Competition among Universities and the Emergence of the Élite Institution*

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Abstract

We consider an environment where two education institutions compete by selecting the proportion of their funding devoted to teaching and research and the criteria for admission for their students, and where students choose whether and where to attend university. We study the relationship between the cost incurred by students for attending a university located away from their home town and the equilibrium configuration that emerges in the game played by the universities. Symmetric equilibria, where universities choose the same admission standard, only exist when the mobility cost is high; when the mobility cost is very low, there is no pure strategy equilibrium. For intermediate values of the mobility cost, only asymmetric equilibria may exist; the final section of the paper provides an example where asymmetric equilibria do indeed exist for a plausible and robust set of parameters.

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1 Introduction

There are surprisingly few theoretical studies devoted to the university system, despite its quantitative and qualitative importance, and researchers' direct interest in it. In this paper, we propose a theoretical model of competition between universities. A sound understanding of this topic is important both as a possible way to explain the considerably different manners in which the sector is organised in different countries, and as an aid to design policies aimed at improving the performance of the sector.

There are several basic features that set the university sector apart from other, better studied, industries. Firstly, the higher education market does not typically clear in the usual sense: notwithstanding the potential existence of a market price for university education, most systems allocate places to students by administrative rationing. Secondly, the performance of a university (measured along the dimension of the quality of the teaching provided) depends positively on the ability of its own students: universities use a customer-input technology (Rothschild and White 1995).[1] Thirdly, the profit maximising behaviour typically assumed for large commercial organisations,[2] as well as for some not-for-profit private institutions,[3] is not likely to be a good proxy for the objective function of individual universities. The model of this paper captures all three of these features: universities set an admission standard, and their performance depends positively on the ability of their students. With regard to their objective function, we choose a very general formulation, and can therefore cover a potentially large range of situations: we assume that the universities aim at maximising a measure of their prestige, which, in turn, we take to be positively affected by the quantity

[1]See Clotfelter (1999) and Winston (1995) for a detailed discussion of the specific characteristics of the higher education sector.

[2]Indeed much theoretical research shows how actions and decisions which would, superficially, appear to indicate that a commercial organisation is not maximising its profit, are in fact, once profit is properly defined and strategic interactions with another agents taken fully into account, part of a profit maximising plan (Tirole 1988).

[3]For example, Dranove and White (1994) offer theoretical arguments, based on the consequences of plausible assumptions on the objective functions of the relevant decision makers inside institutions, suggesting that not-for-profit private hospitals do in fact ultimately behave as if they maximised profit.

and quality of the students enrolled, and by the success of the university's research activities.

The interaction between students and universities is schematically modelled as follows: universities set admissions standards, and those students who qualify for admission choose whether to attend university or not; in addition, if a student achieves the admission standard at more than one university, she chooses which university to attend. To make this choice, students take into account the (current) travel and mobility costs (given by the monetary cost of travel and relocation, by the utility costs of being away from family and friends, and so on), and the (future) labour market income which can be obtained after attending a given university.

We study the game between two ex-ante identical universities. Our main contribution is the characterisation of the relationship between the level of the mobility costs, and the equilibrium configuration. Our results can be summarised as follows.

- If the mobility cost is high, then the equilibrium is symmetric: universities are ex-post, as well as ex-ante, identical. They set the same admission standard, admit the same number of students, and spend the same amount on research.
- At lower levels of the mobility cost, if an equilibrium exists, it must be asymmetric: one university becomes an élite institution: it sets a higher standard and enrols the best students, while the other sets lower admission standards.
- At lower still mobility cost, there is no pure strategy equilibrium.[4]

We believe that our model, though highly stylised, captures appropriately the role of mobility cost in shaping the relationship between institutions within the university sector. For example, the existence of students grants (or cheap loans) and abundant and convenient university accommodation seems likely to make the mobility cost in the UK lower than in the rest of

[4]Del Rey (2000) develops a model related to ours. She obtains *symmetric* equilibria. Her analysis therefore corresponds to the case of high mobility cost in our model.

Europe, enabling an institution to view the entire nation as a potential source of students. Our analysis suggests that the equilibrium configuration which emerges in this case is “hierarchical”, with one institution requiring higher standards than the other, and both institutions recruiting from the entire population: this tallies with the UK situation. In continental Europe, by contrast, the institutions set similar standards and, by and large, students attend their local university.[5] In the US, one may argue that the benefit of attending a leading institution overcomes the mobility cost only for brighter students (and/or that the latter have a lower mobility cost). Therefore, our model is consistent with the evidence of an increase in the stratification of the students population, with leading institutions recruiting an ever increasing number of the brightest students from the whole country, while less bright students attending their local college (see Cook and Frank, 1993).[6]

We present a general model in section 2: this details the behaviour of the universities (subsection 2.1) and of the students (subsection 2.2). In Section 3 we show that if a pure strategy equilibrium exists, then it must be asymmetric, for the mobility cost below a certain threshold. In Section 4 we provide an example with specific functional forms, which shows that such equilibria do in fact exist. Section 5 concludes.

[5] This is a point clearly deserving careful empirical investigation: some preliminary comparison between Italy and England would however support it, suggesting that around 8% of English students live at home while studying for a university degree, when the corresponding proportion is about 67% for Italy; dividing each nation in 9 regions, in England, 18% of students attend a university in the same region, whereas in Italy, at least 80% do (Naylor and Smith, 2001).

