Distributed State Estimation for Uncertain Markov-type Sensor Networks with Mode-dependent Distributed Delays

Jinling Liang*, Zidong Wang and Xiaohui Liu

Abstract

In this paper, the distributed state estimation problem is investigated for a class of sensor networks described by uncertain discrete-time dynamical systems with Markovian jumping parameters and distributed time-delays. The sensor network consists of sensor nodes characterized by a directed graph with a nonnegative adjacency matrix that specifies the interconnection topology (or the distribution in the space) of the network. Both the parameters of the target plant and the sensor measurements are subject to the switches from one mode to another at different times according to a Markov chain. The parameter uncertainties are norm-bounded that enter into both the plant system as well as the network outputs. Furthermore, the distributed time-delays are considered which are also dependent on the Markovian jumping mode. Through the measurements from a small fraction of the sensors, this paper aims to design state estimators that allow the nodes of the sensor network to track the states of the plant in a distributed way. It is verified that such state estimators do exist if a set of matrix inequalities are solvable. A numerical example is provided to demonstrate the effectiveness of the designed distributed state estimators.

Keywords

Distributed state estimation, sensor network, parameter uncertainties, Markovian chain, mode-dependent distributed delays.

I. Introduction

In the past few years, theoretical research and practical applications of sensor networks have received increasing research attention from a variety of areas including military sensing, physical security, air traffic control, distributed robotics, as well as industrial and manufacturing automation. Roughly speaking, a sensor network consists of a number of sensor nodes distributed over a spatial region. Each sensor node has wireless communication capability, some level of intelligence for signal processing and for disseminating data. These sensor nodes collaborate amongst themselves to set up a sensing network [3, 27]. A typical problem in sensor networks, known as the distributed estimation problem, is how to estimate the state of the dynamic process (plant target) from spatially distributed nodes that form a wireless ad hoc network with every node having its own notion of time. In other words, for distributed estimation problems, the individual sensor in a sensor network locally estimates the system state from not only its own measurement but also its neighboring sensors'

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measurements according to the given topology. Fundamentally different from the traditional central filtering techniques [9, 12, 21, 24, 26, 29], the difficulty in designing distributed estimation algorithms stems from 1) the complicated coupling between the sensor nodes according to a given topology; and 2) the network-induced phenomena such as distributed communication delays and Markov-type mode switches.

Owing to its importance in collaborative signal processing to efficiently process distributed information gathered, the distributed estimation problem has gained particular research interests in recent years. For example, in [18,19], a distributed Kalman filtering algorithm has been developed with different sensing models and different consensus strategies. In [22], a distributed estimation algorithm has been proposed to adaptively update the weights for minimizing the estimation error variance. An alternative multiscale approach has been put forward in [11] to accelerate the convergence of the decentralized consensus problem for networks with poor connectivity. In [28], a new type of distributed consensus filter is designed, where each sensor can communicate with the neighboring sensors and filtering can be performed in a distributed way. More recently, in [5, 6], the distributed diffusion filtering algorithm has been designed with the information diffused across the network through a sequence of Kalman iterations and data-aggregation. Despite the popularity of the investigation on various distributed estimation algorithms, the network-induced phenomena that are directly associated with sensor networks have been largely overlooked due probably to the mathematical complexity, which motivates our present research.

For wireless sensor networks, the network size, the communication constraints as well as the stringent energy limit give rise to great challenges to classical estimation techniques that demand enormous storage space and centralized computation. As such, it is not surprising that distributed estimation algorithms have recently stirred considerable research attention. Nevertheless, there remain some open issues outlined as follows that deserve further investigation. 1) A sensor network is often subject to the mode switch according to a Markov chain. In [8], a sensor network has been shown to have *jumping* behavior due to the network's working environment (normal or hazardous) and the mobility of the sensor node. In [10], it has been concluded that the shuffle-exchange networks can model practical interconnection systems due to their size of *switching* (*jumping*) elements and uncomplicated configuration. 2) The distributed delays occur naturally in sensor networks since the wireless information transmission has a spatial nature due to the presence of an amount of parallel pathways of a variety of node sizes and lengths. Note that, for sensor networks, the distributed delay typically takes place in a digital (discrete) way, see [16] for more details about the discrete distributed time-delays. 3) In practice, the actual spatial positions of sensor nodes at the experimentation stage may be uncertain to some extent and randomly fluctuate around some locations specified at the configuration stage [20], and the parameters of the modeled sensor networks are inevitably subject to uncertainties. Unfortunately, up to now, the unavoidable issues of mode switching, distributed delays as well as parameter uncertainties have not yet been investigated for the distributed state estimation problem of sensor networks. It is, therefore, the main aim of this paper to shorten such a gap by launching an initial study on the distributed state estimation for uncertain Markov-type sensor networks with mode-dependent distributed delays.

In this paper, attention is focused on the distributed state estimation problem for a class of sensor networks described by uncertain discrete-time stochastic systems with Markovian jumping parameters and mode-dependent distributed delays. Through available output measurements from each individual sensor, we wish to design distributed state estimators to approximate the states of the networked dynamic system based on the sensor's and its neighboring sensors' measurements according to certain topology. Sufficient conditions are presented to guarantee the convergence of the estimation error systems for all admissible parameter un-

certainties and distributed delays for each mode. The explicit expression of individual estimator is derived to facilitate the distributed computing of state estimation from each sensor. Finally, numerical example is given to verify the theoretical results.

The remaining part of the paper is organized as follows. In Section II, the distributed sate estimation problem is formulated for the sensor network with Markov-type uncertain parameters and distributed delays, and some preliminaries are briefly outlined. In Section III, by resorting to the inequality techniques combining with a novel Lyapunov-Krasovskii functional constructed to reflect the mode-dependent time delays, we conduct the analysis to obtain several sufficient criteria which not only ensure the estimation error systems to be globally asymptotically stable in the mean square sense but also characterize the explicit expression of the estimator gains. In Section IV, a numerical example is given to show the applicability of the acquired conditions. Finally, conclusions are drawn in Section V.

Notations. The notation used throughout this paper is fairly standard. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m\times n$ real matrices, respectively. $\mathbb{N}$ is used to be the set $\{1,2,\ldots\}$. I and 0 represent, respectively, the identity matrix and the zero matrix of appropriate dimensions. The Kronecker product of matrices $A\in\mathbb{R}^{m\times n}$ and $B\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $A\otimes B$. The notation $X>0$ for $X\in\mathbb{R}^{n\times n}$ means that matrix X is real, symmetric and positive definite. $\mathrm{diag}(\cdots)$ and $\mathrm{col}(\cdots)$ stand for, respectively, the block-diagonal matrix and the matrix column with blocks given by the matrices in $(\cdots)$. $\|\cdot\|$ refers to the Euclidean vector norm and the asterisk '$*$' in a symmetric matrix is used to denote the term that is induced by symmetry. For notation $(\Omega,\mathcal{F},\mathscr{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathscr{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{E}\{\alpha_1\}$, $\mathbb{E}\{\alpha_1|\alpha_2\}$ mean, respectively, the mathematical expectation of the stochastic variable α_1 and the expectation of α_1 conditional on α_2. Matrices, if not stated, are assumed to be compatible for algebraic operations.

