Financial contracts and strategic customer exclusion

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Abstract

The paper studies an incentive contract in a monopolistic and duopolistic credit market where borrowers are different in risk. One lender is in an advantaged position with respect to the other due to past relations with the borrowers. The features of the equilibrium contract are investigated. It is shown that the equilibrium contract drastically changes between the monopolistic and the duopolistic situations and are sensitive to other parameters. In some cases, the superior lender strategically yields borrowers, especially the better ones to the opponent lender.

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1 Introduction

A large amount of literature on credit markets has focused upon a perfectly competitive banking environment. In practice, though, a firm has one or a few banks with which it has closer and more regular business relations. The archetype is the Japanese main bank system. The main bank is in an advantaged position with respect to other potential lenders in competition for a credit contract because it has better information on the firm gathered through past repeated relations and thus cost advantages to screen and monitor the firm's project and behaviour. The paper addresses the nature of equilibrium credit contracts in this imperfectly competitive environment.

In literature on competitive adverse selection, Rochet & Stole (1997) analysed two competing sellers in Hotelling's environment and Villas-Boas & Schmidt-Mohr (1999) investigated the credit contract in the same context. Champsaur & Rochet (1989) studied two firms' quality price competition. The present paper analyses the competition described above in a variation of asymmetric Bertrand competition. The two banks, simultaneously, offer a different type of firm an incentive contract composed of repayment and collateral. One lender is in closer relationships with the firm and thus it incurs less cost to screen the firm's investment project. The cost-advantaged bank, unsurprisingly, holds a dominant position in this competition. To win a contract, however, it has to assure the borrower, at least, of the utility level offered by its competitor's contract. Thus, the competition involves of necessity the issue of type-dependent reservation utility. It is known—see Lewis & Sappington (1989), Maggi & Rodriguez-Clare (1995)—that when the reservation utility is type-dependent, there are two countervailing incentives on the part of the agent: those of understating and overstating. It is uncertain in such a case at which type the participation constraint binds whereas in standard adverse selection theory, the participation constraint binds at the worst type. The paper shows that the participation constraint binds at the best type and thus the borrower indifferent between the two banks' contracts is the highest type. The inferior firms have strong reason to resort to their regular bank because the contract terms proposed by the other bank are so disadvantageous. The better firms, on the other hand, feel much freer to switch to the alternative bank since, being aware that the firms are very credit-worthy, it proposes a contract with by far better terms.

There is a distinct feature in the paper from type-dependent reservation utility literature, in which it is assumed that the principal serves every type of agent. We dispense with it; in our context, this assumption is unrealistic. The dominant bank will not necessarily serve all types of firm. Which firm to serve is it's strategic decision and it may voluntarily exclude some types from its contract, yielding them to the competitor. It is shown that in some cases, the regular bank gives up the better types of borrower to the competitor.

In literature on credit contract, the paper is situated in the middle of the two polar cases: a perfectly competitive banking environment and that of a monopolistic lender. Work on the first case concerns in the main credit rationing(Stiglitz & Weiss (1981) for instance) and the roles of collateral in it(Bester (1985), Bester (1987), Besanko & Thakor

(1987)). Work on the second case includes Besanko & Thakor (1987) and Schmidt-Mohr (1997). This paper is new in relation to the above literature in that it deals with a screening contract in imperfect competition.

We also take a close look at the features of incentive credit contracts in different circumstances. Our finding is that a sorting device used in a credit contract is very sensitive to environments, especially, according to the monopolistic or duopolistic situation, the amount of collateralisable resources, the existence of liquidation cost.

The remainder of the article is constituted as follows. In the next section, the model is presented. The monopolistic case is dealt with in the third section. The fourth section treats duopolistic competition. The last section is the conclusion.

2 The model

Let us suppose that there is a firm with a risky investment project. The project requires the investment cost I. The project succeeds or fails respectively with the probability of p or $1-p$. It is assumed that p takes a value of the non-empty interval $[\underline{p}, \overline{p}] \subset (0, 1)$. The investment cost I is identical for all p. In case of success, the risky project yields the return X and in failure, it brings none. The firm is assumed to lack liquidity I necessary to effectuate the investment. To finance its project, the firm must, thus, resort to a bank and there is no other means of financing a project.

The distribution of p is public knowledge. $F(p)$ is the absolutely continuous distribution function and has the density function $f(p) > 0$ on the whole interval $[\underline{p}, \overline{p}]$. The firm knows its success return X as well as p. The lender is also assumed to know the value of X while unaware of p.

In this article, it is assumed that the lender makes a following financial contract with the firm: the lender claims repayment R when the project is successful and collateral C in case of project failure. The lender is assumed to be able to gather money for the investment I with nil interest rate for normalisation, which does not affect the result. The lender incurs cost to examine and screen the borrower's investment project before deciding to grant credit. We write the cost as L such that $0 \leq L$. The borrower is also subject to some cost when soliciting the lender's credit. It must, for instance, prepare some documents or presentations on his financial situations and the investment project for which to demand credit in order to convince the lender of its credit-worthiness. Let us denote the cost by K such that $0 \leq K$. If $L = 0$ and $K = 0$, the model is reduced to the conventional credit contract model.

The firm is supposed to possess collateralisable resources which cannot be liquidated at the start of the investment; for collateralisable assets are often some form of capital which is in operation for productive activity. Let us assume that the value of all possible collateralisable resources is equal to W regardless of the firm's success probability p.

Although unaware of the borrower's success probability p(which is referred to as *type* from now on), the lender can construct a contracting mechanism which induces the borrower to reveal its type p. Let us, as is usually done, focus on a direct mechanism(see Myerson (1979))[1]. Formally, the mechanism is defined as a mapping from $[\underline{p}, \overline{p}]$ to $\mathbb{R}^2$,

$$p \mapsto (R(p), C(p)).$$

If the borrower of type p chooses the contract for type p', the lender's expected profits for the borrower are

$$pR(p') + (1-p)\alpha C(p') - L - I,$$

where $0 \leq \alpha \leq 1$. α indicates that there may be a discrepancy of the valuation of collateral between the lender and the borrower. There may be, for instance, transaction costs in liquidating collateral when the lender seizes it.

The expected profit of type p borrower choosing the contract for type p' is written as

$$u(p, p') := p(X - R(p')) - (1-p)C(p') - K.$$

If the borrower of type p honestly chooses the contract for type p, its profit is,

$$u(p) = p(X - R(p)) - (1-p)C(p) - K. \tag{1}$$

We put to the lender's credit contract a condition that the credit contract induces the borrower to choose the contract for its own type.

Definition 1. *The contract $(R(p), C(p))$ is implementable if and only if, for any $p, p' \in [\underline{p}, \overline{p}]$*

$$u(p) \geq u(p, p').$$

With an implementable contract, it is the borrower's best strategy to choose the contract of its true type.