[6] Riesman (1998) describes the evolution of the American higher education market and raises the concern that competition among institutions for students will lead to lower academic standards and marginal differentiation among universities. Hoxby (1997) studies the effects of the increased competition experienced by the American higher education from 1940 to the present on college prices and quality. She finds empirical evidence that greater competition led to an increase in the average quality and tuition fees as well as in product differentiation between universities. Recently Epple et al (2001) find empirical evidence of a hierarchical stratification of US universities.

2 The model

2.1 The universities.

We consider an environment where a large population of potential students is evenly distributed in two towns, A and B; in both towns the number of students is normalised to 1. In each town there is a university which provides education services to the students it enrols.

We assume that universities are run by a bureaucracy, or management, who are interested in the *prestige* of their institution. Prestige, of course, is a rather vague term, and we let it be given here by a function with three arguments: (i) the number of students, n, (ii) the average ability of the student body, Θ, and (iii) the expenditure on research, R. Formally, we write the objective function of an institution as

$$W(n, R, \Theta). \tag{1}$$

To justify (1), we begin by noting that, clearly, other things equal, a large institution is more prestigious than a smaller one, because, for example, it is more visible in the local community, it features more often in the national press, and so on. Therefore, other things equal, an increase in the number of students improves the value of the objective function of a university. If the quantity of students affects the prestige of a university, so does their quality: to the extent that better students achieve more prestigious social positions, this also enhances the prestige of that institutions.[7] It is also the case that brighter students earn more in the labour market, and this improves the university's long-term potential for financial donations and bequests: if, on average, alumni donate or bequeath a proportion of their income to their alma mater, then an increase in the total earnings of an institution's graduates generates a corresponding increase in that institution's potential for

[7] That the success of an institutions' alumni matters is exemplified in the way university magazines and web pages boast of the success and achievement of their graduates. Another example is given by a typical career progression for the heads of UK institutions: several move from a smaller university, to a larger one, and then to head an Oxbridge college, very small but extremely prestigious institutions. This suggest that size is not all that matters.

donations from alumni. Finally, in addition to teaching, universities do research; the prestige of an institution is clearly affected by the quality of its research. To the extent that expenditure on research increases the quality of research, for example, because it allows institutions to recruit and to retain better researchers, to purchase more expensive equipment, and so on, then, ceteris paribus, an institution prefers to spend more on research. Note also that R could be re-interpreted as expenditure on improving the quality of education: after paying for basic tuition, the university can use the what is left to improve the quality of research (which increases prestige) or the quality of education (which increases the earnings of the alumni). In view of this discussion, the first partial derivatives of W in (1) are all strictly positive. With regard to the second derivative, we assume that $W_{nn}(\cdot), W_{RR}(\cdot), W_{\Theta\Theta}(\cdot) < 0$, where subscripts denotes partial derivatives, and that the second cross derivatives are sufficiently small so that the relevant second order conditions are satisfied. This implies that $W(\cdot)$ is approximately separable.

We assume that each university is assigned the same fixed budget, $b > 0$, by the government agency in charge of the higher education system: universities are unable to affect their revenues, and, in particular, they are not free to choose what students are charged in fees. This is a reasonable approximation of the current practice in most European countries; moreover, from a conceptual point of view, it allows us to analyse the decisions concerning expenditure on research and academic standards separately from decisions about raising revenues. In this setting, each university chooses the required standard necessary to be accepted as a student.[8] We denote by x_i, $i = A, B$, this standard and let it vary in the subset of the real line: $x_i \in X \subseteq \mathbb{R}$, $i = A, B$. This implies that only students who reach at least standard x_i are accepted at institution i. On the financial side, we assume that teaching $n > 0$ students carries a cost of $c(n)$, naturally with $c'(n) > 0$; it is also convenient to assume $c''(n) \geqslant 0$, at least in the relevant range, and that $\lim_{n \to 1} c'(n) = +\infty$.

[8]We do not model explicitly the way in which the admission process operates. We can, however think of university requiring students to have a minimum grade at the school they attend before entering university, or setting a specialist pre-entry exam.

2.2 The students

In addition to their location, students, the potential "customers" of the two institutions, are differentiated according to their ability. This is denoted by $\theta \in [0,1]$ (an innocuous normalisation). In each town, the students' distribution by ability is $F(\theta)$, with $F(0) = 0$, $F(1) = 1$, and density $f(\theta) = F'(\theta)$. We also assume that $f'(x)$ is bounded from above by a sufficiently large (positive) number, again to ensure that the second order conditions are satisfied.

We assume that there is a functional relationship linking the admission standard set by a university, x, and the lowest ability student that can be accepted as a student, $\underline{\theta}$. Let $\psi(x)$ be this function, so that $\underline{\theta} = \psi(x)$. It is natural to assume that there exists $\bar{x}$ such that $\psi(x) = 1$ for $x \geqslant \bar{x}$: a university can set an admission test so tough that nobody can pass it. With this assumption, it is also an innocuous normalisation to measure the admission standard as the lowest ability which is necessary to match that standard: $\psi(x) \equiv x$. It is of course possible to add a stochastic component to this relationship allowing students whose ability is below (above) x to be admitted (rejected) with some probability; this would not alter the nature of the interaction between universities.