II. Problem formulation and some preliminaries

Let the complete probability space $(\Omega,\mathcal{F},\mathscr{P})$ be fixed. Consider the Markov-type uncertain dynamic system (target plant) with n modes described by the following discrete-time model:

$$x(k+1)=D(k,r(k))x(k)+B(k,r(k))f(x(k))+W(k,r(k))\sum_{v=1}^{\tau(r(k))}g(x(k-v)) \tag{1}$$

with N sensors characterized by:

$$y_i(k)=C_i(k,r(k))x(k),\qquad i=1,2,\ldots,N \tag{2}$$

where $k\in\mathbb{N}$; $x(k)\in\mathbb{R}^{n_x}$ is the state of the target; $y_i(k)\in\mathbb{R}^{n_y}$ is the measurement output from sensor i of the target plant; v is the time-delay; $\{r(k):k\in\mathbb{N}\}$ is a discrete-time homogeneous Markov chain which governs the switches among the different system modes and takes values in a finite mode set $\mathcal{S}\triangleq\{1,2,\ldots,n\}$ with mode transition probability matrix $\Pi\triangleq\{\pi_{lm}\}_{n\times n}$ given by

$$\mathscr{P}\{r(k+1)=m|r(k)=l\}=\pi_{lm},$$

where $\pi_{lm}\geq 0$ $(l,m\in\mathcal{S})$ is the transition rate from mode l to mode m and, for every $l\in\mathcal{S}$, $\sum_{m=1}^{n}\pi_{lm}=1$. $\tau(r(k))$ characterizes the distributed time-delay dependent on the system mode $r(k)$; $f(\cdot)$ and $g(\cdot)$ are nonlinear functions; $D(k,r(k))$, $B(k,r(k))$, $W(k,r(k))$ and $C_i(k,r(k))$ $(i=1,2,\ldots,N)$ are time-varying matrices defined

on the Markov chain of the form

$$\begin{aligned} &D(k,r(k)) = D(r(k)) + \Delta D(k,r(k)), \qquad B(k,r(k)) = B(r(k)) + \Delta B(k,r(k)), \\ &W(k,r(k)) = W(r(k)) + \Delta W(k,r(k)), \qquad C_i(k,r(k)) = C_i(r(k)) + \Delta C_i(k,r(k)), \end{aligned} \tag{3}$$

in which the constant matrices $D(r(k))$, $B(r(k))$, $W(r(k))$ and $C_i(r(k))$ are known, while $\Delta D(k,r(k))$, $\Delta B(k,r(k))$, $\Delta W(k,r(k))$ and $\Delta C_i(k,r(k))$ are unknown matrices representing the time-varying parameter uncertainties satisfying the following conditions:

$$\begin{aligned} &[\Delta D(k,r(k)),\ \Delta B(k,r(k)),\ \Delta W(k,r(k))] = E_1(r(k))F_1(k,r(k))[M_1(r(k)),\ M_2(r(k)),\ M_3(r(k))], \\ &\Delta C_i(k,r(k)) = E_{2i}(r(k))F_2(k,r(k))M_4(r(k)); \end{aligned} \tag{4}$$

where $E_1(r(k))$, $E_{2i}(r(k))$ and $M_l(r(k))$ $(l = 1,2,3,4)$ are known real constant matrices and $F_1(k,r(k))$, $F_2(k,r(k))$ are unknown time-varying matrix-valued functions subject to the following conditions:

$$F_l^T(k,r(k))F_l(k,r(k)) \le I, \qquad l = 1,2;\ r(k) \in \mathcal{S},\ k \in \mathbb{N}. \tag{5}$$

Remark 1: In practice, the sensor networks often exhibit the special characteristic of Markov mode switching, that is, the sensor networks sometimes have finite modes that switch from one to another at different times, and such a switching (or jumping) can be governed by a Markovian chain. For example, the bufferless packet switching of trees and leveled networks has been illustrated in [4] to be achievable with certain network topologies. In model (1), the switching signal $\{r(k) : k \in \mathbb{N}\}$ in the target plant could be obtained through statistical experiments. Note that the Markovian jumping system has been well studied for H_∞ control and filtering problems, see e.g. [1] and [21] for discrete- and continuous-time linear systems with mode-dependent delays. For more information about the mode-dependent time-delay systems, we refer to [13–15].

In this paper, consider the sensor network consisting of N sensor nodes characterized by a directed graph $\mathscr{G} = (\mathcal{V}, \mathcal{E})$ with a nonnegative adjacency matrix $\mathcal{A} = (a_{ij})_{N\times N}$ that specifies the interconnection topology (or the distribution in the space) of the sensor network. Here, $\mathcal{V} = \{1,2,\ldots,N\}$ denotes the set of sensor nodes and $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the set of edges. Edge $(i,j) \in \mathcal{E}$ means that there is information transmission from sensor j to sensor i which is represented equivalently by $a_{ij} > 0$. Moreover, it is also assumed in this paper that the sensors are self-connected, i.e., $a_{ii} = 1$ for all $i \in \mathcal{V}$. Node j is called one of the neighbors of node i if $(i,j) \in \mathcal{E}$. According to the given network topology, in the sensor network, node $i \in \mathcal{V}$ can receive information from its neighboring nodes $j \in \mathcal{N}_i$, where $\mathcal{N}_i \triangleq \{j \in \mathcal{V} | (i,j) \in \mathcal{E}\}$.

It should be noted that, in practice, there are a huge amount of sensors that can measure the target state $x(k)$ from measurement outputs $y_i(k)$ $(i = 1,2,\ldots,N)$. However, it is usually impossible to have only one centralized processor to collect the measurements from all the sensors, especially in a remote area. Therefore, it is more appropriate to use local information achieved from neighbors in a distributed way. The objective of this paper, accordingly, is to design a distributed state estimator to track the state $x(k)$ of the target plant with an acceptable accuracy.

Construct the following state estimator on sensor node i:

$$\begin{aligned} \hat{x}_i(k+1) \ = \ & D(r(k))\hat{x}_i(k) + B(r(k))f(\hat{x}_i(k)) + W(r(k)) \sum_{v=1}^{\tau(r(k))} g(\hat{x}_i(k-v)) \\ & + \sum_{j\in\mathcal{N}_i} a_{ij}K_{ij}(r(k))[y_j(k) - C_j(r(k))\hat{x}_j(k)], \end{aligned} \tag{6}$$

where $\hat{x}_i(k) \in \mathbb{R}^{n_x}$ is the estimate for the plant state $x(k)$ on the node i, and $K_{ij}(r(k)) \in \mathbb{R}^{n_x \times n_y}$ is the estimator gain matrix to be determined.

Letting $\tilde{x}_i(k) = x(k) - \hat{x}_i(k)$ be the state estimation error, it follows from (1), (2) and (6) that

$$\begin{aligned}\tilde{x}_i(k+1) &= D(r(k))\tilde{x}_i(k) + \Delta D(k, r(k))x(k) + B(r(k))\tilde{f}(\tilde{x}_i(k)) + \Delta B(k, r(k))f(x(k)) \\ &\quad + W(r(k)) \sum_{v=1}^{\tau(r(k))} \tilde{g}(\tilde{x}_i(k-v)) + \Delta W(k, r(k)) \sum_{v=1}^{\tau(r(k))} g(x(k-v)) \\ &\quad - \sum_{j \in \mathcal{N}_i} a_{ij} K_{ij}(r(k))[C_j(r(k))\tilde{x}_j(k) + \Delta C_j(k, r(k))x(k)], \quad i = 1, 2, \ldots, N \end{aligned} \tag{7}$$

where $\tilde{f}(\tilde{x}_i(k)) = f(x(k)) - f(\hat{x}_i(k))$ and $\tilde{g}(\tilde{x}_i(k-v)) = g(x(k-v)) - g(\hat{x}_i(k-v))$. By using the notation of Kronecker product, the estimation error dynamics governed by (7) can be rewritten in a compact form as follows:

$$\begin{aligned}\tilde{x}(k+1) &= (I_N \otimes D(r(k)) - \bar{K}(r(k))\tilde{C}(r(k)))\tilde{x}(k) + (\Delta\tilde{D}(k, r(k)) - \bar{K}(r(k))\Delta\bar{C}(k, r(k)))x(k) \\ &\quad + (I_N \otimes B(r(k)))F(\tilde{x}(k)) + \Delta\tilde{B}(k, r(k))f(x(k)) \\ &\quad + (I_N \otimes W(r(k))) \sum_{v=1}^{\tau(r(k))} G(\tilde{x}(k-v)) + \Delta\tilde{W}(k, r(k)) \sum_{v=1}^{\tau(r(k))} g(x(k-v)), \end{aligned} \tag{8}$$

where $\tilde{x}(k) = \mathrm{col}(\tilde{x}_1(k), \tilde{x}_2(k), \ldots, \tilde{x}_N(k))$, $\tilde{C}(r(k)) = \mathrm{diag}(C_1(r(k)), C_2(r(k)), \ldots, C_N(r(k)))$,

$$\Delta\tilde{D}(k, r(k)) = [\underbrace{(\Delta D(k, r(k)))^T, (\Delta D(k, r(k)))^T, \ldots, (\Delta D(k, r(k)))^T}_{N}]^T,$$

$$\Delta\tilde{B}(k, r(k)) = [\underbrace{(\Delta B(k, r(k)))^T, (\Delta B(k, r(k)))^T, \ldots, (\Delta B(k, r(k)))^T}_{N}]^T,$$

