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Ugur Akgun and Ioana Chioveanu

Loyalty Discounts

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Loyalty Discounts*

Uğur Akgün[†] and Ioana Chioveanu[‡]

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Abstract

This paper considers the use of loyalty inducing discounts in vertical supply chains. An upstream manufacturer and a competitive fringe sell differentiated products to a retailer who has private information about the level of stochastic demand. We provide a comparison of market outcomes when the manufacturer uses two-part tariffs (2PT), all-unit quantity discounts (AU), and market share discounts (MS). We show that retailer's risk attitude affects manufacturer's preferences over these three pricing schemes. When the retailer is risk-neutral, it bears all the risk and all three schemes lead to the same outcome. When the retailer is risk-averse, 2PT performs the worst from manufacturer's perspective but it leads to the highest total surplus. For a wide range of parameter values (but not for all) the manufacturer prefers MS to AU. By limiting the retailer's product substitution possibilities MS makes the demand for manufacturer's product more inelastic. This reduces the amount (share of total profits) the manufacturer needs to leave to the retailer for the latter to participate in the scheme.

JEL: L42, L12, L13

Keywords: vertical contracts, market share discounts, asymmetric information

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[†] Charles Rivers Associates, 99 Bishopsgate, London EC2M 3XD, UK

[‡] *Corresponding author:* Department of Economics and Finance, Brunel University London, Uxbridge UB8 3PH, UK. Email: ioana.chioveanu@brunel.ac.uk.

1 Introduction

A loyalty discount is the practice that implicitly or explicitly makes discounts conditional on the share of a buyer's purchases made from a supplier within a given period. The discount is typically applied in a rollback format. Once a buyer qualifies, it receives a discount not only on those purchases above the target, but on all purchases in the period. In most cases, it is difficult to link these discount programs to particular instances of economies of scale. The latter can occur at overall production level or in fulfilling a specific order, but are less likely to relate to total purchases of a customer over a period. While loyalty inducing programs directed to final consumers have rarely raised competition concerns[1], the use of rollback rebates in wholesale markets has frequently come under antitrust scrutiny in recent years.[2]

Understanding the motives for the use of these rebates in business to business sales is valuable for policy design. As with related practices of vertical price control and exclusive dealing, firms' use of loyalty discounts has the potential to be both procompetitive and anticompetitive. The major concern with rollback loyalty rebates is that a supplier with substantial market power sets a low price conditional on exclusive (or nearly exclusive) dealing, with the effect that a market is foreclosed to a rival competitor.[3] However, this motive does not seem appropriate for all instances where loyalty rebates are used as in some foreclosure is unlikely. Therefore, it is important to recognize plausible mechanisms that motivate the use of loyalty discounts for reasons other than exclusion. The impact of these practices on competition and welfare can be different depending on the mechanisms that motivate their use.

The present article offers an alternative motivation for dominant firms to offer certain loyalty discount schemes. We assess the effects of the various trading forms on competition and welfare under demand uncertainty. These comparisons of alternative contracts between buyers and suppliers are useful to identify the conditions that lead to the emergence of a specific type of rebate or trading form and their respective effects on the buyers' welfare. In particular, in this study we focus on two-part tariffs and two types of rollback discounts with *quantity* or *market share* targets. We refer to the latter as all-unit quantity discounts (AU) and market share discounts (MS), respectively.[4] The focus of the paper is to understand how different risk attitudes for a

[1] Probably the best known are the "frequent flyer" schemes promoted by airlines and related programs run by supermarkets, cafés, bookstores, or credit card issuers.

[2] The case law related to loyalty inducing rebate programs has developed faster than the economic analysis of the practices. Comprehensive overviews of relevant antitrust cases in US and Europe are presented in Mills (2010) and the UK OFT report 804 (2005). See also Kobayashi (2005), European Commission (2005), European Commission (2009), and Gual et al. (2005).

[3] Lately, European and North American case law have focused on whether loyalty discounts can serve as an exclusionary device that would violate Article 102 of EC Treaty or Section 2 of the Sherman Act. In addition, firms' use of loyalty discounts in the distribution of their products has also been attacked as unlawful primary line price discrimination under the Robinson Patman Act and EC law (Art. 102c).

[4] The term "all-unit" quantity discount is used to emphasize that we deal with rollback rebates. However, our setting also informs on the relative private desirability of "incremental-unit" quantity discounts (that do not rollback to inframarginal units once the target is reached) since they cannot improve upon 2PT under our information/risk setting.

buyer with private information about uncertainty affect comparisons between these contracts.

In the analysis of vertical chains the tension between efficient surplus extraction and maximization of surplus is thoroughly studied as a principal-agent problem where the retailer has private information related to uncertainty. In contrast to the principle-agent literature where different risk attitudes of the two parties play a central role, previous work that studies motives for various pricing schemes assumes that both upstream and downstream firms are risk-neutral. It is quite plausible that a manufacturer that deals with many retailers in different local markets (potentially subject to uncorrelated shocks) behaves as risk-neutral. But, it is less likely that a retailer would agree to bear all the market risk by signing a contract which aims to induce certain level of purchases at no additional cost to the manufacturer. In effect, our analysis suggests that the differences in attitude towards risk across the vertical chain can explain emergence of different types of loyalty inducing contracts.

In this study, we show that in the presence of uncertainty, if the retailer is infinitely risk averse, the manufacturer strictly prefers market share and all-unit quantity discounts to two-part tariffs. Using a linear demand system, we also show that, for a wide range of parameters, the manufacturer strictly prefers market share discounts to all-unit quantity discounts, and that welfare is highest under two-part tariffs. Private incentives for the use of market-share discounts are driven by their ability to induce the retailer to act on a target share. This reduces the elasticity of retailer's demand for manufacturer's product. However, while a market-share discount limits substitution of the manufacturer's product with the competing product, it still allows the retailer to use private information and respond to actual market conditions which affect both products. Even if implementation of market-share discounts requires costly monitoring of rival sales, there is a non-trivial range of costs for which the supplier might still strictly prefer using a market share discount to using two-part tariffs or all-unit quantity discounts. The importance of the retailer's risk attitude is indicated by the fact that, under risk neutrality, the manufacturer is indifferent between two-part tariffs, market share discounts, and all-unit quantity discounts. In that case, the retailer can assume the entire market risk without requiring a compensation from the manufacturer.

Despite the prevalence of loyalty discount schemes, the theoretical literature in economics did not address them specifically until recently. Most of economists' attention was captured by related practices like exclusive dealing or incremental units discounts (see Bernheim and Whinston (1998)). Recent research has identified some market conditions where the use of rollback discounts improves surplus transfer from retailers to manufacturers.