[1] Although it is well known that there is no loss of generality by focusing on the direct mechanism in the single principal case, there arises some tricky question of loss of generality in the multi-principal case. See Martimort & Stole (2002) and Peters (2001) for details.

Furthermore, for the borrower to participate in the contract, the lender needs to assure minimum utility to the borrower,

$$u(p) \geq 0. \tag{2}$$

Here we took reservation utility as zero.
We suppose that for any $p \in [\underline{p}, \overline{p}]$,

$$pX - L - K - I \geq 0. \tag{3}$$

Therefore, in the aggregate, the project is always worth carrying out.

As is usual with the mechanism design problem, we transform the implementability condition into the following condition.

Lemma 1 (incentive compatibility condition). *If the contract $(R(p), C(p))$ is implementable, then hold the following conditions,*

$$u(p) \textit{ is convex and absolutely continuous,} \tag{4}$$

$$\dot{u}(p) = \frac{u(p) + C(p) + K}{p} \quad a.e.^{2} \tag{5}$$

Conversely, if 4 and 5 hold, the implementable contract $(R(p), C(p))$ can be obtained by putting,

$$R(p) = \frac{pX + (p-1)C(p) - K - u(p)}{p}. \tag{6}$$

Proof. The proof is referred to the appendix. □

Before proceeding to the next section, let us notice that in asymmetric information literature, some assumption is usually made on the distribution function $F(p)$ while nothing is supposed on it for the moment.

3 Monopoly

In this section, we examine a contract between a monopolistic lender and a borrower. If managing to make type p borrower choose type p contract $(R(p), C(p))$, the lender has the expected profit for type p borrower,

[2]a.e. stands for almost everywhere.

$$pR(p) + (1-p)\alpha C(p) - L - I.$$

If we eliminate R by use of Definition 1, the lender's expected profit for type p borrower is written as

$$\Pi(p) := pX - K - L - I - u(p) + (\alpha - 1)(1-p)C(p).$$

The lender maximises its expected profit under the incentive compatibility and participation conditions,

$$\max_{u(p),C(p)} \int_{\underline{p}}^{\overline{p}} \Pi(p)f(p)dp \tag{7}$$

$$\text{s. t.}$$

$$4,\ 5,\ 2,$$

$$0 \le C(p) \le W. \tag{8}$$

Notice that u is non-decreasing since the right hand of 5 is non-negative. The constraint 2 is thus satisfied if $u(\underline{p}) \ge 0$. The lender's program is now the one of optimal control and we have the following result.

Proposition 1. *The monopolistic lender's optimal contract exhibits full bunching and is expressed as*

$$R(p) = X - \frac{K}{\underline{p}}, \quad C(p) = 0, \quad u(p) = K\big(\frac{p}{\underline{p}} - 1\big).$$

Proof. We ignore the convexity of u for the time being and see finally if it is satisfied. Let us define the Hamiltonian;

$$H = \Pi(p)f(p) + \lambda(p)\frac{u(p) + C(p) + K}{p}.$$

where λ is an absolutely continuous adjoint variable.
We have $\lambda = -\frac{1}{p}\int_p^{\overline{p}} f(\tau)\tau d\tau \le 0$ from $\dot{\lambda} = f - \frac{\lambda}{p}$ a.e. and the transversality condition $\lambda(\overline{p}) = 0$. Maximisation of H with respect to C leads directly to $C = 0$.
u can be found from 5 as a solution of the differential equation;

$$u(p) = u(\underline{p})\frac{p}{\underline{p}} + K\big(\frac{p}{\underline{p}} - 1\big).$$

$\lambda(\underline{p}) \neq 0$ leads to $u(\underline{p}) = 0$ due to the transversality condition $\lambda(\underline{p})u(\underline{p}) = 0$. The ignored convexity condition is satisfied and R can be retrieved from 6.

□

Notice first that in perfect information with liquidation cost present, the first best efficient contract is:

$$R(p) = X - \frac{K}{p}, \quad C(p) = 0, \quad u(p) = 0.$$

In absence of liquidation cost, the efficient contract is not determined uniquely. Naturally, the existence of liquidation cost leads the lender to refrain from taking collateral.

One notable feature of the result in imperfect information is that the lender demands no collateral whether or not there is liquidation cost. It has no bearing on the fact that collateral may be an inefficient sorting device for rent extraction owing to the cost. Likewise, the repayment R is independent of type p unlike in the perfect information case. The optimal contract, therefore, exhibits full bunching. The lender does not assort different types of borrowers at all.

The only assumption for the result is 3, which might look strong but is not in reference to incentive contract theory, in which the agent is usually assumed to bring a large enough contribution to the principal's utility so that the participation constraint for every type is justified. A general result of the literature is the discrimination of types. Condition 3 is, thus, a standard assumption in the literature and our bunching result highlights a particular feature of the credit contract model in asymmetric information with risk neutral agents.

The following proposition is straightforward from Proposition 1.

Proposition 2. *If K is zero, then*

$$R(p) = X, \quad C(p) = 0, \quad u(p) = 0;$$

hence whole bunching, full rent extraction.

The case where the borrower's cost K for applying for credit is zero is standard in the credit contract model. The lender, there, claims the entire return X and leaves nothing to the borrower. Incentive contract theory does not see, as a rule, either full rent extraction or whole bunching. In our case, the agent possessing exclusive information cannot achieve any informational rent. In perfect information, the monopolistic lender is able to identify a type of borrower and extracts the latter's whole surplus. Thus, there is no difference of welfare allocation between full and asymmetric information in our model. In either case, the borrower enjoys null expected profits and the expected value of the investment projects, $pX - I - L(K = 0)$ wholly accrues to the lender.

Even, without liquidation cost($\alpha = 1$), the lender will not use collateral as a sorting device in our model. To see the reason, suppose that $K = 0$ and that we are in the two type case where $p_B < p_G$ (bad and good risks). Suppose also that there are two distinct optimal contracts such that $(R(p_B), C(p_B)) \neq (R(p_G), C(p_G))$. Since $u(p_B) = u(p_G) = 0$ from optimality, by Equation 1 we have

$$p_i(X - R(p_i) + C(p_i)) = C(p_i) \qquad \text{for } i = B,\, G.$$

It follows from $p_B < p_G$ that

$$\begin{aligned} p_G(X - R(p_B) + C(p_B)) &\geq C(p_B), \\ p_B(X - R(p_G) + C(p_G)) &\leq C(p_G). \end{aligned}$$

From the first inequality, we see that the low risk borrower gains by mimicking the high risk one. By contrast, the high risk borrower never gains by pretending to be less risky. This breaches incentive compatibility. Therefore, the optimal contract bunches all types of borrowers.