$$\Delta\tilde{W}(k, r(k)) = [\underbrace{(\Delta W(k, r(k)))^T, (\Delta W(k, r(k)))^T, \ldots, (\Delta W(k, r(k)))^T}_{N}]^T$$

$$F(\tilde{x}(k)) = \begin{bmatrix} \tilde{f}(\tilde{x}_1(k)) \\ \tilde{f}(\tilde{x}_2(k)) \\ \vdots \\ \tilde{f}(\tilde{x}_N(k)) \end{bmatrix}, G(\tilde{x}(k-v)) = \begin{bmatrix} \tilde{g}(\tilde{x}_1(k-v)) \\ \tilde{g}(\tilde{x}_2(k-v)) \\ \vdots \\ \tilde{g}(\tilde{x}_N(k-v)) \end{bmatrix}, \Delta\bar{C}(k, r(k)) = \begin{bmatrix} \Delta C_1(k, r(k)) \\ \Delta C_2(k, r(k)) \\ \vdots \\ \Delta C_N(k, r(k)) \end{bmatrix}$$

and $\bar{K}(r(k)) = (a_{ij}K_{ij}(r(k)))_{N \times N}$ is a sparse matrix satisfying $\bar{K}(r(k)) \in \mathscr{W}_{n_x \times n_y}$, where $\mathscr{W}_{n_x \times n_y}$ is defined as

$$\mathscr{W}_{n_x \times n_y} = \{\bar{U} = [U_{ij}] \in \mathbb{R}^{n_x N \times n_y N} | U_{ij} \in \mathbb{R}^{n_x \times n_y},\ U_{ij} = 0 \ \text{ if } j \notin \mathcal{N}_i\}. \tag{9}$$

Throughout this paper, the following assumptions are made on the nonlinear functions $f(\cdot)$ and $g(\cdot)$ in the target system (1).

Assumption 1: $f(0) = 0$, $g(0) = 0$.

Assumption 2: There exist matrices Σ_1 and Σ_2 such that

$$\|f(u) - f(v)\| \le \|\Sigma_1(u - v)\|, \quad \|g(u) - g(v)\| \le \|\Sigma_2(u - v)\|; \qquad \forall u, v \in \mathbb{R}^{n_x}.$$

According to Assumption 2, one can easily obtain that the nonlinear functions $F(\cdot)$ and $G(\cdot)$ defined in (8) satisfy the following conditions

$$\|F(\tilde{x}(k))\| \leq \|\tilde{\Sigma}_1\tilde{x}(k)\|, \qquad \|G(\tilde{x}(k)\| \leq \|\tilde{\Sigma}_2\tilde{x}(k)\|; \tag{10}$$

where $\tilde{\Sigma}_1 = \mathrm{diag}(\Sigma_1, \Sigma_1, \ldots, \Sigma_1)$ and $\tilde{\Sigma}_2 = \mathrm{diag}(\Sigma_2, \Sigma_2, \ldots, \Sigma_2)$.

The initial condition associated with (8) is given as

$$\tilde{x}(s) = \varphi(s) \in \mathbb{R}^{n_x N}, \qquad s = -\bar{\tau}, -\bar{\tau}+1, \ldots, 0 \tag{11}$$

where $\bar{\tau} = \max\{\tau(1), \tau(2), \ldots, \tau(n)\}$.

Let $\{\tilde{x}(k;\varphi); k \in \mathbb{N}\}$ denote the corresponding state trajectory of the uncertain error system (8).

Definition 1: The system (6) is said to be a robustly distributed state estimator of the Markov-type dynamic system (1) with measurement outputs (2) if the estimation error system (8) is robustly asymptotically stable in the mean square sense, i.e., for any initial condition $\varphi(\cdot)$ and for all parameter uncertainties satisfying (4)-(5), the following equality

$$\lim_{k\to\infty} \mathbb{E}\{\|\tilde{x}(k;\varphi)\|^2\} = 0$$

holds for the corresponding solution $\{\tilde{x}(k;\varphi); k \in \mathbb{N}\}$ of the error system (8).

Our main aim in this paper is to choose suitable estimator gain matrices $K_{ij}(r(k))$ such that the estimation error system (8) is robustly asymptotically stable in the mean square sense. Before deriving the main results, two useful lemmas are given as follows.

Lemma 1: Let X, Y and F be real matrices of appropriate dimensions with F satisfying $F^TF \leq I$. Then, for any scalar $\varepsilon > 0$,

$$XFY + (XFY)^T \leq \varepsilon^{-1}XX^T + \varepsilon Y^TY.$$

Lemma 2: [16] Let $Q \in \mathbb{R}^{n\times n}$ be a positive semi-definite matrix, $x_i \in \mathbb{R}^n$ and scalar constant $b_i \geq 0$ ($i = 1, 2, \ldots$). If the series concerned is convergent, then the following inequality holds:

$$\left(\sum_{i=1}^{+\infty} b_i x_i\right)^T Q \left(\sum_{i=1}^{+\infty} b_i x_i\right) \leq \left(\sum_{i=1}^{+\infty} b_i\right) \sum_{i=1}^{+\infty} b_i x_i^T Q x_i. \tag{12}$$

III. Main results

In this section, by utilizing the Lyapunov stability theory and the matrix inequality technique, sufficient conditions are to be derived which not only ensure the estimation error system (8) to be robustly asymptotically stable in the mean square sense but also characterize the explicit expression of the estimator gains.

Taking the augmented state vector as $e(k) = \mathrm{col}(x(k), \tilde{x}(k))$, the combination of (1) and (8) leads to the following augmented system:

$$e(k+1) = \mathcal{D}(k, r(k))e(k) + \mathcal{B}(k, r(k))\mathscr{F}(e(k)) + \mathcal{W}(k, r(k)) \sum_{v=1}^{\tau(r(k))} \mathcal{G}(e(k-v)), \tag{13}$$

where $\mathcal{D}(k, r(k)) = \mathcal{D}(r(k)) + \Delta\mathcal{D}(k, r(k))$, $\mathcal{B}(k, r(k)) = \mathcal{B}(r(k)) + \Delta\mathcal{B}(k, r(k))$, $\mathcal{W}(k, r(k)) = \mathcal{W}(r(k)) + \Delta\mathcal{W}(k, r(k))$ and

$$\mathcal{D}(r(k)) = \begin{bmatrix} D(r(k)) & 0 \\ 0 & I_N \otimes D(r(k)) - \bar{K}(r(k))\tilde{C}(r(k)) \end{bmatrix},$$

$$\mathcal{B}(r(k)) = \begin{bmatrix} B(r(k)) & 0 \\ 0 & I_N \otimes B(r(k)) \end{bmatrix}, \quad \mathcal{W}(r(k)) = \begin{bmatrix} W(r(k)) & 0 \\ 0 & I_N \otimes W(r(k)) \end{bmatrix},$$

$$\Delta\mathcal{D}(k,r(k))=\left[\begin{array}{cc}\Delta D(k,r(k)) & 0\\ -\bar{K}(r(k))\Delta\bar{C}(k,r(k))+\Delta\tilde{D}(k,r(k)) & 0\end{array}\right],$$
$$\Delta\mathcal{B}(r(k))=\left[\begin{array}{cc}\Delta B(k,r(k)) & 0\\ \Delta\tilde{B}(k,r(k)) & 0\end{array}\right],\quad \Delta\mathcal{W}(k,r(k))=\left[\begin{array}{cc}\Delta W(k,r(k)) & 0\\ \Delta\tilde{W}(k,r(k)) & 0\end{array}\right],$$
$$\mathscr{F}(e(k))=\left[\begin{array}{c}f(x(k))\\ F(\tilde{x}(k))\end{array}\right],\quad \mathcal{G}(e(k-v))=\left[\begin{array}{c}g(x(k-v))\\ G(\tilde{x}(k-v))\end{array}\right].$$

In terms of Assumptions 1-2 and the properties of nonlinear functions $F(\cdot)$, $G(\cdot)$ characterized in (10), it can be easily concluded that the following inequalities

$$\|\mathscr{F}(e(k))\|\leq\|\widehat{\Sigma}_1e(k)\|,\qquad \|\mathcal{G}(e(k))\|\leq\|\widehat{\Sigma}_2e(k)\| \tag{14}$$

hold, where $\widehat{\Sigma}_1=\mathrm{diag}(\Sigma_1,\tilde{\Sigma}_1)$, $\widehat{\Sigma}_2=\mathrm{diag}(\Sigma_2,\tilde{\Sigma}_2)$ and $\tilde{\Sigma}_1$, $\tilde{\Sigma}_2$ are as defined in (10).

