Sunspots and Monetary Policy*

Jagjit Chadha†
BNP Paribas and University of Brunel

Luisa Corrado‡
University of Cambridge and University of Rome Tor Vergata

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Abstract

A monetary economy subject to expectational sunspots is prone to instability, in the sense of multiple rational expectations equilibria. We show how to modify the policy rule to guarantee stability in the presence of expectational sunspots. The policy-maker must co-ordinate inflation dynamics by targeting each of lagged, current and expected inflation. We show that this solution maps directly into the timeless perspective by Woodford. Finally, we trace the responses in an artificial sunspot economy to the adoption of our rule and illustrate the extent to which macroeconomic persistence is reduced.

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†BNP Paribas, 10 Harewood Avenue, Marylebone, London NW1 6AA. E-mail: jagjit.chadha@bnpparibas.com. Research Professor at Brunel University and Fellow of the Centre for International Macroeconomics and Finance, Cambridge University.

‡Address for Correspondence. Faculty of Economics, Cambridge University, Sedgwick Site, CB3 9DE, Cambridge, UK. e-mail: lc242@econ.cam.ac.uk. Phone +44-1223-335284.

1 Introduction

The question of how to control an economy in which agents base their current behaviour on forward-looking expectations has recently preoccupied monetary theorists and policy-makers. The main stabilization device in this setting appears to be predictable monetary policy rules, which act to contain destabilizing expectations. Michael Woodford (2005) has gone as far to argue that: 'not only do expectations about policy matter, but, at least under current conditions, very little else matters' (p. 3).

There remains, however, an important debate on the specific form of such rules. The literature has considered how active policy rules make economies prone to unintended equilibrium outcomes, such as a liquidity trap (Benhabib *et al,* 2002). It has also shown how forward-looking models of inflation and output may lead to chaotic dynamics and to indeterminacy (for example, Benhabib *et al*, 2001, 2004). To avoid indeterminacy Woodford (2003a) has argued that the authorities should adopt a forward-looking rule where nominal interest rates respond more than equiproportionally to expected inflation (see also Woodford, 1994; Clarida *et al,* 2000; Levin *et al*, 2001; Chari *et al*, 1998, Schmitt-Grohè and Uribe, 2000). From a theoretical point of view such rules are good approximations of optimal feedback rules (Bernanke and Woodford, 1997, Clarida *et al*, 2000). But Batini and Pearlman (2002) show that such a rule may not be sufficient to rule out indeterminacy as reacting to events that lie far in the future may generate multiple equilibria and dynamic instability.[1]

The challenge of designing rules to control expectations has been taken up by policy-makers, for example Mervyn King (2005),[2] Bernanke (2003)[3] and Trichet (2005). Recently Trichet (2005) has taken this point further and looked at the implications for monetary policy of misled inflation expectations, that is expectations that are dislodged from economic fundamentals. The occurrence of such unfounded overreactions in the market may pose serious risk of instability as 'misled market expectations can amplify and prolong the dynamic response of inflation and real activity to an inflationary or deflationary shock of sufficiently great potency' (p. 3).

Given the importance of this ongoing policy debate, in this paper we consider a more general class of policy rules that are not only designed to stabilize

[1] Batini and Pearlman (2002) use interchangeably the term indeterminate equilibria or sunspot equilibria to identify cases where multiple solutions to the model depend either on extraneous random variables (sunspot) or on more fundamental shocks.

[2] The impact of policy rules on expectation formation has been stressed by Mervyn King: 'A key motivation for the study of monetary policy rules was the insight that if economic agents base their decisions on expectations of the future then the way monetary policy is expected to be conducted in the future affects economic outcomes today. Hence it is very important to think about how policy influences the expectations of the private sector' (p. 5).

[3] Bernanke (2003) stresses how in conducting stabilization policy: 'The central bank must also maintain a strong commitment to keeping inflation –and, hence, public expectations of inflation– firmly under control. Because monetary policy influences inflation with a lag, keeping inflation under control may require the central bank to anticipate future movements in inflation and move preemptively. Hence constrained discretion is an inherently forward-looking policy approach' (p. 1).

expectations tied to fundamentals, as in the canonical literature, but also to control for expectational errors that are not: that is sunspots.[4] For our purposes, we may motivate expectational errors from the possibility that agents may have theories of inflation determination that differ from the model used by policy-makers (see Ireland, 2003).

As Cass and Shell (1983) show, sunspots in expectations may lead to indeterminacy of a rational expectations solution for inflation. Indeed, in the real world it may be difficult to distinguish between an economy driven by sunspots shock only and an economy driven by fundamental *and* sunspot shocks. Beyer and Farmer (2003) have shown that they lead to observationally equivalent determinate and non-determinate reduced form models which means that the policy-makers cannot rule out the possibility of sunspots, e.g. stock price bubbles or housing market bubbles.[5] However, Orphanides and Williams (2005) show that when there are expectational errors, but the process for inflation is learnable, a rise of private inflation expectations beyond those implied by perfect knowledge can be resolved by a forecast-based rule implying a more aggressive response than could be expected in normal conditions. In this case expectational errors could gradually be learnt away.[6]

Carlstrom and Fuerst (2001) have also noted that when the process for inflation is not learnable and the economy is subject to expectational errors a backward-looking component in the policy rule could rule out sunspots. Such policy commits the Central Bank to move future policy rates in response to today's price movements. This timing difference is crucial as the monetary authorities do not move until long after the public has moved. So one suggestion is to target backward inflation in order to give an anchor to monetary policy.

Why do pure forward-looking rules ratify sunspots? As (sunspot) inflation expectations raise, the authorities respond by raising nominal interest rates. In the next period agents will then suffer a surprise deflation and a reduction in output but under a sunspot this will not necessarily bring down inflation expectations. If the authorities major concern is to bring back actual inflation (and output) to equilibrium they have to lower nominal interest rates. This will mean that despite the central bank's initial attempt to stabilize inflation, eventually they have to ratify the sunspot.

But a policy-maker who recognizes the dangers of sunspot driven fluctuations will then want to work out the correct policy and act on more than just one of

[4] The idea of sunspots was introduced by Jevons (1875) who posited a structural relationship between commercial crises and the business cycle. Statistical evidence did not support the posited model and so errors in model selection may be thought of as sunspots.

[5] Beyer and Farmer (2003) show how a wide class of forward-looking models lead to observationally equivalent reduced form models. They make the point that the central bank can conceptually respond to current, past or expected variables such as inflation or the output gap. But for an econometrician it is hard to tell from data which is the form of the policy rule adopted. This lack of identification extends also to the other equations of the structural model. So if the central bank responds to lagged inflation the parameter of a hybrid Phillips curve (which incorporates both expected and past inflation) cannot be identified.

[6] For an analysis of the interaction between sunspots, monetary policy and learning in New-Keynesian models see also Honkapohja and Mitra (2004), Carlstrom and Fuerst (2004) and Evans and McGough (2005).

lagged, current and expected inflation. So what is the correct policy response in the face of sunspot shocks? We will show that targeting each of lagged, current and expected inflation can guarantee stability in the presence of inflation sunspot shocks. In this case the interest rate today will respond positively to expected inflation and current inflation but negatively to past inflation. The anchor to past inflation is required to introduce a stationary control in the forward-looking system. The presence of a backward inflation component which enters with an opposite sign with respect to current (and past) inflation has also the effect of mitigating the inflationary spiral of pure forward-looking rules.[7] On average the central bank rises rates less and this means that inflation is closer to its expected value in the following period, therefore avoiding the self-ratifying sunspot of pure forward-looking rules.

To derive the optimal policy response in presence of sunspots, we design the rule in two stages: first, in order to avoid errors in the formation of future inflation expectations, the rule must be dynamically consistent in each time period and hence history-dependent (Woodford, 2000).[8] Woodford (2005) writes on the superiority of history dependent rules over "let bygones be bygones" rules:

> 'In general the most effective policy (the best outcome from among the set of possible rational-expectations equilibria) requires that policy be conducted in a history dependent way, so that policy at any time depends not only on conditions then (and what it is considered possible to achieve from then on), but also on past conditions, even though these no longer constrain what it is possible to achieve in the present' (p.7).

The timeless targeting rule applies the same first order conditions for inflation and output in every time period and ignores any start-up conditions. This feature implies that there is no change in the central bank model and hence no dynamic inconsistency which may invalidate people expectations. However, the timeless criterion will in general not be sufficient to eliminate expectational sunspots. The second stage requires the sunspot error to be expressed as a function of fundamental shocks, which are in turn restricted to eradicate expectational errors. Following the recommendation of Evans and Honkapohja (2006) the monetary authorities should therefore condition their policy action on the structural equations of the model characterizing fundamental shocks and endogenous variables.[9] As we work with forward-looking models whose equations also include expectations of future endogenous variables (possibly subject to expectational errors or sunspots) the rule that we derive ensures that in each time period nominal interest rates are set in such a way that we are

[7] Rotemberg and Woodford (1999) also derive optimal generalized rules where explosive growing response of the funds rate to deviation of inflation from target are 'avoided only if subsequent deviations with the opposite sign eventually counteract the effects of an initial deviation' (p. 47).

