

CONCEPTS AND ANALOGIES IN CYBERNETICS  
MATHEMATICAL INVESTIGATIONS OF THE ROLE OF ANALOGY  
IN CONCEPT FORMATION AND PROBLEM SOLVING; WITH  
EMPHASIS FOR CONFLICT RESOLUTION VIA OBJECT AND MORPHISM ELIMINATIONS

by

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στοις γονεις μου

TO MY PARENTS

## ABSTRACT

We address two problematic areas of cybernetics; nam. Analogical Problem Solving (APS) and Analogical Learning (AL). Both these human faculties do unquestionably require Intelligence. In addition, we point out that shifting of representations is the main unified theme underlying these two intellectual tasks.

We focus our attention on the formulation and clarification of the notion of analogy, which has been loosely treated and used in the literature; and also on its role in shifting of representations.

We describe analogizing situations in a new representational scheme, borrowed from mathematics and modified and extended to cater for our targets. We call it k-structure, closely resembling semantic networks and directed graphs; the main components of it are the so-called objects and morphisms. We argue and substantiate the need for such a representation scheme, by analysing what its constituents stand for and by cataloguing its virtues, the main one being its visual appeal and its mathematical clarity, and by listing its disadvantages when it is compared to other representation systems. Emphasis is also given to its descriptive power and usefulness by implementing it in a number of APS and AL situations.

Besides representation issues, attention is paid to intelligence mechanisms which are involved in APS and AL.

A cornerstone in APS and a fundamental theme in AL is the 'skeletalization of k-structures'. APS is conceived as 'harmonization of skeletons'.

The methodology we develop involves techniques which are computer implemented and extensively studied in theoretic terms via a proposed theory for extended k-structures. To name but a few:

1. 'the separation of the context of a concept from the concept itself', based on the ideas of k-opens and k-spaces;
2. 'object and morphism elimination' of a controversial nature; and
3. 'conflict or deadlock or dilemma resolution' which naturally arises in a k-structure interaction.

The overall system, is then applied to capture the essence of EVANS' (1963) analogy-type problems and WINSTON'S (1970) learning-type situations.

In our attempt not to be too informal, we use basic notions and terminology from abstract Algebra, Topology and Category theory.

We rather tend to be "non-logical" (analogical) in EVANS' and WINSTON'S sense; "non-numeric", in MESAROVIC (1970) terms (we rather deal with abstract conceptual entities); "non-linguistic" (we do not touch natural language); and "non-resolution" oriented, in the sense of BLEDSOE (1977). However, we give hints sometimes about logical deductive axiomatic systems, employing First Order Predicate Calculus (FOPC); and about semiotics, by which we denote syntactic-semantic-pragmatic features of our system and issues of the problem domains it is acting upon.

We believe in what we call: shift from the traditional 'Heuristic-search paradigm' era to the 'Analogy-paradigm' era underlying Artificial Intelligence and Cybernetics. We justify this merely by listing a number of A.I. works, which employ, in some way or another, the concept

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of analogy, over the last fifteen years or so, where a noticeable peak is obvious during the last years and especially in 1977.

Finally, we hope that if the proposed conceptual framework and techniques developed do not straightforwardly constitute some kind of platform for Artificial Intelligence, at least it would give some insights into and illuminate our understanding of the two most fundamental faculties the human brain is occupied with; namely problem-solving and learning.

## AUTHOR'S NOTE

This work makes no pretence of giving to the reader a new theory of our intellectual operations. Its claim to attention, if it possesses any, is grounded on the fact that it is an attempt not to supersede, but to embody, combine and systematize and to look at from another angle at some ideas which have been either promulgated on the subject by speculative writers\*, conformed to by accurate thinkers\*\* in their scientific inquiries or are currently dominant in various research centres\*\*\*.

To cement together the detached fragment of a subject, not usually treated as a whole, and to harmonize the true portions of discordant theories, by supplying the links of thought necessary to connect them, must necessarily require a considerable amount of original speculation.

In the existing state of the cultivation of the sciences and technology and especially in the not yet well established fields of cybernetics and general systems theory, there would be a very strong presumption against anyone who should imagine that he had effected a revolution in the theory of the investigation of human thinking, related intellectual processes and various brain activities, or added any fundamentally new process to the practice of it.

The improvement which remains to be effected in the method of philosophizing can only consist in performing, more systematically,

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\* MINSKY, WINSTON, EVANS, PASK, MITCHIE, EILLENBERG, McLANE, MELTZER.

\*\* BOURBAKI, MESAROVIC, AMAREL, BARNDEN, BLEDSOE, KOWALSKI.

\*\*\* M.I.T., Edinburgh, U.S.S.R.

accurately and methodologically, operations with which, at least in their elementary form, the human intellect in some one or other of its employments is already familiar; and these are, in the writer's opinion, the so-called analogizing activities.

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- (a) The Tutorial on Computational Semantics held in March 1975 at the Institute of Semantic and Cognitive studies in the pleasant atmosphere of Lugano, Switzerland.
- (b) The Summer Conference on Artificial Intelligence and Simulation of Behaviour held at the University of Edinburgh, July 1976.
- (c) The Workshop on Automatic Deduction, held at M.I.T., Cambridge, Mass., August 1977.
- (d) The 5th International Joint Conference on Artificial Intelligence (IJCAI-77) held at M.I.T., August, 1977.

The lectures, talks and especially the valuable suggestions during fruitful discussions with Professors M. Minsky, S. Amarel and B. Meltzer strongly influenced some portions of this thesis.

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CHAPTER 1



*"When we mean to build,  
We first survey the plot, then draw the model;  
And when we see the figure of the house,  
Then must we rate the cost of the erection..."*

WILLIAM SHAKESPEARE

# 1. ON SEMANTIC NETS AND CONCEPTUAL UNIVERSES

## 1.1 ON KNOWLEDGE AND ITS SPECTRUM OF REPRESENTATIONS

### 1.1.1 INTRODUCTION

Knowledge is a collection of 'facts' about some environment. According to the observer's (problem-solver's) viewpoint, the 'facts' we are referring to may be considered and categorized as: conceptual knowledge, events or procedures for doing things.

As far as knowledge Representation is concerned, a lot of efforts have been made to provide a unified schema. Again, here, almost every attempt is tuned according to the target aimed for. An exception, perhaps, is the so-called *frame theory*, MINSKY (1974). WINSTON (1977) defines representation as "a set of conventions about how to describe things". In the case of the above mentioned frame representation, the fundamental concept is that of *frame*. It is a data structure for representing a stereotyped situation; and we can think of it as a network of nodes and relations. A frame's terminals can specify the conditions their assignments must meet. Simple conditions are specified markers that might require a terminal assignment to be a person, an object or a pointer to a subframe. Complex conditions can specify relations among the things assigned to several terminals.

From the whole spectrum of representation alternatives, ranging from *procedural* descriptions to *nonprocedural* representations, we are going to deal with the latter region. There is no unique answer to questions like: how should knowledge be represented? Some knowledge is procedural, other knowledge is factual belonging to the right-hand end of the above spectrum of representational methods ranging from

simple tables to complex frame systems.

In the theory of knowledge there exist, at least, two main attitudes towards knowledge representation; nam. the *declarative* and the *procedural* schools of thought, accompanied, respectively, by two sorts of representation languages thus giving rise to the so-called *declarative-procedural controversy*.

We generally assume that knowledge consists of concepts and relations between them; it is a hypothesis necessary for the development of representational tools towards Conceptual Universes, (conceptual) problem-solving and concept-formation/learning, themes we are going to study.

Issues concerning with acquisition of knowledge are developed in the next chapter, sect. 2.2.1.

### 1,1,2 OTHER SCHEMES OF KNOWLEDGE REPRESENTATION

We are giving a brief overview of representation schemata relevant, more or less, to the one we introduce in a subsequent chapter. The exposition is accompanied by examples, whenever space permits, and in addition, we stress a few pros and cons of the schemata under comparison.

The natural way towards k-structures is, we argue, the one which passes via *semantic nets (SNs) and conceptual universes (CUs)*. These are the two main issues (representation schemes) we are now going to examine in brief.

Semantic nets or relational structures, as they are sometimes called, have been proposed by many people working in various fields,

as knowledge bases, information (rather knowledge) representation schemes or conceptual structures, in a number of systems, with the purpose of performing a variety of cognitive activities. The popularity of semantic nets, is probably due to the fact that their diagrammatic representation 'triggers', somehow, the imagination of users or problem-solvers. Before we proceed with a few informal remarks on semantic nets, we quote from BARNDEN (1975):

*"Conceptual universes are variants of semantic nets specifically designed to cater for the definitions of the meaning of concepts in terms of structures of other concepts".*

Carrying on in this sense, k-structures, the representation system we introduce in chapter 3, may be considered, in some way, as variants of conceptual universes which cater for the study and development of:

- A. Useful mathematical properties about some aspects of the internal structure of CUs; nam. similarity, etc.
- B. Inherent (intrinsic) structural regularities in a CU which may facilitate problem-solving, in a situation which is representable as a CU (k-structure).
- C. Analogy relations between concepts based on, say, internal symmetry of the representation scheme under question.
- D. How problem-solving behaviour is changed by assuming or supplying a specific constructive structure upon the set of relations linking the set of concepts.
- E. Operations between concepts and CUs.
- F. Mappings between CUs.



Remark.- It is this reason\* D we are going to elaborate next, by briefly outlining a similar action initiated by WINDEKNECHT (1964), and adopted by MESAROVIC (1965), in the area of problem-solving and the modelling of a problem; nam. the assumption of a specific constructive structure of set F (functions between problem situations).

### 1.1.3 ON A GENERAL MODEL OF CONTROL SITUATIONS

An outline is given below, of the basic models that BANERJI (1969) is dealing with. In fact, MARINO (1966) proposed the following general model for control systems, problems and games. There are three sets given;

S: *states/situations*; e.g. desirable or winning:  $WS \subset S$ .  
 C: *controls*; elementary controls:  $EC \subset C$ .  
 D: *disturbances*; elementary disturbances:  $ED \subset D$ .

The *control problem* then is, in antecedent-consequent form, as follows:

$\theta$ :	$Y$	Given: a situation $s \in S$
	$\Sigma$	Find: a control $c \in C$ such that for every disturbance $d \in D$ , $c$ is paired with, the resulting situation to be a desirable or winning one.

The control problem is reduced to the so-called "*open-loop controller*" by considering each control as a sequence of elementary controls and each disturbance as a sequence of elementary disturbances.

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\* Dominant motivation underlying the step towards k-structures. (More in the relevant chapter).

\*\*  $\theta$  is what we call theorem schema (we use  $\theta$  quite often); where, Y: antecedent or hypotheses and  $\Sigma$ : consequence.

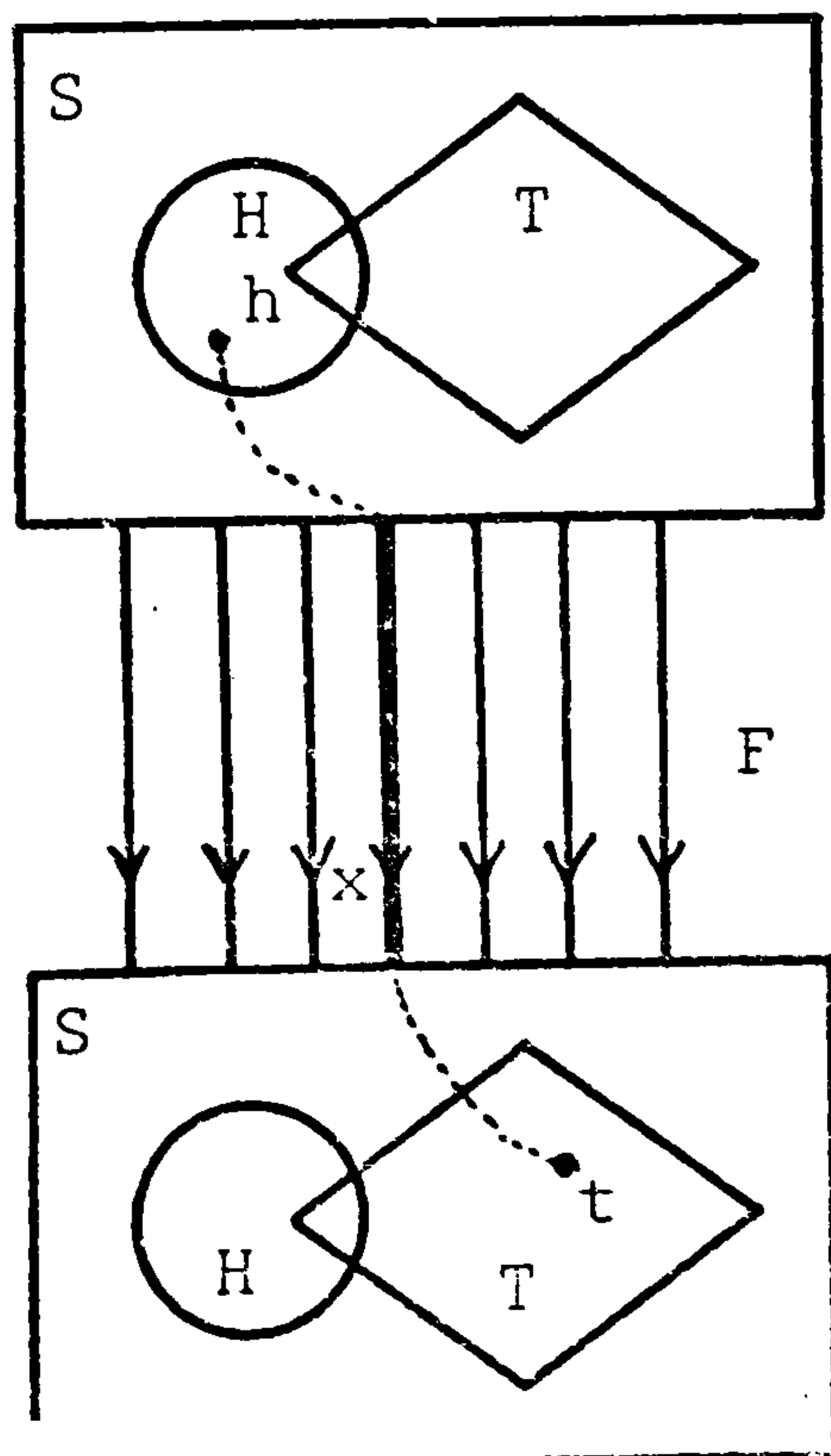


The difficulty which arises, stems from the fact that every elementary control may not be applicable to every situation and this necessitates the shift from the *control sequence* notion to that of *control strategy*; the latter defined as: 'An initial decision on the control to be used at each situation at any time the situation arises'. You may find the concepts of 'decision-making demon', 'strategic advisor' and 'winning strategy' as relevant here. In fact, the above shift results on finding a winning strategy or 'closed-loop controller'.

#### 1.1.4 ON A MODEL OF PROBLEM SITUATIONS

We now move from general models of control situations into special ones, i.e. models of problems situations. One of them, due to MESAROVIC (1965), may be stated, using the above format, as follows:

Y	Given: a set $S$ : problem situations $S \supset T$ : winning situations $S \supset H$ : starting situations; and a set of functions $F \equiv \{ f \in F / f: S \rightarrow S \}$
$\theta$ :	Find: an $F \ni x : H \rightarrow T$ , such that $\exists t \in T$ which : $h \mapsto x(h) \equiv t, \forall h \in H$ .



We are not giving any proof or further analysis of the concepts introduced, for they are beyond the scope of our thesis.

In fact, what we are going to do next, is to link the above models, both the general and the special one, to a previously made remark, sect.1.1.2. MESAROVIC (1965) points out that the set of functions  $F$ , in order to be widely, tractable and controlled should be "constructively defined". In his model for problems, WINDEKNECHT (1964) assumes:

1. A specific constructive structure of  $F$ ; nam. elements of  $F$  are obtained by composing functions from a finite set  $F_0$  of functions.
2. The elements of  $F_0$  are partially defined over  $S$ , so that the composition operator defines a partial semigroup rather than a semigroup\*.
3. The set  $H$  of starting situations is viewed as singleton.

BANERJI (1969) follows the above model with only one modification in part 1; i.e.  $F_0$  is not assumed to be a finite set.

We shall close this section, leaving the details on the nature of problem-solving with the relevant sect. 3.5.4, with four general remarks:

- A. There are clear similarities between Marino's, Banerji's and GPS models of problems; BANERJI (1969), Ch. 2.
- B. Marino's model of problems can be reduced to the model of a two-person game, BANERJI (1969), ch. 3.
- C. Banerji's model of a game is an automaton\* or a labelled directed graph; while VonNeuman's is a tree.
- D. Converting a game with incomplete information into a 'larger' one is analogous to converting a nondeterministic automaton to a deterministic

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\* On "partially defined automata" issue see CHERNYI-SPIVAK (1977).

one. (How?).

### 1.1.5 MORE ON SEMANTIC NETWORKS

In an earlier section, we mentioned that knowledge is organized as a collection of facts, in the general sense of that term. Such factual, so to speak, knowledge can be conveniently represented in the form of semantic networks. A question which naturally arises here is: what is a fact? Let us assume\* that a fact is a tuple  $(R, E)$ , where  $R$  is a relation and  $E$  are entities the relation is connected with, in a meaningful way.

In the same section, we also talked about descriptions. In general, a description will contain information about individual objects as well as information about how they relate to one another. Viewed in this way also, knowledge is nicely accommodated in a semantic network. Before giving an informal analysis of semantic networks, let us list other names given to them; nam. semantic nets, relational structures, relational networks and so on. For our purposes we use the term semantic network (SN).

SNs mean different things to different workers in various fields. They are variously realized as knowledge bases, information or representation schemes, diagrams, abstract sets of  $n$ -tuples, data structures, lists of entities, and also information structures in brains. Generally speaking, a semantic net is a directed or undirected graph\*\* in which nodes

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\* It is very important, from an epistemological point of view, to discuss the consequences of any assumption which is put forward.

\*\* Thus semantic net theory is somehow linked to graph theory; HARARY (1965).



stand for concepts and arcs for relations between concepts. Arcs may be labelled or not. Nodes in an SN, apart from general and particular concepts, may also represent descriptions of named objects, board positions, scenes, situations\*, states, events and so on, found in some environment, eg. linguistic, conceptual, problem-solving, visual, etc. Arc (link) types may be a designator of some relation(s) between objects in the specific environment or the name of some connection between nodes which are considered functionally useful. In a sense the node-arc organization of an SN reflects some environmental structure viewed from some particular standpoint.

Before we give an example of a system, that of WINSTON (1970), which uses an SN\*\* we would like to draw attention, once again, on the importance of choice of representation as a crucial issue, by simply mentioning three similar spectra for descriptive mechanisms. That is,

1. *Procedural-non procedural*; WINSTON (1977).
2. *Analytic-network-axiomatic*; R. BROWN (1977).
3. *State vector-relational structures - predicate calculus*; MITCHIE (1974).

#### 1.1.6 AN EXAMPLE OF A SEMANTIC NETWORK

WINSTON's (1970) *relational structures* (RSs).

We are now going to supply reasons for the choice of WINSTON's method of knowledge representation.

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\* i.e. Control/problem/game situations.

\*\* Actually a Relational Structure (RS).

- A. We want to demonstrate how 'factual' knowledge can be nicely accommodated in an SN.
- B. 2-D Figures and 2-D projections of 3-D scenes, employed (analysed) in his system, intuitively (visually), at least to us, bear similarities with the topology of SN diagrams.
- C. Operations and observations on relational structures in Winston's work have almost nothing intrinsically to do with scene analysis. The same principles (of his system) could be applied to areas such as problem-solving, for which we are mostly interested, as well as to linguistic analysis and so on.
- D. One of our objectives is to focus on a representation scheme in the context of learning simple concepts from examples.
- E. We wish to show that RSs are good for descriptions of definitions (contexts) of concepts, main issue in our investigations.

Finally, we point out that the foregoing 'sketchy' description of Winston's system misses out a lot of its important (for him) points. We shall be highly selective in referring to only those aspects which are of special interest for us.

Winston's system is about how a machine can be taught to see and learn new visual concepts from carefully (for teaching purposes) chosen examples. The core of the system focuses on the problem of analysing scenes consisting of simple visual objects. From such visual scenes, viewed through a TV camera, a set of procedures find the objects of a 3-D environment and determines a family of relations between them. Finally, an SN (RS) is built which facilitates better description of scenes.



Sub-RSs match or partially match internal archetypal RSs, called models\* which are learned rather than being *a priori* knowledge. A model is learned via a 'careful' teaching from examples and 'near misses' scene-RSs. Model-RSs differ from Scene-RSs\*\* only in that arcs in models may bear modifiers.

In the system's mode: 'Learning a model'\*\*\*, viewing an example, it builds the corresponding scene-RS, it then compares it with the current model-RS, it modifies it, and, finally it produces a new model-RS.

In the system's mode: 'Learning a scene'\*\*\*, viewing a complex scene, it builds the corresponding scene-RS, it compares sub-scene-RSs with model-RSs, it modifies them by introducing 'emphatics' WINSTON (1977), it then sequentially builds:

- a: Skeleton-RS
- b: Difference description-RS
- c: Similarity network-RS

Finally, it identifies input scene by identifying its sub-scenes.

Next, a set of examples are given to illustrate the above-mentioned notions and system's modes. WINSTON (1970) and (1977) are the main sources.

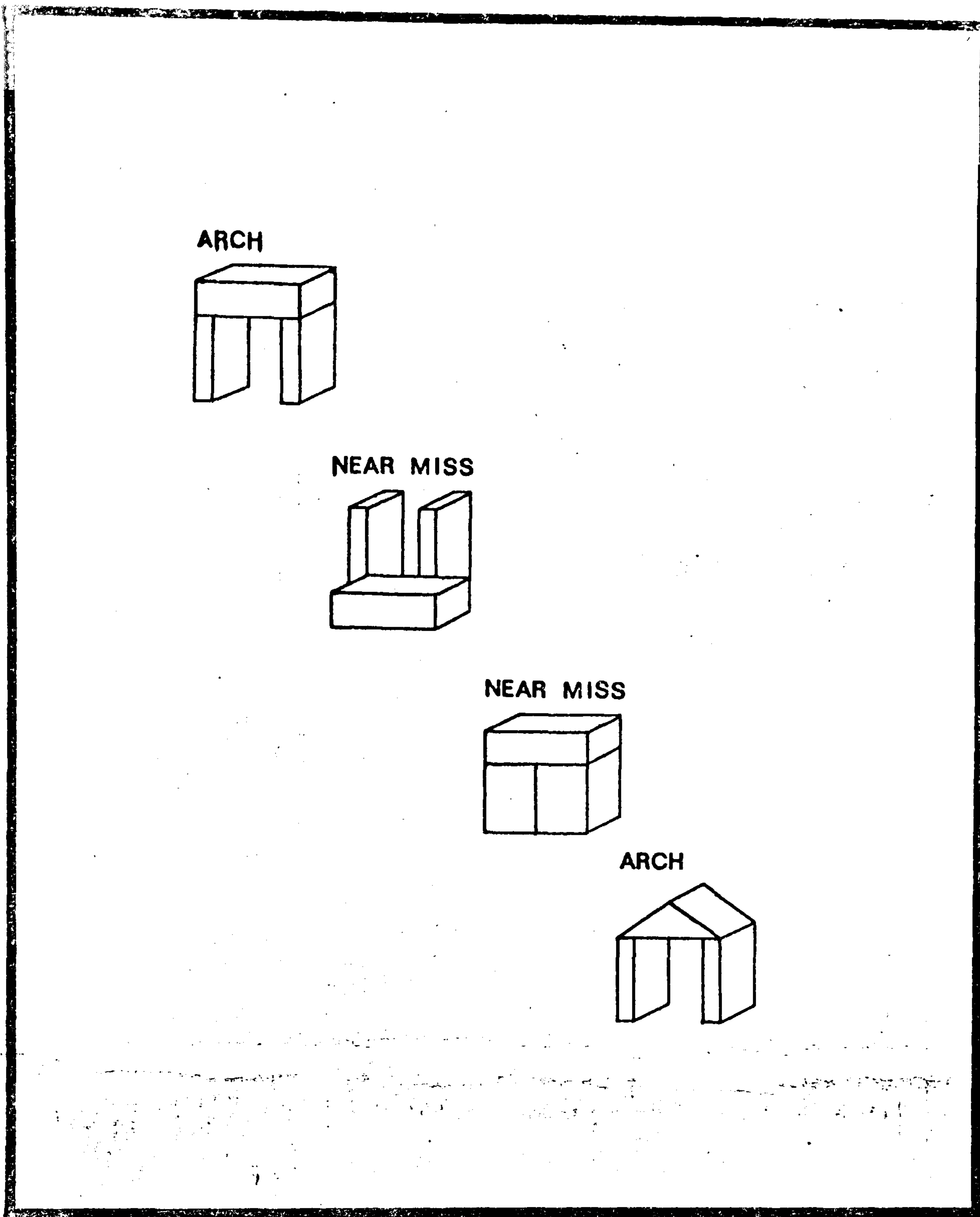
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\* Model-RSs are definitions of entities.

\*\* Scene-RSs are descriptions of scenes.

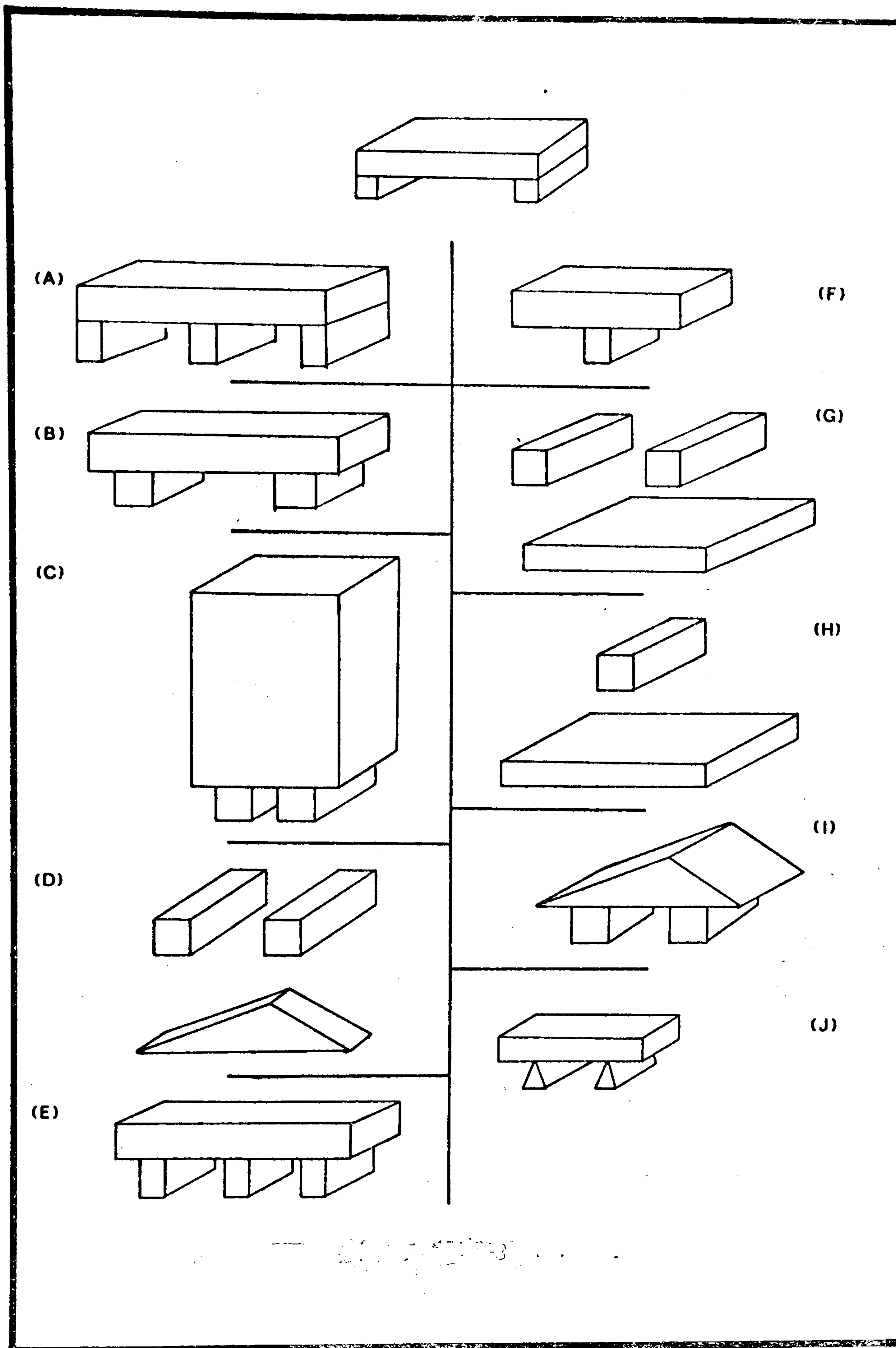
\*\*\* Or 'Learning a simple concept' mode.

\*\*\* Or 'Learning a complex concept' or '(Learn to) identify a scene', since identification power should follow from a successful learning.



Example 1

Learning a model or simple concept, WINSTON (1977)



Example 2

Learning a complex scene, WINSTON (1977).

### 1.1.7 MORE EXAMPLES OF SEMANTIC NET-LIKE REPRESENTATION SCHEMES

Other systems we ought to mention here, being relevant to concepts which we refer to, are:

#### 1. LINDSAY (1973)

He introduces the term of "Relational Contexts", as a way of looking at RSs. He studies: (A) "relation-preserving\* maps\*\*" between RSs (or within an RS; nam. what we distinguish, elsewhere, as shifting-1 and shifting-2, sect. 2.1.3) and (B) regularities\* in an RS as a basis for creating new relations (why not call them new concepts? Thus concept-formation is one result of his investigations).

Lindsay's approach is based on a way of looking 'locally' at a structure; nam. from the 'neighbourhood of a node' standpoint, thus bearing close links with (and a departure into) topological concepts. He also suggests and employs a definition of a context of a node  $x$  by introducing the notion of  $C(x) :=$  "the set of all contexts of  $x$ ". More can be found on "*our topological views of an SN/RS/CU*" in sect.1.2.5.

Remark 1.- Our treatment of 'the context of a concept' bears some sort of similarity to the notions just mentioned; but we are rather viewing them from a different angle. However, they have been, in some way, used as departure points; more in section 3.3.3.

Remark 2.- The above made suggestion from LINDSAY (1973), intuitively led us to link up these notions here, to modules, etc. in abstract algebra, in the sense of HU (1965).

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\* The works of PAVEL (1976), TOURLAKIS-MYLOPOULOS (1973), LUGER (1975), TSICHRITZIS (1976) are very relevant here, for a fruitful departure on theoretical issues. Also, WIENER's (1961) Ch. 2 on invariant transformations.

For a morphism is a function which 'preserves structure'



## 2. MOORE-NEWELL (1973)\*

They build a system, called MERLIN, for understanding and problem-solving, which employs a semantic network for representing knowledge. It is, also, one of the few papers addressing the process of *analogy* and how it can be involved in learning. A mechanism is developed which permits a new piece of knowledge to be constructed by transforming an old one, rather than just by specialization or generalization (WINSTON's (1977) comment). Thus, this work is relevant to some extent, to what we will refer to as 'learning by analogy' and to the role of shifting of representations in concept formation.

3. Finally, the work which is generally credited with developing the concept of an SN is QUILLIAN (1968). It is a system which contains an SN memory for factual knowledge which is used in the comprehension of an English text.

Note.- We are mainly interested for non-linguistic information representation in SNs\*\*; eg. WINSTON's visual knowledge. This is, in fact, the non-linguistic component of a manifold of our attitudes analysed in another section of the thesis under the label "*on our attitudes*".

## 1.2 CONCEPTUAL UNIVERSES

### 1.2.1 ISSUES ON CONCEPTUAL LANGUAGES

In order to talk about concepts, concept formation/learning and related issues, a language is needed. Besides, "knowledge is

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\* More comments are to be found in Sect. 1.2.5.

\*\* SCRAGG (1975) p. 25.



cumulative due to language" GEORGE (1973). A well known language of this type (esp. for conceptual analysis) is due to BANERJI (1969).

GEORGE (1973) argues that:

*"The language of concept formation he made, is designed for internal representation and for internal logical and data processing but it is not necessary convenient for external communication between people".*

BANERJI's language is a set-theoretic one for describing patterns. It is based on the idea of a pattern recognition environment  $\langle U, P \rangle$ , where  $U$  is an abstract set and  $P$  is a family of partitions on  $U$ . We are rather viewing a partition of a set as some kind of *topologizing* a set, in terms of DUGUNDJI(1966); in other words, partitions are the means for supplying an organization\* to a collection of things.

Set-theoretic descriptions are used by various workers; esp. in the field of cognition. However, the issue of recognition, coming back to BANERJI, is mainly set theoretical and logical one.

The various schemes used by workers for concept formation differ, in a sense, in the methods employed, that is, statistics, linear algebra and so on. BANERJI's (1969) description languages have as their motivation the Boolean algebraic structure of a class of concepts\*\*. Such languages are quite distinct from Natural languages.

Finally, we mention that one of the motivations for a conceptual language is the need to transfer knowledge. Examples of such transfers

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\* In ASHBY's (1956) terms.

\*\* Perhaps it is relevant here to mention another interesting language recently introduced by G.S. BROWN (1969) the so-called; laws of forms.

may be the following:

1. Transfer of training; GEORGE (1973).
2. Transfer of problem-solving behaviour; LUGER (1975); BAUER-LUGER (1976); REED (1974) et al.
3. Transfer (learning) of skills; i.e. vehicle/aircraft navigation; GOLSTEIN (1977), KOONCE (1974).

Shifting between knowledge representations, Conceptual Universes or problem-representations which are introduced in Ch. 2, is a closely related issue to the above mentioned transfers.

#### 1.2.2 FIRST ORDER PREDICATE CALCULUS\* (FOPC) vs. SEMANTIC NETWORKS (SNs)

In this section we will make brief remarks contrasting some issues we think of importance between FOPC and SNs. The exposition is at two levels, nam. A: as representation formalisms\*\* and B: as far as translation between the two formalisms is concerned.

A. Generally speaking SN and FOPC are closely akin as forms of representation and in representational power. Many workers in the field of Artificial Intelligence\*\*\* tried to show their near-isomorphism. They usually illustrate it elaborating on a number of examples or, at the most, proving equivalent some of the aspects of both two formalisms\*\*\*\*.

The question which naturally arises at this point is: If FOPC and SNs are, in some way, quasi-equivalent, why study SNs and not

---

\* KOWALSKI (1974) and MELETIS (1975)a.

\*\* And also for certain classes of inference.

\*\*\* From now on referred to as A.I.

\*\*\*\* The argument may be supported by SCHUBERT's (1976) comment: "SNs proposed so far have expresively been weaker than PC".

concentrate on FOPC, which has a well-developed metatheory? BARNDEN (1975).

Next, some points are described which elaborate part A of 'FOPC vs. SNs'.

1. Factual knowledge can be conveniently represented in 'net-like' form as the one provided by SNs. Compared with 'linear-like' form of FOPC encodings of factual knowledge, SNs seem more understandable and natural.
2. SNs and FOPC are, in some way, different views of the same Representation Language (RL); i.e. visual (intuitive) and formal (abstract) respectively.
3. Expressing the RL in a 'net-like' form shows much more clearly, the structural features of a body of knowledge or of a problem, than FOPC does (at least to us).
4. Besides, net form is useful in representation of problems in the sense of sects. 2.3.1 and 3.5.5.
5. An SN representation leads, in some sense, to a straightforward computer implementation.
6. Heuristic programmers found SNs more convenient for factual knowledge used by natural language processing systems; and they are using them as 'graphical analogues of data structures representing facts'.
7. From a psychologist standpoint, an SN is more intuitively appealing than FOPC's well formed formulae.
8. From the computer scientist point of view, SNs aid both in the formulation and exposition of the computer data structures they resemble.



9. In cognitive studies SNs are also used to advantage in the mechanization of forms of understanding as for ex. natural language, WINOGRAD (1972), and scene understanding, WINSTON (1970).
10. Certain kind of deductive inference also appear to be facilitated by network-like representation. It is not surprising to discover that SNs implicitly provide a mechanism for inference for some situations.
11. SNs seem to be quite economical in terms of storage required.
12. One of the facts which gave SNs further impetus was the result of psychological experiments that implied that human information storage might also be of this form, QUILLIAN (1968).
13. There is a lack of functional notation for networks analogous to that of FOPC.
14. 2-D graphs help us perceive the way relations are grouped together.

We now focus in FOPC. FOPC was developed in an attempt to understand exactly what it was that mathematicians were doing. Pure mathematicians prove theorems. FOPC was rigorously defined to help answer questions like: what is a proof? Also questions about correctness of proofs, and facts about FOPC itself. In fact, there is an extensive and well-developed metatheory for FOPC.

Furthermore, FOPC provides a means for representing quantifiers and other concepts, not easily expressed in SNs. In fact, SCHUBERT (1976) develops a network representation which permits the use of n-ary predicates, logical connectives, unrestricted quantification, lambda abstraction and modal operators. We are not dealing with these concepts here.

However, a disadvantage of FOPC as a data representation is that all of the knowledge concerning a particular concept is not necessarily

stored in one place. BARNDEN (1975) argues this criticism of PC as not a well-formed one, since there is nothing to stop an implementation of a PC formalism from having links between all the occurrences of a given concept, so that information about a concept is, after all, tightly bound together, and the implementation thus ends up with some sort of network.

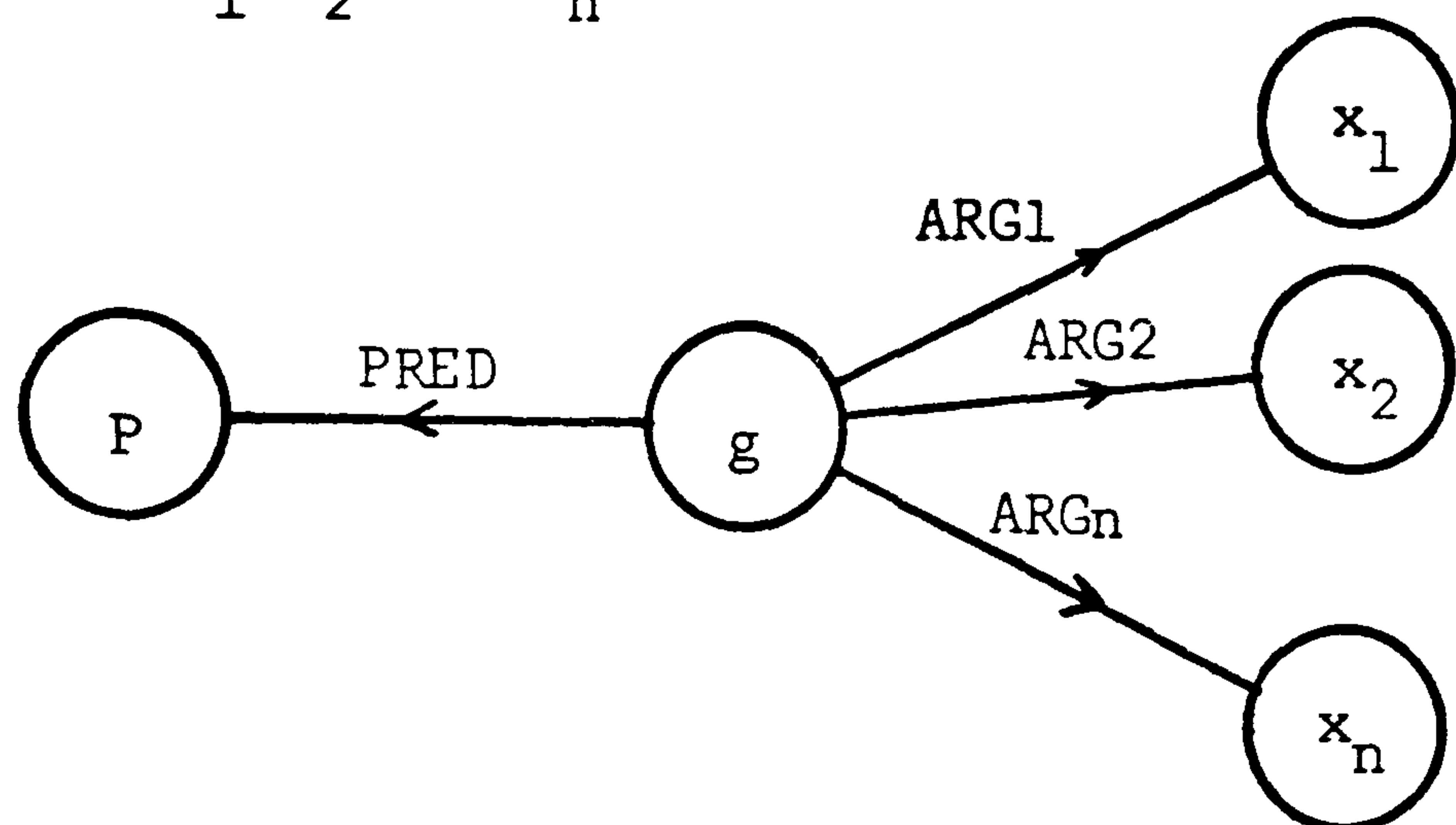
B. As far as the translation process between SNs and FOPC formalisms is concerned, BARNDEN (1975) introduces the notion of a GATE-NODE, which is applied as follows:

1. An  $n$ -ary predicate  $P(x_1, x_2, \dots, x_n)$  is represented by a gate-node  $g$ , with an arc labelled PRED to a node representing the predicate  $P$  itself, and arcs with distinguishable labels to nodes representing the predicate's arguments  $x_i$ ,  $i = 1, 2, \dots, n$ .
2. For a function  $y = f(x_1, x_2, \dots, x_n)$  the representation is similar. The basic role is played by the node  $y$  and gate-node  $g$ .
3. An expression involving logical connectives is also easily represented in a SN form. For the one which specifically represented here, nam.  $X_1 \wedge X_2 \Rightarrow X_3$ , by the gate node  $g_2$ , notice that at the gate-node  $g_1$ , which represents , the condition upon the argument arcs to be distinguishably labelled is weaken.
4. Actions are thought of as predicates, and so are represented as in
  1. The example given here is "John broke the window with a hammer".

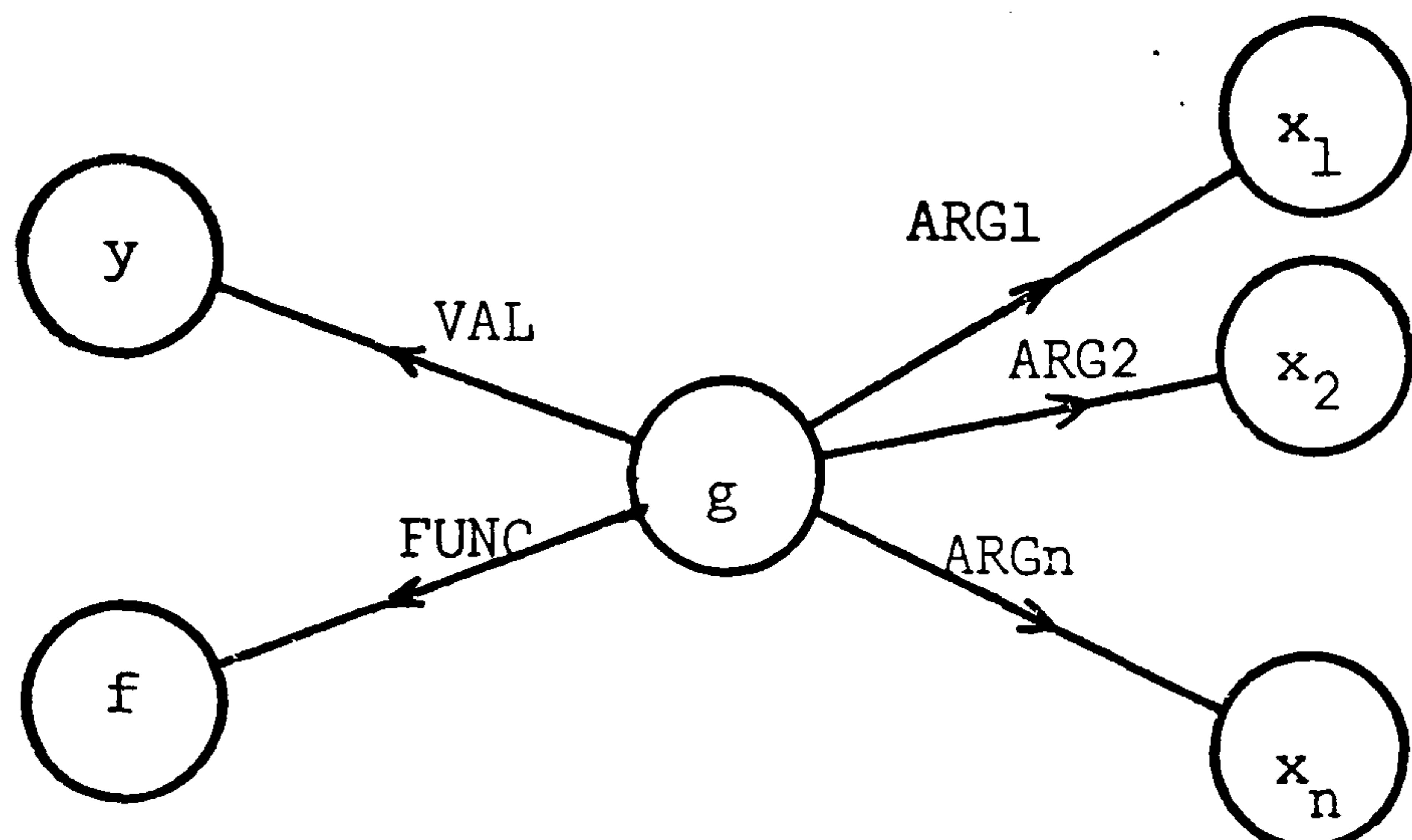
The following four figures F1, F2, F3, F4, give the SN-form of the corresponding cases.



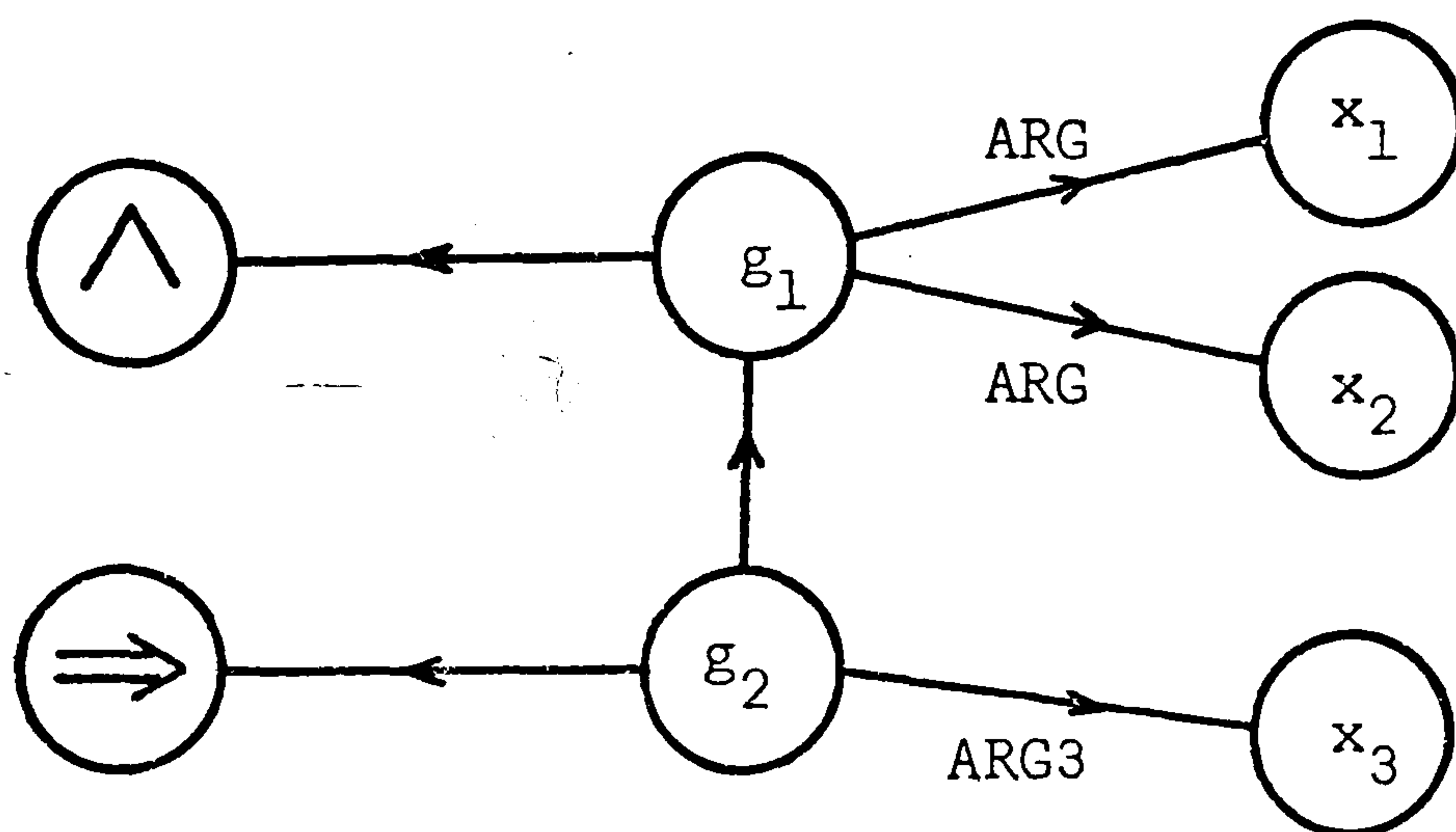
F 1.  $P(x_1, x_2, \dots, x_n)$



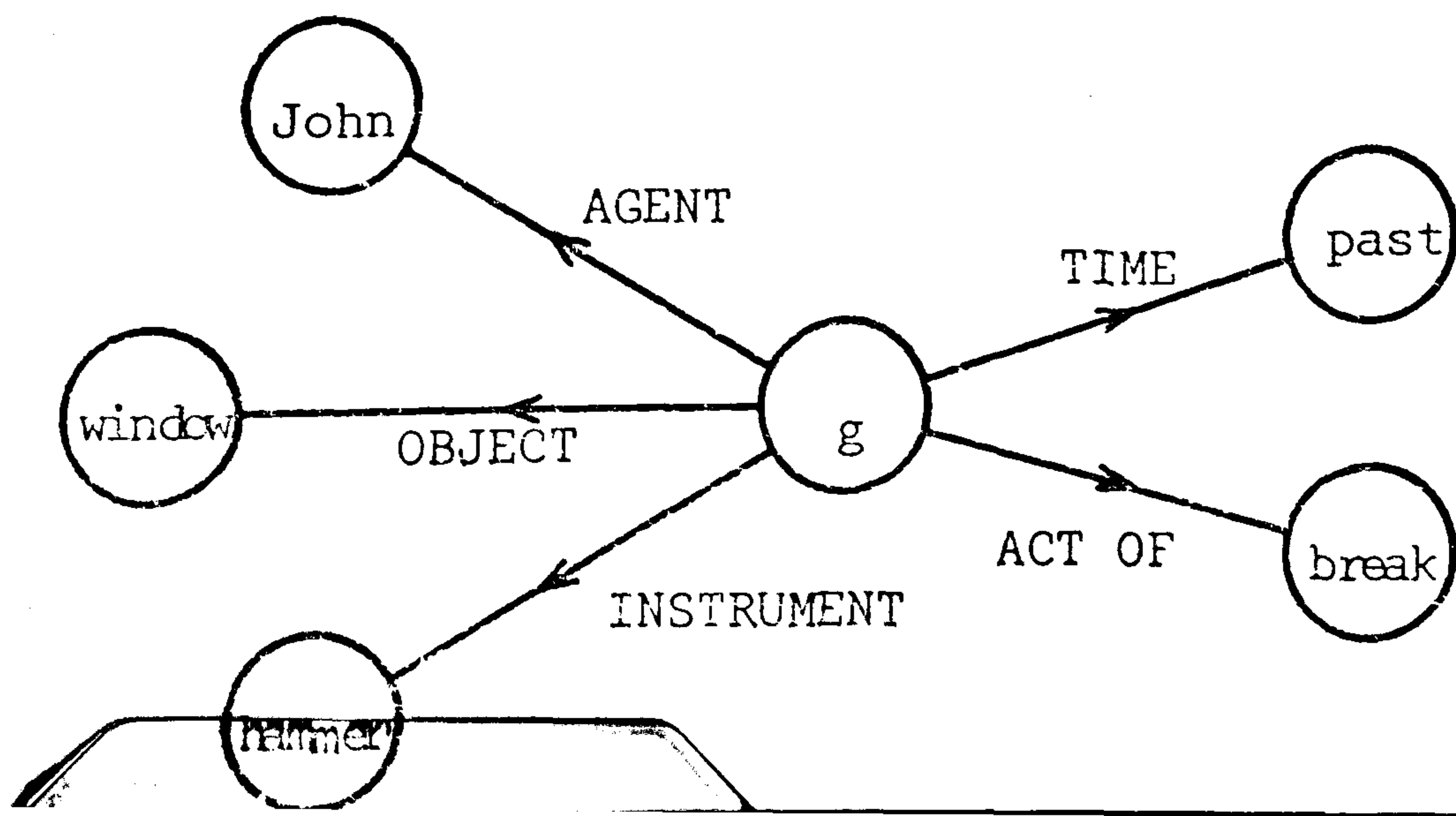
F 2.  $y = f(x_1, x_2, \dots, x_n)$



F 3.  $x_1 \wedge x_2 \Rightarrow x_3$



F 4.



We next give a list\* (by no means exhaustive) of some crucial problems, workers in the field of SNs deal with, as far as net representation is concerned. We do not study them in our thesis.

1. N-ary predicates.
2. Unrestricted quantification.
3. Time.
4. Lambda-abstraction.
5. Modal operators.
6. Variables, definition of relations.
7. Referential opacity.
8. Definite and indefinite descriptions.
9. Definition.
10. Composite objects.
11. Functions.

### 1.2.3 DEFINITION OF A CONCEPTUAL UNIVERSE

A Conceptual Universe (CU) is a quadruple  $\langle \text{Con}, \text{Typ}, \text{Str}, e \rangle$ , where:

Con: finite set of concept-names;

Typ: finite set of arc-types;

Str: set of arc-strengths; and

$e$  : an expansion of Con in  $\langle \text{Con}, \text{Typ}, \text{Str} \rangle$ . In an abbreviated form

CU:=  $\langle \text{Con}, \text{Typ}, e \rangle$ ; where,  $\text{Str} = \emptyset$ .

Informally, a CU is a collection of concepts some or all of which have an image, an SN, under a *definition* map  $e$ .

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\* From BARNDEN (1975).

What a CU simply represents is a set of SNs\* which, together with the corresponding concepts, form the pairs of an explicit or *in extenso* representation of the map  $e$ ; nam.

$$CU_e \equiv \{ SN/SN := e(c), c \in CONCEPTS \}$$

Remark.- Taking into account that:  $CU = \langle Con, Typ, e \rangle$  is a conceptual universe and  $K = (K, M, I)$  is a k-structure (to be defined in Ch. 3 ), a question which intuitively arises is the following: 'does  $e$  have something to do with  $I$ , the identity function of a k-structure?'

#### 1.2.4 CONVERTING A CONCEPTUAL UNIVERSE TO SEMANTIC NETWORK AND VICE-VERSA

##### 1. CUs $\longrightarrow$ SNs

Assertion: Any CU can be transformed into SN form. There are more than one way of doing so, and one is described in BARNDEN (1975) p. 47.

##### 2. SNs $\longrightarrow$ CUs

Here also there are many methods to transform an SN into a CU.

The choice depends on:

A. The type of SN; and

B. What sort of definitional characteristics of the SN, we wish to consider. In other words, from what standpoint are we viewing the SN.

A method of converting an SN  $\longrightarrow$  CU which is of interest for us, is the

\* Finite SNs.

following:

Step 1. Assign distinct concept-names to SN's nodes.

Step 2. For each concept-name  $x$  define  $e(x)$  to be the *context* of the SN node  $n(x)$  labelled  $x$ ; that is, the concept-names in  $e(x)$  are the concept-names for nodes in SN linked to  $n(x)$ ; the arcs attached to  $n(x)$  will induce dangling arcs in  $e(x)$ . The result of this transformation is the so-called *context conceptual universe (CCU)* and it has the important property that all pairs of concepts are *homogeneously* linked throughout the CU.

Winston's (1970) system of model-RSs is a system of definitions which can be expressed simply as a CU.

#### 1.2.5 OUR TOPOLOGICAL VIEWS OF AN SN/RS/CU

The notion of the context of a concept gives us the idea for a departure into the study of the 'environment' of a node of an SN, from a topological standpoint. That is, we want to look at various aspects of the surrounding structure of a node, its features and its role into functions between SNs which may represent some environments. After all, the topological structure of an SN/RS/problem-space, is that which is important in understanding, learning and problem-solving. A full analysis of topological considerations of an RS, and our views (and refinements made) to some existing schemes of context spaces, are given in the following chapters.

We are next hinting at some existing definitions for context. LINDSAY (1973) looks at an RS and interprets regularities, on the basis of similar contexts, for inventing new relations.



If,  $S(X)$  is the set of relations which  $X$  starts; and

$E(X)$  is the set of relations which  $X$  ends;  $X \in \text{NODES}$ ,

then, by looking from this limited aspect the surrounding of each node in the net, he tries to create new relations. He also defines  $C(x)$  to be the set of all contexts of  $X$ ; where, a context is, either, a relation started by  $X$ , plus the end-node of the relation, or a relation ended by  $X$ , plus the start-node of the relation.

Similarity (analogy) relations are inherently connected with ideas such as:

- A. Compare contexts;
- B. Discover maps mapping contexts into each other.

However, we should notice that there are maps between a structure and its substructures or between substructures of a structure and the structure itself and there are relations (maps) between structures representing entities from different worlds (universes of discourse).

MOORE-NEWELL (1973) consider a system for understanding and problem-solving having an SN as knowledge representation. One important point is that each concept is given a set of alternative definitions; where a definition is the set of arcs and nodes a concept-node is attached to\*. Thus entities in their universes are *context-defined*. This gives rise to view a concept  $C_1$  as another one  $C_2$  creating thus a relation:

$$\begin{aligned} V : \text{CON} \times \text{CON} &\longrightarrow \text{'VIEWS'}^{**} \text{ (or rather 'change-view level')} \\ &: (C_1, C_2) \longrightarrow V(C_1, C_2) := \text{'}C_1 \text{ viewed as } C_2\text{'} \end{aligned}$$

---

\* i.e. The concept-node's immediate environment.

\*\* Beliefs; similar/analogical views; are relevant issues.

which is not symmetric. Thus, the important aspects of 'changing view' or 'looking a problem from another standpoint' related to shifting between representations, may be studied via 'contexts'.

CHAPTER 2

### 2.1.1 ON TOPOLOGICAL-ALGEBRAIC NOTIONS RELATED TO CUs, SNs\*

The purpose of this section is to make some informal remarks on the continuity of CUs; functions, and maps between CUs; topology\*\* and organisation\*\*\* of a CU; convergence, limits and so on, in the context of a CU and changes of CUs, i.e. change views, system beliefs, etc. mentioned in a previous section, 1.2.5. Some of the above issues are not extensively studied in BARNDEN's (1975) work.

From topology, we recall that a function  $f: X \rightarrow Y$  is continuous iff  $f^{-1}(U_Y) = U_X$  where  $U_X$  is an open for  $X$ 's topology, i.e.  $U_X \in T_X$  and  $U_Y \in T_Y$ ; that is, iff the  $f^{-1}$ -image of a  $Y$ -open is an  $X$ -open. In other words,  $f: X \rightarrow Y$  is continuous iff it sends members of  $T_Y$  to  $T_X$ 's ones (and v.v. if, in addition, we want  $f^{-1}: Y \rightarrow X$  to be continuous). A Topology  $T_X$ , on a CU  $X$ , is simply an organization of  $X$ 's constituent entities.

In section 3.5.5 we would refer to the notion of ANALOGY between two knowledge representation structures\*\*,  $X$  and  $Y$ , as a function (precisely functor)\*\*\* between them. Analogical Reasoning (AR) may be considered as the form of reasoning based on the use of analogy in its various forms. A reasonable constraint (from a cybernetic point of view, in terms of ASHBY (1956)) imposed on AR is the fact that the function or mapping between two

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\* Abbreviations for Conceptual Universes and Semantic Networks respectively.

\*\* In terms of DUGUNDJI(1966), p. 62.

\*\*\* In BEER's (1972) sense; in fact he reads "cybernetics is effective organization of all available resources".

\*\* SNs, CUs, CCUs or k-structures (see Chapter 3) as the case may be.

\*\* Chapter 2



representation schemes should be kept continuous. That is, some kind of continuity is required during the shifting of representations from which analogizing intellectual activities are fulfilled.

As a natural consequence of the remarks made so far, the notion of continuity of a function or map  $f$  between two structures\*  $X$  and  $Y$  is inherently dealing with the (somehow defined) topologies  $T_X, T_Y$  of the two structures.

If, on the other hand, we consider two topologies on  $X$  and  $Y$ \*\*, nam.  $T_X/R_1$  and  $T_Y/R_2$ , i.e. the so-called "quotient" topologies according to two equivalent or similarity relations  $R_1, R_2$ ; and then we are able to establish a (1-1) correspondence  $\alpha^*$  between the quotient topologies, then  $\alpha^*$  is continuous. (A proof of that may be found in a text on topology for ex. DUGUNDJI (1966). In schematic terms we have:

$$\begin{array}{ccc} \alpha: & T_X & \longrightarrow & T_Y \\ & \downarrow & & \downarrow \\ \alpha^*: & T_X/R_1 & \xrightarrow{1-1} & T_Y/R_2 \end{array}$$

Therefore consistent\*\*\*, complete\*\*\* Analogical Reasoning may be

\* SNS, CUs, CCUs or k-structures (see Chapter 3) as the case may be.