We are now ready to state the main results of this paper as follows.

Theorem 1: Under Assumptions 1-2, for the given $\bar{K}(l)$ ($l\in\mathcal{S}$), the estimation error system (8) is robustly asymptotically stable in the mean square sense if there exist matrices $\mathcal{P}(l)>0$ ($l\in\mathcal{S}$), $\mathcal{Q}>0$ and positive scalars ε_1, ε_2, δ_1, δ_2 such that the following matrix inequalities hold:

$$\left[\begin{array}{cccc}\Upsilon(l) & \mathcal{Z}^T(l)\overline{\mathcal{P}}(l) & 0 & 0\\ * & -\overline{\mathcal{P}}(l) & \overline{\mathcal{P}}(l)\widehat{E}_1(l) & \overline{\mathcal{P}}(l)\widehat{E}_2(l)\\ * & * & -\delta_1I & 0\\ * & * & * & -\delta_2I\end{array}\right]<0,\qquad (l\in\mathcal{S}) \tag{15}$$

where $\overline{\mathcal{P}}(l)=\sum\limits_{m=1}^{n}\pi_{lm}\mathcal{P}(m)$, $\mathcal{Z}(l)=[\mathcal{D}(l)\ \ \mathcal{B}(l)\ \ 0\ \ \mathcal{W}(l)]$, $\Upsilon(l)=\Xi(l)+\delta_1\aleph_1^T(l)\aleph_1(l)+\delta_2\aleph_3^T(l)\aleph_3(l)$,

$$\widehat{E}_1(l)=\mathrm{col}(E_1(l),\tilde{E}_1(l)),\qquad \tilde{E}_1(l)=[\underbrace{E_1^T(l),E_1^T(l),\ldots,E_1^T(l)}_{N}]^T;$$
$$\widehat{E}_2(l)=\mathrm{col}(0,\bar{K}(l)\tilde{E}_2(l)),\qquad \tilde{E}_2(l)=[\underbrace{E_{21}^T(l),E_{22}^T(l),\ldots,E_{2N}^T(l)}_{N}]^T;$$
$$\Xi(l)=\mathrm{diag}\left(-\mathcal{P}(l)+\varepsilon_1\widehat{\Sigma}_1^T\widehat{\Sigma}_1+\varepsilon_2\widehat{\Sigma}_2^T\widehat{\Sigma}_2,-\varepsilon_1I,-\varepsilon_2I+(\tau(l)+\tilde{\tau})\mathcal{Q},-\frac{1}{\tau(l)}\mathcal{Q}\right),$$
$$\aleph_1(l)=\left[\begin{array}{cccc}\widehat{M}_1(l) & \widehat{M}_2(l) & 0 & \widehat{M}_3(l)\end{array}\right],\qquad \aleph_3(l)=\left[\begin{array}{cccc}\widehat{M}_4(l) & 0 & 0 & 0\end{array}\right],$$
$$\widehat{M}_1(l)=\left[\begin{array}{cc}M_1(l) & 0\end{array}\right],\widehat{M}_2(l)=\left[\begin{array}{cc}M_2(l) & 0\end{array}\right],\widehat{M}_3(l)=\left[\begin{array}{cc}M_3(l) & 0\end{array}\right],\widehat{M}_4(l)=\left[\begin{array}{cc}M_4(l) & 0\end{array}\right]$$

and $\tilde{\tau}=\frac{1}{2}(\bar{\tau}-\underline{\tau})(1-\underline{\pi})(\bar{\tau}+\underline{\tau}+1)$, $\underline{\tau}=\min\{\tau(1),\tau(2),\ldots,\tau(n)\}$, $\underline{\pi}=\min\{\pi_{11},\pi_{22},\ldots,\pi_{nn}\}$.

Proof: The Schur complement lemma [2] ensures that condition (15) is equivalent to the following inequality:

$$\left[\begin{array}{cc}\Upsilon(l) & \mathcal{Z}^T(l)\overline{\mathcal{P}}(l)\\ * & -\overline{\mathcal{P}}(l)\end{array}\right]+\delta_1^{-1}\aleph_2(l)\aleph_2^T(l)+\delta_2^{-1}\aleph_4(l)\aleph_4^T(l)<0, \tag{16}$$

where $\aleph_2(l)=\mathrm{col}(0,0,0,0,\overline{\mathcal{P}}(l)\widehat{E}_1(l))$ and $\aleph_4(l)=\mathrm{col}(0,0,0,0,\overline{\mathcal{P}}(l)\widehat{E}_2(l))$. From the special structure of matrices $\aleph_1(l)$ and $\aleph_3(l)$, it is known that (16) is also equivalent to the following condition

$$\left[\begin{array}{cc}\Xi(l) & \mathcal{Z}^T(l)\overline{\mathcal{P}}(l)\\ * & -\overline{\mathcal{P}}(l)\end{array}\right]+\delta_1\tilde{\aleph}_1^T(l)\tilde{\aleph}_1(l)+\delta_2\tilde{\aleph}_3^T(l)\tilde{\aleph}_3(l)+\delta_1^{-1}\aleph_2(l)\aleph_2^T(l)+\delta_2^{-1}\aleph_4(l)\aleph_4^T(l)<0, \tag{17}$$

where $\tilde{\aleph}_1(l) = [\aleph_1(l) \;\; 0]$, $\tilde{\aleph}_3(l) = [\aleph_3(l) \;\; 0]$.

In order to ensure that (6) is a desired distributed state estimator of (1), we just need to show that the augmented error system (13) is robustly asymptotically stable in the mean square sense. To do so, let $\mathscr{X}(k) \triangleq \{e(k), e(k-1), \ldots, e(k-\bar{\tau})\}$ and choose the following Lyapunov-Krasovskii functional for system (13):

$$\begin{aligned} V(k, \mathscr{X}(k), r(k)) &= V_1(k, \mathscr{X}(k), r(k)) + V_2(k, \mathscr{X}(k), r(k)) + V_3(k, \mathscr{X}(k), r(k)) \\ &= e^T(k)\mathcal{P}(r(k))e(k) + \sum_{v=1}^{\tau(r(k))} \sum_{s=k-v}^{k-1} \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) \\ &\quad +(1-\underline{\pi}) \sum_{m=\underline{\tau}}^{\bar{\tau}-1} \sum_{v=1}^{m} \sum_{s=k-v}^{k-1} \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)), \end{aligned} \tag{18}$$

For $l \in \mathcal{S}$, we have

$$\begin{aligned} & \mathbb{E}\{V_1(k+1, \mathscr{X}(k+1), r(k+1))|\mathscr{X}(k), r(k)=l\} - V_1(k, \mathscr{X}(k), l) \\ =\;& e^T(k+1)\overline{\mathcal{P}}(l)e(k+1) - e^T(k)\mathcal{P}(l)e(k) \\ =\;& \xi^T(k,l)\mathcal{Z}^T(k,l)\overline{\mathcal{P}}(l)\mathcal{Z}(k,l)\xi(k,l) - e^T(k)\mathcal{P}(l)e(k), \end{aligned} \tag{19}$$

where $\xi(k,l) = \big(e^T(k), \mathscr{F}^T(e(k)), \mathcal{G}^T(e(k)), \sum_{v=1}^{\tau(l)} \mathcal{G}^T(e(k-v))\big)^T$ and $\mathcal{Z}(k,l) = [\mathcal{D}(k,l) \;\; \mathcal{B}(k,l) \;\; 0 \;\; \mathcal{W}(k,l)]$.