If a student attends a university located in the town where she does not live, she incurs a mobility cost. This is analogous to, but somewhat more general than, the mobility cost in the location model in the horizontal differentiation literature, which measures the petrol cost of going to a shop far away and the cost of the time spent travelling. In our model, to these costs there should be added the inconvenience of being away from one's home (and therefore missing parents, friends and so on), the additional cost incurred in renting a room, and so on.

If a student attends university, she receives a benefit summarised in a payoff function which depends on her own ability, θ, the admission threshold of the university she attended as a student, x, and the mobility cost T:

$$U(x,\theta) - T, \tag{2}$$

where $U_x(x,\theta), U_\theta(x,\theta) > 0$, and $T \in \{0,t\}$, with $T = 0$ if the student attends university in her home town, $T = t > 0$ otherwise.[9]

The function (2) is the reduced form of some underlying process. A way, by no means unique, to obtain (2) is the following. Let the realisation of labour market earnings, w, be distributed in $\mathbb{R}_+$ according to a function $G(w|\cdot)$, so that $G(w|\cdot)$ is the probability that the student's (present discounted value of future) labour market income is below w. Suppose that the shape of $G(w|\cdot)$ depends on the two parameters, θ and x, and naturally, that $\theta_1 > \theta_2$ implies that $G(w|\theta_1, x)$ first order stochastically dominates $G(w|\theta_2, x)$ for every x, and, conversely, that $x_1 > x_2$ implies that $G(w|\theta, x_1)$ first order stochastically dominates $G(w|\theta, x_2)$ for every θ. In words, given the quality of the university she attends a more able student is more likely to receive a higher wage than a less able student, and, a given student is more likely to receive a higher wage if she attends a university with a higher standard. Suppose also that the probability of a student obtaining a degree once admitted be an increasing function of her ability $\zeta(\theta)$. Then, if a student maximises her expected utility, and has a von-Neumann-Morgerstern utility function with argument the present discounted value of future labour market $u(w)$, the first term in (2) would be given by

$$U(x, \theta) = \zeta(\theta) \int\limits_{w \in \mathbb{R}_+} u(w)\, dG(w|\theta, x) + (1 - \zeta(\theta)) \int\limits_{w \in \mathbb{R}_+} u(w)\, dN(w|\theta, x).$$

In the above expression $N(w|\theta, x)$ is the distributions of the future earnings of a student whose ability is $\theta \in \mathbb{R}$ who attends an institution where the admission threshold is x, but is not awarded a degree. It is immediate to show that, if obtaining a degree increases one's earnings, that is, if $G(w|\theta, x)$ first order stochastically dominates $N(w|\theta, x)$, as seems natural, then the assumed signs for the partial derivatives, $U_x(x, \theta), U_\theta(x, \theta) > 0$, obtain.[10]

[9] Del Rey's model (2000) differs from ours in that a student's demand for education at each of the two universities is independent of her own ability: students only take into account mobility costs and the quality of education. Consequently the payoff of a university is independent of the quality of the students it attracts.

[10] In this example, we could allow for the probability of a student obtaining a degree $\zeta(\theta)$, to depend also on the admission threshold level x, which could be due to the fact that a tougher admission test makes a student of a given ability "closer" to the lowest ability of the students admitted.

That ability should influence positively future wages is tautological. It is also reasonable to assume $U_x(\cdot) > 0$: it may be that a stricter admission test at an institution reduces the number of graduates from that institution, and therefore increase their rent in the labour market, or that it improves the reputation of the graduates of the institution. It may also be the case that a higher average ability of the students makes university attendance more productive, for example, via a peer group effect or because it allows more advanced teaching.

Assumption 1 *$U_{x\theta}(x, \theta) > 0$.*

The benefit of an increase in the admission threshold is higher for brighter students. To the extent that x is positively correlated with the quality of teaching it implies that brighter students benefit more from higher quality teaching staff (it may also capture the idea that brighter students, ceteris paribus, need to exert less effort to be successful in their studies). Consequently, a student's *net* benefits from more restrictive admission requirements and/or tougher exams increase with her ability.

We assume that a student who does not attend university has a reservation utility normalised to $U(0, 0)$. Therefore, a student living in town i would surely attend the university in town i if she is admitted; if she is not admitted, she would travel to town j if and only if $U(x_j, \theta) \geqslant t$.

While the following is an almost immediate consequence of Assumption 1, it is worth stating formally, since it facilitates the analysis of the rest of the paper considerably.

Proposition 1 *Let university j set standard x_j, $j = A, B$. Let $x_A > x_B$.*

(a) Let student of ability $\tilde{\theta}_k$ living in town k attend university A. Then all students of ability $\theta \geqslant \tilde{\theta}_k$ living in town k also attend university A.

(b) Let student of ability $\overline{\theta}_k$ living in town k attend university B. Then all students of ability $\theta \geqslant \overline{\theta}_k$ living in town k attend either university A or university B, and all students of ability $\theta \leqslant \overline{\theta}_k$ living in town k either attend university B or do not attend university.

(c) Let the student of ability $\hat{\theta}_k$ living in town k not attend university. Then all students of ability $\theta \leqslant \hat{\theta}_k$ living in town k do not attend university either.