\*\* In fact  $\alpha: T_X \longrightarrow T_Y$  induces two maps; nam.  $\alpha: \mathcal{P}(X) \rightarrow \mathcal{P}(Y)$ ,  $\alpha^{-1}: \mathcal{P}(Y) \rightarrow \mathcal{P}(X)$ ; where  $\mathcal{P}(\cdot)$  is the power set and  $\alpha^{-1}$  is used for the continuity.

\*\*\* Skeleton (topology) represents in some way the 'maximum' topology which has this property of consistency.

\*\*\* On the possibility of complete systems of inductive inference, see MELTZER (1970)c.

somehow achieved via continuous functors between skeletons\*. Where skeleton of  $X$  and  $Y$  is thought of as the substructure commonly shared by  $X$  and  $Y$ . To a skeleton  $S_{XY}$ , corresponds a topology  $ST_{XY}$  - the so-called skeleton topology.

Remark 1.- The above argument (and results) is more or less based on our intuitive reasoning and it led us to a number of speculative results. We are not dealing with them thoroughly in this thesis for a number of mathematical concepts are needed to be introduced which are off the main streamlines of the present work.

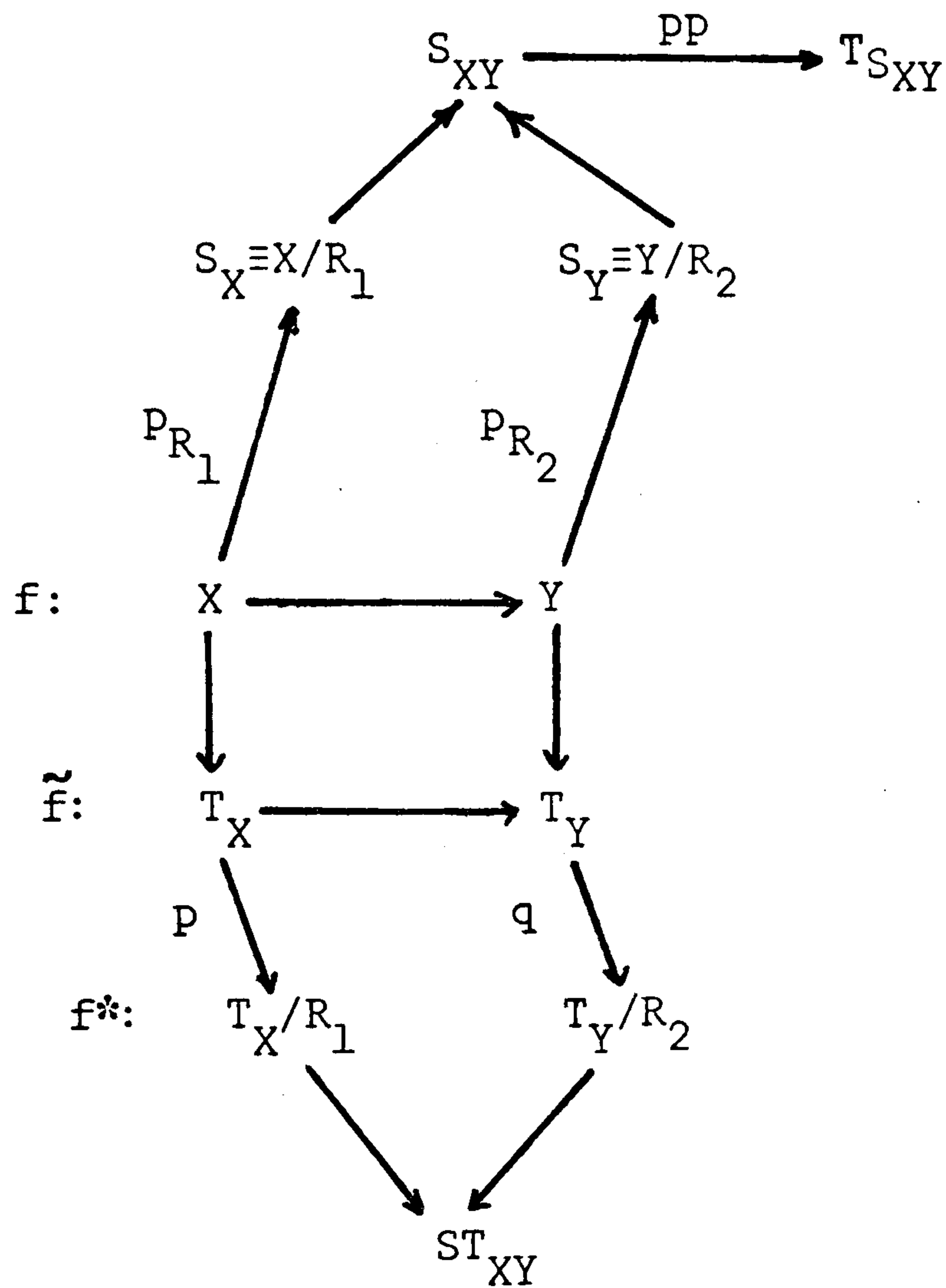
Remark 2.- We are convinced that the 'separation of context' technique, mentioned in section 3.3.3, supplies two representation schemes  $CU_1$ ,  $CU_2$  with such topologies appropriate for an effective study of notions of continuity and so on, introduced at the beginning of this section\*\*. Finally, the topological notions introduced above lead to the important theme of shifting, which is central in our investigations.

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\* The notions of a functor and skeleton are made clear in the next chapter.

\*\* In other words, in order to achieve A.R. we must proceed through 'points' where the 'map' is continuous; i.e. via neighbouring points.

CLARIFYING SCHEMATOLOGY



The question which arises here may be stated as follows: 'is the topology of the XY-skeleton ( $S_{XY}$ ) the same as the "Skeleton topology" of XY?' i.e.  $T_{S_{XY}} = ST_{XY}$  ?

$p, q, PR_1, PR_2, PP$  are projections.

### 2.1.2 SHIFTING BETWEEN CUs

The idea of shifting between CUs is introduced to make the 'static' notion of a CU, in representation terms, more operational and functional. In most of the cognitive (and a No of other) activities, we are familiar with some phenomena and expressions such as the following ones which appear very frequently: transfer of learning, knowledge, skill, methodology; change of opinion, beliefs, definition system; and, change of position, direction, point of view, outlook, attitude, policy or character. The very notion of shifting in its full generality, incorporates, represents and materializes in some way most of the above phenomena and everyday expressions. It thus cat ers for the unification and embodiment of their meaning.

A Conceptual Universe has been informally introduced as a collection of concepts some or all of which have an image, a semantic net, under a *definition* map  $e$ ; nam.  $e: \text{CONCEPTS} \longrightarrow \text{SNs}$ . Thus, we have the following representation of a CU:

$$\text{CU} \equiv \{ (c, e(c)) / c \in \text{CONCEPTS}, e \text{ a definition map} \}$$

The notion of shifting between CUs may be well understood and interpreted via the idea of a map\* or function  $f$  from a CU  $U_1$  to a CU  $U_2$ ; i.e.

$$f: U_1 \longrightarrow U_2$$

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\* The concept of a map and the importance of continuity is stressed in ARBIB (1977), also see maps between spaces in ZEEMAN's (1961) terms. for a connection to brain theory.



The *raison d'être* of this interpretation is the following: it is our intention to correlate the idea of shifting with that of a map, since it was this correlation which led us to conceptualization of shifting between  $k$ -structures, developed in the next chapter, as well as the concept of shifting in problem representations. We may distinguish and categorize two kinds of shifting in the context of CUs, nam. shifting-1 and shifting-2.

#### 2.1.3.1 Shifting-1

Shifting-1 deals with the mapping between images of concepts (under the same definition map  $e$ ), i.e.

$$f^1: U_e \rightarrow U_e$$

Metaphor; an endomorphism in the Geometry world.

#### 2.1.3.2 Shifting-21

It is about the shifting between concept images defined via different definition maps,  $e_1, e_2$  of the same CU, i.e.

$$f^{21}: U_{e_1} \rightarrow U_{e_2}$$

Such sorts of shifting are mainly concerned with changing the context of the same entities (concepts) which we are talking about. That is, two different topologies (organizations) upon the same set of concepts,

Shifting-2 2

It is about the shifting between concept images defined via two completely different definition maps operating upon two different collections of concepts from different Universes of Discourse (CUs  $U^1, U^2$ ), i.e.

$$f^{2\ 2}: U_{e_1}^1 \rightarrow U_{e_2}^2$$

Such types of shifting reflect radical changes in view, or they may represent shifting between two totally different worlds\*:

Remark 1.- The case is interesting when  $e_1$  and  $e_2$  are of the same nature; thus we distinguish a particular subcase of shifting-2 2, nam.

$$f^{2\ 2}_1: U_e^1 \rightarrow U_e^2 .$$

Metaphor of shifting-2.-

$U^1 \equiv G(X) :=$  the set of all (1-1) mapping on a  $X \neq \emptyset$  onto itself,

$\bar{e}_1 :=$  the function composition (which implies some topology (organisation))

$U^2 \equiv Z :=$  the set of integers

$\bar{e}_2 :=$  the operation of addition (which implies a topology on  $Z$ ).

We have chosen the above example to see how an (informal) theory is based on likenesses which one might observe in two familiar mathematical systems. The similarities in these two systems are incorporated in the formal axiomatic theory called GROUP THEORY. The primitive terms are: an unspecified set  $G$ , a binary operation "." on  $G$ , and

---

\* See R. BROWN's (1977) work.

an element  $e \in G$ .

Remark 2.- We just mention here two works of interest and of some relevance to concepts developed herein, although in a different level of abstraction, nam. ZEEMAN (1962) and REDHEAD (1975). We do not elaborate on them, for they are cited only for reference and for a departure into theorizing on the matters involved here.

### 2.2.1 KNOWLEDGE ACQUISITION

It is an issue as important as that of the knowledge representation with which we have already dealt. It is a common belief in the scientific community that knowledge acquisition is acquired generally in a twofold manner, namely:

- A. knowledge by description
- B. knowledge by acquaintance.

Learning by description is, in a sense, 'wider' than learning by acquaintance - at least for humans. Thus, if the ultimate goal of an A.I. community is to build integrated cognitive systems which exhibit some sort of Intelligence, similar to some extent to that of humans, then systems' Artificial Inteligences must have learning abilities, more or less, analogous to the above mentioned ones. Some of the outmost capabilities that humans exercise are concept formation and learning. This naturally leads to the need of mechanization of concept formation/learning cognitive activities. However, there remain two problems, firstly, whether concepts can ever be other than a reorganization, reordering, recombination of existing concepts, and secondly, whether we can derive new concepts by enlarging our knowledge through



acquaintance, GEORGE (1973).

The references cited below\* constitute a compilation of some of the existing approaches and opinions in the literature concerning four important topics, nam. concept formation, concept learning, concept filtration and concept amalgamation. Some of them are taken up and examined later on in the appropriate chapter, from a new outlook from that of k-structures.

### 2.2.2 ON INTELLIGENCE

In this section, we quote general remarks on the vague concept of Intelligence, which have been put forward by a number of people and end up by presenting a summary of our own views on this important notion. We attempt also to link it up with some concepts of primary importance for the present thesis, nam. problem-solving, learning and k-theory.

MINSKY (1961) reads: "*Intelligence is like depth in mathematics*". The notion of Intelligence might be assigned to the capability of: "*model building*", AMAREL (1966), (1970); "*using parables*", PASK (1963); "*steering the exploratory process by a sense of the proximity of a solution*", MACKAY (1959)\*\*; "*synthesizing a metaphor*", LEATHERDALE (1974); "*expressing an allegory*", ARISTOTLE; "*generating establishing and exploiting an analogy*", KLIR-VALACH (1967); "*sharing of experiences*", R. STRAUSS in his opera Capriccio.

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\* BANERJI (1969); CHAVCHANIDJE (1976); BARNDEN (1975); FUNT (1976).

\*\* It also includes an excellent discussion on the distinction between 'Intelligence' and 'Intellect'



It has been conjectured, as TAN (1975) writes in p. 3, that Intelligence is a question of having a "big switch" which "efficiently connects particular domains with special methods", NEWELL (1973) p. 10, each method representing "expertise", R. BROWN (1977), in its own domain.

We now focus into the second part of this section by presenting a variety of parallel, more or less, expressions on the concept of Intelligence. To us, Intelligence denotes *a plegma of performances and their linkages*, or a manifold of methodologies (tactics and strategies in execution) and their morphisms. If, on the other hand, somebody asked us how can we measure Intelligence? then, we should answer by the following (quantative) definition: *"Intelligence might be measured by the degree of flexibility in shifting between appropriate (and relevant for a given task) methodologies and ways of thinking"*.

Finally, to link up the concept of Intelligence with the tasks of analogical problem-solving and learning, investigated in the present work, we very briefly cite some of NILSSON's (1971) notions. For problem-solving, two are the main issues of great importance: (1) Representation of Knowledge (epistemological issue); and (2) Search (heuristic issue). The main problem-solving methods are:

- A. State space
- B. Problem reduction
- C. Theorem proving

Now, Intelligence comes into consideration when we mainly talk about search methodologies, i.e. Intelligence concerns with heuristics involved in problem-solving programmes\*. Simply saying, analogical

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\* See LENAT's (1977) impressive work on these matters.

problem-solving is a matter of attempting to solve a given problem on the basis of an analogy. In fact, analogical problem-solving is largely 'a matter of appropriate selection' based on analogical thinking and reasoning. By Intelligence attributed to an 'analogizer'\*, we mean its power of appropriate selection on the basis of an analogy (or net of analogies). Works which are relevant here are: MELTZER's (1970)b, on power amplification for theorem provers; KLING's (1971) on the power of heuristic search process via analogy; and SACERDOTI's (1974) approach to augment such a power (of the latter kind). The essence of his approach may be summarized by the utilization of a means for discriminating between 'important information' and 'details' in the problem space.

In conclusion, the original question in TURING's (1950) pioneer paper, "*can a machine think*"? may be narrowed down into the following "*can a machine think analogously*"? which according to previously made remarks may be rephrased as "*can a machine select analogously*"? which we think amounts to the mechanization of axiom of choice in mathematics.

### 2.3.1. ON PROBLEM REPRESENTATIONS

We now turn our attention on the issue of how to represent problems. It is widely recognized and accepted from workers in A.I., and related fields, that the main problem in their researches is that of *problem representation*. AMAREL in FEIGENBAUM (1968) p. 1023

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\* Also a useful/practical heuristics.



reads:

*"A man M, facing a problem  $\pi$ , and trying to solve it with the aid of a machine  $\mu$ , he represents problem  $\pi$  to machine  $\mu$  by providing such problem's knowledge which reflects his 'point of view' ".*

The important assumption\* that 'effectiveness in problem-solving and decision-making is facilitated by looking at a problem from a variety point of view', is the main tactic which is used by COLES(1975), (1977); CARTER (1974); TSIPIIS (1976); MASON (1969); SUSSMAN (1975) and other A.I. workers in their strategies and methodologies for problem-solving, conflict resolution, and other tasks in a variety of fields. If, on the other hand, we interpret the expression, 'looking at a problem  $\pi$  from two points of view:  $PV_1, PV_2$ ' as 'shifting between two problem's knowledge representations:  $R_1, R_2$ , reflecting his points of view' then, the above mentioned hypothesis naturally leads\*\* to two crucial A.I. problems, nam. (1) shifting of problem representations, AMAREL (1969), (1970) and (2) its mechanization, AMAREL (1966), (1967), which are introduced in the next section.

Problem Representation\*\*\* (and the language for it) is crucial in the following two senses or contexts:

1. As far as problem solution is concerned, the problem-solver continuously provides various transformations\*\* of the problem in hand, until a successful, adequate matching, quite often to a previously solved problem, occurs.

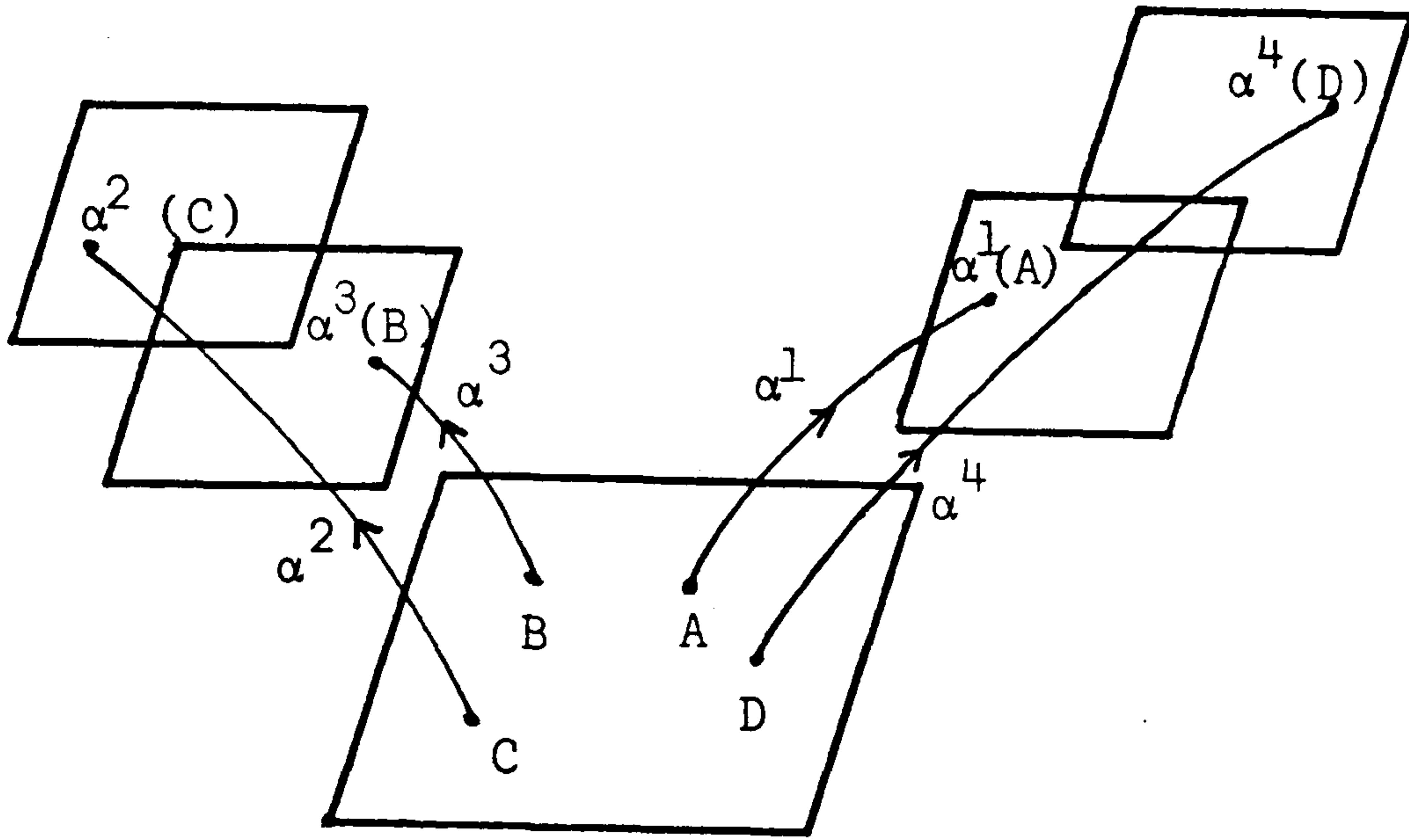
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\* Also a useful/practical heuristic.

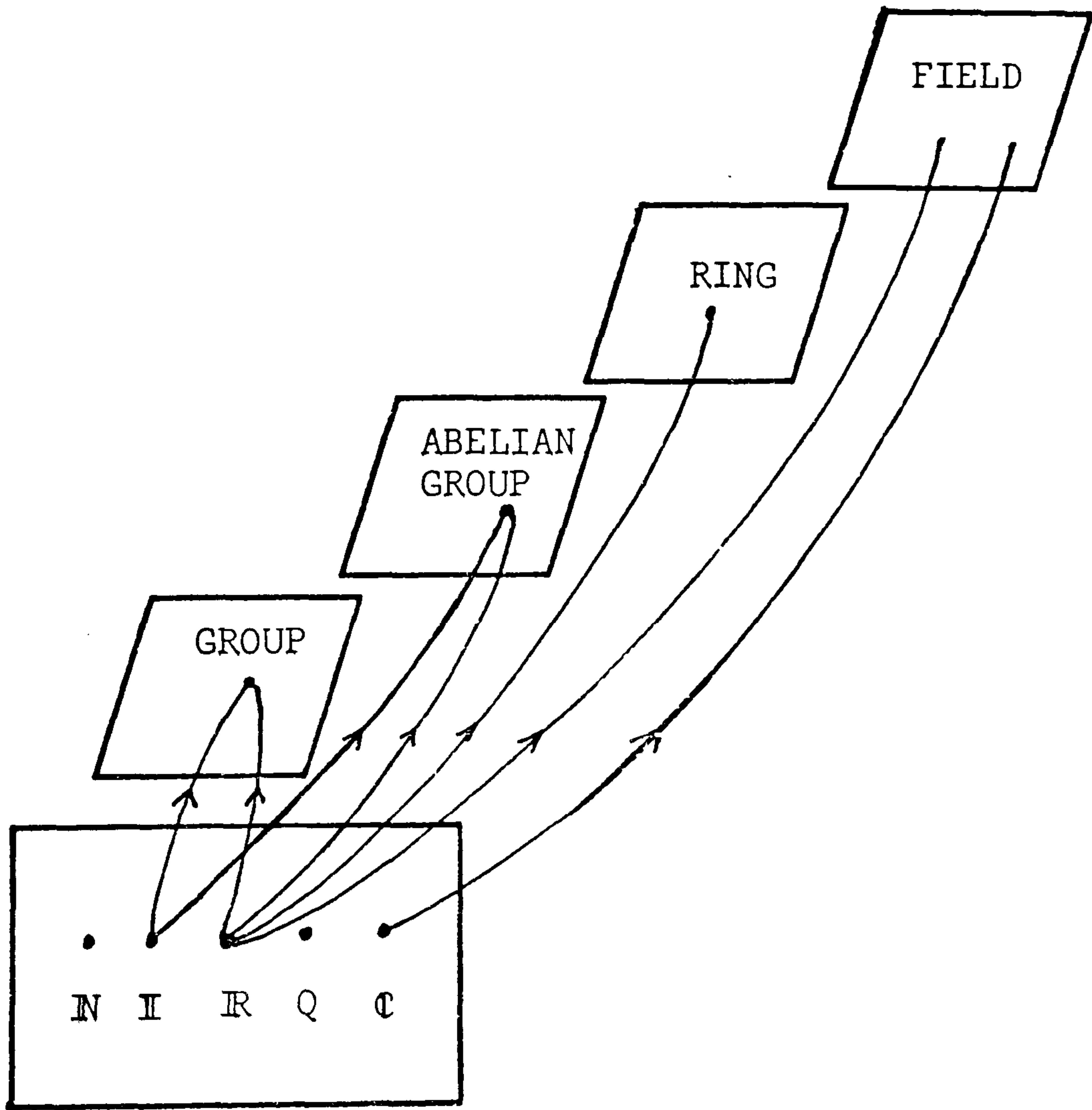
\*\* Our attitude here justifies the main issue in Epistemology, nam. the importance lies on 'the consequences of the assumptions made' in posing (answering) problems (questions)'.  
~~Relaxing hypotheses~~

\*\*\* Generally, representation is a set of conventions, assumptions of how to describe things.

~~Relaxing hypotheses~~ and/or by enlarging existing theories.



Schema 1



Schema 2



2. In a semantic context, we have to distinguish between language and the objects represented by language. Language represents "a degree of abstraction", GEORGE (1973), p 415, from the reality described. And the question, which is naturally raised here, is: 'Are all the objects, involved in a representation, at the same level of abstraction or, otherwise expressed, "equally" abstracted?'. The work of SACERDOTI (1974), (1975) on hierarchies of abstraction spaces might be thought of as relevant here. Also, MINSKY's (1974) frames. The schema 1, is about "Lifted" (abstracted) objects (concepts) A, B, C, D which are mapped in four different levels of abstraction.

Example.- From the world of arithmetic systems, the system of real numbers may be abstracted\*\*, conceptualized in a number of different ways, some of these are illustrated in schema 2. The arithmetic systems themselves, also taking as objects, are abstracted in different algebraic structures/languages.

### 2.3.2 FROM PROBLEM REPRESENTATION TO ITS SOLUTION

The implications of a representation of a problem to its solution is illuminated and elucidated in the so called: *Interaction between problem-structure and problem-solving behaviour* theme of many investigators; WINSTON (1972), LUGER (1975), GAINES (1976),

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\* This question is also linked to epistemological issues.

\*\* Enriched/structured.

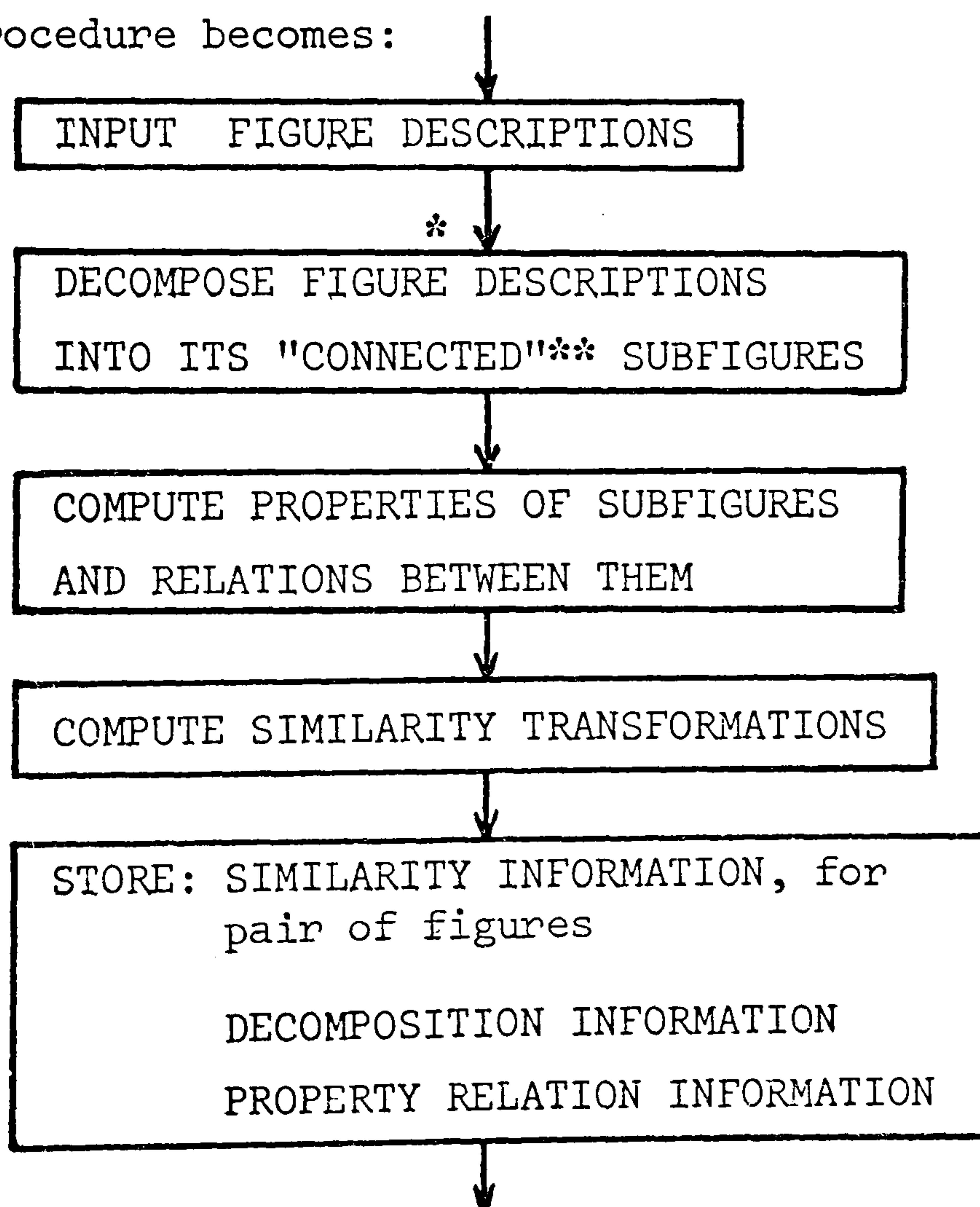
BAUER-LUGER (1975), GOLDING-LUGER (1975), SIMON-HAYES (1976), etc. Let us illustrate some aspect of this interaction taking a concrete and well known example, that of EVANS (1963), on geometric-analogy problems. In this context, a useful heuristic he introduces (pp. 30) is the so-called *intrinsic decomposition heuristic*:

*"choose decompositions of problem representation into sub-figures which have as much as 'internal symmetry' (which can be defined in some precise sense) as possible"*

In other words, what it simply says is, extract, propagate, recognize and detect intrinsic features from a problem by introducing an appropriate topology (organization) to its representation (PAVEL (1976)).

Under this choice, implied by the use of the above heuristic,

EVANS' procedure becomes:



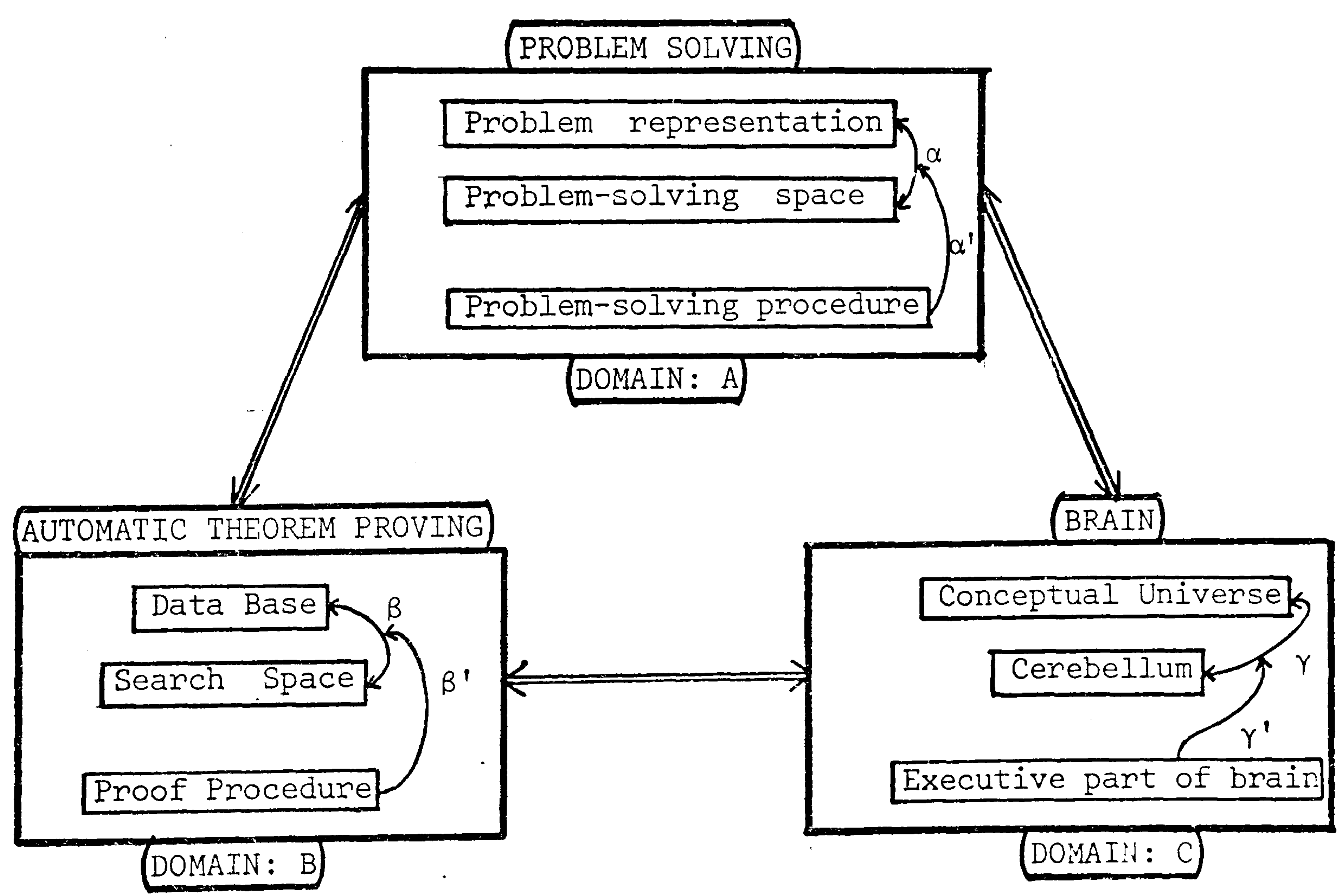
\* This is where *intrinsic decomposition heuristic* is applied

\*\* The *raison d'être* of the topological connectivity here has to do with the notion of continuity, we mentioned earlier, in the following sense: problem's solution is to be found via neighbouring points (subfigures).

As a final remark, we point out that fruitful results, as far as problem-solving is concerned, could be gained by studying the interaction between problem-structure and problem-behaviour, via analogical inductive reasoning; POLYA (1954).

2.3.3 THE SIGNIFICANCE OF INTERACTION BETWEEN PROBLEM-STRUCTURE AND PROBLEM-SOLVING BEHAVIOUR

To stress, in another way, the importance and the wide impact of the above interaction, here is an example: where emphasis is given in some sort of analogy which exists between three A.I. domains (see schema).





Some points of interest are:

1. Symmetry in the structure of the cerebellum influences executive part of the brain and results Intelligence amplification, exhibited as intelligent behaviour; ASHBY (1956), p. 259, MELTZER (1970) a.
2. Symmetry in the structure of search space influences proof procedure and results amplification of theorem-proving power; MELTZER (1970)b.
3. Symmetry in the structure of problem solving space influences problem-solving procedure and results increase in efficiency of problem-solving abilities exhibited as a better problem-solving behaviour; LUGER (1975).
4. The existing symmetry mentioned in point 1 is an exemplary indication of a "redundancy approach" taken from brain when faced with new problems.
5. Also the symmetry in point 3 may consider as a departure for a "Reduction approach to Problem-Solving", NILSSON (1971).
6. In the schema, the interaction between problem-structure and problem-behaviour, indicated as bidirectional arrow  $\alpha$  and monitored (via a P.S. procedure) by  $\alpha'$  can be extended analogously in the other two domains B and C.



#### 2.4.1. INFORMAL INTRODUCTION TO SHIFTING (OF REPRESENTATIONS) PROBLEM

We start straight away with a question which captures the meaning, consequences and scope of the notion under investigation, nam, 'how shifting of representations may be described and constructed during the course of solving a problem by analogy'. To analyse this question, a language  $L$  is required for representing the entities involved in its analysis. In fact,  $L$  may be thought of as a meta-language but we are not dealing here with metalinguistic issues or questions on self-reference languages as for example KLEENE (1952), SMULLYAN (1962) and so on, because such topics are beyond the scope of this work.

Let mathematical structures\*, in the general sense of the term, be, at the moment, the objects (primitives) of the linguistic descriptions we are going to use. Accordingly, then, to this phraseology, a shifting from a representation (structure  $S_1$ ) describing a situation (problematic thesis  $\theta_1$ )\*\*, to another representation (structure  $S_2$ ) describing another situation (problematic thesis  $\theta_2$ )\*\*, may be considered\*\*\* as the job of some kind of correspondance  $\delta$  which relates<sup>\*\*\*</sup>, somehow, the two structures. Nam.

$$\delta: S_1 \rightarrow S_2$$

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\* For a formal definition of a (mathematical) structure see STOLL (1974) p. 146 and for relational structure ENGELER (1968).

\*\* In the case where we are talking in problem-solving terms.

\*\*\* Rather, the shifting is embodied in  $\delta$ .

<sup>\*\*\*</sup> Correlates, in AMAREL's (1967) terms, p. 98.

Before examining in detail  $\delta$ 's anatomy, we wish to mention at this point that the areas which are rich, from this kind of 'shifting activities', are: Analogical Problem-Solving (APS) and Analogical Learning (AL). In these intellectual processes the solver, or learner in the sense of KOCHEN (1974), continuously shifts his attention between analogical representations. This is one reason why we have chosen APS/AL to study, experiment and exercise shifting of representations. We immediately proceed with an analysis of  $\delta$ ; an informal introduction to PS/L and APS/AL is found in the appropriate sections.

Remark.-  $\theta$  stands for  $\theta\epsilon\sigma\iota\varsigma$  (Greek for situation, position);  $\delta$  stands for  $\delta\iota\alpha\lambda\omicron\gamma\omicron\varsigma$ . Question: is  $\delta$  the so-called *heuristic connection* that MINSKY (1961) speaks about?

#### 2.4.2 $\delta$ 's ANATOMY\*

The nature/pattern of  $\delta$ , either explicit or implicit (*in extenso* or *in intenso*) is dependent mainly on the following factors.

A number of constraints or conventions, possibly external to  $(\theta_1, \theta_2, \lambda^{**})$  system, with global, Universal or holistic implications; the otherwise so-called *a priori* choices or environmental impact. e.g., computer storage and time limitations to problem-solver; orthogonal topology of chess board; interval time in a chess tournament.

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\* Analysis/morphology/physiology, MONOD (1970).

\*\* Where  $\lambda$  stands for  $\lambda\acute{\upsilon}\tau\eta\varsigma$  : Greek for problem-solver.

Morphology, PIAGET (1970) of  $\theta_1$ ,  $\theta_2$  structures. That is, what kind are the objects (and the relations between them) involved? This is very important, as it is pointed out for example in DRESHER-HORNSTEIN's (1976) criticism of MINSKY's (1974) frame theory, which is considered to be a unified theory for representation of knowledge and thinking.

Local conditions that  $\theta_1$ ,  $\theta_2$ 's constituents obey or satisfy; TOURLAKIS-MYLOPOULOS (1973) p 440.

Physical characteristics and/or individual, atomic properties of  $\theta_1$ ,  $\theta_2$ 's objects; (FUNT (1976)), e.g. nature of chess pieces.

Value<sup>1</sup>, colour<sup>2</sup>, weight<sup>3</sup>, meaning<sup>4</sup>, strength<sup>5</sup> of  $\theta_1$ ,  $\theta_2$ 's relations:

- 1 either fuzzy or not; GAINES (1976),
- 2 STEPHAN-SIEKMAN (1976),
- 3 LARSON (1974),
- 4 WINSTON (1970),
- 5 BARNDEN (1975) arc strength.

Motivations, beliefs, intentions and indiosyncrasy of problem-solver  $\lambda$ . In other words  $\lambda$ 's philosophy or world model or point of view.

Nature of composition(s) between relations; i.e. how relations can be synthesized.

Nature of operations that are going to be performed by  $\lambda$ , e.g. inference rules in an axiomatic system. Legal moves a chess player can perform on the pieces.

Current state of activity-field affairs during the course of



problem-solving. Namely, form of current problem-situation (problem thesis  $\theta$ ) i.e. the way it is represented or described.

$\lambda$ 's knowledge about the problem  $\pi$  in question or the class of problems that  $\pi$  belongs to.

$\lambda$ 's epistemology (know how). That is, how does  $\lambda$  know what he thinks he knows? This is, perhaps, partly incorporated in what nowadays A.I. community calls: *procedural knowledge*; HEWITT (1971), (1972), i.e. procedures of how to do things.

$\lambda$ 's capabilities, that is:

A. Logical: deductive, inductive, etc.

Non-logical: analogizing; EVANS (1963), KLING (1971), SLOMAN (1971), etc., use of diagrams; FUNT (1976), BROWN, F. (1976), etc. Common-sense; McCARTHY (1959).

Intuitive; SLOMAN (1971).

B. Inherited, hitherto *a priori* knowledge.

C. Learnable, attainable, acquired.

$\lambda$ 's performance history. An aspect which is immediately connected with psychological issues.

$\lambda$ 's Intelligence. To me, *intelligence denotes a plegma of performances and their linkages, or, the degree of flexibility in shifting between methodologies to meet a new situation*. For a discussion on this issue see sect. 2.2.2.

Methodologies available to  $\lambda$ . That is, various *off the shelf* techniques, tactics, policies, heuristic *demons*, advices \*.

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\* LENAT (1977).

\*\* And representations.



Available hint-list relevant to problem  $\pi$  in question.

The level  $\beta$  of detail, complexity that  $\lambda$  is going to pursue.  
 $\beta$  plays important role in the analysis of the interaction between problem-structure and problem-behaviour.

The level  $\alpha$  of abstraction he is going to represent things.

What goal is to be achieved.

Some of the above may be incorporated in our favourite theorem-schema

$\theta:$	$Y$	knowledge problem in question (its hypotheses) assumptions made
	$\Sigma$	goal

### 2.4.3 ON TWO SORTS OF SHIFTING

We may distinguish two kinds of shifting, nam.

1. Shifting between representations, and
2. Shifting between methodologies.

The first one, declarative (epistemological) shifting, is mainly found between knowledge representations, or problem representations describing problem-situations, (problematic theses). In EVANS (1963), geometric analogy problems, for example, we have either

$$\left[ [A : B] \text{ as } [C : X_i] \right] \longrightarrow \left[ [A : B] \text{ as } [C : X_j] \right]$$

or

$$[A : B] \longrightarrow [C : X_k]$$

where ":" symbolizes the expression "...is to..." or "...is related to...", See also sects. 3.5.6., 3.5.8., and 3.5.10,

The second one, heuristic shifting, so to speak, is concerned with methods, tactics, strategies. Here is where the concept of Intelligence (Sect. 2.2.2) really comes in.

The reason underlying this sort of differentiation between the two above described shifts is mainly a picturesque one. It is really difficult, if not impossible, (meaningless) to separate between representations and methodologies.

In some integrated manner, this sort of interlocking (interlinking) between representations and methodologies, may nicely be embodied in what we call *epistemological -heuristic interaction*: <E-H>. <E-H> is the functionality which captures, materializes an epistemologically-heuristically adequate shift \*; McCARTH-HAYES (1969), SLOMAN (1971), SACERDOTI (1974).

We think it appropriate here to intervene the foregoing discussion with some comments on the very issue of Epistemology. Let us put them in question-form, epitomizing what we mean by this term.

A. How do we know what we know?

or to be a little more precise,

A'. How do we know the representation of reality we think we know?

We must leave any further discussion on these issues, otherwise, we will have to enter the vast field of Philosophy.

Finally, we ought to mention two main problems associated with the above categorization, taxonomy of shifting. Namely, the *frame problem* and the *search problem* resp.; see RAPHAEL (1970), NILSSON (1971).

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\* If such a thing exists, which is doubtful.

#### 2.4.4 MORE ON SHIFTING OF (KNOWLEDGE/PROBLEM) REPRESENTATIONS

We analyse some issues by posing a number of questions.

2.4.4.1 Question 1: Why is such sort of shifting necessary to be studied, analysed?

As we mention in section 2.5, 'on our attitudes', one of them is to view induction from a *non-statistical* standpoint. This is a radically different approach contrasting with the traditional probabilistic, statistical one which has been taken in most previous A.I. attempts to tackle induction. This tendency rests upon the *redundancy* assumption due to SIMON-SIKLOSSY (1976). We quote:

*"... In a highly redundant world, coincidences never happen (or hardly ever happen!). If two structures match, and match redundantly, it can be assumed safely that they are related".*

To some extent, the notion of redundancy is loosely associated to *ergonomy* in the broad sense of the term. In the case of Conceptual Universes, where abstract entities (i.e. concepts, thoughts and so on) are involved, we intuitively introduced an analogous notion; that of *skeptonomy* (Greek for "laws of thought = skepsis"). By this term, we mean a set of assumptions, principles and rules, under which a human problem-solver by using, directing the executive part of his brain, he carries out intellectual tasks in the most efficient/economical manner, i.e. using the minimum number of thoughts and thinking power in order to achieve a new goal. In fact, a brain

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\* It is rather important to investigate the implications to A.I. of the *induction without statistics* approach based on this assumption.



(a healthy one) in its employments tends to maintain some kind of stability ZEEMAN (1961), equilibrium GEORGE (1973), homeostasis ASHBY (1956), BEER (1972) of thoughts\*. We generally assume that the brain of an intelligent organism has some sort of tendency to choose the quickest way to solve, get around problems, via intuition, common sense and perhaps other means. A detailed account of various brain's issues, is given in GEORGE (1973). We are not further concerned here with them for they are beyond the scope of our thesis.

To substantiate the argument which follows from question 1, we assume that one of the human aims, during the course of carrying out intellectual activities (even everyday ones) is 'the unification and effective utilization of (relevant to...) thoughts' ( $\equiv$  Cybernetics of thoughts) and that is another interpretation of the term recently introduced and which we label *skeptonomy* (or *skeptonomics*).

Besides, another reason which led us to the introduction of this term is the shift from the notion of mechanical/labour work to that of thought and thinking which underlies WIENER's (1961), pp 27, two types of revolutions, nam. the first industrial revolution which was "the devaluation of the human arm by the competition of machinery" and the second/modern industrial revolution which is "similarly bound to devalue the human brain" in WIENER's terms.

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\* See also "conceptualization of learning as the stable growth of wisdom" by KOCHEN.



2.4.4.2 Question 2: How can such shifting be described (represented) and synthesized (constructed)\*?\*\*\*

The first part of the question may be faced in a two-fold manner, nam, using the same language as the one for representation or using some kind of metalanguage. The latter alternative may be taken as a necessity due to the fact that, dealing with shifting between representations, it is required to talk about entities that do not belong to the representations themselves but it is needed to refer to something in between them. This step, that is from representations to the shifting between them, bears some similarities, loosely speaking, to the step of going from the Propositional Calculus to the Functional Calculus, where constants are replaced by variables and function symbols are introduced.

As far as the second part of the question is concerned, the constructive one so to speak, one is forced to use control concepts. That is, a control/command language is needed to direct the decision making at the 'shifting level'.

The above differentiation of the question in two parts is associated with or, leads naturally to, the current debate in A.I.; that is, whether to separate or not the control language from the representation language, KOWALSKI (1974). The question is also related to the so-called, in the A.I. community, declarative-procedural controversy.

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\* Constructions make use of definitions/axioms/models.

\*\* Carried out/executed/demonstrated/proved.

The methodology we employ to cope with and facilitate such description-construction pair is based on the very notion of analogy; namely, by exploiting a kind of similarity, precisely defined in the next chapter, between representations labelled under the Skeleton name (to be also analysed in the next chapter) and represented with more or less the same means as the parent representations.

The (problem) domains we are dealing with or getting examples from, throughout the work, are mainly the following:

Geometric Structures,

Scenes,

Problem situations,

Conceptual Universes (semantic Nets, Relational Structures).

Question 3: What kind of processes are involved in shifts between representations?

Originally, this question was put forward by AMAREL (1970), pp. 215. Since one of our tendencies is to use a high degree of redundancy, then some processes we can realize, at the moment, are of the nature of eliminating objects (and/or relations). That is, concepts in the case of conceptual Universes; (problem) situations in problem spaces; and (physical) objects in the case of geometric structures or scenes.

The above attitude naturally leads to the notion of:

1. reduction of representation's complexity; AMAREL (1967),
  2. sub-universes; BARNDEN (1975),
  3. sub-spaces; NILSSON (1971), and
- functions (operations) which achieve them.



Our investigations show that, in fact, (as shown in the next chapter) these processes we speak of involve (result on) object/morphism\*-elimination, filtration  $\phi$  (and its variant forms) and amalgamation.

Question 4: How feasible, in a Heuristic-Epistemological sense, is the shifting of representation?

This issue has been discussed by a number of people, McCARTHY-HAYES (1969), SLOMAN (1971), RAPHAEL (1970), SACERDOTI (1974). Most of the question is associated with the well-known (but difficult to cope with) *frame-problem*; RAPHAEL (1970) and other workers discuss it.

An attempt is made here to analyse the feasibility for an Epistemologically-Heuristically (E-H) adequate shift, which is absent from SACERDOTI's (1974) system, as he points out, p. 117. This effort would amount to the discovery of criteria governing Epistemological-Heuristic interactions  $\langle E-H \rangle^{**}$ , e.g. in problem-solving like processes. As such, for example, may be criteria and conditions of disturbing the similarity (symmetry). We should call these disturbances 'noise morphisms, objects'\*\*\* The interesting question here is how can one invoke, detect and cope with such 'disturbance information'. However, one has to be very cautious for , as has been stated (argued on (E-H) adequate shift issue) by some people<sup>\*\*\*</sup>: "there is no way for an adequate representation", BANERJI (1969), p. 167. But, who knows,

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\* Both are the basic constituents of what we call k-structures, defined in the next chapter.

\*\* Notion which is introduced in an earlier section.

\*\*\* "Conflict causing" objects/morphisms.

<sup>\*\*\*</sup> MESAROVIC (1970), p. 174: "there is no best description for a given problem nor there is a best solution process".

at least we cannot prove it, there it might be for shifting between representations!

Question 5: What is a permissible shifting?

A question which rises after a criticism made by DRESHER-HORNSTEIN (1976), that MINSKY (1974) neglects this point in his "theory of frames" when he is talking about "transformations".

Question 6: What are the conditions for shifting?

Question 7: What are the subproblems nested in the shifting of representations problem?

## 2.5 ON OUR ATTITUDES

During the course of developing our ideas, we take the following views:

A. NON-LINGUISTIC FORMS for knowledge representation, PYLYSHYN (1975), WINSTON (1970), (1975), (1977). By this term we mean a representation of a given or wanted situation in which facts, procedures, problems (or whatever) may be involved. The form of representation is not based on grammatic rules, as for example in: First Order Predicate Calculus (FOPC), KOWALSKI (1974); formal languages, ENGELER (1968); natural language, WINOGRAD (1972); and so on. It rather rests upon some sort of diagrammatic\* or network-like type. Here, the dominant

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\* Which is also a kind of language.



features are: nodes and relations between them, or in our actual terms: objects and morphisms.

We think it is worthwhile to put forward the controversial question: "Why is FOPC or any other formal language the appropriate form of knowledge representation, for say, problem-solving?" MESAROVIC (1970); versus: "Do we need images, analogues, diagrams (in general, non-linguistic forms) in representing knowledge?" PYLYSHYN (1975), SLOMAN (1975), FUNT (1976) p. 125. From these questions the latter one implies to ask ourselves the following question: "To analogize or not to analogize?" which we have put forward in a similar manner as PAPAICONOMOY (1975) who asked: "To model or not to model?".

B. NON-LOGICAL reasoning. To that label we assign forms, tools for reasoning other than the traditional formal ones (logical, deductive, etc.), namely, analogical, intuitive, common-sense, use of diagrams and images and so on.

Our main concern is that of analogical mode of non-logical reasoning, similar to the one which has been studied by various workers: EVANS (1963), WINSTON (1970), SLOMAN (1971), (1975), KLING (1972), R. BROWN (1977) and others.

We view the concept of analogy as some kind of heuristic\* aid, in Problem-solving and Learning situations. This attitude towards

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\* Notion which should label the thread used by Theseus to find his way out of the labyrinth and which has been given to him by Ariadni, the 'interface' so to speak, between Theseus and Daidalus, the engineer of King Minos.

the use of analogical forms of reasoning has been partly neglected by A.I. researchers (apart from the above mentioned) in the last twenty years or so, despite a number of suggestions put forward by eminent people in the A.I. field, such as MELTZER (1970)a, MINSKY (1961), AMAREL (1966), FEIGENBAUM (1968), and the urgent suggestion made by BLEDSOE (1977) in his recent paper on what he calls non-resolution theorem-proving.

Recently, there has been growing interest in various A.I. centres in the use of Analogy and a number of researches, in different problem-domains, have emerged. We now list some works, in addition to those mentioned above: BLEDSOE (1977), GOLSTEIN-GRIMSON (1977) WINSTON (1977), LENAT (1977), MUNYER (1977), ULRICH-MOLL (1977); and also, KLING (1971), MOORE-NEWELL (1973), MINSKY-PAPERT (1974) REED (1974) et al., HESSE (1963), LEATHERDALE (1974), MINSKY (1974), and PASK (1963).

Finally, we classify two forms of reasoning from 'proof' point of view. Namely:

logical → demonstrative (resolution-like) POLYA (1954),  
vol. 1, p. VI.

non-logical → plausible (non-Resolution-like) e.g. common-sense  
reasoning, McCARTHY (1959), BLEDSOE (1977).

C. NON-NUMERICAL problem solving attitude, in the sense of MESAROVIC (1970).

D. NON-STATISTICAL INDUCTION. Attitude inspired from SIMON-SIKLOSSY (1976).

## 2.6 MOTIVATIONS

Comparisons\* of structures has always been the main methodology of mathematicians for the study of properties of structures and similarities between them. Because the invariant, inherited\*\* features are those which characterize, underlay and govern the nature, behaviour of a structure and suggest reducibility, BARNDEN (1975), and redundancy (problem-reduction, NILSSON (1971)) which are useful in problem-solving\*\*\*.

In cybernetics, the comparison of structures (viewed as systems organizations, automata or whatever), is widely present and becomes the central theme of it; ASHBY (1956), for example, studies isomorphic, homomorphic machines/systems. Besides, the nature underlying WIENER's well known ingenuity, was based on his ability to compare quickly and effectively various problem-areas and to provide a new problem with (at least) one solution.

Comparisons occur, especially in analogical problem-solving, where an interrelation, "interlocking" GEORGE (1976), of the current problematic thesis (structure  $\theta_{\pi}$ ) to a problem-solving methodology, search or selection strategy  $\mu_{\sigma}$ , continuously takes place. This type of 'problematic-thesis to problem-solving methodology' interaction (interrelationship or communication) may be loosely illustrated in a

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\* Generally, various sorts of mapping between structures.

\*\* Also the internal symmetries.

\*\*\* Detection of invariants/equivalences are to be found in TOURLAKIS-MYLOPOULOS (1973), PAVEL (1976).



manner more or less similar to a schema proposed in MELETIS (1975)a, which involves some kind of feedback\*. That schema was a modification of another one suggested by KOWALSKI (1974), including FOPC notion. However, as it recently turned out, feed-forward is another important cybernetic notion which may, somehow, influence the schema. In that case, the computations now involved may be labelled 'look-ahead computations', KOWALSKI (1975).

Returning to an issue presented earlier in this section regarding the analogical problem solving, the following questions arise:

Question 1: Why has analogy been chosen?

Question 2: How does the problem of shifting of (problem) representations fit into cybernetics?

Question 3: Why analogical mode of reasoning?

Question 4: How does the analogical mode of reasoning fit into cybernetics?

In the argument which follows, we try to give some kind of simultaneous brief answers to these questions.

Firstly, the mode of problem-solving, based on analogical reasoning, provides a fruitful paradigm, appropriate for elaboration in a cybernetic plateau. The main reason is that analogical problem-solving involves shifting of problem representations, AMAREL (1968),

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\* For an additional characteristic of feedback, not found in classical numerical situation, that is, 'the change of problem descriptions' or shifting, see pp 173, 174 MESAROVIC (1970).

MESAROVIC (1970), NILSSON (1971) p. 35, which is one of the main objectives of the present work. This point may be, somehow, implied from the following beliefs (as fairly general comments) about cybernetics, namely:

1. Epistemologically-Heuristically, cybernetics could be thought of as the (meta-) science\* (art) of study (manipulation) of various kinds of morphisms (relations) between various types of conceptual objects (facts)\*\*.
2. Pragmatics of cybernetics (or what is the use of it). Cybernetics' methodologies can be used as an investigating, research tool for the discovery (generation) of parallelisms among the components of the triple: <machines, brains, society> via the use of some (which one?) mathematical language, in order to establish better and more effective functional operations among the triple's parts.

Secondly, following this two-fold view about Cybernetics, we think that the study of analogical reasoning and the related mechanisms, yielding the creations, realization, recognition\*\*\* of analogies, could leads us, hopefully to the correct way of achieving, exhibiting\*\*, attaining\*\*\* this sort of (Artificial) Intelligence, (key theme, by all means, in cybernetics), which is attributed to creative/productive\*\*\* thinking (why? and how?).

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\* Which cuts accross sciences.

\*\* Other characterisations of cybernetics, found in the literature are scattered throughout the thesis.

\*\*\* MINSKY (1963), PAVEL (1976).

\*\* MINSKY (1966).

\*\*\* MINSKY (1961).

\*\*\* WERTHEIMER (1961).

Thirdly, we quote AMAREL (1969):

*"advances in the general area of modelling by a machine will come from a better understanding of processes of reasoning by analogy".*

Finally, we try to give an answer to the question: Why is it worth solving the problem of 'shifting of representations' via analogy? It is our desire to investigate and experiment mainly with analogizing (intellectual) processes. We believe their study might give some insights and hints towards the answer to the above problem of shifting of representations in problem-solving (and learning) cases, with applications not only for the problem-solving (and learning) mechanization, but also for intrinsic themes of Artificial Intelligence. For example, concept formation/generation/learning, inference making by induction, learning (mathematical) structures, and to 'problematique' of cybernetics, nam. design of intelligence robots, automated management systems\*, kybernetes of complex systems and creative problem-solvers.

## 2.7 OBJECTIVES

1. Lack of a clear mathematical description of analogy structure, neglected in KLING (1971), as he points out in p. 177, led us to the need of introducing and devising such a description (developed in Chapters 3 and 4) using a framework of k-structures (cited in Chapter 3).

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\* PULATOV (1972), GEORGE (1976).



2. Clarification and elaboration of the, in some ways, obscure use of objects and transformations referred to by EVANS (1963), attempt to formalize and elucidate his problem-solver for geometric analogy problems, and study the representation and solution of problems from the same domain using k-structures.

3. Another aim is to systematize some aspects of WINSTON's (1970) work on 'learning structural descriptions by analogy'. His ad-hoc skeleton formation process is replaced by a rationalized one.

4. Investigations on mappings between k-structures in order to throw some light on issues, such as mappings between Conceptual Universes (CUs), between Semantic Nets (SNs) and between CUs and SNs, which are only briefly mentioned in BARNDEN's (1975) work on Conceptual Universes. In addition, to manipulate and implement, though in a different mathematical framework, some of his interesting theoretical results.

5. We make an effort to introduce basic notions of Eilenberg-McLane's Category theory\* as it is described in HU (1965), into cybernetics (and its main branch of A.I.) towards 'standardization of a communication language\*\* among cyberneticians' with the ambition of partial resolving the existing 'conversational chaos'. However, serious attempts to insert Category theory into the closely related area of Computer Science, have already been started with a series called "A junction of Computer Science and Category Theory" by GOGUEN et al. (1973).

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\* ARBIB-MANES (1975): "...it is category theory rather than set theory, that provides the proper setting for the study of foundations of mathematics".

\*\* MESAROVIC (1970), p. 174, 175.

6. Attempt to throw some light on the mechanization of shifting of (problem-) representations, for "*it is strongly relevant to the future progress of intelligent problem-solving systems*", AMAREL (1969), p. 97.

7. Effective handling of the phenomenon of morphism/object elimination towards the resolution of conflict, deadlock and dilemma situations, occurring frequently in decision-making and problem-solving processes.