[8] See McCallum and Nelson (2004) clear exposition.

[9] Evans and Honkapohja (2006) show that right amount of response to expectations also yields determinacy and learnability.

on the rational expectation path for inflation. In this setting monetary policy changes as a result of past shocks 'in such a way to bring about the desired evolution that it was desired that people would expect' (p. 101, Woodford 2000).

If the central bank commits itself to set interest rates in accordance with this reaction function, Backward-Current-Expected (BCE) rule, then the rational expectation equilibrium of a forward-looking system with expectational sunspots is necessarily determinate.

The paper is organized as follows. Section 2 shows how different forms of canonical interest rate rules, which target any of current, past or expected inflation lead to indeterminacy of the solution. Section 3 shows how an expectational sunspot can be eliminated by appropriate use of a BCE policy rule. Section 4 examines the local determinacy region of the system solution. Section 5 illustrates policy experiments on a standard New-Keynesian models with inflation sunspots. Section 6 concludes.

2 A sunspot inflation

In this section, we examine the implications of the possibility that there are expectational errors in inflation for the determinacy of a simple forward-looking model. Let us consider a standard forward-looking inflation model, which we make deterministic save for a sunspot shock:

$$\pi_t = \beta E_t \pi_{t+1} + v_t^{\pi}. \tag{1}$$

where the discount factor, β, is less than one and v_t^{π} is a fundamental shock to inflation. Now consider a shock where exogenous non-fundamental errors drive the difference between actual and expected inflation,

$$\pi_t - E_{t-1}\pi_t = \omega_t, \tag{2}$$

where the sunspot shock ω_t affects economic agents' beliefs. Specifically in response to this shock agents will adjust their expectation, $E_{t-1}\pi_t$, which may therefore deviate from its actual value, π_t (Lubik and Schorfheide, 2003; Beyer and Farmer, 2003). As people may have theories of inflation determination that differ from the true model (Ireland, 2003) we could think of (2) as a representation of misled expectation formation where the sunspot can be interpreted as any non-standard theory of inflation determination.

In structural, or companion, form we can write these two equations as:

$$\begin{bmatrix} 1 & -\beta \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix} \tag{3}$$

which in compact form is:

$$\mathbf{A}\mathbf{X}_t = \mathbf{B}\mathbf{X}_{t-1} + \boldsymbol{\Psi}\boldsymbol{\xi}_t \tag{4}$$

and in reduced form becomes:

$$\mathbf{X}_t = \mathbf{A}^{-1}\mathbf{B}\mathbf{X}_{t-1} + \mathbf{A}^{-1}\boldsymbol{\Psi}\boldsymbol{\xi}_t \quad . \tag{5}$$

We can calculate $\mathbf{A}^{-1}$ simply to give:

$$\mathbf{A}^{-1} = \begin{bmatrix} 0/\beta & -\beta/-\beta \\ -1/\beta & 1/\beta \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\beta^{-1} & \beta^{-1} \end{bmatrix} \tag{6}$$

hence,

$$\mathbf{X}_t = \begin{bmatrix} 0 & 1 \\ 0 & \beta^{-1} \end{bmatrix} \mathbf{X}_{t-1} + \begin{bmatrix} 0 & 1 \\ -\beta^{-1} & \beta^{-1} \end{bmatrix} \mathbf{e}_t \quad . \tag{7}$$

In the above model π_t depends on its expected value $E_t\pi_{t+1}$ so it is non-predetermined. In a determinate equilibrium the non-fundamental shock would be endogenously determined as a function of the fundamental shock in a way that it removes the influence of any explosive root. In this case the equilibrium is indeterminate as there are not enough explosive roots to pin down uniquely the two non-predetermined relationships given by (1) and (2). We can see this by verifying whether the conditions for a determinate equilibrium, described in Appendix A.2, hold in this case. Specifically we will evaluate the eigenvalues of $\mathbf{A}^{-1}\mathbf{B}$ which by construction will ensure stability if both roots lie outside the unit circle. As $\det(\mathbf{A}^{-1}\mathbf{B}) = \mathbf{0}$ one of the necessary conditions to have a determinate equilibrium is not satisfied.

Note that the characteristic polynominal of $\mathbf{A}^{-1}\mathbf{B}$ in the simple sunspot case is $\theta^2 - \beta^{-1}\theta = 0$. And so the eigenvalues $\theta_{1,2}$ are β^{-1} and 0 where stability would require both roots to lie outside the unit circle. This system as it stands has no unique Rational Expectations Equilibrium.

We stochastically simulate the reduced form representation of inflation from the non-predetermined system (7) by considering innovations both in the fundamental shock and in the inflation forecast error and compare it with the expected inflation one year ahead for the US. There continues to be much debate as to whether there are bubbles (or sunspots) in asset prices and/or expectations. As Cass and Shell (1983) show sunspots in expectations may lead to indeterminacy of a rational expectations solution for inflation. Figure 1 simulates such an unstable inflation process in the presence of the inflation sunspot given by (2) in which expectational errors in inflation lead to an accumulation of inflation innovations. As it is evident the solution is unstable and cannot track observed data.

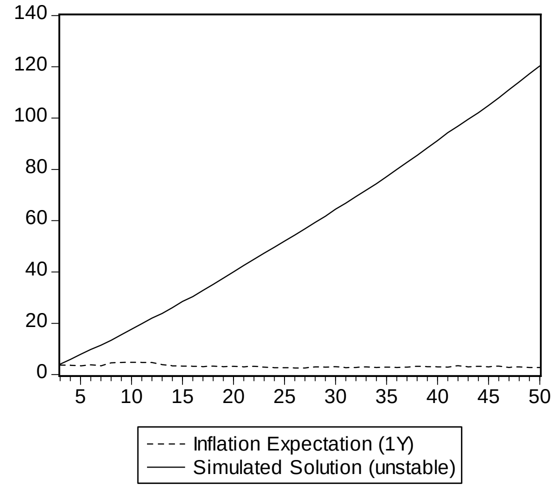

Figure 1: Sunspot Shock and Indeterminacy

2.1 A sunspot inflation with a policy rule

We now examine how policy can act to bring about a unique REE.[10] We model policy as simply acting on either or all of lagged, current or expected inflation.[11] We find that it is possible for policy to introduce stability but that in the presence of sunspot phenomena policy must act against *all* of lagged, current and future inflation.

2.1.1 Current inflation

Let us now consider the forward-looking inflation equation $\pi_t = \beta E_t \pi_{t+1} + k y_t + v_t^{\pi}$ but add a demand equation, $y_t = -(i_t - E_t \pi_{t+1})$, and a policy rule targeting current inflation, $i_t = \phi \pi_t$. Setting $k = 1$ the inflation equation can be rewritten as:

$$\pi_t = (1 + \beta) E_t \pi_{t+1} - \phi \pi_t + v_t^{\pi}. \tag{8}$$

[10] We can show that the dynamics of the economy are observational equivalent under the possibility of sunspots or non-fundamental behaviour as well as fundamental behaviour alone. We can show, it is trivial to complete the exercise, that the reduced forms under cases 1-5 can be observationally identical - available on request.

[11] Bullard and Mitra (2002) also study macroeconomic systems with forward-looking private sector agents and a monetary authority that is trying to control the economy through the use of a linear policy rule. However, they use stability under recursive learning as a criterion for evaluating monetary policy rules in this context.

Let us keep the same sunspot shock driving expectations. In reduced form this system is now:

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & (1+\beta)^{-1}(1+\phi) \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -(1+\beta)^{-1} & (1+\beta)^{-1}(1+\phi) \end{bmatrix} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}. \tag{9}$$

As it stands the system has no predetermined variables. For determinacy we require both roots to lie outside the unit circle. Looking at the necessary and sufficient conditions listed in Appendix A.2 we immediately verify that the first condition $\det(\mathbf{A}^{-1}\mathbf{B}) > \mathbf{1}$ is not fulfilled as the determinant is 0 and hence so must be one of the eigenvalues.