4 Competition

In this section, we consider a financial contract between borrowers and two competing lenders. Usually, a firm, when having an investment project, refers to a bank with which it has steady or long-standing relationships. The bank is more aware of the practice or character of the firm's management and more capable of apprising the risk of the project than other banks having less regular relations with the firm. The bank is also advantaged in cost it incurs to appraise the regular customer's investment project and decide upon the credit-worthiness.

To model this situation, we suppose that there are two lenders: Lenders 1 and 2. Lender 1 has established business relationships with the firm which has the investment project described in Section 2.

As in the previous section, Lender i ($i = 1, 2$) claims repayment R_i in case of the success of the project, and collateral C_i when the project fails.

The borrower knows its success probability, but the lenders do not. They design an implementable contract as in Definition 1. Let us denote Lender i's contract for type p borrower by $(R_i(p), C_i(p))$. We denote by $u_i(p)$ the expected utility of type p borrower honestly choosing Lender i's contract $(R_i(p), C_i(p))$. By definition, therefore,

$$u_i(p) := p(X - R_i(p)) - (1-p)C_i(p) - K_i.$$

We assume

$$0 < K_1 < K_2. \tag{9}$$

This inequality reflects the fact that the borrower has closer regular relationships with Lender 1 and incurs less costs to apply for credit with the lender than the other.

The two lenders' implementable contracts can be expressed by dint of the incentive compatibility condition in Lemma 1. As before, the borrower may refrain from signing a credit contract if it is not interesting enough. Thus, the contracts of the lenders must satisfy the participation condition, $u_i(p) \geq 0$.

The lenders' expected profits for type p borrower, with an implementable contract, are

$$pR_i(p) + (1-p)\alpha C_i(p) - L_i - I,$$

where L_i is a constant such that $0 < L_i$. It is assumed that

$$L_1 < L_2. \tag{10}$$

This indicates that Lender 1 has closer business relationships with the borrower than Lender 2. Past business allows the former lender to learn much about the borrower and it can spend less to screen the borrower's investment project.

We assume

$$0 < W \leq L_2 + I. \tag{11}$$

If $W = L_2 + I$, both lenders' loan can be fully covered by collateral while if $W < L_2 + I$, either lenders may not be able to secure completely their loan. In other words, even though requiring all possible collateral, the lenders cannot totally dissipate the risk of the project's failure.

In addition, the following is assumed in parallel to the monopolistic case,

$$pX - K_2 - L_2 - I \geq 0 \quad \text{for } p \in [\underline{p}, \overline{p}].$$

Thus the project of every type of borrower is worth realising from the social point of view.

As in the previous section, the use of u_i enables us to write Lender i's expected profit for type p borrower as

$$\Pi_i(p) := pX - u_i(p) + (\alpha - 1)(1-p)C_i(p) - K_i - L_i - I.$$

We consider a game in which the two principals maximise their expected profits in competition for a borrower, offering an *implementable* contract: $(R_i(p), C_i(p))$ such that

$$u_i(p) \text{ is convex and absolutely continuous,} \tag{12}$$

$$\dot{u}_i(p) = \frac{u_i(p) + C_i(p) + K_i}{p} \qquad \text{a.e. on } [\underline{p}, \overline{p}], \tag{13}$$

$$0 \leq u_i(p), \tag{14}$$

$$0 \leq C_i(p) \leq W. \tag{15}$$

13 allows us to replace 14 by

$$0 \leq u_i(\underline{p}).$$

If one lender offers a contract for a type which gives less expected profits to the type than the other lender's, it loses the contract for the type and obtains zero expected profits. Therefore, Lender 2's expected profit from type p borrower is

$$\begin{cases} \Pi_2(p) & \text{if } u_2(p) > u_1(p), \\ 0 & \text{if } u_2(p) \leq u_1(p). \end{cases}$$

Here in the tie case, $u_2(p) = u_1(p)$, it has been assumed that Lender 1 wins the contract because in our setting Lender 1 is has closer relationships with a borrower. Another tie-breaker, however, does not affect the following arguments.

In contrast to the asymmetric lenders here, Stole (1995) analysed the competition of this type between two firms with identical production cost. Unlike in his study, we have the constraint on the range of the control variable, 15, which entails the complication that the marginal cost argument in Bertrand competition may not work in our analysis.

Naturally, if his profits from type p borrower are negative, Lender i would rather not lend money to the borrower. Thus Lender i offers type p borrower a loan which brings non-negative profits. Now suppose that in our competitive game Lender 2 offers an implementable contract which satisfies Equations 12 to 15 and brings non-negative profits from all types. Then, having cost advantage, Lender 1 can offer the same contract and have positive profits from all types. Moreover, Lender 1 can propose an implementable contract promising every type of borrower larger expected utility than the opponent lender's. In consequence, Lender 1 can win a contract for all the types. However it is in no way certain that Lender 1 does so. Given Lender 2's implementable contract, Lender 1 proposes a contract maximising the following problem denoted by $\mathcal{O}$.

$$\begin{aligned}\max_{u_1, C_1, M} \int_{\underline{p}}^{\overline{p}} & \Pi_1(p) 1_M(p) f(p) dp \\ & \text{s. t.} \\ & 12,\ 13,\ 14 \text{ and } 15 \text{ for } i = 1, \\ & \text{where } M := \{p | u_2(p) \leq u_1(p)\} \text{ and } 1_M(p) \text{ is an indicator function .}\end{aligned}$$

Notice that the objective function is zero outside set M. Lender 1 may strategically exclude some borrowers from its contract, who will be served by Lender 2. Theoretically, it may serve an interval, disjoint intervals or a more complicated set of types of borrower. It is, however, out of reach to investigate all the possibilities. We assume therefore, as in **??**, that Lender 1's strategy is to win a contract for an interval of types of borrower.

Assumption 1. *Lender 1's strategy in the two-lender competitive game is to win a contract for an interval of types of borrowers $[\grave{p}, \acute{p}]$. In other words, it is assumed that in Problem $\mathcal{O}$, $M = [\grave{p}, \acute{p}]$.*

Later we shall prove that Lender 1 chooses to serve every type of borrowers in the absence of liquidation cost. By contrast, in the presence of the cost, Lender 1's strategic choice of customers is more complicated. In general, the lender may deliberately cede certain types of borrower to the opponent lender.

By means of Assumption 1, we can write Problem $\mathcal{O}$ in the following form that will be used throughout.

$$\max_{u_1(p), C_1(p), \grave{p}, \acute{p}} \int_{\grave{p}}^{\acute{p}} \Pi_1(p) f(p) dp \tag{16}$$

s. t.