Under complete information, Inderst and Shaffer (2010) shows that market-share contracts allow a dominant supplier to dampen competition between the retailers, extract more profit than two-part tariffs or own-supplier contracts do, and deliver the joint-profit maximizing outcome. Hence, even absent exclusionary concerns, market share contracts may harm consumers. Marx and Shaffer (2004) proposes a rent-shifting rationale for rollback quantity discounts when two upstream sellers sequentially contract with one downstream firm. Surplus extraction is better if contracts depend on both sellers' quantities and exclusion is desirable only if the rival is inefficient.[5]

Under asymmetric information, rollback discounts can be used to alleviate the adverse selection problem. Majumdar and Shaffer (2009) reports that by conditioning a discount on *both* quantity and market share thresholds the manufacturer can improve upon rollback quantity discounts. The share threshold reduces the retailer's information rent by decreasing the attractiveness of concealing a high demand state. The authors identify a condition under which the suggested contracts can replicate the full information outcome. Sloev (2008) explores dynamic incentives for the use of market share discounts and shows that they can be employed by a supplier to induce retailers to exert the optimal promotional effort level.[6]

The present study shows that, under incomplete information, the use of loyalty discounts can increase the upstream profits when a risk neutral supplier offers non-contingent contracts to a risk averse retailer. There is a conflict between exploitation of market power and providing insurance to the retailer to secure contracting. From the seller's perspective the desirability of different pricing schemes is determined by their ability to manage these conflicting objectives. In this setting, the preferences of the seller and the buyer over the various pricing schemes may not be aligned. Our information setting is reminiscent of Rey and Tirole (1986). They point to the delegation problem under uncertainty as an essential driver of private incentives to use vertical restraints such as exclusive territories and resale price maintenance.

The rest of this paper is organized as follows. Section 2 lays out the model and introduces the contracts under full information. Section 3 analyzes the contracts in the presence of demand uncertainty. First we analyze the relative performance of contracts when the retailer is risk averse, and then when the retailer is risk neutral. Concluding remarks and directions for future research are collected in the last section.

2 Model

Consider a vertically-related industry where a manufacturer and a competitive fringe operate at the upstream level. The fringe produces an imperfect substitute of the product supplied by the manufacturer. The manufacturer and the competitive fringe supply their products in many independent and identical markets each served by a different retail monopoly.

We explore the relative performance of specific non-linear pricing contracts under uncertainty

[5] In a full information setting where the incumbent faces second period competition by entrants, Feess and Wohlschlegel (2010) shows that all unit discounts shift rents from the entrants. Greenlee, Reitman, and Sibley (2008) considers a monopolist that faces competition in a second market and shows that bundled loyalty discounts (that condition the rebate on the range of products purchased from the monopolist) have ambiguous welfare effects.

[6] Kolay, Shaffer, and Ordover (2004) shows that a discrete menu of rollback quantity discounts generates higher upstream profits than a menu of two-part tariffs in a bilateral monopoly setting, but its welfare effects depend on demand parameters.

and risk aversion. In each local market, the retailer faces an uncertain demand. The retailer and the manufacturer sign a contract prior to the resolution of the demand uncertainty. The order of moves is as follows. First, the manufacturer offers a contract to the retailer. If the retailer rejects, it cannot sell the manufacturer's good. Second, the demand is realized and the retailer chooses the quantities for the two goods. Third, the retailer sells the products to the final consumers at market clearing prices. The manufacturer is assumed to be risk-neutral, as it operates in many independent markets with uncorrelated shocks. The retailer can be either risk averse or risk neutral.

We consider only non-contingent contracts which prevent arbitrage opportunities across local markets. That is, we require that the manufacturer's offer is such that in equilibrium the retailer chooses to purchase at the same wholesale price regardless of the realized demand. If, alternatively, the supplier's offer induced the retailer to buy at different wholesale prices for different realizations of uncertainty, then retailers (in different markets) receiving different demand shocks could profitably trade with each other at the expense of the manufacturer.

In each local market, the retailer purchases the goods and resells them to the final consumers without incurring any additional costs. The retailer buys the competitively supplied product at marginal cost. Prior to downstream final quantity choices, the retailer and the manufacturer sign a non-linear contract that stipulates the terms and conditions of purchase. We examine three types of contracts: standard two-part tariffs, all-unit quantity discounts, and market share discounts. All specify a unit price (w) and a non-negative franchise fee (F). An all-unit quantity discount contract offers a rebate off-the-list price for all the units purchased once a *quantity* threshold is met. Also, a market share discount contract offers a rebate for all units bought if purchases meet a required threshold. However, the threshold is structured differently, a *share* of retailer's purchases must be made from the manufacturer in order to qualify for the rebate.

A retailer who signs a standard two-part tariff (2PT) contract pays to the manufacturer $C^{2PT}(q_1) = wq_1 + F$ for purchasing a quantity $q_1 \geq 0$. An all-unit quantity discount (AU) contract stipulates two wholesale prices ($w_H > w_L$) and a quantity target (q^T) to qualify for the lower price. Then, the payment to the manufacturer is

$$C^{AU}(q_1) = \begin{cases} w_H q_1 + F \text{ if } q_1 < q_1^T \\ w_L q_1 + F \text{ if } q_1 \geq q_1^T \end{cases}.$$

A market-share discount (MS) contract stipulates two wholesale prices ($w_H > w_L$) and a share target, $\tau \in (0, 1)$. The payment with this contract is

$$C^{MS}(q_1) = \begin{cases} w_H q_1 + F \text{ if } q_1 < \tau(q_1 + q_2) \\ w_L q_1 + F \text{ if } q_1 \geq \tau(q_1 + q_2) \end{cases},$$

where q_2 is the quantity of the competitively supplied product.

Marginal costs of production for both products are assumed to be zero. The inverse demand system for the differentiated goods faced by a retailer is given by $P_1(q, \theta)$ and $P_2(q, \theta)$, where

$q = (q_1, q_2)$ is the vector of chosen quantities. $P_i(q) \in C^1$ and $\partial P_i/\partial q_i < 0$ whenever $P_i(q) > 0$ for $i = 1, 2$. The parameter θ is a discrete random variable which captures potential demand uncertainty common to both products. It takes with probability p a low value (θ_L) and with probability $1 - p$ a high value (θ_H). Let $E(\theta)$ be the expectation of θ and $P_i(0, \theta_L) > 0$. Shocks in different downstream markets are assumed to be iid.

A retailer chooses q_1 and q_2 to maximize its profits, $\pi(q, \theta) = R(q, \theta) - wq_1 - F$, where $R(q, \theta) = P_1(q, \theta)q_1 + P_2(q, \theta)q_2$ is its revenue. For a given w, $\pi(q) \in C^2$ is strictly concave and submodular ($\partial^2\pi/\partial q_1\partial q_2 < 0$). The retailer's outside option consists of selling only the competitively supplied variety. Then, if the retailer rejects the manufacturer's offer, it chooses q_2 to maximize $R_O(q_2, \theta) = P_2(0, q_2, \theta)q_2$. Let $R^*_O(\theta)$ be the maximal profit of a retailer that only sells good 2.