2.1.2 Lagged Inflation

Let us now consider a policy rule targeting past inflation. The reduced form system can be expressed as:

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\phi(1+\beta)^{-1} & (1+\beta)^{-1} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -(1+\beta)^{-1} & (1+\beta)^{-1} \end{bmatrix} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}. \tag{10}$$

In this case the dynamics for inflation depends on its expected value and on its past value, π_{t-1}. The model reduces to a linear stochastic rational expectation model, with a predetermined variable. If the equilibrium is unique there must be one unstable root that allows to pin down the non-predetermined variable $E_t\pi_{t+1}$.

Given that $\det(\mathbf{A}^{-1}\mathbf{B}) = \phi(1+\beta)^{-1} > 0$ it must be that the two eigenvalues have the same sign. But as $\operatorname{tr}(\mathbf{A}^{-1}\mathbf{B}) = (1+\beta)^{-1} < 1$ this implies that they are both less than 1, whereas determinacy requires at least one of the two roots to lie outside the unit circle. In addition we can easily verify that the condition (A.3) listed in Appendix A.1 for the existence of a saddle point equilibrium is not satisfied. So in general a backward policy rule will not be able to rule out expectational sunspots.

2.1.3 Expected inflation and current inflation

The policy rule is targeting both current and expected inflation in equal measure, ϕ. The reduced form system is:

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{(1+\phi)}{(1+\beta-\phi)} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ \frac{1}{\phi-1-\beta} & \frac{(1+\phi)}{(1+\beta-\phi)} \end{bmatrix} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}. \tag{11}$$

Again determinacy in this setting would require both roots to be outside the unit circle but again looking at the first necessary condition listed in Appendix A.2 we require $\det(\mathbf{A}^{-1}\mathbf{B}) > \mathbf{1}$ whereas the determinant remains 0.

2.1.4 Backward inflation and current inflation

The policy rule is targeting both current and backward inflation in equal measure, ϕ. The reduced form system is:

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{\phi}{(1+\beta)} & \frac{1+\phi}{(1+\beta)} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -\frac{\phi}{(1+\beta)} & \frac{1+\phi}{(1+\beta)} \end{bmatrix} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}. \tag{12}$$

This does not help either. The model reduces to a linear stochastic rational expectation model, with a predetermined variable. Determinacy in this setting would require at least one of the two roots to lie outside the unit circle. We can easily verify that the condition (A.3) listed in Appendix A.1 for a positive trace is not satisfied.

2.1.5 Lagged, current and expected inflation

We now consider the general form of the problem where the policy acts against lagged, current and expected inflation in equal measure, ϕ. The reduced form system is:

$$\begin{aligned} \begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -\frac{\phi}{(1+\beta-\phi)} & \frac{(1+\phi)}{(1+\beta-\phi)} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \\ & \begin{bmatrix} 0 & 1 \\ -\frac{1}{(1+\beta-\phi)} & \frac{(1+\phi)}{(1+\beta-\phi)} \end{bmatrix} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}. \end{aligned} \tag{13}$$

We note that the feedback rule includes expected inflation; this is one of the conditions that may avoid indeterminacy in the forward-looking model for inflation. However it is not enough to stabilize a system with expectational sunspots. This can be achieved by including in the feedback rule a backward inflation component, which is equivalent to introducing a stationary control in our model and a unique REE. To see this we verify whether the conditions for a saddle point equilibrium exist in this case. We can see that $\det(\mathbf{A}^{-1}\mathbf{B}) = \frac{\phi}{(1+\beta-\phi)}$ and $\mathrm{tr}(\mathbf{A}^{-1}\mathbf{B}) = \frac{(1+\phi)}{(1+\beta-\phi)}$. Since the trace is positive we verify that the condition (A.3) holds implying:

$$\det(\mathbf{A}^{-1}\mathbf{B}) - \mathrm{tr}(\mathbf{A}^{-1}\mathbf{B}) = \frac{-1}{(1+\beta-\phi)} < -1 \text{ if } \phi > \beta,$$

so if the feedback rule coefficient on backward, current and expected inflation is higher than β, the system is regular. In addition also (A.4) must hold implying:

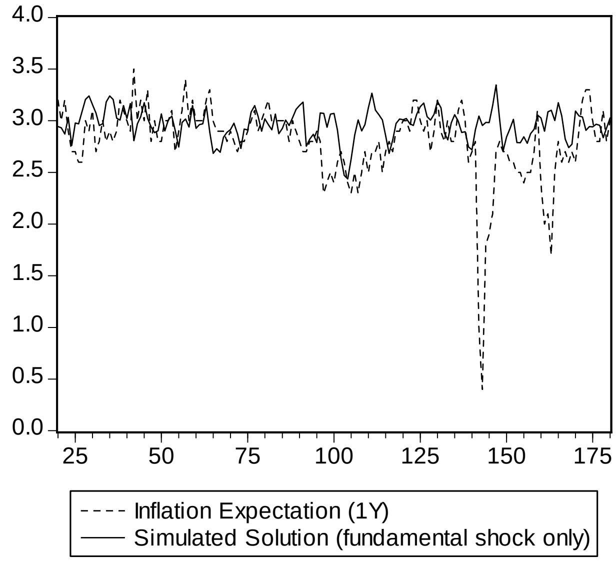

Figure 2: Simulated Solution with Fundamental Shocks and a Stabilizing Policy Rule

$$\det(\mathbf{A}^{-1}\mathbf{B}) + \operatorname{tr}(\mathbf{A}^{-1}\mathbf{B}) = \frac{2\phi + 1}{(1 + \beta - \phi)} > -1,$$

which is always true.

We now analyze how well the reduced form in (13) tracks observed data for inflation expectations in the US. We simulate the data by considering a fundamental shock only, v_t^π, and the fundamental and sunspot shocks (v_t^π plus ω_t). Our findings are in line with Beyer and Farmer (2003) who show that there is an observational equivalence between a world of sunspots plus fundamentals and one of a fundamental alone.

Figure 2 shows the results of the simulation of the same structural model for inflation but where there are no expectational errors and a stabilizing policy rule is in force: here we note that a close correspondence can be found for the one-year expectations of US inflation. The problem for the policy-maker, who cannot know for certain whether there has been a bubble, is to set rule-based policy so that even if there are expectational errors, the inflation process observes a process similar to that contained in Figure 2 and we show such an example in Figure 3: where inflation in the presence of both fundamental and sunspot shocks can still locate a unique rational expectations equilibrium. In the next section we will show how such a rule can be derived.

Proposition 1 *In the presence of sunspot behaviour, the policy rule can*

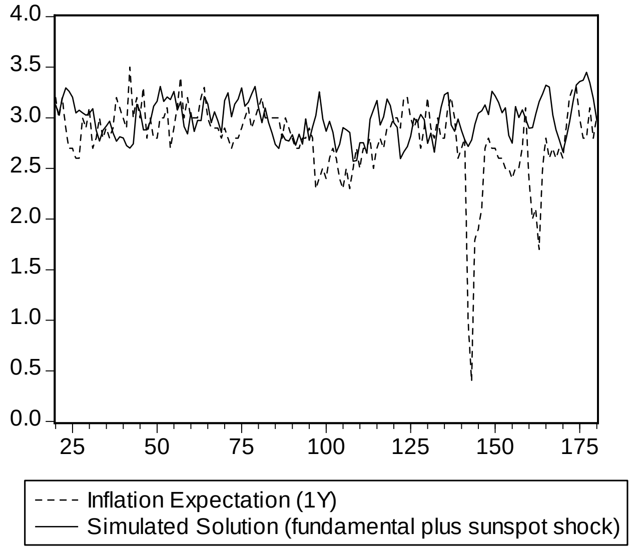

Figure 3: Simulated Solution with Fundamental plus Sunspot Shocks and a Stabilizing Policy Rule

stabilize the economy but may be required to target current, lead and lagged inflation.

In order to avoid self-ratifying sunspots the policy rule must be history dependent. This is equivalent to introduce a stationary-control in the forward-looking system via the backward component of inflation in the feedback rule. This in turn renders the system solution determinate.

In the following section we illustrate how to derive this optimal rule.

3 An Expectation Based Timeless Targeting Criterion

We now show that the rule identified in section 2.1.5 is the optimal response from an expectation based timeless perspective. We first define a loss function for the monetary authority.[12] The central bank's problem at some point in time ($t = 0$) can be expressed as a minimization of the Lagrangian expression:

[12] Evans and Honkapohja (2003) show that a fundamental based policy rule which would be the optimal rule without commitment when private agents have perfectly rational expectations, is unstable if in fact agents follow standard adaptive learning rules. To achieve a monetary policy which both is stable under learning and implements optimal discretionary policy the design of the rule must explicitly take into account of private sector expectations and the economic structure. However the rule derived under learning is not unique.