12, 13 and 15 for $i = 1$,

$$u_2(p) \leq u_1(p) \qquad \text{for } p \in [\grave{p}, \acute{p}], \tag{17}$$

$$u_2(p) > u_1(p) \qquad \text{for } p \notin [\grave{p}, \acute{p}], . \tag{18}$$

Condition 17 is the participation constraint of the borrower faced to Lender 1. Conditions 17 and 18 indicate that Lender 1 chooses to serve only the types $[\grave{p}, \acute{p}]$, giving over the others to the competitor. The borrower simply goes to Lender 2 if Lender 1 offers less utility. We find here an instance of type-dependent reservation utility(see Lewis & Sappington (1989), Maggi & Rodriguez-Clare (1995)). In our problem the participation constraint is more complex than in much of the literature of type dependent reservation. Whereas it is usually assumed that $u_2(p) \leq u_1(p)$ on the whole interval $[\underline{p}, \overline{p}]$, the present paper allows for the exclusion of some types as reflected in 18.

Our story extends to the case of more than two lenders; for it is reduced to the two-lender case as long as the costs K_i and L_i are ordered as in Equations 9 and 10.

We split further analysis into two cases: $W = L_2 + I$ and $W < L_2 + I$. To the former case essentially applies the usual marginal cost argument for Bertrand competition. Contrariwise, one cannot use the same argument in the latter case and needs careful inspection.

4.1 The case of $W = L_2 + I$

All through this subsection, we assume that $W = L_2 + I$. For instance, in Japan where banks are distraught with a huge mount of accumulated bad assets, it is mostly the case that they only lend an amount which collateralisable resources can cover. Therefore, the case of this section is important from a practical point of view as well.

Applying a common argument for Bertrand competition, one can derive the following lemma.

Lemma 2. *In the two lender competitive game, Lender 2 offers the following contract: on* $[\underline{p}, \overline{p}]$*,*

$$C_2(p) = W = L_2 + I, \quad R_2(p) = L_2 + I, \quad u_2(p) = pX - K_2 - L_2 - I.$$

Proof. Note that the expected social value of the project of type p borrower is $pX - K_2 - L_2 - I$ when the project is carried out with Lender 2. This is the largest utility that Lender 2 can offer to type p borrower with an implementable contract. Otherwise, the lender would have negative profits. Suppose that Lender 2 has offered an implementable contract $(R(p), C(p))$ which gives type p buyer utility less than $pX - K_2 - L_2 - I$. Then, Lender 1 offers an implementable contract which gives type p borrower slightly larger utility and does win the contract for the borrower. For fear of losing the contract, Lender 2 offers an implementable contract which gives the same type slightly more utility than

Lender 1. This competition process endures until Lender 2 offers the contract which gives utility $pX - K_2 - L_2 - I$. C_2 can be obtained from 13 and R_2 from 6.

□

As in standard Bertrand competition, Lender 2 gains zero profit. We already see an interesting feature which is distinct from the previous section. While in the monopolistic case the lender demands no collateral, in the competitive case, Lender 2, which is less advantaged in cost, requires the maximum collateral of every type of borrower. As in the monopolistic case, the collateral and repayment are identical among all types of borrower; hence bunching.

Let us seek for the solution of Problem $\mathcal{O}$ by use of the lemma. We call Problem $\mathcal{O}$ with the values of Lemma 2 substituted, Problem $\mathcal{P}$. We solve Problem $\mathcal{P}$ as that of optimal control with free time(see Section 2, Chapter 5 in Neustadt (1976) for instance). We replace Condition 17 by

$$u_2(\acute{p}) \leq u_1(\acute{p}) \tag{19}$$

and verify that Condition 17 is satisfied at the end.[3] We also ignore Condition 18 and verify it ex post.

We can write the Hamiltonian as

$$\begin{aligned} H = \big((\alpha - 1)(1-p)f + \frac{\lambda_1}{p}\big)C_1 + (pX - K_1 - L_1 - I - u_1)f \\ + \lambda_1 \frac{u_1 + K_1}{p}. \end{aligned}$$

The necessary conditions to optimality are

$$\dot{\lambda}_1 = f - \frac{\lambda_1}{p} \quad \text{a.e.} \quad \text{on } [\grave{p}, \acute{p}],$$

$$\lambda_1(\grave{p}) = 0, \quad \lambda_1(\acute{p}) \geq 0, \quad \lambda_1(\acute{p})\big(u_1(\acute{p}) - \acute{p}X + K_2 + L_2 + I\big) = 0,$$

[3] In fact, if we deal with Problem $\mathcal{P}$ while thinking first that $\grave{p}$ and $\acute{p}$ are given and that the problem is set up only on $[\grave{p}, \acute{p}]$ and ignoring Condition 18, then the problem is reduced to that with a state inequality. And we find that Condition 17 is binding only at $\acute{p}$. Hence we conjecture that in Problem $\mathcal{P}$ likewise, Condition 17 is active only at $\acute{p}$ and replace the condition by 19.

$$\Pi_1(\acute{p})f(\acute{p}) + \lambda_1(\acute{p})\frac{u_1(\acute{p}) + C_1(\acute{p}) + K_1}{\acute{p}} - \lambda_1(\acute{p})X \begin{cases} \leq 0 & \text{if } \acute{p} = \underline{p}, \\ = 0 & \text{if } \underline{p} < \acute{p} < \overline{p}, \\ \geq 0 & \text{if } \acute{p} = \overline{p}, \end{cases} \tag{20}$$

$$-\Pi_1(\grave{p})f(\grave{p}) \begin{cases} \leq 0 & \text{if } \grave{p} = \underline{p}, \\ = 0 & \text{if } \underline{p} < \grave{p} < \overline{p}, \\ \geq 0 & \text{if } \grave{p} = \overline{p}. \end{cases} \tag{21}$$

Easily established is the fact that

$$\lambda_1(p) = \frac{1}{p}\int_{\grave{p}}^{p} f(t)tdt \quad \text{on } [\grave{p}, \acute{p}] \tag{22}$$

and therefore

$$u_1(\acute{p}) = \acute{p}X - K_2 - L_2 - I.$$

When $\alpha = 1$, the optimal C_1 is $W = I + L_2$ and also we deduce that

$$u_1(p) = pX - (I + L_2) - \frac{p}{\acute{p}}\big(K_2 - K_1\big) - K_1 \quad \text{on } [\grave{p}, \acute{p}].$$

From Conditions 20 and 21 it follows that $\grave{p} = \underline{p}$ and $\acute{p} = \overline{p}$. Conditions 17 and 18 are obviously satisfied. R_1 can also be retrieved from Equation 6.

