Non-Fragile H_∞ Control with Randomly Occurring Gain Variations, Distributed Delays and Channel Fadings

Zhen'na Li, Zidong Wang, Derui Ding and Huisheng Shu

Abstract

This paper is concerned with the non-fragile H_∞ control problem for a class of discrete-time systems subject to randomly occurring gain variations (ROGVs), channel fadings and infinite-distributed delays. A new stochastic phenomenon (ROGVs), which is governed by a sequence of random variables with a certain probabilistic distribution, is put forward to better reflect the reality of the randomly occurring fluctuation of controller gains implemented in networked environments. A modified stochastic Rice fading model is then exploited to account for both channel fadings and random time-delays in a unified representation. The channel coefficients are a set of mutually independent random variables which abide by any (not necessarily Gaussian) probability density function on $[0, 1]$. Attention is focused on the analysis and design of a non-fragile H_∞ output-feedback controller such that the closed-loop control system is stochastically stable with a prescribed H_∞ performance. Through intensive stochastic analysis, sufficient conditions are established for the desired stochastic stability and H_∞ disturbance attenuation, and the addressed non-fragile control problem is then recast as a convex optimization problem solvable via the semi-definite programme method. An example is finally provided to demonstrate the effectiveness of the proposed design method.

Keywords

Non-fragile H_∞ control; randomly occurring gain variations; channel fadings; infinite-distributed delays.

I. Introduction

In recent years, the study of networked control systems (NCSs) has gradually become an active research area due to the advantages of using networked media in many aspects such as the ease of maintenance and installation, the large flexibility and the low cost. It is well known that the devices in networks are mutually connected via communication cables which are of limited capacity. Therefore, some network-induced phenomena have inevitably emerged in the areas of signal processing and control engineering, of which the most popular one is the communication delays that have attracted a great deal of research effort for the control/filtering problems of NCSs, see e.g. [1–11]. Among various types of time-delays, the distributed delays have recently drawn a growing research interest because of its engineering significance [4, 12, 13], where most corresponding results have been concerned with *continuous-time* systems with continuously distributed delays described by either a finite or infinite integral. Nevertheless, the distributed delays *in the discrete-time setting*

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have not gained adequate research attention yet despite the fact that current digitalized control systems are inherently discrete-time ones, see [14–16] and the references therein. As such, it makes practical sense to focus more attention on the discrete-time NCSs with infinite-distributed delays.

In addition to the communication time-delays, some other network-induced phenomena inevitably emerge in the areas of control engineering and signal processing due to the limited bandwidth of the communication channels with examples including data missing [17–20], quantization [14, 21] and randomly occurring nonlinearities [22, 23]. There is, however, yet another special phenomenon typically induced by wireless networks that has been largely overlooked in the NCS community. Such a phenomenon is commonly known as channel fadings which have a great impact on the wireless channels and therefore constitute one of the most predominant features of wireless communication networks [24]. Generally speaking, two major causes for the fading effects are the multi-path propagation and the shadowing from obstacles. The channel fading phenomenon is widely regarded as a kind of channel unreliability described by a random process reflecting the random changes of phase and amplitude of the transmitted signal [1, 25]. With measurements transmitted through fading channels in a NCS, the overall system performance could deteriorate drastically and, accordingly, it becomes necessary to investigate how the effects from the channel fadings upon the dynamic behaviors of the NCSs can be attenuated. Very recently, some pioneering work has been done on the linear quadratic Gaussian (LQG) control [26] and the Kalman filtering problems [27, 28], and there is still plenty of room for further investigation, say, on the combinational influences from both the distributed delays and the fading channels.

Traditionally, most available controller design approaches rely on the implicit assumption that the designed controller can be accurately implemented [29]. Such an assumption, however, is not always true in reality as the controllers do have a certain degree of imprecisions due to 1) the finite word length in any digital system; and 2) the need for additional tuning of parameters in the final controller implementation. It has been revealed in [29] that a comparatively tiny perturbation in controllers may lead to undesirable oscillatory behavior or even instability, and it is desirable to ensure the insensitivity of the controller to certain parameter perturbations. In the past two decades, considerable research attention has been paid to the *non-fragile controllers* capable of tolerating some level of controller parameter gain variations because of their clear engineering insights in many practical applications (see, e.g. [29–37]). For example, in [30], a non-fragile control strategy has been presented for the output tracking control problem of the longitudinal dynamics of flexible hypersonic air-breathing vehicles (HAVs) model. Moreover, in engineering applications, the prevalence of modern NCSs have resulted in the random nature of the occurrence of the controller gain variations mainly for two reasons: 1) the controller parameters may be randomly varied during transmission or implementation due to network-induced problems such as truncations, saturations, quantizations, disorders or distortions; and 2) certain parameters in the control devices may experience random yet abrupt changes due to unpredictable circumstances such as random network load variations. As such, it is interesting to examine how the randomly occurring gain variations (ROGVs) impact on the overall system performance, where the gain variations of the controller occur probabilistically with certain types and intensity. Note that a similar concept has been proposed in [38] for the synchronization problem. Nevertheless, there is a lack of systematic investigation on the design problem of NCSs with particular emphasis on ROGVs, not to mention the case when both distributed delays and fading channels are also the concerns.