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[\pi_t^2 + \lambda y_t^2 \right] + \varphi_t \left[\pi_t - \beta E_t \pi_{t+1} - \kappa y_t \right] \right\}, \tag{14}$$

subject to the constraint that the evolution for inflation (in presence of sunspots) represents a possible rational-expectations equilibrium, i.e. that satisfies

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + v_t^{\pi} \tag{15}$$

for all periods $t \geq 1$. We assume that there is no welfare loss resulting from nominal interest rate variation as the policy rule is fully used to offset the inflation sunspot and the other fundamental shocks (see also McCallum and Nelson, 2004; Woodford, 1999). Consequently we omit the constraint on the IS relationship which never binds.

We also assume that output is given by:

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + v_t^y, \tag{16}$$

where v_t^y is a temporary supply shock that occurs only at time t. Hence the relevant constraint for the monetary authorities is that the interest rates are set so that deviations of output and inflation from their target - which are assumed to zero in both cases - are minimized given the constraint on inflation expectations as governed by equation (15).

By taking first order conditions of (14) w.r.t. π_t and y_t:

$$\pi_t + \varphi_t - \varphi_{t-1} = 0 \qquad \text{for } t = 1, 2, ... \tag{17}$$

$$\lambda y_t - \kappa \varphi_t = 0 \qquad \text{for } t = 1, 2, .. . \tag{18}$$

That is the central bank ignores the start-up condition $\pi_1 + \varphi_1 = 0$ and applies (17) and (18) in all periods. In this case there will be no dynamic inconsistency in the central bank decision making process as the relationship between π_2 and y_2 chosen in period 2 agrees with the relationship planned in period 1.

So the evolution of inflation must satisfy:[13]

$$\kappa \pi_t + \lambda y_t = \lambda y_{t-1}. \tag{19}$$

According to (19) the optimal rule with commitment must be dynamically consistent in each time period and hence history dependent (Woodford, 2000). One way to interpret the timeless targeting criterion is to say that the control of

[13] Andrew Blake (2001) has studied modifications of optimal rules in a timeless perspective but this would lead to the same need of coordination across time of rule (19).

a forward-looking system is best achieved with a backward-looking rule which acts as a stationary control for the inflation sunspot.[14]

But we know from Section 2 that such a rule will not be sufficient to stabilize a forward-looking model with expectational errors. The second stage requires the sunspot error to be expressed as a function of fundamental shocks, which are in turn restricted to eradicate expectational errors. To do this the monetary authorities should therefore condition their policy action on the structural equations of the model characterizing fundamental shocks and endogenous variables (Evans and Honkapohja, 2006).

In our case the targeting criterion (19) will be expressed as a function of the nominal interest rates, of the exogenous disturbances and of period t and $t-1$ expectations. To do this we take the REE path for inflation and express y_{t-1} as a function of π_{t-1} and $E_{t-1}\pi_t$:

$$y_{t-1} = \frac{[\pi_{t-1} - \beta E_{t-1}\pi_t]}{\kappa}. \tag{20}$$

Substituting (20) and the structural equations for inflation (15) and output (16) in the timeless criterion (19) we obtain the expectations based reaction function:

$$\begin{aligned} i_t \quad = \quad & \frac{1}{\sigma} E_t y_{t+1} + \left(1 + \frac{\beta\kappa}{\sigma(\kappa^2+\lambda)}\right) E_t \pi_{t+1} + \\ & \frac{\lambda}{\kappa\sigma(\kappa^2+\lambda)} \left(\beta E_{t-1}\pi_t - \pi_{t-1} + \frac{\kappa^2}{\lambda} v_t^\pi\right) + \frac{1}{\sigma} v_t^y. \end{aligned} \tag{21}$$

The interest rate targeting rule implied by the optimal targeting criterion (21) acts twofold on sunspots: (i) it introduces a backward inflation component which acts as a stationary control for the formation of future inflation expectations; (ii) it eradicates the sunspot shocks by expressing them as a function of the fundamentals shocks, v_t^y and v_t^π. If the central bank commits itself to set the interest rate in accordance with this flexible reaction function, then the rational expectation equilibrium is necessarily determinate. This will be clearer in the battery of policy experiments presented in section 5.

3.1 Stability Condition of the Timeless Criterion

We now consider the timeless policy rule (21) in the simple case $\kappa = \sigma = 1$:

$$i_t = \left(1 + \frac{\beta}{(1+\lambda)}\right) E_t\pi_{t+1} + \frac{\lambda}{(1+\lambda)} (\beta E_{t-1}\pi_t - \pi_{t-1}) + v_t^y + \frac{1}{(1+\lambda)} v_t^\pi \tag{22}$$

[14] The optimal targeting criterion, as also stressed by Woodford (2003), suggests that with an optimal time consistent rule it is the rate of change rather than the absolute output level (as in the discretionary case) that should determine acceptable deviations from the long-run inflation target.

The implied targeting rule suggests that to pin down a sunspot shock on inflation authorities should give a weight greater than one on forward-looking inflation. However this is a necessary but not sufficient condition as there should be also sufficient weight on backward-inflation, which should be higher than the weight on current inflation.

The model in its reduced form can be rewritten as:

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\beta^{-1} & 1+\frac{(1+\lambda)}{\lambda\beta} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ \frac{1}{\beta} & \frac{1+\lambda}{\lambda\beta} \end{bmatrix} \begin{bmatrix} v_t^\pi \\ \omega_t \end{bmatrix}. \quad (23)$$

We can easily show that the trace is positive and greater than unity $\mathrm{tr}(\mathbf{A}^{-1}\mathbf{B}) = 1 + \frac{1}{\beta} + \frac{1}{\lambda\beta}$ and $\det(\mathbf{A}^{-1}\mathbf{B}) = \frac{1}{\beta}$. Since the trace is greater than 2 and the determinant is greater than one this means that at least one of the root lies outside the unit circle, therefore pinning down the non-predetermined variable (this can also be shown by calculating the roots of the characteristic equation for $\mathbf{A}^{-1}\mathbf{B}$ by hand). As a further proof of the existence of a saddle point equilibrium we verify that both (A.3) and (A.4) hold:

$$\det(\mathbf{A}^{-1}\mathbf{B}) - \mathrm{tr}(\mathbf{A}^{-1}\mathbf{B}) = -1 - \frac{1}{\lambda\beta} < -1 \quad (24)$$

and

$$\det(\mathbf{A}^{-1}\mathbf{B}) + \mathrm{tr}(\mathbf{A}^{-1}\mathbf{B}) = 1 + \frac{2}{\beta} + \frac{1}{\lambda\beta} > -1. \quad (25)$$

In our case both conditions are fulfilled therefore implying that one of the roots is inside the unit circle.

Since the number of roots of $\mathbf{A}^{-1}\mathbf{B}$ which are less than one in absolute value is equal to the number of predetermined variables then the model is said to be regular. We will denote the characteristic roots of $\mathbf{A}^{-1}\mathbf{B}$ as θ^s and θ^u with $|\theta^s| < 1$ and $|\theta^u| > 1$. We note that we have now one predetermined variable since the optimal rule is targeting both current, expected and *past* inflation, therefore introducing an inertial component in the reduced form equation for inflation.

If the model is regular it will be possible to express the non-fundamental shock ω_t as a function of the fundamental shock, v_t^π which makes the process for inflation stationary. This implies that even in the presence of sunspots a timeless policy rule renders the equilibrium of the model determinate.

Since the model (23) is *regular* and *determinate* we can derive an explicit solution for inflation.

Proposition 2 *With a timeless policy rule the rational expectation solution of the system is regular and determinate and can be expressed as a function of lagged inflation and fundamental shocks, which are in turn restricted to eradicate the expectational error*

$$\pi_t = \theta^s\pi_{t-1} + \frac{\lambda}{(1+\lambda)}\left(1 + \frac{\theta^s}{\theta^u - 1}\right)v_t^\pi \quad (26)$$

Appendix B shows how to derive the result in the above proposition following the method of Beyer and Farmer (2003). The result follows as the reduced form (23) assumes the presence of a backward component of inflation in the optimal policy rule which acts as a stationary control in our forward-looking system. This allow us to express the non-fundamental sunspot shock as

$$\omega_t = \frac{\lambda}{(1+\lambda)}\left(1+\frac{\theta^s}{\theta^u - 1}\right)v_t^{\pi} \quad , \tag{27}$$

i.e. as a function of the fundamental shock therefore pinning down any expectational error.[15]

It is straightforward to show that if there is no inertial component $\det(\mathbf{A}^{-1}\mathbf{B}) = \mathbf{0}$ then the system does not meet the conditions for regularity and determinacy.