CHAPTER 3



*"O Aristotle! If you had had the advantage of being 'the freshest modern', instead of the greatest ancient, would you not have mingled your praise of metaphorical speech, as a sign of high intelligence, with a lamentation that intelligence so rarely shows itself in speech without metaphor - that we can so seldom declare what a thing is, except by saying it is something else?"*

GEORGE ELIOT

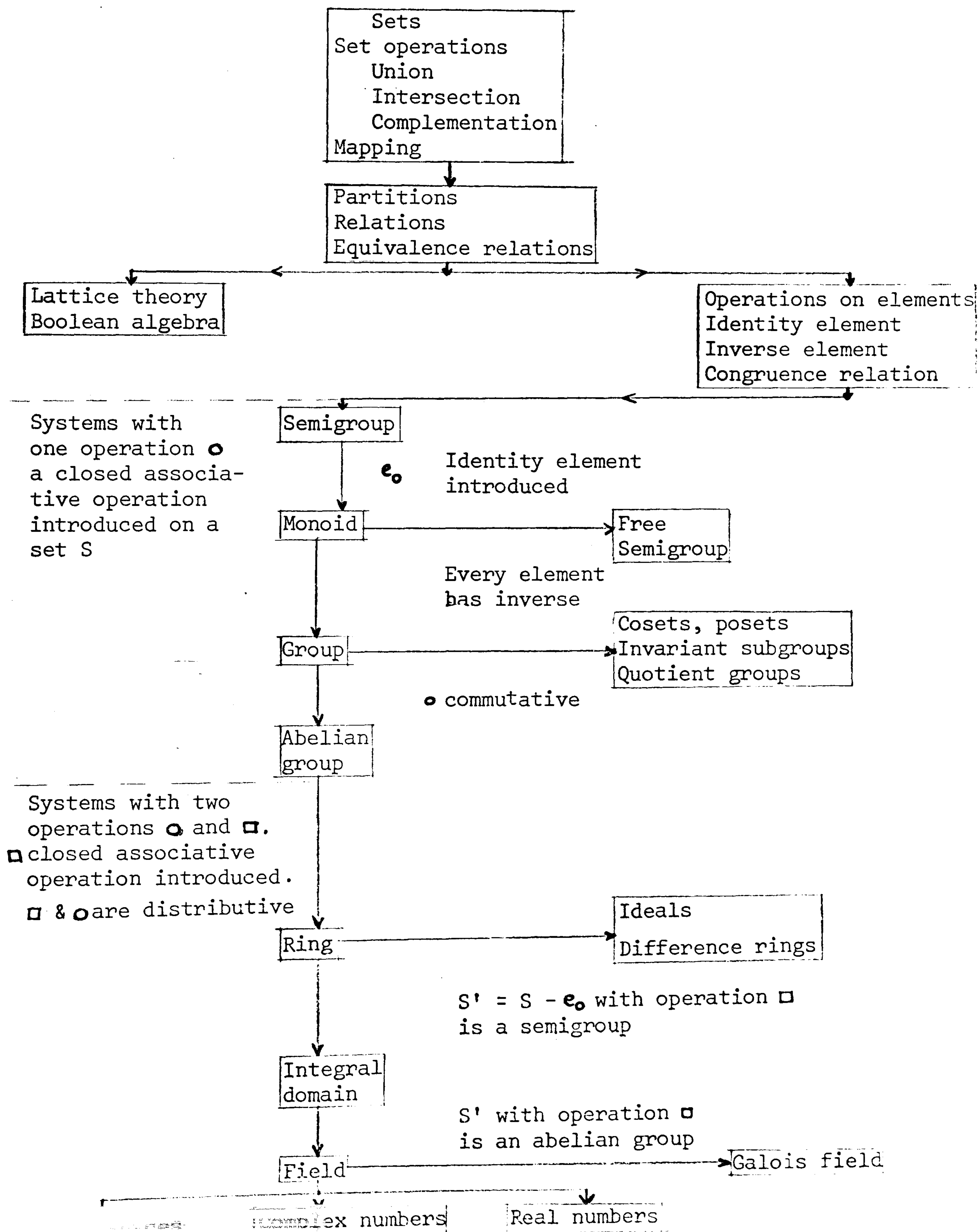
The Mill on the Floss

### 3. METHODOLOGY

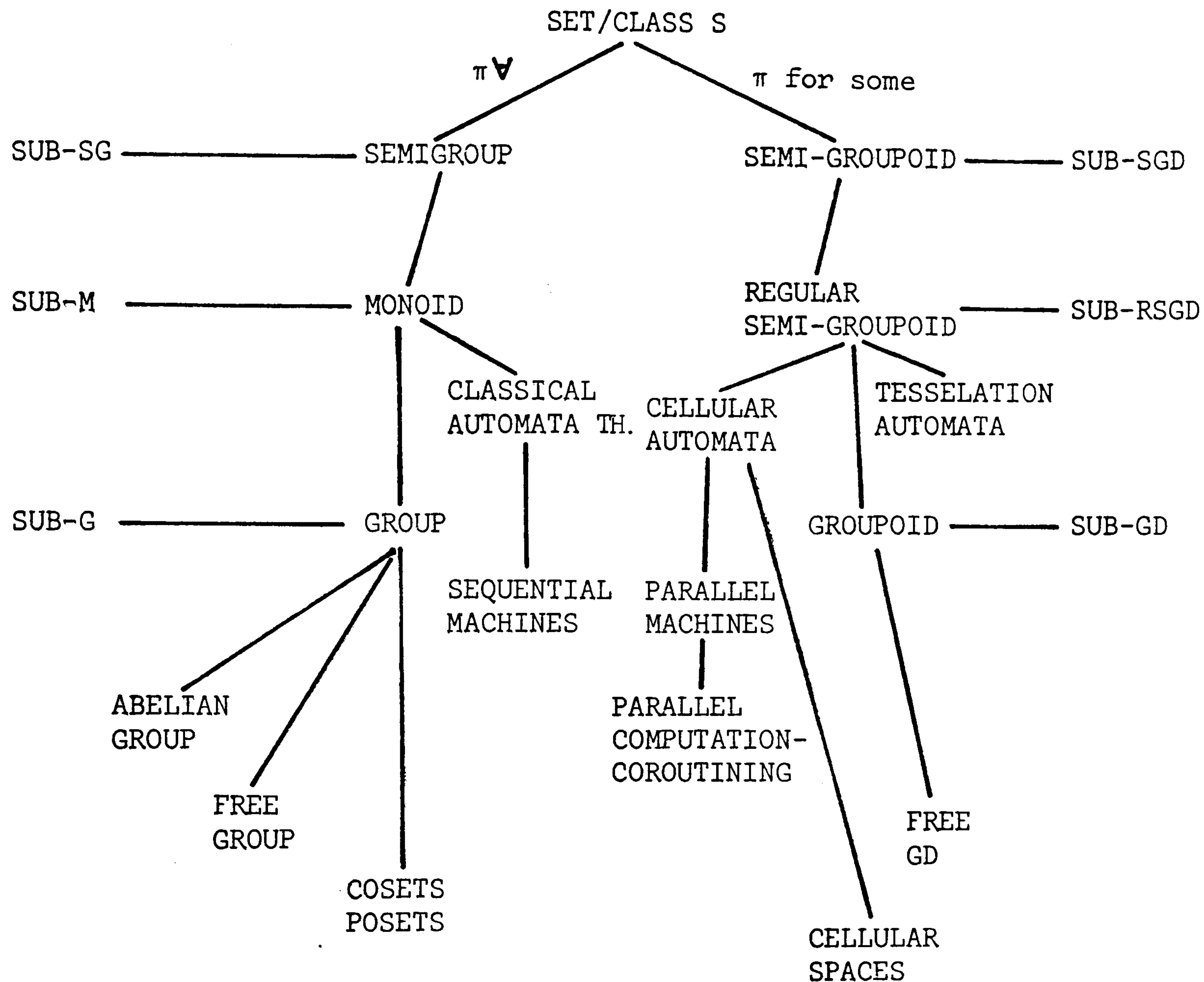
#### 3.1 ON k-STRUCTURES

##### 3.1.1 A LOCALITY OF ALGEBRAIC STRUCTURES (Meletis (1974) p. 152)

Diagram illustrating interrelationships of different algebraic concepts



3.1.2 EXTENDING THE LOCALITY OF ALGEBRAIC STRUCTURES



' $\pi \forall$ ' := operation  $\pi$  is defined  $\forall (x,x) \in S \times S$

' $\pi$  for some' := operation  $\pi$  is defined for some pairs  $(x,x) \in S \times S$



### 3.1.3 ON SEMIGROUPOID\* (SGD)

DEF. 1.- A Semigroupoid (SGD) is a set  $M$  such that for some pairs  $(\alpha, \beta) \in M \times M$  a composition  $\alpha\beta$  is defined which satisfies the following associative properties:

AC1.  $\alpha(\beta\gamma)$  is defined iff  $(\alpha\beta)\gamma$  is defined.

AC2.  $\alpha\beta\gamma$  is defined whenever  $\alpha\beta$  and  $\beta\gamma$  are defined.

The following remark may be considered as *raison d'être* for the SGD notion and the regular SGD which is introduced below.

Remark.- We felt the need to introduce and use a mathematical structure analogous to *monoid* algebraic structure, upon which classical Automata Theory (CAT) rests. This move is due to the fact that CAT have been shown incapable for parallel computation/processing\*\*. We have the intuitive feeling that SGD and category theory are quite promising.\*\*\* Noticeable is also that in a SGD, a relaxation is introduced in the range of composition.

DEF. 2.- Let  $M$  a SGD,  $\alpha \in M$ ,  $\beta \in M$ .

$\xi$  is an *identity* of  $M$  iff  $\xi\alpha = \alpha$  and  $\beta\xi = \beta$ .

DEF. 3.- A SGD  $M$  is *regular* (RSGD) iff  $(\forall \alpha \in M) (\exists$  identities  $\xi, \eta \in M)$  such that  $\xi\alpha, \alpha\eta$  are defined.

PROP. 1.- In a RSGD  $M$  for  $\alpha \in M \exists_1^{***}$  left identity in  $M$ :  $\lambda(\alpha)$  and  $\exists_1$  right identity in  $M$ :  $\rho(\alpha)$  such that  $\lambda(\alpha)\alpha$  and  $\alpha\rho(\alpha)$  are defined.

DEF. 4.- Let  $J(M) \equiv \{\xi \in M / \xi \text{ identity of a RSG } M\}$

\* Details, proofs and examples are found in the appropriate algebraic literature. HU, BOURBAKI, MITCHEL; and MELETIS (1975)b.

\*\* However we are not dealing with such concepts in the present work.

\*\*\* A number of works could justify it; nam. GOGUEN (1973), (1976); ARBIB-MAINES (1975)a, b.

\*\*\*  $\exists_1$  stands for 'there exist only one'.

COL. 1.-  $\forall \xi \in J(M) \Rightarrow \lambda(\xi) = \xi = \rho(\xi)$ .

COL. 2.- For  $J(M) \Rightarrow \lambda(J(M)) = J(M) = \rho(J(M))$ .

LEMMA I.-  $M$  is RSGD.  $\alpha, \beta \in M$ .

$\alpha\beta$  is defined iff  $\rho(\alpha) = \lambda(\beta)$ .

LEMMA 2.-  $M$  is RSGD.  $\alpha, \beta \in M$ .

If  $\alpha\beta$  is defined then  $\lambda(\alpha\beta) = \lambda(\alpha)$ ,  $\rho(\alpha\beta) = \rho(\beta)$ .

DEF. 5.- An *inverse* of  $\alpha \in M$  is a  $\beta \in M$  :  $\alpha\beta = \lambda(\alpha)$  and  $\beta\alpha = \rho(\alpha)$ .

DEF. 6.- If  $\alpha \in M$  has an inverse then  $\alpha$  is called *invertible*.

DEF. 7. - Let  $INV(M) \equiv \{ \xi \in M / \xi \text{ invertible} \}$ .

LEMMA 3.-  $M$  is RSGD.

$\forall \alpha \in INV(M) \Rightarrow \exists_1 x \in M : x \equiv \alpha^{-1} := \text{'inverse of } \alpha \text{'}$ .

PROP. 2.-  $M$  is RSGD.

$\forall \xi \in J(M) \Rightarrow \xi \in INV(M)$  i.e.  $J(M) \subseteq INV(M) \subseteq M$

Remark.- Generally, a RSGD  $M$  has some elements which are not invertible.

DEF. 8.- A *groupoid* (GD) is a RSGD in which every element is invertible.

### 3.1.4 DEFINITION OF A k-STRUCTURE

DEF. 1.- A  $k$ -structure is a triple  $K = (K, M, I)$ , where  $K$  is a class of elements called *objects* of  $K$ .  $M$  is a class of elements called

*morphisms* of  $K$ , with the additional structure of *regular semigroupoid*

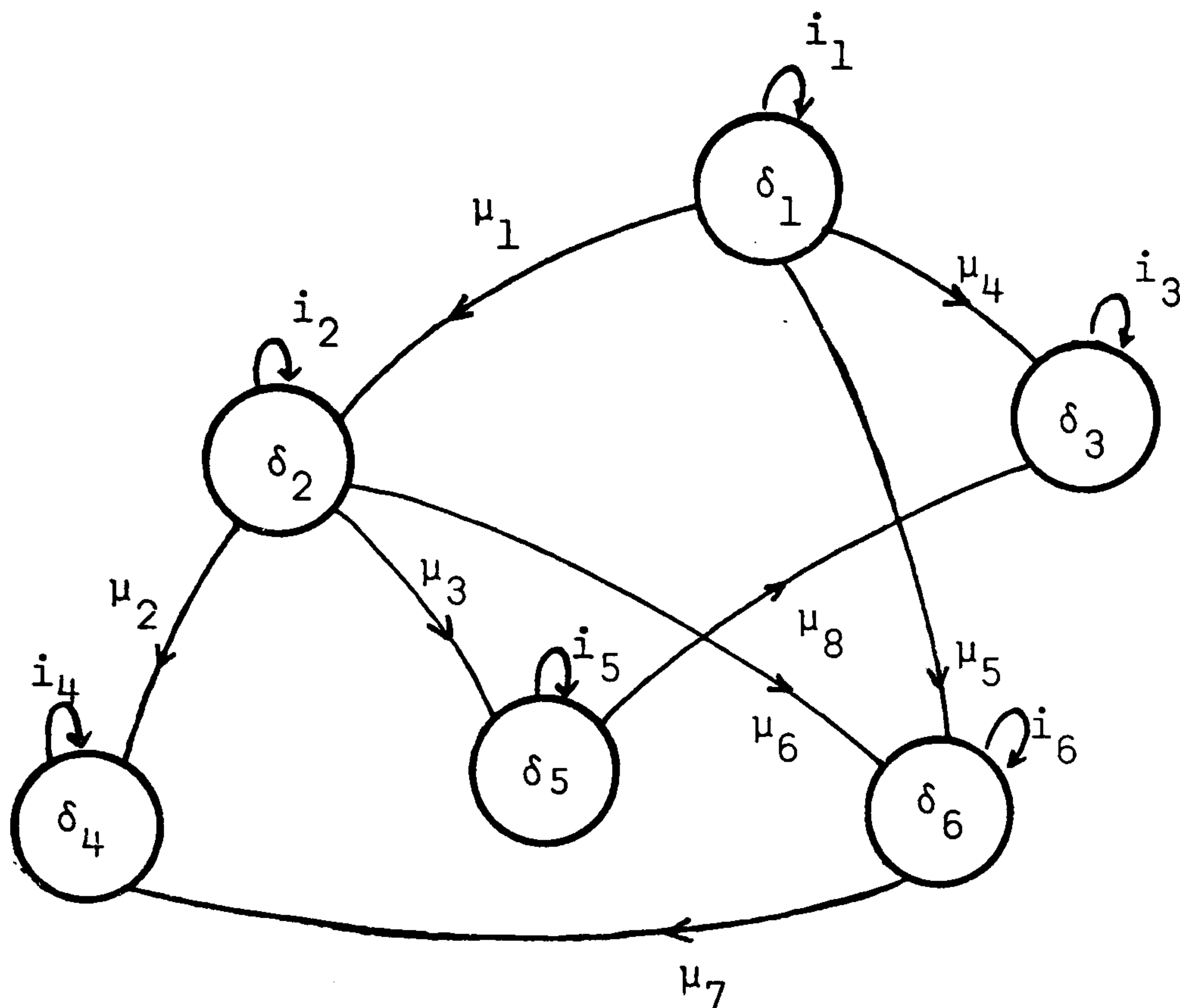
(RSGD).  $I$  is a function of  $K \xrightarrow[\text{on}]{(1-1)} J(M)$

:  $x \mapsto I(x) \equiv i_x := \text{'identity morphism of object } x \text{'}$ \*

\* May be that 'identities form a groupoid (GPD).  $M$  is not a (GPD).

Example.

Let  $K = \{\delta_1, \delta_2, \dots, \delta_6\}$ ,  $M = \{\mu_1, \mu_2, \dots, \mu_8\}$ ,  $I = \{i_1, i_2, \dots, i_6\}$ ,  
 ( $i_1 := i_{\delta_1}$  etc).



SCHEME 1. Representation scheme for a k-structure.

Remark.- We will sometimes omit identity morphisms for the simplicity of the diagram. Thus,  $K = (K, M, I) \equiv (\{\delta_1, \delta_2, \dots, \delta_6\}, \{\mu_1, \mu_2, \dots, \mu_8\}, \{i_1, i_2, \dots, i_6\})$ . We say that:  $(K, M, I)$  is the implicit (or 'in intenso') form, and the r.h.s. is the explicit (or 'in extenso') form, of a k-structure.

DEF. 2.-  $\forall \alpha \in M$  the elements:

$$D(\alpha) \equiv x := i^{-1}(\lambda(\alpha)) \text{ and}$$

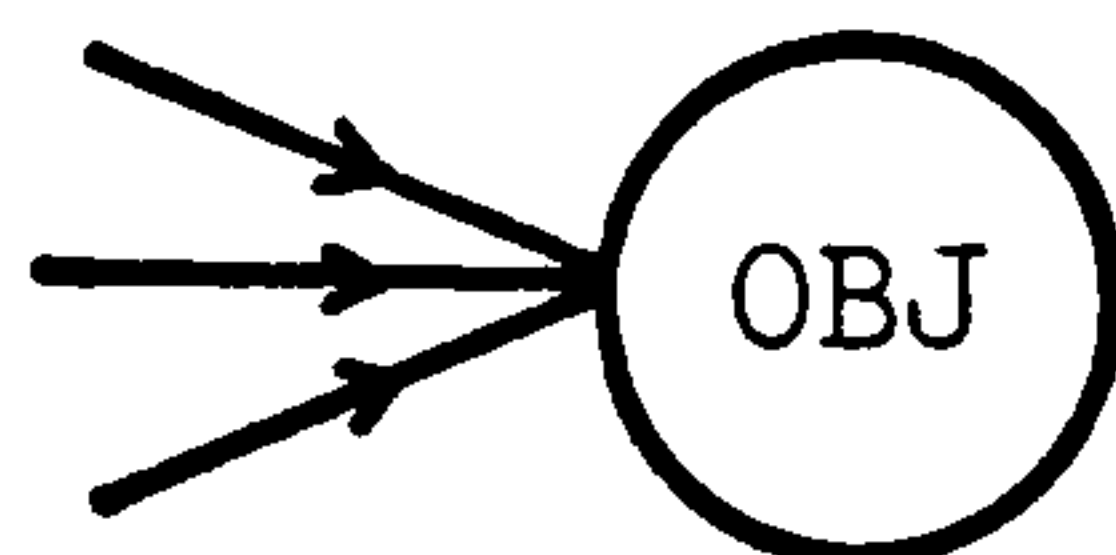
$R(\alpha) \equiv y := i^{-1}(\rho(\alpha))$ , are called *domain* and *range* of morphism  $\alpha$ , resp.



TH.1.- The *product*  $\alpha\beta$  of two morphisms  $\alpha, \beta \in M$  is defined iff  $R(\alpha) = D(\beta)$

Note that:  $R(\alpha) = D(\beta)$  is not always valid for arbitrary SGD. That is the *raison d'être* of imposing a *regular* structure on the SG of morphisms  $M$ .

Remark 1.- In the above example (scheme 1),  $\delta_3, \delta_4$  may be called, according to the current A.I. and cybernetic jargon, *terminal objects*; accordingly,  $\delta_1$  may be designated as *kybernetes objects*. In simple terms, a terminal object is an object from which ~~A~~ departure (genuine) morphism. Diagrammatically



### 3.1.5 REMARKS ON k-STRUCTURES

The following remarks may help for a better understanding and effective utilization of k-structure concept.

1. As far as the *semigroupoid* component of a k-structure is concerned, we have to notice that, the product or composition of any pair of morphisms  $(\mu_1, \mu_2) \in M \times M$ , is not always defined. That is, we can not 'go through' an object (node in the k-structure's scheme) uniquely.
2. The main difference between a *semigroup* (SG) and a *semigroupoid* (SGD) is based on the following fact: the product of a SG is defined for every pair of elements while in a SGD is defined for some elements only. This relaxation (differentiation) illustrates and also embodies the superiority and flexibility of a SGD over a *graph*, say  $G$ , which represents a relation  $R$ , where the product is defined for every pair of elements of a set  $S$ , i.e.  $G \in R := S \times S$ .



3. An interpretation for a  $k$ -structure may be that it represents a superposition of a number of relations.

4. As far as the *regular* components of a  $k$ -structure is concerned, we stress that the assumption of 'regularity' was introduced purely for 'technical' reasons; it is the means to 'pass through' objects (nodes) in a unique manner, i.e.  $\forall \mu \in M \exists_1 \lambda(\mu) \in M$  and  $\exists_1 \rho(\mu) \in M$ ; which says that for every morphism\*  $\mu$ , there uniquely exists a left and right identity morphism such that the products  $\lambda(\mu)\circ\mu$  and  $\mu\circ\rho(\mu)$  are well defined.

5.  $|J(M)| = |K|$  \*\*; i.e. the number of identity morphisms is the same as the number of objects for a  $k$ -structure. This is because  $I:K \longrightarrow J(M)$  is (1-1) and 'on'.

6. The conceptualization and motivation for the schematic representation is borrowed, in some way, from Graph theory and semantic nets. We say, in some way, because the structural assumptions (a  $k$ -structure is based upon) have different character than those of a (directed say) graph.

### 3.1.6 WHY AFTER ALL $k$ -STRUCTURES?

1. To overcome some difficulties which may rise from the use of sets; mind that  $k$ -structure rests upon the notion of class. We are not dealing with contradictions of the set theory for it is beyond the scope of

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\* In the case of identity morphism  $i$ , we have  $\lambda(i) = i = \rho(i)$ .

\*\*  $| |$  denotes the power/cardinality of a set.

the present work. However, we are in favour of "Bernays-Gödel-Von Neuman axiomatics"\* from a number of available axiomatic set theories.

2. We may assume that k-structure constitute a powerful kind of non-numerical mathematics and, "there is no *a priori* reason that they are inferior to other abstract mathematical structure" for problem-solving, for example, as MESAROVIC (1970) points out.

3. We consider k-structures as variants of semantic nets designed to:

α. cater for conflict/deadlock/dilemma resolution in parallel-like processes\*\*,

β. accommodate look-ahead computation\*\*\* and other heuristics,

γ. for communication of (intelligent) parallel automata.\*\*\*

δ. deal with directed nets\* rather than sequences.

4. We have the intuitive feeling that category theory, upon which k-structures rests, is a promising (recent\*\*\* and well established) branch of mathematics, which may suggest 'what a comprehensive theory of human thinking and intelligence looks like' as well as it may help to explain a number of phenomena in human intelligence.

In addition, to the above, one should see the sections dedicated to identity morphisms. Finally, for the usefulness of category theory, see GOGUEN (1973), (1976) and ARBIB-MANES (1975)a, (1975)b.

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\* DUGUNDJI (1966).

\*\* See "parallel realization of systems" based on category theory GOGUEN (1976).

\*\*\* EILENBERG-McLANE (1945).

\*\*\* MITCHIE (1974), KOWALSKI (1975).

\*\*\* PETRI (1965).

### 3.1.7 VIRTUES OF k-STRUCTURES

k-Structures are some kind of Cybernetic language\* for Problem-solving and Skeletization processes. k-Structures offer mathematical treatment(s) which:

1. permits precision;
2. are rigorous;
3. permits generality and unification;
4. allows impossibility and unsatisfiability, eg. undecidability to be shown.

As part of mathematics category theory, which underlies k-structures, has all the above properties; we now turn towards the special nature of category theory itself.

5. Category theory arose as a special part of algebra in response to the need for a conceptual foundation for certain topics in algebraic topology. One way to look at Category theory is as a *language of structure*.
6. A categorical framework provides a more powerful guide to research directions.
7. Being highly abstract, Cat. th. is well suited to eliminate distracting detail from highly structured situations.
8. The abstractness of categorical formulation often permits one to see intriguing similarities between seemingly quite different situations.
9. The general theory of categories is sufficiently well developed to provide rather powerful theorems for use in situations which have

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\* MESAROVIC (1970), PAPAICONOMOU (1975), GOGUEN (1973), (1976);  
ARBIB-MANES (1975)a,b.

\*\* Skeletization is analysed in a subsequent section.



been formulated in its language.

In conclusion, we may say that k-structures could provide a highly abstract framework for a General (mathematical) Systems-theory to rest upon it.

### 3.1.8 ON FUNCTORS\* BETWEEN k-STRUCTURES

Let  $C = \langle K_C, M_C, I_C \rangle$ ,  $D = \langle K_D, M_D, I_D \rangle$  two k-structures. Let  $f$  a function  $F : C \longrightarrow D$  such that:  $f = (f_K, f_M)$ , where

$f_K : K_C \longrightarrow K_D$  i.e. C-objects to D-objects; and

$f_M : M_C \longrightarrow M_D$  i.e. C-morphisms to D-morphisms.

DEF. 1.-  $f : C \longrightarrow D$  is a (covariant) *functor* from C to D iff

CF1 : If  $\alpha : X \longrightarrow Y$  then  $f(\alpha) : f(X) \longrightarrow f(Y)$ .

CF2 :  $f(i_X) = i_{f(X)}$ .

CF3 : if  $\alpha\beta$  is defined then  $f(\alpha\beta) = f(\alpha) f(\beta)$ .

### 3.1.9 ON NATURAL TRANSFORMATIONS\* OF FUNCTORS

Let  $f, g : C \longrightarrow D$  two functors, from C to D.

DEF. 2.- By a *natural transformation* of functor  $f$  into the functor  $g$ , we mean a function  $\phi$  which assigns to each C-object a D-morphism\*\*, i.e.

$\phi : K_C \longrightarrow M_D$   
 $: X \longmapsto \phi(X) : K_D \longrightarrow K_D$ , such that

---

\* Details, examples and interpretations are given in MELETIS (1975)b.

\*\* One may interpret objects (morphisms) as performances (linkages) respectively, which are mentioned in sect. 2.2.2.

$$(NT 1) \quad \forall X \in K_C \Rightarrow \phi(X) : f(X) \rightarrow g(X).$$

$$(NT 2) \quad \forall \alpha \in M_C \Rightarrow \phi(Y) f(\alpha) = g(\alpha) \phi(X);$$

i.e., the following diagram is commutative:

$$\begin{array}{ccc}
 f(X) & \xrightarrow{f(\alpha)} & f(Y) \\
 \downarrow \phi(X) & & \downarrow \phi(Y) \\
 g(X) & \xrightarrow{g(\alpha)} & g(Y)
 \end{array}$$

### 3.1.10 A POSSIBLE CRITICISM\* ON k-THEORY/k-STRUCTURES

One could naturally make some criticisms on k-structures, for example, if objects and morphisms may represent almost anything, then k-theory does say to us nothing at all. Thus, in order for k-theory to pass beyond the realm of metaphor to that of hypothesis, it has to tell us, at least, about three issues:

- A. What sets of possible objects and morphisms are?
- B. What sort of thing transformations among k-structures are ?
- C. What is it that underlies all the areas the theory tries to unify?

Hints to answer the above questions may be found in the following sections, respectively:

- A', 'On objects and morphisms of a k-structure'.
- B', 'On  $\delta$ 's anatomy' and 'on functors and natural transformations',

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\* Similar criticisms are made in DRESHER-HORNSTEIN (1976) on Minsky's quite influential to A.I. community frames theory.

- $C'_1$ . These are various aspects and modes of analogical thinking which are employed or exhibited by humans/machines.
- $C'_2$ . The shifting of representations in learning and problem-solving activities.

### 3.1.11 ON OBJECTS AND MORPHISMS OF A k-STRUCTURE

Objects of a k-structure may represent a number of things; nam. set, class, function, relation, topic, theory, predicate, proposition, word, clause, event, state, scene, situation and so on. Also it may represent some abstract mathematical system/structure; nam. algebraic: that is, group, module, k-structure (!), etc; and topological: that is space, etc. Finally, it may represent a concept; eg. physical objects, qualities, predicates, ways of doing things, moments or intervals of time, locations, distances, numbers, processes, actions, beliefs and so on.

Objects and morphisms are classified as follows:

#### Object types:

01. According to the number of (radiated and/or satellite)\* morphisms as: isolated, terminal, kybernetes, ordinary/typical.
- 02: According to point of view as: global, local.
03. According to topology as: radiated, satellite, centre.

#### Morphism\*\* types:

- M1. According to directionality as: ingoing, outgoing, bidirectional,

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\* Notions which are fully analysed in a subsequent section.

\*\* Sometimes identity morphisms of objects are omitted to make diagrams clear.



non-directed.

- M2. According to point of view or semasiology as: identity, ordinary, global, local, functional.

### 3.1.12 ON IDENTITY MORPHISMS

In the present section, we elaborate, as far as space permits, on nature, role, importance, *raison d'être* and usefulness of identity morphisms of  $k$ -structures.

The *nature* of identity morphisms comes straight forward from the definition of a  $k$ -structure and it may be summarized as follows:

Let  $C = (K, M, I)$  a  $k$ -structure, where the function

$$I : K \xrightarrow[\text{on}]{(1-1)} J(M) \subset M$$

maps  $X \longmapsto I(X) \equiv i_x :=$  "identity morphisms of object  $X$ "

We remind the reader that,

$$J(M) \equiv \{ i_x \in M / i_x : \text{identity morphism of object } X, \forall X \in K \}.$$

As it is stressed in many places throughout the work, morphisms and particularly the identity ones, play the most important *role* in understanding, specification and manipulation of a  $k$ -structure. We are able to see the above made point, in a particular case, that of an organism for example, where relations between subparts of the organism underlay its behaviour; or in the case of an economic organization or an automation and so on. Objects thus, have some sort of secondary role. This is indeed so, for objects can uniquely be determined via the *inverse*  $I^{-1}$  of the identity function  $I$ ; nam.

$$I^{-1} : M \supset J(M) \longrightarrow K$$

$$: i_x \longmapsto I^{-1}(i_x) \equiv X := \text{"object of } k\text{-structure"}.$$

Note that the inverse  $I^{-1}$  does exist and is well defined due to the fact that function  $I : K \longrightarrow J(M) \times M$  is (1-1) and on.

On the *raison d'être* of identity morphisms. Identity morphisms of objects of a  $k$ -structure are introduced for the following reasons:

1. The 'well definedness' in a mathematical sense of the 'morphism component' (i.e. function  $f_M$ ) of a functor  $f = \langle f_K, f_M \rangle$ , which we claim models an analogy between two  $k$ -structures (Universes of Discourse or Conceptual Universes)

$$f : C \longrightarrow D \quad : \quad \begin{array}{l} f_K : K_C \longrightarrow K_D \\ \quad : X \longmapsto f_K(X) \\ f_M : M_C \longrightarrow M_D \\ \quad : \mu \longmapsto f_M(\mu) \end{array}$$

2. To give sense to an 'endomorphism' between a  $k$ -structure and itself, i.e. when  $C \equiv D$ . In this case  $f : C \longrightarrow C$ . This endo-morphism is elaborated in what we call elsewhere (sect. 2.2.3), inter-structure communication, illustrated via shifting-1.

3. For the sake of establishing an 'homomorphism' between  $k$ -structures. In fact,  $D$ 's identity morphisms may serve as  $f_M$ -images for  $C$ 's genuine morphisms between non-isolated  $D$ 's objects. See VOREADOU (1977), BERTZISS (1973).

4. Identity morphisms permit (by virtue of their SGD structure) 'passage via an object' in a 'unique manner' such that the synthesis of morphisms makes sense.

5. Finally, during the course of comparisons (sect. 3.4.3) in skeletization, conflicts may occur; see sect. 3.4. In cases where syntactic or geometric comparisons, carried out for similarity reasons, lead to undecidability or deadlocks, then the objects' additional

qualitative characteristics come into effect\*. Some of those qualities are embodied within identity morphisms.

When we talk in  $k$ -space terms, via  $k$ -opens defined in a subsequent section, then the matrix\*\* element  $X(1,1)$ , which could represent the identity  $i_x$  may be supplied with appropriate information to represent those object's  $X \in K$  qualities, distributed among  $X$ 's adjacent or radiated objects; this is, in fact, a sort of local definition of  $i_x$ , nam. those objects attached to  $X$  relevant to its context.

Furthermore, any matrix element  $X(j,j) := i_{x_j}$ ,  $j \neq 1$ , represents 'the *relative contribution*, from qualitative viewpoint, of  $X_j$  radiated object, to the  $X$ 's quality', or  $X(j,j)$  could be interpreted as "the degree of  $i_{x_j}$ 's membership to  $i_x$ " or "fuzzy value" in ZADEH's (1973) terms; also as 'Local Identity Morphism Emphasis' (LIME) (see Appendix) or "Identity Colour" in STEPHAN-SIEKMAN (1976) terms. The above may be considered as a terminology and a set of concepts for a departure into computational issues on  $k$ -structures.

### 3.1.13 SUMMARY

In the preceding section, we picked up a well known locality of algebraic structure, i.e. Semigroup-Monoid-Group, and we extended it into that of semigroupoid-Regular Semigroupoid-Groupoid by giving briefly basic definitions. Built upon this, as mathematical

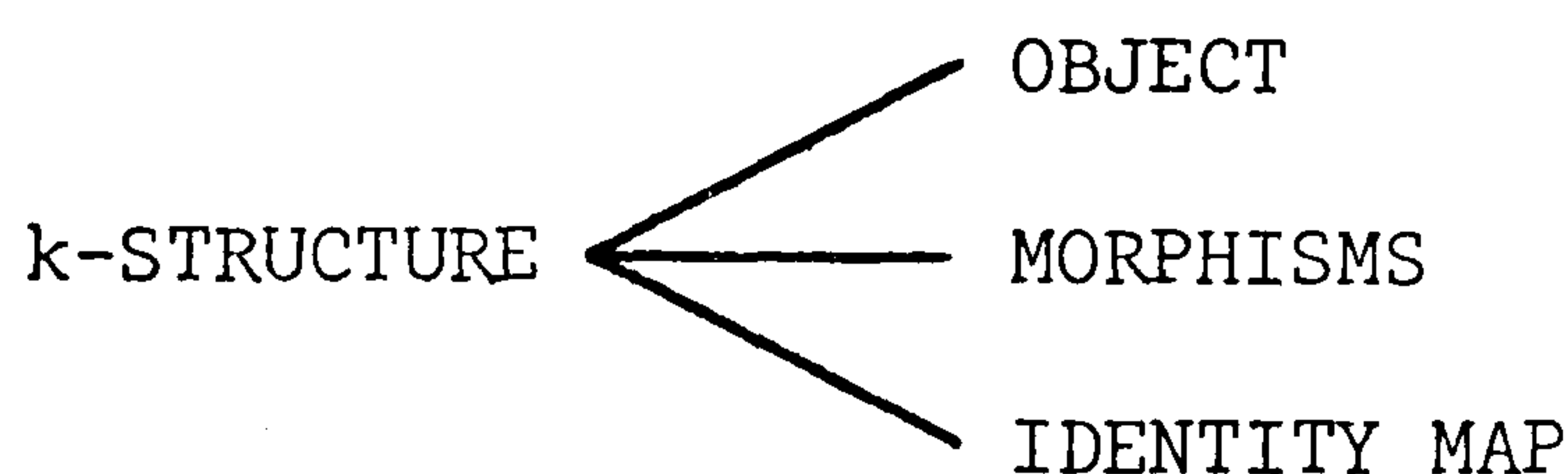
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\* This is made clear in the section 'on conflict resolution'.

\*\* A sort of matrix representation of a  $k$ -structure and  $k$ -open is assumed. More in the Appendix.



background, the notion of  $k$ -structures has been introduced. A few reasons for their introduction were given, followed by some of their virtues. The next important\* issue is that of a functor between  $k$ -structures and of natural transformations between functors. To avoid a reasonable criticism on  $k$ -structures, a clear classification of objects and morphisms, basic ingredients of a  $k$ -structure, is made. Finally, we focus on identity morphisms, a theme which features a  $k$ -structure.



### 3.2 ON $k$ -SPACES

#### 3.2.0. GENERALITIES

As it will be made clear, later on in this chapter, the notion of *shifting of representations* during analogizing intellectual activities\*\* is conceived as a sequence\*\*\* of analogies conceived as transformations or correspondances between skeletons of the previously introduced  $k$ -structures. Special emphasis, therefore, should first be given to skeletons of  $k$ -structures which is the result of *skeletization* or skeleton formation process. Skeletization is based on what we call

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\* For skeletons and shifts of representations.

\*\* e.g. Analogical problem-solving or learning.

\*\*\* Rather a net in a generalized sense, as it is exemplified later on.

*k*-spaces, an important issue in our methodology. Next, *k*-spaces are informally introduced.

### 3.2.1 INFORMAL INTRODUCTION TO *k*-SPACES

Our ultimate aim, before skeletization should take place, is to supply a *k*-structure with some kind of organization\* so that effective utilization of *k*-structure's ingredients could be achieved. To this respect, the organization we supply amounts to the '*separation of the context of a concept\*\* from the concept itself*' technique. This methodology naturally leads to some family or collection of objects and their neighbouring morphisms. We call such a family a *k*-space corresponding to a *k*-structure. We now present a somewhat formal introduction to *k*-spaces. Most of the terminology which follows is borrowed from topology and to a mathematician the process should remind him of a kind of '*topologizing a set*'. We start from the notion of a *k*-open and *k*-spaces.

### 3.2.2 ON *k*-SPACES and *k*-OPENS

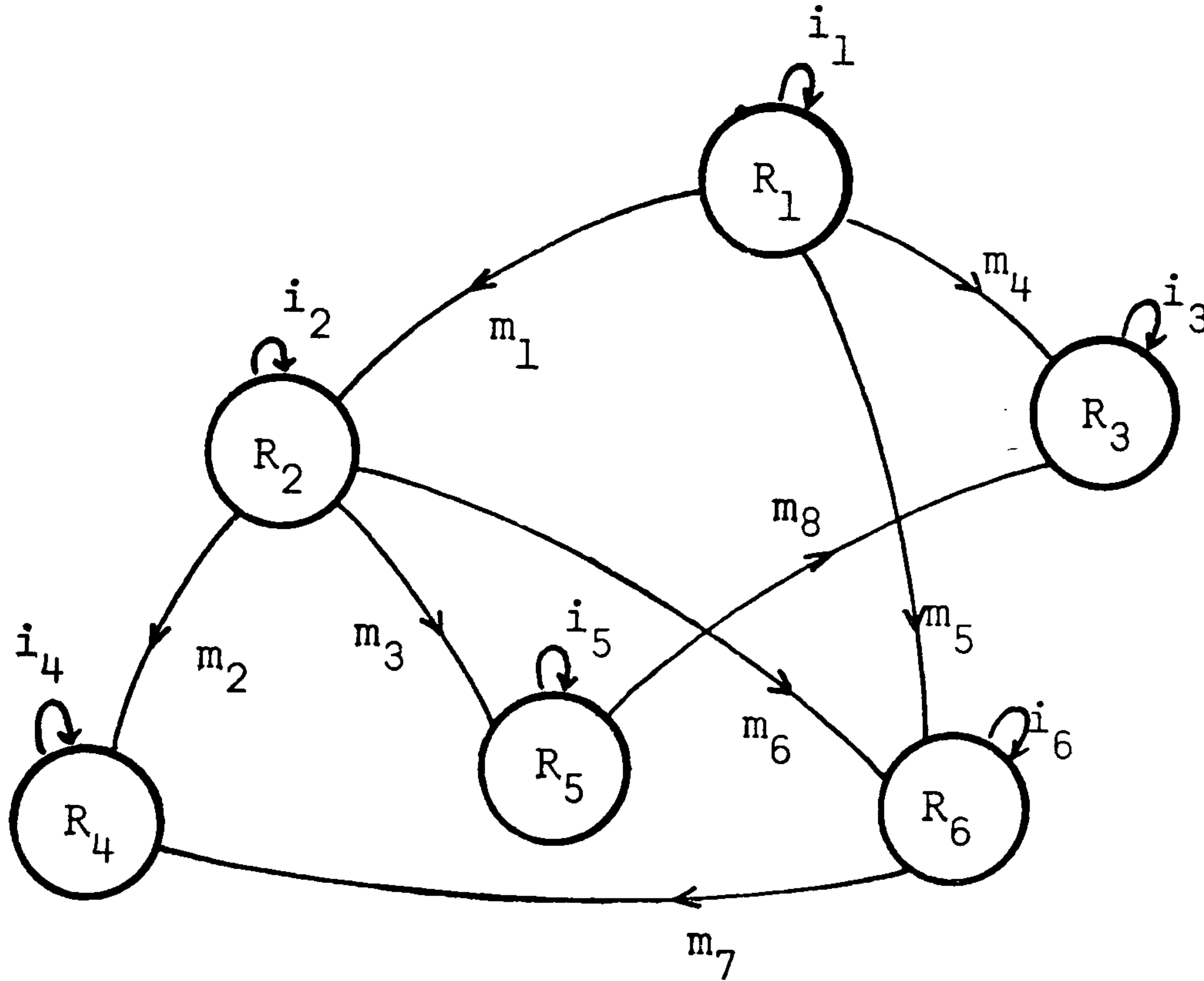
The basic and most fundamental ingredient of a *k*-space is what we call *k*-open. Let a *k*-structure  $K = (K, M, I)$  be the one we used in

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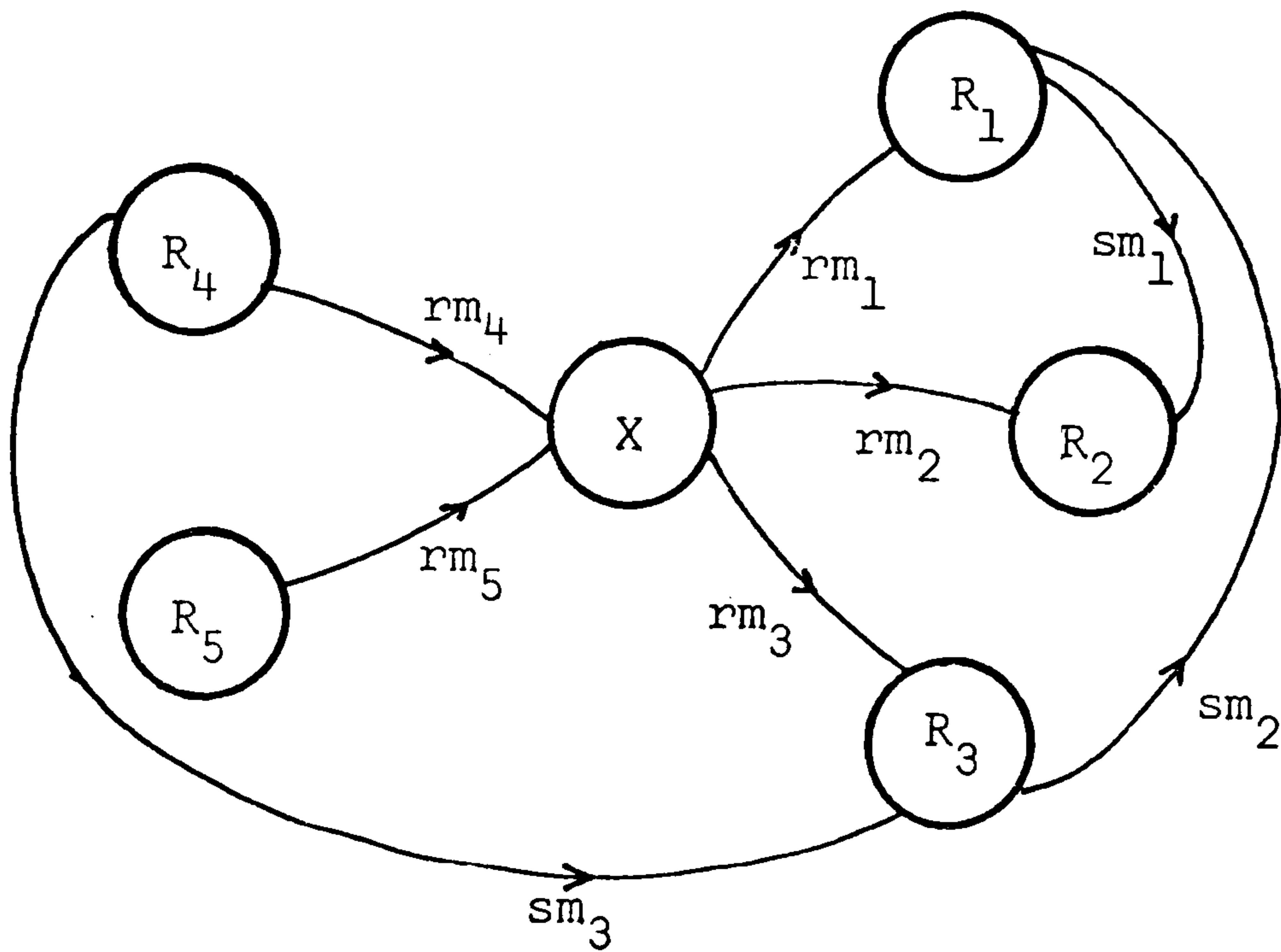
\* Sometimes we refer to it as a '*topology*'; by this term we do not mean it in the strict mathematical sense.

\*\* That is the case when objects of a *k*-structure are concepts.

a previous section; nam.  $K = \{R_1, R_2, \dots, R_n\}$ ,  $M = \{m_1, m_2, \dots, m_m\}$ ,  $I = \{i_1, i_2, \dots, i_n\}$  be its objects, morphisms and identity morphisms resp. In diagrammatic terms ( $n = 6, m = 8$ ) we have the schema 1 shown below.



SCHEMA 1



SCHEMA 2

A morphology for a k-open:  $(S_1, S_2, S_3)$   
 $S_1$  : Centre Object  $X$   
 $S_2$  : Radiated Morphisms and Objects  
 $S_3$  : Satellite Morphisms



### 3.2.2.1 Definition of a k-open

A k-open corresponding to an object  $X \in K$  is defined by a triple  $(S_1, S_2, S_3)$  where:

$S_1$  is the k-open's *Centre or kernel* object  $X$ ,

$S_2$  is the tuple  $(R_0, R_M)$  of *Radiated* arranged Objects and Morphisms (ingoing-outgoing) which are attached (connected) to the centre object,

$S_3$  is the set  $SM$  of *Satellite Morphisms* which link some of the radiated objects.

A possible morphology for a k-open is sketched in schema 2. Notice that in the subsequent diagrams and schemas of this chapter, we omit sometimes the identity morphisms - the illustrations thus becoming rather more clear.

### 3.2.2.2 Definition of a k-space

A *k-space*, for a given k-structure  $K$ , is a collection of k-opens. Examples of k-structure, k-space, k-opens and other notions we refer to in the present section are clearly elaborated in a case study which follows and which treats WINSTON's (1970) Relational Structures from a fresh viewpoint. In the remainder of this section, we examine in detail the concept of a k-open and we start with a few remarks on the triple's  $(S_1, S_2, S_3)$  components.

Remarks.- We are now considering some special cases for the members of the triple  $(S_1, S_2, S_3)$ , such that some meaning may be assigned to the classification schemes we describe in an earlier section entitled 'On objects and morphisms of a k-structure'. In fact, the following

conditions constitute definitions or criteria for the names given to objects and morphisms in that section.

1. If  $RO = \emptyset$ , then the centre object  $X$  is characterized as *isolated*.
2. If the direction of a radiated morphism  $m$  is towards the centre object  $X$ , then it is called *ingoing*.
3. If the direction of a radiated morphism  $m$  is forward from the centre object  $X$ , then the name *outgoing* morphism is assigned to it.
4. If the set of ingoing Radiated Morphisms is empty, i.e.  $IRM = \emptyset$ , then object  $X$  is named as *kybernetes* object.
5. If the set of outgoing Radiated Morphisms is empty, i.e.  $ORM = \emptyset$ , then object  $X$  is called *terminal* object.

### 3.2.3 k-OPEN'S ANALYSIS

A k-open may be analysed from a number of different points of view. We rather choose a picturesque one, shown in schema 3, to emphasize its semasiology and topology. In this context, the representation which is given in that schema reflects two views; nam. a morphism-oriented view and a topological-oriented view.

We next analyse what we mean by T1, T2, T3 shown in schema 3:

Topos T1 : centre object's adjacent objects (syntax)

Topos T2 : centre object's context, expressed mainly via its adjacent morphisms (semantics)

Topos T3 : centre object's environmental\* influence (pragmatics or behaviour area).

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\* To this respect, Topos 3 could also include 'dangling' morphisms emanating from ROs.

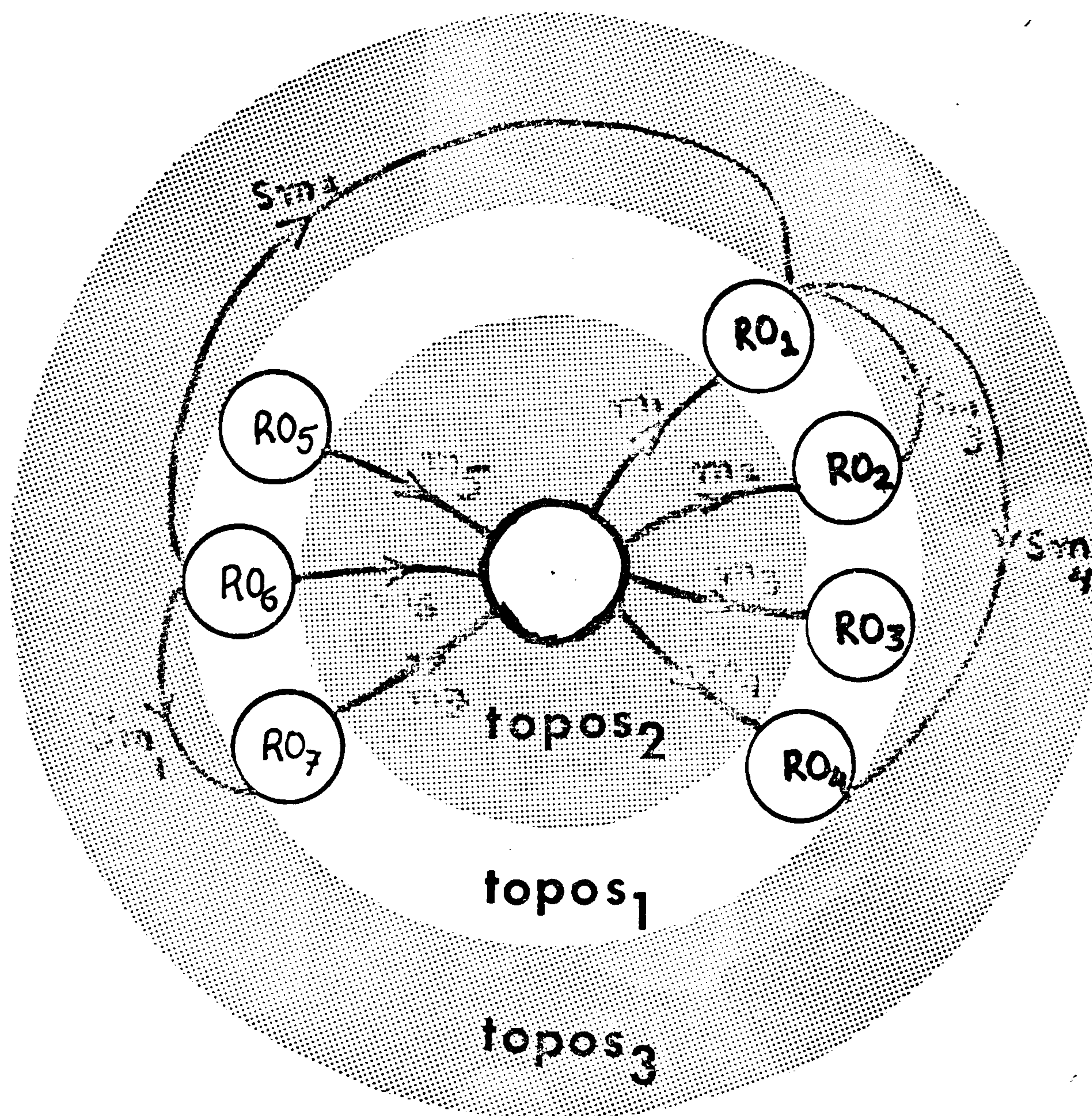


We may thus introduce the following three notions:

$T_1$ -open; eg.  $(X; RO_1, RO_2, \dots, RO_7)$

$T_2$ -open; eg.  $(X; m_1, m_2, \dots, m_7)$

$T_3$ -open; eg.  $(X; sm_1, sm_2, \dots, sm_4)$

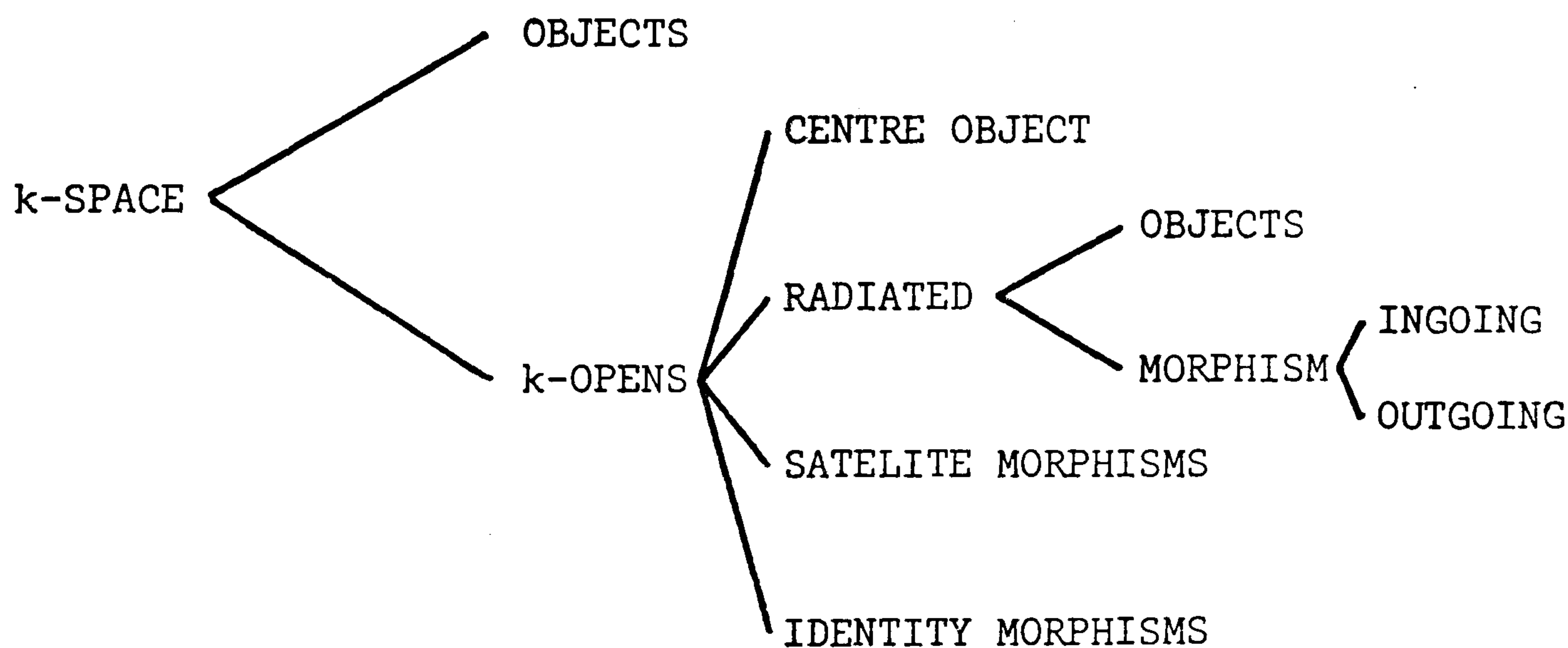


Schema 3



### 3.2.5 SUMMARY

In the preceding section we introduced the notion of a k-space for a given k-structure, as a collection of k-opens. The latter's feature is a triple which captures the (topologically) neighbouring\* objects and morphisms, as well as what we call satellite morphisms; namely the morphisms linking the radiated objects in respect to the central one, X. An analysis also is given which emphasizes the topological features of a k-open. The section ends with a look at a k-open from a number of standpoints; nam. morphism-oriented and topological-oriented. In diagrammatic terms we have:



## 3.3 AN EXAMPLE - CASE STUDY 1

### 3.3.0 INTRODUCTION

Before proceeding further in developing other parts of our theory and methodology in order to tackle questions and problems put forward

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\* To a centre object X which characterizes the k-open in question.

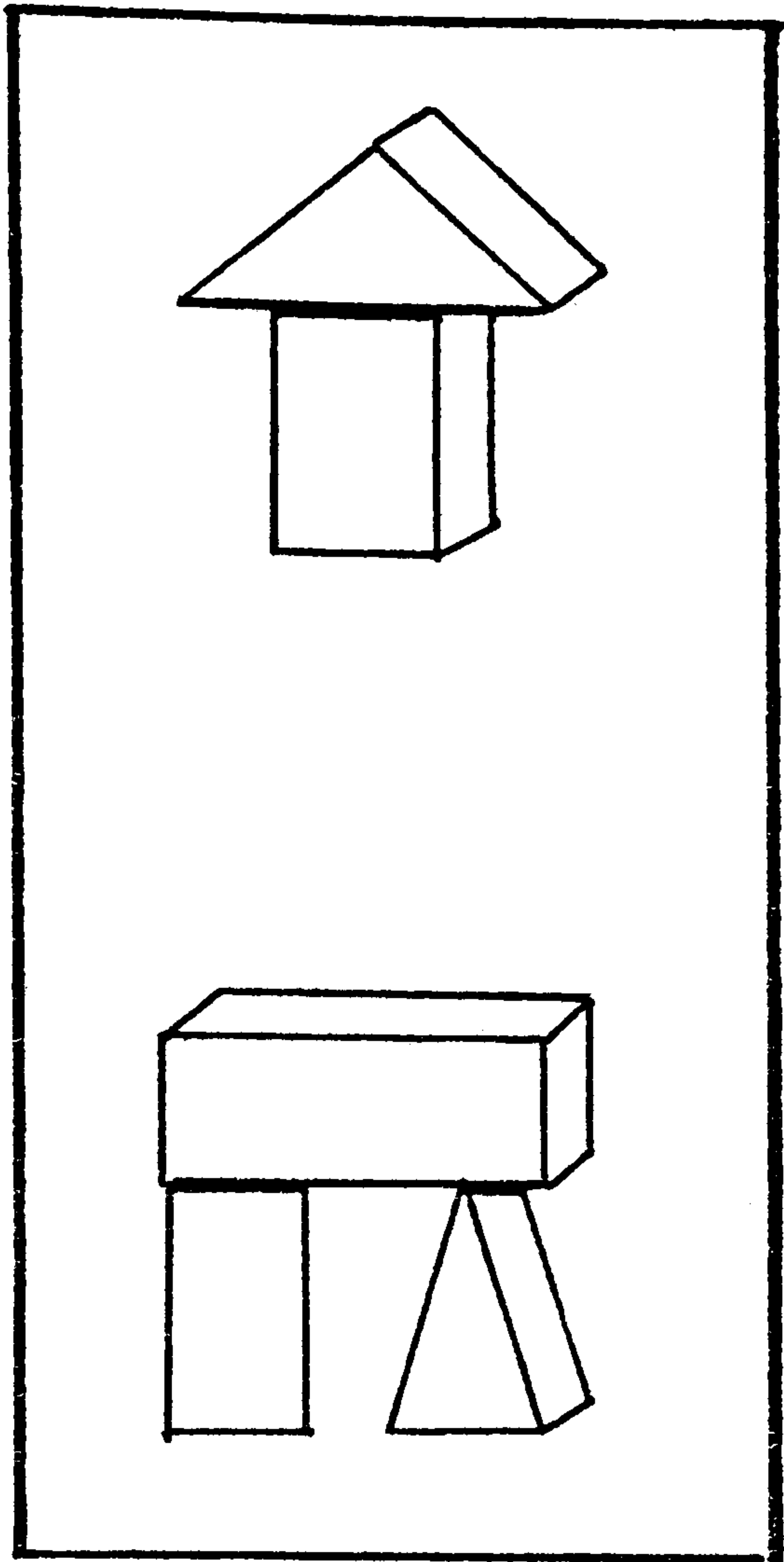


4. Morphisms of MODELS are disjoint from those of K1 or K2.
5. Indefinite objects b and w are given some computer representations.
6. Models for indefinite objects b and w are not identical.
7. A k-structure has only a finite number of objects and morphisms (so-called 'small k-structure').
8. We refer to '*objects*' as 'CONCEPT-names labelling nodes' and to '*morphisms*' as 'ARC-names labelling links between concept names'.
9. K1 or KOSMOS 1 or LEFT are various names given to the same k-structure; likewise, for K2 or KOSMOS 2 or RIGHT.
10. A *context* of an object X is considered as the k-open corresponding to it in the sense of sect. 3.2.