A risk neutral retailer accepts to sign the contract if its expected profit exceeds the expected value of its outside option. In contrast, an infinitely risk averse retailer signs the contract only if its profits under the low (worst) demand realization ($\theta = \theta_L$) are higher than or equal to the value of the outside option for that demand scenario.

Under a deterministic demand the objectives of the manufacturer in designing a vertical contract are maximization of the surplus in the vertical chain and surplus extraction. Market power in vertical chains may lead to a conflict between surplus maximization and extraction. Surplus maximization might fail due to double marginalization. Two part tariffs are an efficacious way to avoid this problem, the product is passed downstream at upstream marginal cost and rent is extracted through the franchise fee. With deterministic demand, two-part tariffs and discounts based on quantity or market share thresholds defined as above are all equally effective tools of replicating the integrated firm's solution. The proof of the following result is straightforward and therefore omitted.[7]

Proposition 1 *With deterministic demand, two part tariff, all-unit quantity discount and market share discount contracts are equivalent both from manufacturer's and social planner's viewpoints. They all maximize surplus in the vertical chain.*

3 Contracts under Demand Uncertainty

3.1 Risk-Averse Retailer

Our main focus is to understand how the relative performance of the different contracts is affected by retailers' risk attitude.[8] We start with the case of a risk averse retailer. With demand uncertainty and retailer risk aversion, vertical contracts need to cater for an additional objective:

[7] In the Appendix we prove a similar equivalence result in the case with uncertainty and a risk-neutral retailer. The deterministic demand version of the result can be recovered from that result by substituting p=1.

[8] Section 3.3 shows that the relative performance of these contracts under uncertainty is still the same when the retailers are risk neutral.

insurance provision to the retailers. Achieving this objective undermines contracts' ability to eliminate the tension between surplus maximization and extraction.

An infinitely risk averse retailer only accepts a contract that under a low demand realization guarantees that its profits are weakly greater than its outside option. This requirement implies that under high demand the manufacturer cannot absorb all surplus via the franchise fee and any attempt to extract more from the retailer requires a wholesale price above the marginal cost. Consequently, the unit price charged exceeds the level that maximizes surplus in the vertical chain. Intuitively, for a given probability p, a larger difference $\theta_H - \theta_L$ makes the retailer's participation constraint more restrictive and increases the manufacturer's need to absorb surplus via the unit price. So, the fact that a risk averse retailer requires some insurance to sign the contract, leads to double marginalization even when two-part tariffs are used.

Under a standard two part tariff, the retailer chooses q_1 and q_2 to maximize

$$\pi_{2PT} = P_1(q,\theta)q_1 + P_2(q,\theta)q_2 - wq_1 - F.$$

The optimal output choices, $q_1^*(w,\theta)$ and $q_2^*(w,\theta)$, satisfy the first-order conditions:

$$\frac{\partial P_1}{\partial q_1}q_1 + \frac{\partial P_2}{\partial q_1}q_2 + P_1 = w \tag{1}$$

$$\frac{\partial P_1}{\partial q_2}q_1 + \frac{\partial P_2}{\partial q_2}q_2 + P_2 = 0. \tag{2}$$

Let $R^*(w,\theta) = R(q_1^*, q_2^*, \theta)$ be the optimal second stage revenue. Concavity and submodularity of retailer's profits imply that $\partial q_1^*/\partial w_{2PT} < 0$ and $\partial q_2^*/\partial w_{2PT} > 0$. With a two-part tariff, double marginalization makes the retailer substitute away from manufacturer's product in favor of the competitively supplied variety.

The upstream manufacturer sets w_{2PT} and F_{2PT} to maximize

$$U_{2PT} = pw_{2PT}q_1^*(w_{2PT},\theta_L) + (1-p)w_{2PT}q_1^*(w_{2PT},\theta_H) + F_{2PT}$$
$$\text{subject to } R^*(w_{2PT},\theta_L) - w_{2PT}q_1^*(w_{2PT},\theta_L) - F_{2PT} - R_O^*(\theta_L) \geq 0.$$

Notice that the outside option (R_O^*) is independent of w_{2PT} and the maximand is increasing in F_{2PT}, thus the constraint binds at the optimum. By the envelope theorem, (1) and (2) imply that the optimal unit price (w_{2PT}^*) satisfies

$$(1-p)(q_1^*(w_{2PT}^*,\theta_H) - q_1^*(w_{2PT}^*,\theta_L) + w_{2PT}^*\frac{\partial q_1^*(\theta_H)}{\partial w}) = -pw_{2PT}^*\frac{\partial q_1^*(\theta_L)}{\partial w}. \tag{3}$$

Manufacturer's profits with the optimal 2PT are given by:

$$\begin{aligned} U_{2PT}^* = {} & pw_{2PT}^*q_1^*(w_{2PT}^*,\theta_L) + (1-p)w_{2PT}^*q_1^*(w_{2PT}^*,\theta_H) \\ & + R^*(w_{2PT}^*,\theta_L) - w_{2PT}^*q_1^*(w_{2PT}^*,\theta_L) - R_O^*(\theta_L). \end{aligned} \tag{4}$$

If a rollback discount does not induce the retailer to act on the threshold, then retailer's quantity choices are still governed by (1) and (2). Thus, a rollback discount can improve upon a

2PT only by inducing the retailer to act on the threshold. If a retailer facing an all-unit quantity discount acts on the threshold, then it optimizes by choosing the quantity of the competitively supplied product to be sold along with the threshold quantity for the manufacturer's product. For example, if the scheme induces the retailer to act on the target when demand is low, then

$$\widehat{q}_2(q_1^T, \theta_L) = \max_{q_2}(R(q_1^T, q_2, \theta_L) - w_L q_1^T - F)$$

where q_1^T is the quantity threshold that qualifies for the discounted unit price w_L. Let $\widehat{R}(q_1^T, \theta)$ denote the optimal second stage revenue when the retailer sells q_1^T units of the manufacturer's product.

Next proposition establishes that when the retailer is infinitely risk-averse there exists an all-unit quantity discount that the manufacturer strictly prefers to the optimal 2PT. In particular, the manufacturer can make the retailer buy the same quantity as in the optimal 2PT at a higher price when demand is low.