4 Numerical Analysis of the System

We now turn to a numerical analysis of our artificial economy by using Sims' (2001) method to derive the system solution. This procedure has the advantage to exploit the notion of the forecast errors introduced in (2). Under determinacy these forecast errors will be a function of the fundamental shocks.

To assess how sunspot shocks are influencing the equilibrium dynamics we introduce the belief shocks ε_t^{π} and ε_t^{y} . As in Lubik and Schorfheide (2003) we assume that sunspots trigger the belief shocks ε_t^{π} and ε_t^{y} that lead to a revision of forecasts. This is possible as we have considered the conditional expectations $E_t\pi_{t+1}$ and $E_t y_{t+1}$ in the vector of endogenous variables. In fact we can always write:

$$\begin{aligned} \pi_t &= E_{t-1}\pi_t + \omega_t^{\pi} \quad , \\ y_t &= E_{t-1}y_t + \omega_t^{y} \quad , \end{aligned} \tag{28}$$

where ω_t is the forecast error between $t-1$ and t. Suppose that based on a sunspot expectations are revised by ε_t^{π} and ε_t^{y} :

$$\begin{aligned} \pi_t &= E_{t-1}\pi_t + \omega_t^{\pi} + \varepsilon_t^{\pi} \ , \\ y_t &= E_{t-1}y_t + \omega_t^{y} + \varepsilon_t^{y} \quad , \end{aligned} \tag{29}$$

Therefore the reduced form described by (15), (16) and (21) can be expressed as:

[15] To derive this result we have used the conditions $\theta^s + \theta^u = \mathrm{tr}(\mathbf{A}^{-1}\mathbf{B})$ and $\theta^s\theta^u = \det(\mathbf{A}^{-1}\mathbf{B})$ which allow us to express $\mathbf{A}^{-1}\mathbf{B}$ as:

$$\begin{bmatrix} 0 & 1 \\ -\theta^s\theta^u & \theta^s + \theta^u \end{bmatrix}$$

$$
\begin{aligned}
E_{t-1}\pi_t &= \beta E_t\pi_{t+1} + \kappa E_{t-1}y_t + v_t^\pi - \omega_t^\pi + \kappa\omega_t^y \quad , \\
E_{t-1}y_t &= E_t y_{t+1} - \sigma i_t + \sigma E_t\pi_{t+1} + v_t^y - \omega_t^y \quad , \\
i_t &= \phi_1 E_t\pi_{t+1} + \phi_2 E_t y_{t+1} + \phi_3 E_{t-1}\pi_t - \phi_4\pi_{t-1} + \phi_2 v_t^\pi + \tfrac{\kappa^2}{\lambda}\phi_4 v_t^y \quad ,
\end{aligned} \tag{30}
$$

where $\phi_1 = (1 + \frac{\beta\kappa}{\sigma(\kappa^2+\lambda)})$, $\phi_2 = \frac{1}{\sigma}$, $\phi_3 = \frac{\lambda\beta}{\kappa\sigma(\kappa^2+\lambda)}$, $\phi_4 = \frac{\lambda}{\kappa\sigma(\kappa^2+\lambda)}$.

The model can be represented as a five dimensional model that includes the conditional expectations $E_t\pi_{t+1}$ and $E_t y_{t+1}$ as endogenous variables. Defining $\mathbf{X}_t = [\pi_t, y_t, i_t, E_t\pi_{t+1}, E_t y_{t+1}]$ the system can be rewritten as:

$$
\mathbf{A}\mathbf{X}_t = \mathbf{B}\mathbf{X}_{t-1} + [\mathbf{\Psi} \;\; \mathbf{\Pi}] \begin{bmatrix} \mathbf{v}_t \\ \boldsymbol{\varepsilon}_t \end{bmatrix} + \mathbf{\Pi}\boldsymbol{\omega}_t \quad , \tag{31}
$$

where the vector of exogenous shocks $\mathbf{e}_t = [v_t^\pi, v_t^y, \varepsilon_t^\pi, \varepsilon_t^y]' = [\mathbf{v}_t \; \boldsymbol{\varepsilon}_t]'$ is composed by the vector of fundamental shocks, $\mathbf{v}_t$, and by the vector of belief shocks, $\boldsymbol{\varepsilon}_t$, and it is serially uncorrelated: in this way the belief shocks are considered as exogenous alongside the fundamental shocks $\mathbf{v}_t$. Finally, the vector $\boldsymbol{\omega}_t = [\omega_t^y, \omega_t^\pi]$ identifies the endogenous sunspot expectational errors. Note that we do not have a vector of constants as all variables are expressed as deviations from steady-state.

The matrices of our system can be expressed as:

$$
\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -\phi_1 & -\phi_2 \\ 0 & 0 & 0 & \beta & 0 \\ 0 & 0 & -\sigma & \sigma & 1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ -\phi_4 & 0 & 0 & \phi_3 & 0 \\ 0 & 0 & 0 & 1 & -\kappa \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\mathbf{\Psi} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \phi_2 & \frac{\kappa^2}{\lambda}\phi_4 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad \mathbf{\Pi} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & -\kappa \\ 0 & 1 \end{bmatrix}.
$$

We now follow the procedure of Sims (2001). The matrix $\mathbf{A}^{-1}\mathbf{B}$ has a Jordan decomposition $\mathbf{P}^{-1}\mathbf{\Theta}\mathbf{P}$ where $\mathbf{P}$ and $\mathbf{P}^{-1}$ are the matrices of eigenvectors whereas $\mathbf{\Theta}$ is the matrix of eigenvalues. Multiplying the system by $\mathbf{P}$ and defining $\mathbf{Z}_t = \mathbf{P}\mathbf{X}_t$ we can rewrite the above expression as:

$$
\mathbf{Z}_{t+1} = \Theta\mathbf{Z}_t + \mathbf{P}\,[\mathbf{\Psi} \quad \mathbf{\Pi}] \begin{bmatrix} \mathbf{v}_t \\ \boldsymbol{\varepsilon}_t \end{bmatrix} + \mathbf{P}\mathbf{\Pi}\boldsymbol{\omega}_t \tag{32}
$$

The solution can be written in decoupled form as:

$$
\begin{bmatrix} \mathbf{Z}_{t+1}^s \\ \mathbf{Z}_{t+1}^u \end{bmatrix} = \begin{bmatrix} \mathbf{\Theta}^s & 0 \\ 0 & \mathbf{\Theta}^u \end{bmatrix} \begin{bmatrix} \mathbf{Z}_{t+1}^s \\ \mathbf{Z}_{t+1}^u \end{bmatrix} + \begin{bmatrix} \boldsymbol{\xi}_{t+1}^s \\ \boldsymbol{\xi}_{t+1}^u \end{bmatrix}, \tag{33}
$$

where s stands for stable and u for unstable and $\boldsymbol{\xi}_{t+1} = \mathbf{P}\left[\boldsymbol{\Psi} \quad \boldsymbol{\Pi}\right]\begin{bmatrix} \mathbf{v}_t \\ \boldsymbol{\varepsilon}_t \end{bmatrix} + \mathbf{P}\boldsymbol{\Pi}\boldsymbol{\omega}_t$.

The unstable block can be iterated forward to give the set of stability conditions for $\mathbf{Z}_t^u = -\sum_{j=0}^{\infty}(\boldsymbol{\Theta}^u)^{-(j+1)}\boldsymbol{\xi}_{t+j}^u$.[16] In order to ensure stability of the forward-looking variables we need to impose the condition $\mathbf{Z}_t^u = 0$:

$$\left[\mathbf{P}_{u.}\boldsymbol{\Psi} \quad \mathbf{P}_{u.}\boldsymbol{\Pi}\right]\begin{bmatrix} \mathbf{v}_t \\ \boldsymbol{\varepsilon}_t \end{bmatrix} + \mathbf{P}_{u.}\boldsymbol{\Pi}\boldsymbol{\omega}_t = 0 \quad , \tag{34}$$

so the sunspot errors are expressed as a function of the fundamental and sunspot shocks (which are exogenous) to ensure stability. Hence the notion of belief shock allows us to generate the full set of stable solutions to the system if (34) is satisfied. This result is isomorphic to (27) derived for the stylized reduced form model (23) where in the stable solution for inflation the sunspot error is expressed as a function of the fundamental shock. If the condition (34) is fulfilled the system is determinate i.e. a solution does *exist* and it is *unique*.