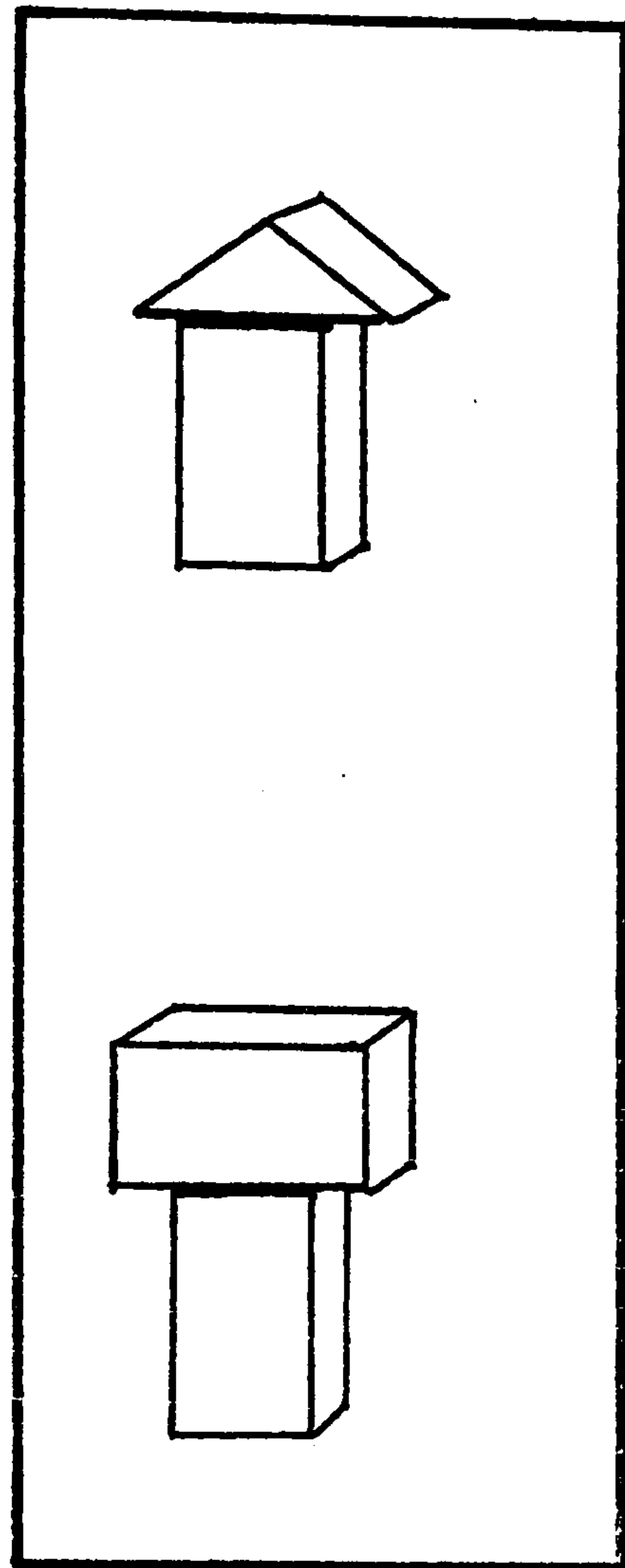
Schema 1 may also be considered as an analogy-type problematic thesis  $\theta$ , in the sense of notions developed in Ch. 2. Scheme 2 gives their representations in k-structure terms; where

OBJECTS	MORPHISMS
l: left object	P: 'part is'
r: right object	S: 'supports'
b: block	K: 'kind of'
w: wedge	H: 'same height'



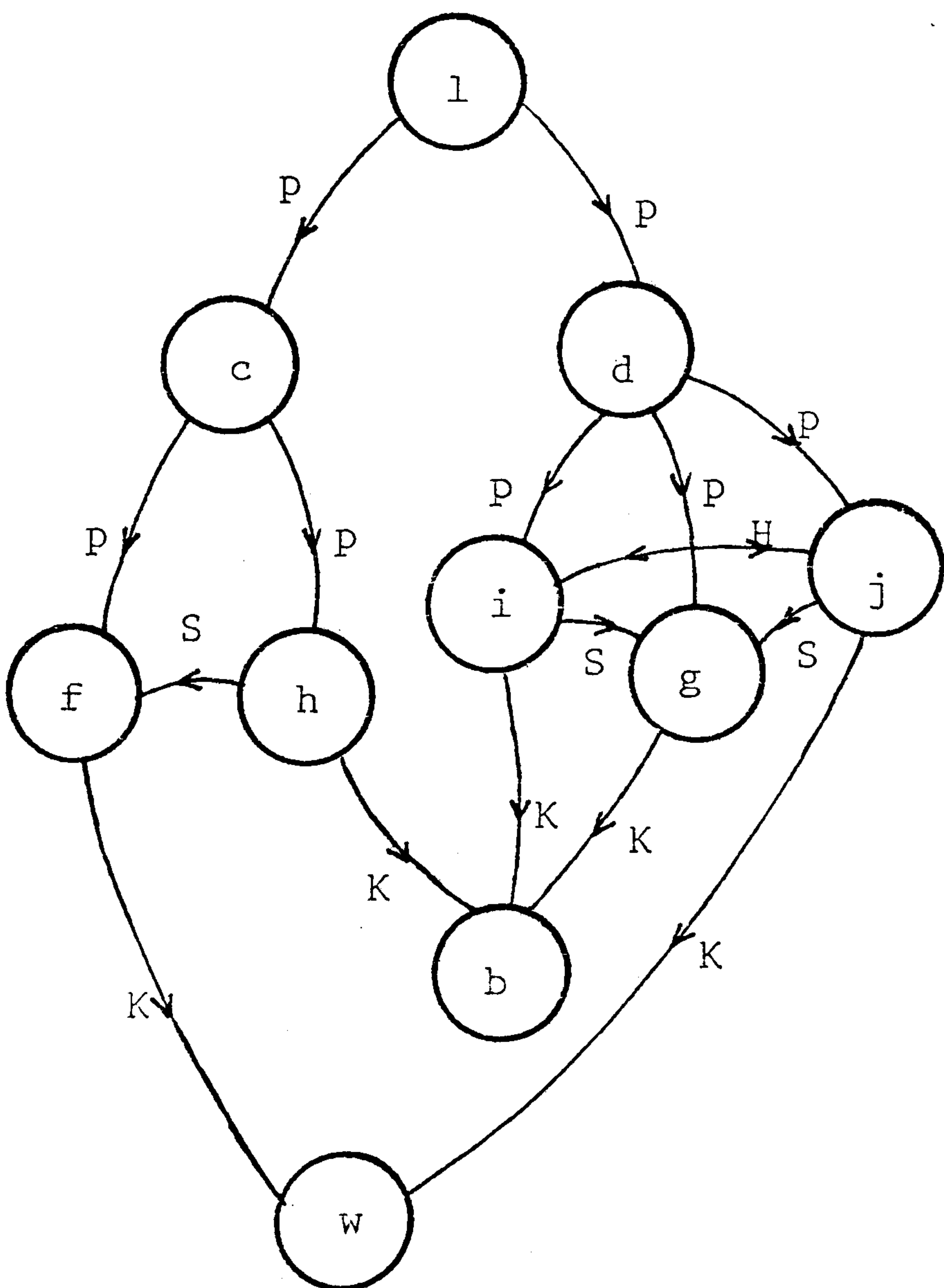


LEFT SCENE

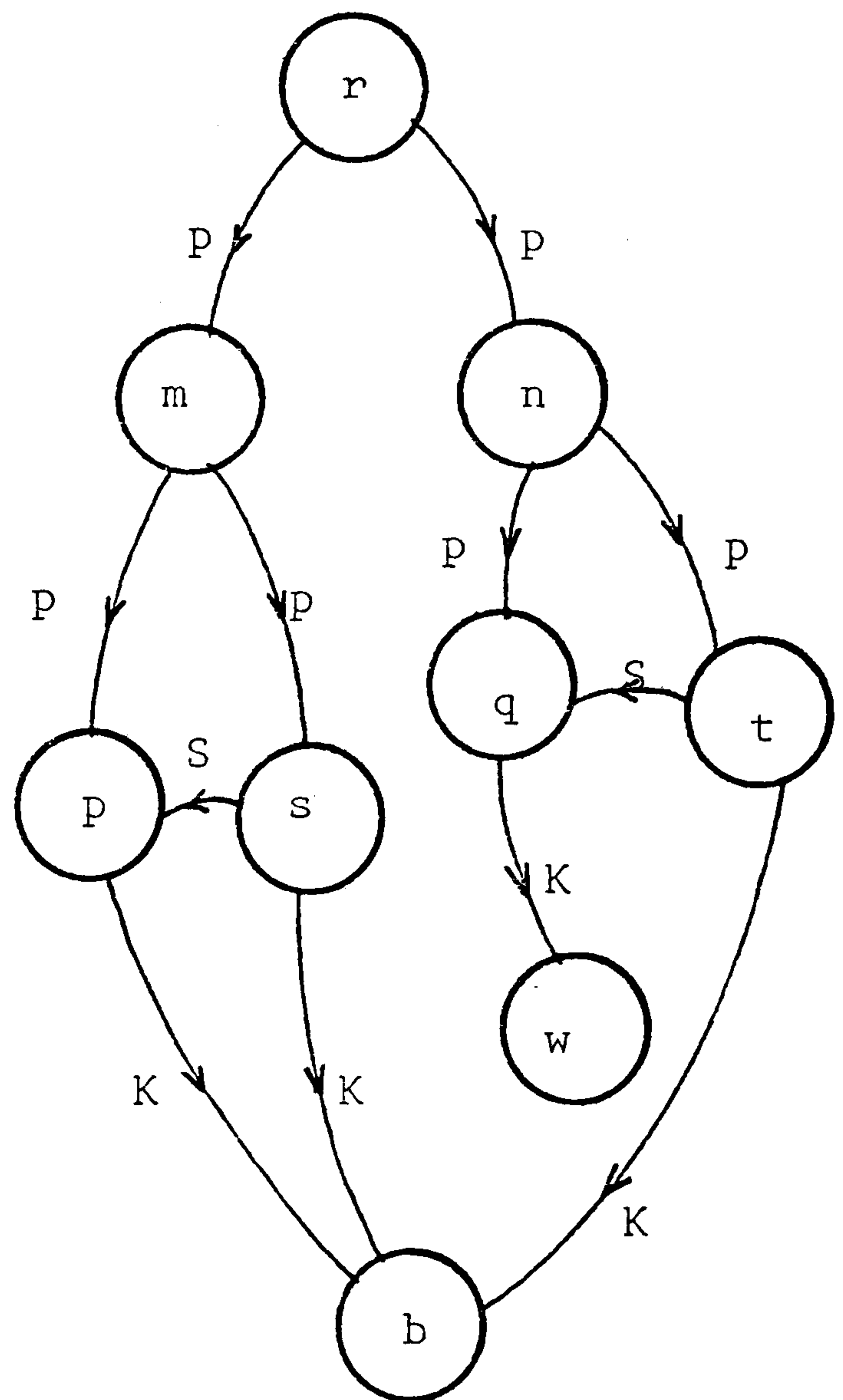


RIGHT SCENE

Schema 1. Winston's Scenes



LEFT STRUCTURE K1



RIGHT STRUCTURE K2

Schema 2. k-Structure representation

### 3.3.2 DEFINITION OF $\pi, \epsilon$ MAPS

The basic idea in the construction of a  $k$ -space is to convert\* LEFT, RIGHT, and MODELS into a single Conceptual Universe with homogeneous constituents, i.e.  $k$ -OPENS.

STEP\_1: Define a map  $\pi$  as follows:

'For every object  $x$ , local or global, in LEFT, RIGHT and MODELS create a new (topological) object  $\pi(x) \in T_X^1$  \*\* i.e.

if LEFT = L =  $\{l, c, d, f, h, i, g, j, w^L, b^L\}$

RIGHT = R =  $\{r, m, n, p, s, q, t, w^R, b^R\}$

MODELS = M =  $\{w, b\}$

X = LURUM

then  $\pi : X \longrightarrow T_X^1$

$: x \longmapsto \pi(x) = \pi_x := k\text{-open for } x$

Let  $T_X^1 = \pi(L) \cup \pi(R) \cup \pi(M)$ ,

where  $\pi(L) = \{\pi(l), \pi(c), \dots, \pi(w^L), \pi(b^L)\}$

$\pi(R) = \{\pi(r), \pi(m), \dots, \pi(w^R), \pi(b^R)\}$

$\pi(M) = \{\pi(w), \pi(b)\}$ , then

STEP\_2: Define a map  $\epsilon$  from  $T_X^1$  into contexts as follows:\*\*\*

For every global/ordinary object  $g$ , set:  $\epsilon(\pi(g)) := \text{'CONTEXT or } g'$

i.e.  $\epsilon : T_X^1 \Big|_{\text{Global}} \longrightarrow C$

$: \pi_g \longmapsto \epsilon(\pi_g) = C_{\pi_g} := \dots$

$T_X^1 \Big|_{\text{Global}} = \{\pi_w^L, \pi_b^L, \pi_w^R, \pi_b^R\} \cup \{\pi_w, \pi_b\}$ .

\* The net effect of this conversion might be thought of, somehow as a manner of 'topologizing X', DUGUNDJI (1966), p. 65.

\*\* In  $T_X^1$  the superscript, nam. 1, signifies some kind of the so-called 'Level-1 environment' in some sense similar to the one KOWALSKI (1975) uses.

\*\*\* Context in the intuitive sense of the term.

For every local/ordinary object, set:

$$\begin{aligned} \varepsilon(\pi_1) &:= \text{'CONTEXT of } l\text{'} \\ \text{i.e. } \varepsilon &: T_X^1 \Big|_{\text{local}} \longrightarrow C \\ &: \pi_1 \longmapsto \varepsilon(\pi_1) = C_{\pi_1} := \dots \end{aligned}$$

### 3.3.3 USING THE SYNTHESIS $\varepsilon \circ \pi$ TO SEPARATE THE CONTEXT OF A CONCEPT FROM THE CONCEPT ITSELF

STEP\_3: Thus the composition  $\varepsilon \circ \pi : X \longrightarrow C$  maps to every object, be it local or global, its context; i.e.

$$\begin{array}{ccc} X & \xrightarrow{\pi} & (X, T_X^1) \\ & \searrow \varepsilon \circ \pi & \downarrow \varepsilon \\ & & C \end{array}$$

The pragmatics of this process is that it provides with a practical distinction: 'Synthesis  $\varepsilon \circ \pi$  separates the context of a concept (object) from the concept itself'.

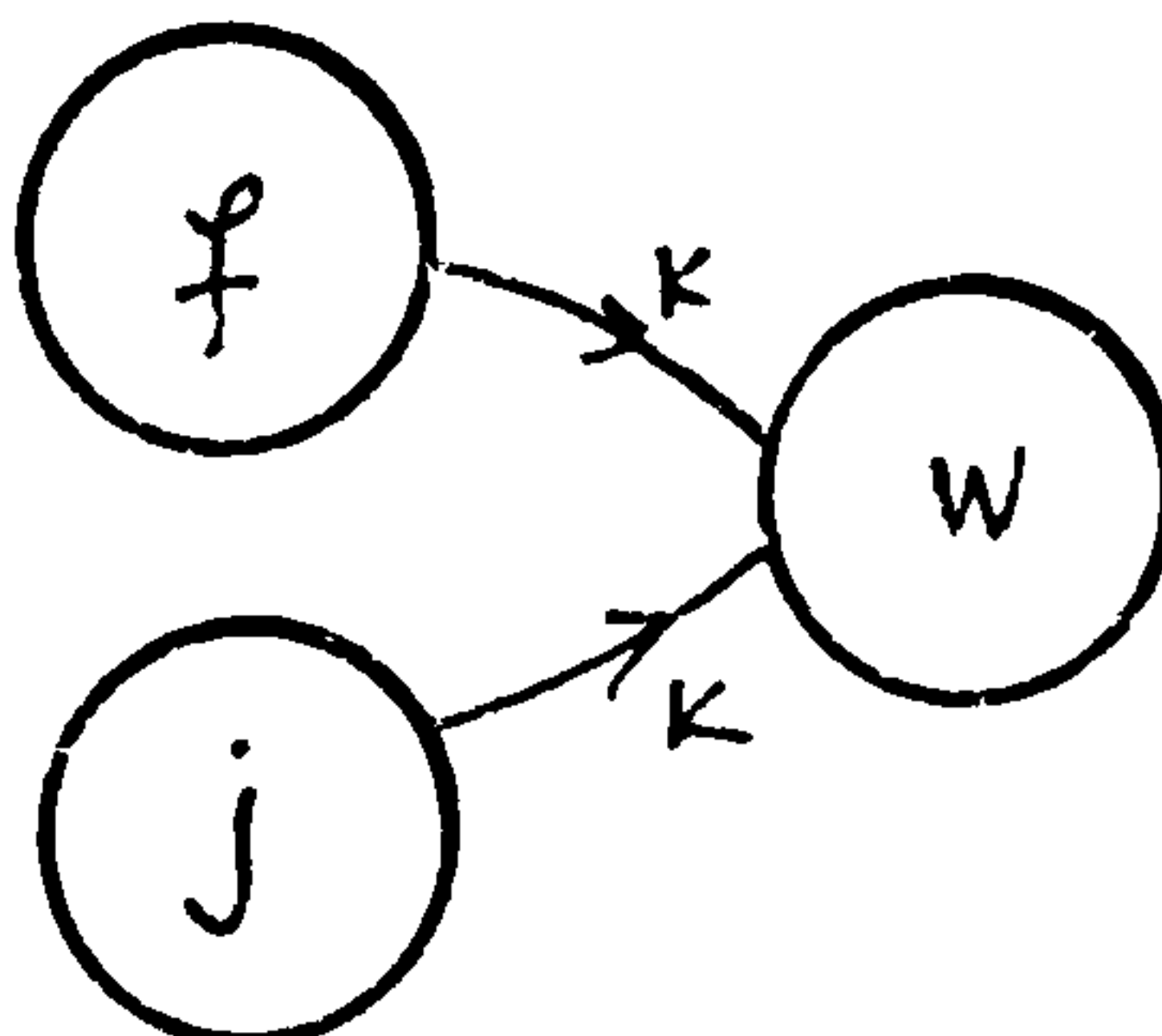
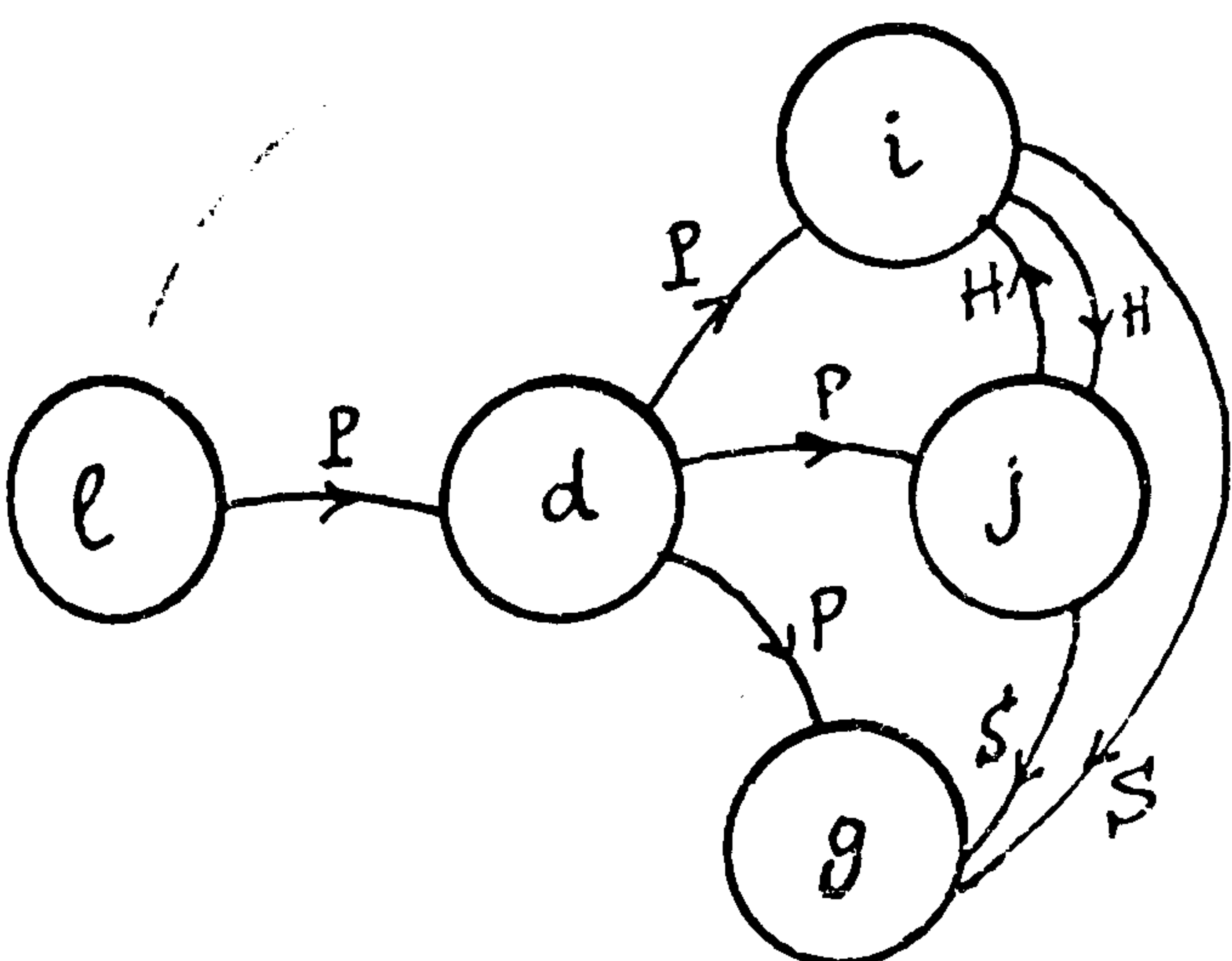
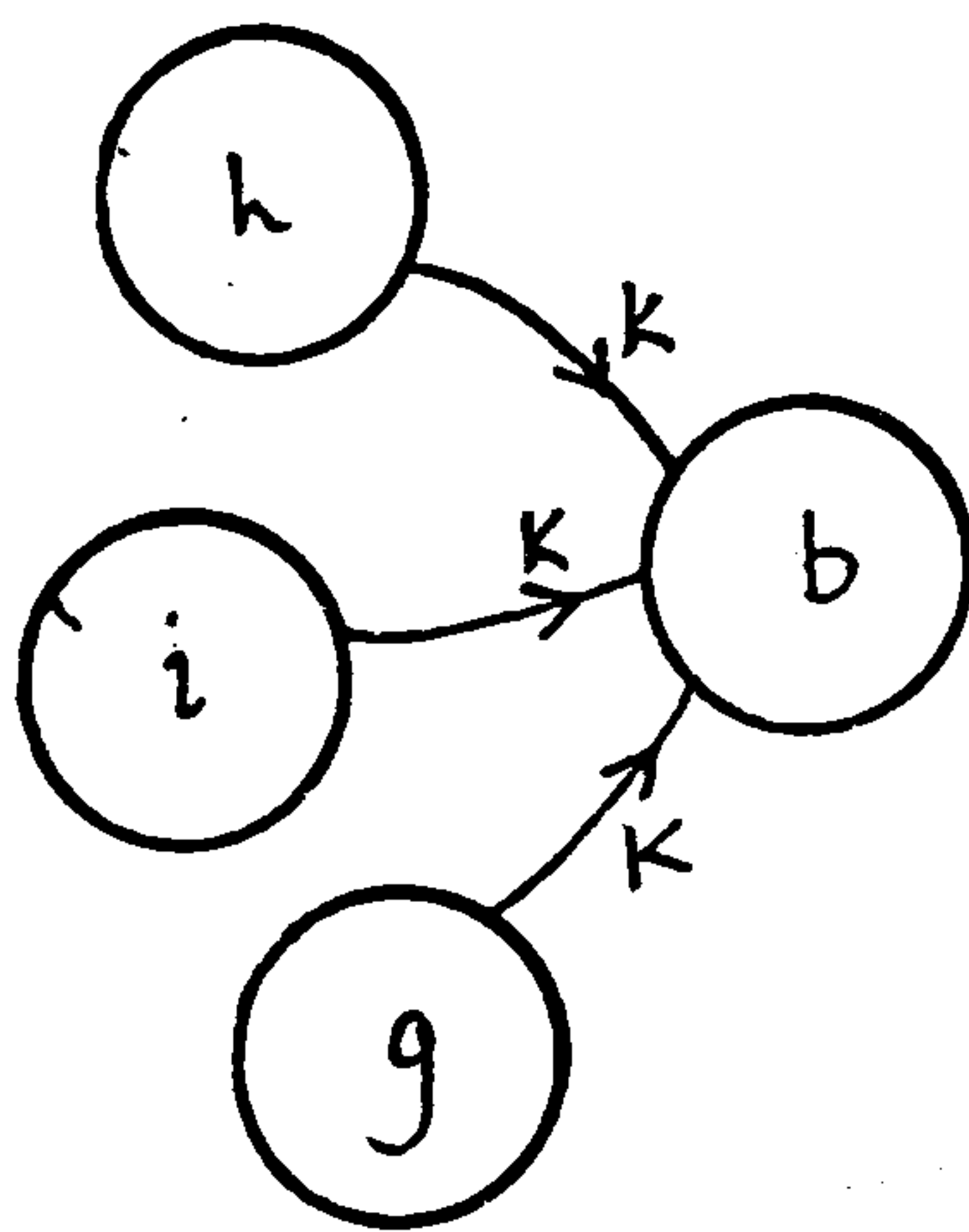
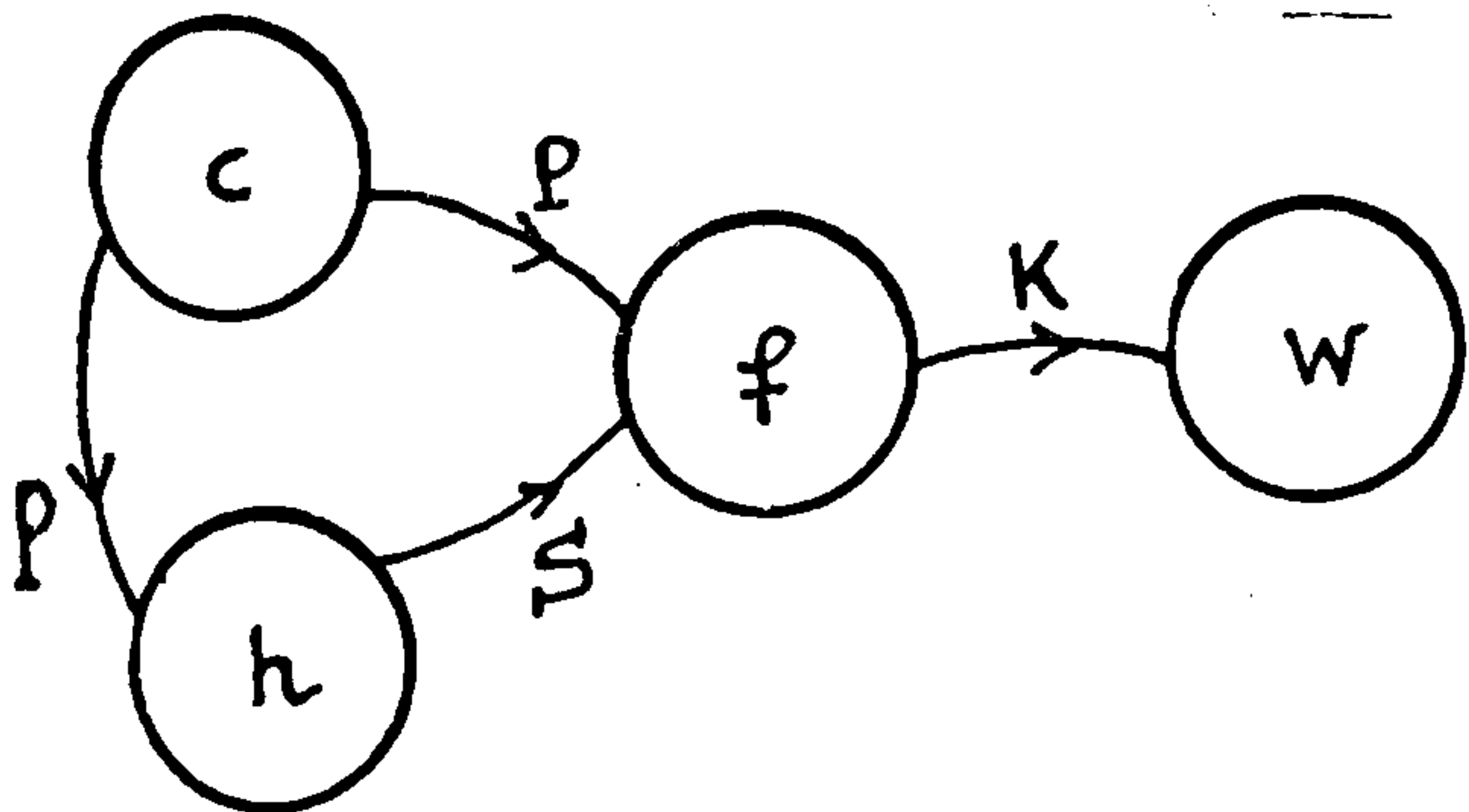
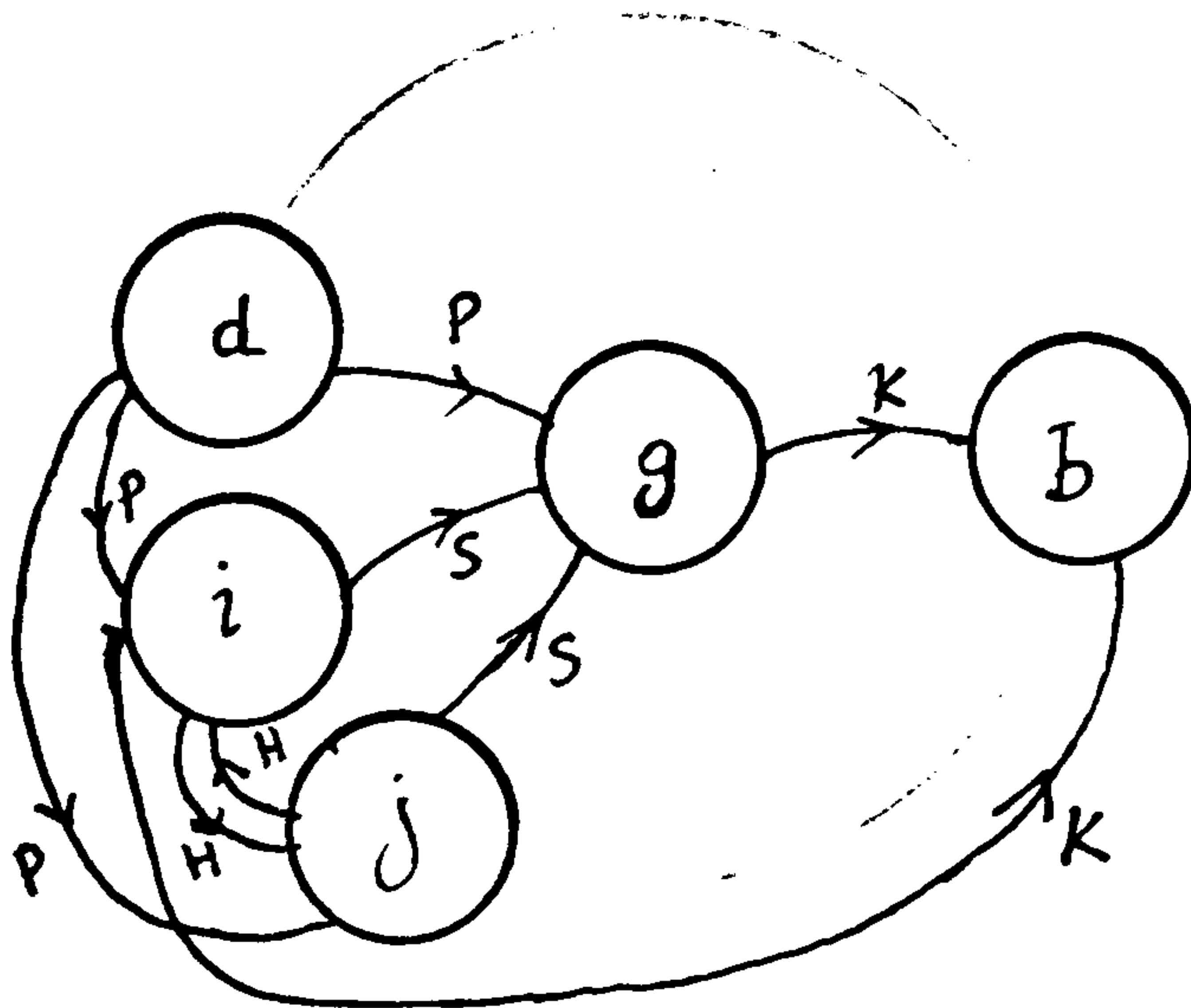
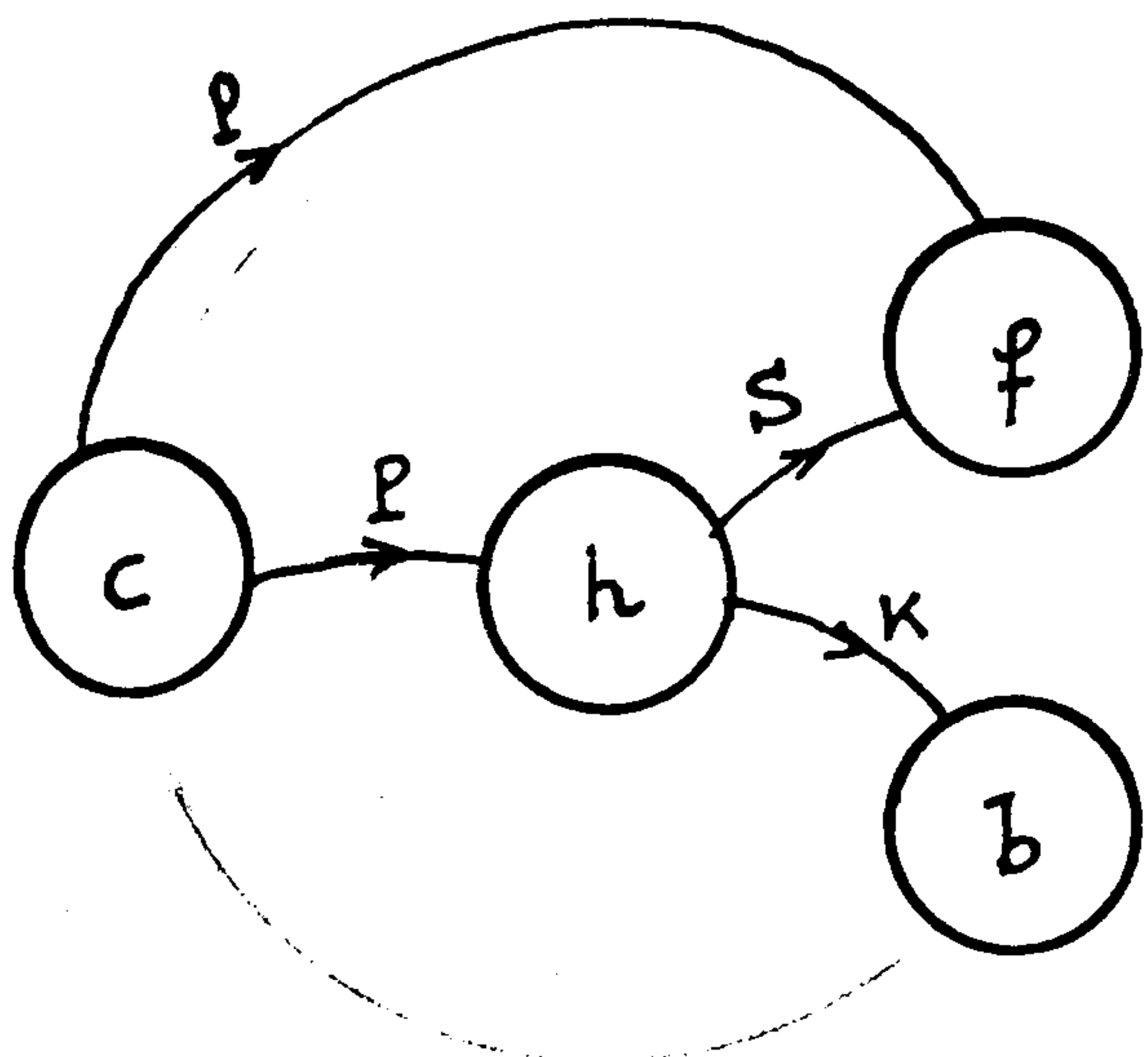
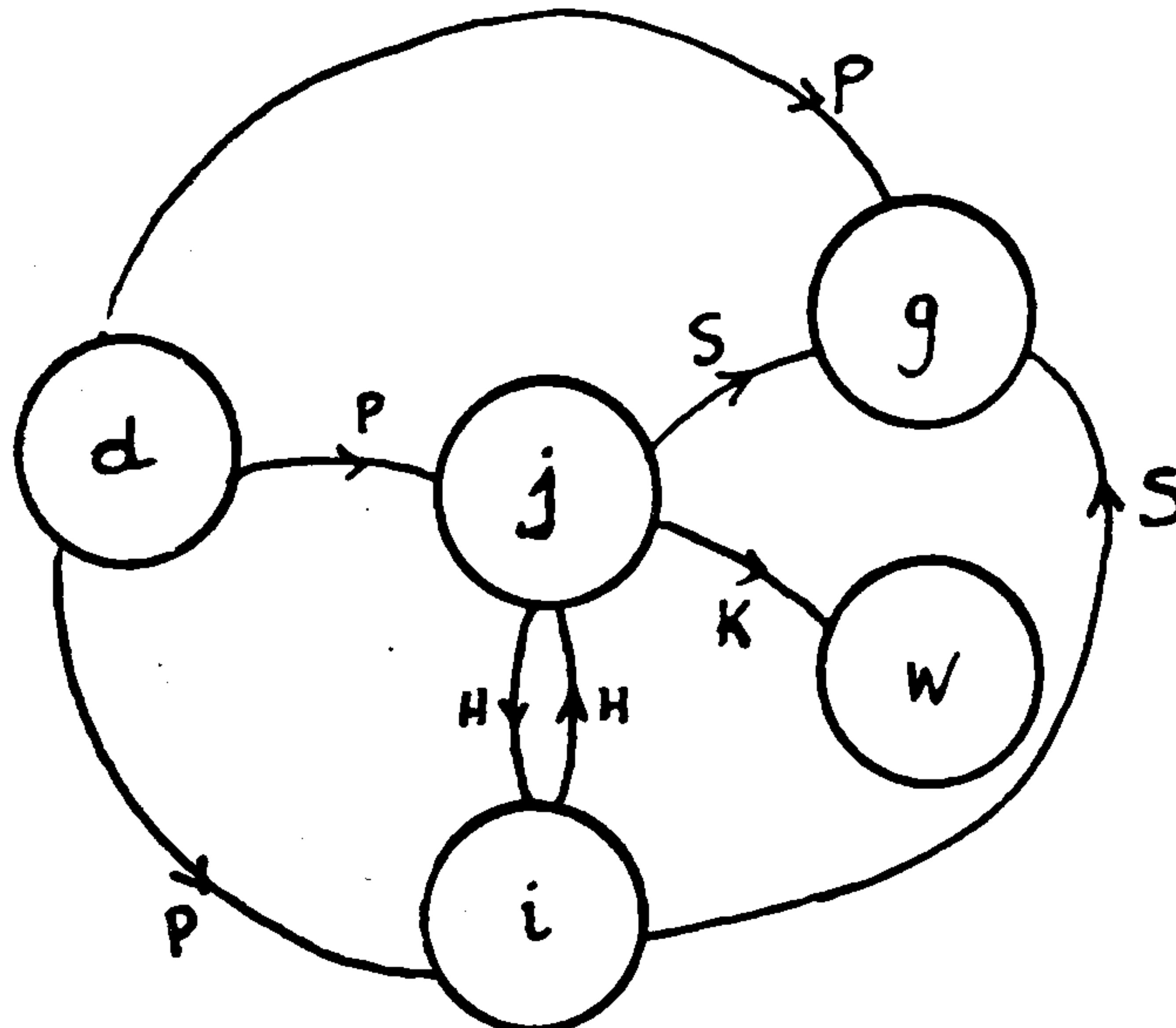
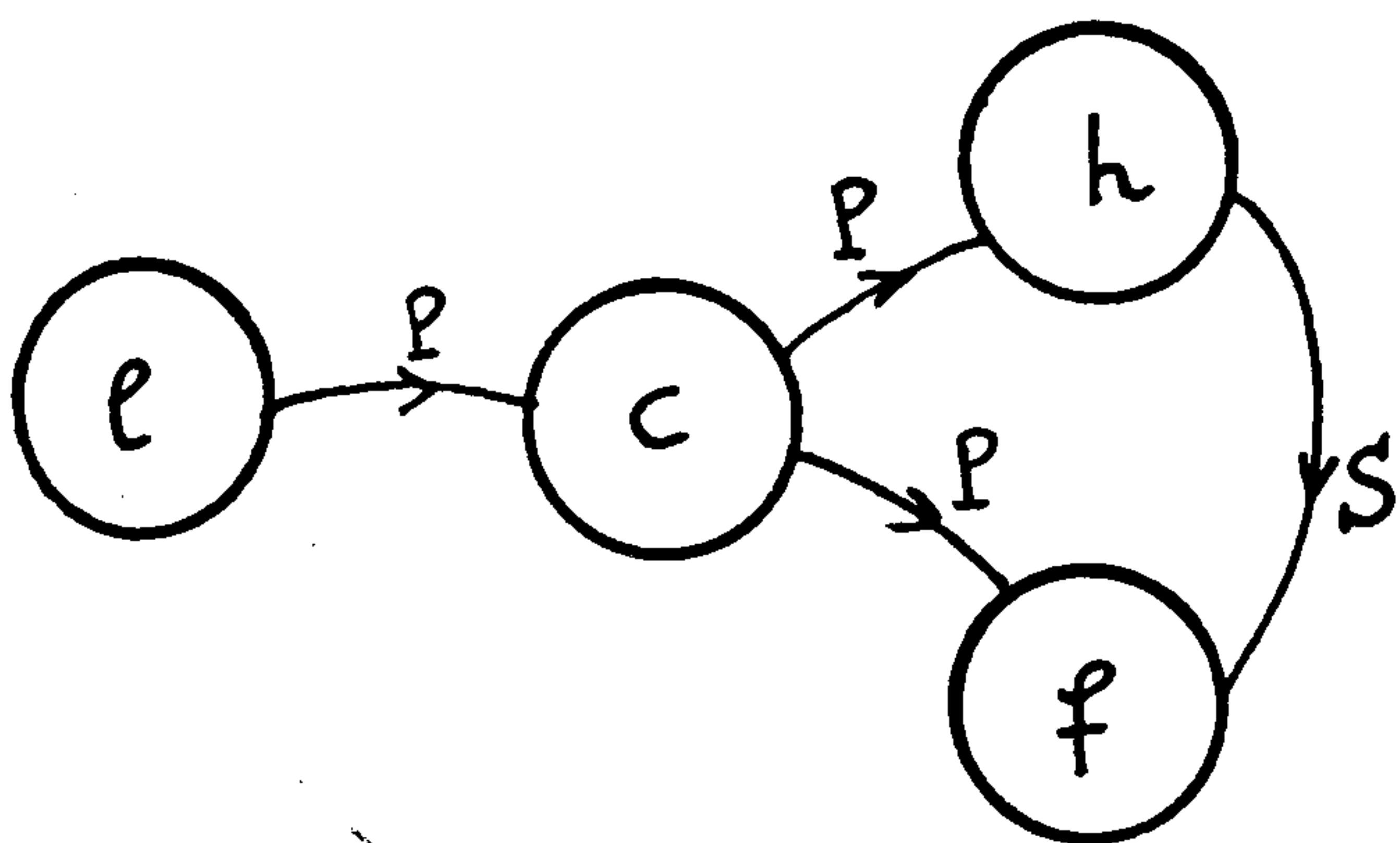
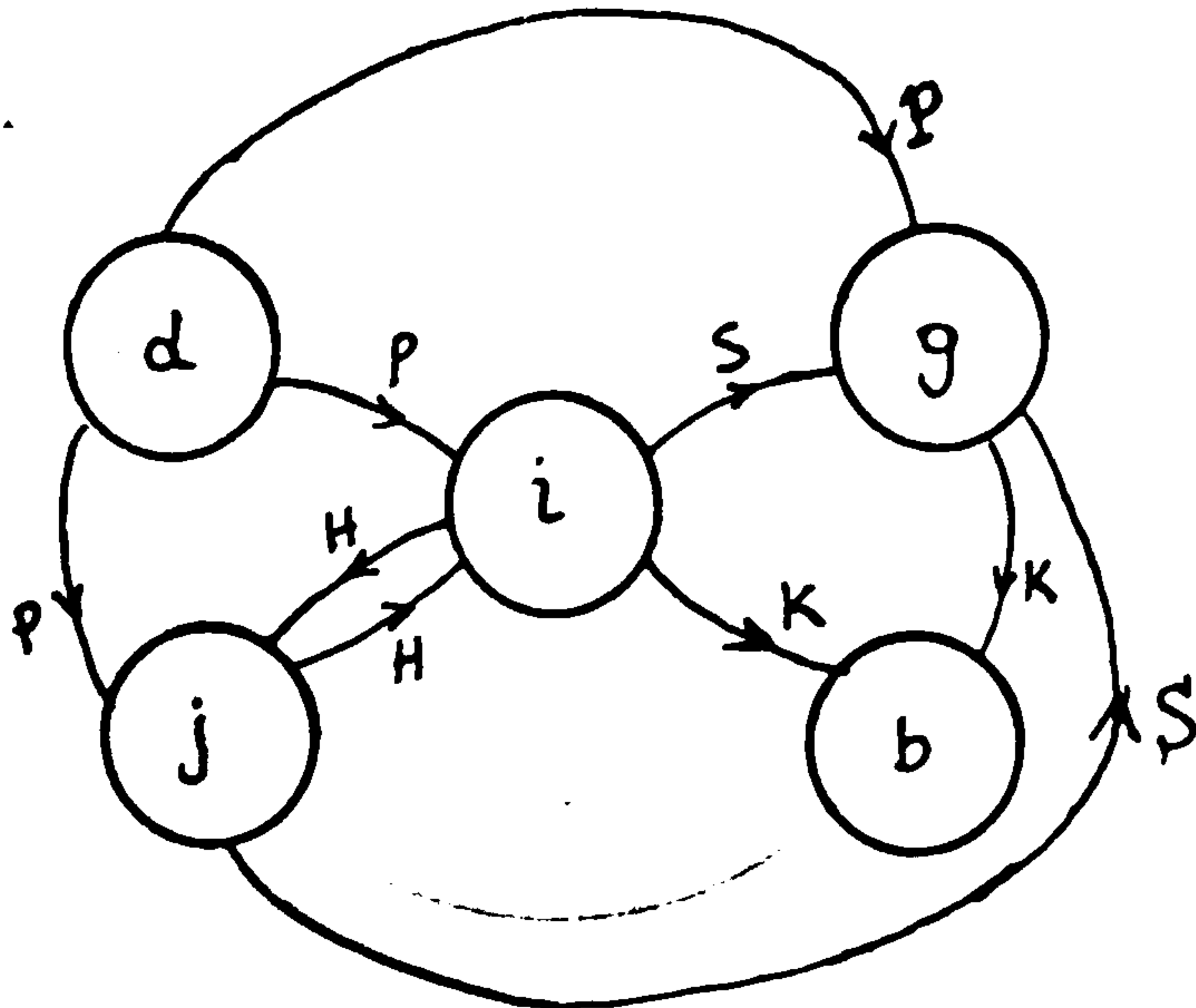
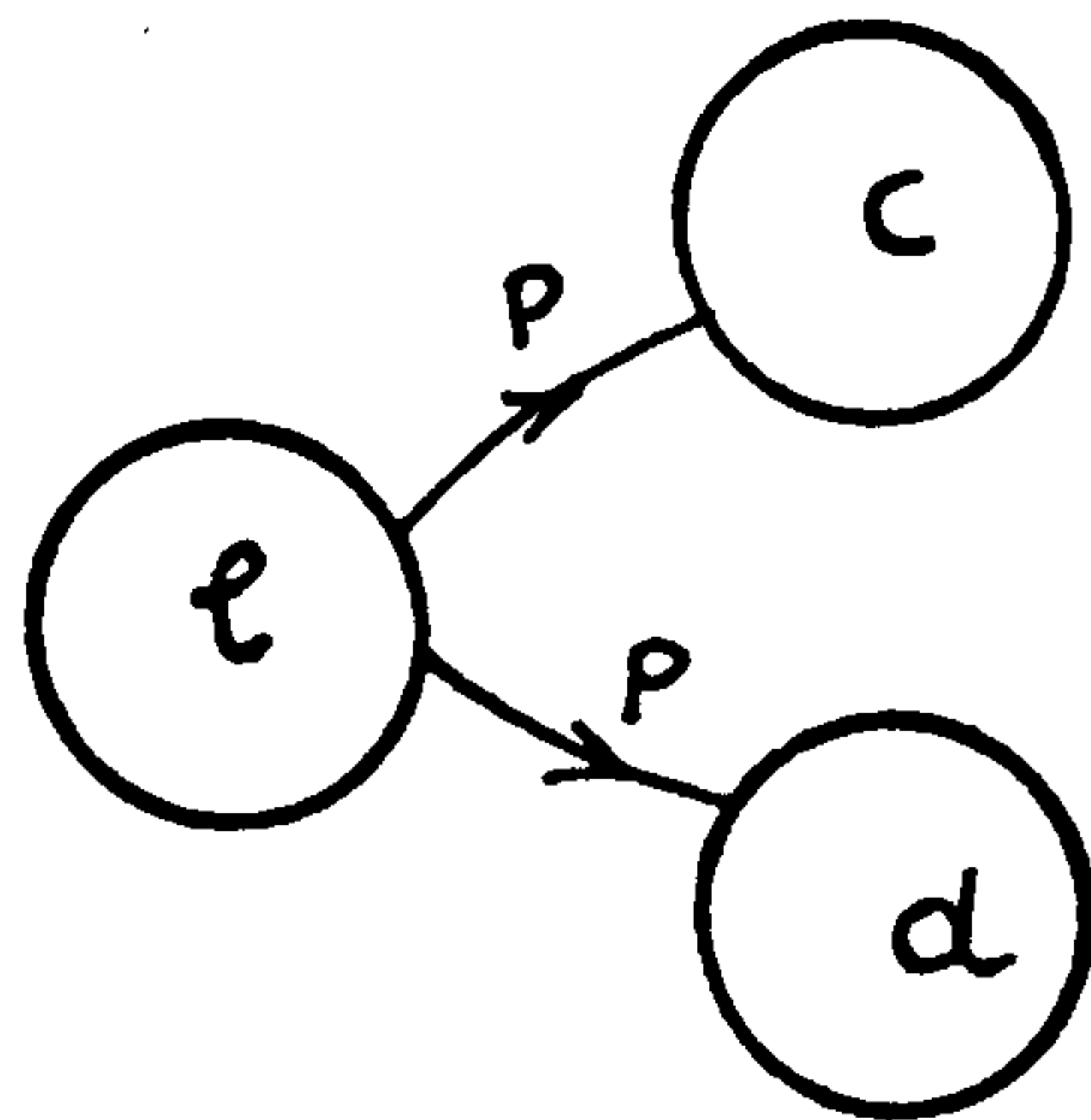
#### 3.3.3.1 Raison d'être for $\varepsilon \circ \pi$ and prolegomena to skeletization