Proposition 3. *Suppose that $\alpha = 1$. Then Lender 1's optimal contract is given as follows. The lender serves all types: $\grave{p} = \underline{p}$ and $\acute{p} = \overline{p}$. On the interval $[\underline{p}, \overline{p}]$,*

$$\begin{aligned} C_1(p) &= I + L_2, \\ R_1(p) &= I + L_2 + \frac{1}{\overline{p}}\big(K_2 - K_1\big), \\ u_1(p) &= pX - (I + L_2) - \frac{p}{\overline{p}}\big(K_2 - K_1\big) - K_1. \end{aligned}$$

As in the monopolistic case, we see full bunching, in which the repayment and collateral are identical across all the types. In contrast to the monopolistic case, though, Lender 1 requires collateral. We have seen in Lemma 2 that Lender 2 also demands the same amount of collateral of every borrower. The existent theoretical literature of credit contract has asserted that a good borrower pledges more collateral than a bad one because the former is more likely to succeed in the investment project and unlikely to lose the collateral. To the best of my knowledge, our result is new in the universal risk-neutral environment.

The empirical work of Berger & Udell (1990) and Berger & Udell (1995) report that riskier borrowers more often pledge collateral than safer ones. This is not in accordance with our result. However, in those studies it is not clear how much collateral banks wanted at the beginning to demand of a particular borrower nor whether banks did not take collateral because there were not collateralisable resources or they did not ask even though there were. Further empirical work is needed to test the coherence of our result with the actual loan contract.

For the purpose of interpreting our result, let us distinguish between borrower risk and loan risk. In asymmetric information, both risks are known to the borrower but neither of them to the lender. Borrower risk is learned over time by the lender through a series of business relationships and borrowers are classified according to the risk. Loan risk varies, in general, according to each investment project with the same borrower. In the present model, K_1 and L_1 can be interpreted as the lender's classification of borrowers according to borrower risk whereas loan risk is indicated by p. The statement in the proposition that the loan contract depends upon K_1 and L_1 but not upon p signifies that the contract is determined by borrower risk but not by loan risk. This implies that the optimal contract is identical to the well employed type of contract, line of credit in which the loan conditions are determined in advance for a borrower and the borrower can draw a loan with the conditions when necessities arise.

Let us take a look at the participation constraint 17. Unlike in the standard incentive theory, it is binding at the best type $\overline{p}$, which is due to type-dependent reservation utility. Indeed, it is seen that $u_1(p) - u_2(p)$ is decreasing. As a result, the worst type obtains positive rent whereas the best type has all rent extracted; the best type is indifferent between Lender 1's and Lender 2's contracts. Lender 1 seeks to extract the borrower's rent but must leave the borrower reservation utility varying with the type. There are, thus, two conflicting incentives for the lender: rent extraction and rent assurance.

Bad borrowers have by far stronger reason to make a contract with their regular bank: the other bank offers them too unfavourable a contract. By contrast, good borrowers are offered by Lender 2 a more favourable contract than bad ones; at the extreme, the best type borrower is indifferent between the two banks. It seems well borne out by the observation of the actual economy in which a seemingly bad firm often has no other option but to go to the regular bank since it is offered a contract of very bad terms or even flatly denied any contract by other banks; an apparently good firm, however, can resort to other banks with much ease to obtain an alternative offer.

Next let us turn to the general case where there is liquidation cost, $\alpha < 1$. We focus upon the uniform distribution. Even in this simple case, it proves complicated to find the solution of $\mathcal{P}$.

The Hamiltonian is linear in C_1 and thus the coefficient of C_1 in it is written as

$$g(p) := (\alpha - 1)(1 - p)\frac{1}{\overline{p} - \underline{p}} + \frac{1}{p^2}\frac{p^2 - \grave{p}^2}{2(\overline{p} - \underline{p})}.$$

It is seen that $g'(p) > 0$ is positive. Notice that u_1 can be written as

$$u_1(p) = u_1(\acute{p})\frac{p}{\acute{p}} + p\int_{\acute{p}}^{p} \frac{C_1(t) + K_1}{t^2} dt.$$

From the value of C_1 we know that there exists $\ddot{u}_1$[4] almost everywhere and $\ddot{u}_1 = 0$ at those points. Accordingly, $\dot{u}_1$ is non-decreasing and the convexity of u_1 is satisfied. One can see directly $g(\grave{p}) < 0$ and thus along with $g'(p) > 0$, we obtain the proposition.

Proposition 4. *Suppose that there is liquidation cost($\alpha < 1$) and that F is a uniform distribution function.*

Then the solution of $\mathcal{P}$ is such that there is $x \in (\underline{p}, \acute{p}]$ and the optimal collateral is given as

$$C_1(p) = \begin{cases} 0 & \textit{on } [\underline{p}, x], \\ W & \textit{on } (x, \acute{p}], \end{cases}$$

$$u_1(p) = \begin{cases} \frac{p}{\acute{p}}\big(\acute{p}X - K_2\big) - \frac{p}{x}W + K_1\big(\frac{p}{\acute{p}} - 1\big) & \textit{on } [\underline{p}, x], \\ \frac{p}{\acute{p}}\big(\acute{p}X - K_2\big) - W + K_1\big(\frac{p}{\acute{p}} - 1\big) & \textit{on } (x, \acute{p}], \end{cases}$$

The proposition states that Lender 1 demands no collateral of the borrower of lower types, in contrast to the no-liquidation-cost case. In the monopolistic case, whether or not there is liquidation cost did not affect the lender's demand for collateral. In competition, collateral demanded by the lender may differ a great deal, dependent upon the existence of liquidation cost. In particular, even the presence of a very tiny liquidation cost leads the lender to relinquish collateral for the lower types.

According to the magnitude of α, there arise two cases. When there is large liquidation cost, the lender does not require collateral of any borrower since it counts for very little after liquidation. When the liquidation cost is not so considerable, the lender differentiates the requirement of collateral between types: no collateral from the lower types. The reason for this— somewhat counter to conventional wisdom— is the same

[4] $\ddot{u}_1$ is a second derivative.

as in the past articles(Bester (1985), Bester (1987), Besanko & Thakor (1987), Chan & Thakor (1987)) which reported the result in varied contexts. The better borrowers are more willing to accept a contract with low repayment and large collateral than the poorer ones: for the former are, due to their high success probability, less likely to lose collateral than the latter.

Intricate is the lender's decision on which types to serve, according to Conditions 20 and 21. They are easily calculated by use of Proposition 4 but it is difficult to draw a clear conclusion. The following factors are inextricably intertwined: the degree of competition between the banks ($L_2 - L_1$, $K_2 - K_1$), the liquidation cost (α) and the distribution of types (f).

One certain thing is that Lender 1 may give up the higher types: $\acute{p} \neq \overline{p}$. The reason is as follows. Taking larger $\acute{p}$, the lender captures new borrowers and makes earnings from them. On the other hand, it may lose some profits for borrowers already captured; for setting larger $\acute{p}$, the lender must satisfy the incentive compatibility and participation constraints for a wider range of types and thus may have to give up some rent to each individual borrower. Lender 1 decides to forego the higher types of borrower when negative effects by rent assurance to already captured borrowers outweigh positive effects brought by gains from new borrowers.

