Experimental and Bond Graph based Sensitivity Calculations for Micro Scale Robust Engineering Design

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Abstract - Bond graph modeling and sensitivity analysis are used to provide a platform for the robust design of a small mechatronic device, a behind-the-ear (BTE) hearing aid. Two key components of the device, namely the telecoil and the receiver, are considered. Experimental measurements, bond graph simulation models and analytic sensitivity analysis are used to investigate the interaction between these components in order to gain insight into the effect of component placement on the robustness of the final product..

I. INTRODUCTION

Robust Engineering Design (RED) refers to the group of methodologies dedicated to reducing the variation in performance of products and processes arising from manufacture and use. In addition to design and analysis of experiments, it now includes computer experiments on CAD/CAE simulation, advanced Response Surface Modelling methods, adaptive optimization methodologies and reliability analysis and should be included within general life-cycle design. RED has been shown to be very effective in improving product or process design through its use of experimental design and analysis methods. However, RED has not been fully developed at the micro-scale [Hsu, 2002] where paradoxically statistical variations become relatively more important.

This paper investigates a behind-the-ear (BTE) hearing aid device using bond graph modeling combined with analytic sensitivity analysis to provide a methodology for the robust engineering design of small mechatronic systems in multiple energy domains, also known as multi-physics modelling. Hearing aids consist of an assembly of very small components such as telecoils, microphones, receivers and amplifiers, shown in Figure 1. In order to fit

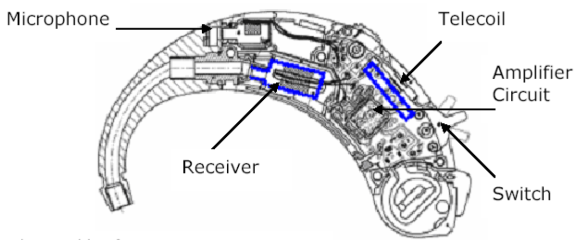

Fig. 1. Assembly diagram of BTE Hearing Aid

within the ear and provide high gain, some of these components are at the limits of commercial miniaturization which can cause unexpected behaviour and variation in performance. Bond graph modelling is used to generate a set of system equations that govern the time dependent behaviour of a multidomain system. These equations are solved numerically to allow for a computer simulation of system operation and, at each step of the simulation, sensitivity terms can be extracted form the governing equations. From such a sensitivity analysis the tolerance of the system output at each point in time to variations in each system parameter is quantified, resulting in immediate identification of the components whose parameter or spatial variations have greatest effect on system performance.

II. BOND GRAPH THEORY AND SENSITIVITY MODELLING

Bond graphs, introduced by Paynter [Paynter, 1961], can be used to generate mathematical models of multienergy domain systems based on electro-mechanical analogues. Notable contributions to bond graph theory have been made by Karnopp [Karnopp, 1985], Rosenberg [Rosenberg, 1987] and Cellier [Cellier, 1990]. Bates and Wynn [Bates and Wynn, 2001] explored sensitivity and validation in the statistical modeling of simulators. Atherton and Bates [Atherton and Bates, 2000] have used bond graph models to perform RED in a complex engineering environment by design of a loudspeaker driver unit and the design of a hedgetrimmer. The causality assignment and model-building associated with bond graphs makes them an interesting proposition for combination with other RED methods.

Dynamic physical systems are concerned with one or more of the following: (i) energy transfer, (ii) mass transfer, and (iii) information (or signal) transfer. Bond graphs are an abstract representation of a system that uses one set of symbols to represent all applicable types of systems in terms of energy transfer [Karnopp, 1990]. In particular, they focus on the exchange of power between components. Figure 2 shows a bond graph model of the hearing aid device.

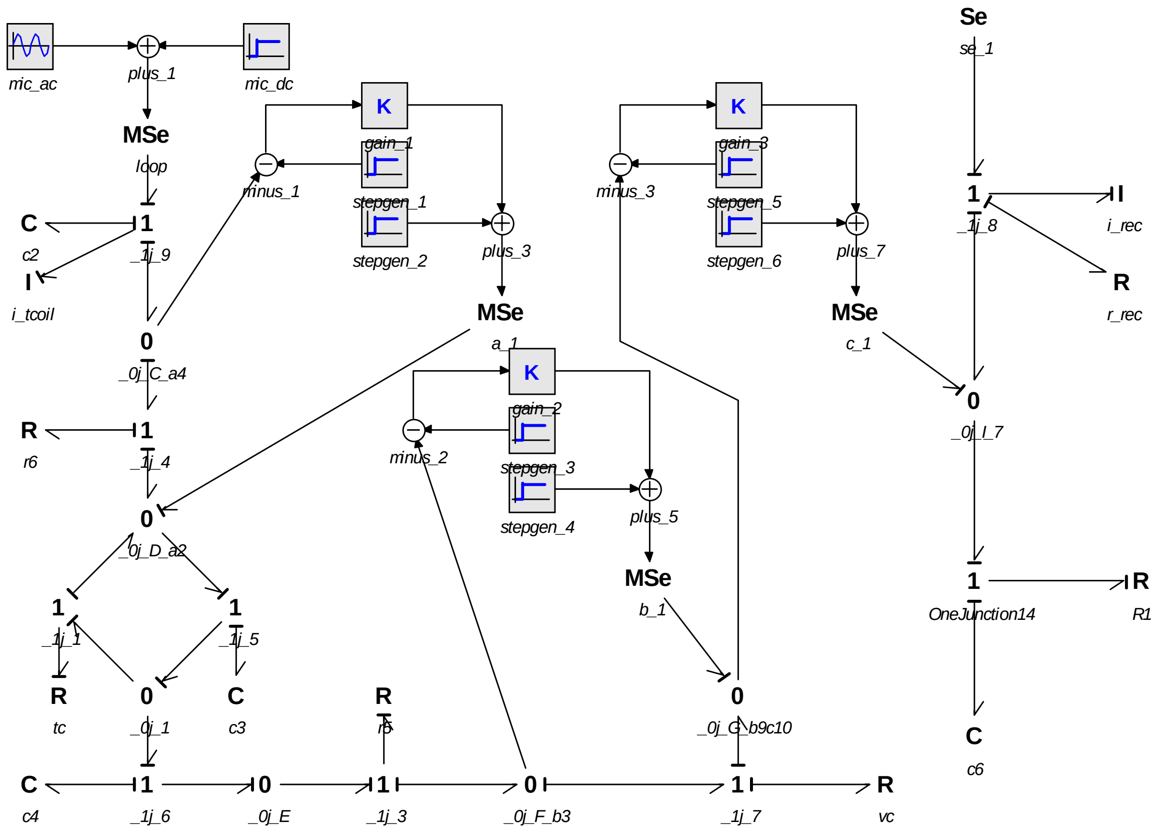

Fig. 2. Bond Graph model of a hearing aid device

From this model, a system of linearized differential equations can be generated that describe the system dynamics. These equations are of the form

$$\frac{dy}{dt} = f(y, \theta, t) \qquad (1)$$

where y and θ are vectors containing the system outputs and parameters respectively. The bond graph system above has 5 system outputs and 10 system parameters, such that

$$y = [e_2, e_3, e_4, f_{rec}, f_{tc}]^T \qquad (2)$$

and

$$\theta = [I_{tc}, R_6, I_{rec}, R_{rec}, C_2, C_4, R_5, R_{vc}, C_3, R_{tc}]^T \qquad (3)$$

where e and f represent efforts and flows respectively [Karnopp, 1990] and R, C, and I represent resistance, capacitance, and inductance of components in the hearing aid.

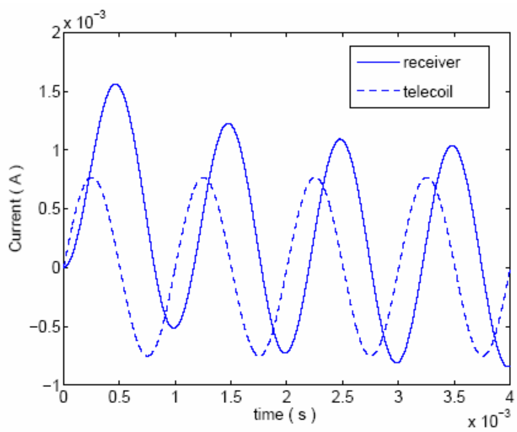

Fig. 3. Simulated dynamic behavior of the current flowing through the telecoil and receiver components of the hearing aid device

Based on the numerical solution to the set of differential equations 1, the direct method for sensitivity analysis [Saltelli et al., 2000] was used to compute the time dependent sensitivity matrix *S(t)* derived from equations 2 and 3 as

$$S(t)=\begin{bmatrix}
\frac{\partial e_2}{\partial I_{tc}} & \frac{\partial e_2}{\partial R_6} & \frac{\partial e_2}{\partial I_{rec}} & \frac{\partial e_2}{\partial R_{rec}} & \frac{\partial e_2}{\partial C_2} & \frac{\partial e_2}{\partial C_4} & \frac{\partial e_2}{\partial R_5} & \frac{\partial e_2}{\partial R_{vc}} & \frac{\partial e_2}{\partial C_3} & \frac{\partial e_2}{\partial R_{tc}} \\
\frac{\partial e_3}{\partial I_{tc}} & \frac{\partial e_3}{\partial R_6} & \frac{\partial e_3}{\partial I_{rec}} & \frac{\partial e_3}{\partial R_{rec}} & \frac{\partial e_3}{\partial C_2} & \frac{\partial e_3}{\partial C_4} & \frac{\partial e_3}{\partial R_5} & \frac{\partial e_3}{\partial R_{vc}} & \frac{\partial e_3}{\partial C_3} & \frac{\partial e_3}{\partial R_{tc}} \\
\frac{\partial e_4}{\partial I_{tc}} & \frac{\partial e_4}{\partial R_6} & \frac{\partial e_4}{\partial I_{rec}} & \frac{\partial e_4}{\partial R_{rec}} & \frac{\partial e_4}{\partial C_2} & \frac{\partial e_4}{\partial C_4} & \frac{\partial e_4}{\partial R_5} & \frac{\partial e_4}{\partial R_{vc}} & \frac{\partial e_4}{\partial C_3} & \frac{\partial e_4}{\partial R_{tc}} \\
\frac{\partial f_{rec}}{\partial I_{tc}} & \frac{\partial f_{rec}}{\partial R_6} & \frac{\partial f_{rec}}{\partial I_{rec}} & \frac{\partial f_{rec}}{\partial R_{rec}} & \frac{\partial f_{rec}}{\partial C_2} & \frac{\partial f_{rec}}{\partial C_4} & \frac{\partial f_{rec}}{\partial R_5} & \frac{\partial f_{rec}}{\partial R_{vc}} & \frac{\partial f_{rec}}{\partial C_3} & \frac{\partial f_{rec}}{\partial R_{tc}} \\
\frac{\partial f_{tc}}{\partial I_{tc}} & \frac{\partial f_{tc}}{\partial R_6} & \frac{\partial f_{tc}}{\partial I_{rec}} & \frac{\partial f_{tc}}{\partial R_{rec}} & \frac{\partial f_{tc}}{\partial C_2} & \frac{\partial f_{tc}}{\partial C_4} & \frac{\partial f_{tc}}{\partial R_5} & \frac{\partial f_{tc}}{\partial R_{vc}} & \frac{\partial f_{tc}}{\partial C_3} & \frac{\partial f_{tc}}{\partial R_{tc}}
\end{bmatrix} \qquad (4)$$

Of particular interest in the scope of the study presented here are the sensitivity matrix elements $\frac{\partial f_{rec}}{\partial I_{tc}}$ and $\frac{\partial f_{rec}}{\partial R_{tc}}$, namely the sensitivity of the current flow through the receiver f_{rec} to the inductance and resistance of the telecoil I_{tc} and R_{tc} respectively. The numerical solution to the time dependent behavior of f_{rec} and f_{tc} due to an system input signal of 6.2 kHz are shown in figure 3.

Preliminary results to the sensitivity analysis calculations of interest are shown in figure 4 which represent the time dependent behavior of the sensitivity terms $\frac{\partial f_{rec}}{\partial I_{tc}}$ and $\frac{\partial f_{rec}}{\partial R_{tc}}$. From these results it is evident that any changes in telecoil Inductance I_{tc} have a significant effect on the receiver current, given that the sinusoid in the plot on the left of figure 4 is of the same order as the system solution of figure 3.

These results highlight the capability of the computational and analytic methodologies that have been put in place to examine the sensitivity of micron system performance to changes, random or otherwise, in the values of micro-scale engineering system parameters.

The sensitivity methodologies presented here rely on a numerical solution to the governing system equations at each time step, such that a simulation of dynamic system performance is achieved in parallel to the sensitivity calculations.

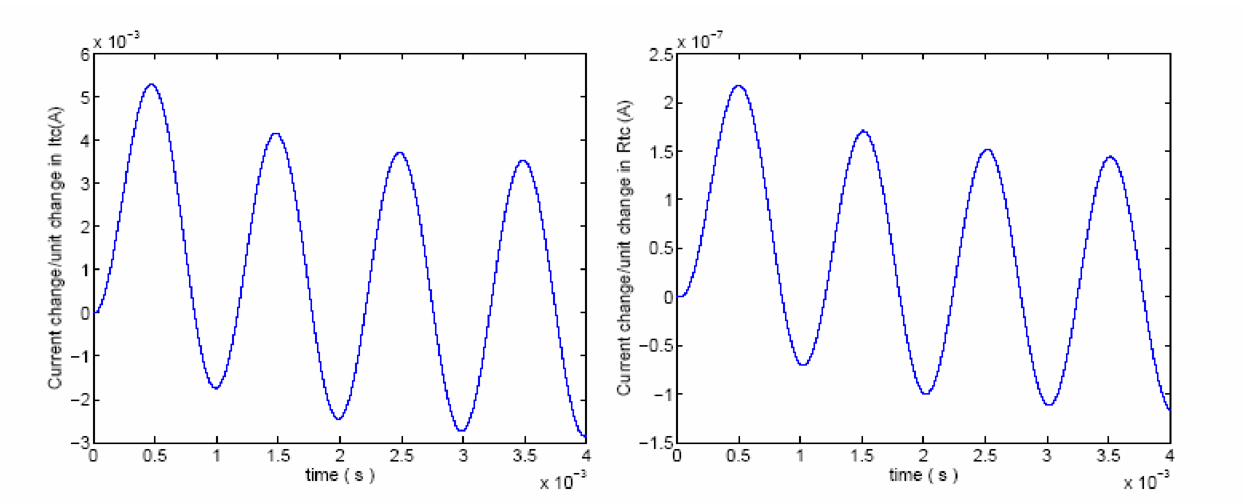

Fig. 4. Simulated dynamic sensitivity of the receiver current to a unit change in the inductance of the telecoil (left) and to a unit change in the resistance of the telecoil (right)

III. EXPERIMENTATION

Figure 5 shows the experimental rig used to measure the interaction between the telecoil and the receiver. A function generator produces a sine signal with a frequency of 1 kHz. The signal goes through an induction-loop with a diameter of 40 cm and generates a magnetic field inside the loop where the telecoil receives the signal. The signal received is amplified and sent to the receiver where the electrical signals are converted into sound waves. The power supply for the amplifier circuit supplies a constant 1.5 V DC Voltage (equivalent to that produced in a hearing aid device). A voltmeter (Multimeter) which has a resolution of 1/10mV is used to measure the voltage variation due to the interaction of the components.

The aim of experimentation is to measure the variation of the signal at the receiver due to the interaction between the telecoil and the receiver. Figure 6 shows the measurement for the voltage in the receiver versus the distance between telecoil and receiver at different operating frequency equal to 0.9 kHz, 1kHz, 1.1kHz, 1.65kHz, 2kHz, and 2.35kHz respectively. It reveals the interaction between telecoil and receiver in the hearing aid device due to the change of distance between the telecoil and receiver. It is clear that the voltage detected in the receiver varies with the distance between the telecoil and receiver. It shows that at close proximity below about 4 mm, the distance between telecoil and receiver has a great influence on the performance of the hearing aid device. The closer the distance between the telecoil and receive, the higher voltage that can be obtained from the receiver. Therefore, the interaction between these two devices at distances less than 4mm will be investigated further.

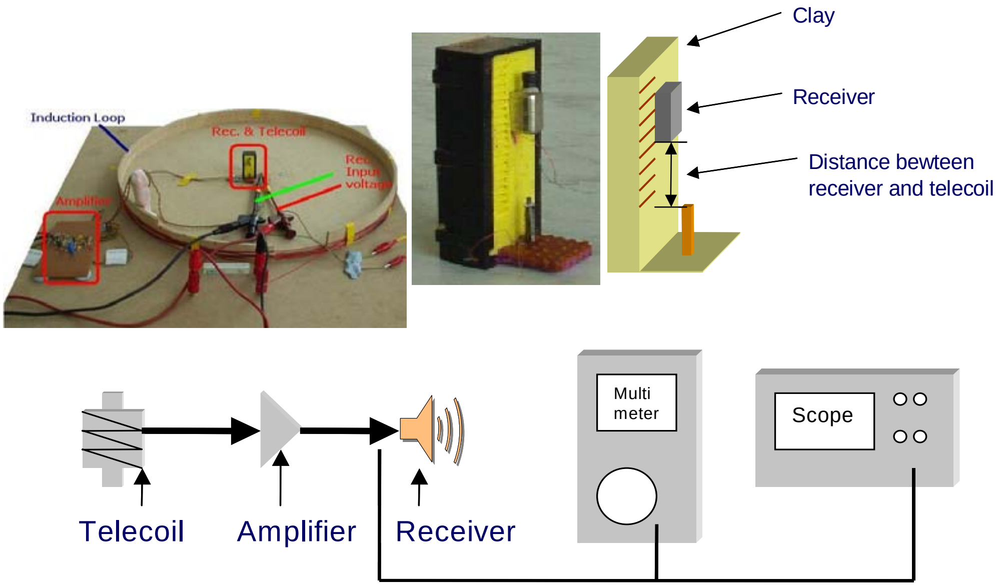

Fig. 5. Experimental Rig and Measuremental Principle

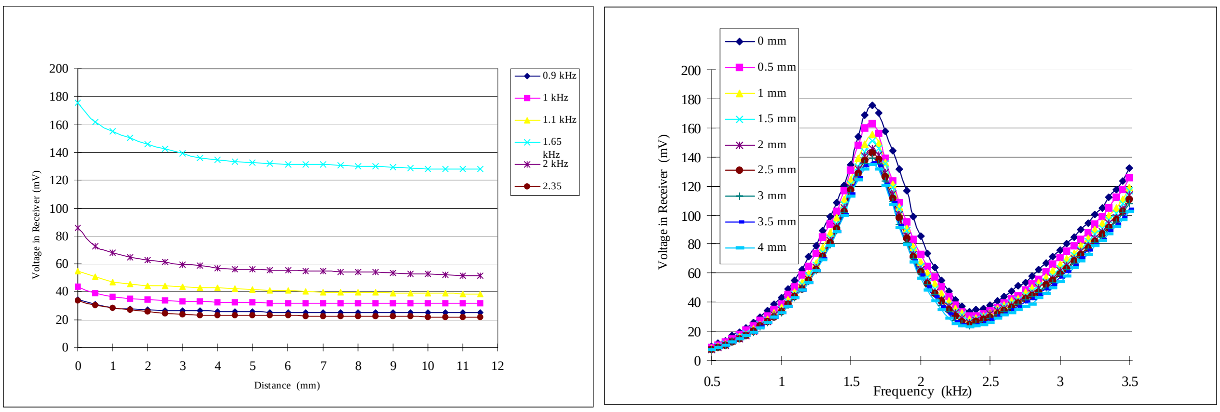

Fig. 6. Voltage vs. distance (left) and voltage vs. frequency (right) between telecoil and receiver

Figure 6 also shows the measurement results of the voltage measured across the receiver versus operating frequency at different distance from 0mm to 4mm between telecoil and receiver respectively. The overall curves of the measured voltage versus operating frequency response characteristics have a peak at 1.65kHz and a trough at 2.35kHz, which can be seen in the same way for different distance between telecoil and receiver from 0 to 4mm with a 0.5mm step. These correspond to the resonant frequency of 1.65 kHz and the anti-resonant frequency of 2.35 kHz respectively.

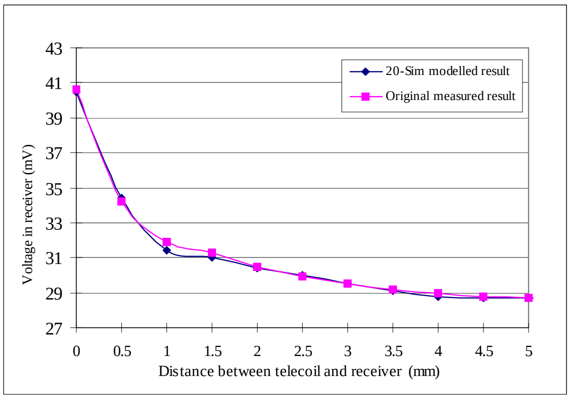

Fig. 7. Comparison of simulation results by 20-Sim and original data

Figure 7 is a comparison of 20-Sim modeled voltage and measured voltage versus the distance between telecoil and receiver. It is clear that there is close agreement between the two modelling approaches and that the 20-Sim model can be used to predict the tendency of the change in the voltage of the receiver against the distance between telecoil and receiver.

The results show that if the receiver is nearer to the telecoil, it is more sensitive to small changes in distance. Future research is planned to investigate this effect in more detail, in order to evaluate the performance of more compact hearing aid designs. In addition to the experimental investigations of the industrial case study discussed in detail here, a differential sensitivity analysis by means of statistical numerical solution to the ODE's governing the hearing aid operation promises to identify other components of the hearing circuit whose parametric or spatial variation may lead to improved system robustness.

IV. CONCLUSIONS

It has been shown that the combination of bond graph modeling and associated mathematical sensitivity analysis can be used as a platform for robust engineering design of micro-scale mechatronic systems in multiple energy domains.

By applying this methodology to an analysis of a BTE hearing aid device, the interaction between the telecoil and receiver of such a system has been revealed.

The variation of electrical voltage detected in the receiver with the driving frequency has been discussed. A comparison of 20-Sim modeled voltage and measured voltage versus the distance between telecoil and receiver shows that there is close agreement, and the average residual is very small. It is therefore reasonable to simulate the behaviour of the hearing aid device by use of bond graph modeling.

By extending the bond graph model to include geometric and other important design parameters, the analytic sensitivity analysis methodoligies outlined here can be used to identify aspects of a design that are important for design robustness. The methods presented can be generally applied, but are especially useful in a multi-energy domain, where an entire system can be modeled and analysed in a single simulation environment.

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