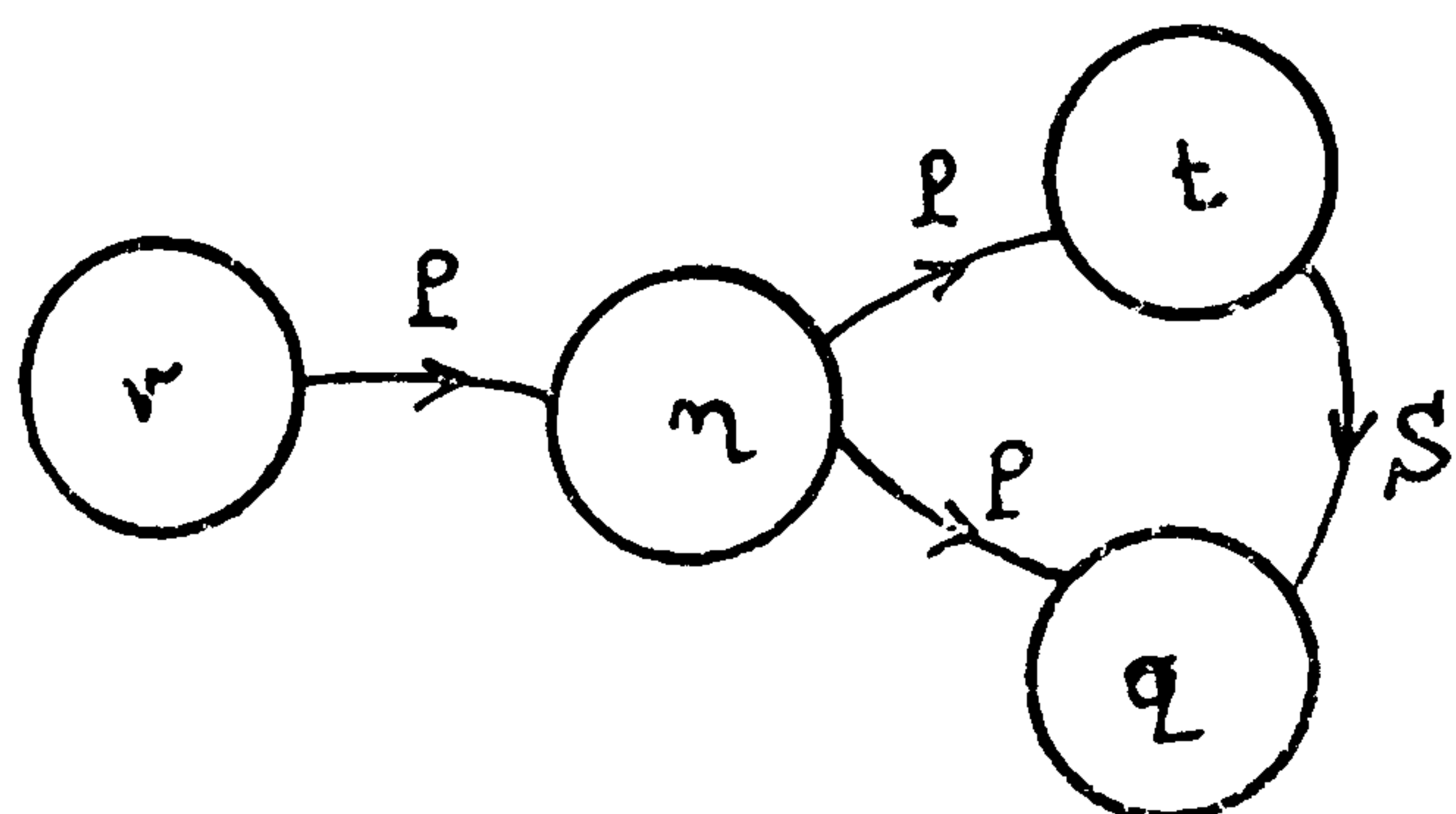
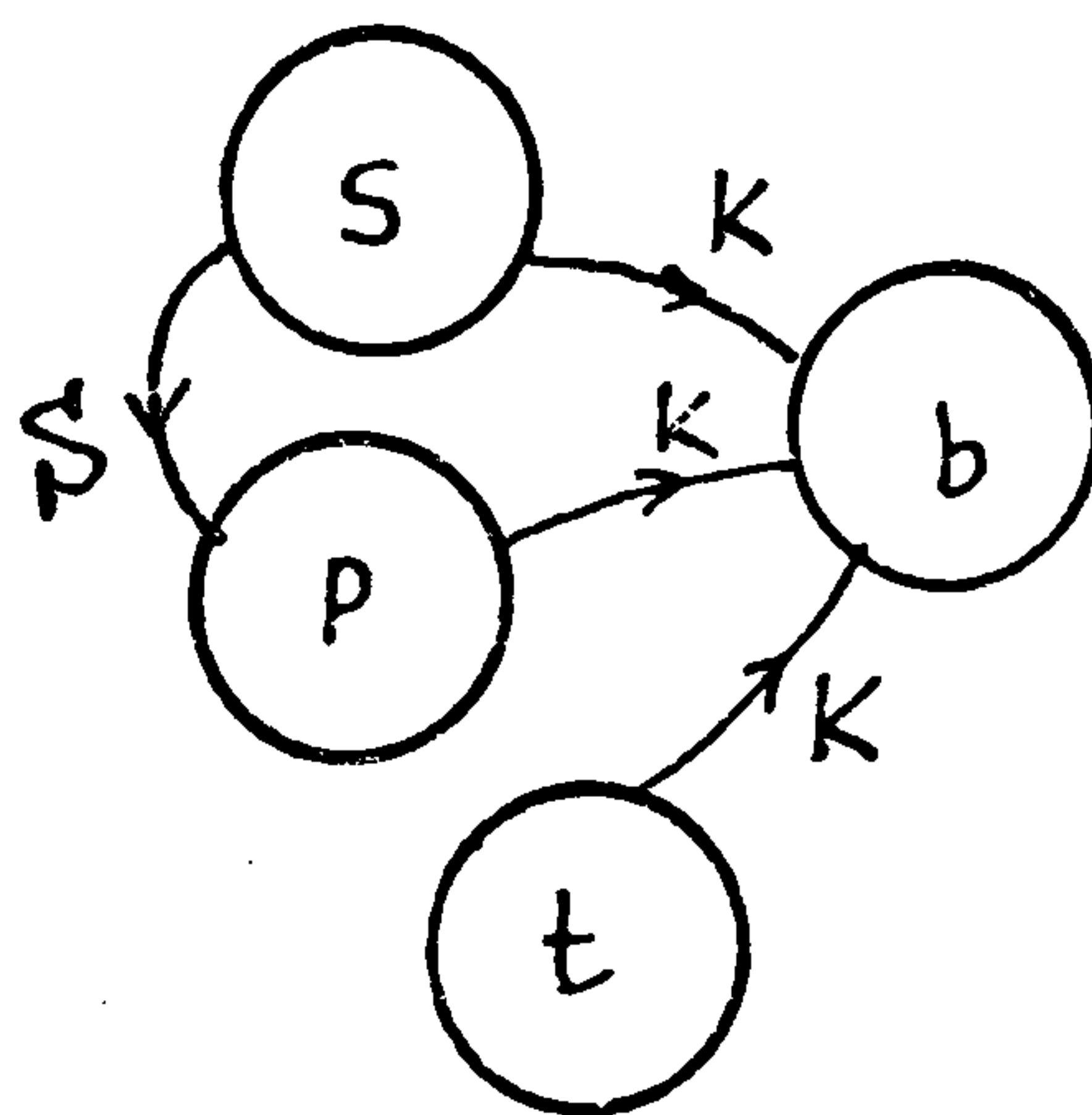
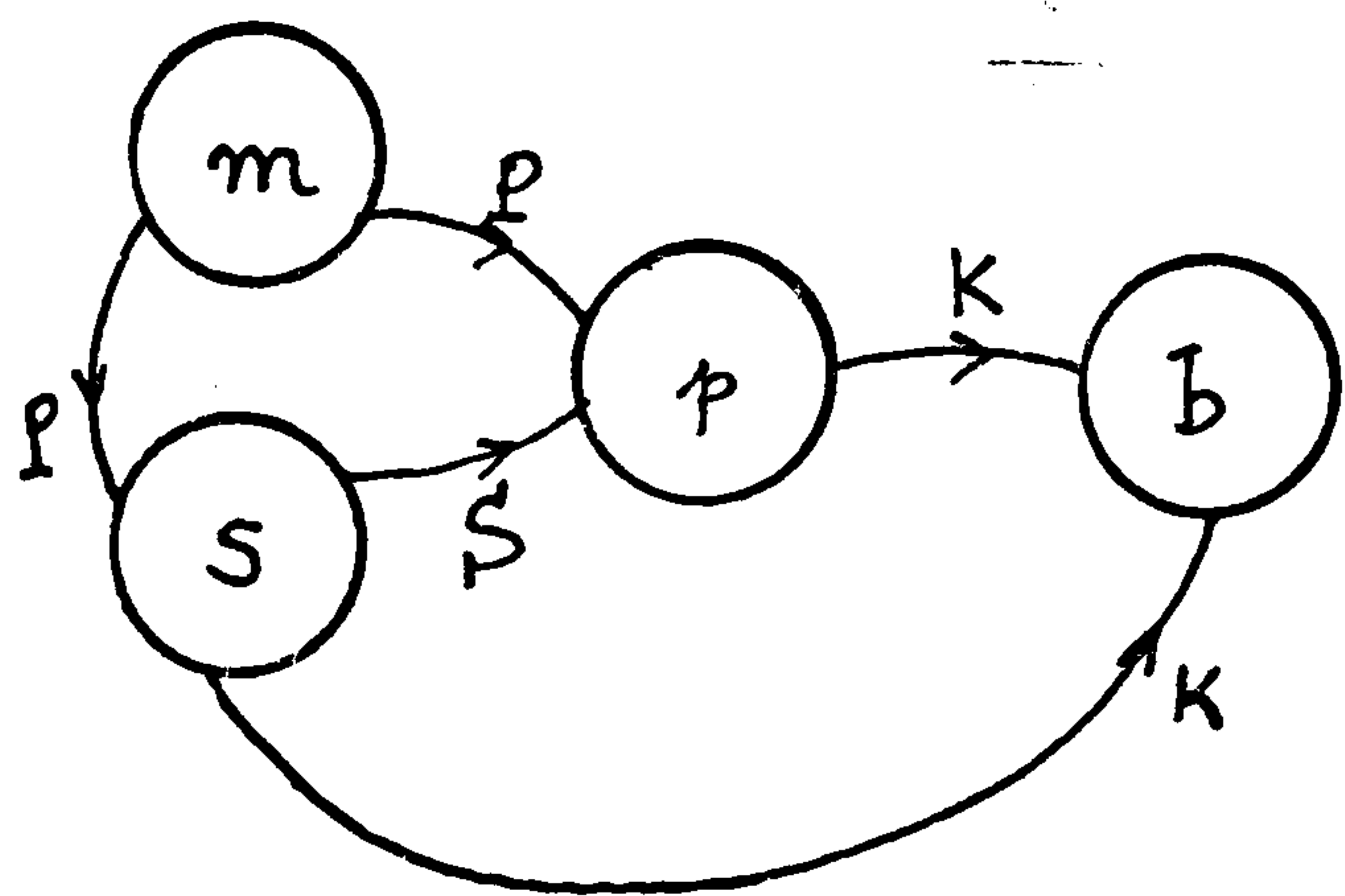
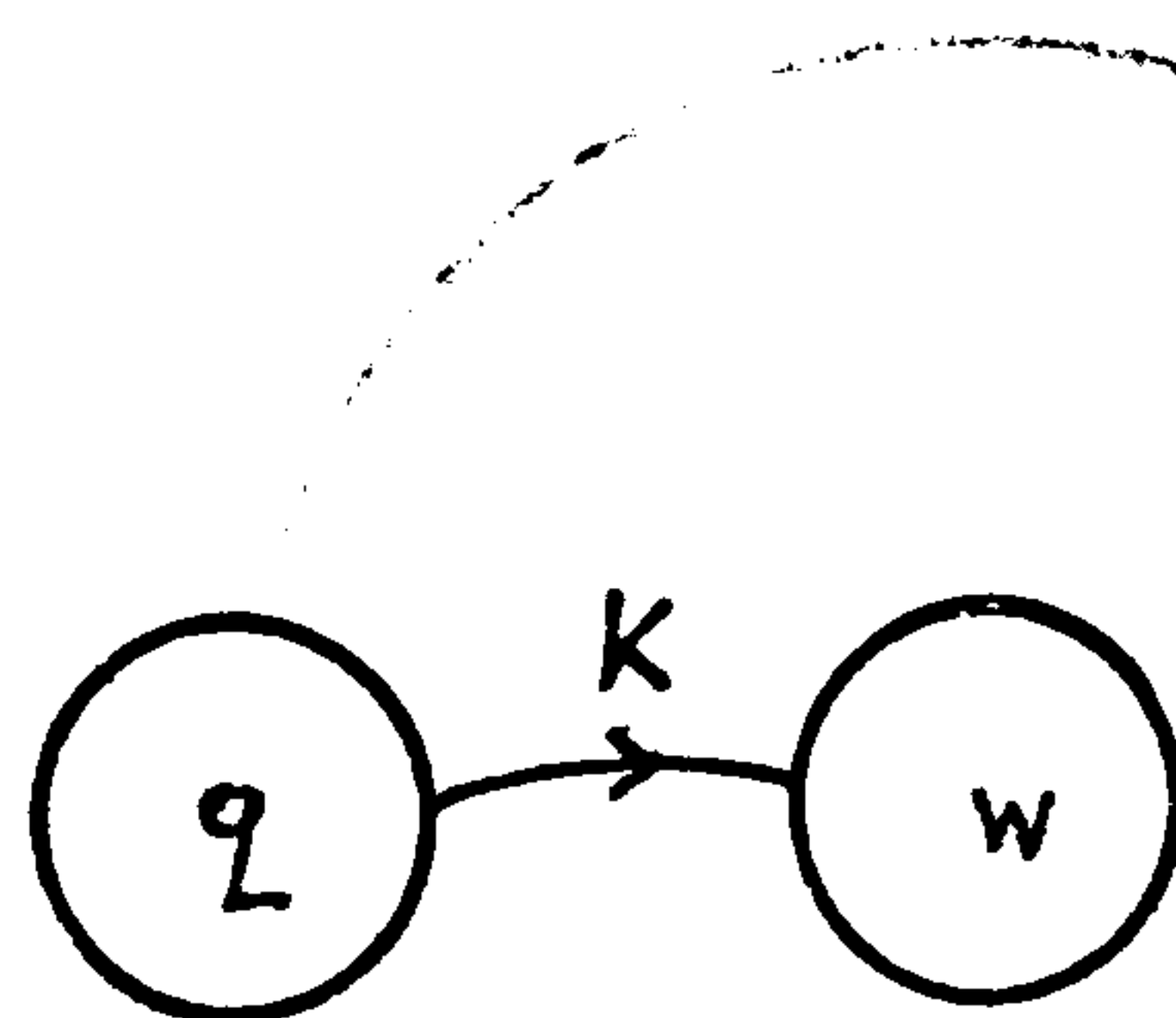
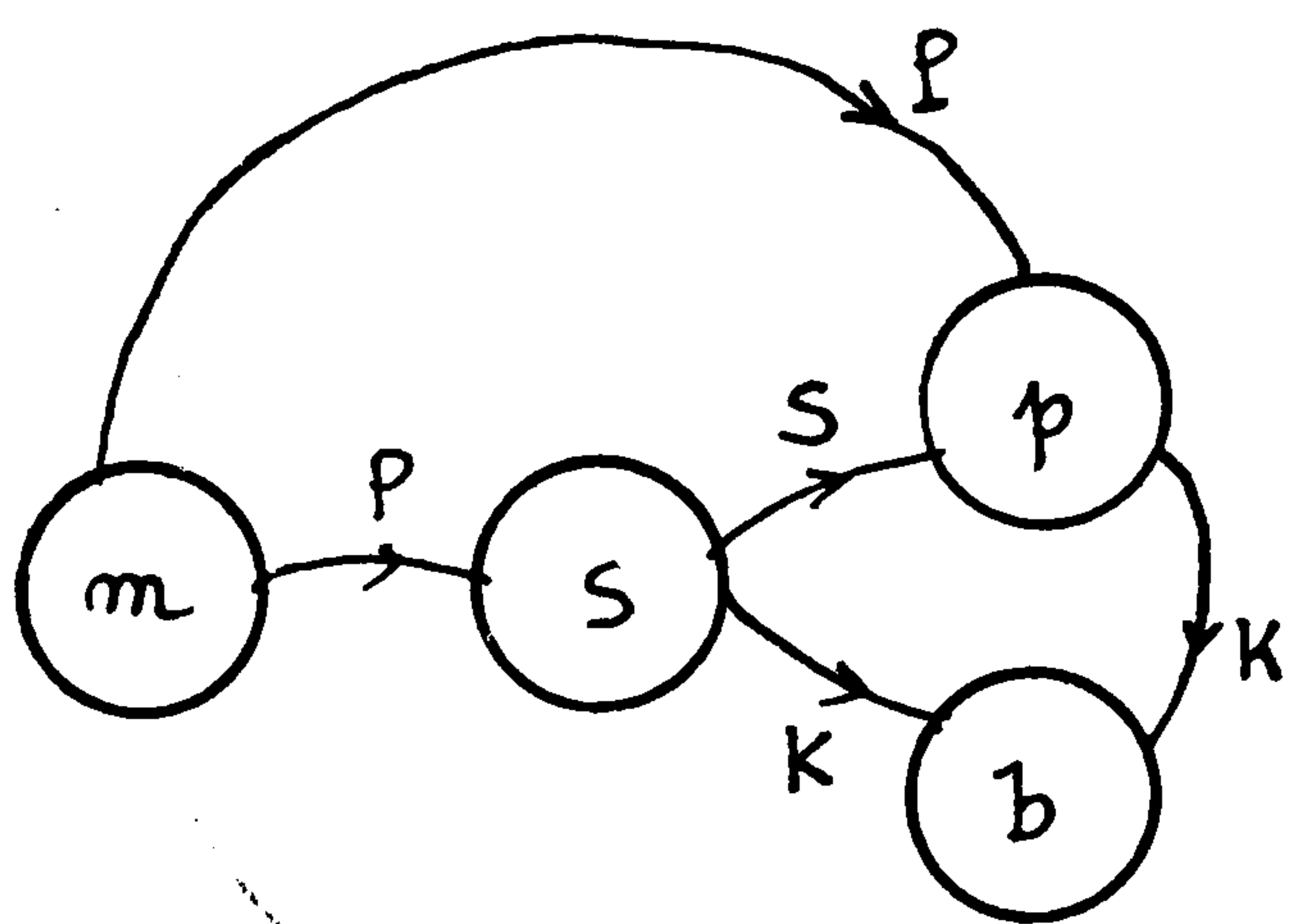
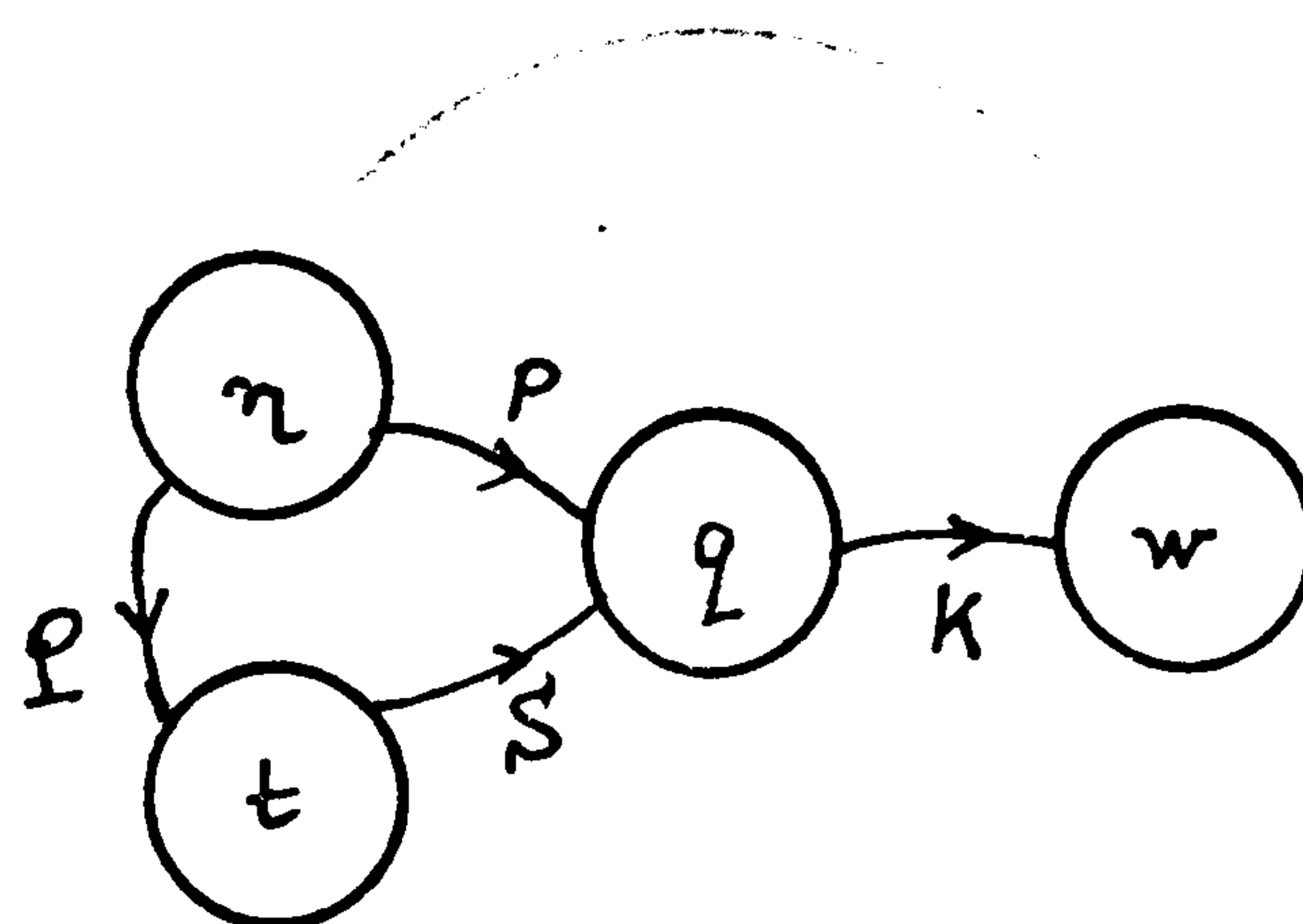
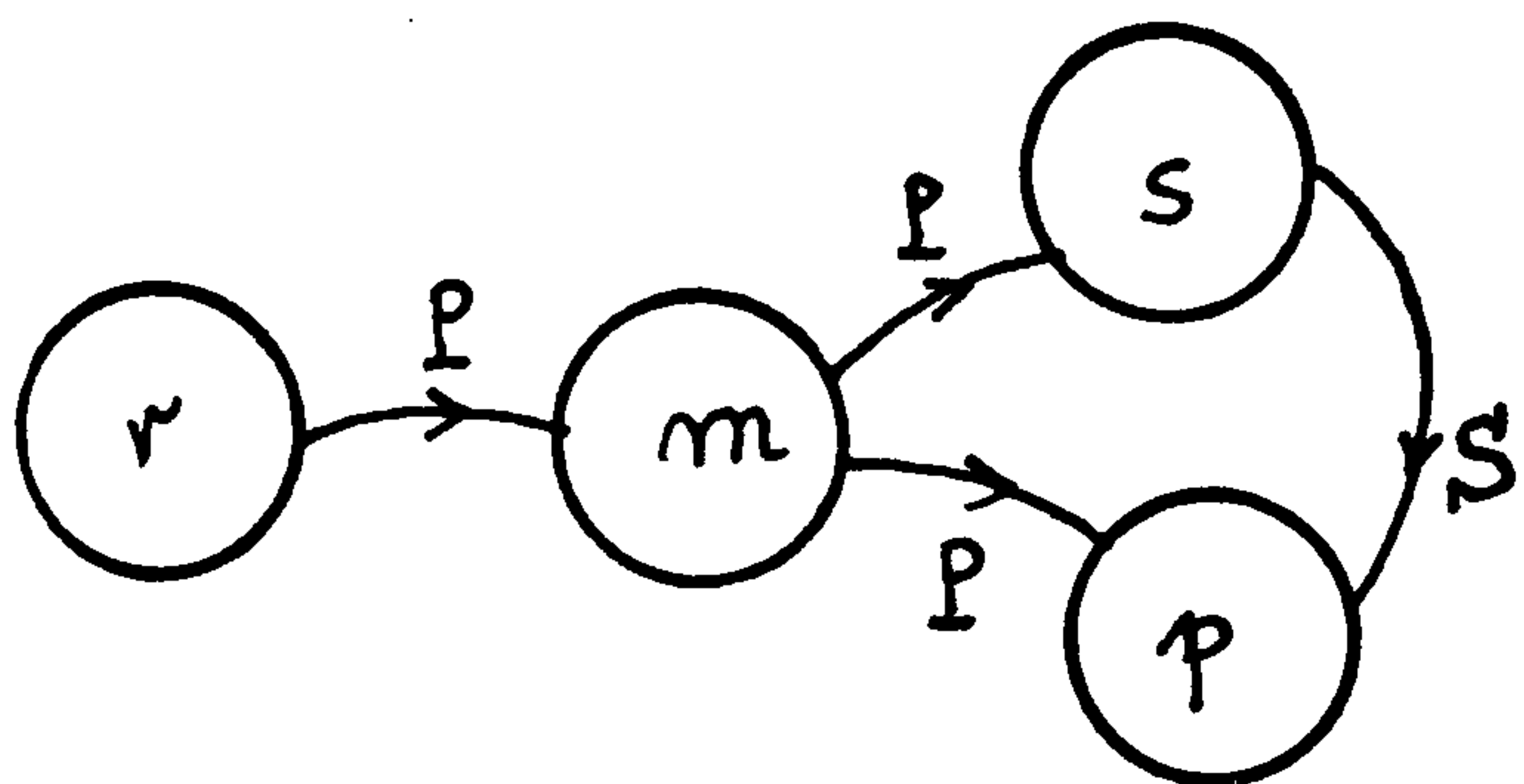
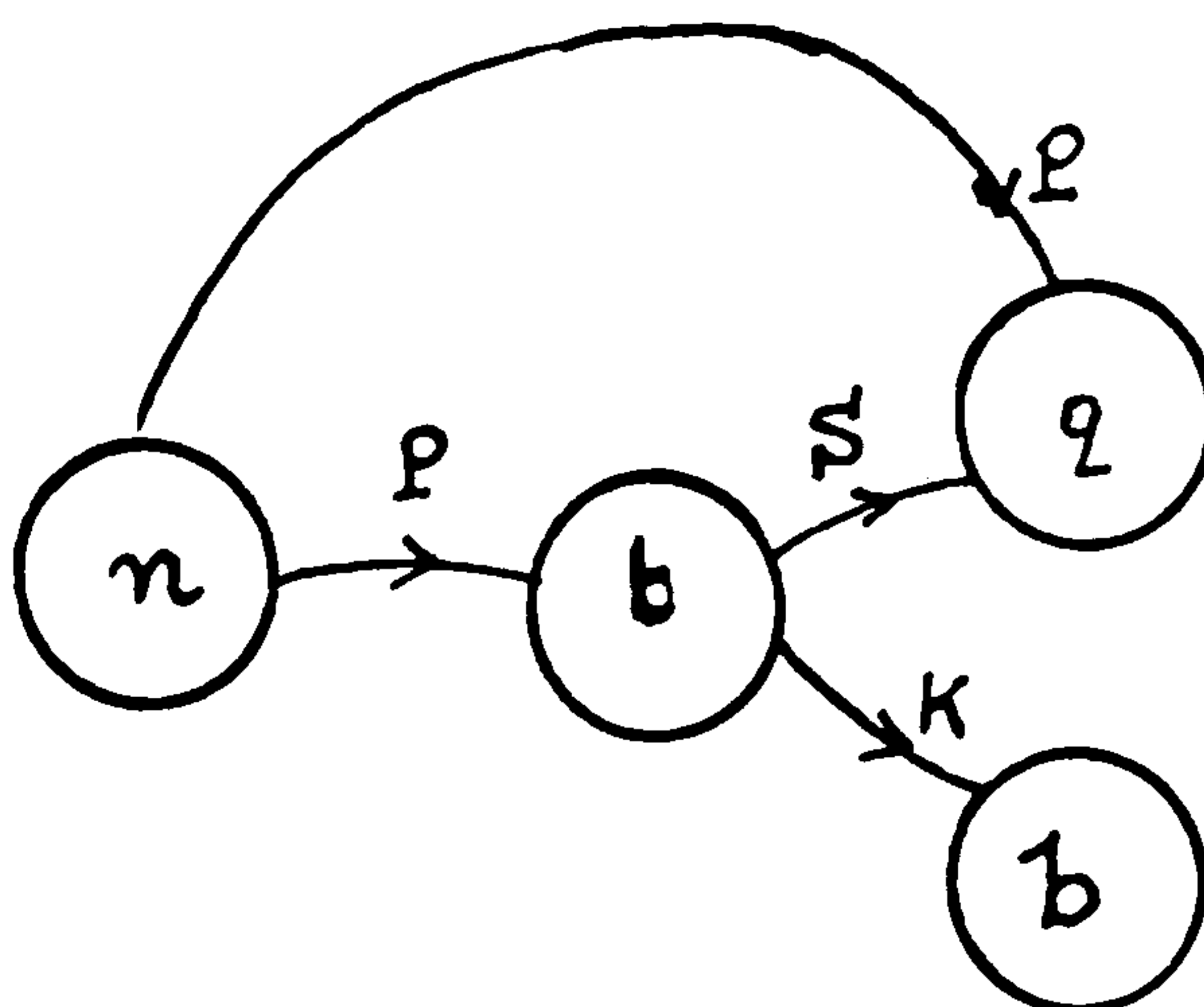
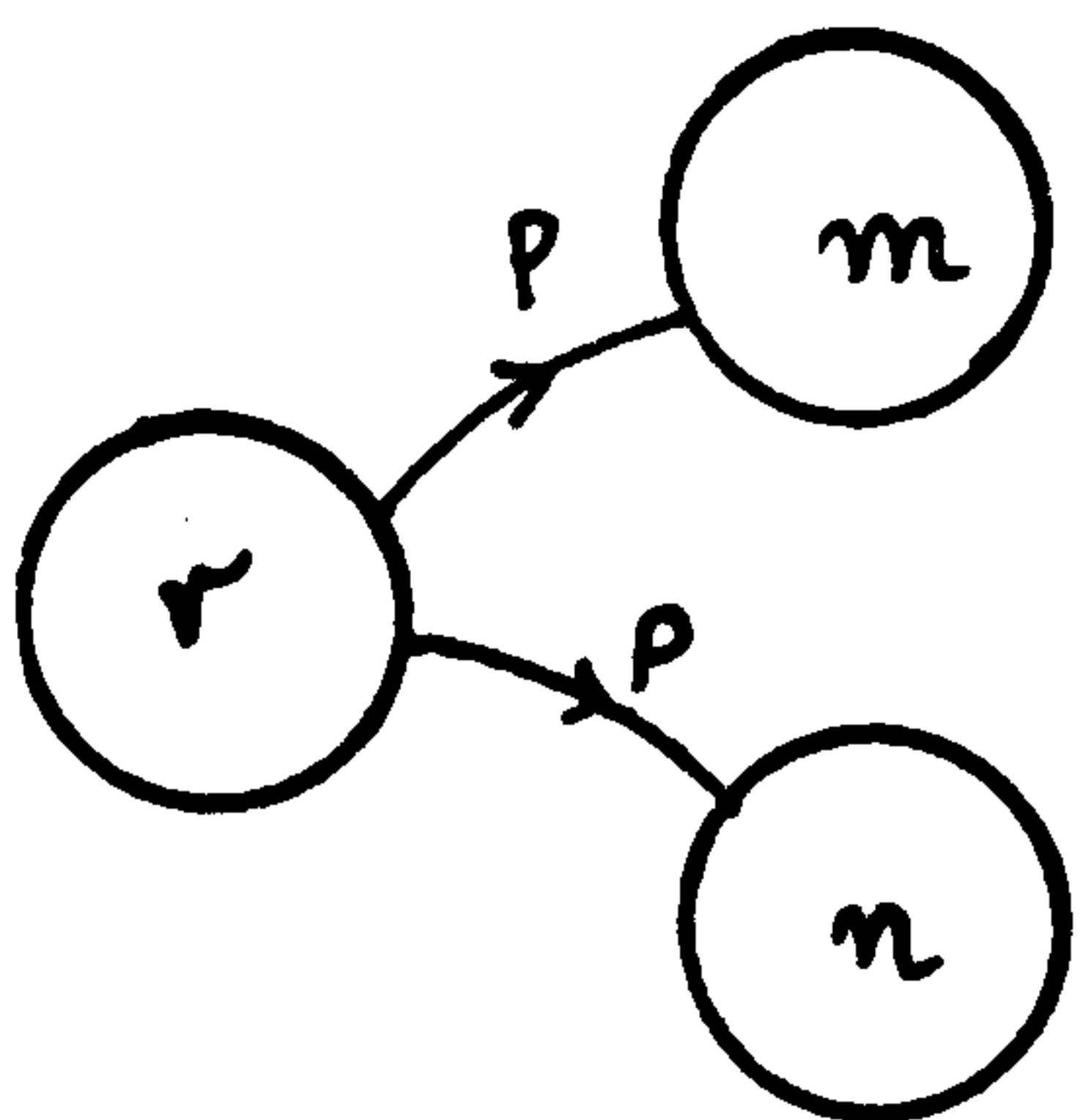
The existence of  $\varepsilon \circ \pi$  is due to the fact that the 'pseudo-semiotic' or 'contextual' similarity, we are going to establish, is about contexts of the concepts involved. *Skeleton formation process* (sect. 3.4) is mainly based on pairing off LEFT's objects to RIGHT's objects, on the basis of similar contexts. Such pairing off yields pairs of similar\*

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\* For similarity relations, etc, see ZADEH (1971) and BARNDEN (1975).







(equivalent) objects, which amounts to the formation of equivalent classes of objects. SKELALG's computer programme, see Appendix, carried on until some kind of 'maximal pairing' is synthesized. The skeletization universe\* then is constructed as 'quotient k-structure', consisting of similar classes of objects (invisible or tacit objects) equipped with their morphisms (those remain after suitable morphism and object eliminations). The skeleton, which might be formed, represents (schema section 3.4.6.2) some sort of common structure shared by the parent structures, LEFT and RIGHT.

### 3.4 ON SKELETIZATION

#### 3.4.0 INTRODUCTION

We now focus our attention on the notion of skeleton and its synthesis amounting to a skeleton formation process, from now on called skeletization and symbolized as SFP. Skeletons play important role on the issue of shifting between representations, as it soon becomes clear from illustrative examples (3.5.10). Skeletons may be conceived as amalgams of k-structures. From a mathematical point of view, skeletons are 'quotient\*\*k-structures'. On the other hand, SFP, main theme of the present work, is elaborated in an algebraic-topological manner; key notions for it are: k-structures, k-spaces, k-opens, and concepts that are going to be developed, as for example, morphism emphasis, set adjacency measures, morphism/object elimination and so on.

---

\* PASK (1975)a.

\*\* The notion of quotient is elaborated in WONG (1974), GOGUEN (1976), DUGUNDJI(1966), ARBIB-MANES (1975)b.



### 3.4.1 INTERPRETING SKELETON

Let us start by giving a number of intuitive interpretations of the notion of a skeleton in its broad sense. The following viewpoints might offer some idea about a skeleton, and perhaps its role in analogical reasoning, in the way we conceive it\*. However, these standpoints represent mere speculations and it might be possible that they reflect partial views on the matter under question. Thus a skeleton may be considered in:

Algebra as a structure commonly shared by the parent structures; HU (1965), BOURBAKI (1951).

Artificial Intelligence and cognition as a sort of abstraction or the result of generalization; MELTZER (1970)a and (1973), LEFAIVRE (1974), WINSTON (1970).

The theory of  $k$ -structures ( $k$ -theory) as a sub- $k$ -structure; in fact, quotient  $k$ -structure.

Finally, in Problem-Solving as a representation of (abstractions of) problematic situations (theses).

Generally speaking, one may think of a skeleton as 'quotient' in the algebraic sense of the term. That is, a set consistent of equivalent classes; where an equivalent class is made up from similar elements in respect to an equivalence relation.

Let us mention a few names which we think are appropriate to label our notion of skeleton and SFP; nam.

---

\* Which is slightly different from that which one can find in an English dictionary.

$K_1, K_2$  quotient generator; BARNDEN (1975).

Equivalence detector; PAVEL (1976), MYLOPOULOS-TOURLAKIS (1973).

Functor constructor; GOGUEN (1970).

Abstractor; MELTZER (1973).

Carrier of a reconciliation\* for  $K_1, K_2$  structures, or Trivial (low level) analogizer.

### 3.4.2 SKELETON FORMATION

We now turn into the skeleton formation process (SFP) or skeletization. In order to skeletize two given  $k$ -structures, say  $K_1$  and  $K_2$ , we are going to link together or compare their  $k$ -spaces. In fact, this process of bringing  $k$ -spaces together is achieved, somehow, in algebraic-topological manner. SFP has been implemented in a computer programme called SKELALG (for SKELeton ALGORITHM); details are given in the Appendix. An outline of SFP follows.

SKELALG receives two  $k$ -structures  $K_1, K_2$  as input and stores them in an internal form (matrix one). It then produces (generates) some sort of topological partitions for  $K_1, K_2$  structures, via the 'separating the context of a concept from the concept itself' technique; sect. 3.3.3. The partitions are what we called in section 3.2  $k$ -spaces, and their basic ingredients are the  $k$ -opens. This step is the job of KOSMOS sub-routine which is outlined in the Appendix.

SKELALG works mainly upon  $k$ -opens. That is why special emphasis has been given to the key notion of a  $k$ -open, detailed analysis for it has been made from a number of standpoints, examples are given and

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\* SLANGLE (1968).

various interpretations are assigned to it; section 3.2.

Skeleton formation then proceeds as follows: a number of computations are carried out among the k-opens, components of the so-formed k-spaces.

One analogy we can think of, at the moment, to illustrate the situation up to now, is the following: imagine a pool, the skeletization universe, and the objects (concepts) of  $K_1$ ,  $K_2$  structures, in addition to the k-opens (contexts) of  $k_1$ ,  $k_2$  spaces, floating around.\* Notice that k-opens (i.e. the objects' contexts) are separated (liberalized) from the objects themselves.

SKELALG's main task now is to compare k-spaces. It is worthwhile here to point out that a shift has been tacitly made for the comparison; nam. the latter is carried out among k-opens, i.e. the constituents of k-spaces, and not between objects, i.e. the ingredients of k-structures. This is, in fact, a very important aspect for the entire skeletization process. During the comparison, a variety of resemblance measures are used in order to match k-opens and group them into equivalent classes made up of similar elements. Notice that, initially, the job of k-structures' comparator partially depends on some sort of *a priori* hint concerning a kind of (external to k-spaces) similarity between the two k-structures involved in the skeletization. In the examples, we are using to illustrate SFP, the above mentioned hint becomes:

LEFT ARCH  $\approx$  RIGHT ARCH, sect. 3.4.7, and LEFT SCENE  $\approx$  RIGHT SCENE, Sect. 3.3.

---

\* The models for  $K_1$ ,  $K_2$  structures are also included.



### 3.4.3 k-SPACES COMPARISON

It is the core of skeletization (SFP). It has been implemented via the KRISIS subroutine, the flavour of which is given below.

Details of its implementation and mathematical forms used in its sub-parts are given in the Appendix. What follows is referred to the k-structures and k-spaces analysed and sketched in sect. 3.3.

Having  $K_1, K_2$  spaces' construction been made, via KOSMOS subroutine, the comparison stage between k-spaces' constituents, i.e. k-opens, takes place via the KRISIS subroutine, the main procedure of SKELALG computer programme, which, as we mentioned earlier, implements skeleton formation.

Comparison of  $K_1, K_2$  spaces leads inevitably to dilemmas\*, conflicts\*\*, or deadlocks\*\*\*, i.e. situations where it is difficult for the decision-maker or problem-solver, in this case programme SKELALG to decide effectively between two directions of action or opinion which are (for it) computationally equal. Elaborating at this point, we may say that SKELALG, after some successful\*\* inferences based on structurally similar pairs of k-opens (contexts) been made, and after carrying them (relations A and B) over to the skeletization universe, it arrives at the following set of similarity relations which are listed below:

$$C_l \approx C_r \Rightarrow C_c \approx C_n, \quad C_d \approx C_m \quad (A)$$

$$C_c \approx C_n \Rightarrow C_h \approx C_t, \quad C_f \approx C_q \quad (B)$$

---

\* RYLE (1949);

\*\* COLES (1975), (1977), LEFEBVRE (1967), SACERDOTI (1975) p. 11.

\*\*\* PETRI (1965), TSIPIS (1977).

\*\* i.e. Where no conflict/dilemma, in the sense we put it, arises.

The above relations are produced by taking into account:

- (a) the structure of  $K_1$ ,  $K_2$  spaces, and
- (b) computations which are carried out via a number of subroutines; nam. BIV, NORMA, MARV, EMARV, LFUZZIF, GFUZZIF and so on, which are outlined in the Appendix.

Notice that initially the hint 'KOSMOS 1 or structure K1 is similar to KOSMOS 2 or structure K2' is supplied.

SKELALG then carries on yielding the following:

$$C_d \approx C_m \Rightarrow \{C_i \approx C_p \text{ or } C_i \approx C_s\} , \{C_j \approx C_p \text{ or } C_j \approx C_s\},$$

$$\{C_g \approx C_p \text{ or } C_g \approx C_s\} \quad (D)$$

At this point, computer programme SKELALG is deadlocked due to the fact that 'distances' between pairs of k-opens are equal, leaving thus SKELALG with no option and therefore unable to decide effectively, i.e. to take an appropriate action\*. The situation which actually arises is as follows:

$$\begin{aligned} \text{ISOL2}(C_i, C_p) &= \text{ISOL2}(C_i, C_s) \\ \text{ISOL2}(C_j, C_p) &= \text{ISOL2}(C_j, C_s) \\ \text{ISOL2}(C_g, C_p) &= \text{ISOL2}(C_g, C_s) \end{aligned}$$

where, ISOL2 is some kind of distance measure between two contexts (k-opens)  $C_n$ ,  $C_y$ ; details in the Appendix.

---

\* In fact, what problem-solving generally amounts to is: "power for an appropriate selection"; AMAREL (1967), ASHBY (1956), MELTZER (1970)b.

### 3.4.4 TACTICAL AND STRATEGIC ADVISORS

#### 3.4.4.0 Introduction

The deadlock thus produced, preventing SFP from any other action, is perhaps because only structural features of k-opens have been taken into consideration. At this moment, when 'structural experts' partially fail, special advisors\* are called into action. That is, SKELALG provides, in cases of undecidability and when dilemmas occur, a set of alternative courses of action controlled by what we call *strategic* and *tactical advisors*. These are some of its, allow me to say, intelligent features, which make skeletization, implemented via SKELALG (and thus analogical problem-solving via HARMONY\*\*) partially superior\*\*\* and to some extent a bit more powerful, computationally, than parts of EVANS' (1963) and WINSTON's (1970) implementations. We are next going to have a closer look on issues concerning the above mentioned advisors.

#### 3.4.4.1 Tactical Advisor

*Tactical Advisor* deals with or is devoted to 'local' structural changes; one of its aims is to bring about a similarity\* via morphism/object elimination\*\*. That is, it discovers, somehow, that a disturbance factor for the contextual similarity, is morphism H in the presence case and it takes the Tactical Action (TA): 'eliminate morphism H' from k-opens (contexts)  $C_1$ ,  $C_j$  and  $C_g$ .

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\* Compare them with the so-called "critics" used by SUSSMAN (1975); SACERDOTI (1975); and LENAT (1977).

\*\* Computer programme to be investigated in the foreseeable future.

\*\*\* As far as the theory underlying their models is concerned.

\*\* A, somehow, 'pseudo-semiotic' one, see sect. 3.5.1.1.

\*\* Object and morphism elimination is elaborated in sect. 3.4.6.



The above used expression 'local' refers to the attributes of the k-spaces involved in the skeletization process. Thus a TA directly affects k-spaces' constituents.

Part of a tactical advisor's task is the analysis and diagnosis of the reasons which cause a deadlock. Thus a tactical advisor is also acting as a diagnostician, so to speak. The discovery of the disturbance factor may be detected in a manner which is fully exemplified in section 3.4.6.

The net effect of a tactical advisor, in the general case, may be summarized in a threefold manner; nam.

- a. morphism-elimination of an appropriate set of morphisms, from an appropriate set of k-opens, directly relevant and involved in a conflict;
- b. reconstruct the appropriate k-opens; thus producing a new k-space; and
- c. reapply subroutine KRISIS to resolve conflict.

#### 3.4.4.2 Strategic Advisor

One of the results of our investigations, via SKELALG computer programme, was that unfortunately the above tactical advisor only partially resolves the occurred dispute in an interaction between k-structures. Thus, skeleton formation process needs something additional. Therefore, SKELALG calls into action its *strategic advisors\**. It takes over and hints to a number of 'Strategic' so to speak, Advices\*\*(SA), which may be described as follows:

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\* Compare it with MacARTHY's (1959) advice taker.

\*\* Relate them with SACERDOTI's (1975) p. 11 "general-purpose critics".

- SA11. eliminate morphism H from  $k_1$ -structure (ME)\*
- SA12. reconstruct  $k_1$ -structure
- SA13. recompute  $k_1$ -space via KOSMOS subroutine
- SA14. refuzzify  $k_1$ -space via GFUZZIF and LFUZZIF
- SA15. reapply KRISIS to resolve conflict

Strategic Advisor, thus, is dealing with some sort of 'global' structural changes in contrast to tactical advisor. That is, strategic advisor affects a k-structure and therefore a k-space.

Another set of strategic advices might be the following one:

- SA21. object-elimination (OE)
- SA22. reconstruction of a k-structure
- SA23. recomputation of its k-space
- SA24. refuzzification of k-spaces' constituents
- SA25. reapplication of *endo-krisis* to resolve an 'internal conflict'.

Notice that in SA25 a variant of KRISIS subroutine is used; namely when  $k_1$ -structure is equal to  $k_2$ -structure. This case of skeletization is linked to inter-shifts (or shifting-1) mentioned in section 2.1.3.; that is, their communication is carried out between a k-structure and itself. The case, though trivial, is interesting as soon will be made clear.

### 3.4.5 APRES TACTICAL AND STRATEGIC ADVISORS - DISCUSSION

From the above stated sets of advices a problem naturally arises. Namely, which sort of action should be taken when conflicts arise?

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\* ME stands for morphism-elimination.

However, which kind or mode of elimination is going to be taken, i.e. morphism-elimination (ME) or object-elimination (OE), is a matter of question open to discussion, and one of the aims of SKELALG is to provide 'freedom of opinion' to that respect. That is, it is left open to it in order to decide, at any particular case, according to the current view that is held about the situation. This is, somehow, elaborated in what follows next and is taken up again in a subsequent section (3.5.1) entitled 'on morphism and object elimination'.

Strategic/tactical advisors, thus skeletization\* SFP, may be supplied with a variety of means to provide the capability of looking from different points of view a deadlock/dilemma/conflict situation. Thus, dilemma resolution may effectively be achieved by considering a situation (k-structure) via various definitions of morphism-emphasis, qualitative features of a k-open; see Appendix for different morphism-emphases. To this respect, a number of alternative sub-routines are available. The repertoire consisting from : RMARVA, ARIADNI, THESEUS, CANTOR and so on.

The net effect of the intervening strategic and tactical advisors, as well as the foregoing 'changing the viewpoint to a situation', amount in fact to a change in the stylistics of KRISIS subroutine's structure, through which SKELALG programme carries out k-structure, k-space comparisons and handles conflicts and dilemmas. This change provides some sort of flexibility in the KRISIS's behaviour.

We have to bear in mind that a strategic advisor is called upon to supply action by KRISIS (or rather invoked) if and only if:

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\* Hence analogical problem-solving.



- A. a dilemma/conflict has occurred; and
- B. a set of morphisms or objects for elimination has to be selected.

We wish to go deeper in the above issue by closing the present section with a final remark that sometimes 'a synergy or combination of Strategic and Tactical Advices (SAT)' should be taken. We think that this combination is the most effective response to resolve a dilemma situation in k-structure communication; i.e. maintaining a global and/or local consideration and simultaneously changing the viewpoints of the situation. Strategic-tactical synthesis illustrates and embodies the dialectic mode of a system, let us call it HARMONY, which may be proved achievable in the foreseeable future, and which would implement the harmonization (sect. 3.5.5) of k-structures, a process or methodology so useful to analogical problem-solving and learning. Next, the rather controversial issue of object/morphism elimination is taken into account.

### 3.4.6 ON MORPHISM AND OBJECT ELIMINATION

#### 3.4.6.0 Introduction

We must now consider a very important issue, mentioned earlier, and which is summarized in the following sentence - there is some kind of freedom of choice supplied to the skeletization process, in order to resolve a conflict/dilemma during the course of k-structure comparison; it is achieved by changing the viewpoint of a situation, and by looking to the k-spaces' constituents from different emphases.

At this point a number of questions naturally arise; nam.

1. How does SKELALG notice that a conflict in k-space communication occurs?
2. How does it discover the disturbance factor?
3. How does it infer which sort of elimination has to be applied?

In what follows we attempt to give some answers to these questions by describing or interpreting a number of results received during the implementation of skeleton formation process (SFP) via SKELALG computer programme which has been partially analysed throughout the sections of the present chapter and for which details are given in the Appendix.

#### 3.4.6.1. Morphsism Elimination

In section 3.4.3 we end up with a number of relations; nam. (A), (B), (D). The members of these relations are the components of k-spaces fully detailed in section 3.3.4. A first suggestion for a conflict is the difference in the numbers of radiated objects of the two k-opens under comparison, nam. d, m. In the present situation, i.e. relation (D), this difference can be easily traced via:

$$LK1OP_d \neq LK1OP_m$$

where  $LK1OP_x$  := the number of radiated objects, adjacent to the central one, i.e. x, or, from schema in sect. 3.2.3, we have:

$$TOPOS1_d \neq TOPOS1_m$$

SKELALG thus notices from the beginning that a disturbing factor\*

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\* During the course of the attempt to establish a similarity between k-spaces.

in the k-space dialogue does exist. Therefore, it activates its 'diagnosticians', which are parts of tactical and strategic advisors (TA) and (SA) we described in a previous section, towards the task of eliminating the disturbing factor. The latter, amounts to what we called object or morphism elimination, (OE) or (ME) resp. Now, how does tactical advisor TA work towards it?

First, TA considers from a topological point of view, the conflict causing k-opens, i.e.  $(C_i, C_j, C_g)$ , appeared in relation (D) section 3.4.3. TA discovers that:

1. Morphism H appears in  $C_i, C_j, C_g$  and not in  $C_s, C_p$ ; this is achieved through scanning over the TOPOS2 area of  $C_i, C_j, C_g, C_s, C_p$ . This event suggests that there is a difference in the number of morphisms; more precisely:

$$\text{LK2OP}_x \neq \text{LK2OP}_y \text{ or } \text{TOPOS2}_x \neq \text{TOPOS2}_y$$

and therefore, a conflict might be caused due to morphism H. Thus, morphism-elimination is a possibility, at the present moment, to remedy the deadlock that skeletization arrives at.

2. Morphism H belongs to TOPOS2 (definition area, sect. 3.2.3), for  $C_i, C_j$ ; also H lies in TOPOS3 (behaviour area, sect. 3.2.3) for  $C_g$ .

Second, if TA eliminates morphism H from  $C_i, C_j, C_g$ , locally (i.e. within k-space), then we found that dilemma occurs again. That is the situation which arises is similar to the one we had in sect. 3.4.3 where:  $\text{ISOL2}(\cdot, \cdot) = \text{ISOL2}(\cdot, \cdot)$ . So, it seems that morphism-elimination does not effectively work in this case.

### 3.4.6.2 Object Elimination

SKELALG thus, performs an alternative, more drastic so to speak,



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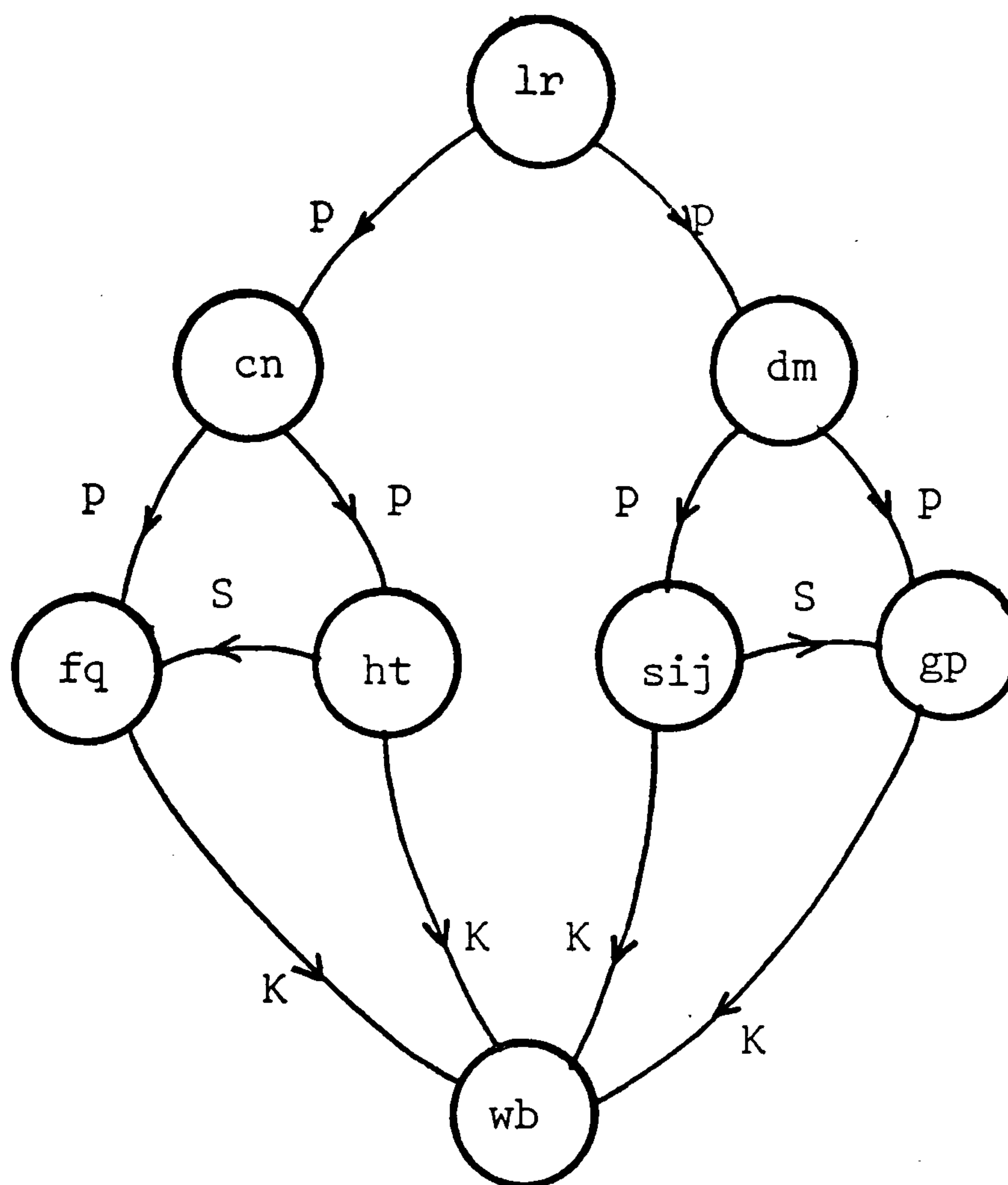
advice, that is object-elimination. This 'opinion' is apparently encouraged from the fact that:  $LKlOP_d \neq LKlOP_m$ , we found earlier on. However, tactical advisor has to find out which object to eliminate.

This selection is effectively achieved by inspecting TOPOS3 for  $C_s, S_p$ . Nam., object-elimination of  $i$  or  $g$  (as radiated objects) yields a TOPOS3-pattern for  $\{C_j, C_g\}, \{C_i, C_j\}$  which is 'much further' from TOPOS3-pattern for  $C_s, C_p$ , than object-elimination of  $j$  yields.  
i.e. TOPOS3 for  $(C_{i-j}, C_{g-j})$  is 'closer' to  
TOPOS3 for  $C_s, C_p$  resp.

Finally, SKELALG after carrying this effective tactical advice, it establishes some kind of isomorphism between the reconstructed  $K_1, K_2$  spaces. The phenomenon may be interpreted as some sort of 'reconciliation' of  $K_1, K_2$  structures'.

If, on the other hand, we take the 'similarity viewpoint' of the situation, then the state where SKELALG arrives after the conflict resolution, via object-elimination and reconstruction of  $k$ -spaces, is a set of equivalent classes of  $k$ -opens, a fundamental theme in the skeletization of  $k$ -structures. The skeleton, which illustrates and materializes the undergoing process (SFP) we may call it 'carrier of reconciliation of the  $k$ -structures involved in the dispute'.

The 'pseudo-isomorphism' captures the commonly shared sub- $k$ -structure and the latter may be illustrated as it is shown in the schema below. The skeleton  $\Sigma$  thus formed is of  $k$ -structure kind which consists of objects and morphisms of the following nature:  
 $\Sigma$ 's objects are equivalent classes of similar  $K_1, K_2$ 's objects; and  
 $\Sigma$ 's morphisms are those morphisms of the parents  $k$ -structures.

SKELETON  $\Sigma$ Schema 3.4.6.2.3.4.7 APRES MORPHISM/OBJECT ELIMINATION - DISCUSSION

In addition to the results we are discussing throughout this chapter, we add the following ones:

1. From the above analysis we can see that a conflict occurs when:
  - A. there is a difference in the number of radiated objects; and
  - B. there is a difference in the nature of radiated morphisms.

Thus, the choice of which sort of elimination, OE or ME, should apply, was not at all clear and easy to decide. There are, however, cases

when only one of A or B holds. In this latter situation, a hint or advice to which elimination has to be performed is more readily available and perhaps, uniquely determined according to which of A or B holds. Therefore, we think that we treat a fairly general case of k-structure communication, amounting to skeletization of relatively speaking, complex scenes.

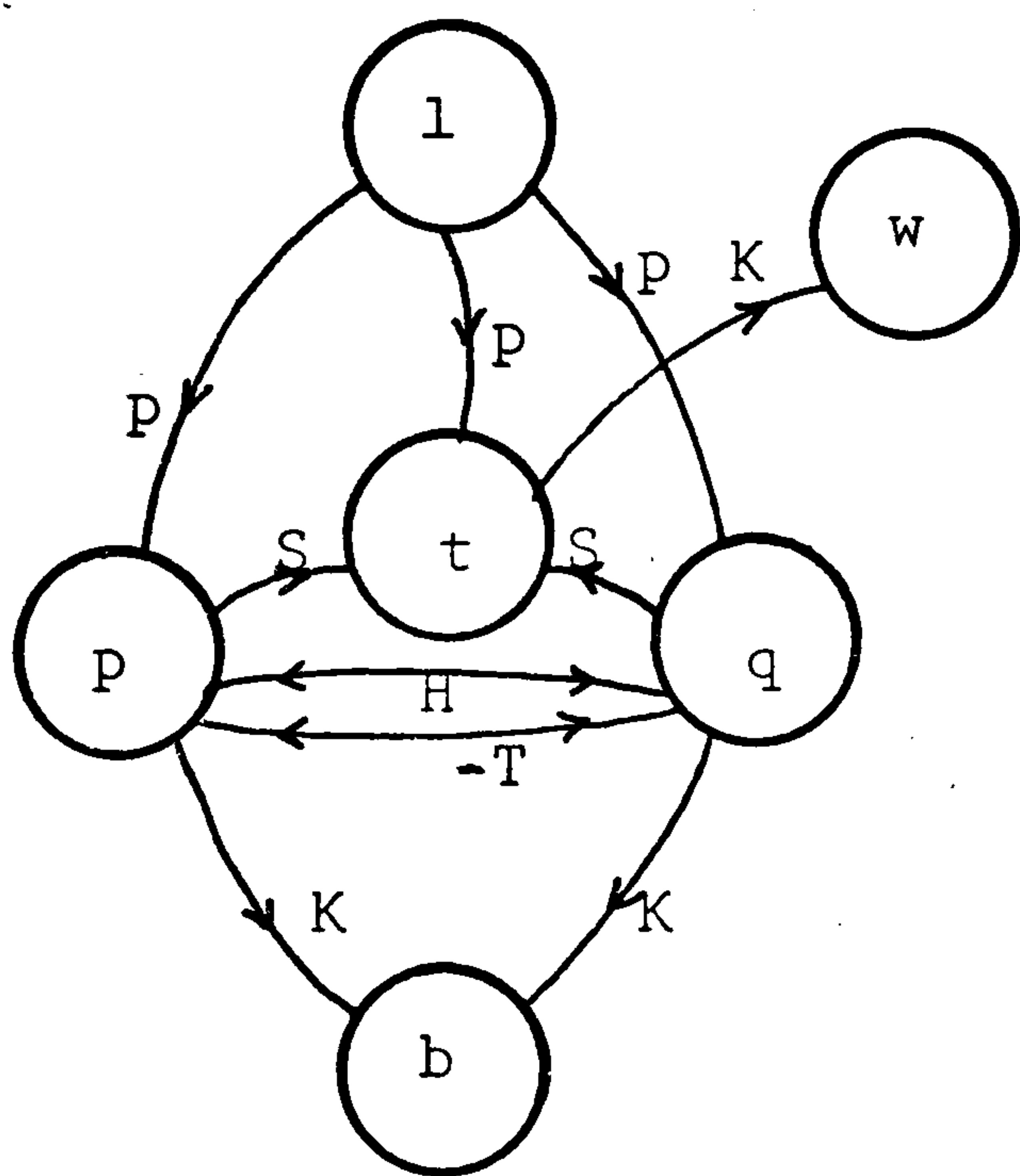
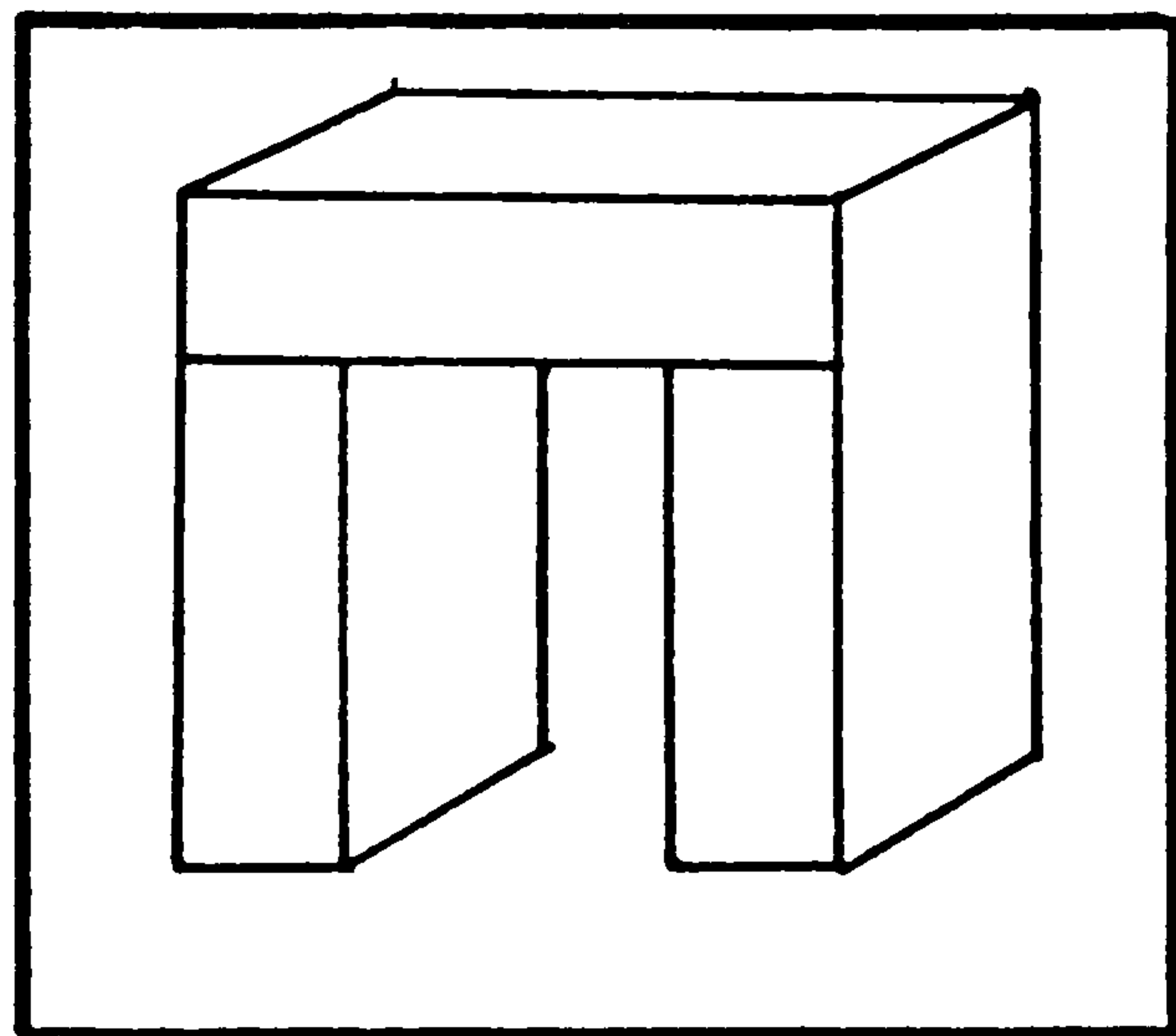
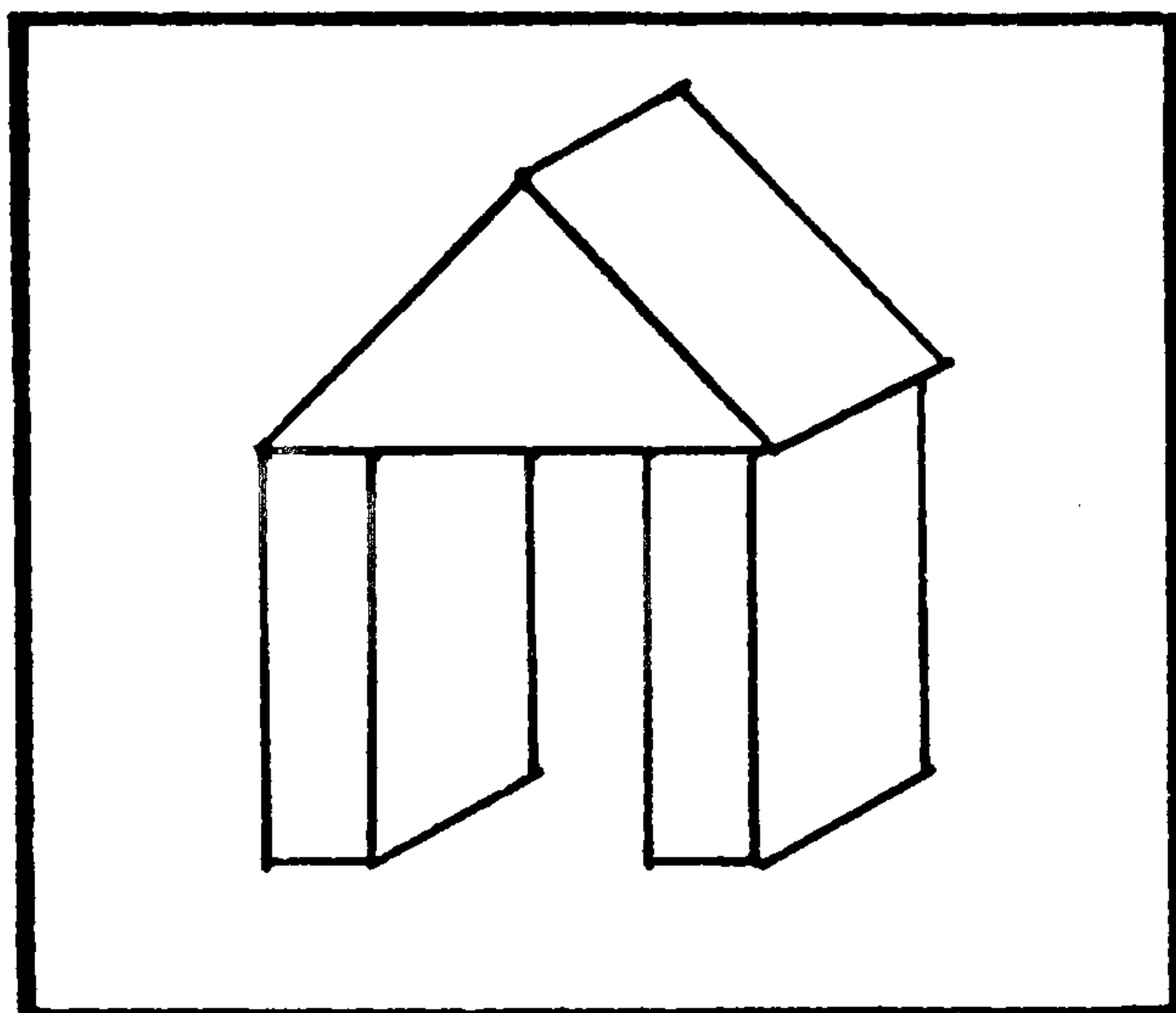
2. Expressions used in the last section, as for example 'much further' or 'closer' are inherently dealing with some kind of *connectivity* or *adjacency* of the k-opens which are involved in the skeletization of k-structures. These notions naturally amounts to the concept of *distance* between k-opens. Morphism emphasis, is mathematically defined in the Appendix. It plays important role in measuring the adjacency of k-opens for the purpose of finding out their resemblance or similarity during the course of comparison stage. Such a connectivity or adjacency *measure* we have used earlier on, nam.  $ISOL2(C_x, C_y)$ , sect. 3.4.3. Others, i.e.  $ISOL1$ ,  $ISOL3$ ,  $D1$ ,  $D15$ ,  $D2$  and so on, are introduced in the Appendix. Most of them are, more or less, algebraic formulae combining the morphism-emphases for the radiated and satellite morphisms. We like to stress a point; that there are various levels of similarity between two k-opens. These may be straightforwardly defined in terms of the different regions into which a k-open can be split; nam.  $TOPOS1$ ,  $TOPOS2$ , and  $TOPOS3$  which have been thoroughly examined in the present chapter. This differentiation gives a departure to an interesting classification scheme of similarity notion, i.e. syntactic, contextual, behavioural, semiotic similarity and so on.

3. The disturbance factor may be, in general, a set of objects or morphisms. This situation looks more complicated but, perhaps, not so difficult to cope with. We do not provide examples of such

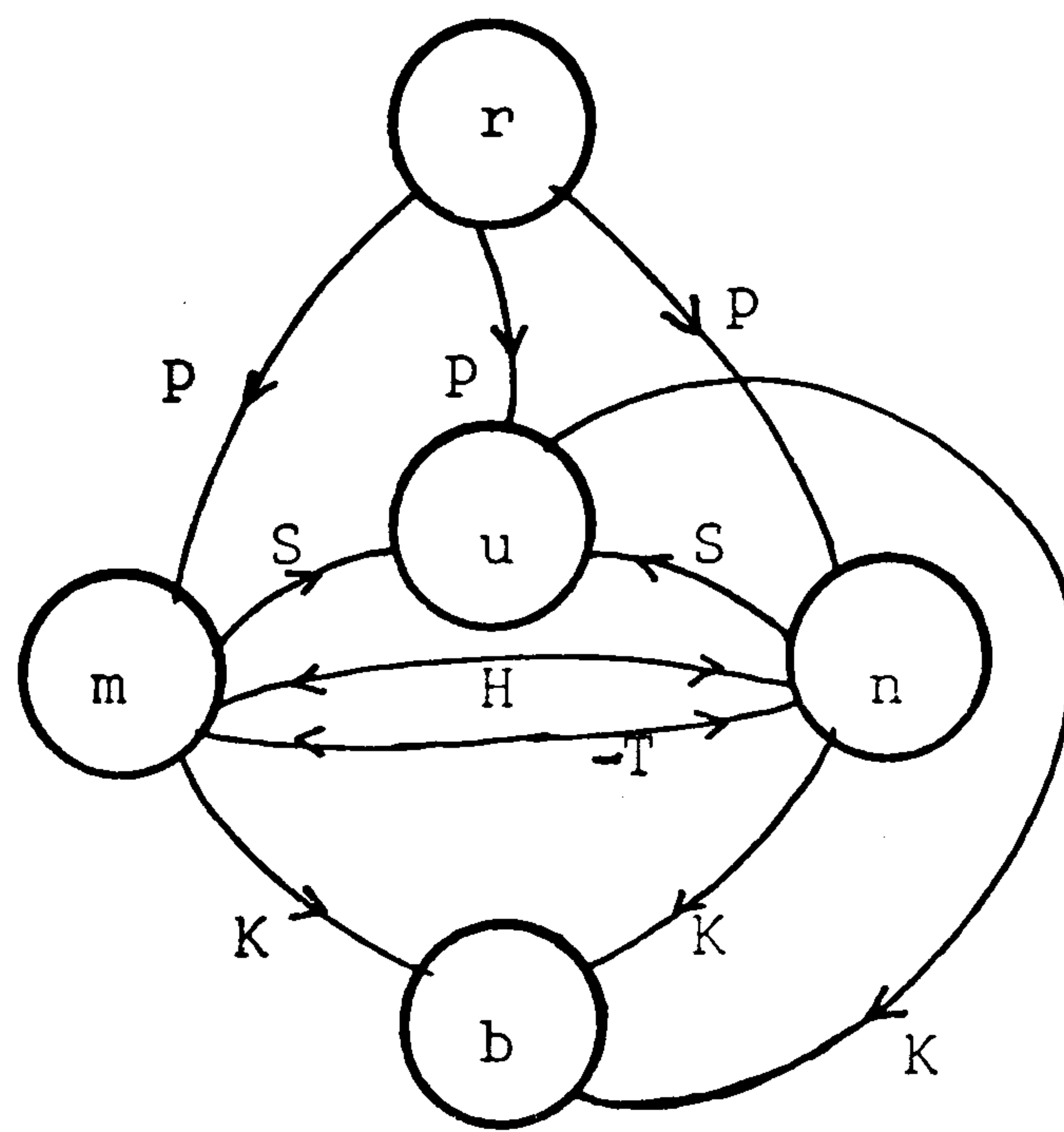


sophisticated cases.