4.2 The case of $W < L_2 + I$

Throughout this subsection, it is assumed that $W < L_2 + I$. To this case, the marginal cost argument having proved Lemma 2 does not apply; for $C_2 = L_2 + I$ is impossible.

Let us write Lender i's non-negative profit condition for borrower p:

$$\Pi_i(p) \geq 0. \tag{23}$$

Then the largest utility which the borrower of type p can obtain with Lender 2's contract is expressed by the following lemma.

Lemma 3. *In the Nash equilibrium of the two-lender competition game, Lender 2's equilibrium contract is the implementable contract* $(u_2(p), C_2(p))$ *such that it satisfies 23 and that* u_2 *is weakly larger at every type of* $[\grave{p}, \acute{p}]$ *for which Lender 1 wins a contract than* $\tilde{u}_2$ *of any other implementable contract* $(\tilde{u}_2(p), \tilde{C}_2(p))$ *which satisfies 23.*

Proof. First it is obvious that Lender 2 offers a contract which satisfies Condition 23. If Lender 2 offers a contract which gives $\tilde{u}_2(p)$ in the lemma, Lender 1 can offer a contract which gives $u_2(p)$ and have positive profits for every type of borrower. The borrowers of all types weakly prefer that contract by Lender 1 and the lender wins them over. Therefore, Lender 2 is bound to propose $u_2(p)$. Faced to this offer by the opponent, Lender 1 will propose a contract giving a bit more utility to the borrower and wins it over. Lender 2 sticks to the contract $u_2(p)$ faced to Lender 1's contract since the former cannot offer a more interesting contract for the borrower on account of the non-negative profit condition 23.

$\square$

There are, however, some problems in the treatment of the equilibrium in the lemma. First, this is essentially a pointwise maximisation problem amongst admissible pairs, i.e, the implementable (u_2, C_2) which satisfies 23. And yet it is not certain that there is u_2 which is at every point weakly larger than any other admissible $\tilde{u}_2$. For instance, one can imagine a situation in which there are two admissible $\hat{u}_2$ and $\bar{u}_2$ which are weakly larger at every point than any other admissible u_2 but between $\hat{u}_2$ and $\bar{u}_2$, at some points $\hat{u}_2$ is larger than $\bar{u}_2$ and at other points contrariwise. Then there is no such an equilibrium pair as is described in the lemma. When the instrument variable C_i is constrained, therefore, the existence of equilibrium may be at issue. Second, even if an equilibrium exists, the maximisation problem is hard to solve directly. Accordingly, we deal with our competitive game by means of optimal control technique. If there exists an equilibrium stated in the lemma, obviously the equilibrium pair is a solution of the following problem denoted by $\mathcal{R}$.

$$\max_{u_2(p), C_2(p)} \int_{\grave{p}}^{\acute{p}} u_2(p) f(p) dp$$

$$\text{s. t.}$$

$$12,\ 13,\ 15,\ 23 \text{ for } i = 2,$$

$$0 \leq u_2(\grave{p}).$$

Before continuing let us note that in this setting there may be a solution even if the pointwise maximisation problem in the above lemma does not possess a solution.

The last constraint comprises the state variable u_2 and the control variable C_2. We cannot therefore refer to such a common technique as transforming the constraint into an end condition. Thus, we have to take the last constraint into explicit consideration. The problem with a mixed state-control constraint is very difficult to deal with. However, when there is no liquidation cost, C_2 disappears and the mixed constraint is reduced to a pure state constraint. We content ourselves with this case on account of the manageability.

Lemma 4. *Suppose that $\alpha = 1$. Then, in the Nash equilibrium of the two competing lenders, Lender 2 proposes the following contract $(R_2(p), C_2(p))$ such that on $[\grave{p}, \acute{p}]$ for which Lender 1 wins a contract,*

$$
\begin{aligned}
R_2(p) &= \frac{1}{\grave{p}}\big(L_2 + I\big) + \big(1 - \frac{1}{\grave{p}}\big)W, \\
C_2(p) &= W, \\
u_2(p) &= pX - \frac{p}{\grave{p}}\big(L_2 + I\big) + \big(\frac{p}{\grave{p}} - 1\big)W - K_2.
\end{aligned}
$$

Proof. See the appendix. □

The same remark made after Lemma 2 applies here. While in the monopolistic case the lender demands no collateral, in the competitive case, Lender 2 requires the maximum collateral of the borrower of all types.

A difference from Lemma 2 is that there, Lender 2 makes no profits for any type of borrower while here, the lender makes profits save for the lowest type $\grave{p}$. It is due to the fact that collateral resources are stringently constrained in the case of this subsection, which makes impossible any implementable contract which gives rise to Lender 2's nil profits.

Let us formulate Lender 1's problem by means of this lemma and call Problem $\mathcal{O}$ with the value of the lemma substituted into it, Problem $\mathcal{Q}$. The problem turns out to be that of free time optimal control. As in Problem $\mathcal{P}$, we replace condition 17 by 19 and make certain of Condition 17 at the end. Condition 18 is also ignored at first and verified ex post. The process is quite parallel to the solution of Problem $\mathcal{P}$. The sole difference in the necessary conditions is that instead of 20 and 21, we have

$$
\Pi_1(\acute{p})f(\acute{p}) + \lambda_1(\acute{p})\frac{u_1(\acute{p}) + C_1(\acute{p}) + K_1}{\acute{p}} - \lambda_1(\acute{p})\big(X - \frac{L_2 + I - W}{\grave{p}}\big)
\begin{cases}
\leq 0 & \text{if } \acute{p} = \underline{p}, \\
= 0 & \text{if } \underline{p} < \acute{p} < \overline{p}, \\
\geq 0 & \text{if } \acute{p} = \overline{p},
\end{cases}
\tag{24}
$$

$$
-\Pi_1(\grave{p})f(\grave{p}) - \lambda_1(\acute{p})\frac{(L_2 + I - W)\acute{p}}{\grave{p}^2}
\begin{cases}
\leq 0 & \text{if } \grave{p} = \underline{p}, \\
= 0 & \text{if } \underline{p} < \grave{p} < \overline{p}, \\
\geq 0 & \text{if } \grave{p} = \overline{p}.
\end{cases}
\tag{25}
$$

The details are deferred to the appendix but the solution is obtained in much the same way as in Problem $\mathcal{P}$ and we can deduce that $\grave{p} = \underline{p}$. A difference from Problem $\mathcal{P}$ is on the determination of $\acute{p}$. Indeed, it follows immediately that Condition 24 becomes

$$\theta(\acute{p}) := f(\acute{p})\big(\frac{\acute{p}}{\underline{p}}(L_2 + I - W) - (L_1 + I - W) + K_2 - K_1\big) - \frac{\lambda_1(\acute{p})}{\grave{p}}\big(K_2 - K_1\big).$$

Hence the following proposition.