$$\begin{aligned} & \mathbb{E}\{V_2(k+1, \mathscr{X}(k+1), r(k+1))|\mathscr{X}(k), r(k)=l\} - V_2(k, \mathscr{X}(k), l) \\ =\;& \sum_{m=1}^{n} \pi_{lm} \sum_{v=1}^{\tau(m)} \sum_{s=k+1-v}^{k} \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) - \sum_{v=1}^{\tau(l)} \sum_{s=k-v}^{k-1} \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) \\ =\;& \sum_{m=1}^{n} \pi_{lm} \left[\sum_{v=1}^{\tau(m)} \sum_{s=k+1-v}^{k} - \sum_{v=1}^{\tau(l)} \sum_{s=k+1-v}^{k} + \sum_{v=1}^{\tau(l)} \sum_{s=k+1-v}^{k} - \sum_{v=1}^{\tau(l)} \sum_{s=k-v}^{k-1} \right] \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) \\ =\;& \sum_{m\neq l} \pi_{lm} \sum_{v=\tau(l)+1}^{\tau(m)} \sum_{s=k+1-v}^{k} \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) + \sum_{v=1}^{\tau(l)} \big(\mathcal{G}^T(e(k))\mathcal{Q}\mathcal{G}(e(k)) - \mathcal{G}^T(e(k-v))\mathcal{Q}\mathcal{G}(e(k-v))\big) \\ \leq\;& (\tau(l) + (\bar{\tau}-\underline{\tau})(1-\underline{\pi}))\, \mathcal{G}^T(e(k))\mathcal{Q}\mathcal{G}(e(k)) - \sum_{v=1}^{\tau(l)} \mathcal{G}^T(e(k-v))\mathcal{Q}\mathcal{G}(e(k-v)) \\ & +(1-\underline{\pi}) \sum_{v=1+\underline{\tau}}^{\bar{\tau}} \sum_{s=k+1-v}^{k-1} \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) \end{aligned} \tag{20}$$

and

$$\begin{aligned} & \mathbb{E}\{V_3(k+1, \mathscr{X}(k+1), r(k+1))|\mathscr{X}(k), r(k)=l\} - V_3(k, \mathscr{X}(k), l) \\ =\;& (1-\underline{\pi}) \sum_{m=\underline{\tau}}^{\bar{\tau}-1} \sum_{v=1}^{m} \left(\sum_{s=k+1-v}^{k} - \sum_{s=k-v}^{k-1} \right) \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) \\ =\;& (1-\underline{\pi}) \sum_{m=\underline{\tau}}^{\bar{\tau}-1} \sum_{v=1}^{m} \big(\mathcal{G}^T(e(k))\mathcal{Q}\mathcal{G}(e(k)) - \mathcal{G}^T(e(k-v))\mathcal{Q}\mathcal{G}(e(k-v))\big) \\ =\;& (1-\underline{\pi}) \left[\frac{1}{2}(\bar{\tau}-\underline{\tau})(\bar{\tau}-1+\underline{\tau})\mathcal{G}^T(e(k))\mathcal{Q}\mathcal{G}(e(k)) - \sum_{m=\underline{\tau}+1}^{\bar{\tau}} \sum_{s=k-m+1}^{k-1} \mathcal{G}^T(e(s))\mathcal{Q}\mathcal{G}(e(s)) \right]. \end{aligned} \tag{21}$$

From Lemma 2, it is not difficult to verify that

$$-\sum_{v=1}^{\tau(l)}\mathcal{G}^T(e(k-v))\mathcal{Q}\mathcal{G}(e(k-v)) \le -\frac{1}{\tau(l)}(\sum_{v=1}^{\tau(l)}\mathcal{G}(e(k-v)))^T\mathcal{Q}(\sum_{v=1}^{\tau(l)}\mathcal{G}(e(k-v))). \tag{22}$$

At the same time, for any positive scalars ε_1 and ε_2, condition (14) ensures that the following two inequalities hold:

$$\varepsilon_1 e^T(k)\widehat{\Sigma}_1^T\widehat{\Sigma}_1 e(k) - \varepsilon_1\mathscr{F}^T(e(k))\mathscr{F}(e(k)) \ge 0, \quad \varepsilon_2 e^T(k)\widehat{\Sigma}_2^T\widehat{\Sigma}_2 e(k) - \varepsilon_2\mathcal{G}^T(e(k))\mathcal{G}(e(k)) \ge 0. \tag{23}$$

With the relationships from (18) to (23), we have

$$\begin{aligned} & \mathbb{E}\{V(k+1,\mathscr{X}(k+1),r(k+1))|\mathscr{X}(k),r(k)=l\} - V(k,\mathscr{X}(k),l) \\ \le\ & \xi^T(k,l)\big(\Xi(l)+\mathcal{Z}^T(k,l)\overline{\mathcal{P}}(l)\mathcal{Z}(k,l)\big)\xi(k,l), \end{aligned} \tag{24}$$

where

$$\mathcal{Z}(k,l) = \mathcal{Z}(l) + \Delta\mathcal{Z}(k,l) \quad \text{and} \quad \Delta\mathcal{Z}(k,l) = \left[\begin{array}{cccc} \Delta\mathcal{D}(k,l) & \Delta\mathcal{B}(k,l) & 0 & \Delta\mathcal{W}(k,l) \end{array}\right].$$

From the Lyapunov stability theory, one knows that if the matrix inequality $\Xi(l)+\mathcal{Z}^T(k,l)\overline{\mathcal{P}}(l)\mathcal{Z}(k,l) < 0$ holds uniformly in k, we could immediately draw our conclusion. Again, from the Schur complement lemma [2], we know that

$$\Xi(l)+\mathcal{Z}^T(k,l)\overline{\mathcal{P}}(l)\mathcal{Z}(k,l) < 0 \Longleftrightarrow \widetilde{\Xi}(k,l) \triangleq \widetilde{\Xi}(l) + \Delta\widetilde{\Xi}(k,l) < 0, \tag{25}$$

where

$$\widetilde{\Xi}(l) = \left[\begin{array}{cc} \Xi(l) & \mathcal{Z}^T(l)\overline{\mathcal{P}}(l) \\ * & -\overline{\mathcal{P}}(l) \end{array}\right], \qquad \Delta\widetilde{\Xi}(k,l) = \left[\begin{array}{cc} 0 & (\Delta\mathcal{Z}(k,l))^T\overline{\mathcal{P}}(l) \\ * & 0 \end{array}\right].$$

By simple computation, it can be obtained that

$$\begin{aligned} \Delta\mathcal{D}(k,l) &= \left[\begin{array}{cc} E_1(l)F_1(k,l)M_1(l) & 0 \\ -\bar{K}(l)\tilde{E}_2(l)F_2(k,l)M_4(l)+\tilde{E}_1(l)F_1(k,l)M_1(l) & 0 \end{array}\right] \\ &= \widehat{E}_1(l)F_1(k,l)\widehat{M}_1(l) - \widehat{E}_2(l)F_2(k,l)\widehat{M}_4(l), \end{aligned} \tag{26}$$

$$\Delta\mathcal{B}(k,l) = \left[\begin{array}{cc} \Delta B(k,l) & 0 \\ \Delta\tilde{B}(k,l) & 0 \end{array}\right] = \left[\begin{array}{cc} E_1(l)F_1(k,l)M_2(l) & 0 \\ \tilde{E}_1(l)F_1(k,l)M_2(l) & 0 \end{array}\right] = \widehat{E}_1(l)F_1(k,l)\widehat{M}_2(l), \tag{27}$$

$$\Delta\mathcal{W}(k,l) = \left[\begin{array}{cc} \Delta W(k,l) & 0 \\ \Delta\tilde{W}(k,l) & 0 \end{array}\right] = \left[\begin{array}{cc} E_1(l)F_1(k,l)M_3(l) & 0 \\ \tilde{E}_1(l)F_1(k,l)M_3(l) & 0 \end{array}\right] = \widehat{E}_1(l)F_1(k,l)\widehat{M}_3(l). \tag{28}$$

Combining the above three equalities together, we can conclude that for any scalars $\delta_1 > 0$ and $\delta_2 > 0$,

$$\begin{aligned} \Delta\widetilde{\Xi}(k,l) &= \aleph_2(l)F_1(k,l)\tilde{\aleph}_1(l) + (\aleph_2(l)F_1(k,l)\tilde{\aleph}_1(l))^T - \aleph_4(l)F_2(k,l)\tilde{\aleph}_3(l) - (\aleph_4(l)F_2(k,l)\tilde{\aleph}_3(l))^T \\ &\le \delta_1\tilde{\aleph}_1^T(l)\tilde{\aleph}_1(l) + \delta_1^{-1}\aleph_2(l)\aleph_2^T(l) + \delta_2\tilde{\aleph}_3^T(l)\tilde{\aleph}_3(l) + \delta_2^{-1}\aleph_4(l)\aleph_4^T(l), \end{aligned} \tag{29}$$

where, in the last step, Lemma 1 has been utilized to derive the inequality. Substituting (29) into the right side of the inequality in (25), we have from condition (17) that the inequality $\Xi(l)+\mathcal{Z}^T(k,l)\overline{\mathcal{P}}(l)\mathcal{Z}(k,l) < 0$ does hold uniformly in k, which means that there exists a constant $\lambda < 0$ such that

$$\mathbb{E}\{V(k+1,\mathscr{X}(k+1),r(k+1))|\mathscr{X}(k),r(k)=l\} - V(k,\mathscr{X}(k),l) \le \lambda\|e(k)\|^2.$$

The remaining part of the proof is similar as that in [13] and therefore omitted here. This completes the proof of the theorem. ■

After conducting the analysis for system (13) to be robustly asymptotically stable in the mean square sense, we are now in a position to design the estimator gains. Note that matrices $\bar{K}(l)$ $(l \in \mathcal{S})$ contain all the information of the desired estimator parameters, to carry on, we still need the following lemma.