Existence problems arise if the exogenous shocks $\mathbf{v}$ and $\boldsymbol{\varepsilon}$ cannot adjust to offset the endogenous sunspot shock $\boldsymbol{\omega}$. We might expect this to happen if $\mathbf{P}^u$ has more rows than columns (Sims, 2001). This is equivalent to the usual notion that there are existence problems if the number of unstable roots exceeds the number of jump variables.[17]

4.1 Local Determinacy of the System

To analyze the local determinacy of the system and its dynamic properties we start by assigning some standard benchmark values to σ, β and κ. Following Clarida *et al* (1999), Rotemberg and Woodford (1998) and most of standard literature we set the (quarterly) discount factor β to 0.99, implying an annual rate of 4%. We also set the coefficient of relative risk aversion, σ, to 1 and the output elasticity of inflation, κ, equal to 1.22 as in Chari *et al* (1996).[18] We also set the relative weight of output with respect to inflation in the loss function to be equal to 0.5.

Figure 4 indicates whether the solution to (30) is determinate. We fix a grid of admissible values for each of the interest rule coefficients on expected, current and backward inflation ϕ_1, ϕ_3 and ϕ_4 and check whether the solution exists and it is unique. In forward-looking systems characterized by forecast errors caused by sunspot shocks a coefficient greater than unity on expected inflation ($\phi_1 > 1$)

[16] Note that

$$\mathbf{Z}_t^u = (\boldsymbol{\Theta}^u)^{-1}\mathbf{Z}_{t+1}^u - (\boldsymbol{\Theta}^u)^{-1}\boldsymbol{\xi}_t^u = (\boldsymbol{\Theta}^u)^{-2}\mathbf{Z}_{t+2}^u - (\boldsymbol{\Theta}^u)^{-2}\boldsymbol{\xi}_{t+1}^u - (\boldsymbol{\Theta}^u)^{-1}\boldsymbol{\xi}_t^u$$

which gives the result in the text.

[17] In order for the solution to be unique (34) has to pin down not only $\mathbf{P}^u\boldsymbol{\Pi}\boldsymbol{\omega}$ but also all the other error terms in the system that are affected by the expectational sunspot error term $\boldsymbol{\omega}$. That is from the knowledge of $\mathbf{P}^u\boldsymbol{\Pi}\boldsymbol{\omega}$ we must be able to determine $\mathbf{P}^s\boldsymbol{\Pi}\boldsymbol{\omega}$ where $\mathbf{P}^s$ includes the rows of $\mathbf{P}^{-1}$ not included in $\mathbf{P}^u$ (Sims, 2001).

[18] Other values usually employed as benchmark values for κ are 0.05 (Taylor, 1980) and 0.3 which is chosen by Woodford (1999) and Clarida Gali and Gertler (1999).

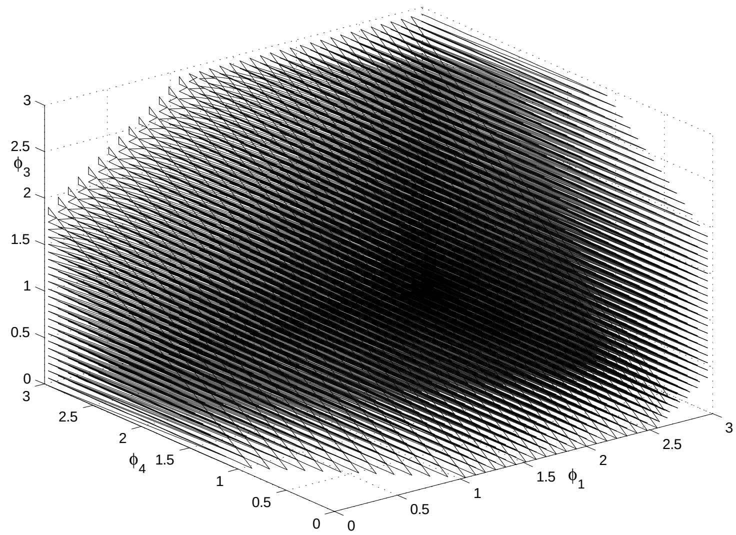

Figure 4: Local Determinacy of System Solutions

is not sufficient to guarantee stability. As proposition 1 states we need all the components of inflation (backward, current and expected) to enter the feedback rule with the coefficients ϕ_1 and ϕ_4 greater than unity. This is equivalent to introduce a stationary-control in the forward-looking system via the coefficient of the backward component of inflation in the feedback rule, ϕ_4. In this case as the darkest region of Figure 4 shows the solution is always determinate.

4.2 Policy Experiments

We now turn to some policy experiments on system (30) by comparing the impulse responses for inflation, output and real interest rates to the fundamental shocks and to the sunspot shock on inflation.[19]

Figure 5 compares the responses of π_t, y_t and $i_t - \pi_t$ to the set of shocks $[v_t^\pi, v_t^y, \varepsilon_t^\pi]$ when we implement the Backward-Current-Expected (BCE) policy rule and when we implement the forward looking Current-Expected (CE) rule.

A cost-push shock, v_t^π, that hits aggregate supply causes a temporary increase in inflation. Nominal interest rates increase sharply in response to the

[19] We solve for $\mathbf{X}_t = \mathbf{G}_1\mathbf{X}_{t-1} + \mathbf{W}_t\mathbf{e}_t$ and therefore can write a VAR in standard MA(∞) form as:

$$\mathbf{X}_t = \sum_{i=0}^{\infty} \mathbf{G}_1^i \mathbf{W}_{t-i}\mathbf{e}_{t-i} = \sum_{i=0}^{\infty} \mathbf{C}_{t-i}\mathbf{e}_{t-i}$$

The responses of $\mathbf{X}_t$ are determined by the rows of G_1^i and $\mathbf{W}_{t-i}$. These are the responses of $\mathbf{X}$ to standard shocks in $\mathbf{e}_t$ i.e. $\mathbf{X}_t = \mathbf{C}_t\mathbf{e}_t + \mathbf{C}_{t-1}\mathbf{e}_{t-1} + \mathbf{C}_{t-2}\mathbf{e}_{t-2}$...where the matrix $\mathbf{C}_k$ has the interpretation $\frac{\partial \mathbf{X}_t}{\partial \mathbf{e}_{t-k}} = \mathbf{C}_{t-k}$ which will be used in our impulse response analysis.

higher levels of current and expected inflation. However because of the higher initial inflation the real interest rates fall after time zero which cause output to increase temporarily in the first two periods. The subsequent deflation real rates will then gradually revert to their long-run level, which in turn will boost output back to equilibrium. The main difference between the BCE rule and the CE rule is that with a BCE rule the inflation dynamics are less persistent and reverts quicker to its long-run equilibrium. Conversely the CE rule is not capable of stabilizing inflation.

In presence of a demand shock, v_t^y, the real interest rate picks up immediately after the shock to avoid inflationary spirals. This causes output to fall temporarily below its long-run level which in turn decreases inflation. The initial shock, which is fully compensated by the increase in interest rates at time zero, is followed by a contraction in output because of higher real interest rates (nominal rates increase more than inflation). The subsequent deflation further amplifies the initial output drop. Output then reverts slowly to its long-run equilibrium as interest rates start decreasing. Note again that with a BCE rule real rates are less persistent and revert very quickly to their long-run equilibrium level. This in turn implies that inflation will respond in a similar fashion and will absorb the effects of the fundamental shock much quicker. So with a history dependent targeting rule the volatility of inflation is lower and the effects of the fundamental shocks less persistent.

The final column of charts in Figure 5, assumes a sunspot shock in inflation, ω_t^π. In this case real rates have to increase sharply in order to offset the inflation sunspot shock. There are clear differences in the responses to a BCE as compared to a CE rule. A BCE rule will respond initially in a more aggressive way to the inflation sunspot (real rates are initially higher with the BCE than with the CE rule). This will cause a sharper fall in inflation which in turn, given the history dependent pattern will cause a reduction in nominal rates (and real rates) in the following periods. In turn the BCE rule will successfully stabilize inflation and given the lower real interest rates in the *interim* period output will temporarily increase (note the hump-shaped response for output in the BCE rule).

5 Conclusions

A pragmatic monetary policy maker may wish to use each of lagged, current and expected information variables when adopting a stabilizing rule. But such an approach cannot be explained by the need to stabilize fundamental shocks alone. It can, however, be explained well when there is the additional possibility of sunspot shocks. Stabilizing leads and lags of inflation may therefore represent a sensible stabilization strategy in the face of sunspots.