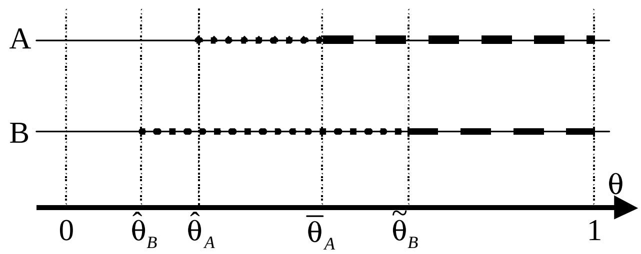

Figure 1: Students' attendance to universities

This follows from the fact that, given the mobility costs, a student of higher ability gains more from attendance to a university with a stricter admission test. An analogue of this is established in Epple and Romano (1998). The proposition implies a straightforward stratification by ability: within each town, the ablest student go to the best university. Of course it may happen that it is not the case that in a given town there are students attending both universities.

Figure 1 illustrates the proposition and shows the marginal values of $\hat{\theta}_k, \overline{\theta}_k$ and $\widetilde{\theta}_k$ for which statements (a), (b) and (c) in Proposition 1 hold. The horizontal axis measures students ability. Students whose ability and location is along the dashed (dotted) line go to university A (B). Note that the lowest $\tilde{\theta}_B$ could be as high as 1, in which case no student from town B goes to university A, and that the lowest $\hat{\theta}_A$ could be as high as the lowest $\bar{\theta}_A$, in which case no student from town A goes to university B.

3 Equilibria

The assumptions of the previous two subsections allow us to simplify considerably the interaction between universities and between universities and students. We can set up a conceptually simple two-person normal form game with universities A and B as the players and with their strategy space given by the admission standards, $x_A \in X$, $x_B \in X$. The payoff functions of the two universities are given by:

$$\pi_j\left(x_A, x_B\right) = W\left(n_j\left(x_A, x_B\right), b - c\left(n_j\left(x_A, x_B\right)\right), \Theta_j\left(x_A, x_B\right)\right), \qquad j = A, B,$$

where $n_j(x_A, x_B)$ and $\Theta_j(x_A, x_B)$ are the number of students admitted to university j, and the average quality of the students at university j ($j = A, B$), respectively, given the admission standards x_A and x_B. We do not determine the exact shape of the functions n_j and Θ_j. It is, however, straightforward to establish that $\partial\Theta_j/\partial x_j > 0$: an increase in a university's admission standards increases the average ability of that university's students. This is obviously plausible. Finally, notice that we have subsituted for $R = b - c(n_j(x_A, x_B))$, where $b > 0$ is the university budget.

A convenient benchmark case is obtained when universities are monopolies. We can think of two cases: a university can be a monopoly in the whole sector, for example because it is the only university allowed, or it can be a monopolist in its own town, for example because the students do not travel.In the first case, if $t = 0$, the number of students it admits if it sets a standard x is $2(1 - F(x))$, in the second it is $(1 - F(x))$. The first order conditions for the two cases are

$$2(W_n(\cdot) - W_R(\cdot)c'(\cdot))f(x) = W_\Theta(\cdot)\Theta_x(\cdot), \quad (4)$$

$$(W_n(\cdot) - W_R(\cdot)c'(\cdot))f(x) = W_\Theta(\cdot)\Theta_x(\cdot). \quad (5)$$

The definition of the function Θ is analogous to that given above for the case where two universities compete. We may denote by x_M and x_m the solution to the above expressions; and note that the assumption that the cross derivatives are sufficiently small ensures that the relevant second order conditions identify x_M and x_m as the unique maxima for the problems.

In general the relationship between x_M and x_m is ambiguous. Consider the optimum choice of x when the distribution of students is given by $\alpha F(\theta)$ (x_M and x_m are the optima for $\alpha = 2$ and $\alpha = 1$, respectively). Total differentiation of the first order conditions (leaving out the cross derivatives) gives:

$$\frac{dx}{d\alpha} = \frac{[W_n(\cdot) - W_R(\cdot)c'(\cdot)]f(\theta) + \left[W_{nn}(\cdot) - W_{RR}(\cdot)c'(\cdot) - W_R(\cdot)c''(\cdot)\right]\alpha f(\theta)(1 - F(\theta))}{d^2W/dx^2}$$

this would be negative if W and $c(\cdot)$ were linear ($W_{nn}(\cdot) = W_{RR}(\cdot) = c''(\cdot) = 0$). Figure 2 depicts the indifference map in the (x, n) plane (using the constraint given by research): the budget set is given by the curves $1 - F(x)$

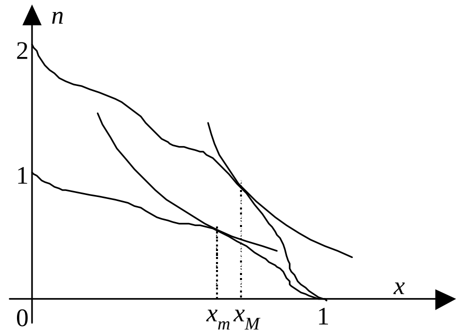

Figure 2: The standard chosen by a monopolist

and $2(1 - F(x))$ for the two cases. In the picture $x_M > x_m$, although it should be obvious that the opposite relationship could well hold. Intuitively, when the population increases the same reduction in the admission threshold is compensated for by a larger increase in the number of students. If $W_n(\cdot)$, $W_R(\cdot)$ and $c'(\cdot)$ are approximately constant, then the cost reduction of this increase in the number of students is approximately constant, but the benefit increases as the population increases. Vice versa, when W is concave, the benefit of an increase in students number is lower, and the cost in reduced research expenditure is higher, for a large university.