Proposition 2 *With demand uncertainty and an infinitely risk-averse retailer, the manufacturer strictly prefers all-unit quantity discount contract to two-part tariff contract.*

Proof. With an all-unit quantity discount that induces only the low type to act on the threshold, supplier's profits are

$$U_{AU}(\widehat{w}_{AU}, q_1^T) = p\widehat{w}_{AU} q_1^T + (1-p)\widehat{w}_{AU} q_1^*(\widehat{w}_{AU}, \theta_H) + \widehat{R}(q_1^T, \theta_L) - \widehat{w}_{AU} q_1^T - R_O^*(\theta_L).$$

Consider now the all-unit quantity discount that offers the rebated price $\widehat{w}_{AU} = w_{2PT}^*$ if $q_1 \geq q_1^T = q_1^*(w_{2PT}^*, \theta_L)$ and a fee $F_{AU} = R^*(w_{2PT}^*, \theta_L) - w_{2PT}^* q_1^*(w_{2PT}^*, \theta_L) - R_O^*(\theta_L)$. Notice that $U_{AU}(w_{2PT}^*, q_1^*(w_{2PT}^*, \theta_L)) = U_{2PT}^*$ (the RHS is defined above in 4). This all-unit discount induces only the low type to act on the threshold so that the marginal variation in supplier's profits when increasing w evaluated at $\widehat{w}_{AU}$ is given below. Note that, by charging a very high price when the retailer buys less than $q_1^*(w_{2PT}^*, \theta_L)$, the manufacturer can guarantee that the retailer keeps buying $q_1^*(w_{2PT}^*, \theta_L)$ when demand is low for a small increase in $\widehat{w}_{AU}$.

$$\frac{\partial U_{AU}}{\partial w} = (1-p)(q_1^*(w_{2PT}^*, \theta_H) - q_1^*(w_{2PT}^*, \theta_L) + w_{2PT}^* \frac{\partial q_1^*(\theta_H)}{\partial w}). \tag{5}$$

By (3) it follows that

$$\frac{\partial U_{AU}}{\partial w} = -p w_{2PT}^* \frac{\partial q_1^*(\theta_L)}{\partial w} > 0.$$

Then, there exists $\epsilon > 0$ such that $U_{AU}(w_{2PT}^* + \epsilon, q_1^*(w_{2PT}^*, \theta_L)) > U_{2PT}^*$. ■

The proof shows that there is an all-unit quantity discount scheme that induces the low-type to buy the same amount as in the optimal two-part tariff and increases the manufacturer's expected profits compared to the two-part tariff. Note that the all-unit quantity discount contract used in this proof induces the retailer to act on the threshold quantity only when the demand is low. Indeed the optimal AU contract has this form. For any AU contract that induces the retailer to act on the threshold both when demand is low and when demand is high, there exist an AU contract that induces the retailer to act on the threshold only when the demand is low which provides higher expected profits to the manufacturer.

Let us now consider market share discount contacts. With this type of contract, the retailer qualifies for a price discount if at least a percentage τ of its purchases are made from the manufacturer. If such a contract induces the retailer to act on the share target, it limits the retailer's substitution of the manufacturer's product with the competitively supplied alternative in response to an increase in price. This reduces the market for substitutes of manufacturer's product and allows it to charge a higher unit price as elasticity of the retailer's demand falls.

A retailer that acts exactly on the share threshold chooses q_1 and q_2 to maximize:

$$P_1(q,\theta)q_1 + P_2(q,\theta)q_2 - wq_1 - F,$$
$$\text{subject to } q_1 = \tau(q_1+q_2).$$

Let $s = \tau/(1-\tau)$ (note that $\tau \in (0,1) \Rightarrow s < \infty$), then the constraint requires that $q_1 = sq_2$. Substituting the constraint, it follows that for a retailer that acts exactly on the threshold, the quantity of good 2, $q_2^{**}(w,s,\theta)$, maximizes:

$$\pi_{MS} = P_1(sq_2, q_2, \theta)sq_2 + P_2(sq_2, q_2, \theta)q_2 - wsq_2 - F.$$

The first order condition of the maximization problem is:

$$\frac{\partial P_1}{\partial q_1}s^2 q_2^{**} + \frac{\partial P_1}{\partial q_2}sq_2^{**} + (P_1 - w)s + \frac{\partial P_2}{\partial q_1}sq_2^{**} + \frac{\partial P_2}{\partial q_2}q_2^{**} + P_2 = 0. \tag{6}$$

Proposition 3 *With demand uncertainty and an infinitely risk-averse retailer, the manufacturer strictly prefers rollback market share discount contract to two-part tariff contract.*

Proof. With a market-share discount that induces only the low type to act on the threshold, manufacturer's profits are

$$U_{MS} = pwsq_2^{**}(w,s,\theta_L) + (1-p)wq_1^*(w,\theta_H) + F_{MS}.$$

Let $s_{2PT} = q_1^*(w_{2PT}^*, \theta_L)/q_2^*(w_{2PT}^*, \theta_L)$ and notice that $s_{2PT}q_2^{**}(w_{2PT}^*, s_{2PT}, \theta_L) = q_1^*(\theta_L)$. Consider a market-share discount that offers the rebated price $\widehat{w}_{MS} = w_{2PT}^*$ if $s \geq s_{2PT}$ and a fee $F_{MS} = R^*(w_{2PT}^*, \theta_L) - w_{2PT}^* q_1^*(w_{2PT}^*, \theta_L) - R_O^*(\theta_L)$. Then, $U_{MS}(\widehat{w}_{MS}, s_{2PT}) = U_{2PT}$ (the RHS is given in 4). The marginal variation in supplier profits when increasing w evaluated at $\widehat{w}_{MS}$ is given by

$$\frac{\partial U_{MS}}{\partial w} = (1-p)(q_1^*(w_{2PT}^*, \theta_H) - s_{2PT}q_2^{**}(w_{2PT}^*, s_{2PT}, \theta_L)) + (1-p)w_{2PT}^*\left(\frac{\partial q_1^*(\theta_H)}{\partial w} - s_{2PT}\frac{\partial q_2^{**}(\theta_L)}{\partial w}\right).$$

It follows by (3) that

$$\frac{\partial U_{MS}}{\partial w} = -w_{2PT}^* s_{2PT}\frac{\partial q_2^{**}(\theta_L)}{\partial w} = -w_{2PT}^*\frac{\partial q_1^*(\theta_L)}{\partial w} > 0.$$

Then, there exists $\epsilon > 0$ such that $U_{MS}(w^*_{2PT} + \epsilon, s_{2PT}) > U^*_{2PT}$. ■

The proof of Proposition 3 shows that there exists a MS contract that results in higher upstream profits than the optimal 2PT. The proof uses an MS contract which induces the retailer to act on the threshold only when the demand is low. However, this is not necessarily true for the optimal MS contract. In the linear demand example presented below, the optimal MS contract actually induces *both* types to act on the share threshold. Notice that a retailer acting on a share target still makes use of its private information on demand. By correctly adjusting its purchases of the competitively supplied product it can meet the threshold while allowing its choices to respond to uncertainty.

The manufacturer prefers MS contracts to 2PT because setting a share threshold limits product substitution and makes retailer's demand for its product more inelastic. Lower price-elasticity allows the manufacturer to charge a higher unit-price and transfer more surplus upstream, so that retailer's participation constraint is met at a lower cost.

A similar effect is in operation under an AU contract when the demand realization is low. With low demand the retailer facing the optimal AU contract purchases at the threshold level for any price leading the effective elasticity of the retailer's demand for manufacturer's product to fall to zero. However, when demand is high, under the optimal AU contract the retailer's purchases of the manufacturer's product is above the threshold level and there is no reduction in the elasticity of retailer's demand for manufacturer's product. Then, increasing the wholesale price decreases the volume of retailer's purchases from the manufacturer faster relative to the optimal MS contract. In summary, elasticity of the retailer's demand for the manufacturer's product is higher under a MS contract than under an AU contract when demand is low, but the opposite is true when the demand is high. In the linear demand example below we derive closed form solutions to illustrate the relative desirability of the three pricing schemes (2PT contract, AU contract and MS contract) from the manufacturer's perspective and also examine the social incentives for their use.

3.2 Linear Demand Example: Contract Comparison and Welfare Analysis

Let the preferences of the representative consumer be given by a quadratic utility,

$$U(q_1, q_2) = (a + \theta)q_1 + (a + \theta)q_2 - \frac{1}{2}(q_1^2 + 2\beta q_1 q_2 + q_2^2),$$

where $\beta \in (0, 1)$ measures product differentiation. Then, the inverse demand functions are given by[9]:

$$P_i(q_i, q_j) = a + \theta - q_i - \beta q_j \text{ for } i, j = 1, 2 \text{ with } i \neq j.$$

Let us consider first a two-part tariff contract. The optimal second stage quantity and profits are given by:

[9] See, for instance, Vives (2001).

$$q_1^* = \frac{(1-\beta)(a+\theta) - w}{2(1-\beta^2)} \text{ and } q_2^* = \frac{(1-\beta)(a+\theta) + \beta w}{2(1-\beta^2)}, \tag{7}$$
$$\pi^* = \frac{2(1-\beta)a(a-w) + w^2}{4(1-\beta^2)} + \theta \frac{2a - w}{2(1+\beta)} + \theta^2 \frac{1}{2(1+\beta)} - F,$$

where $\theta \in \{\theta_L, \theta_H\}$. The retailer's outside option is $R_O^*(\theta) = (a+\theta)^2/4$ and its participation constraint in this case requires that $\pi^*(\theta_L) \geq R_O^*(\theta_L)$.

Manufacturer's equilibrium choices and profits are, respectively:

$$w_{2PT}^* = (1-\beta)(1-p)(\theta_H - \theta_L),\ F_{2PT}^* = \frac{(1-\beta)[a - (1-p)(\theta_H - \theta_L) + \theta_L)]^2}{4(1+\beta)}, \text{ and}$$
$$U_{2PT}^* = \frac{(1-\beta)[(a+\theta_L)^2 + (1-\beta)(1-p)^2(\theta_H - \theta_L)^2]}{4(1+\beta)}.$$

In the Appendix we summarize the equilibrium outcomes when the retailer is infinitely risk averse in the linear demand example.

Consider now an all-unit quantity discount contract. It can be shown that the optimal AU contract induces only the low type to act on threshold, while the high type purchases above the threshold. The optimal second stage revenue of the low type retailer is

$$\widehat{R}(q_1^T, \theta_L) = \frac{(a+\theta_L)^2}{4} - (1-\beta^2)(q_1^T)^2 + (1-\beta)(a+\theta_L)q_1^T.$$

The participation constraint is binding and, in equilibrium,

$$q_1^T = \frac{(a+\theta_L)(1+p) - (1-p)(\theta_H - \theta_L)}{2(1+\beta)(1+p)},\ w_{AU}^* = \frac{(1-\beta)(\theta_H - \theta_L)}{1+p} \text{ and}$$
$$F_{AU}^* = \frac{(1-\beta)[a - (\theta_H - 2\theta_L)]^2}{4(1+\beta)} + \frac{p(1-\beta)(\theta_H - \theta_L)[a - (\theta_H - 2\theta_L)]}{2(1+\beta)(1+p)}.$$

As in the case of a 2PT contract, risk aversion and the related insurance provision to the retailer raise the wholesale price above the level that maximizes total surplus. However, although the wholesale price involves a higher mark-up than in the case of a 2PT contract ($w^{AU} > w^{2PT}$), when demand is low, the retailer purchases more of manufacturer's product than under a 2PT contract due to the incentives provided by the mechanism.[10] Then, the manufacturer is able to absorb more surplus and make higher profits than under a 2PT contract,

$$U_{AU}^* = \frac{(1-\beta)(1+p)(a+\theta_L)^2 + (1-\beta)(1-p)(\theta_H - \theta_L)^2}{4(1+\beta)(1+p)}.$$

However, expected total welfare and consumer surplus are lower under an AU contract than under a 2PT contract,

$$W_{AU} - W_{2PT} = -\frac{(1-p)(1-\beta)p^2(2+3p)(\theta_H - \theta_L)^2}{8(1+p)^2(1+\beta)} < 0 \text{ and}$$
$$CS_{AU} - CS_{2PT} = -\frac{(1-\beta)p^2(2-p-p^2)(\theta_H - \theta_L)^2}{8(1+p)^2(1+\beta)} < 0.$$

[10] The opposite is true when demand is high as the threshold in the optimal AU contract does not constrain the retailer's choice there. If the manufacturer designed a contract that constrains both types, then it would make upstream profits $\overline{U}_{AU} = (1-\beta)(a+\theta_L)^2/4(1+\beta)$. Note that $\overline{U}_{AU} < U_{2PT}^* < U_{AU}^*$. For $\overline{U}^{AU}$ to be well-defined there must be relatively little uncertainty.

There are two underlying effects behind this welfare comparison. First, when demand is low the distortion in the retailer's sales due to double marginalization is lower under the optimal AU contract because sales of the manufacturer's good are determined by the threshold rather than the first order condition.[11] Second, the higher wholesale price increases the negative welfare impact of double marginalization when demand is high and the retailer's sales are governed by the first order condition.[12] The second effect is stronger leading expected welfare to be lower with optimal AU contract than with optimal 2PT contract.[13]

Finally, let us consider a market share discount. A retailer acting exactly on the market share threshold chooses optimally only the quantity of the competitively supplied product.[14] That is,

$$q_2^{**} = \frac{a+\theta+s(a+\theta-w)}{2(1+2\beta s+s^2)} \text{ and } \pi^{**}(w,F,\theta) = \frac{[a+\theta+s(a+\theta-w)]^2}{4(1+2\beta s+s^2)} - F.$$

The optimal MS contract induces both types to act on the share threshold and choose optimally only the quantity of the competitively supplied product. The supplier chooses w, s and F to maximize

$$w_{MS}s\frac{a+E(\theta)+s(a+E(\theta)-w_{MS})}{2(1+2\beta s+s^2)} + F_{MS} \text{ subject to } \pi^{**}(w_{MS},F_{MS},\theta_L) \geq \frac{(a+\theta_L)^2}{4}.$$

The constraint is binding, and it follows that in equilibrium,

$$s_{MS}^* = 1,\ w_{MS}^* = 2(1-p)(\theta_H-\theta_L) \text{ and}$$
$$F_{MS}^* = \frac{[a+\theta_L-(1-p)(\theta_H-\theta_L)]^2}{2(1+\beta)} - \frac{(a+\theta_L)^2}{4}.$$

As in the case of two-part tariff contract and all-unit quantity discount contract, the participation constraint of the risk averse retailer leads to a wholesale price above upstream marginal cost. The resulting upstream profits are given by

$$U_{MS}^* = \frac{(1-\beta)(a+\theta_L)^2+2(1-p)(\theta_H-\theta_L)^2}{4(1+\beta)}.$$

Expected total welfare and expected consumer surplus are lower under MS discounts than under 2PT:

$$W_{MS}-W_{2PT} = -\frac{1}{8}(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+3(1-p)(\theta_H-\theta_L)] < 0 \text{ and}$$
$$CS_{MS}-CS_{2PT} = -\frac{1}{8}(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+(1-p)(\theta_H-\theta_L)] < 0.$$

[11] When demand is low, under the AU contract, the retailer optimizes by choosing the quantity of the competitively supplied good corresponding to the threshold quantity of the manufacturer's good, and although the sales are still distorted in favor of the competitively supplied good the distortion is lower as $1 < q_2^*(w_{AU}^*,\theta^L)/q_1^*(w_{AU}^*,\theta^L) < q_2^*(w_{2PT}^*,\theta^L)/q_1^*(w_{2PT}^*,\theta^L)$.

[12] When demand is high, for both contracts the retailer's choices are governed by the first order conditions and the distortion in relative sales of the two goods is higher with the optimal AU contract $q_2^*(w_{AU}^*,\theta^H)/q_1^*(w_{AU}^*,\theta^H) > q_2^*(w_{2PT}^*,\theta^H)/q_1^*(w_{2PT}^*,\theta^H) > 1$.

[13] Expected total welfare is computed as the probability weighted sum of the representative consumer's utility, and the aggregate retailer and manufacturer profits under the optimal contracts in the two demand states.

[14] When acting on the share threshold, the retailer actually chooses optimally the purchases of only one product. The purchases of the substitute product are determined by the share requirement. Without loss of generality, we let the retailer choose the quantity of the competitively supplied product (q_2).

Proposition 4 *Under demand uncertainty, with an infinitely risk-averse retailer and linear demand, expected total welfare and consumer surplus are highest under 2PT contract. Expected consumer surplus is lowest under MS contract. For* $p^2 < (1+\beta)/2$*, from both private and social viewpoints, MS contract outperforms* AU *contract.*

The intuition underlying Proposition 4 is related to the delegation problem (see, for instance, Rey and Tirole (1986)). Under uncertainty the manufacturer pursues to exploit market power in the vertical chain and to offer insurance to the risk-averse retailer. In our model, the fact that the retailer is a multiproduct firm affects both upstream objectives. An integrated monopolist can exploit market power optimally in the vertical structure. It passes the product downstream at marginal cost and, under uncertainty, it chooses $q_1^{VI} = q_2^{VI} = (a+\theta)/[2(1+\beta)]$. The retail quantity responds to the uncertainty, and the share of manufacturer's product is constant across states. The vertically integrated share of manufacturer's product is $\tau^{VI} = 50\%$ $(= q_1^{VI}/(q_1^{VI} + q_2^{VI}))$.[15]

When dealing with a risk averse retailer, the manufacturer cannot extract the incremental surplus from the retailer through the franchise fee as the retailer requires insurance from market risk, and is forced to sell its product above marginal cost. With a 2PT contract, the retailer chooses quantities $q_1^{2PT} < q_1^{VI}$ and $q_2^{2PT} > q_2^{VI}$ that respond to the uncertainty.[16] Due to the higher unit price, the share of manufacturer's product is lower, $\tau^{2PT}(\theta) = q_1^{2PT}/(q_1^{2PT} + q_2^{2PT}) < 50\%$ (and varies across states, $\tau^{2PT}(\theta_L) < \tau^{2PT}(\theta_H)$), as the retailer purchases more of the substitute product. Under the low demand, the AU contract induces the retailer to act on threshold. This limits retailer's ability to cut down the share of manufacturer's product when facing low demand ($\tau^{AU}(\theta_L) = q_1^T/(q_1^T + q_2^{AU}) > \tau^{2PT}(\theta_L)$). But, it comes at a cost, as $\tau^{AU}(\theta_H) < \tau^{2PT}(\theta_H)$.

Finally, a MS contract allows the retailer's choices to respond to the uncertainty, while at the same time it prevents the retailer from buying more of the substitute product when confronted with a higher unit price for the manufacturer's product, $q_1^{MS} = q_2^{MS} = [a + \theta - (1-p)(\theta_H - \theta_L)]/2(1+\beta)$. Although, $q_1^{MS} = q_1^{2PT}$, retailer's demand for manufacturer's product is more inelastic at levels above the threshold and this allows the manufacturer to absorb more surplus by charging a higher wholesale price. Insurance provision prevents the manufacturer from passing the product downstream at marginal cost and this causes a loss of efficiency. However, a share-based target allows him to restore the vertically integrated share of purchases: $\tau_{MS} = 50\%$ for both demand realizations.

3.3 Risk-Neutral Retailer

When contracting with a risk neutral retailer, the manufacturer designs the contracts to maximize the surplus in the vertical chain and absorb the rents upstream. A risk neutral retailer is willing

[15] The linear demand example presented here rules out vertical differentiation. Due to this symmetry, the share of purchases is 50%.

[16] Note that, $q_1^{2PT} = q_1^*(w_{2PT}^*, \theta) = [a + \theta - (1-p)(\theta_H - \theta_L)]/[2(1+\beta)]$ and $q_2^{2PT} = q_2^*(w_{2PT}^*, \theta) = [a + \theta + \beta(1-p)(\theta_H - \theta_L)]/[2(1+\beta)]$.

to bear all market risk. So, in this case there is no conflict between surplus extraction incentives and insurance provision. Then, the manufacturer passes the product to the retailer at marginal cost ($w_{RN} = 0$) and appropriates surplus through the franchise fee that is equal to retailer's expected profit net of its outside option ($F_{RN} = E_\theta(R^*(0,\theta)) - E_\theta(R^*_O(\theta))$). Two-part tariffs, all-unit quantity discounts, and rollback market share discounts are all equally effective tools to maximize and extract surplus in the vertical chain.

Proposition 5 *With demand uncertainty and a risk-neutral retailer, the manufacturer is indifferent between two-part tariffs, market-share, and all-unit discounts. Private and social incentives are aligned.*

The proof of this result is presented in the Appendix together with a linear demand example. As the contracts are outcome-equivalent, the welfare result directly follows. All contracts generate the same consumer surplus. The retailer's incremental profit is strictly positive under a positive shock, but the retailer makes losses when demand is low. If the implementation of a market share contract requires the manufacturer to monitor at a cost the quantities chosen by the retailer, then this contract should not be observed in related environments when the retailer is risk neutral.

4 Conclusions and Extensions

We have shown that the risk attitude of the retailers plays a role in an upstream manufacturer's choice of contracts and loyalty discounts. In vertical relations, manufacturer's preference over the contracts is affected by their rent extraction and risk sharing properties. This result carries over to the case of loyalty rebate schemes. In a setting where an upstream manufacturer competes with a competitive fringe, a novel driver of our results is a contract's ability to affect product substitution. A market share contract that induces a buyer to act on the share threshold limits retailer's ability to substitute away from manufacturer's product when facing a relatively higher unit price. In addition, a market share contract provides a higher degree of flexibility to the retailer due to the fact that an absolute share threshold is achievable at many different quantity levels. The retailer can still obtain the discount in a bad season as all of its sales would be low. When the retailer is risk averse, this allows the manufacturer to guarantee the participation to the contract at a lower cost than in a two-part tariff and, for a wide range of parameters, than in an all-unit quantity discount. However, there is a conflict between social and private incentives for the use of different contracts as total welfare and consumer surplus are highest under two-part tariffs.

When the retailer is risk-neutral the ability to charge a higher wholesale price does not play a role. The retailer bears all the risk and buys the product at marginal cost. The manufacturer and the social planner are indifferent between two-part tariffs, all unit quantity, and rollback market share discounts.

Amongst possible extensions are generalizations in three directions. The two products sold by the retailer may eventually be vertically differentiated. It is interesting to see if the results extend to more general downward sloping demand functions, or to more general utility functions of the risk averse retailer. So far, we concentrated on non-contingent contracts (used in many identical and independent retail markets where the manufacturer operates). However, such contract comparison might shed light on the use of loyalty rebates also when the contracts are contingent on demand realizations.

5 Appendix

5.1 Risk-Neutral Retailer

Proof of Proposition 5. With uncertain demand, the risk-neutral retailer makes quantity choices after observing the realized demand. Hence, the second stage optimizations presented in subsection 3.1 still apply. But, since contracts are agreed upon before the resolution of uncertainty, a different risk attitude changes the first stage optimization. When the manufacturer faces a risk neutral retailer, the participation constraint requires retailer's expected profit to be at least equal to retailer's expected outside option.

Under a 2PT contract, the upstream manufacturer chooses w and F to maximize

$$pwq_1^*(w,\theta_L) + (1-p)wq_1^*(w,\theta_H) + F \text{ subject to}$$
$$p(R^*(w,\theta_L) - wq_1^*(w,\theta_L)) + (1-p)(R^*(w,\theta_H) - wq_1^*(w,\theta_H)) - F \geq pR_O^*(\theta_L) + (1-p)R_O^*(\theta_H).$$

The constraint is increasing in the franchise fee so the supplier chooses the unit price to maximize $pR^*(w,\theta_L) + (1-p)R^*(w,\theta_H)$. It follows that the optimal unit price w_{RN} satisfies the first order condition

$$p(\frac{\partial R(\theta_L)}{\partial q_1}\frac{\partial q_1^*(\theta_L)}{\partial w} + \frac{\partial R(\theta_L)}{\partial q_2}\frac{\partial q_2^*(\theta_L)}{\partial w}) + (1-p)(\frac{\partial R(\theta_H)}{\partial q_1}\frac{\partial q_1^*(\theta_H)}{\partial w} + \frac{\partial R(\theta_H)}{\partial q_2}\frac{\partial q_2^*(\theta_H)}{\partial w}) = 0. \quad (8)$$

Using (1), (2), and envelope theorem, (8) becomes $pw_{RN}(\partial q_1^*(\theta_L)/\partial w) + (1-p)w_{RN}(\partial q_1^*(\theta_{HL})/\partial w) = 0$, and $(\partial q_1^*(\theta)/\partial w) < 0$ implies that, in equilibrium,

$$w_{RN} = 0 \text{ and } F_{RN} = E_\theta(R^*(0,\theta)) - E_\theta(R_O^*(\theta)). \quad (9)$$

Let us consider an AU contract which induces the retailer to act on the quantity target only under a low demand. Then, the supplier chooses w, q_1^T and F to maximize

$$pwq_1^T + (1-p)wq_1^*(w,\theta_H) + F \text{ subject to}$$
$$p(\widehat{R}(q_1^T,\theta_L) - wq_1^T) + (1-p)(R^*(w,\theta_H) - wq_1^*(w,\theta_H)) - F \geq pR_O^*(\theta_L) + (1-p)R_O^*(\theta_H).$$

The constraint is increasing in the franchise fee, so the supplier chooses w and q_1^T to maximize $p\widehat{R}(q_1^T,\theta_L) + (1-p)R^*(w,\theta_H)$. It follows that the optimal unit price w_{RN} satisfies the first order condition $(1-p)(\frac{\partial R(\theta_H)}{\partial q_1}\frac{\partial q_1^*(\theta_H)}{\partial w}+\frac{\partial R(\theta_H)}{\partial q_2}\frac{\partial q_2^*(\theta_H)}{\partial w})=0$. By a similar argument as in the case of a 2PT, it follows that the optimal unit price and franchise fee are given by (9). In addition, $q_1^T=\arg\max p\widehat{R}(q_1^T,\theta_L)=q_1^*(0,\theta_L)$. Clearly the manufacturer cannot improve upon this contract. The optimal 2PT and AU contracts result in the same output levels. ■

Finally, consider a MS contract that induces the retailer to act on the threshold always. Then, the supplier chooses w, s and F to maximize

$$pwsq_2^{**}(w,s,\theta_L)+(1-p)wsq_2^{**}(w,s,\theta_H)+F \text{ subject to}$$
$$p(R^{**}(w,s,\theta_L)-wsq_2^{**}(w,s,\theta_L))+(1-p)(R^{**}(w,s,\theta_H)-wsq_2^{**}(w,s,\theta_H))-F\geq$$
$$pR_O^*(\theta_L)+(1-p)R_O^*(\theta_H).$$

The constraint is increasing in the franchise fee, so the supplier chooses w and s to maximize $pR^{**}(w,s,\theta_L)+(1-p)R^*(w,s,\theta_H)$. Then, the unit price satisfies

$$p(\frac{\partial R}{\partial q_1}s\frac{\partial q_2^{**}(\theta_L)}{\partial w}+\frac{\partial R}{\partial q_2}\frac{\partial q_2^{**}(\theta_L)}{\partial w})+(1-p)(\frac{\partial R}{\partial q_1}s\frac{\partial q_2^{**}(\theta_H)}{\partial w}+\frac{\partial R}{\partial q_2}\frac{\partial q_2^{**}(\theta_H)}{\partial w})=0. \tag{10}$$

From (6), using envelope theorem, (10) becomes $p(\partial q_2^{**}(\theta_L)/\partial w)sw+(1-p)(\partial q_2^{**}(\theta_H)/\partial w)sw=0$. Then, the optimal unit price and franchise fee are given by (9). Again, the quantities purchased by the retailer at equilibrium are the same as in the optimal 2Pt contract and the manufacturer's expected profit is the same. We conclude that the manufacturer is indifferent between these contracts and the same level of aggregate welfare is obtained under all three contracts.

5.2 Closed-Form Solutions with Linear Demand

Using the linear demand specification presented in subsection 3.2, we can derive closed-form solutions for the quantities and prices of the two products (q_i and P_i for $i\in\{1,2\}$), retailer's profit (R), manufacturer's profit (U), welfare (W), and consumer surplus (CS) for all contracts. Table 1 and 2 present the equilibrium outcomes under uncertainty when the retailer is, respectively, risk-neutral and risk-averse.

Table 1:	**Uncertainty and Risk Neutrality**		
Contract	2PT	AU	MS
$q_1=q_2$	$\frac{a+\theta}{2(1+\beta)}$		
$P_1=P_2$	$\frac{a+\theta}{2}$		
R	$R_L=-\frac{(1-\beta)(1-p)(\theta_H-\theta_L)(2a+\theta_L+\theta_H)}{4(1+\beta)}+\frac{(a+\theta_L)^2}{4}$ $R_H=\frac{(1-\beta)p(\theta_H-\theta_L)(2a+\theta_L+)}{4(1+\beta)}+\frac{(a+\theta_H)^2}{4}$		
w	0		
$F=U$	$\frac{(1-\beta)[(1-p)(a+\theta_H)^2+p(a+\theta_L)^2]}{4(1+\beta)}$		
$E(CS)$	$\frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$		
$E(W)$	$\frac{3[p(a+\theta_L)^2+(1-p)(a+\theta_H)^2]}{4(1+\beta)}$		

Table2: Uncertainty and Infinite Risk Aversion

Contract	2PT	AU	MS
q_1	$\frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$	$q_1^L = \frac{(1+p)(a+\theta_H)-2(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$ $q_1^H = \frac{(1+p)(a+\theta_H)-(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$	$\frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$
q_2	$\frac{a+\theta+\beta(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$	$q_2^L = \frac{(1+p)(a+\theta_L)+\beta(1-p)(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$ $q_2^H = \frac{(1+p)(a+\theta_H)+\beta(\theta_H-\theta_L)}{2(1+\beta)(1+p)}$	$\frac{a+\theta-(1-p)(\theta_H-\theta_L)}{2(1+\beta)}$
P_1	$\frac{a+\theta+(1-\beta)(1-p)(\theta_H-\theta_L)}{2}$	$p_1^L = \frac{(1+p)(a+\theta_L)+(1-\beta)(1-p)(\theta_H-\theta_L)}{2(1+p)}$ $p_1^H = \frac{(1+p)(a+\theta_H)+(1-\beta)(\theta_H-\theta_L)}{2(1+p)}$	$\frac{a+\theta+(1-p)(\theta_H-\theta_L)}{2}$
P_2	$\frac{a+\theta}{2}$	$\frac{a+\theta}{2}$	$\frac{a+\theta+(1-p)(\theta_H-\theta_L)}{2}$
R_L	$\frac{(a+\theta_L)^2}{4}$	$\frac{(a+\theta_L)^2}{4}$	$\frac{(a+\theta_L)^2}{4}$
R_H	$\frac{(1-\beta)(\theta_H-\theta_L)[2(a+E(\theta))-(\theta_H-\theta_L)]}{4(1+\beta)} + \frac{(a+\theta_H)^2}{4}$	$\frac{(1-\beta)(\theta_H-\theta_L)[(2a+\theta_H)(1+p)^2-(3+4p)(\theta_H-\theta_L)]}{4(1+\beta)(1+p)^2} + \frac{(a+\theta_H)^2}{4}$	$\frac{(\theta_H-\theta_L)[(1-\beta)(2a+\theta_H+\theta_L)-4(1-p)(\theta_H-\theta_L)]}{4(1+\beta)} + \frac{(a+\theta_H)^2}{4}$
w	$(1-\beta)(1-p)(\theta_H-\theta_L)$	$\frac{(1-\beta)(\theta_H-\theta_L)}{1+p}$	$2(1-p)(\theta_H-\theta_L)$
F	$\frac{(1-\beta)[a+\theta_L-(1-p)(\theta_H-\theta_L)]^2}{4(1+\beta)}$	$\frac{(1-\beta)[a+\theta_L-(\theta_H-\theta_L)][(1+p)(a+\theta_L)-(1-p)(\theta_H-\theta_L)]}{4(1+\beta)(1+p)}$	$\frac{2[a+\theta_L-(1-p)(\theta_H-\theta_L)]-(1+\beta)(a+\theta_L)^2}{4(1+\beta)}$
U	$\frac{(1-\beta)[(a+\theta_L)^2+(1-p)^2(\theta_H-\theta_L)^2]}{4(1+\beta)}$	$\frac{(1-\beta)[(1+p)(a+\theta_L)^2+(1-p)(\theta_H-\theta_L)^2]}{4(1+\beta)(1+p)}$	$\frac{(1-\beta)(a+\theta_L)^2+2(1-p)^2(\theta_H-\theta_L)^2}{4(1+\beta)}$
$E(W)$	$\frac{A}{8(1+\beta)} + \frac{3p(a+\theta_L)^2+3(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{A}{8(1+\beta)} - \frac{(1-p)(1-\beta)p^2(2+3p)(\theta_H-\theta_L)^2}{8(1+p)^2(1+\beta)}$ $+\frac{3p(a+\theta_L)^2+3(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{A}{8(1+\beta)} - \frac{(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+3(1-p)(\theta_H-\theta_L)]}{8}$ $+\frac{3p(a+\theta_L)^2+3(1-p)(a+\theta_H)^2}{4(1+\beta)}$
$E(CS)$	$\frac{B}{8(1+\beta)} + \frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{B}{8(1+\beta)} - \frac{(1-\beta)p^2(2-p-p^2)(\theta_H-\theta_L)^2}{8(1+p)^2(1+\beta)}$ $+\frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$	$\frac{B}{8(1+\beta)} - \frac{(1-p)(\theta_H-\theta_L)[2(a+\theta_L)+(1-p)(\theta_H-\theta_L)]}{8}$ $+\frac{p(a+\theta_L)^2+(1-p)(a+\theta_H)^2}{4(1+\beta)}$

$A = (1-\beta)(1-p)(\theta_H-\theta_L)(-2a+\theta_L-3E(\theta))$, $B = (1-\beta)(1-p)(\theta_H-\theta_L)[2a+\theta_L+E(\theta)]$

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