Lemma 3: Let $\mathbf{S} = \mathrm{diag}(\mathbf{S_{11}}, \mathbf{S_{22}}, \ldots, \mathbf{S_{NN}})$ with $S_{ii} \in \mathbb{R}^{n_x \times n_x}$ $(i = 1, 2, \ldots, N)$ being invertible matrices. If $X = \mathbf{S}\bar{\mathbf{U}}$ for $\bar{U} \in \mathbb{R}^{n_x N \times n_y N}$, then we have $\bar{U} \in \mathscr{W}_{n_x \times n_y} \Leftrightarrow X \in \mathscr{W}_{n_x \times n_y}$.

Theorem 2: Under Assumptions 1-2, the system (6) is a robustly distributed state estimator of the Markov-type dynamic system (1) with measurement outputs (2) if there exist matrices $P_i(l) > 0$ $(i = 1, 2, \ldots, N+1; l \in \mathcal{S})$, $Q_{11} > 0$, $\mathcal{Q}_{22} > 0$, Q_{12}, $\mathcal{X}(l) \in \mathscr{W}_{n_x \times n_y}$ $(l \in \mathcal{S})$ and positive scalars ε_1, ε_2, δ_1, δ_2 such that the following matrix inequalities hold:

$$\mathcal{Q} \triangleq \begin{bmatrix} Q_{11} & Q_{12} \\ * & \mathcal{Q}_{22} \end{bmatrix} > 0 \tag{30}$$

and

$$\Phi(l) \triangleq \begin{bmatrix} \Phi_{11}(l) & \Phi_{12}(l) & 0 & \Phi_{14}(l) & \Phi_{15}(l) & 0 & 0 \\ * & \Phi_{22}(l) & 0 & \Phi_{24}(l) & \Phi_{25}(l) & 0 & 0 \\ * & * & \Phi_{33}(l) & 0 & 0 & 0 & 0 \\ * & * & * & \Phi_{44}(l) & \Phi_{45}(l) & 0 & 0 \\ * & * & * & * & \Phi_{55}(l) & \Phi_{56}(l) & \Phi_{57}(l) \\ * & * & * & * & * & -\delta_1 I & 0 \\ * & * & * & * & * & * & -\delta_2 I \end{bmatrix} < 0, \qquad (l \in \mathcal{S}) \tag{31}$$

where

$$\begin{aligned}
&\Phi_{11}(l) = \mathrm{diag}(\Upsilon_1(l), -\mathcal{P}_2(l) + \varepsilon_1 \tilde{\Sigma}_1^T \tilde{\Sigma}_1 + \varepsilon_2 \tilde{\Sigma}_2^T \tilde{\Sigma}_2), \quad \Phi_{12}(l) = \mathrm{diag}(\delta_1 M_1^T(l) M_2(l), 0), \\
&\Upsilon_1(l) = -P_1(l) + \varepsilon_1 \Sigma_1^T \Sigma_1 + \varepsilon_2 \Sigma_2^T \Sigma_2 + \delta_1 M_1^T(l) M_1(l) + \delta_2 M_4^T(l) M_4(l), \\
&\Phi_{14}(l) = \mathrm{diag}(\delta_1 M_1^T(l) M_3(l), 0), \quad \Phi_{15}(l) = \mathrm{diag}(D^T(l) \bar{P}_1(l), (I_N \otimes D(l))^T \bar{\mathcal{P}}_2(l) - \tilde{C}^T(l) \mathcal{X}^T(l)), \\
&\Phi_{22}(l) = \mathrm{diag}(-\varepsilon_1 I + \delta_1 M_2^T(l) M_2(l), -\varepsilon_1 I), \quad \Phi_{24}(l) = \mathrm{diag}(\delta_1 M_2^T(l) M_3(l), 0), \\
&\Phi_{25}(l) = \mathrm{diag}(B^T(l) \bar{P}_1(l), (I_N \otimes B(l))^T \bar{\mathcal{P}}_2(l)), \quad \Phi_{45}(l) = \mathrm{diag}(W^T(l) \bar{P}_1(l), (I_N \otimes W(l))^T \bar{\mathcal{P}}_2(l)), \\
&\Phi_{33}(l) = \begin{bmatrix} -\varepsilon_2 I + (\tau(l) + \tilde{\tau}) Q_{11} & (\tau(l) + \tilde{\tau}) Q_{12} \\ * & -\varepsilon_2 I + (\tau(l) + \tilde{\tau}) \mathcal{Q}_{22} \end{bmatrix}, \quad \Phi_{56}(l) = \begin{bmatrix} \bar{P}_1(l) E_1(l) \\ \bar{\mathcal{P}}_2(l) \tilde{E}_1(l) \end{bmatrix}, \\
&\Phi_{44}(l) = \begin{bmatrix} -\frac{1}{\tau(l)} Q_{11} + \delta_1 M_3^T(l) M_3(l) & -\frac{1}{\tau(l)} Q_{12} \\ * & -\frac{1}{\tau(l)} \mathcal{Q}_{22} \end{bmatrix}, \quad \Phi_{57}(l) = \begin{bmatrix} 0 \\ \mathcal{X}(l) \tilde{E}_2(l) \end{bmatrix}
\end{aligned}$$

and $\Phi_{55}(l) = \mathrm{diag}(-\bar{P}_1(l), -\bar{\mathcal{P}}_2(l))$, $\mathcal{P}_2(l) = \mathrm{diag}(P_2(l), P_3(l), \ldots, P_{N+1}(l))$, $\bar{P}_i(l) = \sum_{m=1}^{n} \pi_{lm} P_i(m)$ $(i = 1, 2, \ldots, N+1; l \in \mathcal{S})$, $\bar{\mathcal{P}}_2(l) = \mathrm{diag}(\bar{P}_2(l), \bar{P}_3(l), \ldots, \bar{P}_{N+1}(l))$ $(l \in \mathcal{S})$, the other symbols are the same as defined in Theorem 1. Furthermore,

$$\bar{K}(l) = \bar{\mathcal{P}}_2^{-1}(l) \mathcal{X}(l); \qquad (l \in \mathcal{S}) \tag{32}$$

accordingly, the state estimator gains $K_{ij}(l)$ $(i = 1, 2, \ldots, N, j \in \mathcal{N}_i, l \in \mathcal{S})$ can be derived from (9).

Proof: With the special structure of $\bar{\mathcal{P}}_2(l)$ and the restriction on matrix $\mathcal{X}(l) \in \mathscr{W}_{n_x \times n_y}$, it follows from Lemma 3 that $\bar{\mathcal{P}}_2^{-1}(l)\mathcal{X}(l) \in \mathscr{W}_{n_x \times n_y}$. By taking $\mathcal{P}(l) = \mathrm{diag}(P_1(l), \mathcal{P}_2(l))$, $\overline{\mathcal{P}}(l) = \mathrm{diag}(\bar{P}_1(l), \bar{\mathcal{P}}_2(l))$ in Theorem 1 and noticing the condition $\bar{\mathcal{P}}_2(l)\bar{K}(l) = \mathcal{X}(l)$, after some computation it can be shown that

$$\begin{aligned}
&\overline{\mathcal{P}}(l)\mathcal{D}(l) = \mathrm{diag}(\bar{P}_1(l)D(l), \bar{\mathcal{P}}_2(l)(I_N \otimes D(l)) - \mathcal{X}(l)\tilde{C}(l)),\\
&\overline{\mathcal{P}}(l)\mathcal{B}(l) = \mathrm{diag}(\bar{P}_1(l)B(l), \bar{\mathcal{P}}_2(l)(I_N \otimes B(l))), \quad \overline{\mathcal{P}}(l)\mathcal{W}(l) = \mathrm{diag}(\bar{P}_1(l)W(l), \bar{\mathcal{P}}_2(l)(I_N \otimes W(l))),\\
&\overline{\mathcal{P}}(l)\widehat{E}_1(l) = \begin{bmatrix} \bar{P}_1(l)E_1(l) \\ \bar{\mathcal{P}}_2(l)\tilde{E}_1(l) \end{bmatrix}, \qquad \overline{\mathcal{P}}(l)\widehat{E}_2(l) = \begin{bmatrix} 0 \\ \mathcal{X}(l)\tilde{E}_2(l) \end{bmatrix}.
\end{aligned}$$

And the corresponding result is derived easily from Theorem 1, the proof is then completed. ■

Remark 2: Our main results are based on the linear matrix inequality (LMI) conditions. The LMI Control Toolbox implements state-of-the-art interior-point LMI solvers. While these solvers are significantly faster than classical convex optimization algorithms, it should be kept in mind that the complexity of LMI computations remains higher than that of solving, say, a Riccati equation. For instance, problems with a thousand design variables typically take over an hour on today's workstations. However, research on LMI optimization is a very active area in the applied math, optimization and the operations research community, and substantial speed-ups can be expected in the future.