We can now begin the analysis of competition between universities. Our first result may appear surprising.

Proposition 2 *A symmetric equilibrium in pure strategies exists if and only if* $t \geqslant U(x_m, x_m)$.

Proof. At a symmetric equilibrium in pure strategies, it must be the case that $x_A = x_B = x_m$: any value of common threshold $x_A = x_B$ different from x_m necessarily violates the first order condition (5). Notice moreover, that, when t is high enough, $x_A = x_B = x_m$ does indeed identify a Nash equilibrium, in view of the definition of x_m and of the assumptions we made on the second derivatives of the function W.

Now suppose, by contradiction, that there is in fact a symmetric equilibrium with $t < U(x_m, x_m)$. We now show that one university, say B, has an incentive to deviate from $\{x_m, x_m\}$. Let δ_L be such that

$$U(x_m - \delta_L, 1) = U(x_m, 1) - t$$

That is, if a university, say university A, reduces its standard by δ_L, it makes the brightest student living in town A indifferent between attending university A and travelling to B. Clearly $\delta_L > 0$, and by Assumption 1, $U(x_m - \delta_L, \theta) > U(x_m, \theta) - t$ for every $\theta \in [x_m, 1)$. Now let university B choose $x_m - \varepsilon$, for some $\varepsilon \in (0, \delta_L)$. Its change in payoff is approximated by:

$$-\varepsilon \times \left(\left(W_n(\cdot) - W_R(\cdot)\, c'(\cdot)\right) \left.\frac{\partial n_B(x_m, x_m)}{\partial x_B}\right|_{x_B < x_m} + W_\Theta(\cdot)\, \Theta_x(\cdot) \right) \quad (7)$$

where

$$\left.\frac{\partial n_B(x_m, x_m)}{\partial x_B}\right|_{x_B < x_m} = \lim_{h \to 0^-} \frac{n_B(x_m, x_m + h) - n_B(x_m, x_m)}{h}$$

is the left partial derivative of $n_B(x_m, x_m)$ Note that, for $x_B < x_m$, $n_B(x_m, x_B) = 1 - F(x_m) + 2(F(x_m) - F(x_B))$ and, therefore

$$\left.\frac{\partial n_B(x_m, x_m)}{\partial x_B}\right|_{x_B < x_m} = \lim_{h \to 0^-} \frac{2(F(x_m) - F(x_m + h))}{h} = -2f(x_m)$$

and (7) becomes:

$$-\varepsilon \times \left(-2\left(W_n(\cdot) - W_R(\cdot)\, c'(\cdot)\right) f(x_m) + W_\Theta(\cdot)\, \Theta_x(\cdot)\right)$$

using (5) this is:

$$\varepsilon W_\Theta(\cdot)\, \Theta_x(\cdot) > 0$$

Therefore a small reduction from x_m increases university B's payoff; this establishes the Proposition.

On the other hand, if $t \geqslant U(x_m, x_m)$, then this deviation is not possible, because, if a student of ability below x_m travelled to the other town to attend university, she would get a negative utility. ■

The argument underlying the proof of the Proposition may be described informally as follows. If $x_A = x_B$ and x_A, say, is increased marginally, then the objective function for university A is the same as when there is local monopoly since no one would travel to town A. This implies that if $x_A = x_B < x_m$ we are not at an equilibrium. Moreover, if $x_A = x_B$ and again x_A is decreased marginally, then either there is no travel to town A ($t \geqslant U(x_A, x_B)$), or there is some travel to town A by those who now have the opportunity to go to a university ($t < U(x_A, x_B)$). In the former case, if initially $x_A = x_B < x_m$, then one can check that university A's payoff rises more rapidly as x_A is decreased marginally than under local monopoly. Hence, $x_A = x_B = x_m$ is necessary for a symmetric equilibrium.[11] However, a symmetric equilibrium does not exists for low mobility costs. This is because of the asymmetric effect of a small reduction and a small increase in a university's choice of x. The reasoning is as follows. When $x_A = x_B$, all students located in town $k = A, B$ with ability $\theta \geqslant x_A$ go to the university in their home town; while all students with ability $\theta < x_A$ do not attend university. Local optimality implies that the beneficial effects of an increase in x by $\varepsilon > 0$ (given by the improvement in the quality threshold) are exactly compensated by its negative effect of the reduction in size (dampened by the resources freed for research). However, when the admission threshold is lowered by ε, it is possible to obtain an increase in the number of students whose positive effects have the same absolute value as an increase by 2ε.

Mixed strategy equilibria, in the present situation, lack appeal; we therefore prefer to investigate whether asymmetric pure strategy equilibria exist. For the sake of definiteness, we consider the case $x_A > x_B$ (so that A and B are mnemonics for 'Alto' (high) and 'Basso' (low)). Clearly, if $\{x_A, x_B\}$ is an equilibrium, then the mirror strategy pair $\{x_B, x_A\}$ is too.

