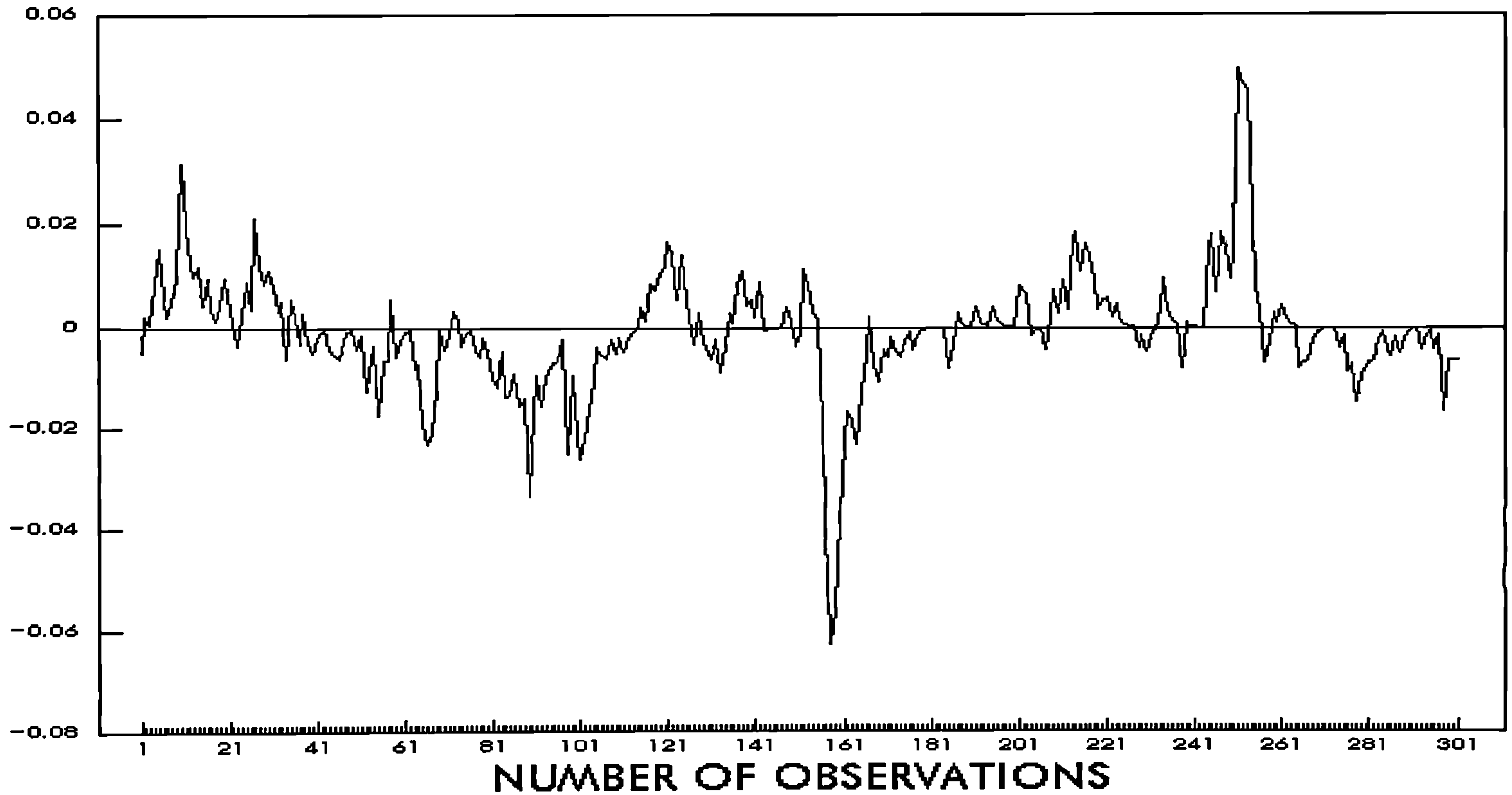
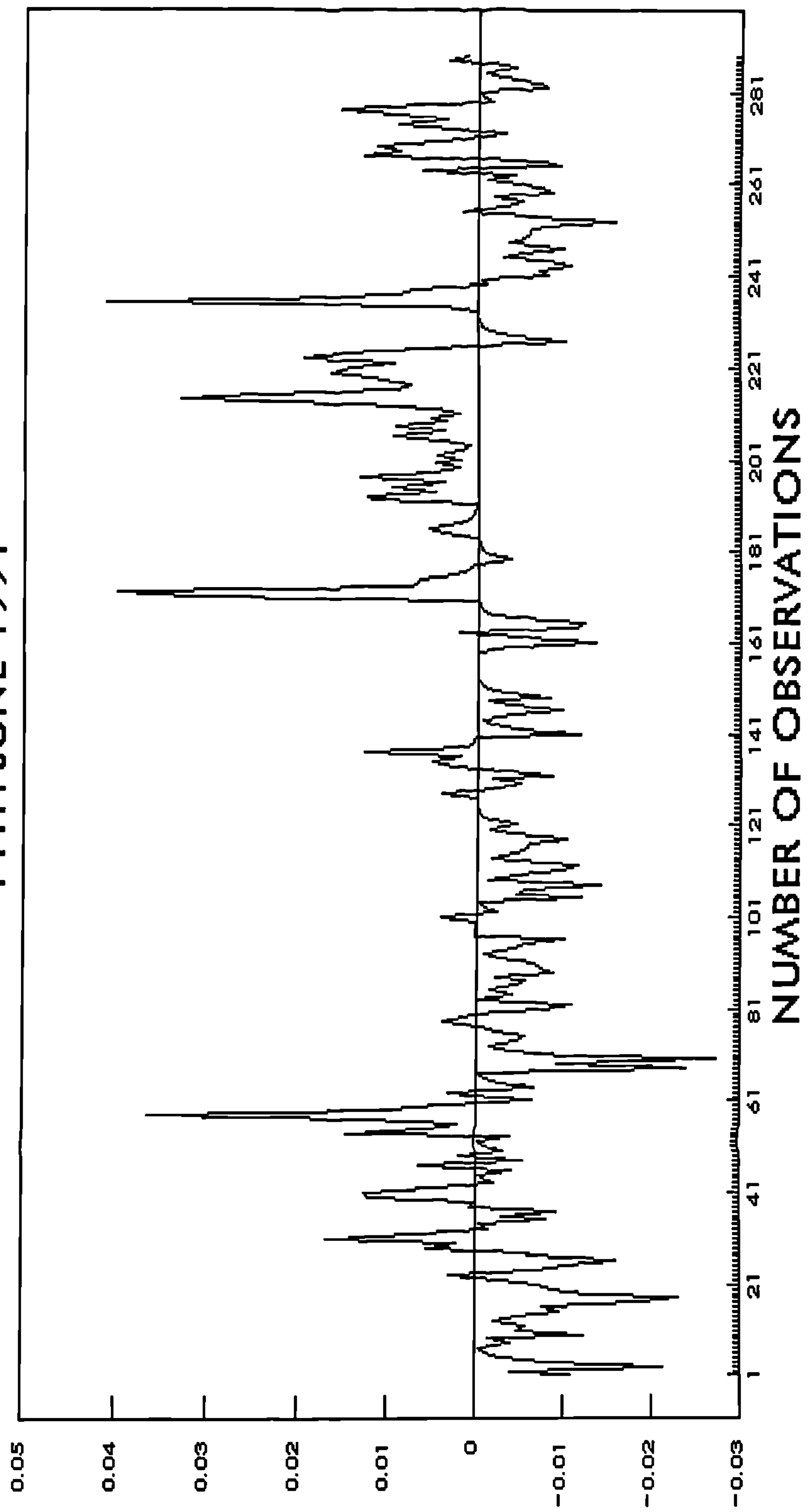


FIGURE 4.7 : NONSYNCHRONOUS TRADING ADJUSTMENT
14TH JUNE 1991



smaller the Index, the less severe the nonsynchronous trading problem is.[6]

More interesting are the graphs of the percentage changes in futures prices for each of the trading days, plotted in figures 8 through 12. They reveal that the futures price fluctuations are roughly of the same magnitude, whether they be positive or negative. Moreover, this pattern of behaviour is consistent across all of the trading days in the sample, suggesting that whilst the futures is not thinly traded, it is most certainly not heavily traded.

Let us now turn our attention to modelling the intra-daily pricing relationship between the two markets. The interesting issue is whether the results found on a daily basis carry over to an intra-daily basis. We consider first of all whether the adjusted Index and futures price cointegrate on each of the trading days and whether or not the cointegrating vector is the basis. There is an extra testable restriction on the cointegrating vector on an intra-daily basis : the cost of carry is constant on an intra-daily basis and therefore a natural hypothesis to test is whether this is zero, which it could be for two reasons.

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[6] More recent evidence from the US shows that, not surprisingly, the nonsynchronous trading problem is much worse for the S&P 500 Index as opposed to the MMI, which only comprises twenty securities.

FIGURE 4.8 : PERCENTAGE CHANGE IN FUTURES PRICE 10TH JUNE 1991

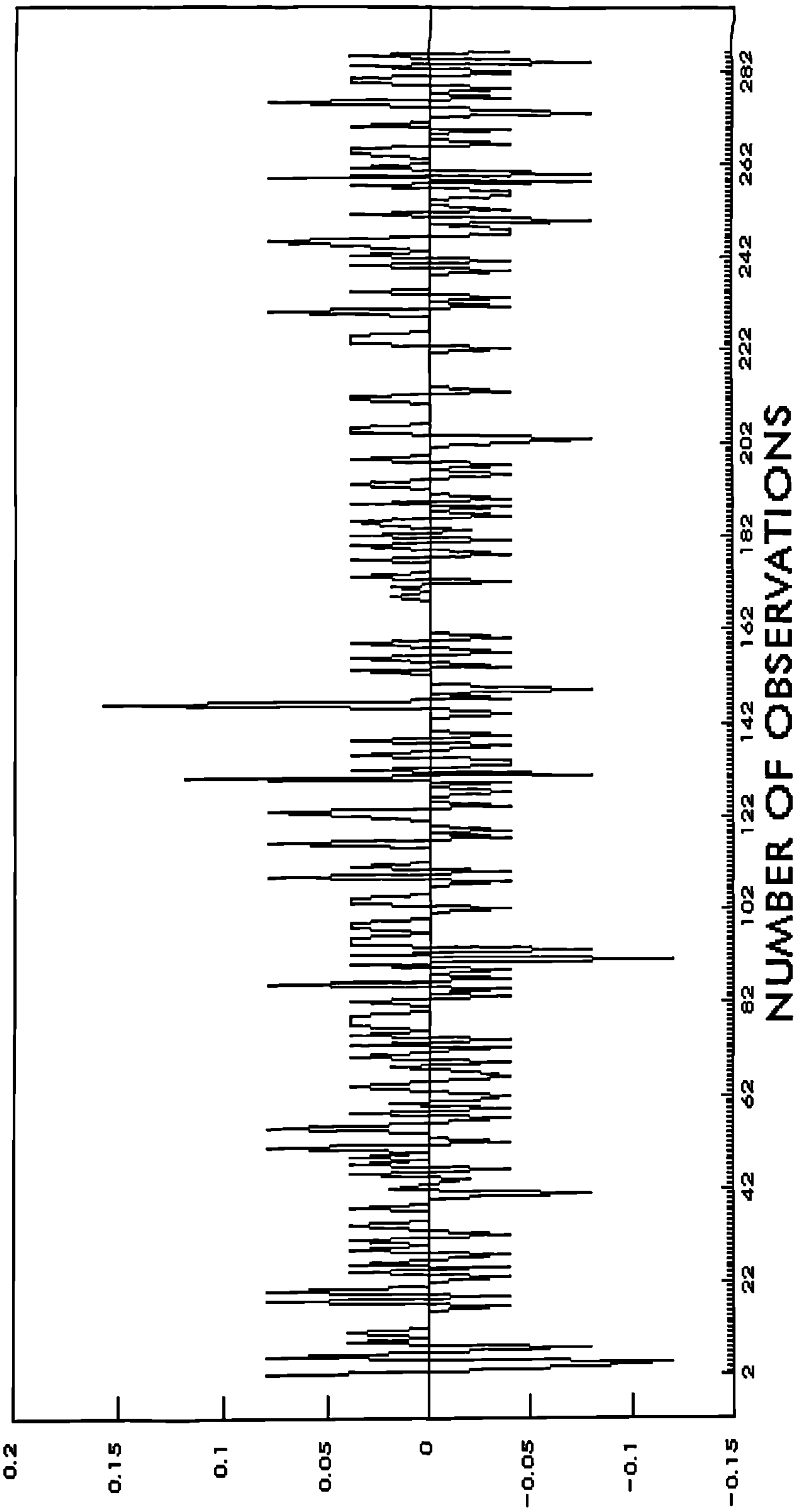


FIGURE 4.9 : PERCENTAGE CHANGE IN FUTURES PRICE
11TH JUNE 1991

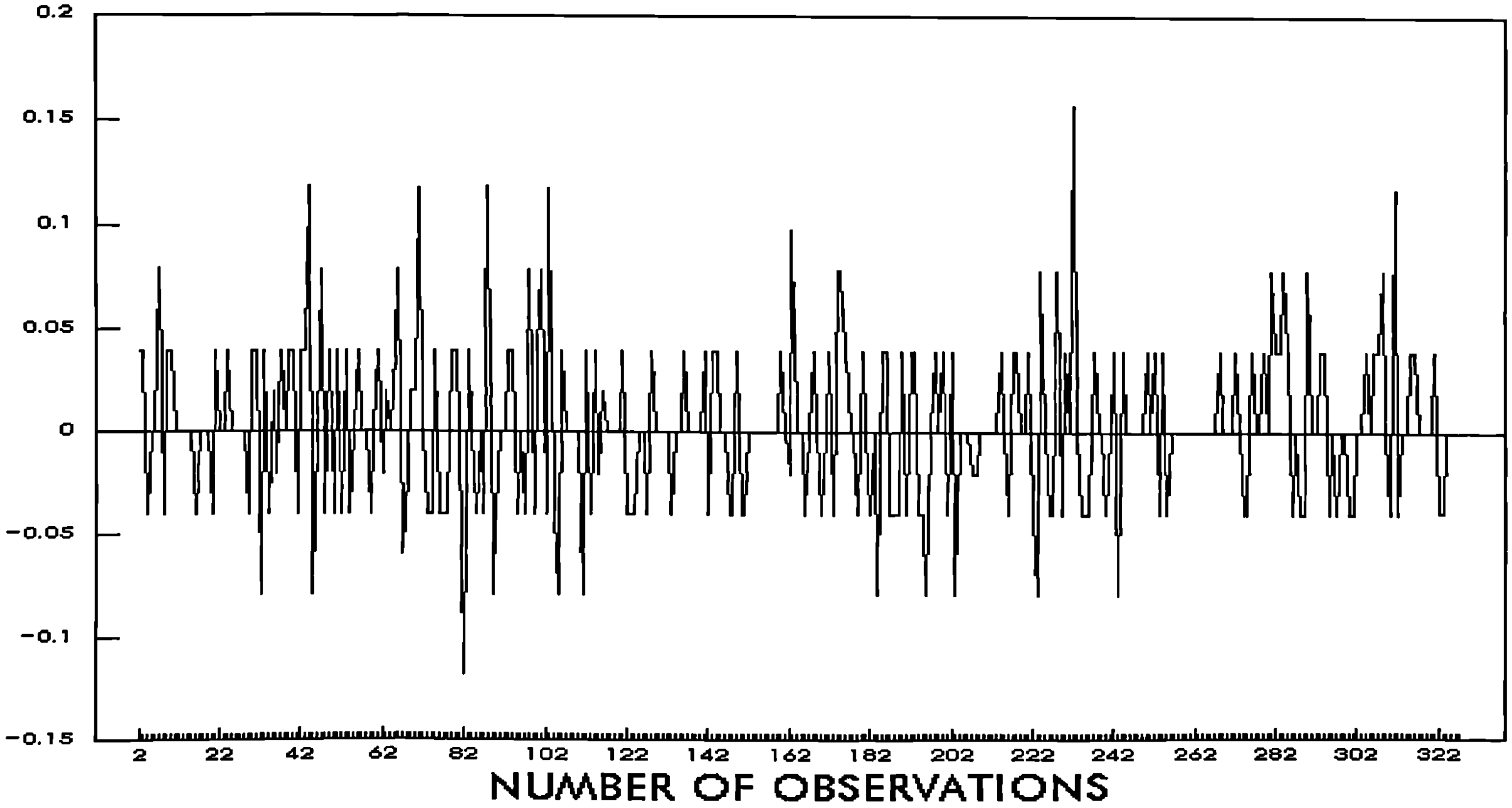


FIGURE 4.10 : PERCENTAGE CHANGE IN FUTURES PRICE
12TH JUNE 1991

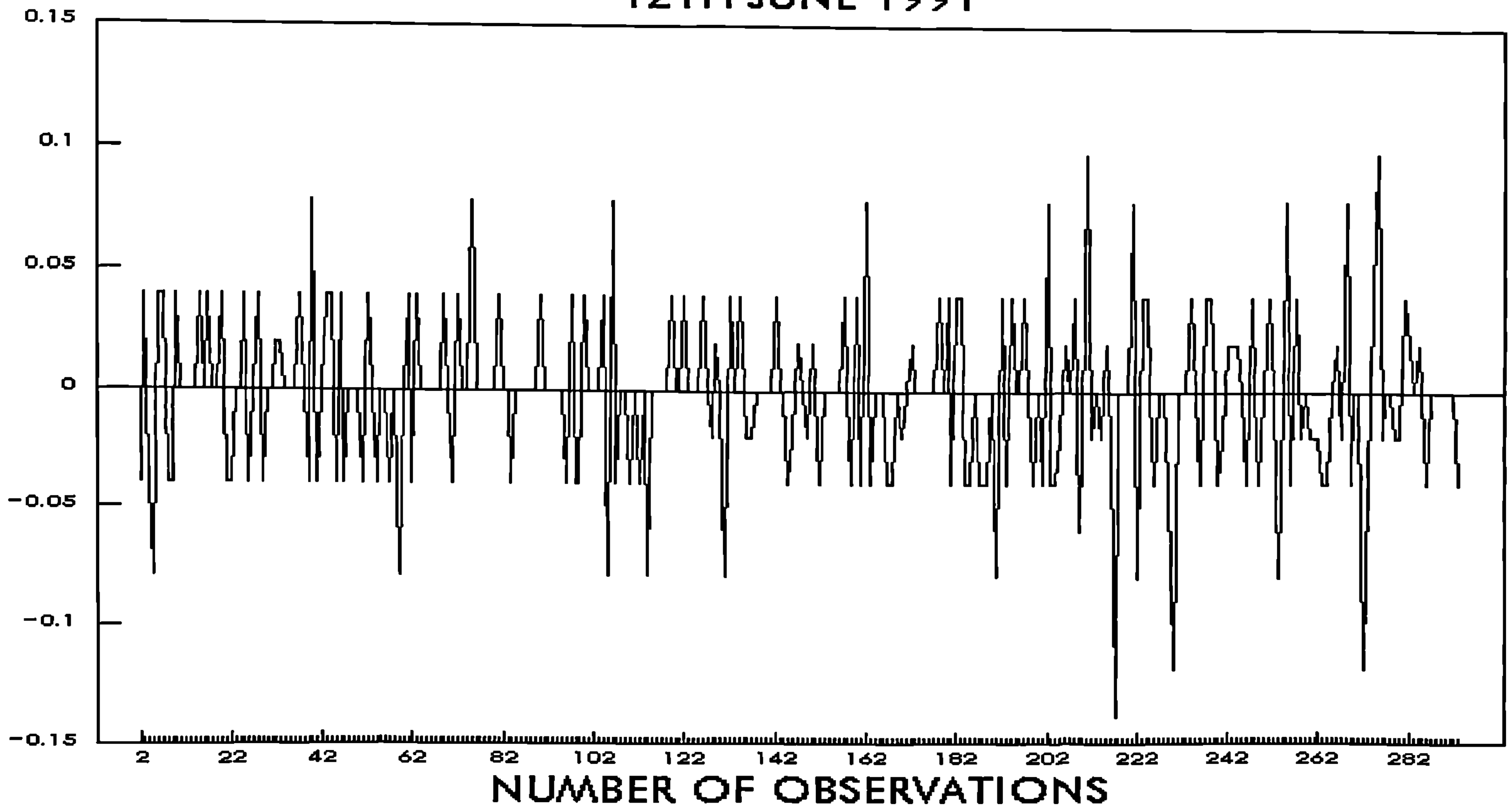


FIGURE 4.11 : PERCENTAGE CHANGE IN FUTURES PRICE
13TH JUNE 1991

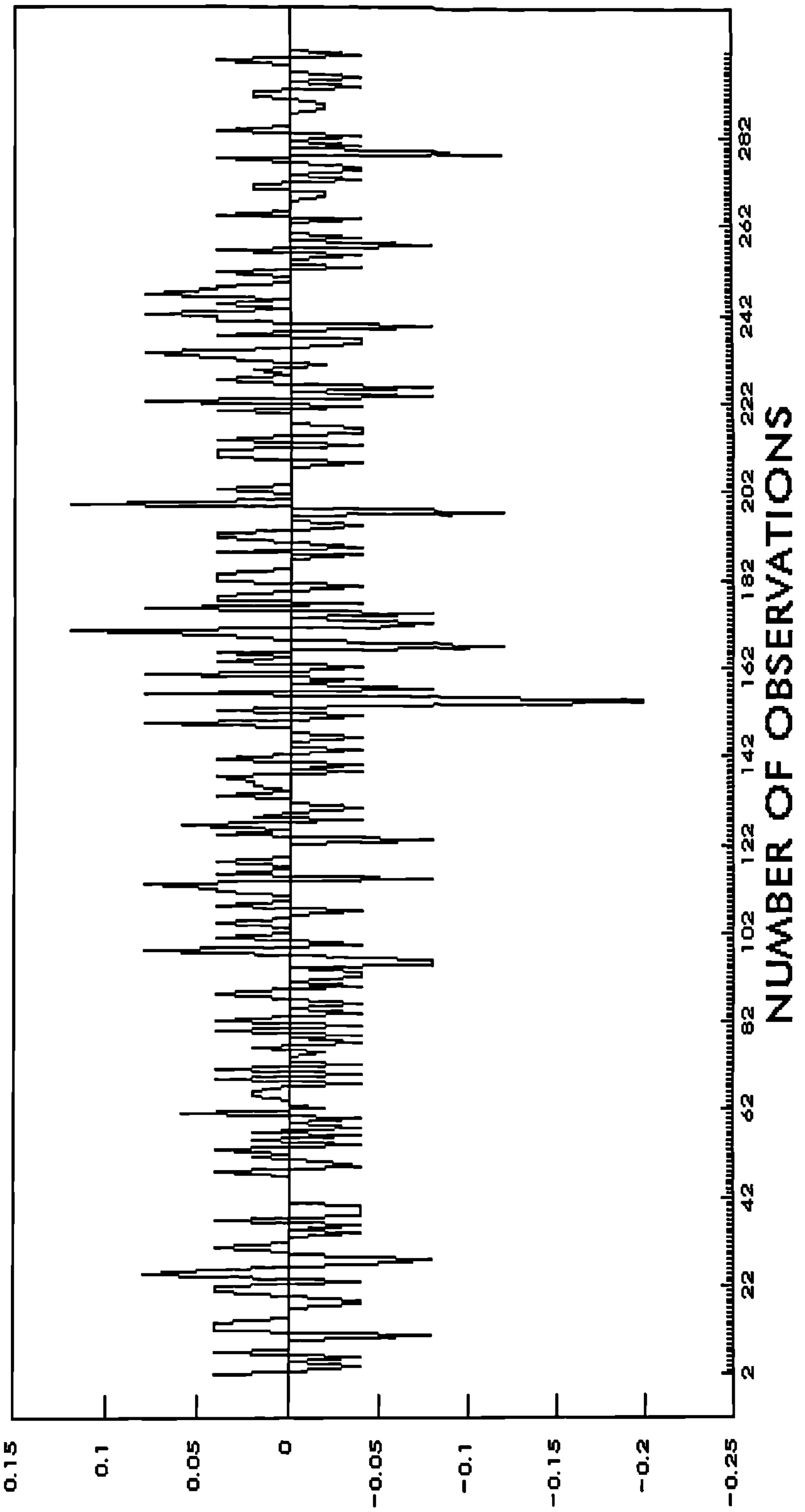
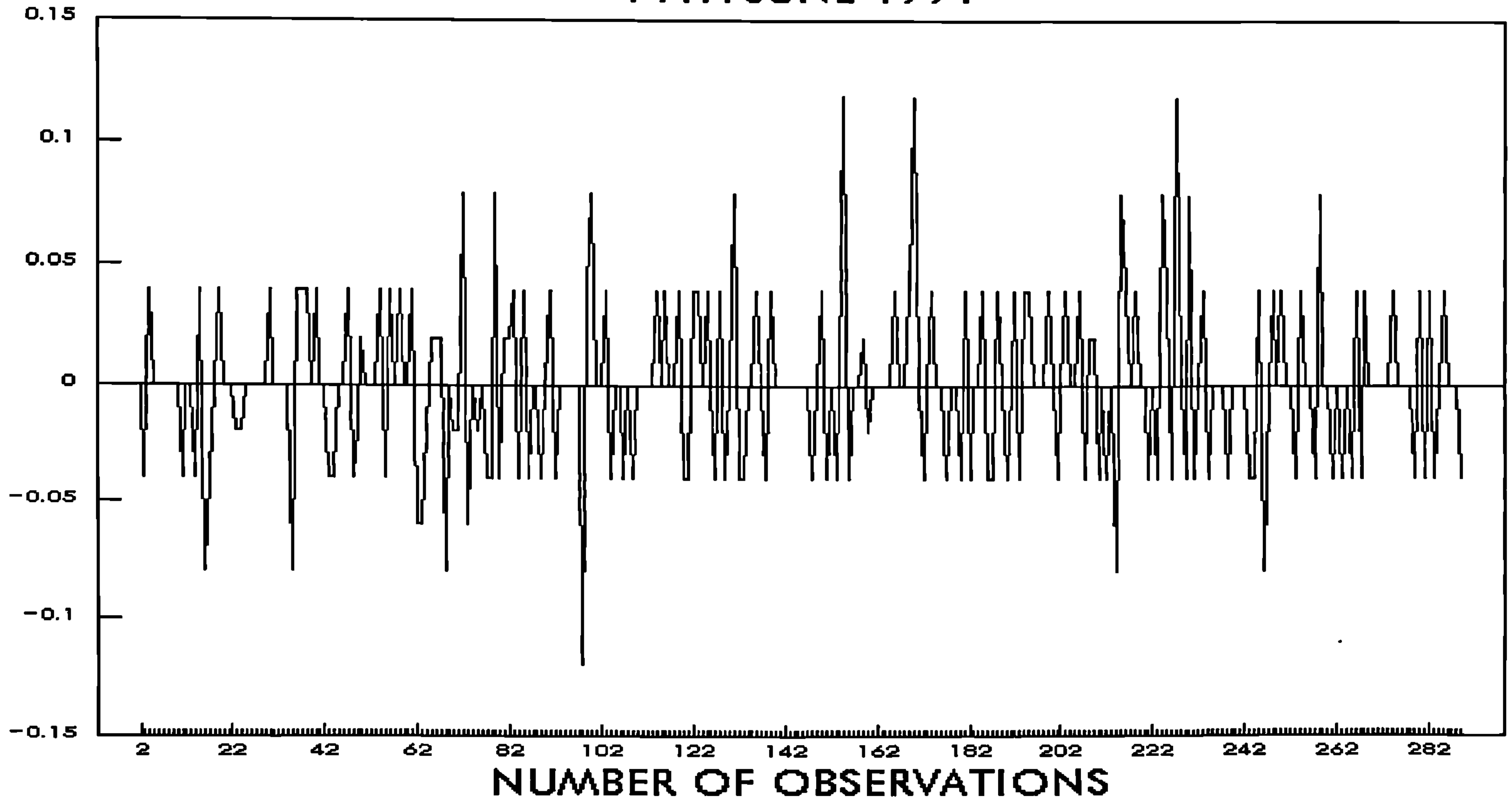


FIGURE 4.12 : PERCENTAGE CHANGE IN FUTURES PRICE
14TH JUNE 1991



First, the cost of carrying the spot portfolio to maturity may be trivial. This may be the case because the dividend yield from holding a portfolio that mimics the FTSE 100 may not be great. Second, as the futures contract approaches maturity, so the carrying cost must approach zero. Clearly, we are analysing prices for the June 1991 contract in the expiry month. Therefore, in the context of the cointegrating regression $f_t = \gamma_0 + \gamma_1 s_t$, the two restrictions are that $\gamma_0 = 0$ and $\gamma_1 = 1$.

Tests for the appropriate lag length of the VAR are presented in table 4.2. Through the nature of intra-daily data, and to allow for a general enough specification of the model, we test down from ten lags, reducing the order of the VAR by one each time until we obtain a rejection at the 1% level.[7] The table shows that for the 10th, 11th and 14th the appropriate lag length in the VAR is two, whilst the 11th requires three and the 13th five.

Tests for the number of cointegrating vectors and tests of the restrictions placed on them are reported in table 4.3. As in the previous chapter, the

[7] The reason for doing this is that we have not made a small sample correction because of the size of the sample. However, it is likely that a statistic that is significant at 5% would not be if the small sample correction were made.

TABLE 4.2 : LIKELIHOOD RATIO TESTS FOR VAR LENGTH (DISTRIBUTED $\chi^2(4)$)

	10TH JUNE	**11TH JUNE**	**12TH JUNE**	**13TH JUNE**	**14TH JUNE**
10 to 9	0.34	1.28	7.16	3.40	9.42
9 to 8	5.16	2.60	4.60	1.90	1.00
8 to 7	0.42	9.08	6.16	3.08	5.40
7 to 6	2.38	7.24	1.66	12.36*	7.28
6 to 5	3.42	4.62	1.06	2.74	2.22
5 to 4	3.94	4.74	2.78	18.84**	2.44
4 to 3	8.38	5.54	5.14		3.50
3 to 2	10.56*	9.52*	21.98**		7.32
2 to 1	221.06**	320.32**			130.54**

* denotes significant at 5%
** denotes significant at 1%

TABLE 4.3 : TESTS FOR THE NUMBER OF COINTEGRATING VECTORS AND TESTS OF THE RESTRICTIONS ON THE VECTORS (H_o^a:r=0, H_1^a:r=1 ; H_o^b:r≤1, H_1^b:r=2)

	10TH JUNE		11TH JUNE	
	H_o^a	H_o^b	H_o^a	H_o^b
λ_{max}	29.82	2.901	25.87	3.509
λ_{trace}	32.72	2.901	29.38	3.509

RESTRICTIONS :

	10TH JUNE	11TH JUNE
$\gamma_0 = 0 : \chi^2(1)$	2.848	4.817*
$\gamma_1 = 1 : \chi^2(1)$	2.851	4.822*

Notes :

* denotes significant at 5%
** denotes significant at 1%

Critical Values (Johansen and Juselius (1990) Table A.3.)

		90%	95%
λ_{max}	r=0	7.563	9.094
	r=1	13.78	15.75
λ_{trace}	r=0	7.563	9.094
	r=1	17.96	20.17

TABLE 4.3 CONT...

(H_o^a: r=0, H_1^a: r=1 ; H_o^b: r≤1, H_1^b: r=2)

	12TH JUNE		13TH JUNE	
	H_o^a	H_o^b	H_o^a	H_o^b
λ_{max}	28.73	2.745	21.67	3.701
λ_{trace}	31.47	2.745	25.37	3.701

RESTRICTIONS :

	12TH JUNE	13TH JUNE
$\gamma_0 = 0 : \chi^2(1)$	9.062**	2.896
$\gamma_1 = 1 : \chi^2(1)$	9.081**	2.902

Notes :

* denotes significant at 5%
** denotes significant at 1%

Critical Values (Johansen and Juselius (1990) Table A.3.)

		90%	95%
λ_{max}	r=0	7.563	9.094
	r=1	13.78	15.75
λ_{trace}	r=0	7.563	9.094
	r=1	17.96	20.17

TABLE 4.3 CONT...

(H_o^a: r=0, H_1^a: r=1 ; H_o^b: r≤1, H_1^b: r=2)

14TH JUNE

	H_o^a	H_o^b
λ_{max}	16.87	3.520
λ_{trace}	20.39	3.520

RESTRICTIONS :

$\gamma_0 = 0 : \chi^2(1)$	0.355
$\gamma_1 = 1 : \chi^2(1)$	0.356

Notes :

* denotes significant at 5%
** denotes significant at 1%

Critical Values (Johansen and Juselius (1990) Table A.3.)

		90%	95%
λ_{max}	r=0	7.563	9.094
	r=1	13.78	15.75
λ_{trace}	r=0	7.563	9.094
	r=1	17.96	20.17

Johansen procedure is used to test for the number of cointegrating vectors (Johansen (1988) and Johansen and Juselius (1990)). In all cases the null of zero cointegrating vectors is rejected whilst the null of one cointegrating vector is not.[8] Therefore, the two variables are I(1), with a linear combination of them being stationary. The interesting question is whether the linear combination is the basis, as required for effectively functioning equity markets.[9] In all cases but the 12th, both the proportionality and zero cost of carry restrictions cannot be rejected at the 1% level. For the 12th, the cointegrating vector is given by $f_t = 1.2713s_t - 2.1229$.

With the exception of the 12th, then, the foundations for the effective functioning of both markets on an intra-daily is there. However, in moving to the identification problem, the equations for both markets on each day could be identified. The final equations are reported in table 4.4 overleaf. $Q_{SC}(.)$ is the Box-

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[8] For the 13th June, we also tested for the number of cointegrating vectors using seven lags in the VAR, given that the reduction of the VAR from seven lags to six is rejected quite strongly at the 5% level, although it is not rejected at the 1% level. The null of zero cointegrating vectors was marginally accepted. The estimated cointegrating vector and the statistics testing the restrictions were not altered.

[9] Alternatively, if the proportionality restriction is accepted but the zero cost of carry restriction is not, we have the basis adjusted for the cost of carry as the cointegrating vector. The same arguments with regard to effectiveness still apply.

TABLE 4.4 : MODELLING THE INTRA-DAILY PRICING RELATIONSHIP.

10TH JUNE 1991

Number of Observations = 286

FUTURES :

$$\Delta f_t = \underset{(-7.067)}{-0.359}\Delta f_{t-1}$$

$$Q_{SC}(10) = 8.532 \qquad Q_H(10) = 5.360$$

INDEX :

$$\Delta s_t = \underset{(17.65)}{0.566}\Delta s_{t-1} + \underset{(5.548)}{0.014} ecm_{t-1}$$

$$Q_{SC}(10) = 9.532 \qquad Q_H(10) = 13.88$$

Notes :

ecm_{t-1} denotes the error correction term which is the basis if the restrictions are valid. If not, it is as given in the text.

TABLE 4.4 CONT....

11TH JUNE 1991

Number of Observations = 327

FUTURES :

$$\Delta f_t = -0.204\Delta f_{t-1}$$
$$(-4.016)$$

$$Q_{SC}(10) = 6.353 \qquad Q_H(10) = 9.892$$

INDEX :

$$\Delta s_t = 0.786\Delta s_{t-1} + 0.007 ecm_{t-1}$$
$$(29.88) \qquad (4.676)$$

$$Q_{SC}(10) = 10.92 \qquad Q_H(10) = 22.93$$

Notes :

ecm_{t-1} denotes the error correction term which is the basis if the restrictions are valid. If not, it is as given in the text.

TABLE 4.4 CONT....

12TH JUNE 1991

Number of Observations = 293

FUTURES :

$$\Delta f_t = -0.211\,\Delta f_{t-1} - 0.145\,\Delta f_{t-2} + 2.053\,\Delta s_{t-1} - 1.943\,\Delta s_{t-2}$$
$$(-3.876) \qquad (-2.663) \qquad (4.181) \qquad (-3.965)$$

$$Q_{SC}(10) = 4.280 \qquad Q_H(10) = 7.301$$

INDEX :

$$\Delta s_t = 0.786\,\Delta s_{t-1} - 0.147\,\Delta f_{t-2} + 0.007\,ecm_{t-1}$$
$$(29.88) \qquad (-2.418) \qquad (4.676)$$

$$Q_{SC}(10) = 18.29 \qquad Q_H(10) = 14.19$$

Notes :

ecm_{t-1} denotes the error correction term which is the basis if the restrictions are valid. If not, it is as given in the text.

TABLE 4.4 CONT....

13TH JUNE 1991

Number of Observations = 302

FUTURES :

$$\Delta f_t = -\ 0.140\Delta f_{t-1} - 0.135\Delta f_{t-3} - 0.103\Delta f_{t-6} - 0.652\Delta s_{t-5}$$

$$\qquad (-2.768) \qquad (-2.685) \qquad (-2.047) \qquad (-2.202)$$

$$Q_{SC}(10) = 3.798 \qquad Q_H(10) = 12.07$$

INDEX :

$$\Delta s_t = 0.663\Delta s_{t-1} + 0.014\Delta f_{t-3} + 0.144\Delta f_{t-4} + 0.011 ecm_{t-1}$$

$$\qquad (20.96) \qquad (2.900) \qquad (3.053) \qquad (5.226)$$

$$Q_{SC}(10) = 18.22 \qquad Q_H(10) = 13.94$$

Notes :

ecm_{t-1} denotes the error correction term which is the basis if the restrictions are valid. If not, it is as given in the text.

TABLE 4.4 CONT....

14TH JUNE 1991

Number of Observations = 289

FUTURES :

$$\Delta f_t = -\underset{(-4.878)}{0.261}\Delta f_{t-1} - \underset{(-2.596)}{0.138}\Delta f_{t-2}$$

$$Q_{SC}(10) = 5.592 \qquad Q_H(10) = 5.583$$

INDEX :

$$\Delta s_t = \underset{(11.69)}{0.554}\Delta s_{t-1} + \underset{(4.338)}{0.009}ecm_{t-1}$$

$$Q_{SC}(10) = 11.22 \qquad Q_H(10) = 5.033$$

Notes :

ecm_{t-1} denotes the error correction term which is the basis if the restrictions are valid. If not, it is as given in the text.

Pierce test for serial correlation in the residuals, distributed $\chi^2(.)$ under the null of no serial correlation, and $Q_H(.)$ tests for serial correlation in the *squared* residuals, that is, it is a test for ARCH, distributed $\chi^2(.)$ under the null hypothesis of no ARCH effects.

Examining the diagnostics for the models, all are insignificant at the 1% level and thus the models are generally well specified. The interesting point to note is that whilst contemporaneous values are not significant in either equation on all of the days, a 'structural' model can still be identified. The implication of this is that equity markets are neither effectively functioning nor efficient when examined intra-daily. Each of the equations can be identified and therefore prices in both markets can be forecast using past information. Clearly, this is the source of both the ineffectiveness and the inefficiency.

It is interesting to note how this situation may arise. One of the findings in the literature on cointegration is that if the error correction term (the cointegrating vector) is significant in only one of the equations, then there is weak exogeneity. In this case, in no instance is the cointegrating vector significant in the futures equation. Therefore, the change in the futures price is weakly exogenous for the change in the Index price. Moreover, on the 10th, 11th and 14th June the futures price is *strongly* exogenous, that is, it is weakly exogenous for Index returns, plus Index returns do not Granger cause movements in the futures price : on these days the movement in the Index futures price is best described by an autoregressive process. For the futures market to be effectively independent of the stock market implies something is not functioning effectively. That the basis is insignificant in the futures equation suggests that the source of the ineffectiveness and inefficiency is improper functioning of arbitrage : it would appear to be uni–directional whereas it should be bi-directional.

Thus, again by analysing the pricing relationship within the context of the framework developed in chapter two it is possible to infer likely sources of ineffectiveness. It certainly appears to be the case that from the evidence presented above, the blame lies with ineffective functioning of arbitrage through the fact that the stock index futures market is exogenous to the stock market when in actual fact it should be endogenous.

A final point worthy of note is the absence of (G)ARCH effects in the models estimated above. This finding singularly contrasts with findings in the US. For example, Chan, Chan and Karolyi (1991) examine the relationship between intraday price changes and price change volatility for the S&P 500 Index and S&P Index futures contract. They specify returns as a vector autoregressive process with vector GARCH errors and find significant GARCH effects that persist. However, consider their specification of the vector autoregression for returns in both markets : it is a VAR specified in first differences. Thus the critique of tests of lead-lag relationships presented in chapter two is equally applicable in this context, that is, their model is misspecified. In this case, then, the GARCH effects may not be genuine : they may be more indicative of misspecification, something which we have avoided here by utilising the framework presented in chapter two for the analysis of pricing relationships between spot and futures markets.

4.4. Conclusions

In this chapter we have used minute by minute data on the FTSE 100 Index and the FTSE 100 Index futures contract for one week in June 1991 to analyse the pricing relationship on an intra-daily basis to determine if equity markets still function effectively. A problem with the Index (and portfolios in general) is that on an intra-daily time scale it may suffer from nonsynchronous trading effects. The upshot of this is that the observed value of the Index does not reflect its true value and therefore any inferences based on it are misleading.

To overcome this problem, we proposed a new method to estimate the nonsynchronous trading adjustment, formulating the problem as one of signal extraction, where the signal is extracted using the Kalman Filter. This method overcomes problems with other models proposed in the literature, in particular by modelling nonsynchronous trading not as an autoregressive process, which may in actual fact be genuine correlation, but as a moving average process. That nonsynchronicity generates a moving average error has intuitive appeal since one can then recast the problem of nonsynchronous trading as one of slow adjustment to new information.

Utilising this model, we estimated the nonsynchronous trading adjustment for the Index on each of the days and found it to be small. Armed with the adjusted Index, we then proceeded to examine whether both the stock index futures market and the stock market could be said to function effectively on an intra-daily time scale. We find that the first necessary condition, that the cointegrating vector be the basis, holds on all days bar one. However, the second necessary condition, that of under identification of the system, does not hold. Indeed, one can develop models that can be used in forecasting prices in *both* markets. In addition, an at first sight puzzling result emerges : the futures market is exogenous to the stock market, as evidenced by the insignificance of the basis in the equation for movements in the futures price. By implication, arbitrage is only uni-directional and therefore the markets can not possibly function effectively. It is tempting to blame the stock index futures market for this situation. However, one must be careful for it may be equally valid that the differing nature of the trading systems, in particular the pure dealership system operated on the International Stock Exchange, discourages arbitrage trades.

If this scenario is the correct one, then the blame lies with the stock market. Moreover, the significance of the error correction term then represents the stock market adjusting to information provided by the futures market. We investigate these possibilities in a little more detail in the next chapter to see if it is possible to pin down the cause of the crash to one particular market.

CHAPTER FIVE : THE OCTOBER 1987 STOCK MARKET CRASH

5.1. INTRODUCTION

Events in the international financial markets on the days surrounding the so-called Black Monday have variously been labelled a panic, a debacle, a long-overdue price correction, the burst of a speculative bubble and so on, the list seems potentially endless. Indeed, there was such concern about the speed with which prices fell and the sheer volume of shares that were traded that the then President of the United States formed a task force to investigate the role of market mechanisms in financial markets and in particular to determine whether the market mechanism ceased to serve its proper function on the days in question. Moreover, such was the concern with which the crash was viewed that the Task Force was ordered to report within sixty days.

It is apparent then that some form of investigation into the crash is necessary in order to determine what happened to the market mechanism in financial markets on the 19th and 20th October 1987.[1]

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[1] Note that here we are not interested in the cause(s) of the crash but rather what turned the initial downward pressure into the alarming decline in

There is no shortage of proposed explanations for why the market break took place. Indeed, as Roll (1988) notes, the one industry that has positively flourished since the crash is the production of explanations as to why it took place in the first place. Most of these 'explanations' seem to lay the blame fairly and squarely at the door of the US, particularly the size of its trade deficit, with the initial downward pressure supposedly being exacerbated by such factors as concurrent trading in stock index futures, computer-assisted trading, portfolio insurance sales and a whole host of other institutional characteristics that comprise the world financial system.

It is obviously of concern to determine the cause(s) of the crash (see Roll (1988) for one explanation), if for no other reason than to prevent it happening again given the profound effect it had on the confidence of world markets. However, of perhaps greater importance is the determination of what actually caused the initial downward pressure to be converted into the alarming decline in prices that followed.

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prices that followed. Roll (1988) has investigated the cause(s) of the crash, though for a critique of the method he uses see Garrett (1991a,b).

Attempting to explain what happened becomes even more important when one considers the fact that strong selling pressures generated by the market break threatened, and could easily threaten again, the short-term liquidity and long-term solvency of the financial markets so very important in the effective functioning of a modern economy. This point is elegantly summarised on the first page of the report of the Presidential Task Force (1988) :

> 'The significance of this decline lies in the role that the stock market plays in a modern industrial economy...Stock price levels can have an important effect on the confidence and hence the behaviour of both businesses and households...Equity markets are inextricably linked to the wider financial system through the structure of banks and other financial institutions. Given the importance of equity markets to the public, effectively structured and functioning equity markets are vital.'

Clearly the crash provoked widespread concern over the notion of effectively functioning stock markets and, by association, stock index futures markets.[2] Various aspects of the crash have been examined in some detail in the US (see, for example, Blume, MacKinlay and Terker (1989), Furbush (1989), Harris (1989) and Netter and Mitchell (1989)). These studies have

[2] Indeed, one only has to examine the speed with which the then Chancellor of the Exchequer, Nigel Lawson, reacted by lowering interest rates to avoid the feared recession that would follow the crash.

focused on the role of stock index futures in the US market decline, following on from the discussion of their role in the Presidential Task Force Report (1988). As yet, however, there has been little systematic empirical investigation of the crash in the UK. We aim to rectify this here.

In previous chapters, the daily pricing relationship was examined and an important point that emerges is the importance of the basis in the pricing relationship given its role its role in the arbitrage process and also its natural interpretation as the error correction mechanism which prevents prices in the two markets drifting apart without bound. In this chapter, we investigate the pricing relationship between the stock market and the stock index futures market using minute by minute values of the Index and minute by minute transaction prices for the December 1987 stock index futures contract.

We examine the pricing relationship between the two markets because this should allow us to determine whether the link between the two markets deteriorated to such an extent that the two markets effectively functioned as separate entities rather than acting as if they were one market. This latter scenario is the one that should occur given the inextricable links between derivative and underlying spot markets.

The rest of the chapter is organised as follows. The next section provides an overview of events surrounding the stock market crash to set the scene. In section three we briefly review the nonsynchronous trading problem and consider the extent to which nonsynchronous trading contributed to the observed behaviour of the markets on the 19th and 20th October 1987. In section four we model the minute by minute pricing relationship to determine whether or not the arbitrage link, so crucial to the effective functioning of equity markets broke and if it did, what precipitated the break. Section five concludes.

5.2. THE CRASH

During the months preceding the worldwide market break of October 1987 the performance of stock markets differed quite markedly from country to country. However, by October 1987 all stock markets were generally moving in the same direction : downwards with most markets suffering falls in the region of 20% (Roll (1988) p.21). This co-movement of all major stock markets appears to provide support to the belief that for too long stock markets had been overvaluing equity and a major price correction was due,[3] a point

[3] In terms of equity prices in relation to fundamentals, the well-known dividend valuation model (or, to use the terminology of Shiller (1981), the efficient markets model) did not hold, that is, equity

that is confirmed by figures 3.1 and 3.2 in chapter three. It is clear from these graphs that the crash returned price levels to their pre-Big Bang trend.

The relatively sharp upward trend in prices that commenced in the UK with Big Bang had begun to falter by April 1987 in the New York and London markets with Tokyo following suit in June. Despite this potential early warning signal, by September all three markets had embarked upon another sharp upward swing.

Such was the extent of the bull market that prevailed pre-October 1987 that at its peak, the London market was experiencing share price levels 46% higher than those at the beginning of 1987, with New York and Tokyo experiencing peaks of 44% and 42% respectively (Bank of England (1988) p.51). With the benefit of hindsight, it is not surprising that some form of price correction was overdue.

The downturn in share prices commenced on October 6th and prices fell almost continuously over the next two trading weeks. The most telling evidence of what was to come can be found by examining the New York market on the 14th, 15th and 16th October, where the Dow Jones Index fell by 95 points, 58 points and 108

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prices did not reflect the expected present value of future dividends.

points on respective days (Bank of England (1988)). This substantial downturn signalled the worldwide collapse that was to follow,[4] the FTSE 100 Index opening 138 points down and closing 250 points down on October 19th[5] (Bank of England (1988)). What emerges from this, however, is an apparently surprising difference in attitudes between the US and UK authorities as to the role derivative markets, and stock index futures in particular, played in the decline.

The Presidential Task Force Report (1988) pays considerable attention to the importance of stock index futures in the decline. This singularly contrasts with the view taken by the Bank of England that

[4] It is interesting that in attempting to explain the cause of the crash, Roll (1988) intimates that the crash cannot be traced back to the US since the American markets are the last to trade on any given trading day. In examining the transmission of international stock market movements over the period 1980-1985, however, Eun and Shim (1989) found that innovations in non-American markets had very little effect on the US markets whereas innovations in the US markets were rapidly transmitted to markets in other countries. This seems to concur with the pattern of events surrounding the crash.

[5] The International Stock Exchange in London did not open on the 16th October due to severe storms in the south of England. Consequently the collapse in share prices seems all the more bewildering.

> '...the interaction of the cash and derivative products markets seems to have played a very limited direct role in the crash in London.' (Bank of England (1988) p.57).

Whilst this may be true, figures for the daily trading volume and open interest of the December futures contract on the 19th and 20th appear to tell a different story, with approximately 10,000 contracts traded on *each* day, approximately double that of any other near-maturity contract in 1987.[6] This reinforces the fact that we cannot overlook the importance of stock index futures in the market decline and in particular the change (if any) in the pricing relationship on these two crucial days in stock market history.

5.3. NONSYNCHRONOUS TRADING AND THE CRASH

In this section, we address the issue of nonsynchronous trading and whether this contributed to the observed behaviour of the markets on the 19th and 20th October 1987. Harris (1989) investigated this issue for the US markets and concluded that nonsynchronous trading could explain some of the

[6] At the time of the crash, the December contract was nearest to maturity. Recall that the other expiration months for the FTSE 100 stock index futures contract are March, June and September. The expiration date is the last trading day of the expiration month.

observed behaviour of the basis, but not all. However, how much confidence we can have in Harris's results is unsure since he draws this conclusion solely on the basis of graphical analysis. We investigate the issue in a more rigorous fashion in the next subsection.

5.3.1. THE DATA AND THE NONSYNCHRONOUS TRADING ADJUSTMENT

The data we use to model the pricing relationship are minute by minute values of the FTSE 100 Stock Index and minute by minute transactions prices[7] for the December 1987 FTSE 100 Stock Index futures contract for the 19th and 20th October 1987. The data are for the period 09.05 to 16.05 on both days. The data were kindly provided by the International Stock Exchange and LIFFE and are plotted in figures 1 and 2 overleaf.

One of the interesting features of the data is the fact that the futures appears to have traded at a discount which was at times substantial. This point is confirmed in figures 3 and 4 overleaf, where the minute by minute basis is graphed for both days. The

[7] There are a few minutes during both days where transactions never took place. In this case, we use an average of the bid-ask quotes for that minute. These periods of no trading are, however, very few and far between.

FIGURE 5.1 : FTSE 100 FUTURES AND INDEX PRICES
19TH OCTOBER 1987

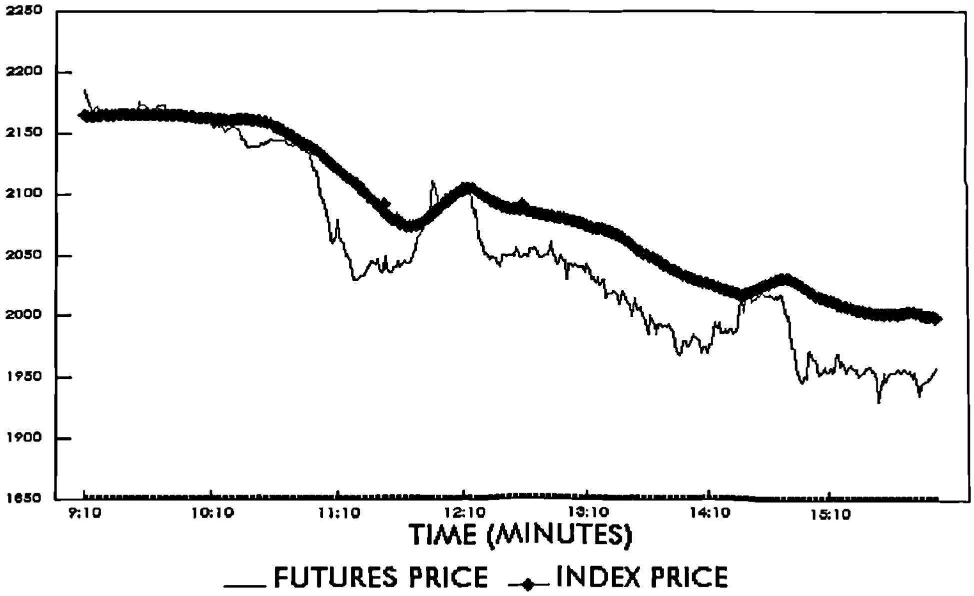


FIGURE 5.2 : FTSE 100 FUTURES AND INDEX PRICES
20TH OCTOBER 1987

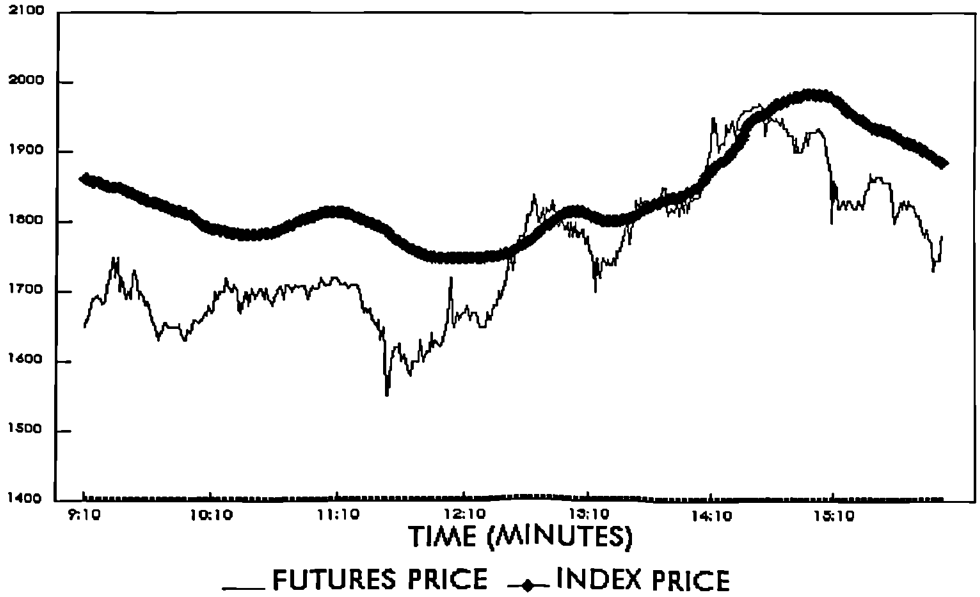


FIGURE 5.3 : THE TIME PATH OF THE BASIS
19TH OCTOBER 1987

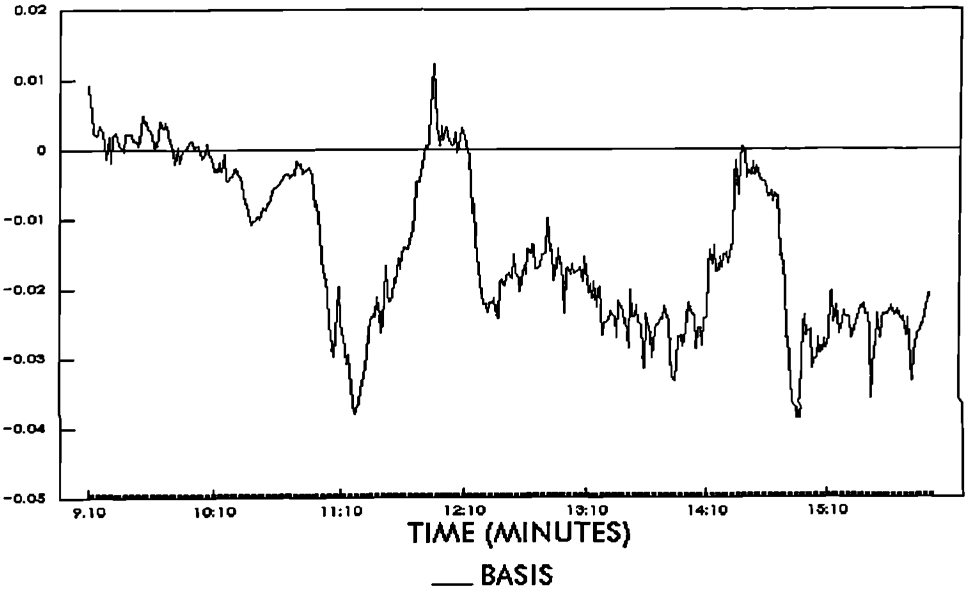
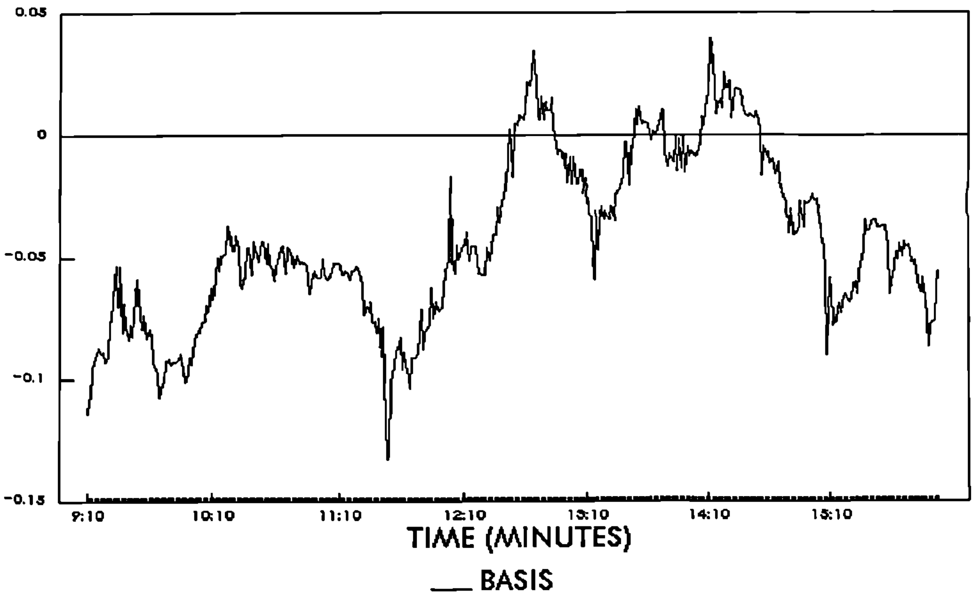


FIGURE 5.4 : THE TIME PATH OF THE BASIS
20TH OCTOBER 1987



size of the basis on both days *would* appear to be indicative of the presence of arbitrage opportunities. An interesting issue to be analysed is why these apparent opportunities for arbitrage persisted.

However, as we know from chapter four, caution must be exercised at this point for, as we have seen previously, one must be careful in uncritically using the data for the FTSE 100 Index since the recorded value is unlikely to reflect its true value. This arises because not all shares within the Index will necessarily trade in any one given minute. Some will react to new information with a time lag, leading to the so-called problem of nonsynchronous trading whereby the reported value of the Index contains old, or stale, prices.

Removing the effects of nonsynchronous trading, then, is important if we are to obtain an accurate measure of the basis. Recall from chapter four that the nonsynchronous trading problem to be *considered can be* stated as follows :

$$S_t = S_t^* + u_t \tag{5.1}$$

where S_t is the observed value of the Index, S_t^*, which is unobservable, is the true value of the Index and u_t

is the nonsynchronous trading adjustment. Following Garrett (1991), and as discussed in chapter four, (5.1) should be treated as an unobserved components model. By doing this, we can extract S^* using the Kalman Filter.[8] The specification of the model that we use is the local linear trend model (Harvey (1987)) which is given by

$$S_t = S_t^* + u_t \tag{5.2a}$$

$$S_t^* = S_{t-1}^* + \beta_{t-1} + v_t \tag{5.2b}$$

$$\beta_t = \beta_{t-1} + \zeta_t \tag{5.2c}$$

The system given by (5.2) was estimated using (the log of the) minute-by-minute recorded value of the FTSE 100 Index to generate the nonsynchronous trading adjustment. Graphs of the nonsynchronous trading adjustment to the Index[9], minus the first five observations lost through the initialisation of the Kalman Filter are presented in figures 5 and 6 overleaf.

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[8] We will not repeat the econometrics of the Kalman Filter as they were discussed in chapter four and are not essential to the argument here.

[9] The figures are multiplied by 100 for readability.

FIGURE 5.5 : NONSYNCHRONOUS TRADING ADJUSTMENT
19TH OCTOBER 1987

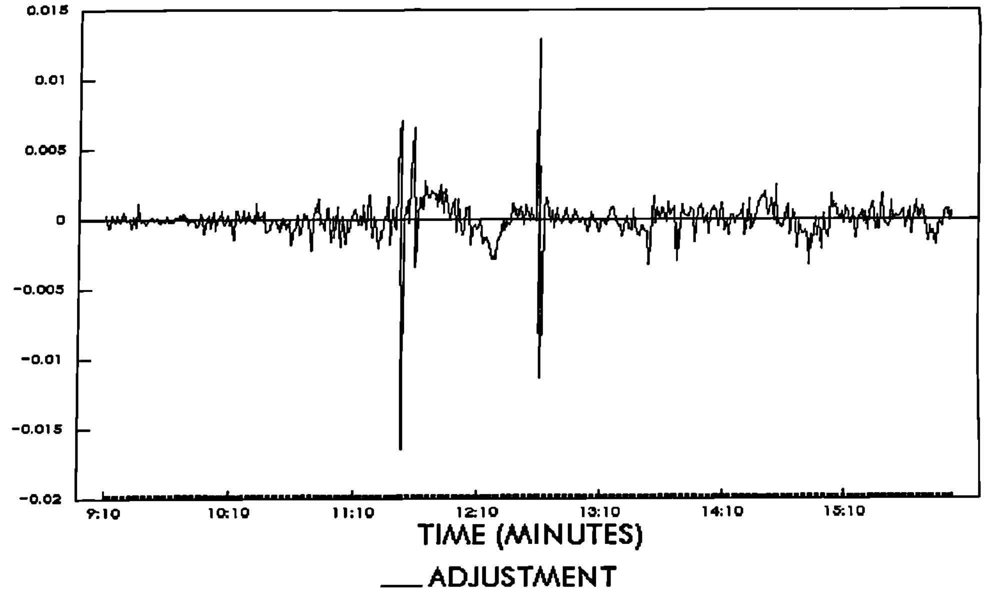
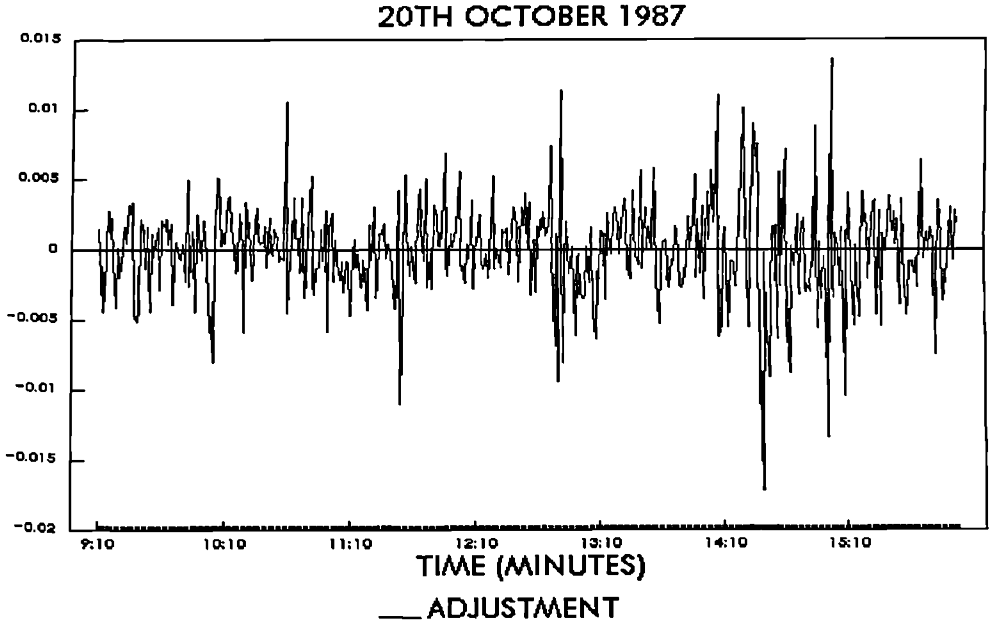


FIGURE 5.6 : NONSYNCHRONOUS TRADING ADJUSTMENT
20TH OCTOBER 1987



As can be seen, the adjustment is relatively small on both days, perhaps reflecting the fact that the Index only comprises 100 shares.[10] The fact that the nonsynchronous adjustment is small implies that nonsynchronicity alone cannot explain why prices in both markets fell so dramatically and in unison. In order to test this proposition, we regress the basis on the nonsynchronous trading adjustment estimated from the system (5.2). However, before we report the results of these models for the 19th and 20th, one point worth commenting on is the behaviour of the basis.

The time series of the basis on both the 19th and the 20th October, plotted in figures 3 and 4, appears to have quite substantial variation, more than we would expect. This would appear to suggest that the variance of the basis changes over time and as such any attempt to model the effect the nonsynchronous trading adjustment has on the basis must take this time variation in the variance into account.

That the variance of financial time series can change over time is not a new concept (see Mandlebrot (1963) and Fama (1965) for example). However, it is only in

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[10] This seems consistent with results emerging from the US showing that, as one would expect, those indices comprised of more shares suffer more from the problems of nonsynchronous trading.

recent years, with the advent of ARCH (Engle (1982)) and its extensions (Bollerslev (1986), Engle, Lilien and Robins (1987), Bollerslev, Engle and Wooldridge (1988), Nelson (1991) to name but a very few) that changing variance in a series has been modelled explicitly.

The ARCH model, first introduced by Engle (1982) is based on the rather simple observation that large price changes tend to be followed by large price changes and small price changes tend to be followed by small price changes, but of unpredictable sign. In this situation, whilst the unconditional variance of the returns series will be constant, the conditional variance will not. The argument here is that this happens because large price changes generate increased uncertainty and, as price changes fluctuate between large and small, so will uncertainty. If uncertainty is measured by the conditional variance, then this must change as well. In particular, it will be autoregressive, hence ARCH (AutoRegressive Conditional Heteroscedasticity). To capture the effects of a changing conditional variance, Engle (1982) proposed that the conditional variance of a series $E(\varepsilon_t^2|\Omega_t)=\sigma_t^2$, where Ω_t is the information set at time t, be parameterised as

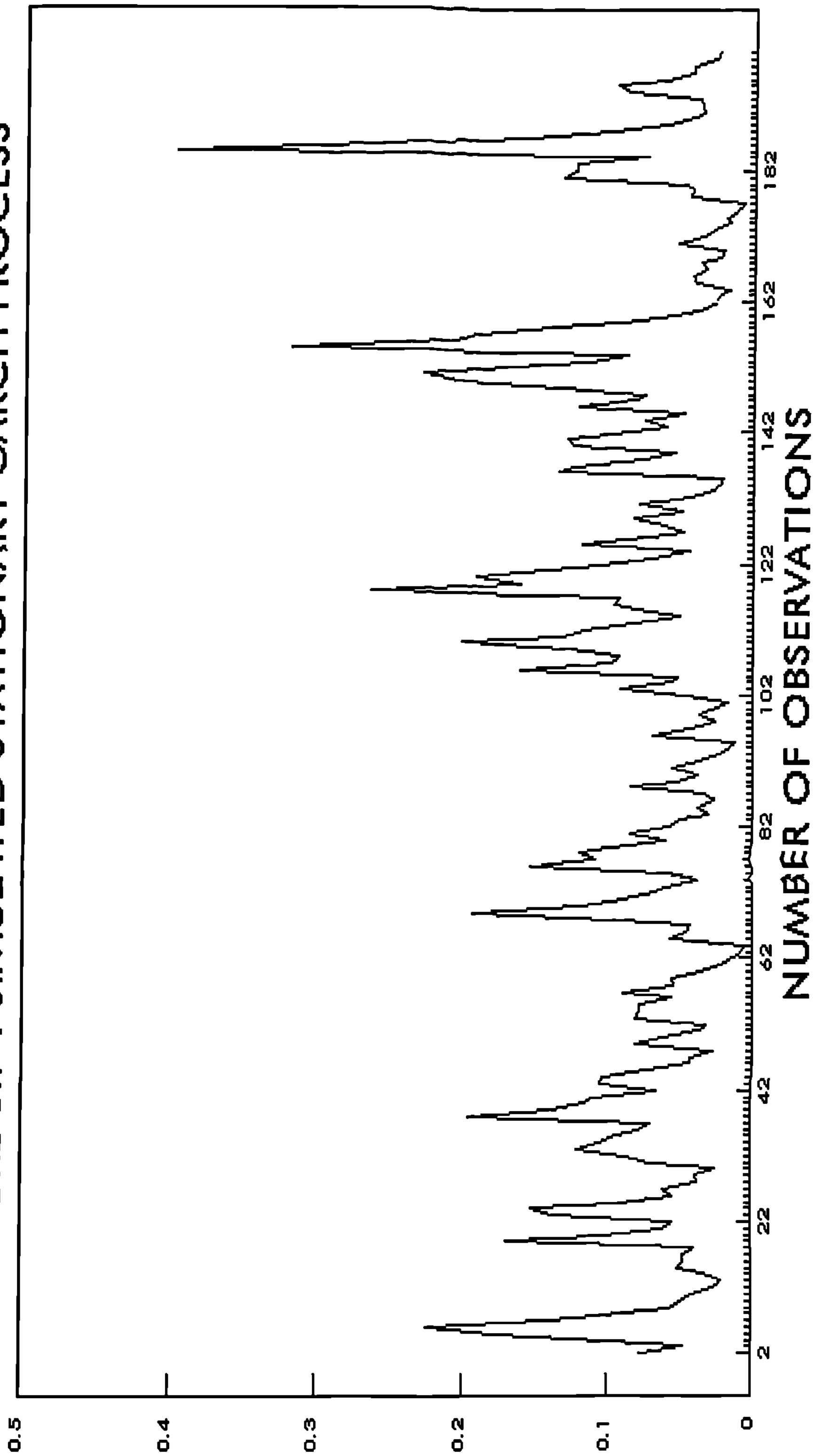
$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 \tag{5.3}$$

where $\omega > 0$ and $\alpha_i \geq 0$. This model is the linear ARCH(q) model. The problem with this model is that often q is chosen to be quite large and thus, to ensure that more distant shocks have a smaller impact, an *ad hoc* linearly declining lag structure has to be imposed. Bollerslev (1986) overcame this problem by generalising Engle's (1982) ARCH model. The model proposed by Bollerslev (1986) is the GARCH(p,q) model and can be thought of as the ARCH equivalent to the ARMA(p,q) model (Box and Jenkins (1970)). The GARCH(p,q) model is given by

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2 \tag{5.4}$$

This model has proved to be extremely popular in modelling financial time series (see the bibliography in Bollerslev, Chou and Kroner (1992)), primarily because it has two attractive features. First, the GARCH(1,1) specification almost invariably best describes the behaviour of the conditional variance and second, it allows for a unit root in the conditional variance such that shocks to the conditional variance are permanent. If the latter is

FIGURE 5.7 : SIMULATED STATIONARY GARCH PROCESS



the case, then the series has an integrated GARCH (IGARCH) representation. This will occur if the sum of the α_i's and β_i's is equal to one. To give some indication of how the conditional variance may change, a simulated stationary GARCH(1,1) process is plotted in figure 5.7.[11]

To examine whether nonsynchronous trading contributed to the observed behaviour of the markets, we formalise Harris (1989) and estimate the following model for both the 19th and 20th, with the conditional variance evolving as a GARCH(1,1) process

$$(f-s)_t = \alpha_0 + \alpha_1(f-s)_{t-1} + \alpha_2\hat{u}_t + \alpha_3\hat{u}_{t-1} + \epsilon_t \qquad (5.5)$$

where $\hat{u}_t$ is the estimated nonsynchronous trading adjustment and lower case letters denote variables in natural logarithms. Estimation of (5.5) yields (standard errors in parentheses, T=415)

[11] For figure 7,

$$\sigma_t^2 = 0.25\epsilon_{t-1}^2 + 0.6\sigma_{t-1}^2$$

i) 19th October

$$(f-s)_t = \underset{(0.00)}{0.00} + \underset{(0.01)}{0.96}(f-s)_{t-1} - \underset{(0.07)}{0.18}\hat{u}_t + \underset{(0.08)}{0.20}\hat{u}_{t-1}$$

$$\sigma_t^2 = \underset{\left(0.26^{10^{-6}}\right)}{0.13^{10^{-5}}} + \underset{(0.04)}{0.20}\epsilon_{t-1}^2 + \underset{(0.05)}{0.61}\sigma_{t-1}^2$$

ii) 20th October

$$(f-s)_t = \underset{(0.00)}{0.00} + \underset{(0.01)}{0.96}(f-s)_{t-1} + \underset{(1.03)}{0.45}\hat{u}_t - \underset{(1.26)}{0.15}\hat{u}_{t-1}$$

$$\sigma_t^2 = \underset{\left(0.63^{10^{-5}}\right)}{0.31^{10^{-4}}} + \underset{(0.08)}{0.38}\epsilon_{t-1}^2 + \underset{(0.11)}{0.23}\sigma_{t-1}^2$$

Likelihood ratio tests testing the restrictions $\alpha_2=\alpha_3=0$ yield $\chi^2(2)=13.14$ for the 19th and $\chi^2(2)=0.24$ for the 20th. The restrictions are clearly rejected for the 19th, but are accepted on the 20th. Therefore, nonsynchronous trading did explain some of the variation in the basis on the 19th, although it explained none of the variation on the 20th. However, the effect of the nonsynchronous trading adjustment is small relative to the effect of the previous period's basis and it can by no means explain the behaviour of the basis. One possible explanation of this is that, as Harris (1989) found for the US, the relationship between the two markets actually broke down completely on at least one of the days. We turn attention to examining this possibility in the next section of the chapter.

5.4. DID THE LINK BREAK?

From the framework presented in chapter two, any analysis of the pricing relationship should be conducted in the context of a VAR, taking into account the arbitrage link between the two markets which can be identified as the basis. The basis provides this link theoretically through its role in index arbitrage and econometrically through its role as the error correction mechanism which ensures that the two prices do not drift apart without bound, that is, it ensures that in the long-run $f=s$. Thus, as in previous chapters, we begin our analysis in the context of the VAR reparameterised in error correction form (Johansen (1988) and Johansen and Juselius (1990))

$$\Delta \boldsymbol{y}_t = \boldsymbol{\mu} + \boldsymbol{\Gamma}_1 \Delta \boldsymbol{y}_{t-1} + \ldots\ldots + \boldsymbol{\Gamma}_{k-1} \Delta \boldsymbol{y}_{t-k+1} + \boldsymbol{\Pi} \boldsymbol{y}_{t-k} + \boldsymbol{u}_t \qquad (5.6)$$

where Π represents the long-run response matrix. Writing the long-run response matrix as $\Pi=\alpha\beta'$, then the linear combinations $\boldsymbol{\beta}'\mathbf{x}_{t-k}$ will be I(0) if there is cointegration, with α being the adjustment coefficients, and the long-run response matrix will be of reduced rank. The Johansen test for cointegration is then based on testing the rank of the matrix Π. Denoting rank (Π) by r, recall that there are three possibilities. First, r=0 in which case all of the variables are I(1) and there are no cointegrating vectors. Second, r=N in which case all of the variables are I(0) and there will be N cointegrating vectors given that any linear combination of stationary variables will also be stationary. Finally, 0<r<N in which case there will be r linear combinations of the nonstationary variables that are stationary, that is, there will be r cointegrating vectors or, equivalently, N-r common stochastic trends.

The advantage of using the Johansen procedure for our purposes here is that it is possible to test restrictions on the cointegrating vectors, the statistics being χ^2 distributed. This is particularly useful in this case since we know the form the cointegrating vector should take.

For the basis to be the cointegrating vector, we require proportionality to hold, that is, in terms of the equation $f_t = \gamma_0 + \gamma_1 s_t + e_t$, we require γ_1 to be equal to one. γ_0 can be interpreted as the cost of carry in this case since on an intra-day basis it will be constant and if the futures contract is near maturity, it should be near zero. Table 5.1 overleaf reports the test statistics discussed in Johansen and Juselius (1990) for the number of cointegrating vectors and also tests the restrictions on the cointegrating vectors.

The null hypothesis of zero cointegrating vectors is rejected at the 5% level on both days whilst the null of one cointegrating vector cannot be rejected.[12] It is clear, then, that both variables are I(1), a point which is confirmed graphically in figures 8 through 11, with the linear combination being I(0).

What is interesting from these results is the form of the cointegrating vector for the two days. This provides us with a first idea as to what happened on these two days. The important restriction here is the proportionality restriction, though the cost of carry restriction does have minor interest.[13] The

[12] For critical values see Johansen and Juselius (1990), table A3.

[13] The zero restriction is not so important for if the homogeneity restriction were valid but the zero cost of carry restriction were not, the error correction term would simply be the basis adjusted for the cost of carrying and this is what arbitrageurs would compare with transactions costs to see if arbitrage opportunities were available.

TABLE 5.1 : TESTS FOR THE NUMBER OF COINTEGRATING VECTORS AND TESTS OF THE RESTRICTIONS ON THE VECTORS (H_o^a:r=0, H_1^a:r=1 ; H_o^b:r≤1, H_1^b:r=2)

	19TH OCTOBER		20TH OCTOBER	
	H_o^a	H_o^b	H_o^a	H_o^b
λ_{max}	21.71	3.606	18.51	5.318
λ_{trace}	25.31	3.606	23.83	5.318
RESTRICTIONS :				
$\gamma_0 = 0 : \chi^2(1)$	14.17		2.339	
$\gamma_1 = 1 : \chi^2(1)$	14.13		2.233	

Notes :

Critical Values (*Johansen and Juselius (1990)* Table A.3.)

		90%	95%
λ_{max}	r=0	7.563	9.094
	r=1	13.78	15.75
λ_{trace}	r=0	7.563	9.094
	r=1	17.96	20.17

FIGURE 5.8 : PERCENTAGE CHANGE IN FUTURES PRICE 19TH OCTOBER 1987

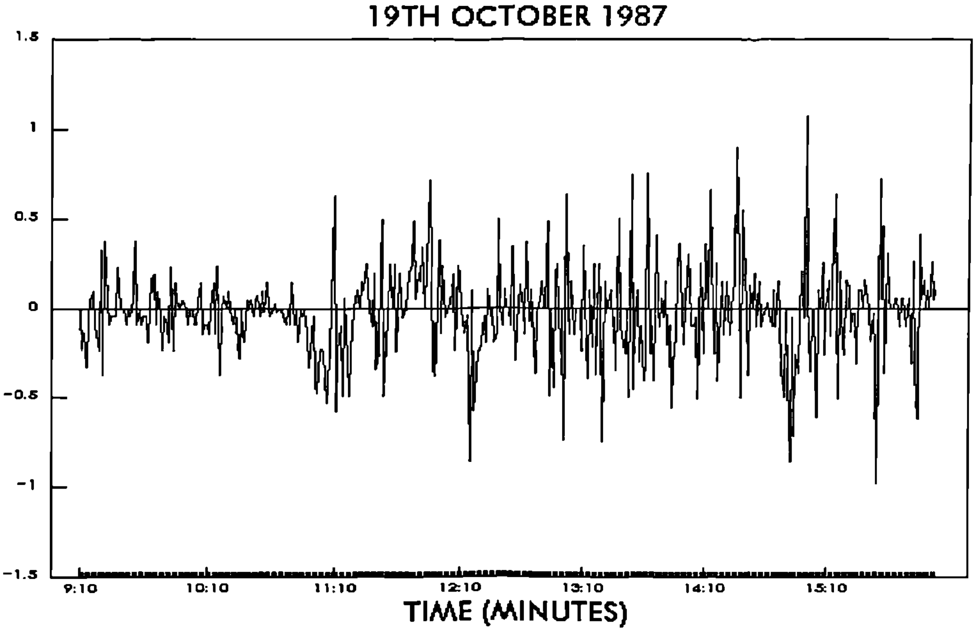


FIGURE 5.9 : PERCENTAGE CHANGE IN ADJUSTED INDEX PRICE 19TH OCTOBER 1987

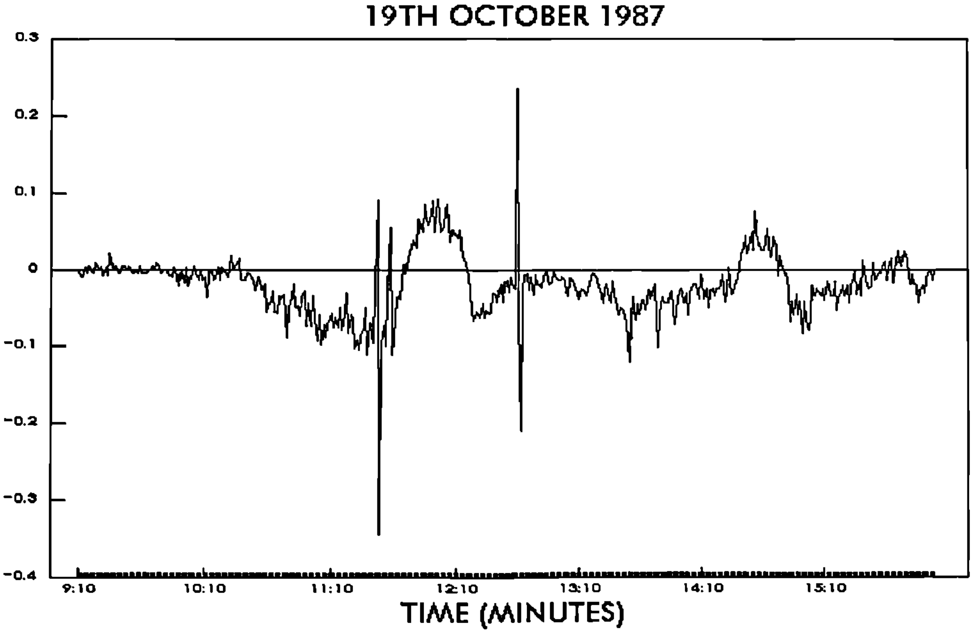


FIGURE 5.10 : PERCENTAGE CHANGE IN FUTURES PRICE
20TH OCTOBER 1987

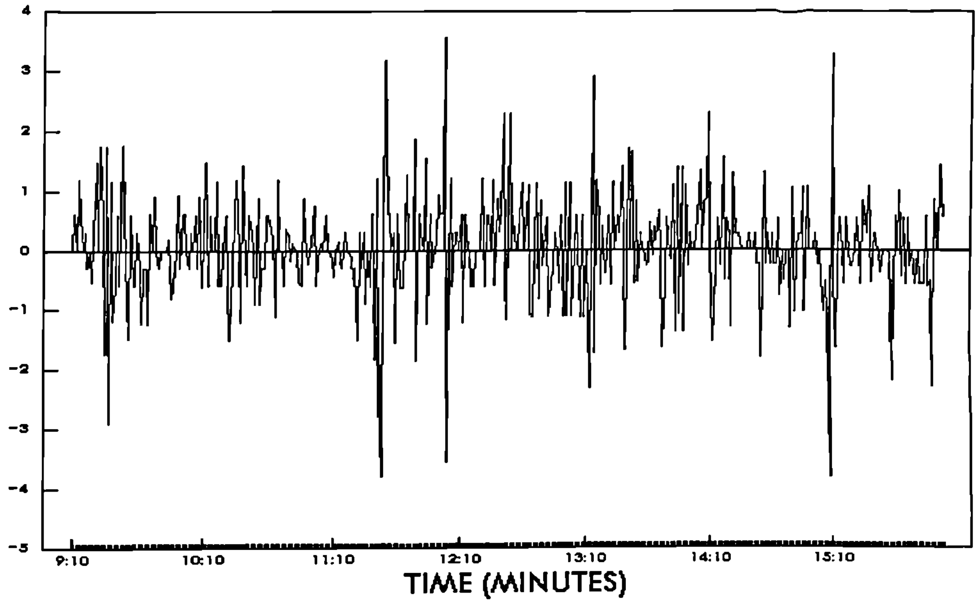
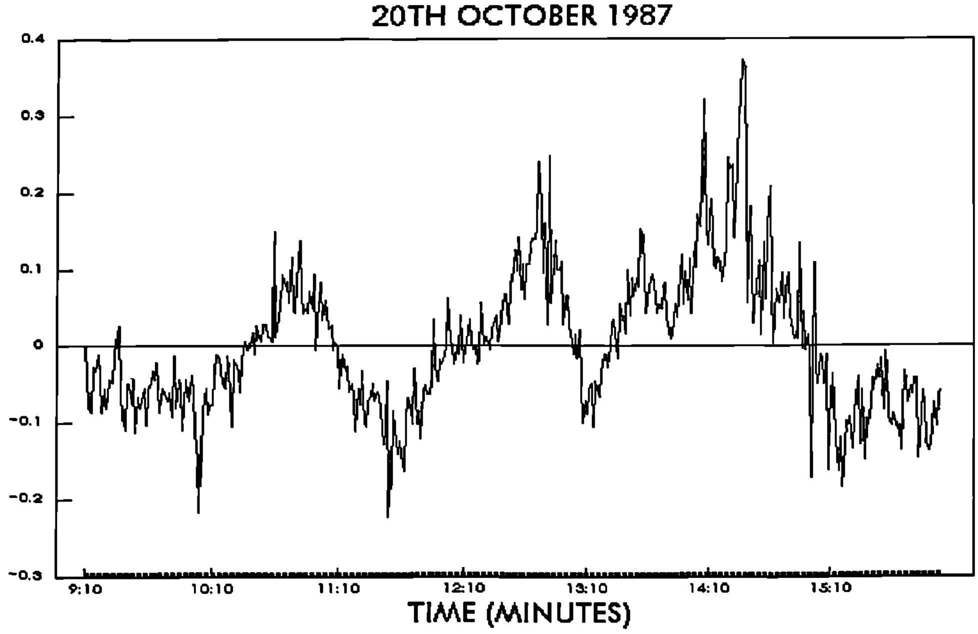


FIGURE 5.11 : PERCENTAGE CHANGE IN ADJUSTED INDEX PRICE
20TH OCTOBER 1987



proportionality restriction is strongly rejected for the 19th October, as, less importantly, is the zero cost of carry restriction.

The implication of this is that whilst the two markets were linked on the 19th, that link was not the basis (adjusted for the cost of carry) and therefore, by implication, the link was not the one provided by index arbitrage. Indeed, for the 19th, the cointegrating vector is given by $f_t=1.2704s_t-2.0765$.

The evidence, then, suggests that the arbitrage link did not operate effectively on the 19th October : the important link between the two markets broke down. The implication of this is that the mechanism that serves to stabilise prices in both markets, index arbitrage, would not serve its purpose. If stock index futures prices were falling such that the futures price was below its fair value and outside of the no-arbitrage window then arbitrageurs would buy futures and sell stock. Initial selling pressure would then be transmitted from the stock index futures market to the stock market. If the futures price then rose so that it lay outside the upper no-arbitrage window, then the reverse trade would be initiated and buying pressure would be transmitted from the stock index futures market. Thus, the futures price would fluctuate around its equilibrium value and the basis would be stationary.

However, the basis on the 19th is best described as a martingale process such that arbitrage trades based on the basis would be incorrect trades since the basis was nonstationary and thus of little guide in determining the existence of arbitrage opportunities. In other words, the markets could not possibly have been effectively functioning on the 19th because the first necessary condition does not hold.

This was not the case on the 20th, when the link between the two markets was the basis and again, by implication, the arbitrage link was restored. We will return to the question of how this might have occurred later. What we appear to observe, then, is different behaviour by the markets on the two different days. On the 20th, the error correction term was the basis whilst on the 19th it was not.

It must be *noted, however, that some link did still* exist on the 19th because both prices continued to fall in unison. The implication of this is that we should observe differences in the behaviour of the pricing relationship between the two markets on both days.

From the discussion in earlier chapters, and the cointegration results, we know that the pricing relationship should be modelled in the context of the error correction representation of the VAR. Given the evidence of significant ARCH effects documented by Antoniou and Garrett (1989), we estimate the models using the GARCH(1,1) specification (see Bollerslev (1986)). Starting from a model with 10 lags of each variable (except the error correction term, which is lagged once),[14] we obtained the parsimonious models reported overleaf. The models seem to be adequately specified with none of the diagnostic tests being significant at the 1% level.[15]

Turning our attention to the results, an interesting scenario emerges. If both the equity and futures markets were effectively functioning then the basis should be significant in explaining price movements in both markets. However, we observe something very different.

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[14] We realise of course that the choice of lag length is somewhat *ad hoc*. However, 10 minutes does not seem an unreasonable starting point for our analysis given the extraordinary events that took place on those two fateful days.

[15] The standardised residuals used in the construction of the Box-Pierce-type tests are given by $\varepsilon_t\sigma_t^{-1}$. The test for heteroscedasticity is based on the autocorrelation function of the squared standardised residuals.

TABLE 5.2 : ESTIMATED EQUATIONS

19TH OCTOBER 1987

$$\begin{bmatrix}\Delta f_t\\ \Delta s_t\end{bmatrix}=\begin{bmatrix}0 & 1.198\\ (0.000) & (3.083)\\ 0.010 & 0.194\\ (2.768) & (3.613)\end{bmatrix}\begin{bmatrix}\Delta f_{t-1}\\ \Delta s_{t-1}\end{bmatrix}+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.014 & 0.211\\ (5.021) & (3.613)\end{bmatrix}\begin{bmatrix}\Delta f_{t-2}\\ \Delta s_{t-2}\end{bmatrix}+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.011 & 0.143\\ (3.367) & (4.446)\end{bmatrix}\begin{bmatrix}\Delta f_{t-3}\\ \Delta s_{t-3}\end{bmatrix}$$

$$+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.015 & 0.146\\ (4.423) & (3.497)\end{bmatrix}\begin{bmatrix}\Delta f_{t-4}\\ \Delta s_{t-4}\end{bmatrix}+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.011 & 0.049\\ (2.760) & (2.022)\end{bmatrix}\begin{bmatrix}\Delta f_{t-5}\\ \Delta s_{t-5}\end{bmatrix}+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.007 & 0\\ (2.045) & (0.000)\end{bmatrix}\begin{bmatrix}\Delta f_{t-6}\\ \Delta s_{t-6}\end{bmatrix}$$

$$+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.003 & 0\\ (0.934) & (0.000)\end{bmatrix}\begin{bmatrix}\Delta f_{t-7}\\ \Delta s_{t-7}\end{bmatrix}+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.012 & 0\\ (4.101) & (0.000)\end{bmatrix}\begin{bmatrix}\Delta f_{t-8}\\ \Delta s_{t-8}\end{bmatrix}+\begin{bmatrix}0 & 0\\ (0.000) & (0.000)\\ 0.022 & 0\\ (8.146) & (0.000)\end{bmatrix}\begin{bmatrix}\Delta f_{t-9}\\ \Delta s_{t-9}\end{bmatrix}+\begin{bmatrix}-0.05\\ (-2.51)\\ 0\\ (0.000)\end{bmatrix}\begin{bmatrix}ecm_{t-1}\end{bmatrix}$$

$$\begin{bmatrix}\sigma^2_{f,t}\\ \sigma^2_{s,t}\end{bmatrix}=\begin{bmatrix}0.132\\ (5.294)\\ 0.002\\ (6.765)\end{bmatrix}+\begin{bmatrix}0.218 & 0\\ (3.973) & (0.000)\\ 0 & 0.988\\ (0.000) & (12.27)\end{bmatrix}\begin{bmatrix}\varepsilon^2_{f,t-1}\\ \varepsilon^2_{s,t-1}\end{bmatrix}+\begin{bmatrix}0.605 & 0\\ (10.99) & (0.000)\\ 0 & 0.046\\ (0.000) & (1.021)\end{bmatrix}\begin{bmatrix}\sigma^2_{f,t-1}\\ \sigma^2_{s,t-1}\end{bmatrix}$$

Futures : $Q_{SC}(10)=15.86$ $Q_H(10)=5.06$

Index : $Q_{SC}(10)=16.35$ $Q_H(10)=1.10$

Notes :

Figures in parentheses are t ratios
Constants in the variance equation are multiplied by 10^5 for readability
$Q_{SC}(.)$ is a portmanteau test for serial correlation in the standardised residuals.
$Q_H(.)$ is a portmanteau test for heteroscedasticity in the standardised residuals.

20TH OCTOBER 1987

$$\begin{bmatrix}\Delta f_t \\ \Delta s_t\end{bmatrix} = \begin{bmatrix} -0.29 & 0 \\ (-4.60) & (0.000) \\ 0.004 & 0.484 \\ (2.708) & (9.190) \end{bmatrix} \begin{bmatrix}\Delta f_{t-1} \\ \Delta s_{t-1}\end{bmatrix} + \begin{bmatrix} 0 & 0 \\ (0.000) & (0.000) \\ 0 & 0.293 \\ (0.000) & (5.154) \end{bmatrix} \begin{bmatrix}\Delta f_{t-2} \\ \Delta s_{t-2}\end{bmatrix} + \begin{bmatrix} 0 \\ (0.000) \\ 0.004 \\ (3.574) \end{bmatrix} \begin{bmatrix} ecm_{t-1} \end{bmatrix}$$

$$\begin{bmatrix}\sigma^2_{f,t} \\ \sigma^2_{s,t}\end{bmatrix} = \begin{bmatrix} 2.490 \\ (4.076) \\ 0.002 \\ (2.969) \end{bmatrix} + \begin{bmatrix} 0.464 & 0 \\ (6.087) & (0.000) \\ 0 & 0.105 \\ (0.000) & (3.557) \end{bmatrix} \begin{bmatrix}\varepsilon^2_{f,t-1} \\ \varepsilon^2_{s,t-1}\end{bmatrix} + \begin{bmatrix} 0.272 & 0 \\ (2.328) & (0.000) \\ 0 & 0.760 \\ (0.000) & (11.93) \end{bmatrix} \begin{bmatrix}\sigma^2_{f,t-1} \\ \sigma^2_{s,t-1}\end{bmatrix}$$

Futures : $Q_{SC}(10)=15.25$ $Q_H(10)=10.38$

Index : $Q_{SC}(10)=12.71$ $Q_H(10)= 9.46$

Notes :

Figures in parentheses are t ratios
Constants in the variance equation are multiplied by 10^5 for readability
$Q_{SC}(.)$ is a portmanteau test for serial correlation in the standardised residuals.
$Q_H(.)$ is a portmanteau test for heteroscedasticity in the standardised residuals.

The first indication of breakdown is the fact that, as was discussed earlier, the basis was not the error correction term on the 19th. The results also tend to support those found in studies of the US markets (for example, Kawaller, Koch and Koch (1987), Stoll and Whaley (1990)) in the sense that the structure of feedback between the two markets is asymmetric, with the futures playing the predominantly greater role.

However, what is of interest here is the extent of the feedback from the stock market to the stock index futures market on the 19th October for whilst some of the feedback occurs through the error correction term the interesting point to note is the coefficient on Δs_{t-1} : it is approximately 1.2. Thus, whilst the stock market was reacting to declines in the stock index futures market, the stock index futures market was reacting (indeed one could argue overreacting) to declines in the stock market. Therefore, a vicious downward spiral in prices ensued. Moreover, given the evidence presented earlier that the arbitrage link effectively broke on the 19th, there was nothing to counteract the fall.

This conclusion of market breakdown is reinforced when we consider the behaviour of the conditional variance for the markets on both days. An interesting aspect of the interaction between conditional variances is the notion of cointegration in variance (for a brief discussion, see Bollerslev, Chou and Kroner (1992)). There is a great deal of evidence (see the review and bibliography in Bollerslev, Chou and Kroner (1992)) showing that for many financial time series the restriction that $\alpha+\beta=1$ in the conditional variance equation cannot be rejected such that the conditional variance has a unit root (IGARCH).

This obviously raises the question about whether the conditional variances of two similar series cointegrate such that a linear combination of them shows no persistence. Through the similarity of the stock index and stock index futures prices one would expect their conditional variances to cointegrate, and this is indeed the case on the 20th since shocks to the conditional variances for both *markets are not* persistent[16] and therefore a linear combination would not be persistent. However, the 19th again shows an altogether different state of affairs, with the conditional variance for the index exhibiting I(1) behaviour as opposed to the apparent I(0) type

[16] Given the evidence in Antoniou and Foster (1992) that shocks to volatility do not persist when one allows for the effects of futures markets and also the evidence in Lamoureux and Lastrapes (1990) that persistence may in actual fact be caused by structural shifts in volatility (which in itself may be indicative of misspecification), one would expect that the conditional variances for the stock and stock index futures prices are indeed not persistent.

behaviour of the conditional variance for the futures. Given that cointegration requires the same order of integration in the individual series, it is apparent that the two are not cointegrated in variance and this is a further indication of market breakdown.

Thus, there is clear evidence that there was a breakdown. Moreover, the overreaction of the futures price would appear to provide *prima facie* evidence that the anti-futures lobby is correct : futures destabilise. However, this may not be the case, as we shall see.

5.4.1. Why Did The Link Break And Why Was It Restored ?

It would seem that the initial downward pressure on prices manifested itself in the decline that followed because the link between the two markets broke down. The important question is why this should be the case. Given that, with the futures being undervalued, the appropriate arbitrage strategy would be to buy futures and sell stock it would appear that indeed arbitrage broke down, certainly on the 19th.

Consider now the 20th. We see a partial reversal of what occurred on the 19th : we see that the stock index futures market leads the stock market, both through Δf_{t-1} and through the error correction term, with no feedback from the stock market to the futures, even though the futures sold at a discount. This also coincides with the restoration of the basis as the arbitrage link. It would appear, then, that the reaction of participants in the stock index futures market on the 20th was to effectively ignore price movements in the stock market whilst the stock market utilised the information provided by price movements in the index futures market.

There are several points to note about the behaviour of the markets which may explain why this apparent 'reversal' took place. Consider first of all the conclusions reached in the Quality of Markets Report (see the discussion in Kleidon and Whaley (1992)) about why sellers were willing to trade futures at a discount. They argue that two factors were at work : sellers did not believe they could transact immediately in the stock market at quoted prices and second, sellers may not have believed that the prices quoted were the correct ones.

Consider now the nonsynchronous trading adjustment plotted in figures 5 and 6. These show that the nonsynchronous trading problem was more severe on the 20th and therefore, by implication, there was less trading in the stock market on the 20th relative to the 19th. These results appear to confirm the conclusions reached in the Quality of Markets Report.

This situation may also help to explain why the basis was restored as the link. One possible reason for the break in the link on the 19th is the argument put forward in the Quality of Markets Report. The Quality of Markets Report suggests that sellers came to believe that they could not transact in the stock market. As a result, sellers moved away from the stock market to the futures market. By implication, there was less trading in the stock market on the 20th relative to the 19th, thereby alleviating the selling pressure, allowing the link to be restored. Drawing all these points together, the evidence seems to point to the stock market as the cause of the breakdown.

5.5. CONCLUSIONS

In this chapter, we have set out to analyse the pricing relationship between the FTSE 100 Index and the FTSE 100 stock index futures contract on the 19th and 20th October 1987, the period of the stock market crash. In particular we have set out to investigate the extent to which the FTSE 100 futures contract contributed to the crash.