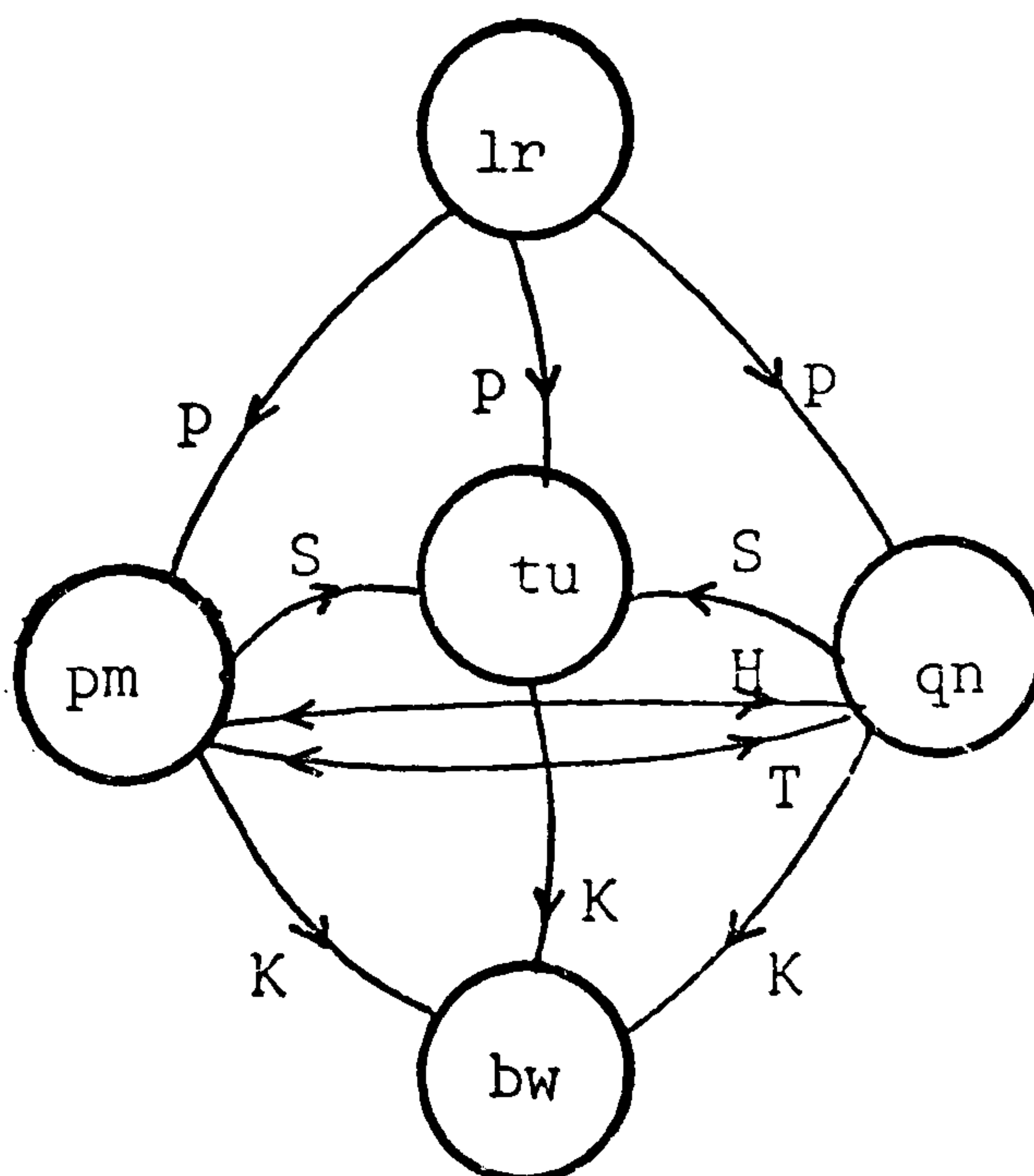
3,4,7 ANOTHER EXAMPLE - CASE STUDY 2



STRUCTURE K1



STRUCTURE K2



SKELETON

- l: left object
- r: right object
- b: block
- w: wedge
- P: "part is"
- S: "supports"
- H: "same height"
- K: "kind of"
- T: "not touching"

### 3.5 ANALOGICAL INTELLECTUAL ACTIVITIES

#### 3.5.0 INTRODUCTION

One of the aims of the present work is to describe some insights of an algebraic model\* (and its mathematical background) of the concept of analogy between k-structures; the latter might be loosely considered as the case may be, as universes of discourse, conceptual universes or even problem-spaces and generally as some sort of nets representing knowledge.

Fundamental to problem-solving, productive/creative thinking and cognitive learning are analogical thinking and reasoning. It is the conceptualization, discovery, realization, generation and exploitation of an analogy\*\* between two (or more) different domains of knowledge, fields of activities (phenomena or interpretations) or universes of discourse which: underlays the solution of an unsolved or new problem, forms the basis for finding out a new theorem or proof, and suggests innovation and the basis for theory (and methodology) unification; providing thereby the means for economizing and effectively utilizing thought; the latter being, what we call elsewhere, 'skeptomomy'\*\*\*, sect. 2.4.4.

Root and basic motivation in the conceptualization and development of the present model and also in the formulation of the analogy concept as thinking/learning/problem-solving tool, has been the very problem of shifting of representation in the human brain, when we

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\* Lacking in KLING's (1971) work as he clearly points it out, p. 177.

\*\* HESSE (1963).

\*\*\* Or 'intellectual ergonomics'.

face new (for example, problem-solving) situations.

We claim, intuitively, that the discovery and utilization of analogies will hopefully lead our ambitions in the correct way of achieving this sort of (artificial) intelligence which is attributed to creative\*, productive\*\*, innovative\*\*\*, not merely mechanical\*\*\* thinking which crucially characterizes and differentiates humans from other creatures. To support this intuitive feeling, a number of suggestions, could be cited. To name but a few, AMAREL, EVANS, KLING, SLOMAN, MINSKY, BLEDSOE and so on.

What our model of analogy amounts to is captured in the following: 'given two k-structures, an analogy between them is understood as a functor between them'.

Finally, as MacCARTHY recently\*\*\* suggested and stressed, "a conjectural approach to AI and real world problem-solving is what we should look forward to in the future". We next present some ideas on conjectures and a conjecture generator.

### 3.5.1 ON CONJECTURES

Closely connected with analogies are arguments\*\*\* by analogy, the so-called conjectures\*\*\*, which mark and underlie major or

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\* AMAREL (1966), AMOSOV (1975), POINCARÉ (1913), POLYA (1962)

\*\* WEIRTHEIMER (1961). \*\*\* PASK (1975)b, (1976).

\*\*\* MCKAY (1952), (1959)

\*\*\*\* IJCAI (1977) conference, held at M.I.T., August 1977.

\*\*\*\* LLOYD (1958)

\*\*\*\*\* For conflicting conjectures, see POLYA (1954) Vol. 2, p. 20.



minor innovations throughout the historical development of mankind. Many examples could be cited here which we shall not discuss due to lack of space; POLYA's (1954) Vols. 1 and 2 are full of mathematical examples; see also: BURGESS (1969), HESSE (1963), ARBIB (1972), LEATHERDALE (1974). The following conjecture-generator may be used as a guiding tool for an innovation-generator working in analogizing manner. In summary, it is given below:

1. Scan, observe input data;
2. Establish, create, compute analogy;
3. Conceive, describe, formulate conjecture;
4. Test, validate conjecture;
5. Accept or reject conjecture.

Next a more detailed sequence of instructions for a conjecture generator is given, based in the so-called 'inductive analogical reasoning for conceiving conjectures'; (psychology of invention, HADAMARD (1945)); also POLYA (1954), EVANS (1963). As far as the possibility of complete systems of inductive inference is concerned, see MELTZER (1970)c.

Remarks.-

1. Comparing the above sequence 1 to 5 with the flow-charts on "steps in a simulation study" and 'Development of a simulation model', MELETIS (1974) App. 3A, 3B, a number of similarities can be drawn.
2. Link conjectures to the concept of "*imitational semiotic simulation*" (recently used in the Institute of Economics, Odessa branch, Acad. Sci. Ukr. USSR) as a basis for the development of an A.I. approach to the problem of sea economics and ecology.

### 3.5.1.1 A Conjecture Generator

*Suggestive points* of contact during an analogizing or conversational\* situation.

- 1.1 Scan, observe existing relations, properties and extract special features among given (geometric say) objects.
- 1.2 Notice, detect resemblances, symmetries, similarities, equivalences, invariants among observed relations, properties, etc.
- 1.3 Form partitions, classify relations and properties, order objects, recognize relations which bear similarities, isolate relations, analogous to each other by finding correspondances between them and common characteristics.
- 2.1 Generalize from analogous relations.
- 2.2 Abstract to the most possible general relation, (upon what criteria?).
- 2.3 Formulate clearly the general relation as conjecture.

*Supporting points.*

- 3.1 Try various cases to validate conjecture (by giving, say, other geometric objects or questioning appropriately the system).
- 3.2 For every verification increase truth-membership of conjecture. Add to its plausibility.
- 3.3 Try extreme cases; i.e. minimal or special elements of appropriately chosen special sets of objects.
- 3.4 Adapt conjecture to reality; i.e. find more 'suggestive points of contact' of conjecture with the real world.

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\* PASK (1975)a, (1976).

- 3.5 Carry out 'quasi-experiments'; i.e. find other favourable signs to increase conjecture's credibility.
4. If conjecture is not accepted go to end; otherwise:
- 4.1 State conjecture.
- 4.2 Find and apply formal methods to prove or disprove conjecture.
- 4.3 If conjecture doesn't pass theoretical tests, go to end; otherwise:
- 4.4 State conjecture as some sort of general rule (or even a theorem, if you like; thus theorem-generator).
5. END.

### 3,5.2 ON HARMONIZATION

We assume that one of human aims, during the course of carrying out various sorts of intellectual tasks or even everyday common sense activities, is 'the unification/economization and effective utilization of thoughts' which we call 'skeptomomy' for reasons we give in sect. 2.4.4. We claim that this target is, in some way, epistemologically and heuristically, at least for a class of problems\*, feasible via appropriate 'harmonization of skeletons' (HS). The latter amounts to some sort of intelligent navigation\*\* of skeletons which are amalgams of k-structures.

In the case of problem-solving, skeletons could be considered or interpreted as representations of abstractions of problem situations; o

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\* Nam, geometric-analogy type; EVANS (1963).

\*\* Or search through a net of analogies.



as a reduction or 'quotient of problematic theses' notions, to which we refer in sects. 2.4.1., 3.4.1., and are made clear later on in this chapter. Harmonization of skeletons could become a general cybernetic technique, and it may be characterized as a model of (analogical) problem-solving. It provides\* a methodology \*\* towards the choice of a desired target problem-situation (problematic thesis  $\theta$ ), via appropriately chosen metrics or distances of k-spaces

In fact, the notion of a k-structure is further analysed and a knowledge representation scheme is presented in a subsequent chapter entitled 'Prolegomena to a theory of k-structures', for a departure into theoretical issues.

### 3.5.3 SHIFTS IN ANALOGICAL PROBLEM-SOLVING

In the section 3.2, it was mentioned that we conceive the notion of shifting of representations during analogizing intellectual activities as a sequence (rather a net) of analogies between (skeleton of) k-structures. As it turned out from sect. 3.4, a skeleton, the result of skeletization, is some kind of k-structure. Loosely speaking, it is a sub-k-structure commonly shared by the parent k-structures\*\*\*. The results gathered in that section were exemplified via WINSTON's (1970) structures, and they were mainly concerned with some issues from the domain of learning. We are now going to focus our attention in

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\* Rather: it is a framework for ...

\*\* Based on skeletization for reduction purposes.

\*\*\* May be possible that the parents are the same k-structures.

the area of problem-solving. We assume, however, that both learning and problem-solving

1. are genuine intellectual activities;
2. they do unquestionably require intelligence (sect. 2.2.2);
3. they could be carried in analogizing manner (or both they can undertake the analogical mode); and
4. these domains are rich in shifts of representations.

We finally accept that mechanization of shifting of representation is a crucial, long term target of A.I. research; AMAREL (1966), (1967), (1970) and others.

Hoping that the present thesis would result some hints towards that goal, we next give an informal, brief account from our ideas on the matter; leaving thus the details to be developed in the future.

In sect. 2.4.1, we informally introduced the notion of shifting from a representation, describing a situation,  $S_1$ , to another representation, describing a situation  $S_2$ , as a kind of correspondance  $\delta$ , map if you want, between  $S_1$  and  $S_2$ . Furthermore, a detailed account of the factors that  $\delta$  may depend on, was given in 2.4.2. Here, we take up again this notion, restricting it, in some sense, into the area of analogical problem-solving and wherever that is possible, we provide examples which are concerned with EVANS's (1963) work.

A number of questions that naturally come up to our mind are: what do we mean, in the present context, by:

1. problem-representation,
2. shifting of problem-representation, and
3. mechanization of shifting of problem-representation and additionally.

4. what the concept of analogy has to do with them?
5. what role could it play?
6. can a formal account(model) be devised for analogy?

We take up this questions again in sects. 3.5.5.5 and 3.5.5.6.

In the next section, we present an informal description of our understanding of problem-solving.

#### 3.5.4 A GENERAL DESCRIPTION OF PROBLEM-SOLVING

Let  $\pi$  be a given problem to be solved. In a very general sense, the course of problem-solving may be loosely or informally described, following the same, more or less, phraseology of section 2.4.1, as follows:

A variety of assumed, known, realizable or deducible structures  $\Sigma$ , with various degrees of complexity and relevancy to the problem  $\pi$  in question, pass\* through the mind of problem-solver, say  $\lambda$ . Then  $\lambda$  is trying to choose, with some epistemological adequacy, that appropriate structure which 'pseudo-semiotically' is related (or match in a stronger sense) to the current problem-situation or problematic thesis, say structure  $\theta_{\pi}^x$ ; ( $\theta\epsilon\sigma\iota\varsigma$ : Greek for situation). This correlation or matching may be conceived as a map between the structure  $\theta_{\pi}^x$  and one of the structures  $\Sigma$ .

Thus,  $\lambda$  proceeds constructing, heuristically, a partially reasonable sequence (generally net) of correspondances  $\delta_i$  between  $\theta_{\pi}^x$  - structures; nam,



$$\begin{aligned} \delta_i^\lambda & : \theta_\pi^\lambda \longrightarrow \theta_\pi^\lambda \\ & : \theta_\pi \longmapsto \delta_i^\lambda(\theta_\pi^x), \quad i \in I \subset \mathbb{I} \end{aligned}$$

where,  $\theta_\pi^\lambda$  is a collection of  $k$ -structures, the set of problematic situations or theses the problem-solver  $\lambda$  is going through, trying to solve problem  $\pi$ .

$I$  is an index set, subset of integers  $\mathbb{I}$ .

$\lambda$  indicates an arbitrary problem-solver and it may be omitted from  $\delta_i^\lambda$ .

$\theta_\pi$  may consider the set of all possible problematic situations for  $\pi$ .

If sequence  $\delta_i$ ,  $\alpha$ -approximately converges (if at all) to some stable desirable target structure  $\theta_\pi^g$ , hopefully the required one, then we may say that 'problem  $\pi$  is solved by  $\lambda$  with tolerance  $\alpha$ '.

In order to make the above general problem-solving activity a little more concrete, and also to get an idea how a trip into the 'labyrinth of cybernetics' problematique' looks like, we felt the need to investigate some aspects of intellectual activities involved in problem-solving (PS) and especially its mode under the title of analogical problem-solving (APS). Another reason was that we wanted to focus our attention on the shifts involved in PS and, in fact, its analogical mode, nam. APS which involves a large number of shifts.

Finally, we close this section by a brief sentence which might give the flavour of PS; nam., "problem-solving may be considered as the process of acquiring an appropriate set of responses to a situation"; GEORGE (1976).

### 3.5.5 A MODEL OF ANALOGY

#### 3.5.5.1 On Analogical Problem Solving

In the present section, a rather brief account of results is given on matters involving the six-tuple of questions which we have put forward in section 3.5.6. Thus, the following should be more or less considered as a (naive or preliminary if you wish) platform for a departure into more rigorous investigations and theoretical conceptualizations.

We rather concentrate on analogical problem-solving (APS) and we elaborate on some possible answers to the above questions by giving mathematical descriptions for the concepts involved (wherever that is feasible) based on the rather extensive theoretical framework provided by the first sections (3.1, 3.2) the present chapter.

We exemplify these notions by presenting a reformulation of EVANS (1963) descriptions in a concise manner. For this was one of our objectives, described in section 2.7; nam. the clarification of the obscure, in some ways, use of 'objects' and 'transformations' referred to by EVANS (1963).

#### 3.5.5.2 Applications in Evans' problem-domain

To begin with, we understand a representation of problem  $\pi$  as a  $k$ -structure  $\theta_{\pi}$  which may describe  $\pi$  or capture its meaning. For example, in EVANS's case, a problem situation, which we call 'problematic thesis' is a snapshot/instance of activity field of affairs and we describe it in terms of  $k$ -structures. To be more precise Evans's

---

\* It is tacitly assumed that this is feasible.

classic A.I. programme solved geometric-analogy problem which may have the following forms: (use schemata  $\Sigma_1, \Sigma_2, \Sigma_3$ )

1. "figure A is to figure B as figure C is to which of the given answer figure x,  $x \in X$ ?"

2. briefly: "A is to B as C is to x",  $x \in X$ ;

3. diagrammatically:

A	is to	B	as	C	is to	x
---	-------	---	----	---	-------	---

$, x \in X,$

4. correct analogizing schema:

A	is related to	B	As	is related to	C	is related to	x
---	---------------	---	----	---------------	---	---------------	---

5. symbolically:  $\{A:B::C:x\}_{x \in X}$ ; or

A	→	C
↓		↓
B		x

6. as a problematic thesis:

A	is to	B	as	C	is to	x
---	-------	---	----	---	-------	---

$, x \in X$

7. or  $L_{\theta \pi}^x$  as  $R_{\theta \pi}^x$

L: for left k-structure

R: for right k-structure

and via the notion of a functor  $f: K_1 \rightarrow K_2$  between two k-structures, developed in sect. 3.1.8. we may describe or capture the above (in 7.) 'as' relation or correspondence as follows:



$$f_{\pi}^x : L_{\theta_{\pi}}^x \longrightarrow R_{\theta_{\pi}}^x, \quad x \in X.$$

8\*. Metalinguistic level. Let us take into account two of the assumptions made so far;

- I. In section 3.1.11 we point out that k-structures may play the role of objects in a k-structure\*.
- II. The general algebraic meaning of 'a morphism', sect. 3.1.4. is that of 'a function which preserves structure'.

According to I and II and by intuitively interpreting the word 'as' in the above schemata, as some sort of similarity (resemblance\*\*) between the two corresponding 'members' which might somehow capture or embody the preservation of a structure, then we may consider  $f_{\pi}^x$  in 8, (above) as a morphism between  $L_{\theta_{\pi}}^x$  and  $R_{\theta_{\pi}}^x$ ; the latter being now considered as objects. This move actually changes the level of description, nam. we are dealing with a 'higher' k-structure which objects are k-structures and morphisms are functors.

Furthermore, in section 3.1.4, the notions of domain  $D(m)$  and range  $R(m)$  of a morphism  $m$  were introduced. Therefore, in a metalinguistic level the expression (8) may be simply written:

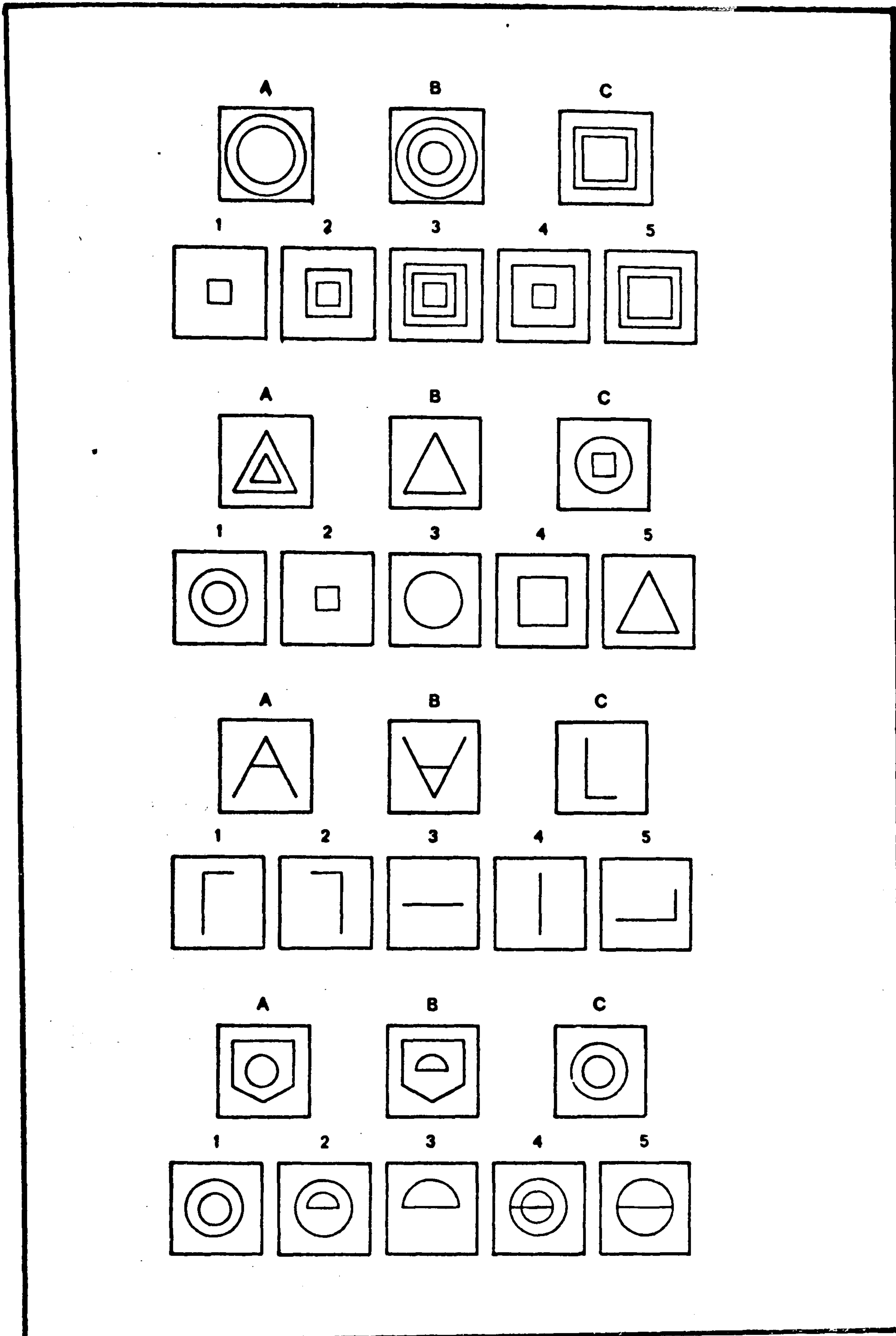
$$f : D(f) \longrightarrow R(f)$$

The *raison d'être* of this departure into metalinguistic issues, it will soon be made clear.

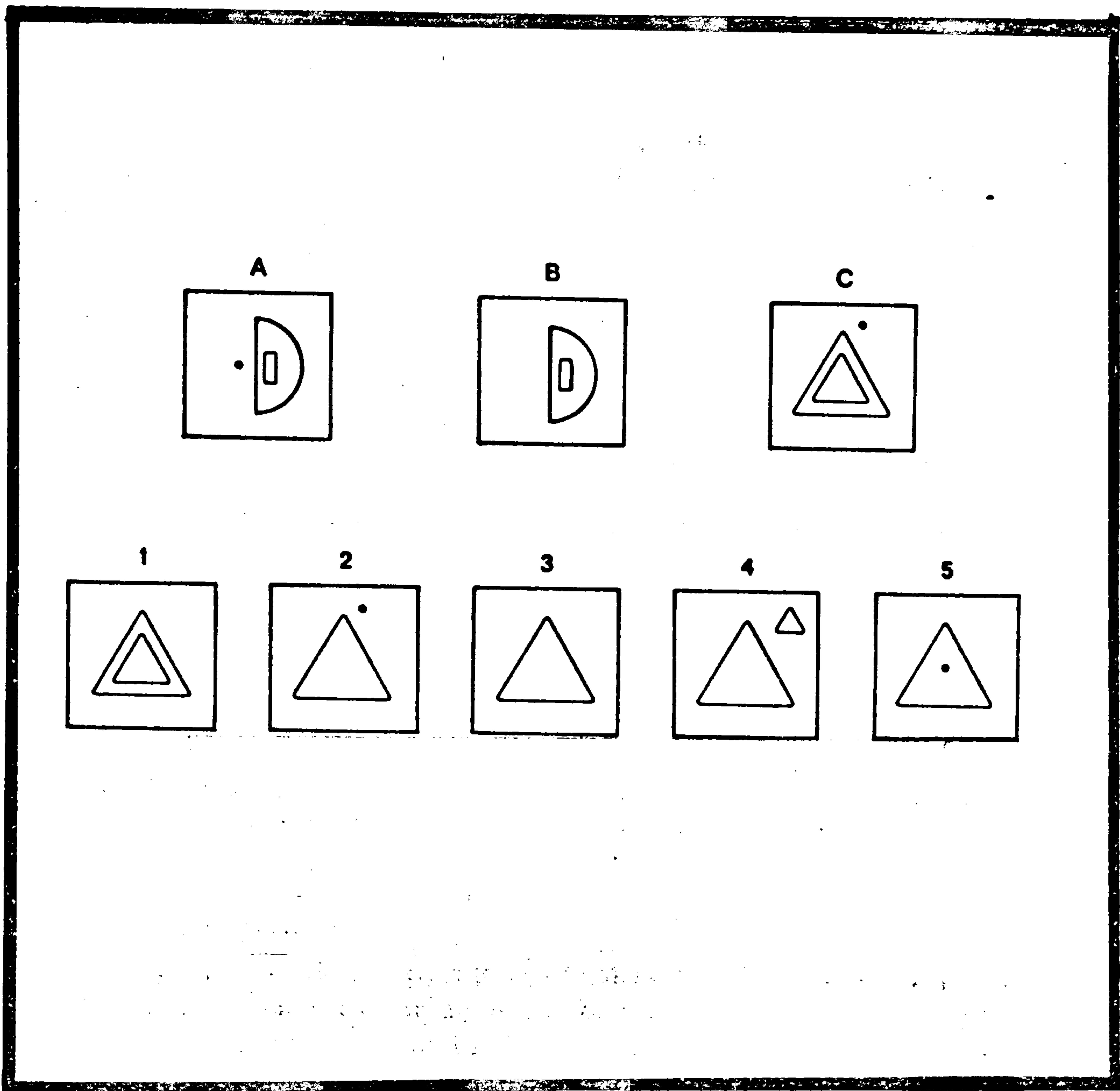
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\* This is perhaps not always true; discussion on this issue is of great challenge (why?). The question is partially taken up again in sect. 2.4.4.2.

\*\* Correlation or even analogy.

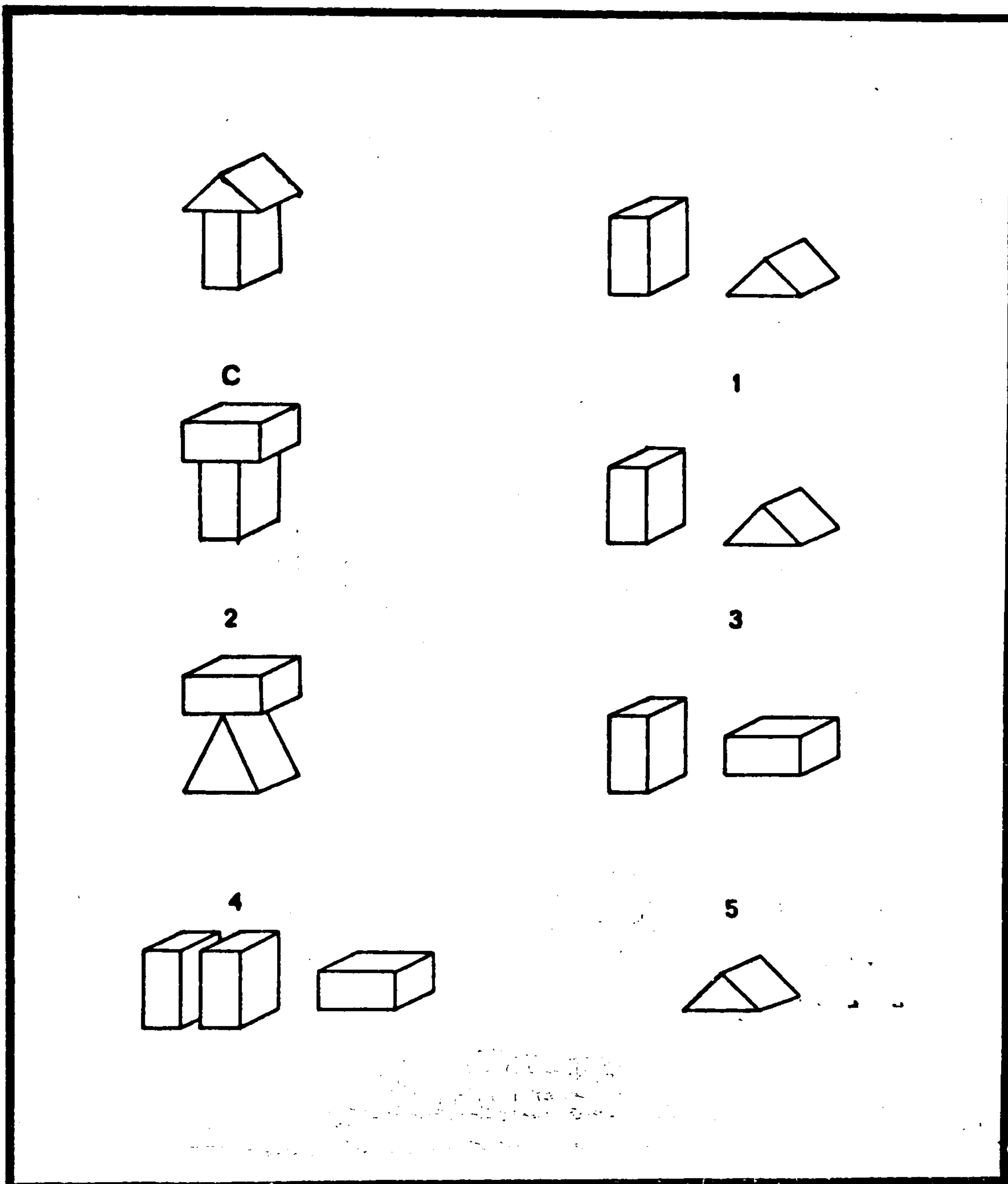


Schema  $\Sigma_1$



Schema  $\Sigma_2$





Schema  $\Sigma_3$

3.5.3.3 Some Theorem Schemata on Analogical Problem-Solving

According to the previously described schematology, therefore, Analogical Problem Solving, for EVANS's problem-domain, may be summarized in the following theorem-like schemata:

$\theta_1$ :	Y	Given : figures A,B,C and $x_i \in X$
	$\Sigma$	Find : $x \in X$ such that $d\{A:B::C:x\} = \min_i d\{A:B::C:x_i\}$

or, via (8) of the previous section,

$\theta_2$ :	Y	Given : $f_{\pi}^{x_i} : L_{\theta}^{x_i} \longrightarrow R_{\theta}^{x_i}$ set of problematic theses
	$\Sigma$	Find : $f_{\pi}^x$ such that $\forall \epsilon > 0 \exists \delta(\epsilon) > 0 : d_{11}(f_{\pi}^x, f_{\pi}^{x_i}) < \epsilon, \forall R_{\theta}^{x_i} :$ $d_{12}(R_{\theta}^{x_i}, R_{\theta}^x) < \delta(\epsilon)$

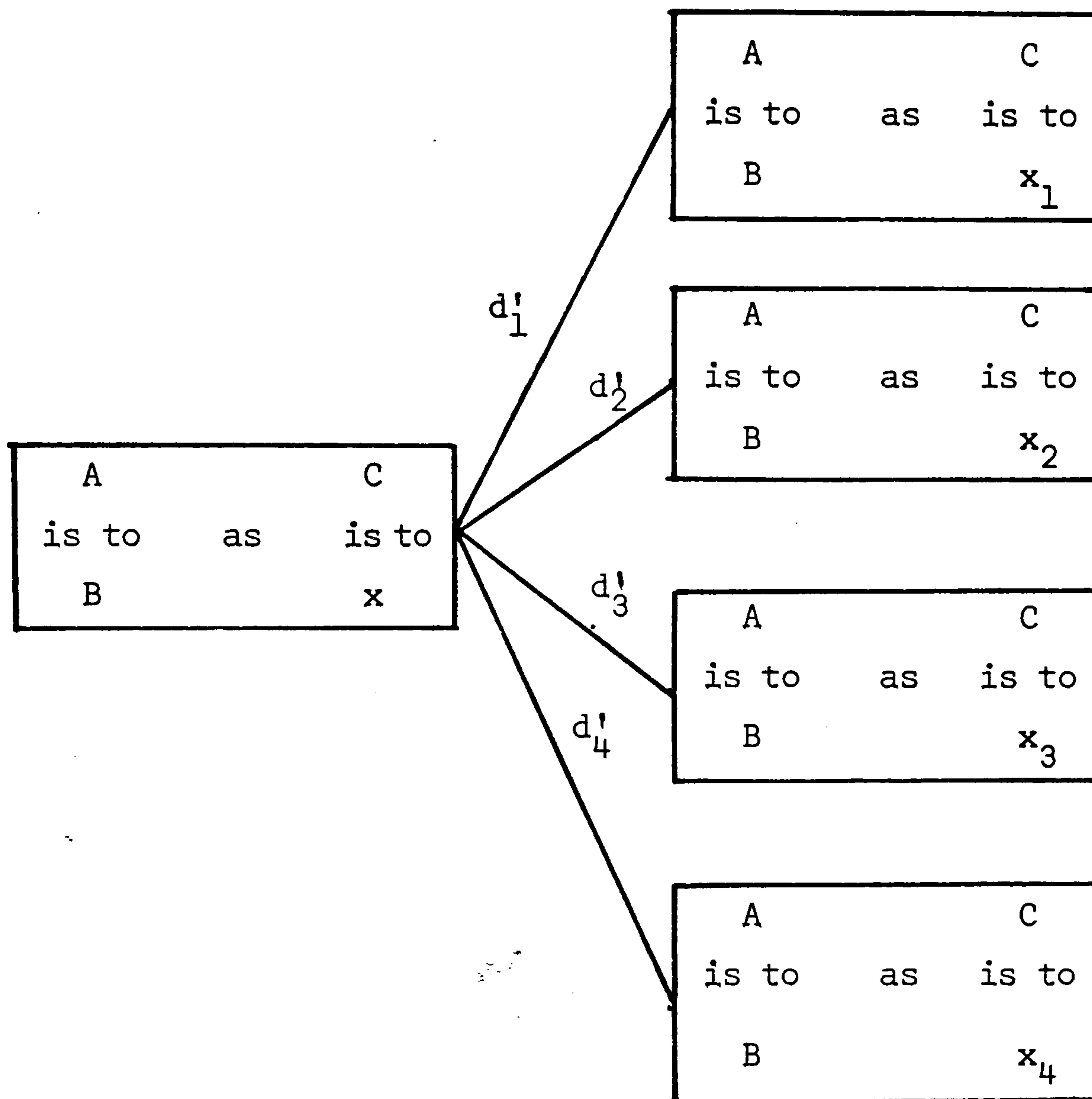
or, via (8\*) of the previous section,

$\theta_3$ :	Y	Given : $f_{\pi}^{x_i} : D(f_{\pi}^{x_i}) \longrightarrow R(f_{\pi}^{x_i})$ , set of morphisms
	$\Sigma$	Find : $f_{\pi}^x$ such that $\forall \epsilon^* > 0 \exists \delta^*(\epsilon^*) > 0 : d_{21}(R(f_{\pi}^x), R(f_{\pi}^{x_i})) < \epsilon^*, \forall f_{\pi}^{x_i} :$ $d_{22}(f_{\pi}^{x_i}, f_{\pi}^x) < \delta^*(\epsilon^*)$

where  $d$ ;  $d_{11}$ ,  $d_{12}$ ,  $d_{21}$ ,  $d_{22}$  are appropriately defined metrics/measures

3.5.5.4 Interpreting  $\theta_1, \theta_2, \theta_3$ 

To put  $\theta_1, \theta_2, \theta_3$  in more illustrative terms, we may say;  
for  $\theta_1$  that: from the next patterns or problem-situations:

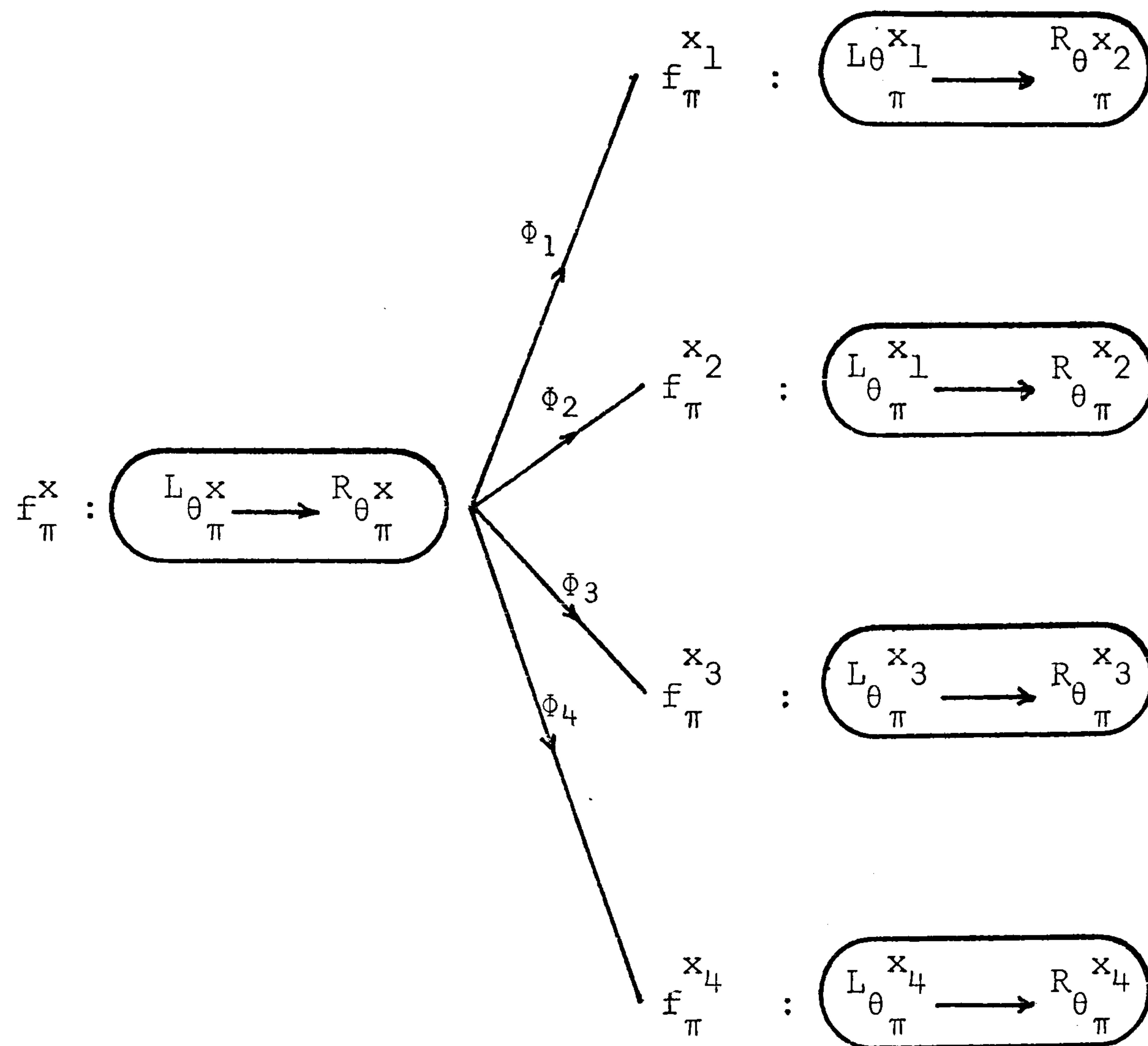


choose the one which makes  $d'$  minimum;

where  $d'$  is a metric in problem-space.



In the same way for  $\theta_2$ , that from the following problematic theses:

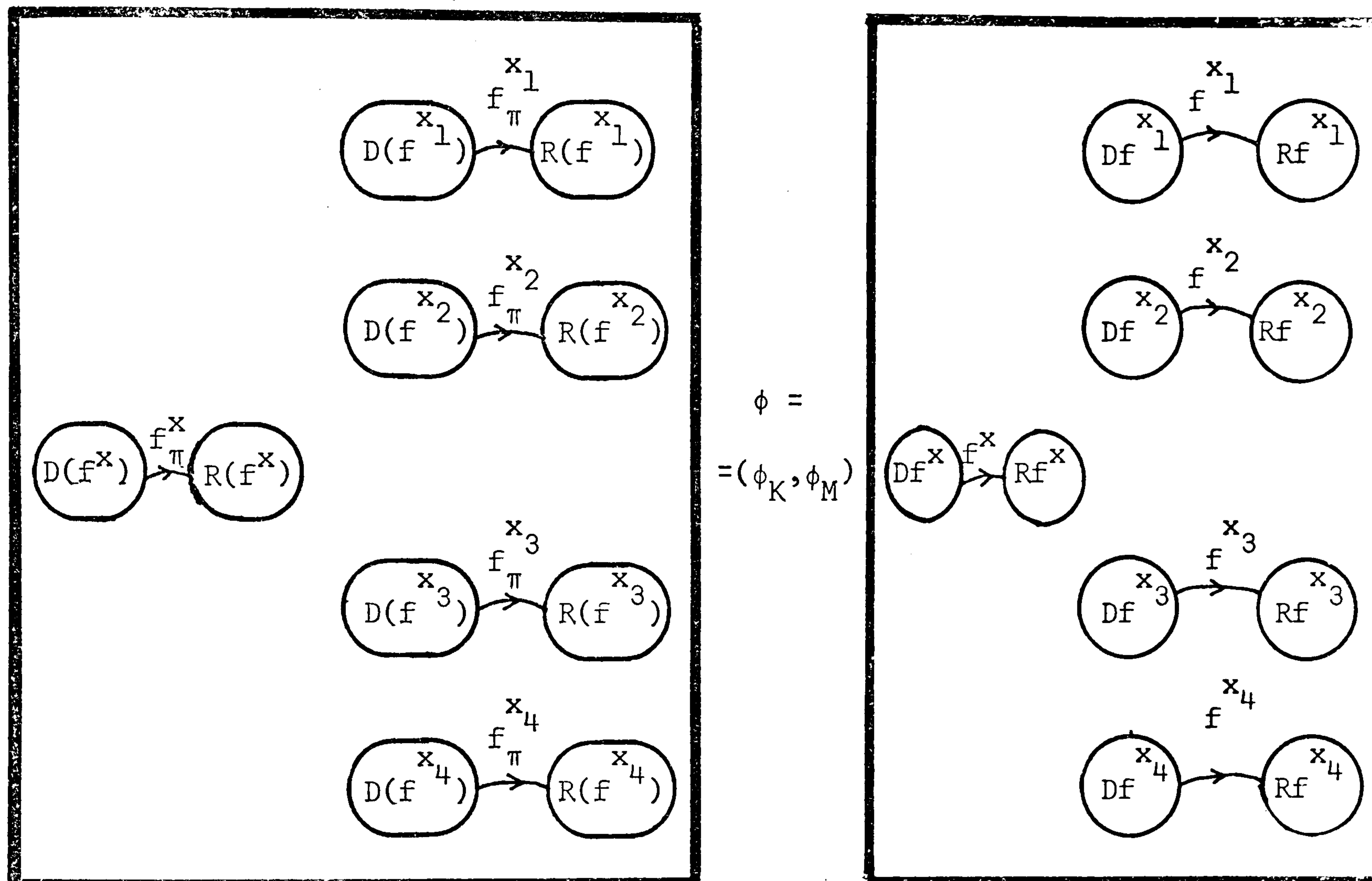


SCHEMA  $S_2$

which are described by functors  $f_s$ ,  
 choose the one, via appropriate selection of a natural transformation  $\phi$  (section 3.1.9), which makes  $\phi$  a "natural equivalence" of the functors  $f_{\pi}^x$  and the answer  $f_{\pi}^a$ ; in other words, choose that which brings them 'very close'.

Finally, we have for  $\theta_3$  that: from the following problematic theses:

SCHEMA  $S_3$



Which are described by the domains\* Ds and ranges\* Rs of morphisms\*\*  
 fs, choose that morphism, via appropriate selection of a functor\*\*\*  $\phi$   
 which makes  $\phi_M$ , the morphism component of the functor  $\phi$  (Sect. 3.1.8),  
 to bring 'as close as possible' the two morphisms  $f_{\pi}^x$  and the  
 answer one  $f_{\pi}^a$ .

(Note that  $\pi$  is sometimes omitted for clarification)

\* k-Structures.

\*\* Functors between k-structures.

\*\*\* Endo-functors, i.e.  $\phi : C \rightarrow C$ , where C is a k-structure made up from Ds, Rs as objects and fs as morphisms.

### 3.5.5.5 Applying theorem schemata to the shifting of representations and its mechanization.

Taking into account the previous schematology, we are now going back to section 3.5.3 to provide some answers to the rest of the sextuple of questions.

The shifting of representation has to be divided into two parts. The first is some kind of internal shifting, let us call it horizontal shifting, and it may be described via the functors  $f_{\pi}^{x_i}$ , schema  $S_2$ , which map left  $k$ -structures to right  $k$ -structures. This shifting may be controlled by a second type of shifting which is describable as a correspondence  $\Phi$  between two  $f$ s. Let us call it methodology or vertical shifting.

Thus what the mechanization of shifting amounts to is an appropriate choice of a map  $\Phi$ , i.e. method, which it would assign a transformation  $f$  to a 'much better' one, and eventually into the solution. The formalization of the mechanization of shifting may be captured via the theorem-schema  $\theta_2$ , sect. 3.5.1.0.

### 3.5.5.6 On analogy and its role in Shifting of Representations

As far as the notion of analogy is concerned, we say that it amounts to the following: given two  $k$ -structures, an analogy between them may be described and understood as some kind of map/function between them. In fact, it may be illustrated as a functor (sect. 3.1.8) between them\*.

---

\* Furthermore, a skeleton may be considered as an embodiment of an analogy.



The role of analogy in shifting is to bring about the maximum resemblance/similarity between the two parents k-structures.

Controlling a family of analogies, i.e. fs, is to choose\* an appropriate transformation  $\Phi$ , see theorem-schema  $\theta_2$  and schema  $S_2$ , which it would bring two fs 'very close' as it is formalized by  $\theta_2$ .

---

\* Which is the main job of harmonization (Sect. 3.5.2),

CHAPTER 4

*"La scolastique, qui produisit dans la logique, comme dans la morale, et dans une partie de la métaphysique, une subtilité, une précision d'idées, dont l'habitude inconnue aux anciens, a contribué plus qu'on ne croit au progrès de la bonne philosophie"*

CONDORCET

Vie de Turgot



#### 4. PROLEGOMENA TO k-THEORY

##### 4.1 'SLOW MOTION' CONSTRUCTION OF A k-STRUCTURE

We start with a construction which stems from a basic fact, mentioned throughout our work; that is, the morphisms are the most important constituents of a k-structure; the latter has been introduced in 3.1.4. We are next giving an informal construction for a k-structure (and thus for a skeleton\* section 3.4, considered as made up from attributes of a nature similar to those of a k-structure). A k-structure may be viewed as a sort of 'Superposition of many morphisms' (say for ex. relations). This, in set theoretic terms, may be rephrased as 'superposition of the cartesian products (graphs) of relations'. Thus, a k-structure's realization/derivation could be based on 'consecutive embedding of morphisms' technique, which is achieved via the following two (macro-) steps:

MSTEP 1 : Embed identities; that is, identify morphisms of objects of a k-structure. An interpretation of that MSTEP could be: supplying a 'pool' with objects; a metaphor also used in Ch. 3.

MSTEP 2 : Embed genuine morphisms; i.e. filling the 'pool' with pure morphisms.

The whole business of MSTEP 1 and MSTEP 2 might be captured in the following sentence: "Establishing links between objects". Next, we give a 'slow-motion' or piecewise construction of a k-structure via morphisms. Schemata 1-10 and 11-14\*\* constitute MSTEP 1 and MSTEP 2 respectively.

---

\* Yielding the schema  $\Sigma$ , shown later on, which illustrates the result for a construction of a skeleton for analogy-type problematic thesis, that of sect. 3.3.1.

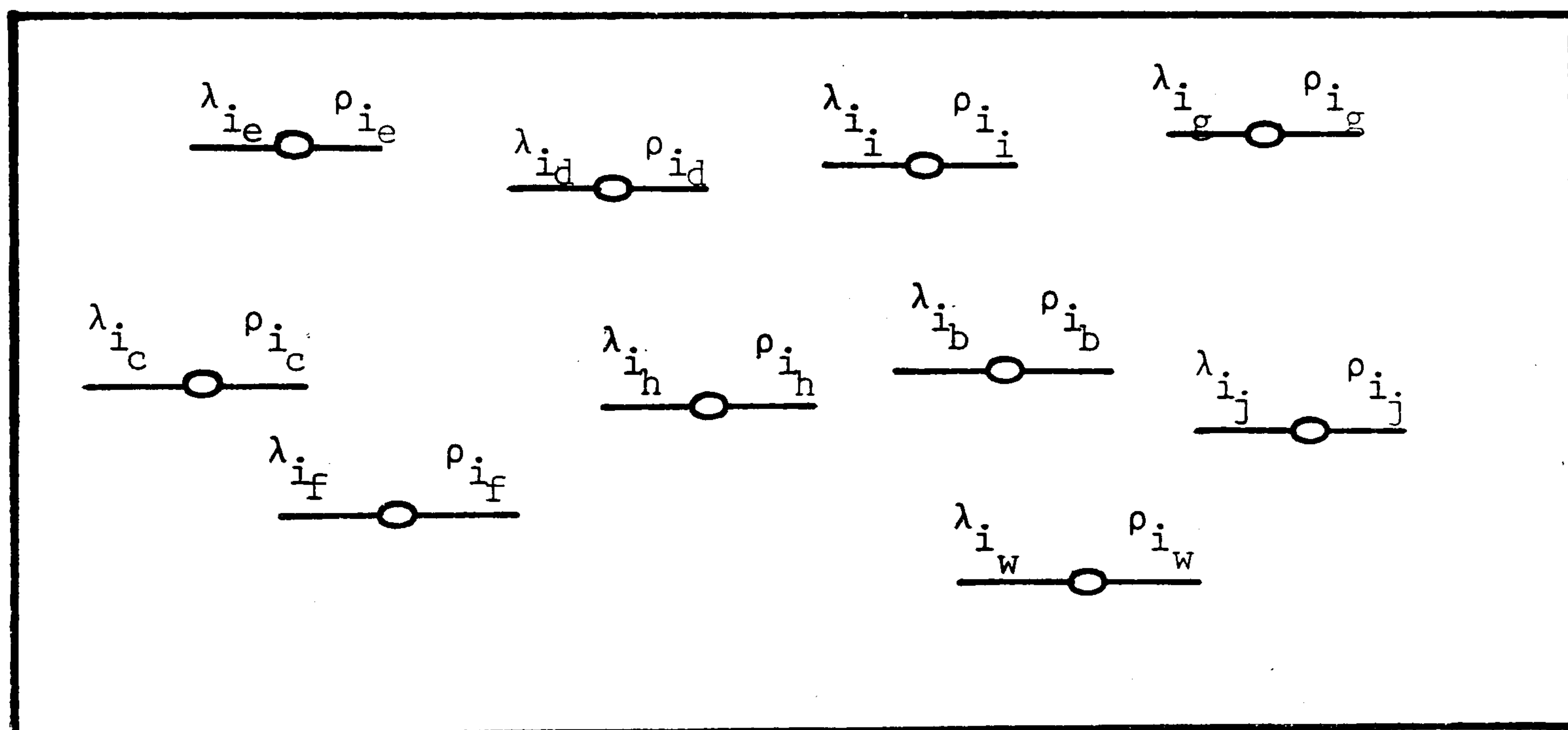
\*\* We omit the labelling of identities to make them clear.

Note that:

$$\begin{aligned} \{\#steps\} &= \{\#objects\} + \{\#morphisms\} \\ &= \{\#identity\ morphisms\} + \{\#genuine\ morphisms\} \end{aligned}$$

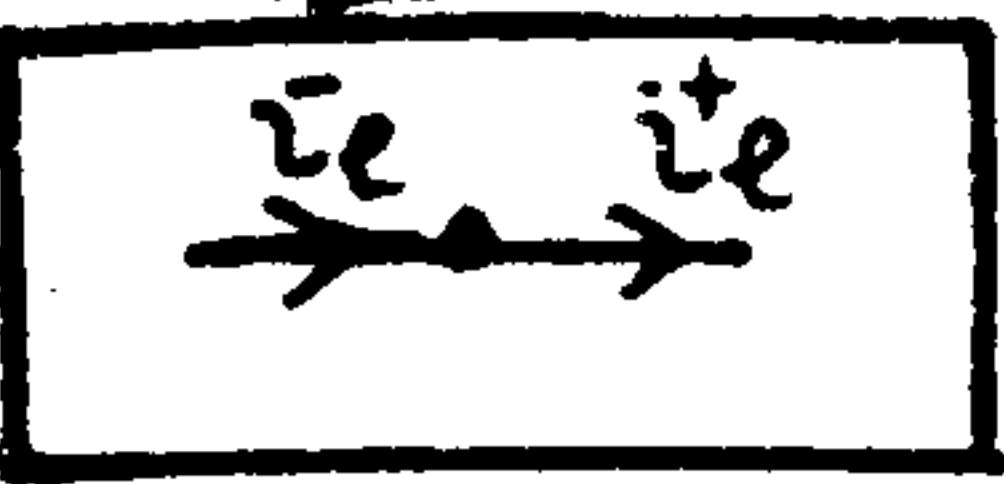
where  $\#$  denotes 'the number of...'

The so-described 'slow-motion' construction, reminds one some sort of 'cellular-like organism in evolution'\* so to speak. Each figure may be considered as representing a snapshot of the above mentioned 'pool', a picture of which is given below, at the end of MSTEP 1.

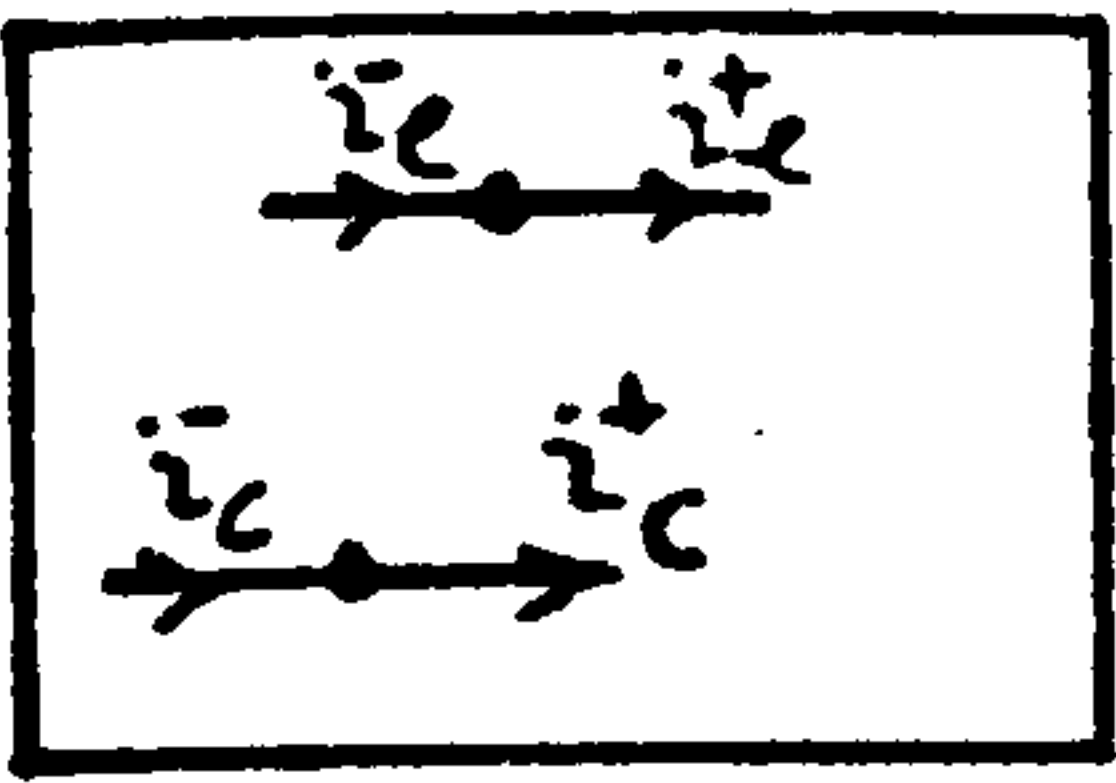


Left and right identity morphisms ( $\lambda_{i..}$ ,  $\rho_{i..}$ ) are 'floating', randomly around in the pool, until a genuine (invoked) morphism eventually links some of them up. This 'eventuality' depends in the case of a skeleton's construction, on the current content of the skeletization Universe. The so-characterized random movements in the pool, gives rise to talk about 'pending objects' and 'dangling morphisms'. Other names of the latter situation are: "uninstantiated (objects)" used by SUSSMAN (1975) and SACERDOTI (1975); and in the skeleton case, embedding

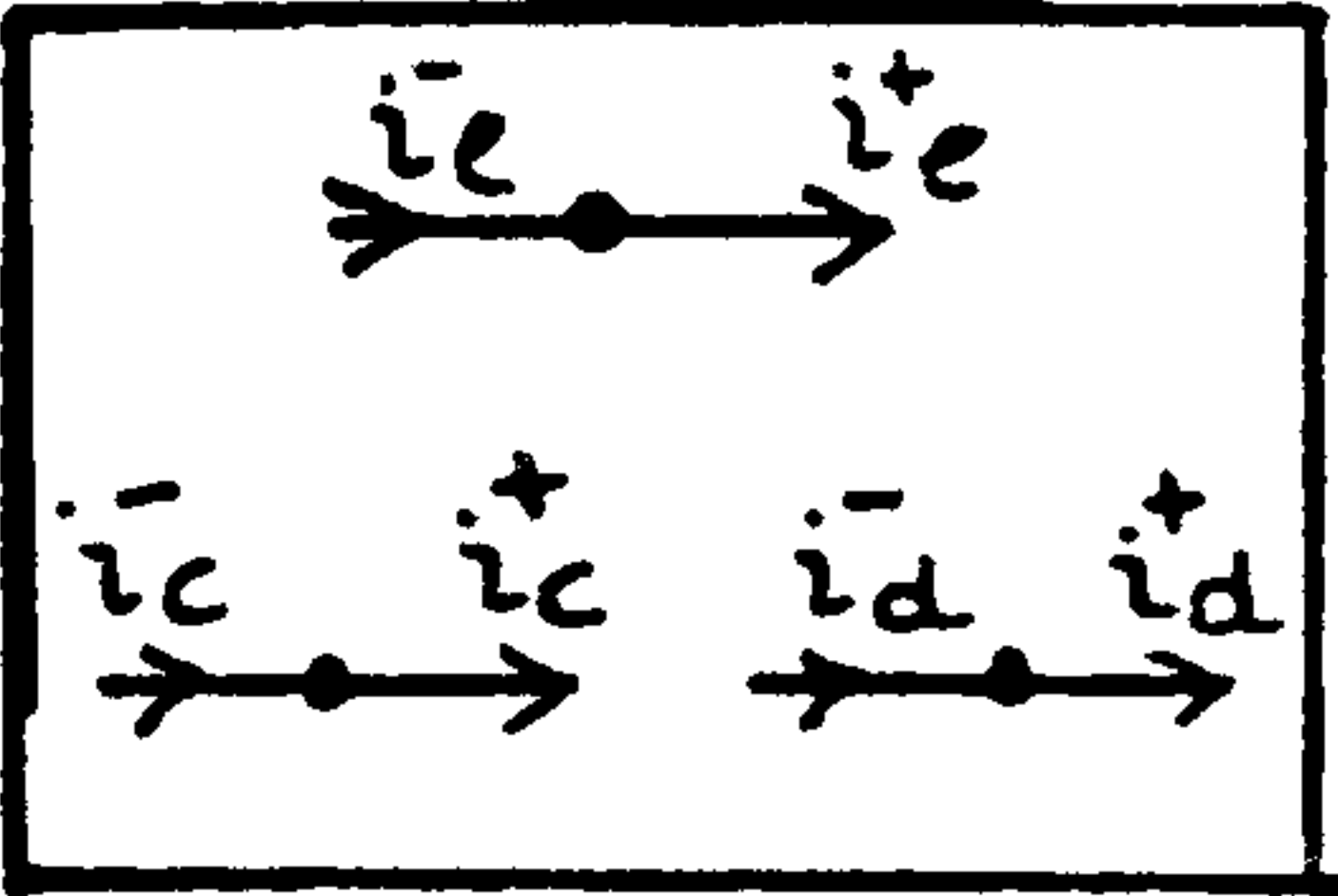
\* Which is linked to issues on cellular, Von Neuman (1966), and tessellation automata, for self-reproduction and evolution.