An important concept in the rest of the paper is the function $\widetilde{\theta}\,(x_A, x_B, t)$, which is the ability level such that the student with that ability who lives in town B (the low admission university) is indifferent between going to her local university and travelling to town A, where she incurs travel cost t, but can expect higher future labour market income. $\widetilde{\theta}\,(x_A, x_B, t)$ is therefore the solution in θ of

$$U\,(x_A, \theta) - t = U\,(x_B, \theta)\,, \tag{8}$$

if there is a solution to (8) in $(x_A, 1)$. If there is no such solution, then

[11] We thank an anonymous referee for suggesting this argument.

$\widetilde{\theta}(x_A, x_B, t)$ is the appropriate extreme of the interval $(x_A, 1)$: $\widetilde{\theta}(x_A, x_B, t) = 1$ if $t > U(x_A, 1) - U(x_B, 1)$, and $\widetilde{\theta}(x_A, x_B, t) = x_A$ if $t < U(x_A, x_A) - U(x_B, x_A)$. Note that, by Assumption 1, there is at most one value of θ which satisfies (8). Moreover, denote by $\widehat{\theta}(x_B, t)$ the ability level such that the student with that ability who lives in town A is indifferent between travelling to town B and not attending the university. Then, $\widehat{\theta}(x_B, t)$ is the solution of

$$U(x_B, \theta) - t = U(0, 0) \tag{9}$$

if there exists $\theta \in [x_B, x_A]$ satisfying the above. Otherwise, if the solution to (9) is such that $\theta < x_B$ then $\widehat{\theta}(x_B, t) = x_B$, and if the solution to (9) is such that $\theta > x_A$, then the $\theta(x_B, t) = x_A$, and no student from university A attends the university in town B.

An immediate consequence of Proposition 1 and of the definitions of $\widetilde{\theta}(x_A, x_B, t)$ and $\widehat{\theta}(x_B, t)$ is the following.

Corollary 1 *At any equilibrium where $x_A > x_B$: students from town A attend university A if $\theta \in [x_A, 1]$ and university B if $\theta \in \left[\widehat{\theta}(x_B, t), x_A\right)$. Students from town B attend university A if $\theta \in \left[\widetilde{\theta}(x_A, x_B, t), 1\right]$ and university B if and only if $\theta \in \left[x_B, \widetilde{\theta}(x_A, x_B, t)\right)$.*

The next Proposition is the main result of this section. It characterises how the equilibrium configuration changes as the mobility cost changes. This is important: casual empiricism suggests that the parameter t varies considerably from country to country; moreover, education policies targeted to the university sector are likely to affect this parameter, perhaps even unintentionally.

Proposition 3 *There exist $t_1 < U(x_m, x_m)$, such that:*

For $t \in [0, t_1)$ there is no pure strategy equilibrium.

For $t \in [t_1, U(x_m, x_m)]$ there are, at most, asymmetric pure strategy equilibria.

For $t > U(x_m, x_m)$ the pure strategy equilibrium is symmetric, with $x_A = x_B = x_m$.

Proof. The third statement has been proved in Proposition 2, and the second follows trivially from it. Consider the first statement. We show that there is no pure strategy equilibrium for $t = 0$, using an argument which can be extended by continuity to a (sufficiently) small $t > 0$. By contradiction, let $\{x_A, x_B\}$ with $x_A > x_B$ be a Nash equilibrium when $t = 0$. In this case all students with $\theta \in [x_B, x_A)$ attend university in town B, and all students with $\theta \geqslant x_A$ attend university in town A. Note that the payoff of university B must be at least as big as university A's:

$$\pi_B(x_A, x_B) \geqslant \pi_A(x_A, x_B). \tag{10}$$

The reason is that, otherwise, university B could deviate to $x_A + \varepsilon$, take (almost) all the students from university A, and therefore improve its payoff. Notice also that $\hat{\theta}(x_B, t) = x_B < 1$, because of $t = 0$. Now let $\tilde{x}$ be given by: $1 - F(\tilde{x}) = F(x_A) - F(x_B)$: $\tilde{x}$ is such that there are, overall, $2(F(x_A) - F(x_B))$ students of ability $\tilde{x}$ or higher. Clearly if university A chose $\tilde{x}$, it would have the same number of students as university B has at the candidate equilibrium, and therefore the same research expenditure as university B at this candidate equilibrium. However, since $\tilde{x} > x_B$ it would have a bigger payoff:

$$\pi_A(\tilde{x}, x_B) > \pi_B(x_A, x_B) \tag{11}$$

(10) and (11) together imply $\pi_A(\tilde{x}, x_B) > \pi_A(x_A, x_B)$ but this in turn implies that x_A cannot be university A's maximizing choice. This establishes the first statement. ∎

4 An example

In this section we specify the functional relationships of the model in order to show that asymmetric equilibria do in fact exist for plausible and robust functional forms and parameter sets. We modify the model as may be necessary to achieve explicit and not too cumbersome solutions (and therefore some of the results obtained in the general set-up considered above may not hold). Specifically, we assume the following

$$U(x,\theta) = wx + \theta \tag{12}$$

$$W(n,R,\Theta) = wxn + R \tag{13}$$

$$c(n) = cn^2 \tag{14}$$

$$F(\theta) = \theta \tag{15}$$

Note that (12) implies $U_{x\theta} = 0$ against Assumption 1. This simplifies the analysis, without altering the qualitative features of the solution. (14) and (13) are specific functions which allow the determination of an explicit solution. According to (13), a university's payoff is an increasing function of the number of graduates, n, and of its own admission standard, as well as the amount spent on research. This simplify the analysis, and is justified on the ground that an increase in the admission standard increases the average ability of its graduates. Finally, (15) says that the distribution of abilities is uniform.