Motivated by the above discussion, in this paper, we consider the non-fragile output feedback H_∞ control problem for discrete-time stochastic systems involving ROGVs, channel fadings and infinitely distributed time-delays. The main contributions of this paper can be highlighted as follows. 1) In the context of output feedback

H_∞ control, the phenomenon of ROGVs is considered that is governed by a sequence of random variables with a known conditional probability. 2) A modified Rice fading model, whose coefficients are mutually independent stochastic variables with the probability density function on the interval $[0, 1]$, is employed to describe the wireless communication networks. 3) This paper represents one of the first few attempts to cope with the non-fragile H_∞ control problem, within a unified framework, for discrete-time stochastic systems subject to simultaneous presence of ROGVs, channel fadings and distributed time-delays.

The rest of this paper is organized as follows. In Section II, a class of discrete-time stochastic systems with ROGVs, channel fadings and infinite-distributed delays are presented. In Section III, some sufficient conditions are established to guarantee the stability and the H_∞ performance of the closed-loop control system. In Section IV, an example is presented to demonstrate the effectiveness of the results obtained. Finally, conclusions are drawn in Section V.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n\times m$ real matrices. The set of all non-positive integers is denoted by $\mathbb{Z}^-$. $l_2([0,\infty);\mathbb{R}^n)$ is the space of square-summable n-dimensional vector functions over $[0,\infty)$. I denotes the identity matrix of compatible dimension. The notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of M. $\mathbb{E}\{x\}$ stands for the expectation of stochastic variable x. $||x||$ describes the Euclidean norm of a vector x. The shorthand $\mathrm{diag}\{M_1, M_2, \cdots, M_n\}$ denotes a block diagonal matrix with diagonal blocks being the matrices M_1, ..., M_n. The symbol $\otimes$ denotes the Kronecker product. In symmetric block matrices, the symbol $*$ is used as an ellipsis for terms induced by symmetry. $B^\perp$ denotes an orthogonal basis for the null space of B^T.

II. Problem Formulation and Preliminaries

Consider the following class of discrete time-delay systems:

$$\begin{cases} x_{k+1} = Ax_k + A_d \sum_{d=1}^{\infty} \mu_d x_{k-d} + Bu_k + Dv_k \\ y_k = Cx_k + Ev_k \\ z_k = Lx_k \\ x_k = \phi_k, \quad \forall k \in \mathbb{Z}^- \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$, $y_k \in \mathbb{R}^{n_y}$, $z_k \in \mathbb{R}^{n_z}$ and $u_k \in \mathbb{R}^{n_u}$ are the state vector, the *transmitted* measurement output (without fading), the controlled output vector and the input vector, respectively. $v_k \in l_2([0,\infty);\mathbb{R}^q)$ is the exogenous disturbance signal. ϕ_k ($\forall\ k \in \mathbb{Z}^-$) is an initial sequence. A, A_d, B, C, D, E and L are known real-valued matrices with appropriate dimensions. The constants $\mu_d \geq 0$ ($d = 1, 2, \cdots, \infty$) satisfy the following convergence condition:

$$\bar{\mu}_d := \sum_{d=1}^{\infty} \mu_d \leq \sum_{d=1}^{\infty} d\mu_d < +\infty \tag{2}$$

and the term $\sum_{d=1}^{\infty} \mu_d x_{k-d}$ represents the so-called infinitely distributed delays in the discrete-time setting.

Remark 1: Distributed time delays have been widely recognized and intensively studied for continuous-time systems [12, 13]. However, the corresponding results for discrete-time systems have been very few due mainly to the difficulty in formulating the distributed delays in a discrete-time domain. The delay term $\sum_{d=1}^{\infty} \mu_d x_{k-d}$

in (1) is the the so-called infinitely distributed delay in the discrete-time setting, which was first proposed in [16] and can be regarded as the counterpart of the infinite integral form $\int_{-\infty}^{t} k_{t-s}x_s ds$ for the continuous-time system. Note that the constants μ_d $(d = 1, 2, \cdots)$ are assumed to satisfy the convergence condition (2) so as to make sure that the terms of the distributed time delays (i.e. $\sum_{d=1}^{\infty} \mu_d x_{k-d}$) as well as the Lyapunov-Krasovskii functional to be constructed later are convergent.

In a networked environment, it is quite common that the measurements y_k are affected by channel fadings during the signal transmission. According to Lth-order Rice fading model in [24], the *received* signal $\tilde{y}_k$ is expressed by

$$\tilde{y}_k = \sum_{s=0}^{\ell} \beta_k^s y_{k-s} + E_y \xi_k \tag{3}$$

where ℓ is a given positive scalar and β_k^s $(s = 0, 1, \cdots, \ell)$ are the channel coefficients which are mutually independent. Furthermore, β_k^s have the probability density function on the interval $[0,\ 1]$ with mathematical expectation $\bar{\beta}_s$ and variance $(\tilde{\beta}_s)^2$. $\xi_k \in l_2([0, \infty); \mathbb{R}^{n_y})$ is an external disturbance and E_y is a known real-valued matrix with appropriate dimension.

Remark 2: The research on the phenomenon of channel fadings has started to gain a momentum for its theoretical and practical significance in the area of signal processing. