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Quasi-static stationary-periodic model of percussive deep drilling

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ABSTRACT: In the percussively deep drilling, the rock is modeled by an infinite elastic medium with a semi-infinite cylindrical bore-hole having a curvilinear bottom. First, the stationary indentation is formulated as a non-classical non-linear free-boundary contact problem with unknown rupturing and non-rupturing parts of the bore-hole boundary. Then the stationary-periodic percussive drilling problem is reduced to the stationary one on the rupture progression stage of the cycle and to the classical contact problem on the reverse and progression-before-rupture stages of the cycle. This provides a nonlinear progression - force diagram for the bit dynamics prediction.

1 Introduction

The progression in the percussive drilling is caused by a material rupture under the action of a drilling bit applied at the bore-hole boundary points $y(t)$ moving in time t due to rupture. This boundary loading generates stress $\sigma_{ij}(y,t)$ and strain $\varepsilon_{ij}(y,t)$ tensors at all material points y. Let a material point y has Cartesian coordinates (y_1, y_2, y_3) in the non-deformed state. The radius-vector of the same material point y in a deformed state at a time t is $\tilde{y}(y,t) = y + u(y,t)$, where $u(y,t)$ is the displacement vector. We will use all equations in terms the non-deformed (reference) coordinates y and refer the boundary conditions to the non-deformed boundary surfaces (Lagrange approach).
Let us consider stationary-periodic percussive drilling of a half-infinite bore-hole, $\Omega_H(t)$, spreading to $y_3 = \infty$ in an infinite elastic space. Let y_3-axis of the coordinate system coincide with the bore-hole axes, and the drill bit progressive-periodic motion occurs only in the y_3 direction. Let $\Omega(t) = \mathbb{R}^3 \backslash \Omega_H(t)$ be the domain occupied by the material (i.e. the infinite space with drilled bore-hole) and $\partial\Omega(t)$ be the bore-hole surface in the non-deformed state (or, the same, in the reference coordinates y), while $\tilde{\Omega}_H(t)$, $\tilde{\Omega}(t) = \mathbb{R}^3 \backslash \tilde{\Omega}_H(t)$ and $\partial\tilde{\Omega}(t)$ be their counterparts in the deformed state. If the rupture front $\partial_F\Omega(t)$ constitutes only a finite part of the boundary $\partial\Omega(t)$, then the borehole is a semi–infinite (not necessarily circular) cylinder with a curvilinear bottom being the rupture front $\partial_F\Omega(t)$. Otherwise, the bore-hole has a monotonously widening shape. If the bit is axially-symmetric then the bore-hole is axially symmetric as well. Let $B(t)$

be the domain occupied by the bit at the instant t, and $\partial B(t)$ be its surface.

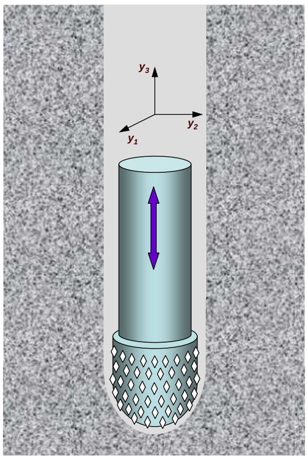

Figure 1: Stationary-periodic percussive drilling

To describe the material strength for a point y, we will use an instant strength condition at a point y at an instant t written as

$$\Lambda(\sigma(y,t)) < 1, \quad y \in \Omega(t), \tag{1}$$

where the function $\Lambda(\sigma)$ is associated with the von Mises, Coulomb–Mohr, Drucker–Prager or another appropriate strength condition.
We suppose that the rupture appears in the form of a rupture front $\partial_F\Omega(t)$. The rupture front is a part of the bore-hole boundary $\partial\Omega(t)$. The rupture front equation can be taken as

$$\Lambda(\sigma(y),t) = 1, \quad y \in \partial_F\Omega(t). \tag{2}$$

To formulate the simplest model, let us make the following

Model assumptions:

(i) The deformation gradient is small.

(ii) The material is homogeneous, i.e. its elastic moduli $C_{ijkl} = const$.

(iii) The bit is rigid.

(iv) The borehole surface is loaded by the bit at the contact surfaces and is free of tractions at all other points.

(v) The bit action can be reduced to a pressure $p(x)$.

(vi) The ruptured material, for which strength condition (1) is violated, disappears (is washed away) thus leaving the fresh rupture front (the bottom of the bore-hole) either free of tractions or in contact with the bit bottom.

Under the model assumptions, the bore-hole boundary $\partial\Omega$ generally consists of four non-overlapping parts: a free of traction non-rupturing part $\partial_{00}\Omega$, a contact non-rupturing part $\partial_{c0}\Omega$, a free of traction part of the rupture front $\partial_{0F}\Omega$, and a contact part of the rupture front $\partial_{cF}\Omega$.

2 Stationary indentation model

Let us consider in this section an auxiliary problem of stationary indentation of an infinite elastic space by a rigid indentor with a constant progression rate $\dot{h}_3 < 0$ in the y_3 direction. Then $\dot{h} = (0, 0, \dot{h}_3)$ is the progression rate vector.
In the stationary problem, the displacements, strains, and stresses are the same at the corresponding points at the corresponding instants,

$$u_i(y,t) = u_i(y - t\dot{h}, 0),\ \varepsilon_{ij}(y,t) = \varepsilon_{ij}(y - t\dot{h}, 0),\ \sigma_{ij}(y,t) = \sigma_{ij}(y - t\dot{h}, 0),\ y \in \Omega(t). \quad (3)$$

This implies

$$\dot{u}_i(y,t) = -\dot{h}_3 u_{i,3}(y,t),\ \dot{\varepsilon}_{ij}(y,t) = -\dot{h}_3 \varepsilon_{ij,3}(y,t),\ \dot{\sigma}_{ij}(y,t) = -\dot{h}_3 \sigma_{ij,3}(y,t), \quad (4)$$

$$\Lambda(\sigma(y,t)) = \Lambda(\sigma(y - t\dot{h}, 0)),\ y \in \Omega(t). \quad (5)$$

The dot over $u(y,t)$, $\varepsilon(y,t)$ and $\sigma(y,t)$ means partial derivative with respect to t, which for the chosen Lagrange approach coincides with the material derivative in time, while the subscript after comma means partial derivative with respect to the corresponding coordinate.
The boundary $\partial\Omega$ moves with the velocity $\dot{h}_3$ in the y_3 direction, the corresponding moving boundary points are related as

$$y(t) = y(0) + t\dot{h}, \quad y(t) \in \partial\Omega(t), \quad y(0) \in \partial\Omega(0).$$

This prescribes the normal velocities of the material points on the non-rupturing part of the boundary,

$$\dot{u}_i(y,t)\tilde{\eta}_i(y) = \dot{h}_3 \tilde{\eta}_3(y), \quad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t), \quad (6)$$

where $\tilde{\eta}_j(y)$ is a unit outward (i.e. directed inward the bore-hole) normal vector to the boundary $\partial\tilde{\Omega}(t)$ in the *deformed* state.
On the other hand, the normal movements of the material points on the rupture front should be slower than the normal movement of the front surface itself,

$$\dot{u}_i(y,t)\tilde{\eta}_i(y) > \dot{h}_3 \tilde{\eta}_3, \quad y \in \partial_{0F}\Omega(t) \cup \partial_{cF}\Omega(t), \quad (7)$$

since otherwise there will be creation of material instead of its rupture there, to fill the gap if the material movement is faster, or there will be no rupture if the movements coincide.
Taking in mind the first relation in (4), the time derivative can be replaced with the space derivative, reducing (6) and (7) to the form independent of $\dot{h}_3$,

$$u_{i,3}(y,t)\tilde{\eta}_i(y) = -\tilde{\eta}_3, \qquad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t), \quad (8)$$

$$u_{i,3}(y,t)\tilde{\eta}_i(y) > -\tilde{\eta}_3, \qquad y \in \partial_{0F}\Omega(t) \cup \partial_{cF}\Omega(t). \quad (9)$$

Relation (8) can be rewritten as

$$\tilde{y}_{i,3}(y,t)\tilde{\eta}_i(y) = 0, \quad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t). \quad (10)$$

Equation (10) is satisfied if y belongs to a cylindrical surface parallel to the y_3 axis, since the vector $\tilde{y}_{i,3}(y,t)$ is then tangent to the deformed surface which $\tilde{y}_i$ belongs to. This implies that condition (9) can not be satisfied on any cylindrical part of $\partial\Omega$, that is, *there is no rupture on* *the cylindrical part of* $\partial\Omega$. On the other hand, this means that we may replace condition (8) by the condition of the cylindrical non-rupturing surface,

$$\eta_3(y) = 0, \quad y \in \partial_{00}\Omega(t) \cup \partial_{c0}\Omega(t), \tag{11}$$

where $\eta_j(y)$ is a unit outward boundary normal vector to the non-deformed boundary $\partial\Omega(t)$.
On the contact surfaces we have generally the boundary inclusion $\tilde{y} = y + u(y) \in \partial B$, $y \in \partial_{c0}\Omega \cup \partial_{cF}\Omega$. Assuming the displacements $u(y)$ are small, the boundary condition can be linearized as

$$u_i(y)\eta_i(y) = d(y), \quad y \in \partial_{c0}\Omega \cup \partial_{cF}\Omega, \tag{12}$$

where $d(y)$ is a (positive or negative) distance between the point y and the bit boundary ∂B in the $\eta(y)$ direction in the non-deformed state, and it is known if the contact surface is known and is to be determined otherwise.
To solve the stationary indentation problem, it is sufficient to consider it only for $t = 0$. Thus, taking into account relations (3)-(12) and dropping the argument $t = 0$ for brevity, we arrive at the following non-classical non-linear free boundary problem,

$$\sigma_{ij,j}(y) = 0, \qquad \Lambda(\sigma(y)) < 1, \qquad y \in \Omega; \tag{13}$$

$$\begin{aligned} &\sigma_{ij}(y)\eta_j(y)\xi_i(y) = 0, \quad \sigma_{ij}(y)\eta_j(y)\zeta_i(y) = 0, \quad u_i(y)\eta_i(y) = d(y), \\ &\eta_3(y) = 0, \quad \sigma_{ij}(y)\eta_j(y)\eta_i(y) < 0, \quad \Lambda(\sigma(y)) < 1, \quad y \in \partial_{c0}\Omega; \end{aligned} \tag{14}$$

$$\begin{aligned} &\sigma_{ij}(y)\eta_j(y)\xi_i(y) = 0, \quad \sigma_{ij}(y)\eta_j(y)\zeta_i(y) = 0, \quad u_i(y)\eta_i(y) = d(y), \\ &\Lambda(\sigma(y)) = 1, \quad \sigma_{ij}(y)\eta_j(y)\eta_i(y) < 0, \quad u_{i,3}(y)\tilde{\eta}_i(y) > -\tilde{\eta}_3(y), \quad y \in \partial_{cF}\Omega; \end{aligned} \tag{15}$$

$$\begin{aligned} &\sigma_{ij}(y)\eta_j(y) = 0, \\ &\eta_3(y) = 0, \quad u_i(y)\eta_i(y) < d(y), \quad \Lambda(\sigma(y)) < 1, \quad y \in \partial_{00}\Omega; \end{aligned} \tag{16}$$

$$\begin{aligned} &\sigma_{ij}(y)\eta_j(y) = 0, \\ &\Lambda(\sigma(y)) = 1, \quad u_i(y)\eta_i(y) < d(y), \quad u_{i,3}(y)\tilde{\eta}_i(y) > -\tilde{\eta}_3(y), \quad y \in \partial_{0F}\Omega; \end{aligned} \tag{17}$$

$$u_i(y) \to 0, \qquad y \to \infty. \tag{18}$$

Here

$$\sigma_{ij}(y) = \sigma^0_{ij} + C_{ijkl}\varepsilon_{kl}(y), \quad \varepsilon_{kl}(y) = (u_{k,l} + u_{l,k})/2, \tag{19}$$

and the constant stiffness tensor C_{ijkl} and the constant stress σ^0_{ij} in the rock without the drill hole are know; $\xi_j(y)$, $\zeta_j(y)$ are unit vectors orthogonal to to the normal vector $\eta_j(y)$ and to each other. Condition (18) is understood on almost any straight ray originating from $y = 0$, thus permitting a non-zero limit of the displacements as $y \to \infty$ parallel to the bore-hole.
All the four boundary parts $\partial_{00}\Omega$, $\partial_{0F}\Omega$, $\partial_{c0}\Omega$, $\partial_{cF}\Omega$, and consequently $d(y)$, are generally unknown in this setting and the corresponding "excessive" boundary equalities and inequalities are provided in (14) and (17) to allow their determination.
Different strategies can be chosen to solve this problem, see e.g. Eliot & Ockendon (1982), Crank (1984, Section 8), Johnson (1985). One of the possibilities is the iteration algorithm described below. It consists of iterations, each solving a linear mixed boundary value problem (13)-(18), (19) with some *fixed* boundaries, $\partial_{00}\Omega$, $\partial_{0F}\Omega$, $\partial_{c0}\Omega$, $\partial_{cF}\Omega$, and consequently $d(y)$.

Then conditions (13)-(17) are checked and the boundaries are changed to alleviate the violation of inequalities, and the next iteration starts.
On the first iteration one can reasonably assume that the rupture front coincides with the contact part of the bit, $\partial_c B$, which in turn coincides with the bit bottom, $\partial_b B$, (consisting of the bit surface points with algebraically smallest y_3 coordinate, over the points with the same (y_1, y_2) coordinates), i.e. $\partial_{cF}\Omega = \partial_c B = \partial_b B$, $\partial_{0F}\Omega = \emptyset$, and there is no contact without rupture, i.e. $\partial_{c0}\Omega = \emptyset$. Those assumptions imply that the borehole free boundary $\partial_0\Omega$ is the semi-infinite cylindrical surface ended by the bit bottom, on the first iteration.
After the iterations converge, the integration of the component $\sigma_{3j}(y)\eta_j(y)$ of the contact traction gives the total axial force $P(t)$ applied to the bit during the progression,

$$\mathcal{P}(t) = \int_{\partial_c\Omega} \sigma_{3j}(y)\eta_j(y)\, dS(y_c). \tag{20}$$

In the case when condition (iii) is not satisfied, the rupture front, the contact surface and the pressure are unknown and we need to take into account elasticity of the bit. If condition (v) is violated, that is, the bit interacts with the material not only by pressure, one has to introduce some friction contact, describing it e.g. by the Coulomb–More law.
Note that in all the cases, the obtained solution and particularly the total force $\mathcal{P}$ is independent of the progression rate $\dot{h}_3$ or the progression itself.

3 Stationary-periodic indentation model

Let the instant bit progression $h_3(t)$ be the lowest y_3 coordinate of the bit boundary. Let T be the cycle period, $dh_3(t)/dt = \dot{h}_3(t) = -|\dot{h}(t)| \le 0$ be instant progression rate and $\dot{h}_{T3} = [h_3(t) - h_3(t-T)]/T = -|\dot{h}_T| \le 0$ be average progression rate over cycle, in the y_3 direction, where $\dot{h} = (0, 0, \dot{h}_3)$ and $\dot{h}_T = (0, 0, \dot{h}_{T3})$ are the instant and average progression rate vectors, respectively. In the stationary-periodic problems, $\dot{h}_T$ does not depend on t, the strains, stresses and boundary tractions are independent of the cycle number m in the corresponding points at the corresponding instants,

$$\varepsilon_{ij}(y + mT\dot{h}_T, t + mT) = \varepsilon_{ij}(y, t), \quad y \in \Omega(t); \tag{21}$$

$$\sigma_{ij}(y + mT\dot{h}_T, t + mT) = \sigma_{ij}(y, t), \quad y \in \Omega(t) \tag{22}$$

for any positive or negative integer m.
Rupture in this model does not in fact depend on the natural time t but depends only on the actual state in the considered material point.
In addition to the Model assumptions (i)-(vi), let as make the following
Reverse assumption

(vii) The rupture do not proceed during the reverse stage of the bit motion, i.e. the borehole boundary consists of the same material points until the load reaches the extremum value during the next cycle.

Due to assumption (vii), the stress and strain return to the same states during the reverse and the following progressive stages of the bit motion up to the rupture restarts. This implies the reverse and the following progressive stages can be considered as some interruptions of the stationary progression process, analyzed in the previous section, and moreover, the interruptions do not influence the material rupture. This means the relation between the total force and the bit progression looks as on Fig. 2, that is the elastic loading stage is followed by the rupture stage followed by the elastic unloading stage. Generally the elastic stages are non-linear on the loading and unloading stages due to the changing contact surface between the bit and the drilled material, and the force is constant on the rupture stage. The extremum force and strains in the process coincide with those obtained in the stationary indentation analysis in the previous section.

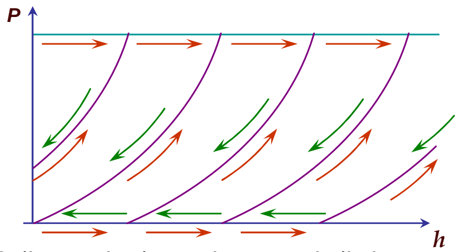

Figure 2: $h - P$ diagram in the stationary-periodic instant rupture model.

Then solution of the stationary indentation problem from the previous section fixes the bore-hole boundary for the reverse stage and gives the extremum values of the contact distribution $p(y, t^{ex})$, whose integral (20) gives the maximum force $\mathcal{P}^{ex} = \mathcal{P}(t^{ex})$ on the bit.
To predict the curvilinear part of the $h-P$ diagram, one has to solve the classical linearly elastic contact problem with a material boundary $\partial\Omega$ known in the non-deformed state, for both the progressive (before rupture restart) and reverse stages of the cycle (which do coincide). The problem consists of the following equations,

$$\sigma_{ij,j}(y) = 0, \qquad y \in \Omega; \tag{23}$$

$$\begin{aligned} \sigma_{ij}(y)\eta_j(y)\xi_i(y) &= 0, & \sigma_{ij}(y)\eta_j(y)\zeta_i(y) &= 0, & & \\ u_i(y)\eta_i(y) &= d(y, h_3), & \sigma_{ij}(y)\eta_j(y)\eta_i(y) &< 0, & y &\in \partial_c\Omega(h_3); \end{aligned} \tag{24}$$

$$\sigma_{ij}(y)\eta_j(y) = 0, \qquad u_i(y)\eta_i(y) < d(y, h_3), \qquad y \in \partial_0\Omega(h_3); \tag{25}$$

$$u_i(y) \to 0, \qquad y \to \infty. \tag{26}$$

The overall boundary $\partial\Omega$ here is known from the end of the previous progression-rupture stage, although the boundary partition into the free and contact parts is to be determined for each h_3. Classical conforming contact problem (23)-(26), (19) can be solved by any of the well known methods, see e.g. Johnson (1985). Particularly, one can use the iteration algorithm similar to the on on the progression-rupture stage described above, that is, to choose some reasonable partition of $\partial\Omega$ onto $\partial_0\Omega$ and $\partial_c\Omega$, solve mixed elasticity problem (23)-(26), (19), modify the partition of $\partial\Omega$ to alleviate the violation of inequalities in (24)-(26) and start the next iteration.

4 Conclusions

A stationary-periodic quasi-static model of percussive drilling is obtained. The cycles of the bit progression – force diagram consist of three stages: elastic loading, constant-force rupture progression, and elastic unloading parallel to the loading. The problem is split into a stationary free-boundary non-linear problem for the rupture stage of the cycle, and a classical contact problem for the rest of the cycle. Some iteration algorithms are described reducing the solution to a sequence of linear problems of elasticity. Those problems are to be solved by a general numerical method, e.g. the FEM or the Boundary Integral Equation Method. As a result, this provides a nonlinear progression-force diagram, which is to be used in the bit dynamic motion prediction.

5 Acknowledgement

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