Since our aim is to show that asymmetric equilibria exist, we further simplify the model by assuming that the utility from obtaining a university degree is always greater than any mobility costs the student might have to incur in order to attend the university. That is, for every $x, \theta \in (0,1]$

$$U(x,\theta) - t \geqslant U(0,0)$$

Hence U has a discontinuity at $(0,0)$, with a jump greater than t. Given the normalisation $\psi(x) = x$, this implies that $\widehat{\theta}_B = \widehat{\theta}_A = x_B$: students from both towns with ability at least x_B will be admitted to university and will be willing to attend.[12]

Towards the analysis of the equilibrium, note that students in town A attend university A, if their ability is at least x_A, and university B, if their ability is less than x_A, but at least x_B. Students in town B, on the other hand, attend university B, if their ability is less than x_A, but at least x_B;

[12]This assumption is made for simplicity and implies the non-existence of symmetric equilibria. It is therefore a special case of the result in Proposition 2, which holds because of $U(x_m, x_m) - t > U(0,0)$ for all t.

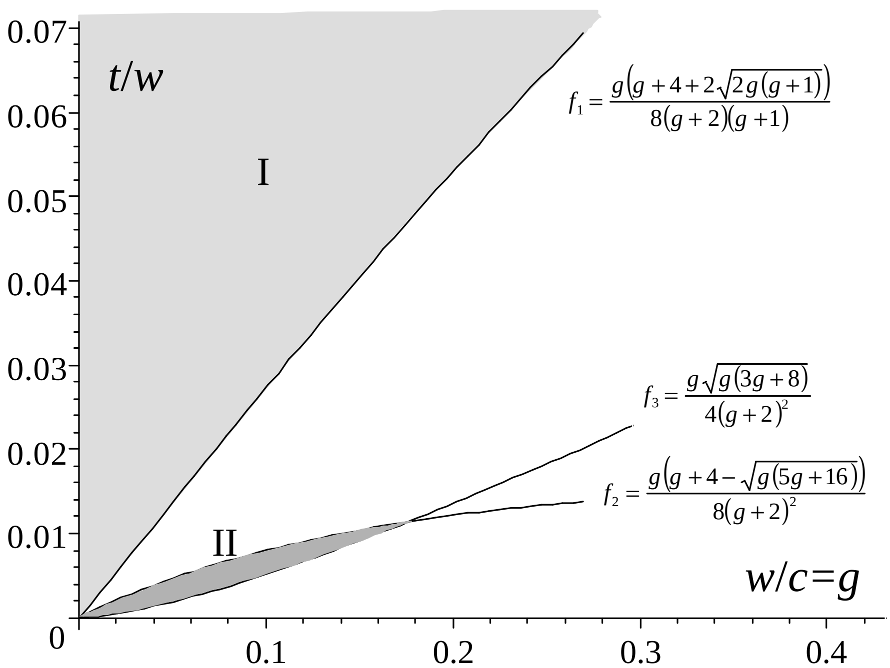

Figure 3: Equilibria in the parameter space

and if their ability is x_A or above, attend university A if $x_A - x_B \leqslant t/w$, university B in the opposite case.

Note that the parameter space (t, w, c) can be simplified by defining $g = \frac{w}{c}$ so that the equilibrium can be fully characterised in a Cartesian diagram $\left(\frac{t}{w}, \frac{w}{c}\right)$. This we do in figure 3. In the diagram, the lines are given by: $f_1 = \frac{g\left(g+4+2\sqrt{2g(g+1)}\right)}{8(g+2)(g+1)}$, $f_2 = \frac{g\left(g+4-\sqrt{g(5g+16)}\right)}{8(g+2)^2}$, $f_3 = \frac{g\sqrt{g(3g+8)}}{4(g+2)^2}$.

These lines identify a number of regions and the following proposition describes the nature of the equilibrium in each of them.

Proposition 4 *In the white areas in the figure, there is no pure strategy equilibrium. In the remaining areas there exists an asymmetric equilibrium.*

In the areas labelled I, university A and B choose respectively

$$x_A = \frac{g+2}{2(g+1)} = x_m \tag{16}$$

$$x_B = \frac{(4+3g)(g+4)}{8(g+1)(g+2)} \tag{17}$$

In the areas labelled II

$$x_A = \frac{g+4}{2(g+2)} = x_M \tag{18}$$

$$x_B = \frac{(g+4)^2}{4(g+2)^2} \tag{19}$$

The proof of this result is analytically very cumbersome, and is relegated to an appendix available on request from the authors or from `http://www-users.york.ac.uk/~gd4/elite.htm`. Note also that area I extends beyond the boundary of the drawing.