We can summarize our findings as follows: first along the lines of Beyer and Farmer (2003) we have shown that there is an observationally equivalence between a world of sunspots *and* fundamentals and one of fundamentals alone, hence sunspots cannot be ruled out. If the process for inflation is learnable,

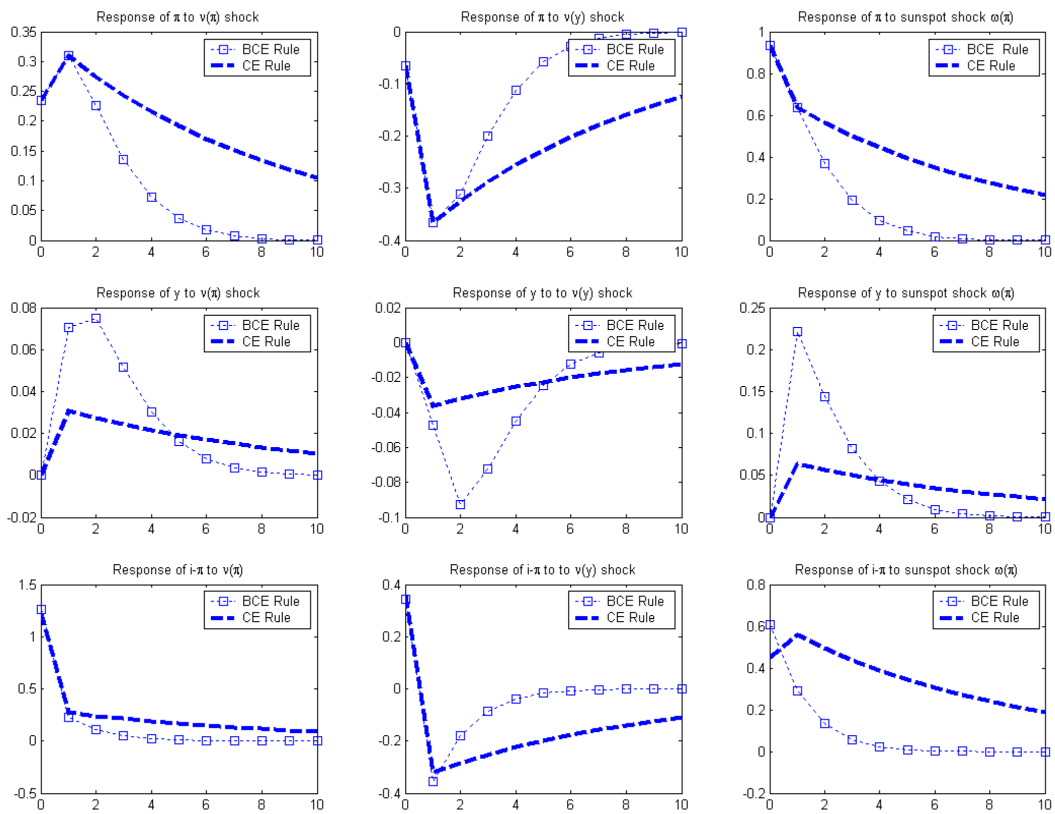

Figure 5: Responses of π_t, y_t and $i_t - \pi_t$ to the fundamental and sunspot shocks $[v_t^\pi, v_t^y, \varepsilon_t^\pi]$

inflation forecast errors could be ruled out by a simple forecast-based rule as suggested by Orphanides and Williams (2005) and Evans and Honkapohja (2001). However, if the process for inflation is not learnable and there are inflation expectational errors we prove that the intuition of Calmstrom and Fuerst (2000, 2001) is that a rule with a backward inflation component is more likely to produce stability than a forward-looking rule.

In this paper we take this point further and try to derive a more general class of policy rules able to deal with forecast errors when the process for inflation is not learnable. The policy implication of the optimal rule derived - the BCE rule - is that the central bank should coordinate its stabilization effort by targeting *all* of lagged, current and expected inflation as this brings about stability in the presence of a sunspot. The BCE rule derived in the paper is isomorphic to the timeless policy rule of Woodford (2003b) and to the expectational rule of Evans and Honkapohja (2006). Hence an optimal policy rule will stabilize a forward-looking system prone to expectational errors or sunspots, if it is inertial in the sense that the rule is contained by past behaviour; and if this self-same rule is 'learnable' i.e. meets the supplementary conditions for stability and uniqueness.

Our paper sums up these results to think about the policy-maker's problem in the possible presence of sunspot shocks. In particular we show that if sunspots cannot be ruled out then co-ordinating stabilization policy across time will contain any deleterious effects.

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A Some Simple Rules for Establishing Stability in 2x2 Matrix for a Difference Equation

Let us define the polynomial function $p(\lambda)$

$$p(\lambda) = \lambda^2 - \lambda \mathrm{tr}(\mathbf{A}^{-1}\mathbf{B}) + \det(\mathbf{A}^{-1}\mathbf{B}) = 0, \tag{A.1}$$

where λ will be a root of the equation when the polynomial equals zero. We therefore draw this polynomial in $\mathrm{tr}(\mathbf{A}^{-1}\mathbf{B}) = \mathrm{tr}(\bullet)$, $\det(\mathbf{A}^{-1}\mathbf{B}) = \det(\bullet)$ space for λ as a root:

$$\det(\bullet) = -\lambda^2 + \lambda \mathrm{tr}(\bullet). \tag{A.2}$$

So for $\lambda = 1$, this line be positive and for $\lambda = -1$ the line will be negative. The trace and determinant of a given 2x2 matrix will then determine how many eigenvalues will lie inside or outside the unit circle.

A.1 Case 1 - saddle point: $\lambda_1 < |1|$ and $\lambda_2 > |1|$.

No restrictions of either trace or determinant, *per se.*

For a positive trace:

$$\det(\bullet) < -1 + tr(\bullet) \tag{A.3}$$

$$\det(\bullet) > -1 - tr(\bullet). \tag{A.4}$$

As in (Woodford, 2003a) we observe that $\text{tr}(\bullet) = \lambda_1 + \lambda_2$ and $\det(\bullet) = \lambda_1\lambda_2$. Conditions (A.3) and (A.4) imply respectively that $(\lambda_1 - 1)(\lambda_2 - 1) < 0$ and $(\lambda_1 + 1)(\lambda_2 + 1) > 0$. Hence the two roots are on the same side of 1 but one is greater than 1 and the other is less than 1.

For a negative trace:

$$\det(\bullet) > -1 + tr(\bullet) \tag{A.5}$$

$$\det(\bullet) < -1 - tr(\bullet). \tag{A.6}$$

This implies that $(\lambda_1 - 1)(\lambda_2 - 1) > 0$ and $(\lambda_1 + 1)(\lambda_2 + 1) < 0$. Hence the two roots are on the same side of -1 but one is less than -1 and the other is greater than -1.

A.2 Case 2: $\lambda_{1,2} > |1|$ i.e. eigenvalues both outside the unit circle

For a positive trace:

$$\det(\bullet) > 1 \tag{A.7}$$

$$tr(\bullet) > 2 \tag{A.8}$$

$$\det(\bullet) > -1 + tr(\bullet) \tag{A.9}$$

$$\det(\bullet) > -1 - tr(\bullet). \tag{A.10}$$

Proof. Conditions (A.7)-(A.10) imply:

$$\lambda_1\lambda_2 > 1 \tag{A.11}$$

$$\lambda_1 + \lambda_2 > 2 \tag{A.12}$$

$$(\lambda_1 - 1)(\lambda_2 - 1) > 0 \tag{A.13}$$

$$(\lambda_1 + 1)(\lambda_2 + 1) > 0. \tag{A.14}$$

In case of real roots (A.13) and (A.14) imply that both roots are on the same side of 1. Considering the positive trace condition in (A.12) they will be both outside the unit circle. ■

For a negative trace:

$$\det(\bullet) > 1 \tag{A.15}$$

$$tr(\bullet) < -2 \tag{A.16}$$

$$\det(\bullet) > -1 + tr(\bullet) \tag{A.17}$$

$$\det(\bullet) > -1 - tr(\bullet). \tag{A.18}$$

Conditions (A.17) and (A.18) are equivalent to (A.13) and (A.14). Considering the negative trace this implies that both roots are on the same side of -1.

In the final case, where the determinant is less than -1, the eigenvalues take opposite signs:

$$\det(\bullet) < -1 \tag{A.19}$$

$$\det(\bullet) < -1 + tr(\bullet) \tag{A.20}$$

$$\det(\bullet) < -1 - tr(\bullet). \tag{A.21}$$

Conditions (A.20) and (A.21) imply that $(\lambda_1 - 1)(\lambda_2 - 1) < 0$ and $(\lambda_1 + 1)(\lambda_2 + 1) < 0$ so the roots must lie on opposite sides of both -1 and 1. Thus one root must lie below -1 while the other is above 1.