IV. Numerical Examples

Consider a Markov-type system plant with 3 modes described by the discrete-time model (1) with

$$\begin{aligned}
&D(1) = \begin{bmatrix} -0.3 & 0.2 \\ 0.1 & -0.2 \end{bmatrix}, \quad D(2) = \begin{bmatrix} -0.1 & 0.2 \\ -0.1 & -0.1 \end{bmatrix}, \quad D(3) = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.1 \end{bmatrix};\\
&B(1) = \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & -0.2 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 0.2 & 0.2 \\ 0 & 0.1 \end{bmatrix}, \quad B(3) = \begin{bmatrix} 0.1 & 0 \\ 0.2 & 0 \end{bmatrix};\\
&W(1) = \begin{bmatrix} 0.2 & -0.2 \\ 0.1 & 0.2 \end{bmatrix}, \quad W(2) = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad W(3) = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}
\end{aligned}$$

and $\tau(1) = 3$, $\tau(2) = 2$, $\tau(3) = 4$; the mode transition probability matrix is given by

$$\Pi = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.2 & 0.1 & 0.7 \end{bmatrix},$$

which means $\bar{\tau} = 4$, $\underline{\tau} = 2$ and $\underline{\pi} = 0.6$. The nonlinear functions $f(\cdot)$ and $g(\cdot)$ in (1) are assumed to satisfy Assumptions 1-2 with

$$\Sigma_1 = \begin{bmatrix} 0.2 & 0.3 \\ -0.1 & 0.2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 0.1 & -0.2 \\ 0.3 & 0.2 \end{bmatrix}.$$

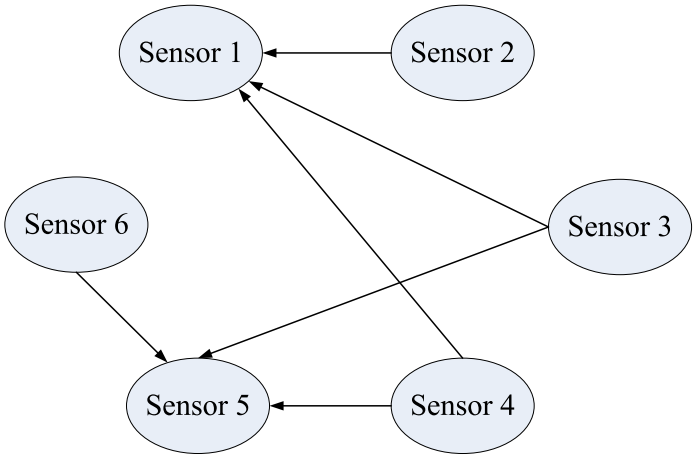

Fig. 1. Topological structure of the sensor network.

The sensor network shown in Fig. 1 consists of 6 nodes with interconnection topology characterized by the adjacency matrix

$$\mathcal{A} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

which illustrates that Sensor 1 receives information from Sensors 2-4, Sensor 5 receives information from Sensors 3,4,6, but Sensors 2,3,4,6 receive no information from other sensors. The dynamics of the 6 sensors are described by equation (2) with

$$C_1(1) = \begin{bmatrix} -0.2 & 0.1 \\ -0.1 & 0.2 \end{bmatrix}, \quad C_1(2) = \begin{bmatrix} 0 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \quad C_1(3) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix};$$

$$C_2(1) = \begin{bmatrix} 0.2 & -0.2 \\ 0.1 & 0.2 \end{bmatrix}, \quad C_2(2) = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix}, \quad C_2(3) = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & 0.2 \end{bmatrix};$$

$$C_3(1) = \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, \quad C_3(2) = \begin{bmatrix} 0.2 & -0.1 \\ 0 & 0.1 \end{bmatrix}, \quad C_3(3) = \begin{bmatrix} -0.1 & 0 \\ 0.2 & 0.1 \end{bmatrix};$$

$$C_4(1) = \begin{bmatrix} 0.2 & 0.2 \\ 0 & 0.3 \end{bmatrix}, \quad C_4(2) = \begin{bmatrix} 0.1 & 0 \\ -0.1 & 0.2 \end{bmatrix}, \quad C_4(3) = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix};$$

$$C_5(1) = \begin{bmatrix} -0.2 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \quad C_5(2) = \begin{bmatrix} -0.1 & 0.2 \\ 0 & 0 \end{bmatrix}, \quad C_5(3) = \begin{bmatrix} 0 & 0 \\ 0.1 & -0.2 \end{bmatrix};$$

$$C_6(1) = \begin{bmatrix} 0.2 & 0.1 \\ -0.2 & 0 \end{bmatrix}, \quad C_6(2) = \begin{bmatrix} 0.2 & -0.2 \\ 0.1 & -0.1 \end{bmatrix}, \quad C_6(3) = \begin{bmatrix} 0.1 & 0.2 \\ -0.2 & 0 \end{bmatrix}.$$

The parameter uncertainties satisfy conditions (4)-(5) with

$$E_1(1)=\begin{bmatrix}0.1\\-0.1\end{bmatrix}, E_1(2)=\begin{bmatrix}0.1\\0.2\end{bmatrix}, E_1(3)=\begin{bmatrix}0\\0.1\end{bmatrix}; E_{21}(1)=\begin{bmatrix}-0.1\\0.1\end{bmatrix}, E_{21}(2)=\begin{bmatrix}0.1\\0\end{bmatrix},$$
$$E_{21}(3)=\begin{bmatrix}0\\0.2\end{bmatrix}; E_{22}(1)=\begin{bmatrix}0\\0.2\end{bmatrix}, E_{22}(2)=\begin{bmatrix}0.1\\-0.1\end{bmatrix}, E_{22}(3)=\begin{bmatrix}-0.1\\0.1\end{bmatrix};$$
$$E_{23}(1)=\begin{bmatrix}0.1\\-0.1\end{bmatrix}, E_{23}(2)=\begin{bmatrix}0\\0.2\end{bmatrix}, E_{23}(3)=\begin{bmatrix}0.1\\0.2\end{bmatrix}; E_{24}(1)=\begin{bmatrix}0.2\\0.1\end{bmatrix},$$
$$E_{24}(2)=\begin{bmatrix}0.1\\0.2\end{bmatrix}, E_{24}(3)=\begin{bmatrix}0.1\\0\end{bmatrix}; E_{25}(1)=\begin{bmatrix}0.1\\0.2\end{bmatrix}, E_{25}(2)=\begin{bmatrix}-0.1\\0\end{bmatrix},$$
$$E_{25}(3)=\begin{bmatrix}0.2\\-0.1\end{bmatrix}; E_{26}(1)=\begin{bmatrix}0.1\\0\end{bmatrix}, E_{26}(2)=\begin{bmatrix}0.1\\0.2\end{bmatrix}, E_{26}(3)=\begin{bmatrix}0.1\\0.1\end{bmatrix};$$
$$M_1(1)=\begin{bmatrix}0.1&0.2\end{bmatrix}, M_1(2)=\begin{bmatrix}-0.1&0.2\end{bmatrix}, M_1(3)=\begin{bmatrix}0&0.1\end{bmatrix}, M_2(1)=\begin{bmatrix}0.2&0\end{bmatrix},$$
$$M_2(2)=\begin{bmatrix}0.1&-0.1\end{bmatrix}, M_2(3)=\begin{bmatrix}0&-0.1\end{bmatrix}, M_3(1)=\begin{bmatrix}-0.1&0.1\end{bmatrix}, M_3(2)=\begin{bmatrix}0.2&-0.1\end{bmatrix},$$
$$M_3(3)=\begin{bmatrix}0.1&0.1\end{bmatrix}; M_4(1)=\begin{bmatrix}-0.1&0.1\end{bmatrix}, M_4(2)=\begin{bmatrix}0.1&-0.2\end{bmatrix}, M_4(3)=\begin{bmatrix}0.2&0\end{bmatrix}.$$