However, a verbose description of the two type of equilibria can be given relatively straightforwardly: when the parameter combination is represented by a point in area labelled I, students' attendance to universities is described in Figure 4: students who can be admitted to university A go there only if they live in town A; all other students, of ability x_B or above go to university B. The admission standard set by university B is sufficiently close (relative to the mobility cost) to make it impossible for university A to capture all the high ability students: those living in town B go to town B. So university A chooses the optimal location given distribution $F(x)$, x_m, determined in (5). Consider now university B: could it profitably deviate from standard x_B? Suppose that university B lowers its standard so as to lose its high quality students and gain more low quality students from both towns. Since the higher the mobility costs the lower the level of x_B that induces the high quality students in town B to move to town A, it is clear that (given g) for high mobility costs, this deviation is too expensive in terms of lost prestige. A different type of deviation is a sufficiently large increase in admission standard, so as to leapfrog university A. This, again, will not be profitable if, given g, the mobility cost is high, since the higher the mobility cost the higher the level of x_B that is necessary to attract the high quality students

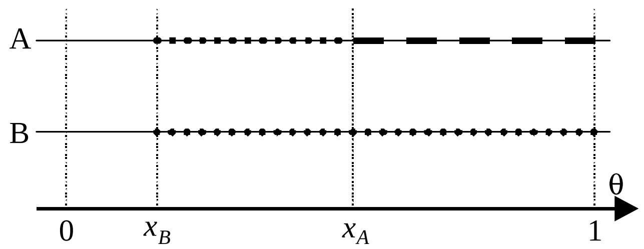

Figure 4: Type I Equilibrium

and the lower the number of students. If a parameter combination is an equilibrium of type I, then a sufficient reduction in the mobility cost will destroy it by making it profitable for university B to deviate in one of the two ways described above.

Consider now parameter combinations represented by points in the area labelled II. Here, all the students who can attend the high quality university go to university A. This situation is described in figure 5, where, as before, the dashed (dotted) line denotes students whose location and ability is such that they attend university A (B). To understand why this constitutes an equilibrium, note first of all that, given that university B has lower quality than university A, the latter will choose the admission standard that an "undisturbed" monopolist would choose, x_M. Consider university B next. It is at a local optimum: given that it cannot attract students of ability x_A or above, x_B is the preferred standard. However, it could choose an admission standard sufficiently close to x_A so that all the students living in town B prefer to go to university B. This has the benefit of attracting $(1 - F(x_A))$ students (the high ability students who attend university A in the candidate equilibrium), but at the cost of losing low ability students from both towns, who are no longer capable of passing the admission test. When the mobility cost is low, in order to attract the high ability students, university B needs to choose an admission standard quite close to university A, implying a high cost of the deviation: thus an increase in the mobility cost would destroy this equilibrium by allowing the university B to position its standard below x_A but sufficiently close to attract the high ability students living in town A. Instead of attracting only the high ability students in its own town, university

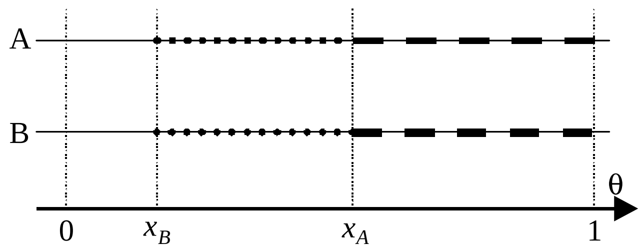

Figure 5: Type II Equilibrium

B could deviate from the candidate equilibrium trying to attract *all* the high ability students: it can do so by leapfrogging university A, choosing an admission standard sufficiently higher than x_A. For this deviation to be profitable, it must be the case that the mobility cost is sufficiently low that university B's deviation standard is close enough to x_A (otherwise there are not enough students in the two towns).[13] To sum up, when an equilibrium in area II exists, it is destroyed both by an increase in the mobility cost –which makes a deviation to a standard below x_A profitable– and by a decrease in the mobility cost –which makes a deviation to an admission standard above x_A profitable.

Some further features of the above equilibria may be worth mentioning. First, in the equilibrium in area I, the number of students at each of the two universities is $n_A = \left(1 - x^A\right)$ and $n_B = (1 - 2x_B + x_A)$ and, therefore, $n_B > n_A$: that is, university A is an élite institution: it has fewer students and higher admission standards. Instead, in the equilibria in areas II the number of students at each of the two universities is: $n_A = 2\,(1 - x_A)$ and $n_B = 2\,(x_A - x_B)$. A simple calculation shows that, in this case, $n_A > n_B$; this is be robust to change in the functional forms. It is also the case that, in our example, the degree of differentiation between the two universities (as captured by the difference between x_A and x_B) tends to increase with the level of competition (as captured by a reduction in the mobility costs t (relative to w)).

[13] This is the same type of deviation we discussed for the previous equilibrium. Notice though that the mobility cost does not need to be as high as in the previous case since the level of x_A is lower.

5 Conclusion

In this paper we use a simple industrial organisation framework to analyse the strategic interaction between two non-profit education institutions. They compete by unilaterally choosing their own admissions standard, and each aims to maximise prestige, which we let depend on the number of students enrolled, their ability, and the success of its research.

We concentrate on the relationship between the equilibrium configuration and the mobility cost for students. When this cost is low, there is no pure strategy Nash equilibrium. When the cost is high there are pure strategy equilibria; for intermediate values of the mobility cost, if equilibria exist, then they are asymmetric. The example in Section 4 shows that asymmetric equilibria do in fact exist for reasonable parameter combinations.

Throughout the paper we have restricted attention to the case where the universities are allocated a fixed and identical budget from the government and where tuition fees play no role in the determination of the allocation of students between the institutions. Further research should extend the model to the more general and realistic case where universities are also able to charge a tuition fee to their students and the government optimally designs the subsidy to be given to the universities, taking into account their interaction.

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