To address this question, we examine the pricing relationship between the stock market and stock index futures market on those two fateful days, using the foundation provided in chapter two for analysing problems of this sort. Before modelling the pricing relationship, however, we address the nonsynchronous trading problem prevalent in high frequency price data on indices. We find that nonsynchronicity explains little of the observed behaviour of the markets, a result consistent with Harris' (1989) findings for the US.

Despite the fact that the futures traded at a discount, which is indicative of arbitrage opportunities, we find that the link between the two markets on the 19th was not the link provided by arbitrage. We also find that the futures price strongly leads the Index with some weaker evidence of feedback from the Index to the futures on the 19th, a result apparently consistent with evidence from the US for stable time periods.

However, in this turbulent period we observe the futures price on the 19th overreacting to information contained in the previous minute's Index price. On the basis of this evidence, it is tempting to conclude that the futures market was to blame. This conclusion, however, may be a premature one. What we observe on the 19th is a situation where apparently arbitrage trades could not be executed effectively. As a result, the arbitrage link broke down, the outcome being a vicious downward spiral in prices in both markets.

For the 20th, the futures continued to trade at a large discount, again pointing to the presence of unexploited arbitrage opportunities. In addition, we also observe a change in the nature of the pricing relationship with the restoration of the basis as the link between the two markets, the futures still leading the spot but this time with no feedback from the spot to the futures. As the Quality of Markets Report suggests, sellers did not believe they could transact immediately in the stock market, driving sellers away from the stock market to the futures market. It would appear that this action is precisely what restored the basis as the link.

What seems clear, then, is that the futures market did not serve its purpose on the 19th. Indeed, it helped exacerbate the downward movement in prices. However, the blame for this does not necessarily lie with the futures market, for the initial source of the problem may have been the stock market.

The message from this is clear. Looking towards further regulation of the futures market as a separate entity may be premature because the futures market may not be the source of the problem. To further regulate the futures market may be to alleviate the symptoms without curing the illness. Regulating the two markets as a single entity, as recommended in the Presidential Task Force Report (1988) is only part of the solution. In addition, it is necessary to consider the trading practices in both markets. Kleidon and Whaley (1992) suggest that the solution for the US is more efficient trading systems for the stock market. Similar conclusions may apply for the UK, with reforms of trading practices bringing trading systems in both markets closer together.[17] We cannot know for sure, but we suspect that had this been the case the crash might never have taken hold in the way it did.

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[17] For example, in the UK there are two markets closely linked yet with different trading systems. The stock market is a purely screen-based system whilst the futures market has trading based on open outcry. Whilst in theory there is no reason as to why purely screen-based systems shouldn't execute trades efficiently, in practice human and technical factors will ensure this isn't the case in times of market turbulence.

CHAPTER SIX : SUMMARY AND CONCLUSIONS

t the start of this thesis, we observed that the raditional notion of the behaviour of prices in asset arkets, and the argument that the introduction of utures contracts traded on these underlying assets akes matters worse, still persists because the traditional' analysis of pricing relationships etween spot and futures markets is separated into two holly independent areas. As such, no framework xists to allow the appraisal of the anti-futures arket lobby argument that futures provide no enefits. Rather, they destabilise prices in spot arkets already populated by emotional and irrational raders.

ı chapter two we developed a framework within which ıe validity of this argument can be appraised. In ıapter two, it was shown that treating the analysis lead-lag relationships and mispricing as two ɘparate issues is not only incorrect, it can generate ɔnclusions which are potentially false. The reason ıy is these issues are very much interdependent, with ıe latter contributing greatly to any analysis of the ɔrmer.

unifying these two areas into a coherent error ɔrrection framework, in which the error correction term is the basis, which is also a measure of the degree of mispricing, we argued that they become the single question of whether equity markets can be said to function effectively. Further, by analysing the pricing relationship in a systems error correction framework, it is possible to have two categories of effective functioning : a strong categorisation and a weak categorisation. Necessary conditions, which together provide a necessary and sufficient condition, where proposed for each of these categories.

By categorising effective functioning in this way, it is then possible to pinpoint any source of ineffectiveness through seeing which necessary condition is violated. This is obviously of interest to regulators for it allows regulation to be directed at the correct market. A further advantage generated by this framework is that it is also possible to conjointly analyse the efficiency of markets in an objective manner, the necessary conditions, which again taken together provide a necessary and sufficient condition, for efficiency following on directly from the conditions for effective functioning. In chapter three, we used this framework to analyse the functioning of the UK stock and stock index futures markets. We find that not only are they strongly effectively functioning, they are efficient as well. Therefore, at least on a daily basis, the stock index futures market does serve its prescribed role(s), as discussed in chapter one. The conclusion here is that the stock index futures market does provide direct benefits and is useful. Moreover, if it functions effectively, as it does at least on a daily time scale, it should stabilise the stock market rather than destabilise it.

This analysis was extended to an intra-daily time period in chapter four, where we proposed and utilised a model that expunges nonsynchronous trading effects from the Index. We argued that extant models of nonsynchronous trading are flawed because they predict that nonsynchronous trading generates autoregressive properties in observed returns. The problem is that a true inefficiency may be wrongly interpreted as evidence of nonsynchronous trading. By redefining nonsynchronous trading, we are able to argue that generates moving average behaviour in observed returns. To capture this moving average behaviour, the appropriate model to use is then an unobserved components model, with the unobserved component being extracted by the Kalman Filter. This model not only has the attractive feature of capturing moving average behaviour. It is intuitively appealing in its formulation, easy to estimate and implement, and justifies previous *ad hoc* methods of removing nonsynchronicity by simply including a moving average error term in the model.

Using the indices adjusted for nonsynchronous trading, we analysed the effectiveness of both the stock and stock index futures market using minute by minute data using the framework developed in chapter two. We found that whilst the foundations for effectiveness are present, they do not function effectively, with the probable source of the ineffectiveness being the stock market.

Using both the method proposed for removing nonsynchronous trading in chapter four and the framework for analysing effective functioning proposed in chapter two, chapter five analysed claims that stock index futures were at fault during the stock market crash of October 1987. Specifically, the claim we investigate is that they (stock index futures) did not serve their purpose on the 19th and 20th October 1987 and as a consequence prices spiralled downward not only in the stock index futures market but also in the stock market. Using the framework of effective functioning we find that indeed stock index futures did not serve their purpose. However, to argue that the futures market was at fault is possibly premature since the blame could lie fairly and squarely at the door of the stock market, particularly in terms of its pure dealership trading system. This is reinforced by the conclusions from chapter four.

The policy implications of the analysis in this thesis are clear. If markets are not functioning effectively, it may be unwise to assume that the cause of the ineffectiveness lies with the futures market. It may lie with the spot market, certainly in the case of the relationship between the stock market and the stock index futures market. Moreover, this situation will arise because the two markets are regulated as entirely separate entities when in fact they are one. Thus, whilst stock index futures will serve their purpose well in tranquil periods, they will function as a separate entity in a crisis period such as the stock market crash *not because they are not useful, but because they are treated as a separate entity from a regulatory standpoint.* It is hoped that the analysis presented in this thesis provides some of the necessary insights to be used in correcting this state of affairs.

In addition to the usefulness of the framework for analysing the effective functioning of markets and, in the case of stock markets, the usefulness of the model of nonsynchronous trading, proposed, several extensions of this work naturally suggest themselves. A first area for future research is the rather obvious one : what insights does this framework provide in the nalysis of other markets, for example the elationship between spot, forward and futures foreign xchange markets. Second, for those markets where olatility is genuine, the framework for analysing ffective functioning can be extended to incorporate olatility using the multivariate GARCH model iscussed in Bollerslev, Engle and Wooldridge (1988). n interesting issue worthy of further investigation ere would be the analysis of the cointegration roperties of the conditional variances of spot and itures prices.

s far as the model of nonsynchronous trading is oncerned, this can also be generalised to allow for olatility in the nonsynchronous trading adjustment sing the model presented in a very recent paper by arvey, Ruiz and Sentana (1992).

l of these extensions represent very real and teresting areas worthy of investigation in the ture.

Appendix : Construction Of The Data Used In Chapter Three

In this appendix, we discuss the construction of the data series used in the empirical analysis in chapter three. To recap briefly, one of the interesting empirical issues raised in chapter three is that of the behaviour of both the stock and stock index futures markets as time to maturity varies. Specifically, does(the equations of the system) alter as time to maturity alters? Conceptually, under rational expectations, the structure of the model will differ, the difference between periods being a cumulative moving average term. In practice, this may not happen because of factors such as, for example, aggregation of futures contracts, or the true model is backward looking. Therefore, it is of interest to see if the structure of the model alters as time to expiration alters.

In order to analyse this issue, the data were split by contract and according to time to maturity. To illustrate, consider the following series :

$$\boldsymbol{X} = (X_1 , X_2 , X_3 ,\ldots, X_n)'$$

Define **X** as the futures price (or spot price, the ırgument is applicable to both), constructed as a ·ollover. Thus, we define the vectors $\mathbf{X}_1$, $\mathbf{X}_2$ and $\mathbf{X}_3$ as he daily futures price quoted for the March 1985 ontract in January, February and March 1985 espectively; $\mathbf{X}_4$, $\mathbf{X}_5$ and $\mathbf{X}_6$ are defined as the daily utures price for the June contract 1985 quoted in pril, May and June 1985, respectively; $\mathbf{X}_7$, $\mathbf{X}_8$ and $\mathbf{X}_9$ re defined as the daily futures price for the eptember 1985 contract, quoted in July, August and eptember 1985, respectively; $\mathbf{X}_{10}$, $\mathbf{X}_{11}$ and $\mathbf{X}_{12}$ are efined as the daily futures price for the December 985 contract, quoted in October, November and ecember 1985 respectively. The procedure continues ı this fashion, such that $\mathbf{X}_{13}$, for example, is the aily price of the March 1986 contract, quoted in anuary 1986. The data series ends with $\mathbf{X}_n$ being equal ɔ the daily futures price for the December 1990 ɔntract, quoted in December 1990.

ıe construction of the data series for the empirical ıalysis then proceeds as follows. Consider what has ɜen termed the March 3 series. This consists of the :ices quoted in January for the March futures ɪntract. Thus, the series for March 3 is given by

$$M_3 = (X_1 , X_{13} , X_{25} ,\ldots, X_{n-12})'$$

Similarly, for March 2,

$$M_3 = (X_2 , X_{14} , X_{26} ,\dots, X_{n-11})'$$

The series for the other contracts are constructed in the same way.

Of course, constructing the data in this fashion introduces breaks into the levels of the series. To combat this, dummy variables were used in the cointegration regressions to capture the breaks which are not a feature of the data but a feature of the way the data are constructed. In taking first differences, the nature of the construction of the data induces large outliers where, for example, the March 85 data meets the March 86 data. These outliers were removed since, as already mentioned, they arise as a result of the way the data are constructed rather than being properties of the data itself.

REFERENCES

Anderson, R.W. and J-P. Danthine (1981). 'Cross Hedging.' *Journal of Political Economy 81*, 1182-1196.

Antoniou, A. (1986). *Futures Markets : Theory and Tests.* Unpublished DPhil dissertation, University of York.

Antoniou, A. and A. Foster (1991). 'Efficient predictions of inefficient markets : an analysis of commodity spot and futures markets.' *CERF Discussion Paper 91-03*, Department of Economics, Brunel University.

Antoniou, A. and A. Foster (1992). 'Futures trading and spot price volatility.' *Journal of Business Finance and Accounting 19*, 473-484.

Antoniou, A. and I. Garrett (1989a). 'To what extent did stock index futures contribute to the 1987 stock market crash?' *Brunel University Discussion Papers in Economics, 8905.*

Antoniou, A. and I. Garrett (1989b). 'Long-run equilibrium and short-run dynamics : the case of the FTSE 100 Share Index and futures contract.' *Brunel University Discussion Papers in Economics, 8906.*

Antoniou, A. and P.R. Holmes (1992). 'The relationship between futures trading and spot price volatility: distinguishing between the message and the messenger.' *CERF Discussion Paper 92-02*, Department of Economics, Brunel University.

Arrow, K.A. (1964). 'The role of securities in the optimal allocation of risk-bearing.' *Review of Economic Studies 31*, 91-96.

Bank of England (1988). 'The equity market crash.' *Bank of England Quarterly Bulletin, February*, 51-58.

Baillie, R.T. and T. Bollerslev (1989). 'Common stochastic trends in a system of exchange rates.' *Journal of Finance 44*, 167-181.

Baillie, R.T. and T. Bollerslev (1990). 'A multivariate generalised ARCH approach to modelling risk premia in forward foreign exchange markets.' *Journal of International Money and Finance 9*, 309-324.

Baillie, R.T. and T. Bollerslev (1990). 'Intra-day and inter market volatility in foreign exchange rates.' *Review of Economic Studies 58*, 565-585.

Baillie, R.T., R.E. Lippens and P.C. McMahon (1983). 'Testing rational expectations and efficiency in the foreign exchange market.' *Econometrica 51*, 553-563.

Beach, C. and J. MacKinnon (1978). 'A maximum likelihood procedure for regression with autocorrelated errors.' *Econometrica 46*, 51-58.

Black, F. (1972). 'Capital market equilibrium with restricted borrowing.' *Journal of Business 45*, 444-455.

Black, F. (1975). 'Fact and fantasy in the use of options.' *Financial Analysts Journal 31*, 36-41 and 61-72.

Black, F. (1976). 'The pricing of commodity contracts.' *Journal of Financial Economics 3*, 167-179.

Black, F. and M.S. Scholes (1973). 'The pricing of options and corporate liabilities.' *Journal of Political Economy 81*, 637-659.

Blume, M.E., A.C. MacKinlay and B. Terker (1989). 'Order imbalances and stock price movements on October 19 and 20 1987.' *Journal of Finance 44*, 827-848.

Bollerslev, T. (1986). 'Generalised autoregressive conditional heteroscedasticity.' *Journal of Econometrics 31*, 307-327.

Bollerslev, T., R.Y. Chou and K.F. Kroner (1992). 'ARCH modelling in finance : a review of theory and empirical evidence.' *Journal of Econometrics 52*, 5-59.

Bollerslev, T., R.F. Engle and J. Wooldridge (1988). 'A capital asset pricing model with time-varying covariances.' *Journal of Political Economy 96*, 116-131.

Brennan, M.J. and E.S. Schwartz (1990). 'Arbitrage in stock index futures.' *Journal of Business 63 (supplement)*, S7-S31.

Box, G.E.P. and G.M. Jenkins (1970). *Time Series Analysis, Forecasting and Control*. San Francisco: Holden-Day.

Box, G.E.P. and D.A. Pierce (1970). 'Distribution of residual autocorrelations in autoregressive integrated moving average time series models.' *Journal of the American Statistical Association 65*, 1509-1526.

Campbell, J.Y. and R.J. Shiller (1988). 'Interpreting cointegrated models.' *Journal of Economic Dynamics and Control 12*, 505-522.

Chan, K. (1992). 'A further analysis of the lead-lag relationship between the cash market and the stock index futures market.' *Review of Financial Studies 5*, 123-152.

Chan, K., K.C. Chan and G.A. Karolyi (1991). 'Intraday volatility in the stock index and stock index futures markets.' *Review of Financial Studies 4*, 657-684.

Chatfield, C. (1989). *The Analysis of Time Series : An Introduction*, fourth edition. London : Chapman and Hall.

Chung, Y.P. (1991). 'A transactions data test of stock index futures market efficiency and index arbitrage profitability.' *Journal of Finance 46*, 1791-1809.

Clements, M.P. (1991). 'Testing structural hypotheses by encompassing: US wages and prices. Is the mark-up pricing hypothesis dead?' *Applied Economics Discussion Paper no.114*, University of Oxford.

Clements, M.P. and G. Mizon (1991). 'Empirical analysis of macroeconomic time series : VAR and structural models.' *European Economic Review 35*, 887-932.

Cooley, T.F. and S.L. LeRoy (1985). 'Atheoretical macroeconomics.' *Journal of Monetary Economics 16*, 283-308.

Copeland, T.E. and J.F. Weston (1988). *Financial Theory and Corporate Policy*, 3rd edition. New York : Addison Wesley.

Cornell, B. (1981). 'The relationship between volume and price variability in futures markets.' *Journal of Futures Markets 1*, 303-316.

Cornell, B. (1985). 'Taxes and the pricing of stock index futures.' *Journal of Futures Markets 5*, 89—101.

Cornell, B. and K.R. French (1983a). 'The pricing of stock index futures.' *Journal of Futures Markets 3*, 1-14.

Cornell, B. and K.R. French (1983b). 'Taxes and the pricing of stock index futures.' *Journal of Finance 38*, 675-694.

Cornell, B. and M. Reinganum (1981). 'Forward and futures prices : evidence from the foreign exchange markets.' *Journal of Finance 36*, 1035-1045.

Cox, J.C., J.E. Ingersoll and S.A. Ross (1981). 'The relation between forward and futures prices.' *Journal of Financial Economics 9*, 321-346.