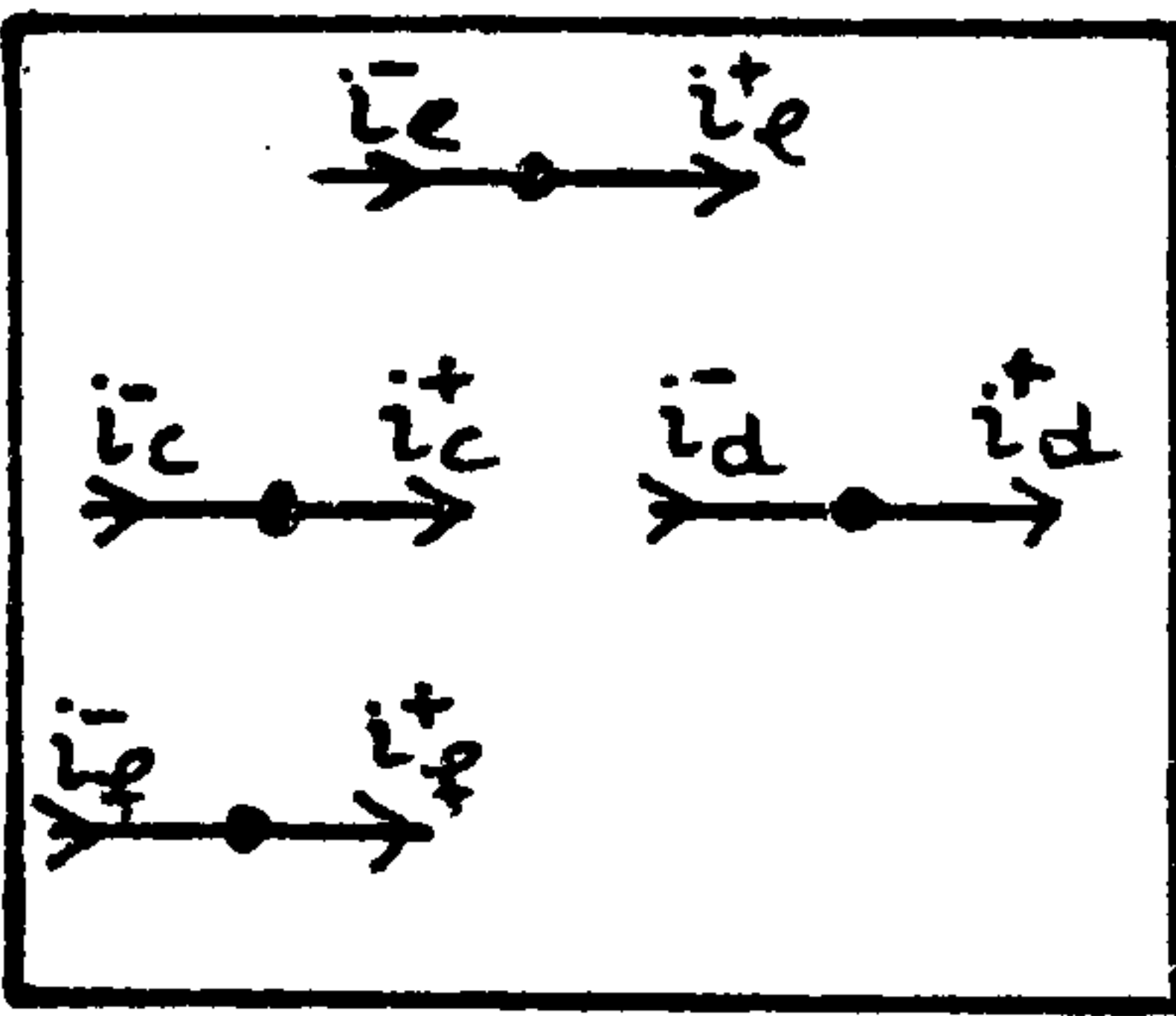
Sch. 1



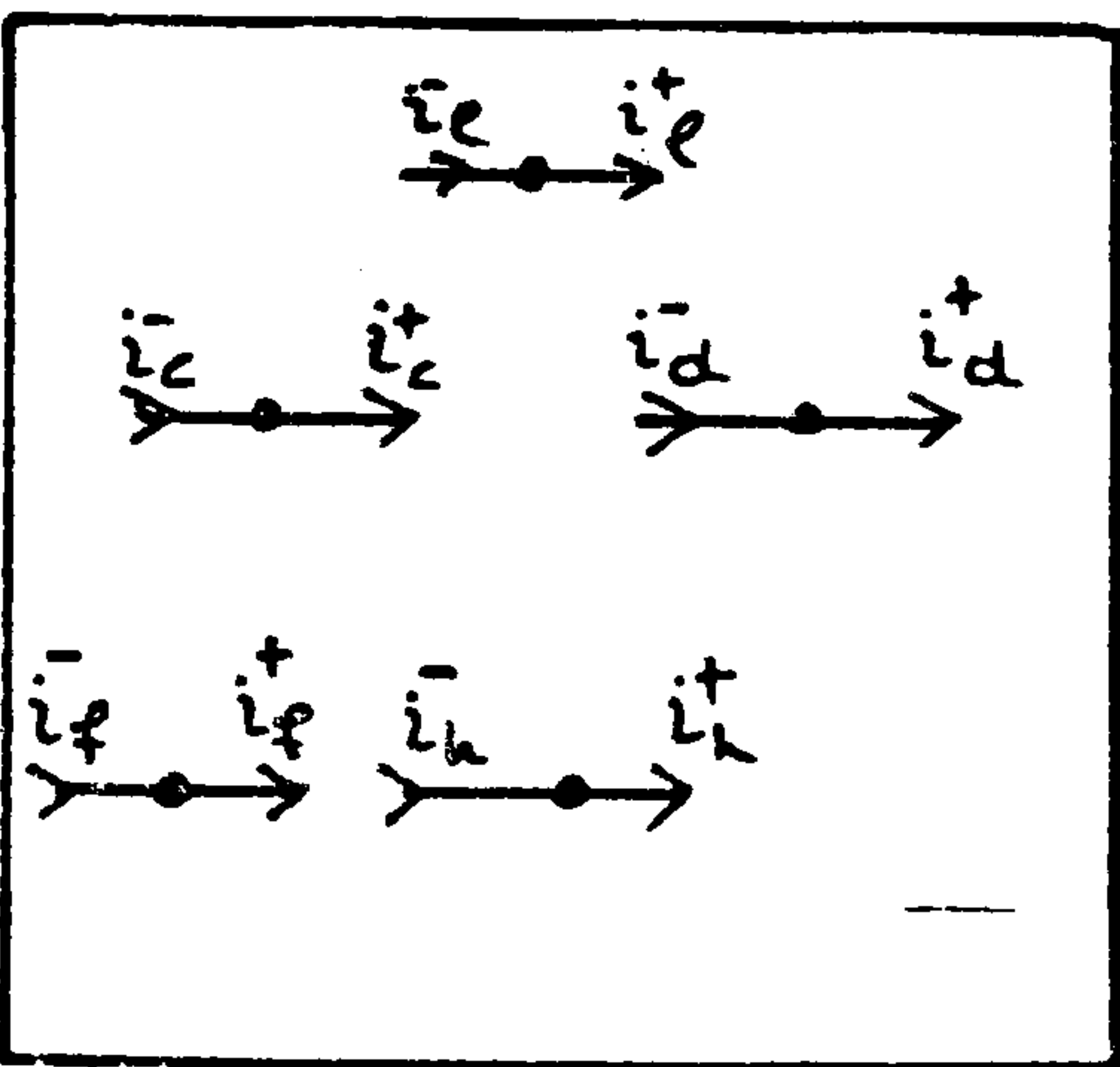
Sch. 2



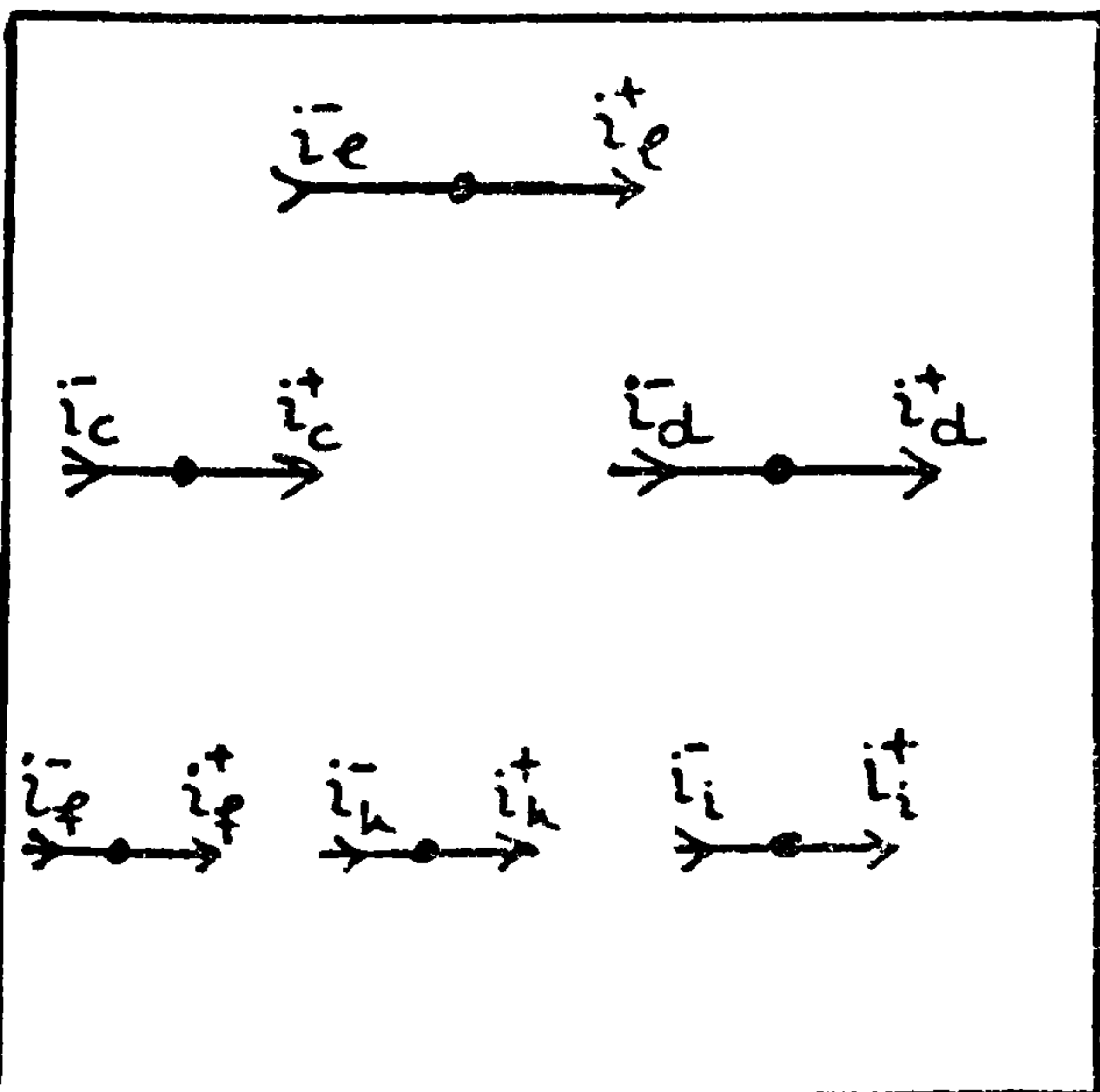
Sch. 3



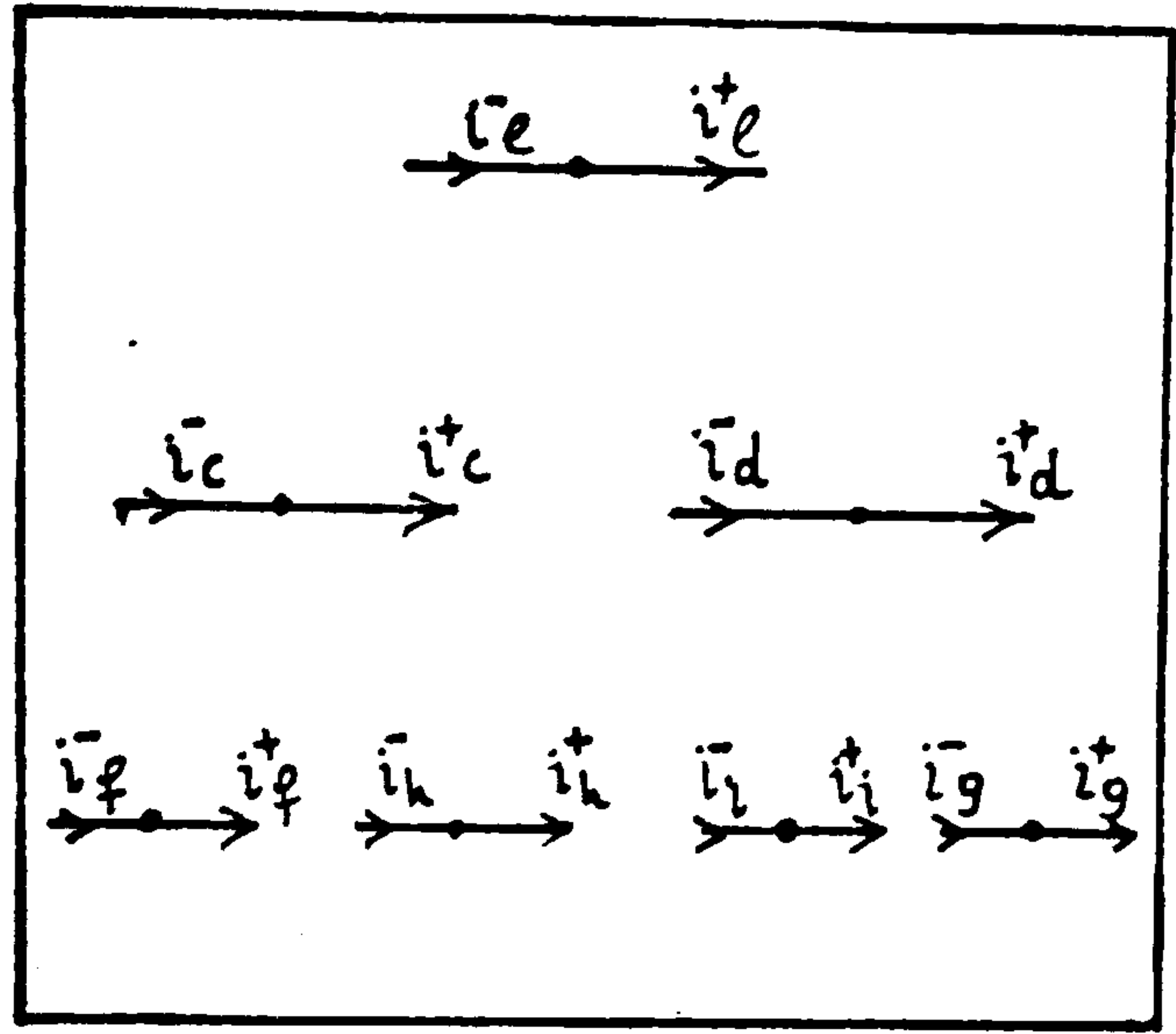
Sch. 4



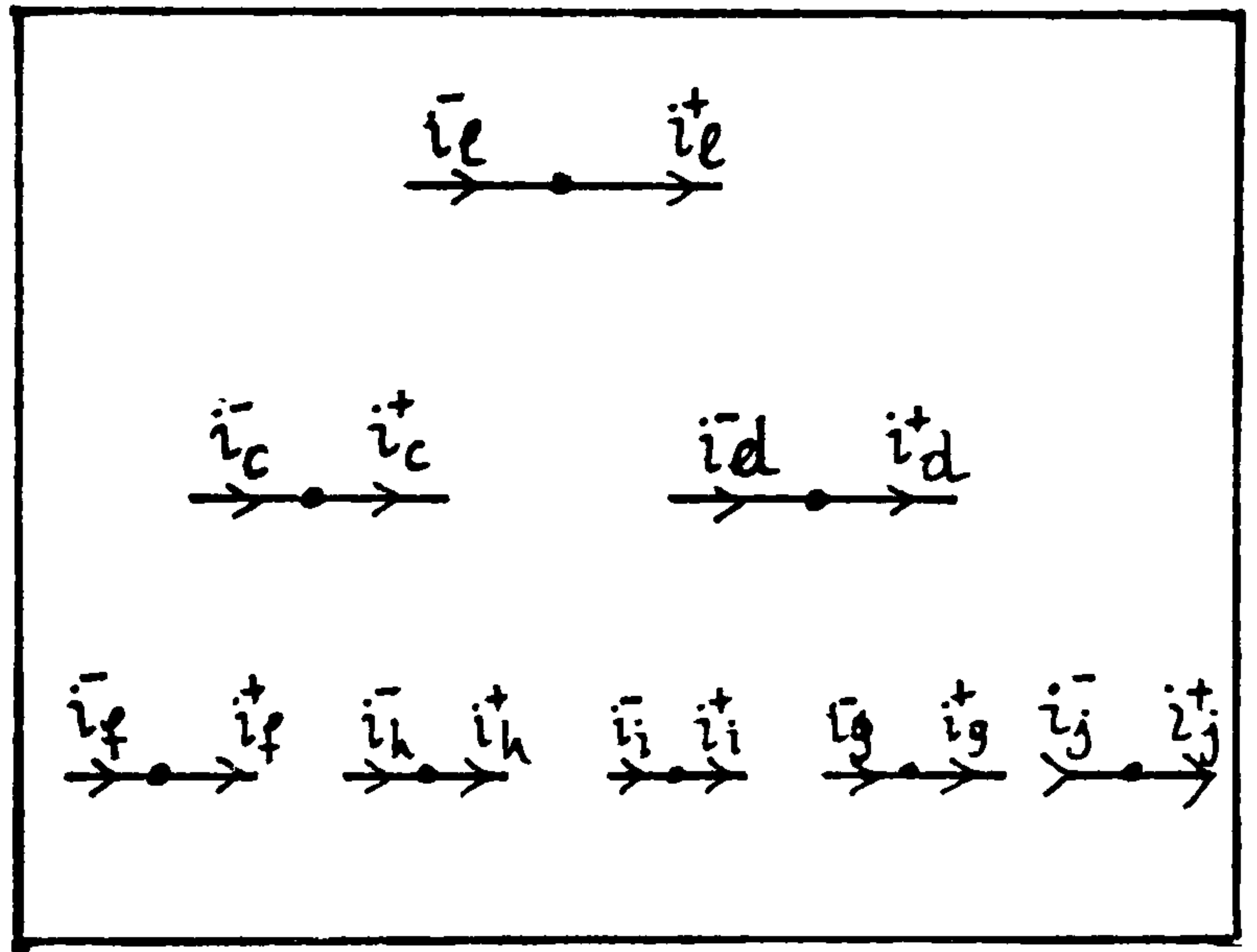
Sch. 5



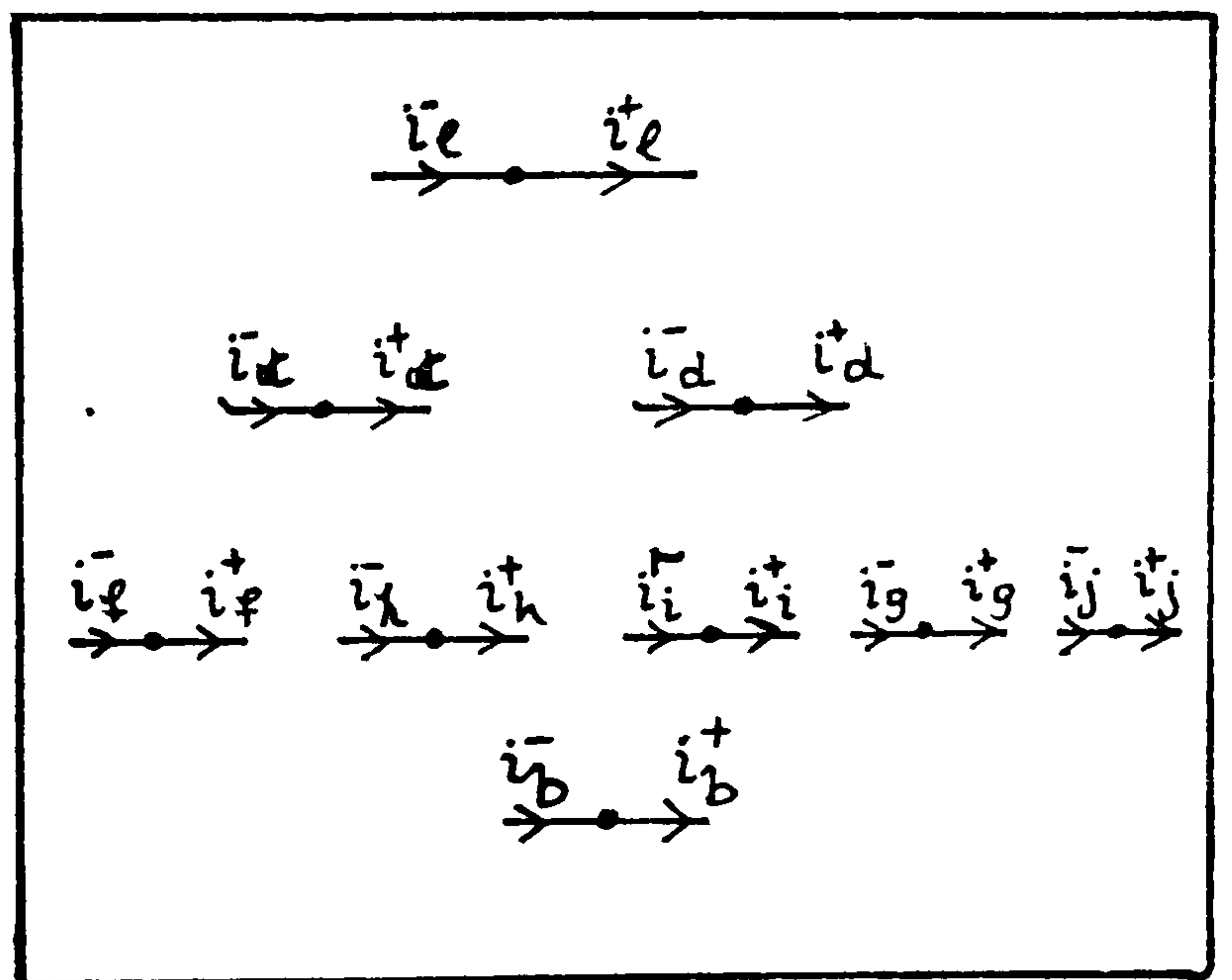
Sch. 6



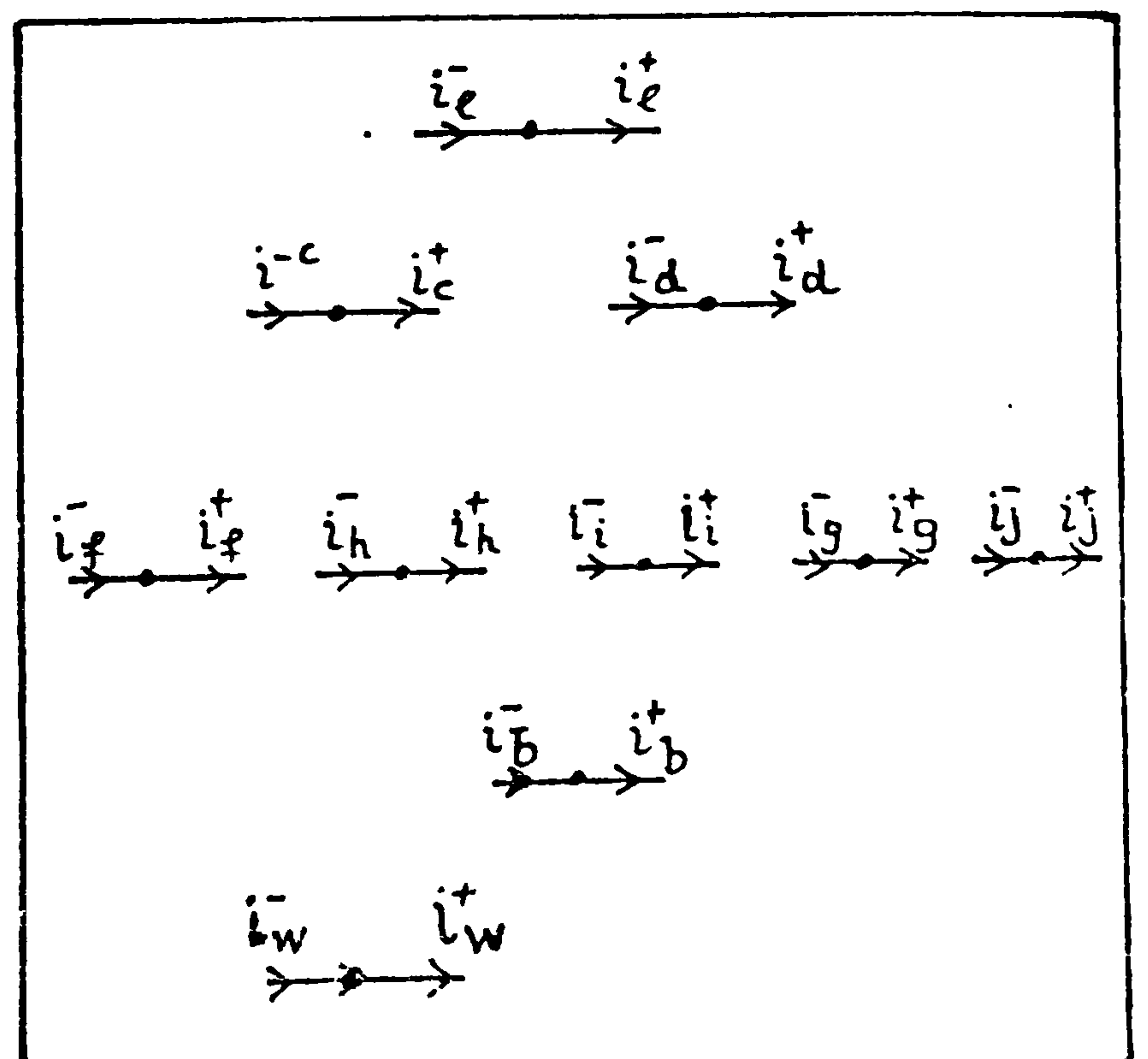
Sch. 7



Sch. 8

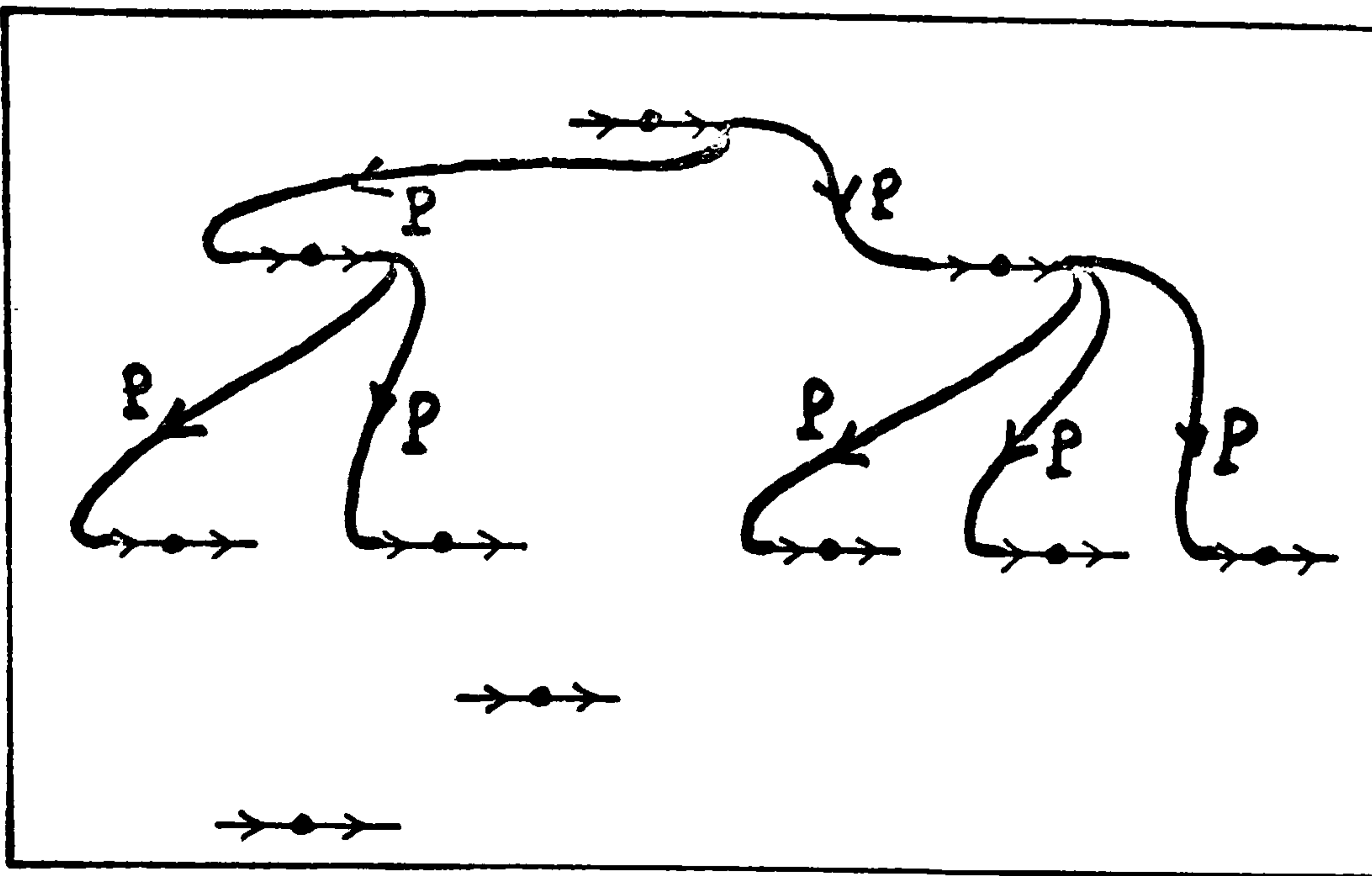


Sch. 9

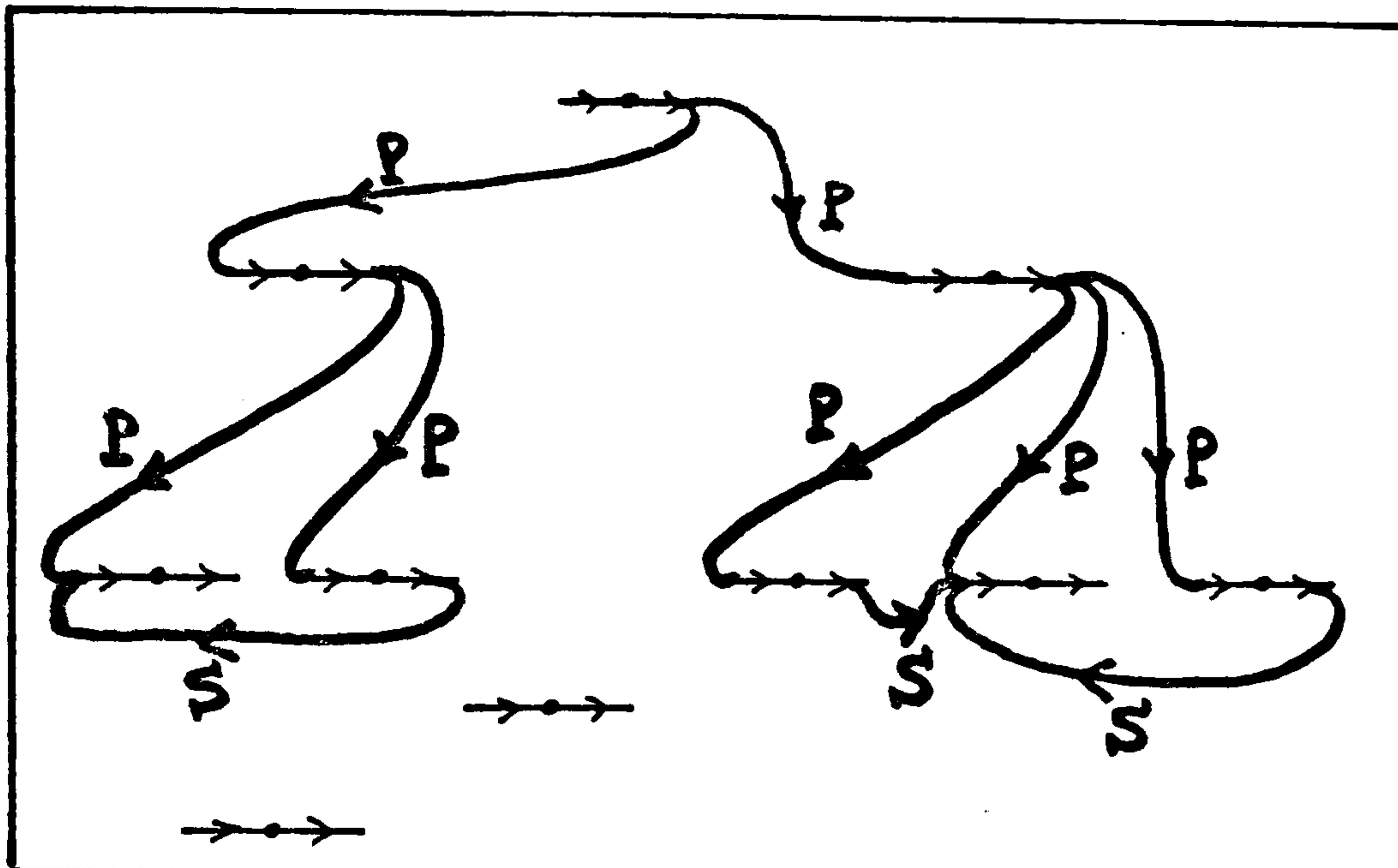


Sch. 10

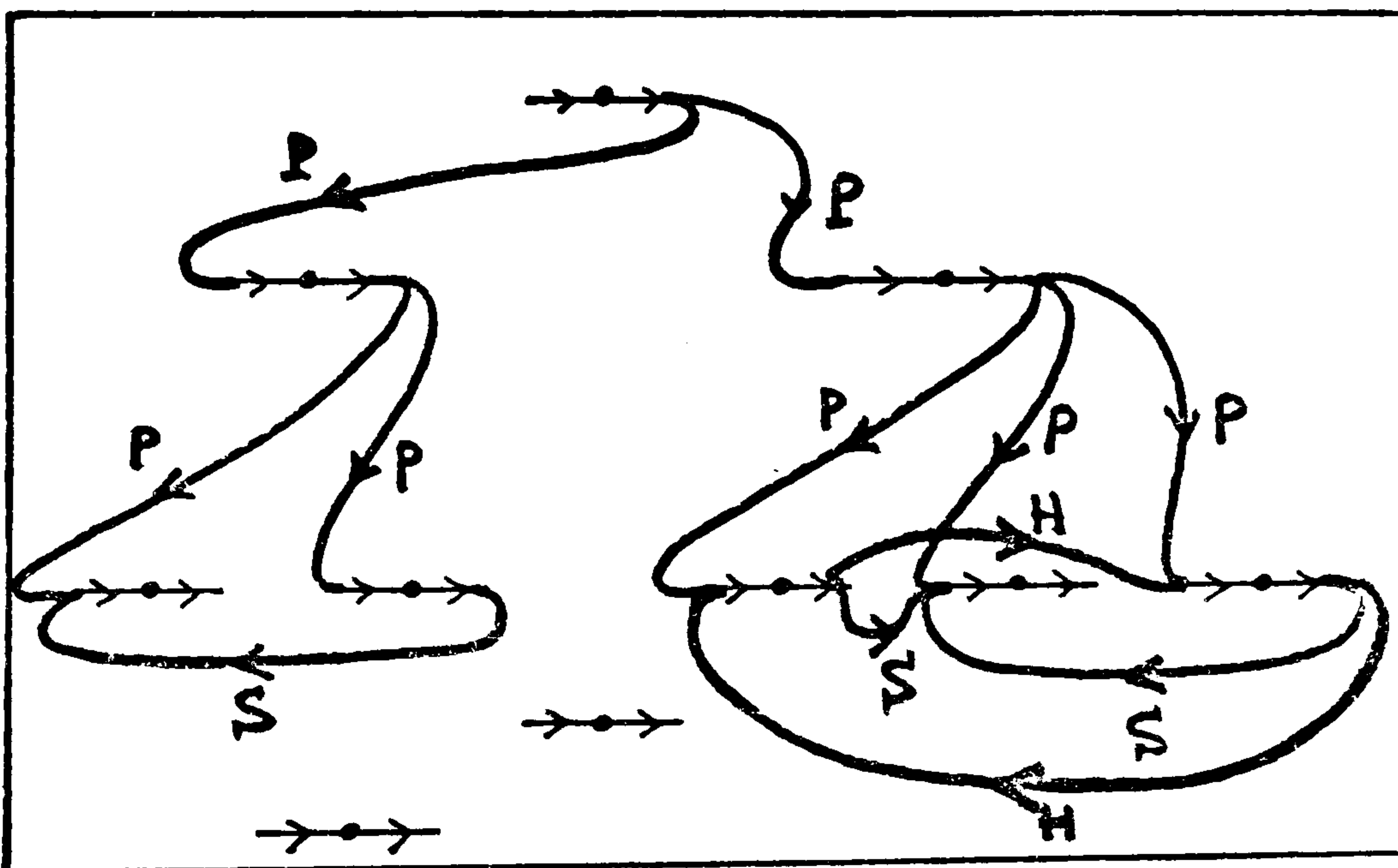




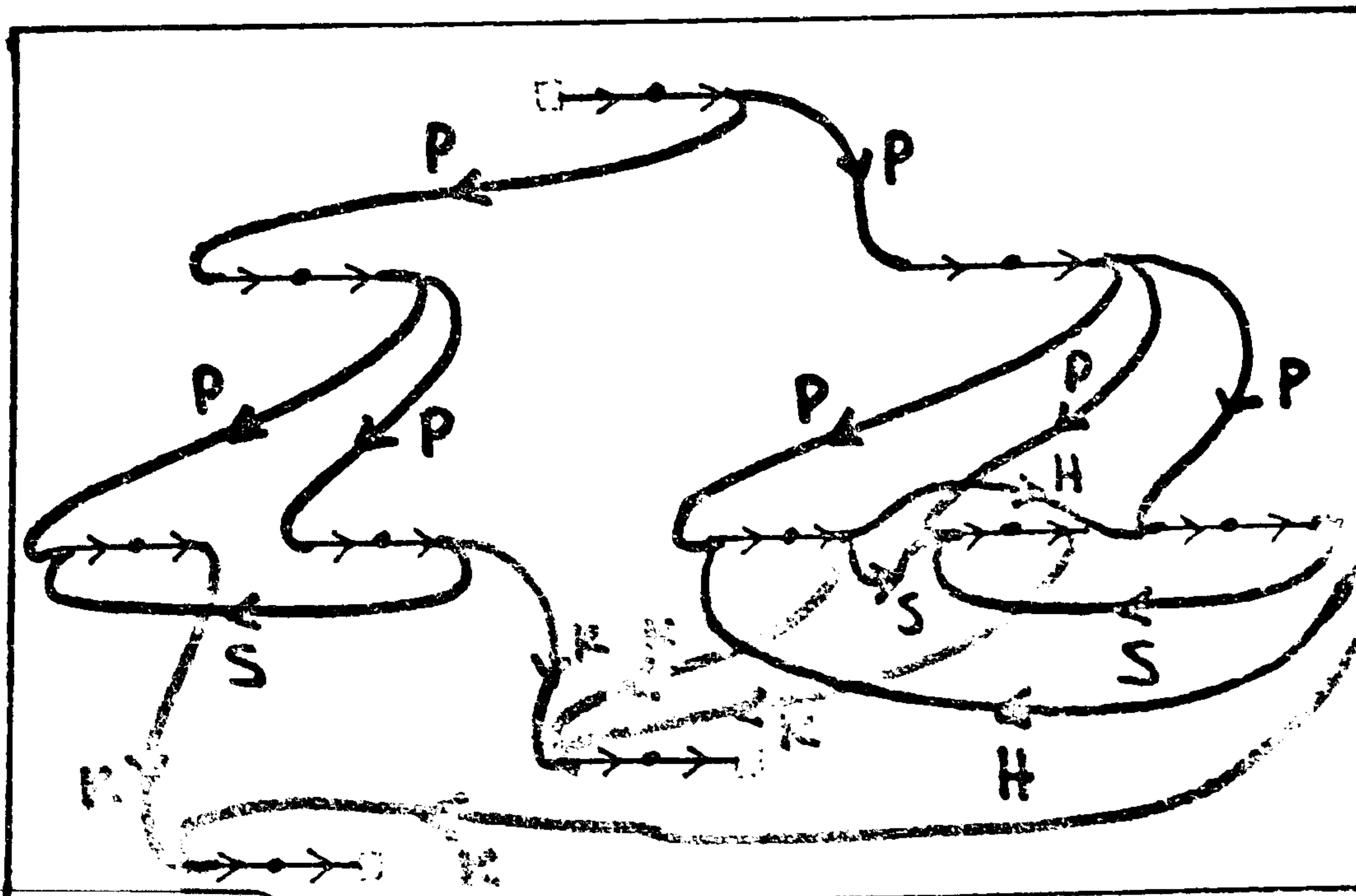
Sch.11



Sch.12



Sch.13



Sch.14



morphisms over 'invisible' (skeleton's) objects.

Finally, we may consider the present section as some kind of interface between k-structures and the Knowledge Representation Structures (KRS) which we are going to analyse next.

## 4.2 KRS STRUCTURES

### 4.2.0 INTRODUCTION

Knowledge Representation Structures (KRS) are variants of k-structures in their full generality. That is, a KRS:

1. Incorporates/materializes all the mathematical features of a k-structure.
2. Accommodates, nicely, the very concept of '*drasis*'\*; to be introduced later for conflict resolution in interaction of k-structures.
3. In a KRS, objects may be assumed as some sort of tacit knowledge; in their place, the so-called *reverse type m-mixer*\*\* is positioned, resulting thus in a structure consisting mainly from:
  - A. morphisms; and
  - B. morphism-mixers.

Finally, in a KRS, the 'invisible' objects and morphisms may be almost anything. On a possible criticism of that statement, see section 3.1.1.0.

---

\* Which is introduced in Sect. 4.2.3.

\*\* Introduced in Sect. 4.2.2.



4.2.1 THE SHIFT FROM k-STRUCTURE TO KRS

We brief here on the shifting from a k-structure of objects and morphisms to a knowledge representation structure which is simply a representational extension of a k-structure; some reasons for doing this shifting are given later, sect. 4.2.3 .

Let  $K = (K, M, I)$  be a k-structure, of objects  $K$ , and morphisms  $M$ . Where  $I$  is its identity function, defined as follows: details in sects. 3.1.4 and 3.1.12.

$$I : K \rightarrow M$$

$$: X \mapsto I(X) \equiv i_X := \text{'identity morphism of object X'}. \quad (R)$$

Furthermore, from the sect. 3.1.3 on semigroupoids, and k-structures, sect. 3.1.4., we have that  $M$  is a regular semigroupoid (RSGD); thus, for every genuine morphism  $\mu \in M$ , there is only one left and right identity morphism such that:  $\lambda_{\mu} \circ \mu = \mu = \mu \circ \rho_{\mu}$ .

Finally, from (R) above, we have:  $\forall X \in K \Rightarrow I(X) \equiv i_X \in M$ ;  
thus,  $\forall i_X \in M \Rightarrow$  there is only one left and right identity morphism:


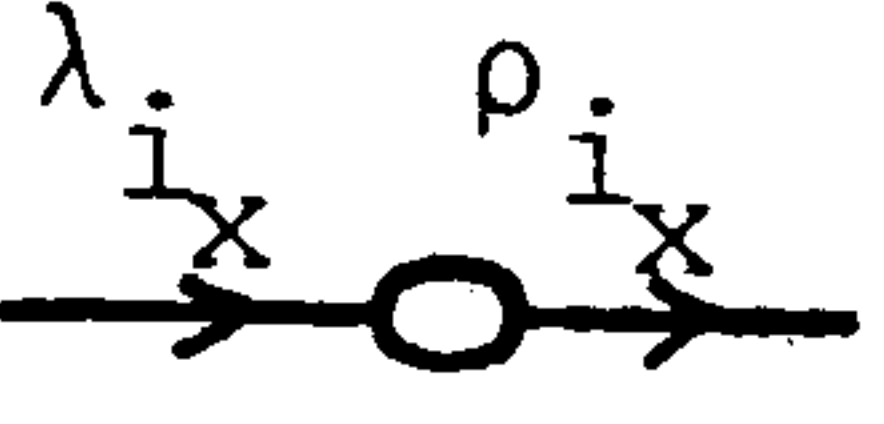



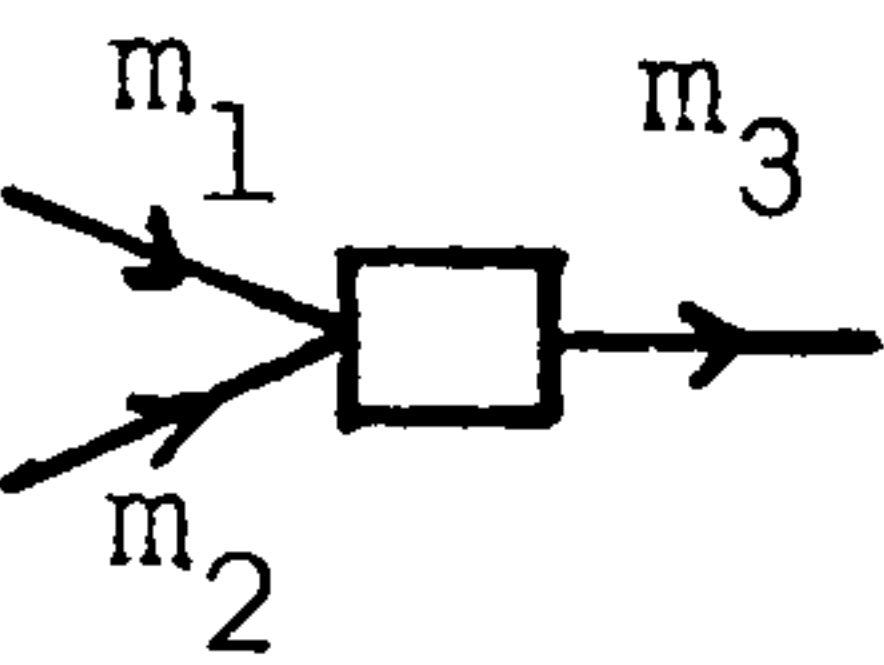
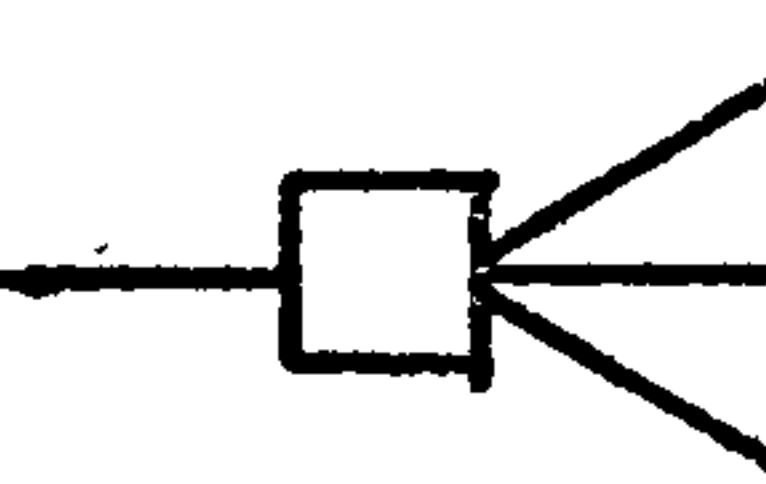
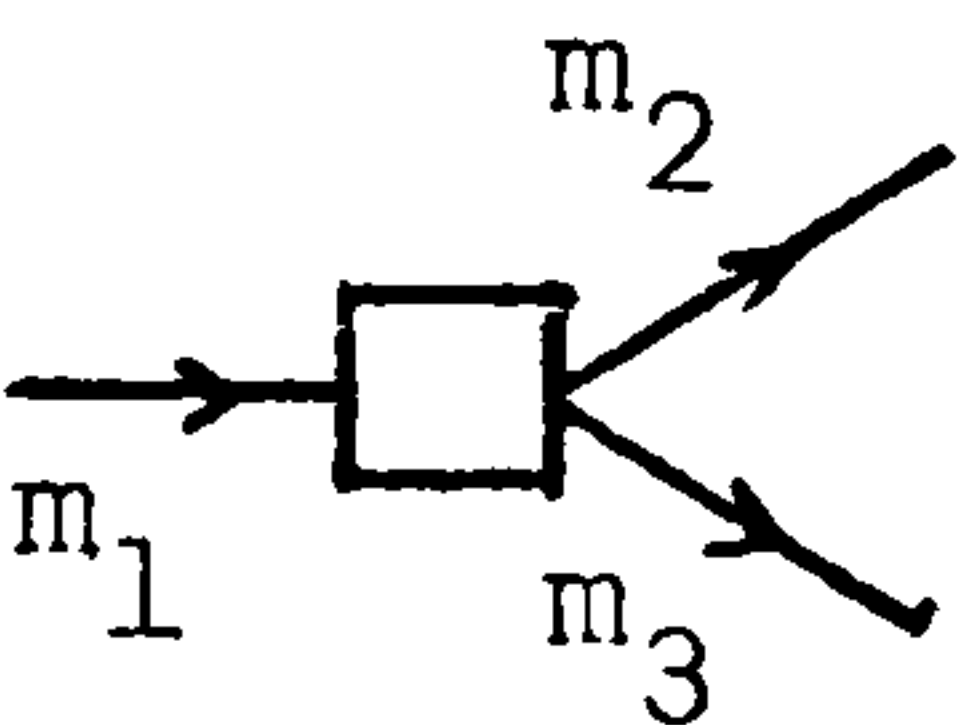

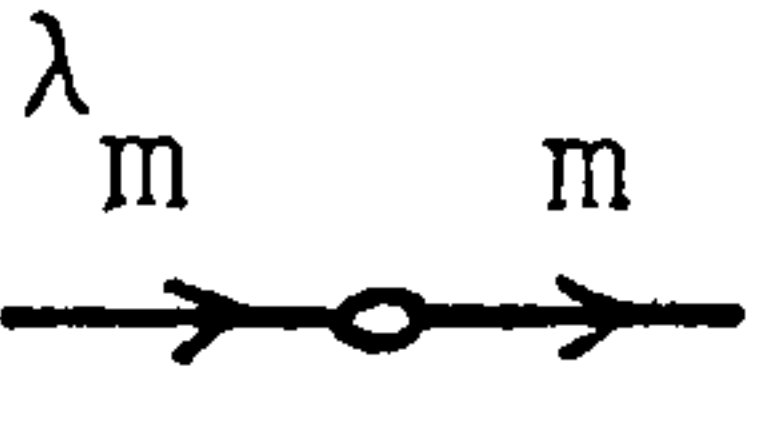
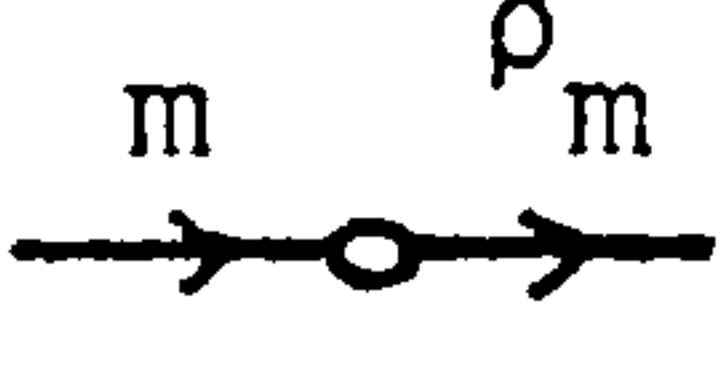

$$\lambda_{i_X} \circ i_X = i_X = i_X \circ \rho_{i_X}^*.$$

These additional (micro-)features of a k-structure led us to the introduction of the following Schematology as some kind of phraseology for KRS.

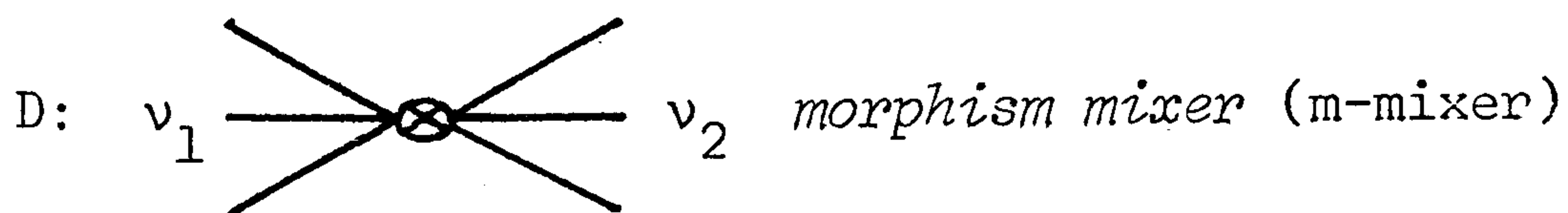
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\*  $\lambda_{i_X} = \rho_{i_X}$ ; we differentiate them for schematic reasons.

4.2.2 A SCHEMATOLOGY FOR KRS

DIAGRAM	NAME*	EXAMPLE
 $D_1$	identity-composer (i-mixer) or reverse** type m-mixer, see diagram D)	 Common  Kybernetes  Terminal
 $D_2$	m-synthesizer	
 $D_3$	m-analyser	
 $D_4$	(genuine) m-composer	 
 $D_5$	drasis point	examples are given in sects. 4.2.3 and 4.3.

$D_i, i = 1, \dots, 5$  may be unified in the following diagram:



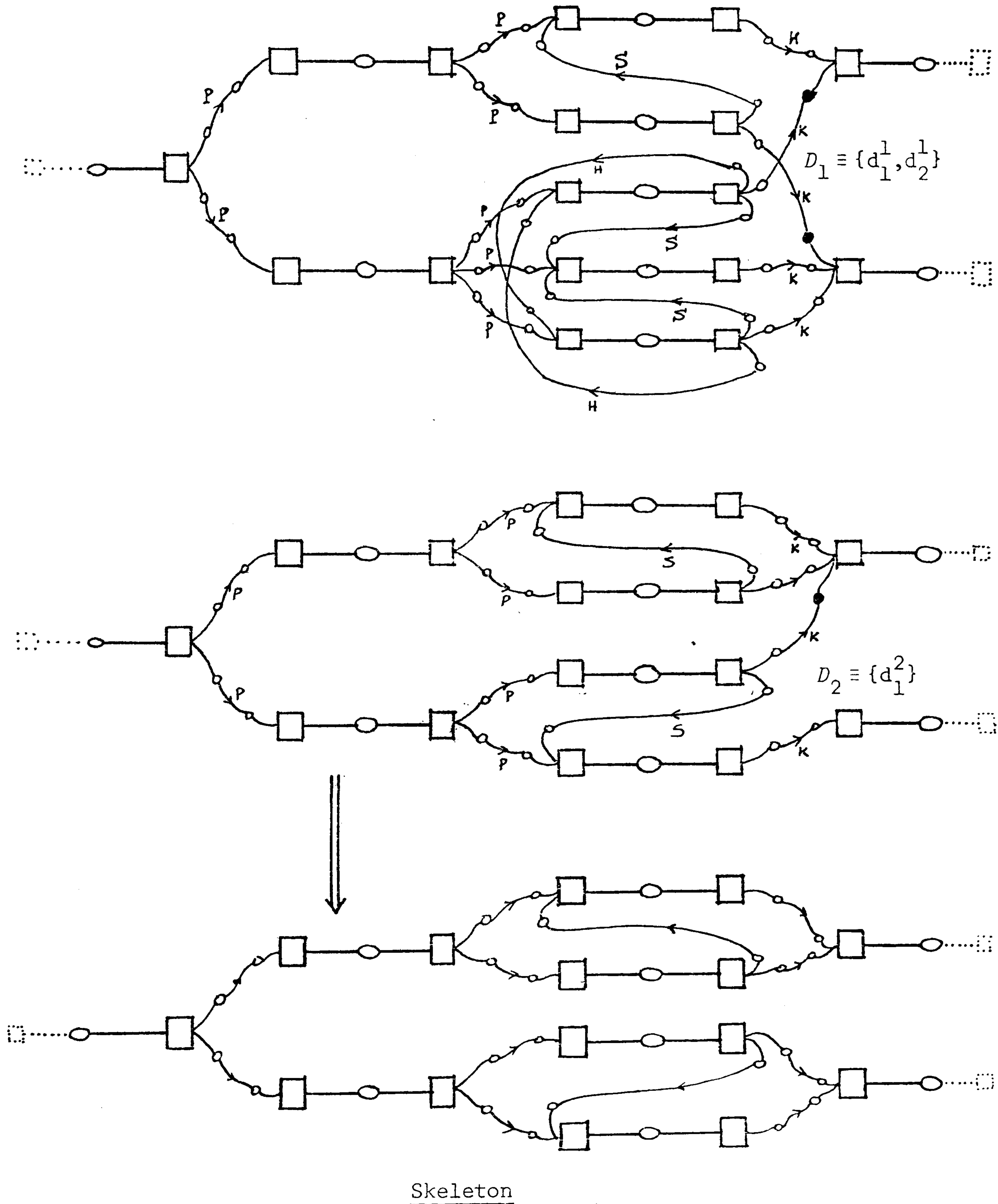
where:  $v_1 \geq 0, v_2 \geq 0$  is the number of in(out) going morphisms.

\* 'm-' stands for 'morphism-'.

\*\* In the case where morphisms are allowed to be stored as objects also, then i-mixer takes the form of an m-reverse which somehow reverses a morphism. See, for a similar case, in p. 23-24 WINSTON (1970) when relations are considered as nodes.

4.2.2.1 KRS Schemata

In order to get the flavour of a Knowledge Representation Structure (it is just an extended form of a k-structure), an analogy-type problematic thesis (that of sect. 3.3.1) is sketched in the following schema: (objects are somehow suppressed to emphasize morphisms)





REMARKS

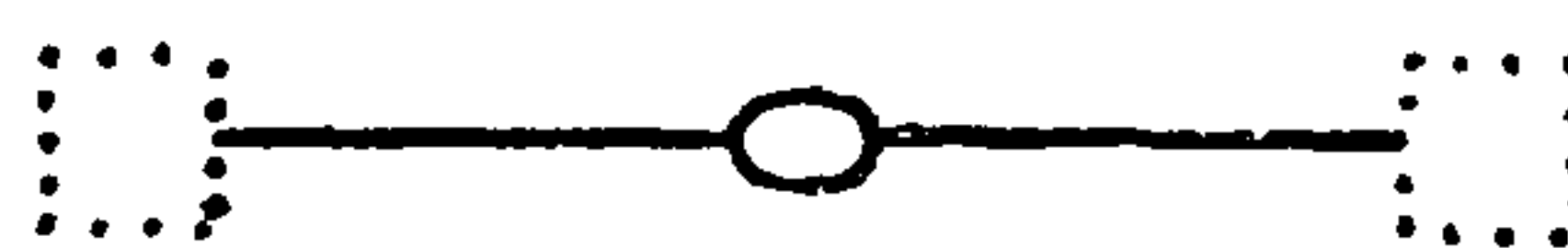
1. In KRS pharaselology, an object X may be represented as:

$$X = \langle \tau\sigma; \lambda_{i_x}, \rho_{i_x}; \sigma, \alpha \rangle \quad \text{or} \quad \begin{array}{c} \lambda_{i_x} \quad \rho_{i_x} \\ \sigma \quad \tau\sigma \quad \alpha \end{array}$$

- where:
- $\lambda_{i_x}$  : left identity of morphism  $i_x$
  - $\rho_{i_x}$  : right identity of  $i_x$
  - $\sigma$  : m-synthesizer
  - $\alpha$  : m-analyser
  - $\tau\sigma$  : (X's) identity composer; (i.e. X's qualities, attributes, features, etc., see sect. 3.1.12).

2.  $v_1 \geq 0, v_2 \geq 0$  (see diagram D, sect. 4.2.2).

We assume that  $v_1, v_2$  are not both equal to zero, simultaneously. In this case, i.e. when  $v_1 = 0$  and  $v_2 = 0$ , we have the so-called *isolated* (non-interactive) object.

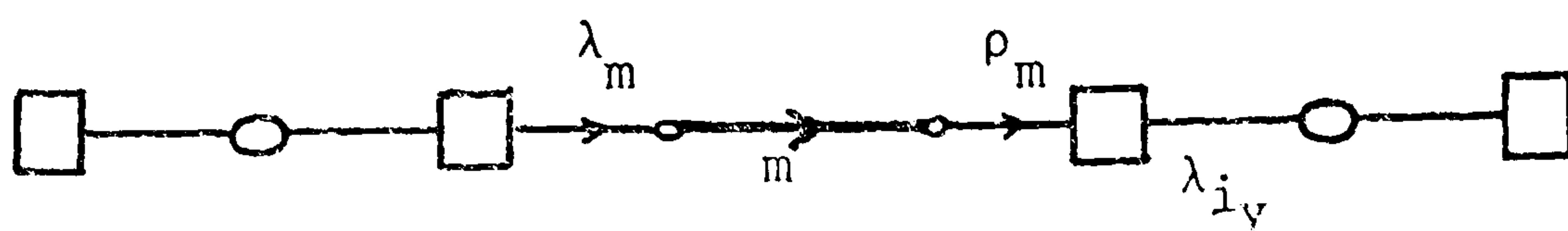


kybernetes  $\sigma$

terminal  $\alpha$

3. Thus, a condition for 'isolation' comes out quite straightforward according to the above representations.

4. A (genuine) morphism m is never composed directly to an identity morphism of an object Y. This is done via  $(\lambda_m), \rho_m$ ; i.e.



Thus  $\lambda_m, \rho_m$  are acting as some kind of 'interface' so to speak, morphisms.