B Solution for Inflation

In the simple case $\kappa = \sigma = 1$, by replacing the interest rate rule (22) and the demand equation (16) in (15) we derive the reduced form equation for inflation. By combining the latter with the sunspot shock (2):

$$\underbrace{\begin{bmatrix} 1 & -\frac{\lambda\beta}{(1+\lambda)} \\ 1 & 0 \end{bmatrix}}_{A} \begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\lambda}{(1+\lambda)} & -\frac{\lambda\beta}{(1+\lambda)} \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{\lambda}{(1+\lambda)} & 0 \\ 0 & 1 \end{bmatrix}}_{\Psi} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}. \tag{B.1}$$

Multiplying by A^{-1}:

$$\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} = A^{-1}B \begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix} + A^{-1} \begin{bmatrix} \frac{\lambda}{(1+\lambda)} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}, \tag{B.2}$$

where $A^{-1} = \begin{bmatrix} 0 & 1 \\ -\frac{(1+\lambda)}{\lambda\beta} & \frac{1+\lambda}{\lambda\beta} \end{bmatrix}$ and $A^{-1}B = \begin{bmatrix} 0 & 1 \\ -\beta^{-1} & 1+\frac{(1+\lambda)}{\lambda\beta} \end{bmatrix}$.

The eigenvalues of $A^{-1}B$ are related to the parameters by the equations:

$$\begin{aligned} \theta^s + \theta^u &= 1 + \frac{1+\lambda}{\lambda\beta} \\ \theta^s\theta^u &= \beta^{-1}. \end{aligned} \tag{B.3}$$

So we can rewrite the above expression as:

$$A^{-1}B = \begin{bmatrix} 0 & 1 \\ -\theta^s\theta^u & \theta^s + \theta^u \end{bmatrix}. \tag{B.4}$$

We have shown in the text that one of the roots lies outside the unit circle and another inside. Therefore we assume that $\theta^u > 1$ and $\theta^s < 1$.

The two eigenvectors of $A^{-1}B$ are equal to:

$$\begin{aligned} \begin{bmatrix} 0 & 1 \\ -\theta^s\theta^u & \theta^s+\theta^u \end{bmatrix} \begin{bmatrix} 1 \\ v_1 \end{bmatrix} &= \theta^s \begin{bmatrix} 1 \\ v_1 \end{bmatrix} \Longrightarrow v_1 = \theta^s \\ \begin{bmatrix} 0 & 1 \\ -\theta^s\theta^u & \theta^s+\theta^u \end{bmatrix} \begin{bmatrix} 1 \\ v_2 \end{bmatrix} &= \theta^u \begin{bmatrix} 1 \\ v_2 \end{bmatrix} \Longrightarrow v_2 = \theta^u. \end{aligned} \tag{B.5}$$

The matrix $A^{-1}B$ can be decomposed as $Q\Theta Q^{-1}$ where Q is the matrix of eigenvectors and Θ is a diagonal matrix which contains the eigenvalues of $A^{-1}B$:

$$\underbrace{\begin{bmatrix} 0 & 1 \\ -\theta^s\theta^u & \theta^s+\theta^u \end{bmatrix}}_{A^{-1}B} = \underbrace{\begin{bmatrix} 1 & 1 \\ \theta^s & \theta^u \end{bmatrix}}_{Q} \underbrace{\begin{bmatrix} \theta^s & 0 \\ 0 & \theta^u \end{bmatrix}}_{\Theta} \underbrace{\begin{bmatrix} \frac{\theta^u}{\theta^u-\theta^s} & -\frac{1}{\theta^u-\theta^s} \\ -\frac{\theta^s}{\theta^u-\theta^s} & \frac{1}{\theta^u-\theta^s} \end{bmatrix}}_{Q^{-1}} \tag{B.6}$$

Using the definition $A^{-1}=\begin{bmatrix} 0 & 1 \\ 1-(\theta^s+\theta^u) & (\theta^s+\theta^u)-1 \end{bmatrix}$ we derive:

$$Q^{-1}A^{-1}=\begin{bmatrix} \frac{(\theta^s+\theta^u)-1}{\theta^u-\theta^s} & -\frac{\theta^s-1}{\theta^u-\theta^s} \\ -\frac{(\theta^s+\theta^u)-1}{\theta^u-\theta^s} & \frac{\theta^u-1}{\theta^u-\theta^s} \end{bmatrix}. \tag{B.7}$$

Premultiply (B.2) by Q^{-1}:

$$\begin{aligned} Q^{-1}\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix} &= Q^{-1}(Q\Theta Q^{-1})\begin{bmatrix} \pi_{t-1} \\ E_{t-1}\pi_t \end{bmatrix}+Q^{-1}A^{-1}\begin{bmatrix} \frac{\lambda}{(1+\lambda)} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix} \\ Z_{t+1} &= \Theta Z_t+\varsigma_t, \end{aligned}$$

where $Z_t=Q^{-1}\begin{bmatrix} \pi_t \\ E_t\pi_{t+1} \end{bmatrix}$ and $\varsigma_t=Q^{-1}A^{-1}\begin{bmatrix} \frac{\lambda}{(1+\lambda)} & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} v_t^{\pi} \\ \omega_t \end{bmatrix}$.

The solution can be written in decoupled form as:

$$\begin{bmatrix} Z_{t+1}^s \\ Z_{t+1}^u \end{bmatrix}=\begin{bmatrix} \theta^s & 0 \\ 0 & \theta^u \end{bmatrix}\begin{bmatrix} Z_t^s \\ Z_t^u \end{bmatrix}+\begin{bmatrix} \varsigma_t^s \\ \varsigma_t^u \end{bmatrix}, \tag{B.8}$$

where

$$\begin{aligned} Z_t^s &= \frac{\theta^u}{\theta^u-\theta^s}\pi_t-\frac{1}{\theta^u-\theta^s}E_t\pi_{t+1} \\ Z_t^u &= -\frac{\theta^s}{\theta^u-\theta^s}\pi_t+\frac{1}{\theta^u-\theta^s}E_t\pi_{t+1} \\ \varsigma_t^s &= \frac{\lambda}{(1+\lambda)}\frac{(\theta^s+\theta^u)-1}{(\theta^u-\theta^s)}v_t^{\pi}-\frac{\theta^s-1}{\theta^u-\theta^s}\omega_t \\ \varsigma_t^u &= -\frac{\lambda}{(1+\lambda)}\frac{(\theta^s+\theta^u)-1}{(\theta^u-\theta^s)}v_t^{\pi}+\frac{\theta^u-1}{\theta^u-\theta^s}\omega_t. \end{aligned} \tag{B.9}$$

The stable block is iterated backwards to derive the stability conditions for the predetermined variables Z^s:

$$Z_{t+1}^s=\theta^s Z_t^s+\zeta_t^s=\sum_{j=0}^{\infty}(\theta^s)^j\,\zeta_{t-j}^s, \tag{B.10}$$

so it can be iterated backwards only if the eigenvalues in θ^s lie inside the unit circle.

The unstable block can be iterated forward to give the set of stability condition for Z:

$$E_tZ_t^u=\frac{1}{\theta^u}E_tZ_{t+1}^u=\left(\frac{1}{\theta^u}\right)^n E_tZ_{t+n}^u. \tag{B.10}$$

If the roots lie outside the unit circle then $\lim_{n\to\infty}\left(\frac{1}{\theta^u}\right)^n=0$ which implies $Z_t^u=0$ and from (B.9):

$$\frac{\theta^s}{(\theta^u - \theta^s)}\pi_t - \frac{1}{\theta^u - \theta^s}E_t\pi_{t+1} = 0 \tag{B.11}$$

hence $E_t\pi_{t+1} = \theta^s\pi_t$. Using this result and the definition of Z_t^s from (B.9) it follows that $Z_t^s = \pi_t$.

Given the definition of Z_{t+1}^u in (B.8) if $Z_t^u = 0$ it must also be $\varsigma_t^u = 0$ which from (B.9) implies:

$$\omega_t = \frac{\lambda}{(1+\lambda)}\left(1 + \frac{\theta^s}{\theta^u - 1}\right)v_t^{\pi}. \tag{B.12}$$

Knowing that $Z_t^s = \pi_t$, considering the definition of ς_t^s from (B.9) and the relationship between ω_t and v_t^{π} as derived in (B.12), the solution for inflation is:

$$\begin{aligned} Z_{t+1}^s &= \theta^s Z_t^s + \varsigma_t^s \\ \pi_t &= \theta^s\pi_{t-1} + \frac{\lambda}{(1+\lambda)}\frac{(\theta^s+\theta^u)-1}{(\theta^u-\theta^s)}v_t^{\pi} - \frac{\theta^s - 1}{\theta^u - \theta^s}\omega_t \\ \pi_t &= \theta^s\pi_{t-1} + \frac{\lambda}{(1+\lambda)}\left(1 + \frac{\theta^s}{\theta^u - 1}\right)v_t^{\pi}. \end{aligned} \tag{B.13}$$