Cuthbertson, K., S.G. Hall and M.P. Taylor (1992). *Applied Econometric Techniques*. London : Harvester Wheatsheaf.

Diamond, D.W. and R.E. Verrechia (1987). 'Constraints on short-selling and asset price adjustment to private information.' *Journal of Financial Economics 18*, 277-311.

Dickey, D.A. and W.A. Fuller (1981). 'Likelihood ratio statistics for autoregressive time series with a unit root.' *Econometrica 49*, 1057-1072.

Dickey, D.A., W.R. Bell and R.B. Miller (1986). 'Unit roots in time series models : tests and implications.' *American Statistician 40*, 12-26.

Dimson, E. (1979). 'Risk measurement when shares are subject to infrequent trading.' *Journal of Financial Economics 7*, 197-226.

Domowitz, I. and C.S. Hakkio (1985). 'Conditional variance and the *risk premium in the foreign* exchange market.' *Journal of International Economics 19*, 47-66.

Dwyer, G.P. and M.S. Wallace (1990). 'Cointegration and market efficiency.' mimeo, Clemson University.

Edwards, F.R. and C.W. Ma (1992). *Futures and Options*. McGraw Hill.

Engle, R.F. (1982). 'Autoregressive conditional heteroscedasticity with estimates of the variance of UK inflation.' *Econometrica 50*, 987-1008.

Engle, R.F. and C.W.J. Granger (1987). 'Cointegration and error correction : representation, estimation and testing.' *Econometrica 55*, 251-276, reprinted in R.F. Engle and C.W.J. Granger (1991).

Engle, R.F. and C.W.J. Granger (eds.) (1991). *Long-run Economic Relationships : Readings in Cointegration*. Oxford : Oxford University Press.

Engle, R.F., D.F. Hendry and J-F. Richard (1983). 'Exogeneity.' *Econometrica 51*, 277-304.

Engle, R.F, D.M. Lilien and R.P. Robins (1987). 'Estimating time varying risk premia in the term structure : the ARCH-M model.' *Econometrica 55*, 391-407.

Ericsson, N.R. (1992). 'Cointegration, exogeneity and policy analysis : an overview.' *Journal of Policy Modelling 14*, 251-280.

Eun, C.S. and S. Shim (1989). 'International transmission of stock market movements.' *Journal of Financial and Quantitative Analysis 24*, 241-256.

Fama, E.F. (1965). 'The behaviour of stock market prices.' *Journal of Business 38*, 34-105.

Fama, E.F. (1970). 'Efficient capital markets : a review of theory and empirical work.' *Journal of Finance 25*, 383-417.

Fama, E.F. (1984). 'Forward and spot exchange rates.' *Journal of Monetary Economics 14*, 319-338.

Figlewski, S. (1984). 'Hedging performance and basis risk in stock index futures.' *Journal of Finance 39*, 657-669.

Figlewski, S. (1985). 'Hedging with stock index futures : applications in a new market.' *Journal of Futures Markets 5*, 183-199.

Fisher, L. (1966). 'Some new stock market indexes.' *Journal of Business 39*, 191-225.

French, K.R. (1983). 'A comparison of futures and forward prices.' *Journal of Financial Economics 12*, 311-342.

Froot, K. and A. Perold (1990). 'New trading practices and short-run market efficiency.' *NBER Working Paper No. 3948.*

Furbush, D. (1989). 'Program trading and price movement : evidence from the October 1987 stock market crash.' *Financial Management 19*, 68-81.

Garbade, K.D. and W.L. Silber (1983). 'Price movements and price discovery in futures and cash markets.' *Review of Economics and Statistics 65*, 289-297.

Garman, M. and M. Klass (1980). 'On the estimation of security price volatilities from historical data.' *Journal of Business 53*, 67-78.

Garrett, I. (1991a). 'Using the market model : a note.' *CERF Discussion Paper 91-01*, Department of Economics, Brunel University.

Garrett, I. (1991b). 'On the implementation and interpretation of the market model : an alternative approach.' *CERF Discussion Paper 91-05*, Department of Economics, Brunel University.

Garrett, I. (1991c). 'Nonsynchronous trading and the October 1987 stock market crash.' mimeo, Centre for Empirical Research in Finance, Department of Economics, Brunel University.

Garrett, I. and R. Priestley (1991). 'Miller-Modigliani, behavioral models of dividend policy and the dividend behaviour of the aggregate stock market : some evidence for the UK.' *CERF Discussion Paper 91-06*, Department of Economics, Brunel University.

Geweke, J.F. and E.L. Feige (1979). 'Some joint tests of the efficiency of markets for foreign exchange.' *Review of Economics and Statistics 61*, 334-341.

Gilbert, C.L. (1986). 'Professor Hendry's econometric methodology.' *Oxford Bulletin of Economics and Statistics 48*, 283-307, reprinted in C.W.J. Granger (1990).

Grammatikos, T. and A. Saunders (1986). 'Futures price variability : a test of maturity and volume effects.' *Journal of Business 59*, 319-330.

Granger, C.W.J. (1969). 'Investigating causal relations by econometric models and cross-spectral methods.' *Econometrica 37*, 424-438, reprinted in R.E. Lucas and T.J. Sargent (1981).

Granger, C.W.J. (1986). 'Developments in the study of cointegrated variables.' *Oxford Bulletin of Economics and Statistics 48*, 213-228, reprinted in R.F. Engle and C.W.J. Granger (1991).

Granger, C.W.J. (ed.) (1990). *Modelling Economic Series : Readings in Econometric Methodology.* Oxford : Oxford University Press.

Granger, C.W.J. and P. Newbold (1974). 'Spurious regressions in econometrics.' *Journal of Econometrics 2*, 111-120.

Grossman, S.J. and J.E. Stiglitz (1976). 'Information and competitive price systems.' *American Economic Review 66*, 246-253.

Grossman, S.J. and J.E. Stiglitz (1980). 'On the impossibility of informationally efficient markets.' *American Economic Review 70*, 393-408.

Hall, B.H. (1990). *TSP User's Guide*, version 4.2.

Hall, S.G. (1991). 'The effects of varying length VAR models on the maximum likelihood estimates of cointegrating vectors.' *Scottish Journal of Political Economy 38*, 317-323.

Hakkio, C.S. (1981). 'Expectations and the forward exchange rate.' *International Economic Review 22*, 663-678.

Hakkio, C.S. and M. Rush (1989). 'Market efficiency and cointegration : an application to the Sterling and Deutschemark exchange markets.' *Journal of International Money and Finance 8*, 75—88.

Haldane, A.G. and S.G. Hall (1991). 'Sterling's relationship with the Dollar and the Deutschemark: 1976-1989.' *Economic Journal 101*, 436-443.

Hamao, Y., R.W. Masulis and V. Ng (1990). 'Correlations in price changes and volatility across international stock markets.' *Review of Financial Studies 3*, 281-307.

Hansen, L.P. (1982). 'Large sample properties of Generalised Method of Moments estimators.' *Econometrica 50*, 1029-1054.

Hansen, L.P. and R.J Hodrick (1980). 'Forward exchange rates as optimal predictors of future spot rates : an econometric analysis.' *Journal of Political Economy 88*, 829-853.

Harris, L.H. (1989). 'The October 1987 S&P 500 stock-futures basis.' *Journal of Finance 44*, 77-99.

Harvey, A.C. (1984). 'Dynamic models, the prediction error decomposition and state space.' In *Econometrics and Quantitative Economics* (ed. D.F. Hendry and K.F. Wallis). Oxford : Basil Blackwell.

Harvey, A.C. (1987). 'Applications of the Kalman filter in econometrics.' In *Advances in Econometrics : Fifth World Congress*, vol. 1 (ed. T.F. Bewley). Econometric Society Monograph no. 13. Cambridge : Cambridge University Press.

Harvey, A.C., E. Ruiz and E. Sentana (1992). 'Unobserved component time series models with ARCH disturbances.' *Journal of Econometrics 52*, 129-157.

Hendry, D.F (1989). *PC-GIVE : An Interactive Econometric Modelling System.* Oxford : Oxford Institute of Economics and Statistics.

Hendry, D.F. and N.R. Ericsson (1991a). 'An econometric analysis of U.K. money demand in *Monetary Trends in the United States and the United Kingdom* by Milton Friedman and Anna J. Schwartz.' *American Economic Review 81*, 8-38.

Hendry, D.F. and N.R. Ericsson (1991b). 'Modelling the demand for narrow money in the United Kingdom and the United states.' *European Economic Review 35*, 833-886.

Hendry, D.F. and G. Mizon (1978). 'Serial correlation as a convenient simplification, not a nuisance: a comment on a study of the demand for money by the Bank of England.' *Economic Journal 88*, 549-563.

Hendry, D.F. and G. Mizon (1990). 'Evaluating dynamic econometric models by encompassing the VAR.' *Applied Economics Discussion Paper no.102*, University of Oxford.

Hendry, D.F., A.R. Pagan and J.D. Sargan (1984). 'Dynamic specification.' Ch. 18 in Z. Griliches and M.D. Intriligator (eds), *Handbook of Econometrics*, North-Holland.

Hendry, D.F. and J-F. Richard (1982). 'On the formulation of empirical models in dynamic econometrics.' *Journal of Econometrics 20*, 3-33, reprinted in C.W.J. Granger (1990).

Hendry, D.F. and J-F. Richard (1983). 'The econometric analysis of economic time series.' *International Statistical Review 51*, 111-163.

Johansen, S. (1988). 'Statistical analysis of cointegration vectors.' *Journal of Economic Dynamics and Control 12*, 231-254, reprinted in R.F. Engle and C.W.J. Granger (1991).

Johansen, S. and K. Juselius (1990). 'Maximum likelihood estimation and inference on cointegration - with applications to the demand for money.' *Oxford Bulletin of Economics and Statistics 52*, 1-40.

Johnson, L.L. (1960). 'The theory of hedging and speculation in commodity futures.' *Review of Economic Studies 27*, 139-151.

Junkus, J.C. and C.F. Lee (1985). 'Use of three stock index futures in hedging decisions.' *Journal of Futures Markets 5*, 201-222.

Karpoff, J.M. (1987). 'The relation between price changes and trading volume : a survey.' *Journal of Financial and Quantitative Analysis 22*, 109-126.

Kawaller, I.G., P.D. Koch and T.W. Koch (1987). 'The temporal price relationship between S&P 500 futures and the S&P 500 Index.' *Journal of Finance 42*, 1309-1329.

Kawaller, I.G., P.D. Koch and T.W. Koch (1988). 'The relationship between the S&P 500 Index and S&P 500 Index futures prices.' *Federal Reserve Bank of Atlanta Economic Review*, May/June 2-10.

Kleidon, A.W. and R.E. Whaley (1992). 'One market? Stocks, futures and options during October 1987.' *Journal of Finance 47*, 851-877.

Kunitomo, N. (1992). 'Improving the Parkinson method of estimating security price volatilities.' *Journal of Business 65*, 295-302.

Lamoureux, C.G. and W.D. Lastrapes (1990a). 'Persistence in variance, structural change and the GARCH model.' *Journal of Business and Economic Statistics 8*, 225-234.

Lamoureux, C.G. and W.D. Lastrapes (1990b). 'Heteroscedasticity in stock return data : volume versus GARCH effects.' *Journal of Finance 45*, 221-230.

LeRoy, S.F. (1989). 'Efficient capital markets and martingales.' *Journal of Economic Literature 27*, 1583-1621.

Levich, R.M. (1985). 'Empirical studies of exchange rates : price behaviour, rate determination and market efficiency.' In volume 2 of R.W. Jones and P.B. Kenen (eds), *Handbook of International Economics*, New York : Elsevier Science.

Lo, A.C. and A.C. MacKinlay (1990). 'An econometric analysis of nonsynchronous trading.' *Journal of Econometrics 45*, 181-211.

Lucas, R.E. and T.J. Sargent (eds.) (1981). *Rational Expectations and Econometric Practice.* London : George Allen and Unwin.

MacDonald, R. and M.P. Taylor (1988). 'Metals prices, efficiency and cointegration : some evidence from the London Metal Exchange.' *Bulletin of Economic Research 40*, 235-239.

MacDonald, R. and M.P. Taylor (1989). 'Rational expectations, risk and efficiency in the London Metal Exchange : an empirical analysis.' *Applied Economics 21*, 143-153.

MacKinlay, A.C. and K. Ramaswamy (1988). 'Index-futures arbitrage and the behaviour of stock index futures prices.' *Review of Financial Studies 1*, 137-158.

Mandlebrot, B. (1963). 'The variation of certain speculative prices.' *Journal of Business 36*, 394-419.

Markowitz, H.M. (1952). 'Portfolio selection.' *Journal of Finance 7*, 77-91.

Modest, D.M. and M. Sundaresan (1983). 'The relationship between spot and futures prices in stock index futures markets : some preliminary evidence.' *Journal of Futures Markets 3*, 15-41.

Monfort, A. and R. Rabemananjara (1990). 'From a VAR model to a structural model with an application to the wage-price spiral.' *Journal of Applied Econometrics 5*, 203-227.

Nelson, C.R. and C.I. Plosser (1982). 'Trends and random walks in macroeconomic time series.' *Journal of Monetary Economics 10*, 139-162.

Nelson, D.B. (1990). 'Conditional heteroscedasticity in asset returns : a new approach.' *Econometrica 59*, 347-370.

Netter, J.M. and M.L. Mitchell (1989). 'Stock repurchase announcements and insider transactions after the October 1987 stock market crash.' *Financial Management 19*, 84-96.

Newey, W.K. and K.D. West (1987). 'A simple, positive semi-definite, heteroscedasticity and autocorrelation consistent covariance matrix.' *Econometrica 55*, 703-708.

Park, H. and A. Chen (1985). 'Differences between futures and forward prices : a further investigation of the marking-to-market effect.' *Journal of Futures Markets 5*, 77-88.

Parkinson, M. (1980). 'The extreme value method for estimating the variance of the rate of return.' *Journal of Business 53*, 61-65.

Perron, P. (1988). 'Trends and random walks in macroeconomic time series : further evidence from a new approach.' *Journal of Economic Dynamics and Control 12*, 297-332.

Pesaran, M.H. and B. Pesaran (1989). *Microfit : An Interactive Econometric Software Package.* Oxford: Oxford University Press.

Phillips, P.C.B. (1987). 'Time series regression with unit roots.' *Econometrica 55*, 277-302.

Phillips, P.C.B. and P. Perron (1988). 'Testing for a unit root in time series regression.' *Biometrika 75*, 335-346.

Presidential Task Force (1988). *Presidential Task Force Report on Market Mechanisms.* Submitted to the President, the Secretary of the Treasury and the Chairman of the Federal Reserve Board, January 1988.

Rolfo, J. (1980). 'Optimal hedging under price and quantity uncertainty : the case of a cocoa producer.' *Journal of Political Economy 88*, 100—116.

Roll, R. (1977). 'A critique of the asset pricing theory's tests, part 1 : on past and potential testability of the theory.' *Journal of Financial Economics 4*, 129-176.

Roll, R. (1988). 'The international stock market crash of October 1987.' *Financial Analysts Journal 44*, 19-35.

Ross, S.A. (1976). 'Options and efficiency.' *Quarterly Journal of Economics 91*, 75-89.

Ross, S.A. (1976). 'The arbitrage theory of capital asset pricing.' *Journal of Economic Theory 13*, 341-360.

Scholes, M.S. and J. Williams (1977). 'Estimating beta from nonsynchronous data.' *Journal of Financial Economics 5*, 309-327.

Shanken, J. (1987). 'Nonsynchronous data and the covariance factor structure of returns.' *Journal of Finance 42*, 221-232.

Sims, C.A. (1972). 'Money, income and causality.' *American Economic Review 62*, 450-552, reprinted in R.E. Lucas and T.J. Sargent (1981).

Stein, J.L. (1961). 'The simultaneous determination of spot and futures prices.' *American Economic Review 51*, 1012-1025.

Stoll, H.R. and R.E. Whaley (1990). 'The dynamics of stock index and stock index futures returns.' *Journal of Financial and Quantitative Analysis 25*, 441-468.

Subrahmanyam, A. (1991). 'A theory of trading in stock index futures.' *Review of Financial Studies 4*, 17-51.

Taylor, S.J. (1986). *Modelling Financial Time Series*. New York : John Wiley.

Tucker, A.L., J. Madura and T.C. Chiang (1991). *International Financial Markets*. New York : West Publishing Company.

Working, H. (1953). 'Hedging reconsidered.' *Journal of Farm Economics 35*, 59-76.

Yadav, P.K. and P.F. Pope (1990). 'Stock index futures arbitrage : international evidence.' *Journal of Futures Markets 10*, 573-603.

Yadav, P.K. and P.F. Pope (1992). 'Intraweek and intraday seasonalities in stock market risk premia : cash and futures.' *Journal of Banking and Finance 16*, 233-270.

Zeckhauser, R. and V. Niederhoffer (1983). 'The performance of market index futures contracts.' *Financial Analysts Journal 39*, 59-65.