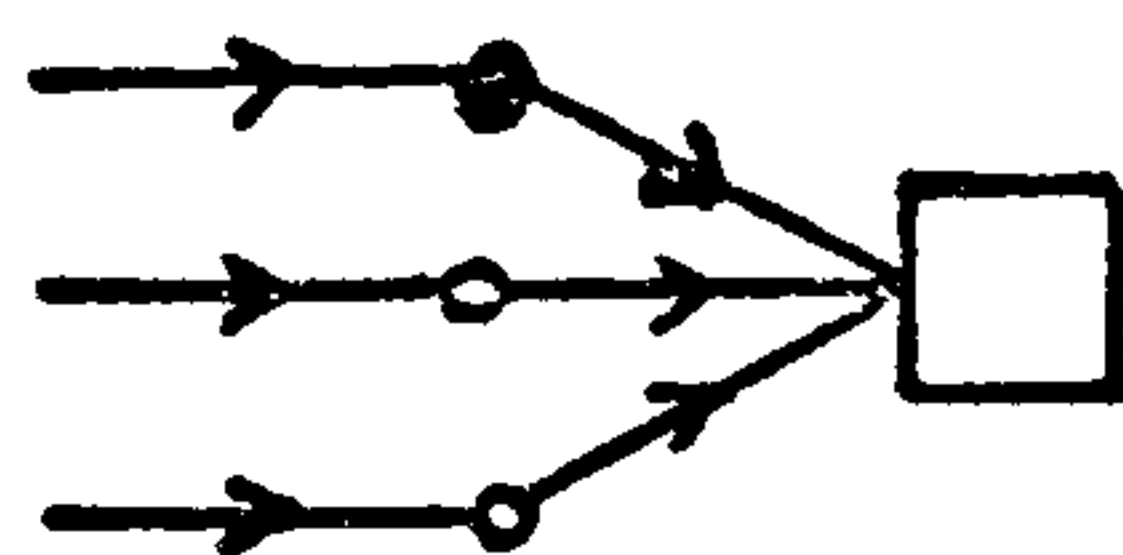
#### 4.2.3 ON THE NOTION OF DRASIS

##### 4.2.3.0 Introduction

The drasis concept invoked from k-structures interaction. It has been created, besides other possible uses, to cure and remediate a conflict situation in such an interaction. It is catered for accommodation of decision-making and organization mechanisms. Drasis facilitates Analogizing for it helps in dilemma resolution. It underlies and illuminates a shifting between two k-structures. In fact, the conceptualization of drasis came up after a thorough investigation of a k-open's components, from a number of standpoints; i.e. algebraic, topological, operational and mainly structural.

##### 4.2.3.1 Role, representation, nature, utilization

Drasis plays the role of some sort of *conflict-resolving clue*. It is a corner-stone in a communication (considered as interaction of k-structures) when the latter is in the analogizing mode. In an extended k-structure (KRS), drasis is a conflict-causing m-composer.



From the k-open's point of view, sect. 3.2.1, drasis is a fundamental (procedural) feature of a k-open in a k-space. Generally, it is a morphism mixer (m-mixer) in the sense of the earlier given schematology.

Drasis may be utilized as catalyst (effector) for elimination of semiotic differences between k-structures or to decrease some dissimilarity between them. That is, it brings together k-structures or parts of them.

The introduction of Drasis concept may be served as a useful framework for a better conceptual understanding and effective operational use of another vital notion, that of Local Control Quantum (LCQ) during the course of skeletization process. LCQ is also fundamental in the study of non-hierarchical systems via what we call Local Kybernetes; a theme for future investigation. Drasis is, in some sense, the materialization/realization of LCQ and LCQ is the conceptualization of Drasis.

#### 4.2.3.2 Properties

The main property of drasis is that of conflict/dilemma/deadlock resolution. Furthermore, it facilitates type-2 shifting, that is shifting between tactics or methodologies. More on shifting-2 in sect. 2.4.3. and sect. 2.2.2 'on intelligence'. Drasis is an m-composer with the property of interlinking parallel\* (sub-)tasks or processes which may be represented as (sub-) k-structures; and in more technical terms: communicating task<sub>1</sub>'s m-composers and m-synthesizers to those of task<sub>2</sub>.

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\* This is a tentative suggestion of parallel computation.

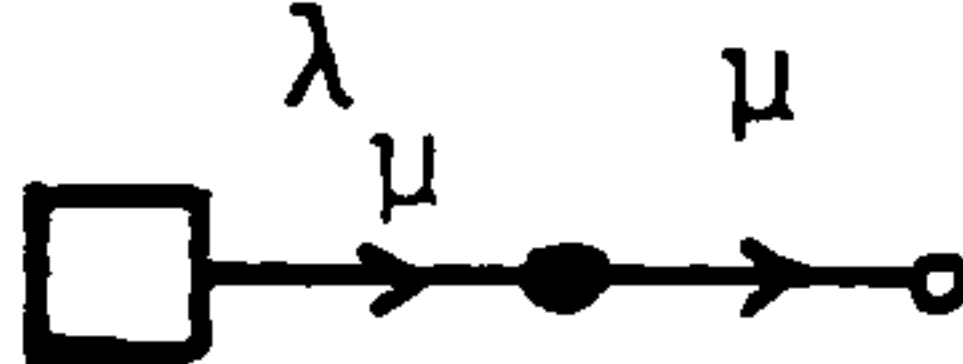
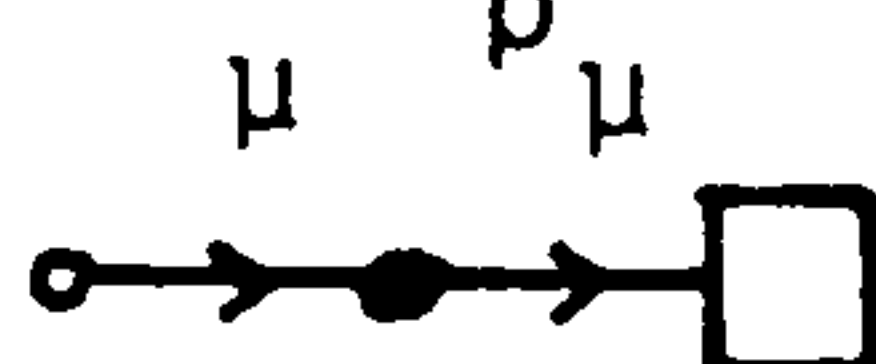


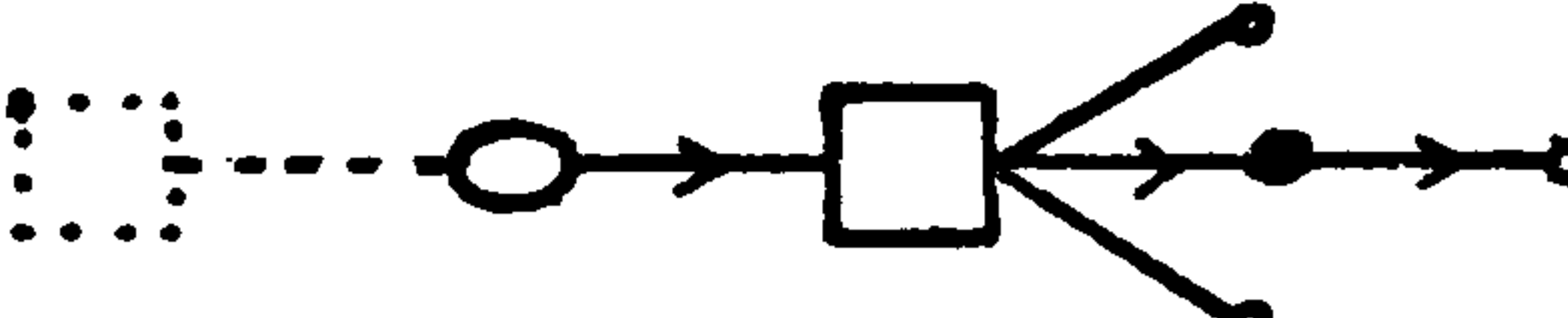
### 4.2.3.3 Functions

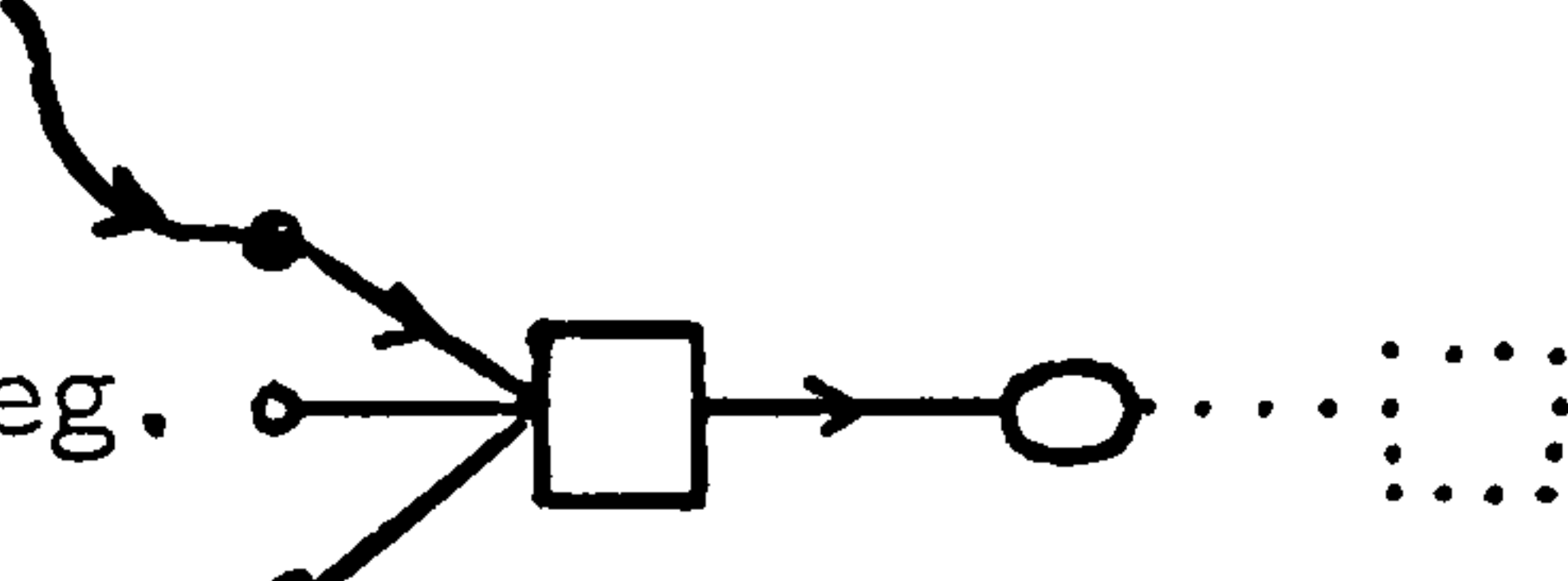
1. Drasis considered as an m-composer, it composes a morphism coming into an m-synthesizer (ingoing morphism) with its right identity morphism or it composes a morphism coming out from an m-analyser (outgoing morphism) with its left identity morphism.
2. It generally joins or composes morphisms.

### 4.2.3.4 Tanonomy of drasis

#### I. Types of drasis

1. *Genuine* or pure or non-terminal drasis; eg.  

2. *Kybernetes* drasis, eg. 

3. *Terminal* drasis, eg. 

#### II. Remarks

1. Drasis is a sort of *action* or *decision making* point in an extended k-structure (KRS).
2. It is a *place* in a problem-space or control space, where a vital decision is going to be taken, with immediate implications on the structural (and semiotic) features of the conceptual Universe, or k-structure under investigation.
3. It is a *noise* point, from the fact that its morphism environment

disturbs the (semiotic) similarity of two Universes of Discourse, k-structures or Knowledge structures.

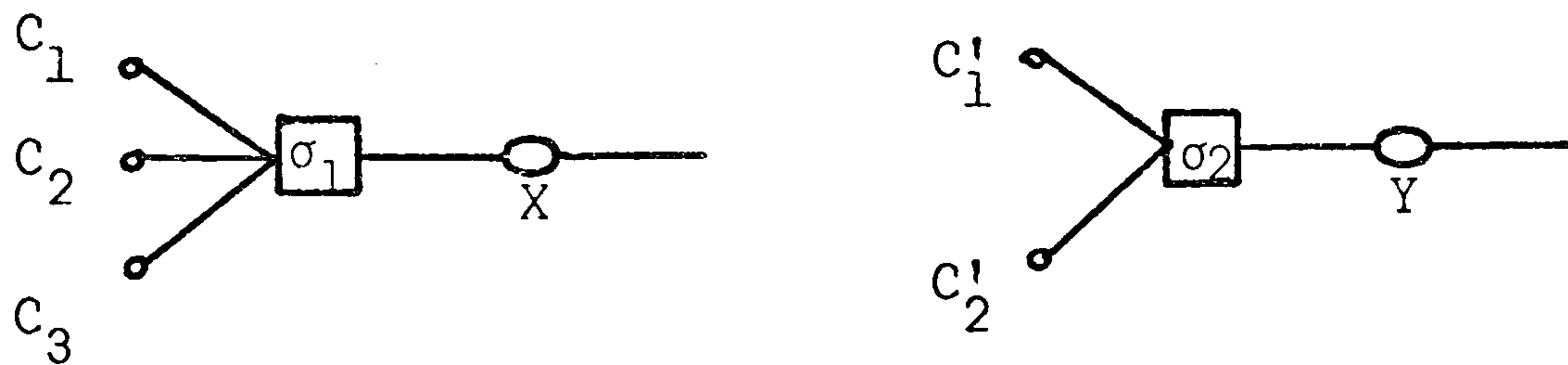
4. It is a *choice* point, ASHBY (1956); or a *distinction* point.
5. Finally, it is a *disturbance* point because it interferes between two parallel processes or tasks causing conflicts.
6. Dijkstra's semaphor is a loose analogy of a drasis point.

### 4.3 CONFLICT RESOLUTION

#### 4.3.0 INTRODUCTION

A question which naturally arises at this point, from what has been said so far; nam., How can we distinguish or differentiate an m-composer as being a drasis? (Q). We, therefore, need to devise some *criteria for drasis point identification*.

Up to now, the following suggestions have been detected, allowing for computable criteria to be developed; that is, structural (semiotic) dissimilarity in terms of neighbouring m-synthesizers, i.e.



$$\text{if } X \approx Y \Rightarrow \sigma_1 \approx \sigma_2 \iff |\{C_1, C_2, C_3\}| \stackrel{\text{must}}{=} |\{C'_1, C'_2\}|, \\ \text{be}$$

That is, from m-composers  $C_1, C_2, C_3$ , there is one which might play the role of a drasis point. Where,  $||$  denotes the power or cardinality of a set.

A suggestive point for the existence of a drasis point is a jumping/crossover in a (2-D) planar arrangement of a k-structure, representing, say, a task structure or a project network or a planning net or even a circuit in the sense of ROSE's(1970) work: Computer-aided circuit design, and SUSSMAN's (1977) : Electrical Design, a problem for A.I. research.

The above question (Q) may be restated in the terminology of parallel computation/programming: nam., 'How can we identify parallel processes or coroutines?' (the latter in KNUTH's(1961) terms; see also KOWALSKI (1975) p. 591.)

#### 4.3.1 REMARK (m.e.d.)

We don't deal directly with this question in the present work. We leave it for the future. However, we think that such an identification is somehow achievable if one would follow the work of DASGUPTA-JACKSON (1973). Finally, in a similar manner, perhaps, an answer might be given to the question: 'How can we identify conflicting parallel tasks represented as k-structures and more general as knowledge representation structures?' . We rather deal with questions like: 'Given two (or more) analogous or parallel k-structures with the phenomenon of dilemma/conflict/deadlock upon them, due to some sort of (semiotic) dissimilarity, what can we do to resolve it?'. This question gives rise to compute the set of drasis points and develop or rather impose upon it some measure of effectiveness or significance of drasis points which may amount to the creation of some notion of *most effective drasis* (m.e.d.)\* which actually plays important role

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\* The term reminds one of the notion of 'most general unifier' (m.g.u.) KOWALSKI (1975), and 'most common divisor' in arithmetic. Someone going deeper in these matters could find similarities.



in conflict resolution as we shall see next. The procedure of the following section is one of the main results of our investigations.

#### 4.3.2 A METHOD TO RESOLVE CONFLICTS IN KRS

The following procedure is a suggestion for conflict resolution in k-structures' interaction using:

- A. their extended representations, i.e. KRS;
- B. the above introduced concepts of drasis points and most effective drasis (m.e.d.); and
- C. operations like morphism and object eliminations (ME or OE) introduced in sect. 3.4.6.

The procedure may be illustrated using the KRS schemata of section 4.2.2.1.

We next give an outline of its sequence of steps:

- STEP 1: Identify terminal i-mixers via their right dead ends, i.e. right identity morphisms.
- STEP 2: Locate terminals' m-synthesizers by inspection or scanning of terminals' left identity morphisms.
- STEP 3: Compare m-synthesizers carrying out syntactic or geometric comparisons based on numerical computations.
- STEP 4: Take notes of any dissimilarity or differences during the above comparisons.
- STEP 5: Identify terminal drasis points; i.e. m-mixer locations where conflicts happen.
- STEP 6: Identify the most effective drasis (m.e.d.)  $\Delta$  via some sort of 'look-ahead' computations and reasoning.

- STEP 7: Eliminate m.e.d.  $\Delta$  and its morphism-like environment.
- STEP 8: Carry on by eliminating  $\Delta$ 's 'closest' m-analyser which is apparently uniquely determined.
- STEP 9: Proceed with relevant morphism eliminations (ME), eliminating thus the 'noise' morphisms; and object-eliminations OE (rather i-mixer eliminations), eliminating the 'noise' objects.
- STEP 10: Filtrate the remaining terminals\*.
- STEP 11: Amalgate filtrations.
- STEP 12: Validity, correctness. Check the remainder m-composers for validation; for ex. is  $\{K_1\text{'s m-composers}\} = \{K_2\text{'s m-composers}\}$ ?
- STEP 13: Proceed towards synthesizing skeleton in k-structure form.

Remark 1.- Both filtration and amalgamation operations ( $\phi$  and  $\alpha$  resp.) are defined in subsequent sections of the present chapter .

Remark 2.- The steps 9 up to 13 are useful when apart from conflict resolution, skeletization is also needed.

#### 4.4.1 ON FILTRATION

The nature of filtration ( $\phi$ ), in the present work, is understood as a binary operation on a k-space. It takes place between two k-opens. A formal definition of  $\phi$  is given below:

$\phi : \text{KSP} \times \text{KSP} \longrightarrow \text{SKEL}$

$:(O_1, O_2) \longmapsto \phi(O_1, O_2) \equiv \phi_{O_1, O_2} :=$

$:= \{ \{RO_{O_1} \cup RO_{O_2}\}, \{RM_{O_1} \cup RM_{O_2}\}, \{SM_{O_1} \cup SM_{O_2}\} \} \cup \{AM\}$

---

\* As well as the remaining genuine objects (i-mixers in KRS terms) in an appropriate manner.

where,  $O_1, O_2$  are k-opens;

RO, RM are the radiated objects and morphisms resp.

SM are the satellite morphisms

AM are the 'additional morphisms', and

SKEL is the skeletization universe.

For an example, see schema 4.4.1.

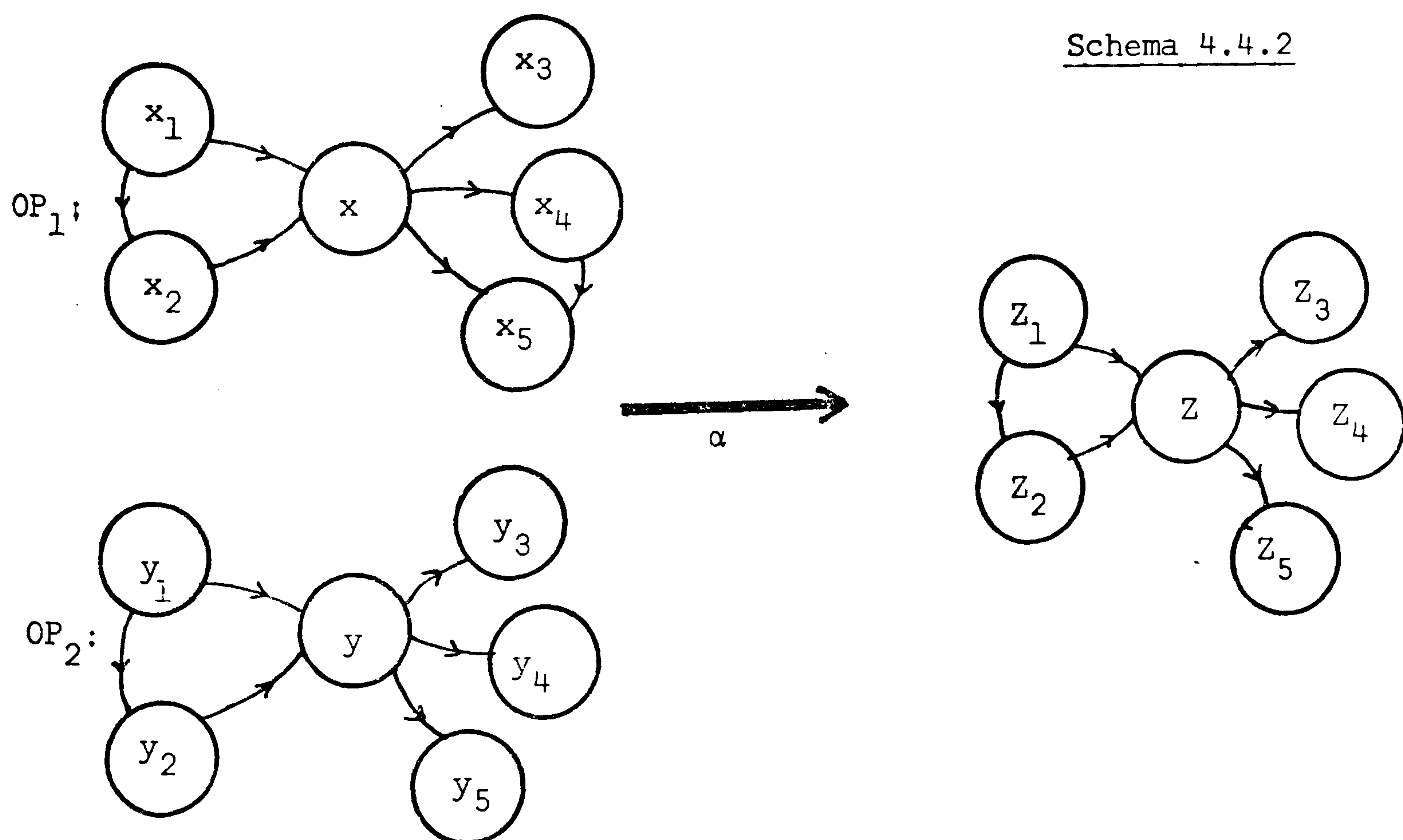
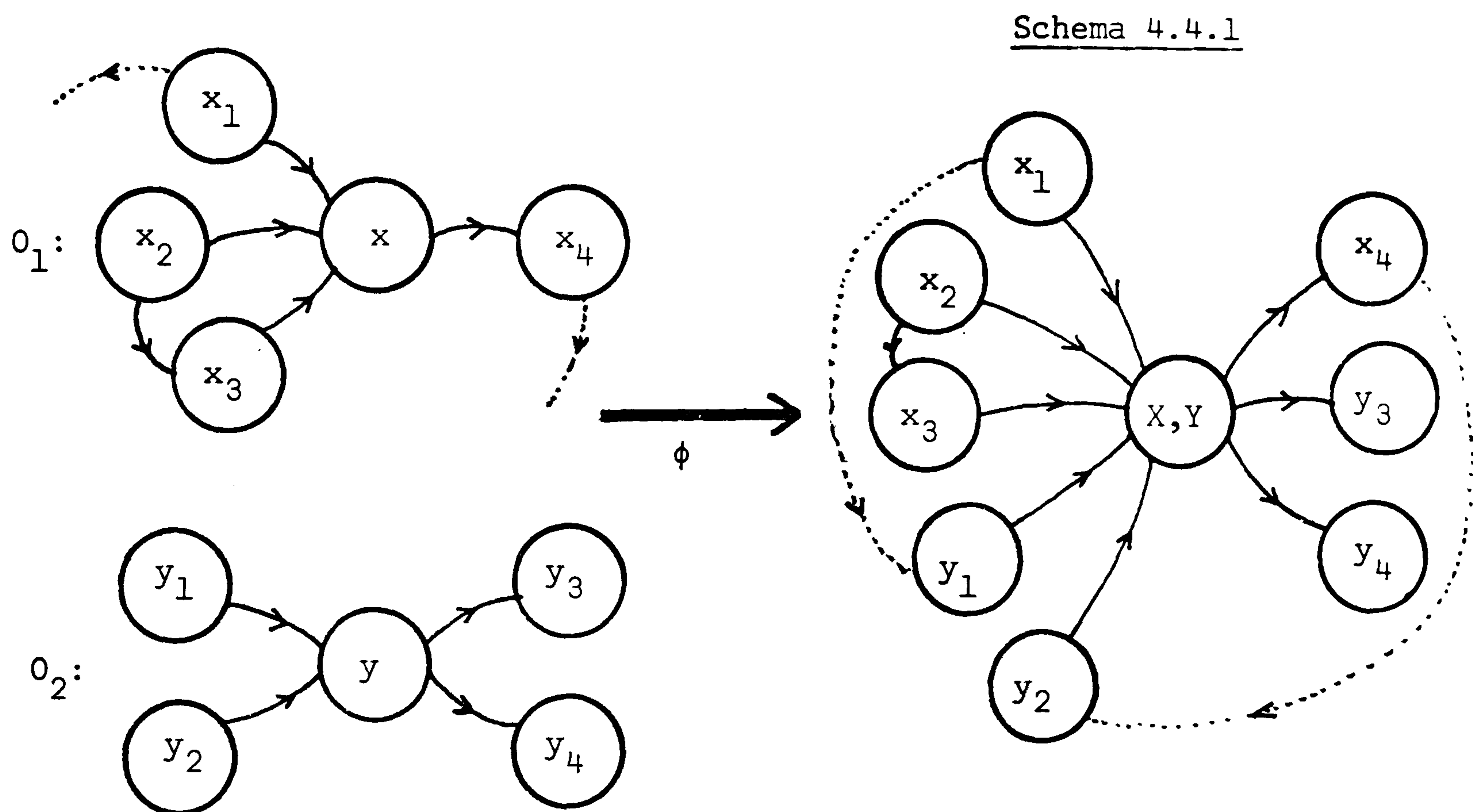
#### Remarks

1. We refer to k-structures (hence to k-spaces) and not to KRS so that definitions and schemata become simpler.
2.  $\phi$  might be generally defined as an n-ary operation on k-spaces.
3. In schema 4.4.1, the dotted satellite morphisms in the result of  $\phi$ 's application are examples of what we called in the definition of  $\phi$ , 'additional morphisms' AM.
4. The *raison d'être* of an additional morphism is the existence of a dangling morphism which may spring off a radiated object in one of the  $\phi$ 's operands.
5. A k-space is not closed (in set theoretic terms) under  $\phi$  operation.

In the case where  $\phi$  is applied between terminals (e.g. w's and b's section 3.3), then we call it 'low-level concept filtering process' or 'terminals'  $\phi$ ' or 'elementary  $\phi$ '. To give an interpretation of the act of filtration in the way we understand it, we say that when  $\phi$  is applied then filtration of a k-open through another k-open takes place, as for example in a 'permeable filter' in brain theory terms. It is important to notice here that some radiated objects of the k-opens which take part in  $\phi$ , may be sources of dangling morphisms which are invisible in a k'open's schema. Thus, after  $\phi$  has been applied, it may be possible that some radiated objects, in the resulting k-open, are linked with



morphisms which are not explicitly referred to in the parent k-opens which take part in the filtration. Therefore, new (dotted) satellite morphisms, called additional morphisms (AM), have to be created and established.



4.4.2 ON AMALGAMATION

Informal introduction. Amalgamation ( $\alpha$ ) is a binary operation on k-spaces.  $\alpha$  Takes place between two k-opens\*, either ordinary (typical) or terminal ones. We are at the moment interested for terminal's amalgamation, because terminals play the most important role in conflict resolution and concept formation via filtrations (otherwise concept filtering).

Amalgamation  $\alpha$  may be considered, somehow, similar to the well known unification; the latter in the sense of NILSSON (1971), KOWALSKI (1975). The binary nature of  $\alpha$  can be straightforward generalized into an n-ary operation.

Formally, amalgamation operation  $\alpha$  is defined as:

$$\begin{aligned} \alpha & : K_1SP \times K_2SP \rightarrow SKEL \\ & : (OP_1, OP_2) \mapsto \alpha(OP_1, OP_2) \equiv \alpha_{OP_1, OP_2}^{**} \end{aligned}$$

where KSP stands for a k-space. For an example see schema 4.4.2.

We next put forward two important questions:

Question 1: How can amalgamation be achieved?

Question 2 : What are the criteria and conditions under which amalgamation  $\alpha$  is feasible?

The following remarks build upon terminology introduced in sects. 3.2. and 3.2.3, mainly elaborate on question 2, that is,

let two k-structures  $C_1 = (K_1, M_1, I_1)$ ,  $C_2 = (K_2, M_2, I_2)$

---

\* In fact, filtrated k-opens in the case of conflict resolution.

\*\* is external operation on KSPs; i.e. the result of  $\alpha$  is not in KSP or KSPs is not closed under  $\alpha$ .

and also two k-opens  $q_1, q_2$  ingredients of  $C_1, C_2$ -spaces; nam.,

$$q_1 = (T_1^1, T_2^1, T_3^1) \equiv (S_1, M_1, P_1)$$

$$q_2 = (T_1^2, T_2^2, T_3^2) \equiv (S_2, M_2, P_2)$$

Then, amalgamation between two k-opens is feasible iff the following

'Amalgamation Conditions' (AC) are true:

$$AC1 : T_1^1 \approx T_1^2$$

$$AC2 : T_2^1 \approx T_2^2$$

$$AC3 : T_3^1 \approx T_3^2$$

Where,  $T_i$  is the  $TOPOS_i$  in the topological sense mentioned in sect. 3.2 and  $S_j, M_j, P_j$  may be, loosely, considered as some sort of syntactic, semantic and pragmatic environment respectively. These conditions are equivalent to the following ones: (in terms defined in section 3.2).

$$AC'_1 : \{ \text{centre object for } q_1 \} \overset{R}{\underset{\alpha_1}{\approx}} \{ \text{centre object for } q_2 \}$$

$$AC'_{21} : \{ \#q_1 \text{'s radiated objs.} \} \underset{\alpha_{21}}{=} \{ \#q_2 \text{'s radiated objs.} \}$$

$$AC'_{22} : \{ \#q_1 \text{'s radiated morphisms} \} \underset{\alpha_{22}}{=} \{ \#q_2 \text{'s radiated morphisms} \}$$

$$AC'_3 : \{ \#q_1 \text{'s satellite morphisms} \} \underset{\alpha_3}{=} \{ \#q_2 \text{'s satellite morphisms} \}$$

where:  $R$  is some external posited similarity, in other words *a priori* hint. In the case study we refer to (sect. 3.3.1),  $R$  is 'LEFT  $\approx$  RIGHT'.

$\alpha = (\alpha_1, \alpha_{21}, \alpha_{22}, \alpha_3)$  is a quantitative tolerance; and  $(q_1, q_2) \in K_1^{SP} \times K_2^{SP}$ .



#### 4.4.3 APRES AMALGAMATION $\alpha$ AND FILTRATION $\phi$

(General remarks; discussion; and partial conclusions)

1. Amalgamation of two k-opens intuitively leads to and is inherently connected to the notions of abstraction and generalization, i.e. we may say that a machine abstracts or generalizes via amalgamation operation.
2. Amalgamation takes place between objects and morphisms of a number of 'homogeneous' k-opens; this is one of the reasons that in most cases we first 'filtrate' k-opens, via  $\phi$ , i.e. to become topologically comparable and compatible.
3. Talking in algebraic terms, i.e. equivalences, quotient set, etc., we may say that the actual result amalgamation operation amounts to is a representative of an equivalent class of similar objects or k-opens (in k-structure or k-space terms respectively).
4. Generally speaking, there are, or at least we arrive at, two main modes of amalgamation and filtration operations; nam.,
  - A. amalgamation or filtration with morphism elimination (ME); and
  - B. amalgamation or filtration with object elimination (OE).
 Both (ME) and (OE) have been studied and elaborated in section 3.5.1. As far as questions like: 'which of (ME) or (OE) is better to be carried out first' is an open question which we investigate in sect. 3.4.6.
5. As far as the future is concerned, we intuitively believe that 'efficient combination of amalgamation and filtration operations' (with object or morphism eliminations) with the so-called "n-level look ahead computation" in terms of KOWALSKI (1975), may lead ourselves to an intelligent search strategy. In fact, Kowalski's connection graphs utilize syntactic similarity to facilitate look-ahead.

6. By combining amalgamation and filtration appropriately, we may well find, in the future, what kind of network relations (morphisms) are adequate for describing a model of "human functional thinking" referred to by WINSTON (1970) p. 172. Also AMOSOV (1967) provides an obscure model of thinking processes for a departure into future investigations.
7. The identity morphisms, such as  $i_X, i_Y \dots$  of terminal objects are quite important and they play a vital role in the filtration operation.
8. Filtration is the operation which, in many instances, acts as a fundamental requirement for amalgamation. Filtration yields the appropriate 'Domain'  $D$  and 'Range'  $R$  for a quasi- or pseudo-isomorphism between two  $k$ -spaces  $K_1, K_2$ ; while amalgamation is the effectual carrier of  $K_1; K_2$  similarity vital to analogizing.
9. Worthwhile, here, I think is to point out the recent attempt of CHAVCHANIDJE (1976), reviewed by SCHUKIN (1977) on "concept filtering" and "a conceptual model of A.I.", which despite it being a little obscure, at least to us, it emphasizes the transition from cybernetics ideas into Brain theory; a striking difference between the Georgian Cybernetic Institute and Western A.I. centres.
10. From the Analysis-Synthesis viewpoint, we conclude that during Analysis phase (say, for example in skeletization), we are carrying out filtrations; while, in synthesis stage, what mainly dominates is amalgamation. It is also interesting to apply the above conclusion in a conflict resolution situation viewed from the Analysis-Synthesis standpoint.

4.5.1 ON CONCEPTS

A k-open is introduced in section 3.2 and somehow interpreted as the context of a concept. Furthermore, what was really achieved from constructing a k-space (sect. 3.2) amounts to the separation of the context of a concept from the concept itself.

From the other hand, we may say that the previously introduced operation of filtration  $\phi$  concentrates or condenses two or more k-opens while amalgamation  $\alpha$  unifies two or more (filtrated) k-opens.

Let us consider a *concept* as 'a sequence\* of (similar) contexts', i. e.  $\{C_i\}_{i \in I}$  \*\*. In terms of k-opens, the above sentence becomes: a sequence of (similar) k-opens. More precisely, and taking into account  $\phi$  and  $\alpha$  operations, *concept formation* amounts to: 'the construction of a (directed) sequence (or net) of amalgamations of (similar) k-opens  $OP_i$ '. The role of filtration here is emphatically present. To elaborate we simply say that k-opens  $OP_i$  usually are the result of filtrations.

The obscure speculations made loosely so far, lead on to the notion of understanding or learning of a concept if we intuitively assume that: 'we say we learn or understand something new if our minds stabilize to some conception which more or less represents a class of similar conceptions'.

4.5.2 DEFINITION

We may say that R understand (learns or forms) a concept  $\sigma$

---

\* Rather a net.

\*\* This expression can be fruitfully used for a departure into topological notions in k-structures, as for ex., convergence, continuity, etc.



iff 'a sequence (rather a net)  $C_i^R$  of similar contexts converges';  
in other words, iff

$$\exists \sigma : \lim_i C_i^R = \sigma \Leftrightarrow \forall \epsilon > 0 \exists \delta(\epsilon) > 0 : d(C_i^R, \sigma) < \epsilon, \forall i : i > \delta(\epsilon)$$

where  $d$  is some metric.

Remark.-  $C_i^R$  may be characterized as 'point of view' or 'partial opinion' of  $R$ .

Let  $R_1, R_2$  understand a concept  $\sigma$ . As we are interested in analogizing intellectual activities, we are curious to see whether  $R_1, R_2$  may have a common understanding of a concept. The intuitive appeal of the notion of a sequence leads us to conceive of it as two equally converging sequences of contexts, as the following proposition asserts:

#### 4.5.3 PROPOSITION

If  $R_1, R_2$  understand separately a concept  $\sigma$  then  $R_1, R_2$  have a common understanding of concept  $\sigma$  iff

$$\forall \epsilon > 0 \exists \delta(\epsilon) > 0 : d(C_i^{R_1}, C_j^{R_2}) < \epsilon, \forall i, j : i, j > \delta(\epsilon)$$

#### Proof

From hypothesis we have:

$$\forall \epsilon_1 > 0 \exists \delta_1(\epsilon_1) > 0 : d(C_i^{R_1}, \sigma) < \epsilon_1, \forall i > \delta_1(\epsilon_1)$$

$$\forall \epsilon_2 > 0 \exists \delta_2(\epsilon_2) > 0 : d(C_j^{R_2}, \sigma) < \epsilon_2, \forall j > \delta_2(\epsilon_2)$$

Thus,

$$d(C_i^{R_1}, C_j^{R_2})^* \leq d(C_i^{R_1}, \sigma) + d(C_j^{R_2}, \sigma) < \epsilon_1 + \epsilon_2 \equiv \epsilon$$

$$\forall i, j : i, j > \delta(\epsilon) \equiv \max \{ \delta_1(\epsilon_1), \delta_2(\epsilon_2) \}$$

Therefore, if  $\epsilon := \epsilon_1 + \epsilon_2$  and  $\delta(\epsilon) := \max \{ \delta_1(\epsilon_2), \delta_2(\epsilon_2) \}$ , then

$$\forall \epsilon > 0 \exists \delta(\epsilon) > 0 : d(C_i^{R_1}, C_j^{R_2}) < \epsilon, \forall i, j : |i-j| < \delta(\epsilon)$$

i.e. the two sequences of partial opinions of  $R_1, R_2$  are equally converging.

#### 4.6. SUMMARY DISCUSSION ON OPERATIONS INTRODUCED SO FAR

The following operations have been introduced:

1. Object elimination (OE) or o- $\epsilon^{**}$ ;
2. Morphism elimination (ME) or m- $\epsilon$ ;
3. Filtration  $\phi$ ;
4. Amalgamation  $\alpha$ .

OE and ME are unary operations, while  $\phi$  and  $\alpha$  are binary ones.

Furthermore, in sect. 4.2.3.2 we introduced the concepts of conflict causing m-synthesizer and m-composer which led to the following operation:

5.  $\tau$ : identification of conflict-causing parallel or analogous tasks or concepts.

In some sense, the above operations have been introduced in such

---

\* Where d is a metric on the sets of concepts satisfying the triangle inequality.

\*\*  $\epsilon$  on its own, stands for 'object or morphism elimination'.

a way as to accommodate an 'additional function in the non-numeric feedback processes, not found in the classic numeric situation; and that is the change of the problem description'; MESAROVIC (1970) p. 173-74. The latter amounts to the shifting of (problem) representation, an issue discussed in the previous chapters.

One changes a description by

- A. selecting new properties and/or
- B. generating new concepts.

We are mainly dealing in this work with the second one, otherwise called concept formation.

Besides the above five operations, we next introduce a few 'synthetic' ones, namely:  $\phi^*, \phi, \phi'$ .

$$6. \quad \phi^* = \tau\epsilon\phi$$

$$7. \quad \tilde{\phi} = \epsilon\phi$$

$$8. \quad \phi' = \tau\phi$$

MATRIX<sub>1</sub> shows a priority schema during the execution of a synthesis of  $\epsilon, \phi, \tau$  operations. MATRIX<sub>2</sub> shows the complete situation in skeletization when amalgamation is taken also into account.

$\#$	$\exists C$	$\exists C$
=	$\phi$	$\phi'$
$\neq$	$\tilde{\phi}$	$\phi^*$

$\#$	$\exists C$	$\exists C$
=	$\phi\alpha$	$\tau\phi\alpha$
$\neq$	$\epsilon\phi\alpha$	$\tau\epsilon\phi\alpha$

where  $\#$  : number\* or radiated objects in a k-open involved in the conflict.  
 $\exists C$  : there exist conflict.

---

\* This is the same as the power of a drasis-set  $D$ , see sect. 4.3 and 4.2.2.1.



$\#C$  : there is no conflict.

$\phi, \phi', \phi \circ \tilde{\phi}$  : various filtrations given earlier.

$\varepsilon$  : elimination.

$\alpha$  : amalgamation.

$\tau$  : identification.

#### 4.7 UTILISATION OF $\alpha, \phi$ TO SKELETIZATION

k-Structures skeletization is described in sect. 3.4, and skeleton formation process SFP) in sect. 3.4.2. Here, we deal again with skeletization, emphasizing in (A) the role of previously introduced  $\alpha$  and  $\phi$  operations; and (B) the Drasis point(s) and most effective drasis (m.e.d.) notions.

A. The most important feature in the synthesis stage for  $K_1, K_2$ -skeletization, is the set of operations is based upon. These operations are the means by which skeletization, a k-structure interaction, could be thought of as some sort of 'k-space dynamic interaction'. Thus, we may characterize operation  $\tau$  as 'exogenous' activity and  $\alpha, \phi, \varepsilon$  as 'endogenous' ones, respectively.

The outcome of repetitive application of  $\varepsilon, \phi$  and especially  $\alpha$ , among  $K_1, K_2$ 's ordinary opens and  $K_1, K_2$ 's fundamental terminals (e.g. b, w in sect. 3.3.1 case study) yield the *domain* ( $D$ ) and the *Range* ( $R$ ) of a function: (prec. functor, set 3.1.8)  $f: D \rightarrow R$  which might be thought of as an embodiment of  $K_1, K_2$ -skeleton, the corner-stone for an analogy. We conceive skeletizations as some sort of (concept)-filtering in higher-level\*.

---

\* In fact, a skeleton may be thought of as a context of a higher abstraction-level concept.

Successive applications of  $\alpha$  and  $\phi$  lead to the formation of the fundamental conceptual stones for skeleton; that is, abstracted objects\* and generalized morphisms\* between them.

$\phi$  Takes places between opens

and when  ~~$\#$~~  conflict situation in which they are involved, i.e. opens that they do not belong to a conflict C.

$\phi^*$  Takes place between opens having different number of objects, and which they do belong to a conflict.

During skeleton synthesis a dilemma occurs: Which course of action should be taken first, amalgamation or filtration? We think that the correct order is, first filtration and then amalgamation.

During the (semiotic) similarity matching of two k-opens another dilemma appears; nam. between what pair of k-opens should amalgamation and/or filtration take place? We believe in this case, that the priority is as follows:

1. object/morphism-elimination (if such an operation is required);
2. filtration;
3. amalgamation;

and the appropriate pairs are found via computational methods described in section 3.4.

B. In order to achieve skeletization when conflict occurs, we have to 'synthesize' (unify)  $D_1, D_2$ ; i.e. the sets of drasis points for  $K_1, K_2$  structures respectively, see also sect. 4.2.2.1. For the case

---

\* Are they of the same nature as the parent ones, or another semantics is required? This is an open question.

study we treat in sect. 4,2,2.1,

$$D_1 \equiv \{d_1^1, d_2^1\} \quad \text{and} \quad D_2 \equiv \{d_1^2\}$$

The above unification inevitably requires elimination. In addition, the unification between:

$D_1$  i.e.  $K_1$ 's terminal drasis points and

$D_2$  i.e.  $K_2$ 's terminal drasis points

in order to be consistent to initially external posited similarities (equivalence relation: LEFT  $\approx$  RIGHT) implies the elimination of morphisms attached to the most effective drasis (m.e.d); notions which are extensively analysed in sect. 4.3.1. Notice that, as we have seen, terminal drasis points are the most important; I think this situation is analogous to the following one in arithmetic, nam. 'when comparison of two factorizations is attempted, particular attention is given to the prime numbers involved in the factorizations'; see also footnote in sect. 4.3.1.

Finally, we point out that the above eliminated morphisms may be considered or characterized as 'irrelevant to analogizing' or 'no contributing'\* or 'redundant' or 'noise' morphisms. Also, the skeleton is consistent of: {amalgamations of non-conflicting (filtrated) opens} + {amalgamations of conflicting (filtrated) opens}; results which can be illustrated via the schemata of sect. 4.2.2.1.

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\* An example on the elaboration of 'contributing...' is given in MELETIS (1975) b.



## SUMMARY

## SUMMARY

In our attempts to investigate the role of analogy in problem-solving, learning and concept formation, we arrive at a number of interesting results which amount to the development of a new representational tool and a methodology involving various techniques, which are applied to different problem domains.

As far as the representation scheme is concerned, we devised, what we call, k-structure to symbolise a variety of situations and to represent knowledge. A k-structure consists of objects and morphisms and it closely resembles semantic networks and directed graphs. Its mathematical background is described in algebraic, topological and category theory terms. Among its virtues, are its visual appeal, its mathematical clarity and soundness, and the high degree of abstraction it offers for conceptualizations between similar and even quite distinct Universes of Discourse. Furthermore, a number of interpretations are given to its objects and morphisms to fit currently existing problem areas in the fields of Cybernetics and Artificial Intelligence. Finally, comparisons of k-structure vs. existing representational schemes are made; a list of its advantages and disadvantages is given, and its computer representations are outlined. Furthermore, open questions for future investigations are put forward.

On the other hand, the methodology we devised to meet our objectives incorporates the following points:

1. 'separation of the context of a concept from the concept itself' technique. It is based on the additional organization supplied on a k-structure, which amounts to the notion of a k-space founded on

the idea of what we call k-open; the latter constituents' being the centre object, radiated (in/out-going) morphisms and satellite morphisms. An analysis of a k-open is made from various viewpoints to emphasize its importance in the comparisons of k-structures.

2. 'Skeleton Formation Process' (SFP). It is the cornerstone in Analogical Problem-Solving (APS) especially for those which are in favour of a reduction approach to PS. In addition, SFP is a fundamental issue to Analogical Learning (AL). A skeleton is conceived of as the commonly shared substructure of two parent k-structures in algebraic terms. Thus, 'skeletalization' becomes a unified theme underlying both these two intellectual activities the brain is occupied with.

3. In our efforts to compare k-structures quite often we arrive at conflicts and dilemmas, situations which are overcome by devising a flexible 'conflict resolution' technique based, among other things, on object and morphism eliminations. Its flexibility is based on tactical and strategic advisors which may, somehow, be considered as the intelligent characteristics of the conflict resolution.

4. A rather formal account of conflict resolution is provided after the introduction of Knowledge Representation Structure (KRS) which is considered as an extended k-structure incorporating all of its mathematical features. The shift from k-structure to KRS is given and a schematology for KRS's utilization is provided.

5. The need to devise a 'conflict resolving clue' led us to the development of what we call 'drasis points', a fundamental attribute of a KRS, and to the 'most effective drasis' which actually resolves a dilemma after a suitable elimination of its morphism environment.



6. The need to compare k-spaces resulted in the introduction of a number of operations between their constituents. Among the operations we arrived at are amalgamation  $\alpha$  and filtration  $\phi$ . Then utilization of  $\phi$  and  $\alpha$  is made in skeletization.
7. We have considered a 'concept' as a sequence of similar contexts and we proved a proposition about the common understanding of a concept from two conversationalists or learners.
8. Particular attention is also given to shifting between representations (considered as k-structures). Various kinds of shifting are proposed and the role of analogy in such shifts is emphasized.
9. The significance of interaction between problem structure and problem solving behaviour is stressed. It is captured in what we call 'epistemological-heuristic interaction' which may lead us in the future to an epistemologically heuristically adequate shift of representations which has not yet been achieved.
10. A description of problem-solving is then given in terms of k-structures following by a detailed analysis of (APS) for which a number of theorem schemata and their interpretation is provided.
11. We conceive of an analogy as a functor between k-structures. The role of analogy in (the mechanization of)shifting of representations may be captured in what we call 'Harmonization of skeletons' a fundamental cybernetic technique to be developed in the future for mechanizing analogical intellectual activities.

Finally, representation and methodology are employed in WINSTON's (1970) and EVANS' (1963) problem domains to capture and illustrate the issues developed in the present thesis.

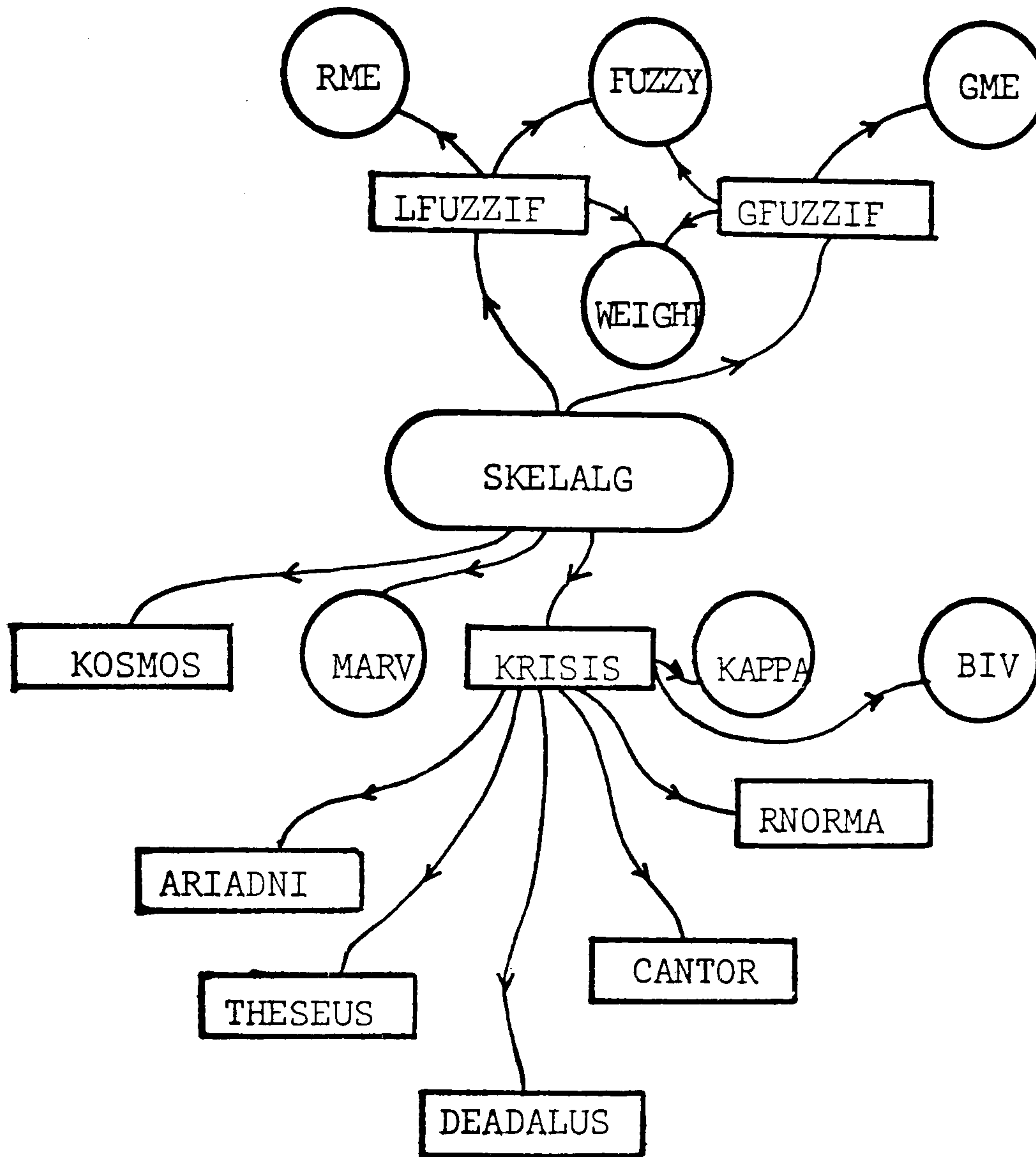
Further hints and suggestions for future investigations are scattered throughout the work but we avoid here their repetition.

**APPENDIX**

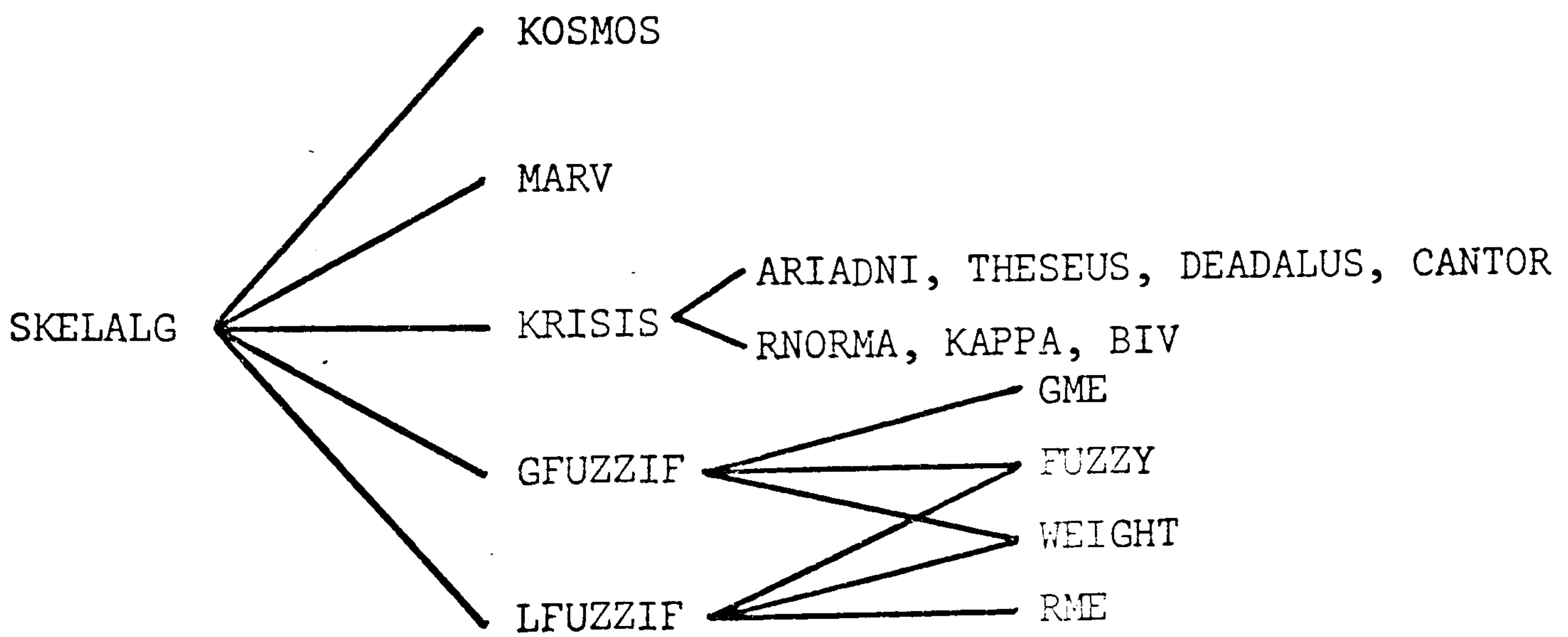


APPENDIX

0.1 SKELALG - COMPUTER PROGRAMME IN NET FORM



0.2 SKELALG - IN TREE FORM



## 1. ON SUBROUTINES

KOSMOS subroutine. Its purpose is the construction of the (Level-1) context of every object, i.e.  $\epsilon\pi$  composition. KOSMOS implements step 3 in the discussion on pairing off objects (Sect. 3.3.3). It receives a  $k$ -structure and outputs contexts, which we called *k-opens*, with some extra information used in the comparison stage. KOSMOS' output, called *k-space* serves for the comparison step in skeleton formation process (SFP).

## 2. TECHNICAL POINTS

### 2.0 Introduction

The following short notes are about the nature of concepts GME, LME, MARV, EMARV, BIV, RNORMA, and LAC matrix. Some of them constitute the basis for a variety of similarity measures for  $k$ -opens in a  $k$ -space. Their mathematical formula is given and an account for their *raison d'être* is outlined. The main references for this section are: WINSTON (1970), GLUSHKOV (1966), HALPERN (1975), ZADEH (1973), KOWALSKI (1975), EVANS (1963), BARNDEN (1975) and MELETIS (1977)a,b.

Next, an analysis is given of the implemented, via SKELALG computer programme, concepts:

1. Global Morphism Emphasis:  $GME \in (0,1)$
2. Local Morphism Emphasis :  $LME \in (0,1)$
3. Means for Adjacency Representation Vector:  $MARV \in R^3$
4. Extended MARV :  $EMARV \in R^4$
5. Boundary Index Vector: BIV
6. RNORMA: BIV's norm.

This note is a detailed account for *dilemma 1*, outlined in MELETIS (1977)a, p. 14. Having the purpose of bringing about a connection between k-opens, i.e. k-spaces' ingredients, we found that hints (as guides for k-opens' similarity/adjacency/connectivity) might be devised from the contribution of radiated objects and morphisms, and of satellite ones, that is, the constituents for a k-open. GME, LME, MARV, and EMARV offer the basis for introducing some kind of variety of similarity measures, which is, in some sense, absent in WINSTON (1970) implementation; and as he points out, "the creation of a manifold of similarity hints, would provide with greater flexibility the decision making process", during the course of comparison, i.e. "off the cuff" alternative similarity measures would be in "off the shelf" manner available, facilitating thus the resolution for conflicts, during various processes, especially amalgamation and filtration, which are vital to skeletization (and harmonization). Noteworthy, EVANS (1963) uses some, but limited, variety of alternative similarity measures. Finally, the flavour of GME and LME is to give a distinct colour to each morphism. That is, they might be thought of as interpretations, nam. global and local resp.

## 2.1 GME

Global Morphism Emphasis is defined as

$$\begin{aligned} \text{GME} : M &\longrightarrow (0,1) \subset \mathbb{R} \\ &: m \longmapsto \text{GME}(m) := \frac{O(m)}{\text{TNOM}} \end{aligned}$$

where,  $M$ : morphisms;  $m \in M$ ,  $O(m)$ : occurrences of morphism  $m$  in a  $k$ -structure; and  $\text{TNOM}$ : total number of morphisms;  $\mathbb{R}$ : real numbers.

For WINSTON's analogy type example we found:

<u>STRUCTURE 1</u>	<u>STRUCTURE 2</u>
GME(m1)=.412	GME(M1)=.500
GME(m2)=.176	GME(m2)=.167
GME(m3)=.118	GME(m3)=.0
GME(m4)=.294	GME(m4)=.333

For normalisation is valid:  $\sum_{i,k} GME(m_i) = 1, i=1,2,3,4, k=1,2.$

*raison d'etre*: the values of GME function may be taken as some kind of colour to a morphism or morphism's weight (MW) or fuzzy value (MFV), by means of which fuzzification (from global point of view) of k-space's opens is feasible, and therefore assignment of quantitative, thus computable, features to k-opens is achievable. One of them, for example, is the weight of a k-open given by:

$$B = \frac{W}{TNOM} + \frac{SW}{TNOM}$$

where, W : weight of radiated morphism

SW : weight of satellite morphisms.

Via GME the fuzzy k-space for some of WINSTON's structures is computed.

## 2.2 LME

Local morphism emphasis is defined as

$$\begin{aligned} \text{LME: } M &\longrightarrow (0,1) \subset R \\ ; m &\longmapsto \text{LME}(m) := \frac{1}{2} \left( \frac{R}{TR} + \frac{S}{TS} \right) \end{aligned}$$

where, R: occurrences of radiated morphism m

TR: total no. of radiated morphisms



TS : total no. of satellite morphisms

S : satellite occurrences of m.

Nature of LME: relative contribution of radiated and satellite morphisms to a k-open.

*Raison d'etre*: LME is served as a kind of hint for similarity comparisons. As from its nature implies, LME leads to a local feature for each k-open. That is the open's weight from local point of view.

Implications: LME is used for computing connectivity/adjacency measures D1, D15, D2, see MELETIS (1977)a,b.

Via LME the local fuzzification of k-space leads to a fuzzy k-space computable by SKELALG computer programme.

### 2.3 MARV, EMARV, BIV, RNORMA

The above are defined as:

$$\begin{aligned} \underline{\text{BIV}} : \text{OPENS} &\longrightarrow \mathbb{R}^3 \\ \text{...} : X &\longmapsto \text{BIV}(X) := (I1, I2, I3), \text{ for every non-isolated} \\ &\text{object} \end{aligned}$$

where,

$$I1 := \frac{RO}{TNO}, \text{ RO : radiated objects, TNO: total no. of objects in the k-open}$$

$$I2 := \frac{RM}{TNOM}, \text{ RM : radiated morphisms, TNOM: total no. of morphisms}$$

$$I3 := \frac{SM}{TNOM}, \text{ SM : satellite morphisms.}$$

An object is called *isolated* iff it does not have radiated ingoing or outgoing morphisms.

$$\begin{aligned} \underline{\text{MARV}} &: \text{ OPENS} \longrightarrow \mathbb{R}^+ \\ &: X \longleftarrow \longrightarrow \text{MARV}(X) := \text{NORMA}(\text{BIV}), \end{aligned}$$

$$\text{where, } \text{NORMA}(\text{BIV}) := \sqrt{(I1)^2 + (I2)^2 + (I3)^2}$$

$$\text{EMARV} := \text{NORMA}(\text{EBIV})$$

where,  $\text{EBIV} := (I1, I21, I22, I3)$ .  $I21, I22$  symbolise resp. the no. of ingoing and outgoing morphism.

MARV may be interpreted as 'the object's distance from isolation', which features a k-open (qualitative feature). Via MARV or EMARV, various fuzzifications for a k-space of a given k-structure are computationally feasible.

MARV, BIV, NORMA are used for expressing ISOL1, ISOL2, ISOL3, which is another triple for structural/quantitative set adjacency of k-opens; for example

$$\begin{aligned} \text{ISOL2}(C_X, C_Y) &:= \sum_{x_i} \sum_{y_j} |\text{MARV}_{x_i} - \text{MARV}_{y_j}|, \\ & \quad x_i \in C_X, y_j \in C_Y. \end{aligned}$$

### 3.11 GLOBAL FUZZIFIER

Computes the global emphasis of each morphism, for all k-opens. Then it fuzzifies k-space's constituents, assigning to each morphism its global emphasis. GFUZZIF uses FUZZY, GME, WEIGHT subroutines.

### 3.2 LOCAL FUZZIFIER

Computes the local emphasis of each morphism for each open. Then it fuzzifies k-space, assigning to each morphism its local emphasis, which generally varies from open to open. LFUZZIF uses FUZZY, RME, WEIGHT subroutines.

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