Proposition 5. *Suppose that $\alpha = 1$. Then always hold $\grave{p} = \underline{p}$.*

1. *If $\theta(p) \neq 0$ on $[\underline{p}, \overline{p}]$, Lender 1 wins a contract for all types, that is, $[\grave{p}, \acute{p}] = [\underline{p}, \overline{p}]$ and the solution of Problem $\mathcal{Q}$ is: on $[\underline{p}, \overline{p}]$,*

$$\begin{aligned}
C_1(p) &= W, \\
R_1(p) &= (1 - \frac{1}{\underline{p}})W + \frac{1}{\underline{p}}\big(L_2 + I\big) + \frac{1}{\overline{p}}\big(K_2 - K_1\big), \\
u_1(p) &= pX + \big(\frac{p}{\underline{p}} - 1\big)W - \frac{p}{\underline{p}}\big(L_2 + I\big) - \frac{p}{\overline{p}}K_2 + \big(\frac{p}{\overline{p}} - 1\big)K_1.
\end{aligned}$$

2. *If there is $\acute{p}$ such that $\theta(\acute{p}) = 0$ on $[\underline{p}, \overline{p}]$, Lender 1 may yield the higher types of borrower to Lender 2, that is, $[\grave{p}, \acute{p}] = [\underline{p}, \acute{p}] \subsetneq (\underline{p}, \overline{p}]$ and the solution of Problem $\mathcal{Q}$ is such that on $[\underline{p}, \overline{p}]$,*

$$\begin{aligned}
C_1(p) &= W, \\
R_1(p) &= (1 - \frac{1}{\underline{p}})W + \frac{1}{\underline{p}}\big(L_2 + I\big) + \frac{1}{\acute{p}}\big(K_2 - K_1\big), \\
u_1(p) &= pX + \big(\frac{p}{\underline{p}} - 1\big)W - \frac{p}{\underline{p}}\big(L_2 + I\big) - \frac{p}{\acute{p}}K_2 + \big(\frac{p}{\acute{p}} - 1\big)K_1.
\end{aligned}$$

Proof. See the appendix. □

The qualitative features are identical to the case of $W = L_2 + I$ and all the interpretations there apply here too. One difference worthy of remark is that here Lender 1 may yield the higher types of borrower to the opponent lender. The more types Lender 1 seizes, the more borrowers it contracts with. In this regard, the capture of a larger number of borrowers has positive effects on the lender's profit. However, when there are more borrowers to contract with, the lender is obliged to satisfy the participation constraint over a larger interval of types. The participation constraint becomes more stringent. It acts in a negative way on the lender's profit. Altogether, there are situations in which the lender gives up some borrowers and compensates the loss by extracting more rent from the rest of borrowers.

Let us take a look at $\theta(\acute{p})$. $\lambda_1(\acute{p})$ is known from 22 to be non-negative. Therefore, Lender 1 gives up the higher types the larger $K_2 - K_1$ is. This implies that when borrowers find it costly and hard to accost Lender 2, Lender 1 yields the higher types to the other lender. Intuition behind this is that when having predominant power compared to the rival, Lender 1 attempts to extract more rent from borrowers. Then the higher types switch to lender 2 on the strength of their good prospect of success whereas the lower types do not since they are unlikely to succeed and find Lender 2's contract more unfavourable than Lender 1's.

5 Conclusion

The issue of the principal's strategic exclusion of customers has been relatively ignored. In mechanism design literature, the custom has been adopted that the agent brings so large contribution to the principal's utility that the principal always finds it in its interest to include all types of agent in its contract.

One of the situation in which this assumption becomes unrealistic is where there is a principal's competitor, in which case it has no reason any longer to serve every type of customer. The competitor may assure some customers of high utility and the principal may prefer not to serve them but instead to extract more rent from the other customers.

In duopolistic competition, if the lender seeks to extract too much rent of the borrower, this simply goes to the opponent lender. The lender always has to give the borrower at least what is proposed by the other. It faces a conflict of incentives between the extraction of a borrower's rent and the attraction of the borrower by giving more than the opponent lender.

The paper shows that the participation constraint is binding at the highest. The less effective firms have every reason to turn to to their regular lender since they find the contract by the other too unfavourable. By contrast, the better firms are more disposed to change lenders since the opponent lender proposes a far better contract to the credit-worthy borrowers.

The article has also examined how environments affect the way that collateral is used as a sorting device. In the monopolistic setting, the lender never demands collateral as a sorting device; in addition, borrowers have their whole surplus extracted in the standard setting. These features are independent of whether or not collateral entails liquidation cost.

In our competitive game, if there are enough collateralisable resources to cover a loan, there holds the usual marginal cost argument in Bertrand competition. However, in the opposite case, a more careful treatment is required.

In absence of liquidation cost, the lender demands the fixed amount of collateral and reimbursement regardless of a borrower's project risk. In practice, banks often grant their borrowers a line of credit, in which the conditions of the loan are specified in advance. Banks do not examine each and every loan application of the borrowers. The conditions of a line of credit is, therefore, specific to borrowers but not to their loans. Our result confirms this.

By contrast, in presence of liquidation cost, the lender's collateral requirement differs according to a borrower's type. As opposed to the monopolistic case, the magnitude of liquidation cost also affects the lender's collateral requirement.

It is widely accepted that relationship banking is an essential feature of relations between a firm and a financial intermediary. Sharpe (1990) showed that even perfectly competitive banking provides an initial lender with monopolistic power over time since the lender can learn about the borrower's characteristics. Rajan (1992) examined pros and cons between relationship lending and capital market debt for a borrower. Boot & Thakor (2000) investigated the effects of increasing competitive pressure from capital markets upon relationship lending and demonstrated that in some situation, banks enhanced relationship lending activity.

The existent literature on relationship banking has not considered a situation in which a bank builds an incentive mechanism as a loan contract to distinguish different borrowers; often on the assumption that after the first lending, a bank learns all information about the borrower. This article analyses a situation in which a bank, even in established relationships with a borrower, is subject to asymmetric information with it and constructs an incentive mechanism to overcome the asymmetry.

There is very little study which incorporated collateral in study on relationship banking. Boot & Thakor (1994) is probably the only work to have done it in the context of moral hazard but not adverse selection. Further study is expected in this area.

A The proof of Lemma 1

Proof. The proof refers to Rochet (1987). Let us suppose that the mechanism is implementable. First, by definition, $u(p)$ is the superior envelope of the convex(in fact, linear) functions of p which are

$$p(X - R(p')) - (1-p)C(p') - K,$$

so that $u(p)$ is convex. At the same time, it is deduced that u is absolutely continuous from the fact u is the superior envelop of linear functions. Next, we have, by definition of the implementable mechanism: for any p, p',

$$p(X - R(p)) - (1-p)C(p) - K \geq p(X - R(p')) - (1-p)C(p') - K.$$

If we transform this inequality using $u(p)$, it follows that for any p, p',

$$u(p) \geq u(p') + (p - p')(X - R(p') + C(p')).$$

By the definition of subdifferential(see Rockafellar (1970)),we have thus

$$X - R(p) + C(p) \in \partial u(p).$$

The convex function is almost everywhere differentiable so that

$$X - R(p) + C(p) = \dot{u}(p) \quad a.e.$$

Eliminating $R(p)$ by use of the definition of $u(p)$, we obtain 5.

On the contrary, suppose that the conditions 4, 5 hold. Since $\partial u(p) = \dot{u}(p)$ if the right side exists,

$$\partial u(p) = \frac{u(p) + C(p) + K}{p} \qquad \text{a.e.}$$

There is G dense in $[\underline{p}, \overline{p}]$ in which $\dot{u}$ exists. Take a point $\tilde{p} \notin G$ and a sequence $p_n \in G$ converging to $\tilde{p}$. Since u is convex and thus locally Lipschitzian(see p.86 in Rockafellar (1970)), by convexity we can take a subsequence of $\dot{u}(p_n)$ converging to some y. u is an absolutely continuous function and thus by theorem 24.4 in Rockafellar (1970) it follows that $y \in \partial u(\tilde{p})$. We have deduced that on $[\underline{p}, \overline{p}]$,

$$\partial u(p) \ni \frac{u(p) + C(p) + K}{p}.$$

Finally, we have by the definition of subdifferential,

$$\forall p, p', \quad u(p') \geq u(p) + (p' - p)\frac{u(p) + C(p) + K}{p}. \tag{26}$$

Let us define R by 6. If we eliminate u from the fractional term of 26,

$$\forall p, p', \quad u(p') \geq u(p) + (p' - p)(X + C(p) - R(p)).$$

It is the very condition of the implementable mechanism. $\square$

B The proof of Lemma 4

It is evident that Lender 2's contract as a solution of problem $\mathcal{R}$ can be offered by Lender 1 and that the latter lender obtains non-negative profits from it because of the cost advantage. Moreover, Lender 1 can offer a contract giving the borrower more utility than the solution contract of Problem $\mathcal{R}$. Consequently, Lender 1 wins a contract on the interval of $[\grave{p}, \acute{p}]$.

Let us solve Problem $\mathcal{R}$ while ignoring condition $0 \leq u_2(\grave{p})$ and replacing 23 by $\Pi_2(\grave{p}) \geq 0$. At the end, we shall verify that the original two conditions are satisfied.

The Hamiltonian is written as;

$$H = u_2 f + \lambda \frac{u_2 + C_2 + K_2}{p}$$

where λ is an absolutely continuous adjoint variable. We have

$$\dot{\lambda} = -f - \frac{\lambda_1}{p} \quad \text{a.e.}$$

The transversality conditions are

$$\lambda_1(\grave{p}) \geq 0, \qquad \lambda_1(\grave{p})\Pi_2(\grave{p}) = 0, \qquad \lambda_1(\acute{p}) = 0.$$

We have immediately

$$\lambda = \frac{1}{p} \int_p^{\acute{p}} f(t) t dt > 0 \quad \text{ on } [\grave{p}, \acute{p}).$$

Maximising the Hamiltonian, we obtain $C_2 = W$. u_2 can also be found if we notice that $\lambda(\grave{p}) \neq 0$ and thus $\Pi_2(\grave{p}) = 0$. R_2 can be retrieved as well. It is easily seen that the two original constraints ignored are satisfied.

Now we have found C_2, R_2 and u_2 on $[\grave{p}, \acute{p}]$ but the implementability conditions 12 to 15 require us to find them on the whole interval $[\underline{p}, \overline{p}]$.

Note that although C_2, R_2 and u_2 found above are defined only on $[\grave{p}, \acute{p}]$, Lender 2 has no means to prevent borrowers of types outside $[\grave{p}, \acute{p}]$ from choosing the contract

$(C_2, R_2) = (W, \frac{1}{\grave{p}}\big(L_2 + I\big) + \big(1 - \frac{1}{\grave{p}}\big)W)$ since the lender cannot identify the borrowers' types. Therefore, the borrowers outside $[\grave{p}, \acute{p}]$ also take the same (C_2, R_2) and come by the utility

$$u_2(p) = pX - \frac{p}{\grave{p}}\big(L_2 + I\big) + \big(\frac{p}{\grave{p}} - 1\big)W - K_2.$$

In this way we have naturally extended the contract to the whole interval. It is seen that if $\grave{p} \neq \underline{p}$, Lender 2's non-negative profit condition 23 is breached. However, it does not occur, for later we will see that $\grave{p} = \underline{p}$.

C The proof of Proposition 5

Everything goes in the same way as in $\mathcal{P}$. We have therefore on $[\grave{p}, \acute{p}]$,

$$\begin{aligned} &C_1(p) = W, \\ &u_1(p) = pX + \big(\frac{p}{\grave{p}} - 1\big)W - \frac{p}{\grave{p}}\big(L_2 + I\big) - \frac{p}{\acute{p}}K_2 + \big(\frac{p}{\acute{p}} - 1\big)K_1. \end{aligned} \tag{27}$$

Now let us turn to Conditions 24, 25 and determine $\grave{p}$ and $\acute{p}$. It is easily seen that

$$-\Pi_1(\grave{p})f(\grave{p}) - \lambda_1(\acute{p})\frac{(L_2 + I - W)\acute{p}}{\grave{p}^2} < 0$$

and thus we have $\grave{p} = \underline{p}$.

Condition 24 becomes

$$\theta(\acute{p}) \ := \ f(\acute{p})\big(\frac{\acute{p}}{\underline{p}}(L_2 + I - W) - (L_1 + I - W) + K_2 - K_1\big) - \frac{\lambda_1(\acute{p})}{\grave{p}}\big(K_2 - K_1\big).$$

Therefore, if $\theta(p) \neq 0$ on $[\underline{p}, \overline{p}]$, $\acute{p} = \overline{p}$ follows from 24. On the other hand, if there is $\acute{p}$ such that $\theta(\acute{p}) = 0$ on $[\underline{p}, \overline{p}]$, it is not necessarily that $\acute{p} = \overline{p}$ but that $\acute{p} \in (\underline{p}, \overline{p}]$.

To complete the solution of $\mathcal{Q}$, we are bound to find the implementable contract C_1 and u_1 on the whole interval $[\underline{p}, \overline{p}]$ We only extend 27 to the whole interval and it can be easily verified that Conditions 17 and 18 are satisfied.

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