With the above parameters, by resorting to the Matlab toolbox, the matrix inequalities in (30)-(31) are solvable with feasible solution as follows (here only some of the matrices are listed for space saving):

$$\varepsilon_1=13.3632,\quad \varepsilon_2=50.4805,\quad \delta_1=14.4284,\quad \delta_2=13.4711,$$
$$P_1(1)=\begin{bmatrix}14.5824&1.7032\\1.7032&15.5435\end{bmatrix}, P_2(2)=\begin{bmatrix}14.4964&0.8935\\0.8935&14.8581\end{bmatrix}, P_3(3)=\begin{bmatrix}14.8398&1.4817\\1.4817&14.8786\end{bmatrix},$$
$$P_4(1)=\begin{bmatrix}14.5694&1.5955\\1.5955&14.9914\end{bmatrix}, P_5(2)=\begin{bmatrix}14.4554&0.9990\\0.9990&14.8456\end{bmatrix}, P_6(3)=\begin{bmatrix}14.8805&1.4923\\1.4923&14.8647\end{bmatrix},$$
$$P_7(1)=\begin{bmatrix}14.5396&1.6435\\1.6435&15.0343\end{bmatrix}, Q_{11}=\begin{bmatrix}8.9461&-0.0047\\-0.0047&8.9307\end{bmatrix}.$$

From Theorem 2, we know that system (6) is a robustly distributed state estimator of the Markov-type dynamic system (1) with measurement outputs (2). The estimation gain matrices can be obtained from (32) as follows

$$K_{11}(1)=\begin{bmatrix}1.1380&0.5299\\-0.1974&-0.8014\end{bmatrix}, K_{12}(1)=\begin{bmatrix}0.0211&0.0869\\0.0213&0.0877\end{bmatrix}, K_{13}(1)=\begin{bmatrix}0.0407&-0.0113\\0.0402&-0.0111\end{bmatrix},$$
$$K_{14}(1)=\begin{bmatrix}0.0648&-0.0235\\0.0690&-0.0250\end{bmatrix}, K_{22}(1)=\begin{bmatrix}-1.3023&-0.2051\\0.6984&-0.2023\end{bmatrix}, K_{33}(1)=\begin{bmatrix}1.4388&0.1756\\-0.8127&-0.3919\end{bmatrix},$$
$$K_{44}(1)=\begin{bmatrix}-1.2707&1.5833\\0.5028&-1.0010\end{bmatrix}, K_{53}(1)=\begin{bmatrix}0.0532&-0.0147\\-0.0883&0.0244\end{bmatrix}, K_{54}(1)=\begin{bmatrix}0.0853&-0.0309\\-0.1433&0.0520\end{bmatrix},$$
$$K_{55}(1)=\begin{bmatrix}2.3527&-1.5978\\-1.2534&1.3268\end{bmatrix}, K_{56}(1)=\begin{bmatrix}0.2769&0.2854\\-0.4593&-0.4735\end{bmatrix}, K_{66}(1)=\begin{bmatrix}1.1286&2.6005\\-1.1294&-1.6011\end{bmatrix};$$

$$K_{11}(2)=\begin{bmatrix}0.9209 & -1.0391\\ -1.8212 & -0.9115\end{bmatrix}, K_{12}(2)=\begin{bmatrix}-0.1467 & 0.3593\\ 0.2878 & -0.7048\end{bmatrix}, K_{13}(2)=\begin{bmatrix}-0.0176 & -0.1056\\ 0.0371 & 0.2207\end{bmatrix},$$
$$K_{14}(2)=\begin{bmatrix}-0.0714 & -0.0329\\ 0.1442 & 0.0663\end{bmatrix}, K_{22}(2)=\begin{bmatrix}0.2918 & 0.8373\\ -0.4807 & -0.2707\end{bmatrix}, K_{33}(2)=\begin{bmatrix}-0.6959 & 0.3358\\ -0.3032 & -0.3285\end{bmatrix},$$
$$K_{44}(2)=\begin{bmatrix}-0.7166 & 0.6706\\ -0.5977 & -0.0851\end{bmatrix}, K_{53}(2)=\begin{bmatrix}-0.0009 & -0.0051\\ -0.0021 & -0.0113\end{bmatrix}, K_{54}(2)=\begin{bmatrix}-0.0047 & -0.0021\\ -0.0104 & -0.0046\end{bmatrix},$$
$$K_{55}(2)=\begin{bmatrix}1.0006 & 0\\ -0.2424 & 0\end{bmatrix}, K_{56}(2)=\begin{bmatrix}-0.3429 & 0.6859\\ 0.0602 & -0.1203\end{bmatrix}, K_{66}(2)=\begin{bmatrix}-1.0012 & 0.5051\\ -0.0103 & 0.0150\end{bmatrix};$$

and

$$K_{11}(3)=\begin{bmatrix}-0.4937 & 0.8411\\ 0.4937 & -0.8392\end{bmatrix}, K_{12}(3)=\begin{bmatrix}0.0387 & -0.0080\\ -0.0381 & 0.0079\end{bmatrix}, K_{13}(3)=\begin{bmatrix}-0.7443 & -0.3208\\ 0.7445 & 0.3208\end{bmatrix},$$
$$K_{14}(3)=\begin{bmatrix}-0.1473 & -0.0732\\ 0.1491 & 0.0741\end{bmatrix}, K_{22}(3)=\begin{bmatrix}-0.7439 & -0.0522\\ 0.7429 & 0.0524\end{bmatrix}, K_{33}(3)=\begin{bmatrix}0.4997 & -0.0776\\ -0.4970 & 0.0787\end{bmatrix},$$
$$K_{44}(3)=\begin{bmatrix}-0.0001 & -1.0000\\ 0.0040 & 1.0020\end{bmatrix}, K_{53}(3)=\begin{bmatrix}0.0001 & 0.0001\\ -0.0112 & -0.0049\end{bmatrix}, K_{54}(3)=\begin{bmatrix}0.0000 & 0.0000\\ -0.0005 & -0.0003\end{bmatrix},$$
$$K_{55}(3)=\begin{bmatrix}-0.2948 & -0.5892\\ 0.3097 & 0.5892\end{bmatrix}, K_{56}(3)=\begin{bmatrix}0.0000 & 0.0000\\ -0.0009 & -0.0011\end{bmatrix}, K_{66}(3)=\begin{bmatrix}0.2764 & 0.4779\\ -0.2757 & -0.4770\end{bmatrix}.$$

Remark 3: In this paper, the sensor network consists of n modes and the network switches from one mode to another according to a Markov chain with known transition probability. In our future work, we could further consider two independent Markovian jumping modes for *node switching* as well as *network topology switching*, which would reflect more features of the complexity. On the other hand, the topology for many sensor networks in a real world can be described by a random graph and the study about random dynamical networks is therefore quite interesting and useful. One of our future research topics would be the extension of our results to random dynamical networks.

V. Conclusions

In this paper, we have studied the state estimation problem for a class of Markov-type uncertain dynamical systems with mode-dependent distributed delays. Through the measurement outputs obtained by the sensors in sensor networks, a distributed state estimator has been designed to estimate the states of the dynamical plant. By resorting to the Lyapuonv stability theory and the matrix inequality techniques, sufficient criteria have been obtained to ensure the estimation error system to be robustly asymptotically stable in the mean square sense which in turn show that the designed estimator is a robustly distributed state estimator of the original plant system. Furthermore, the estimation gains have been explicitly characterized by the solution of the matrix inequality conditions. Finally, an illustrative example has been given to demonstrate the effectiveness of our results. Future research topics include the extension of the present results to other systems such as fuzzy systems [30, 31] and networked control systems [17, 17, 23, 25].

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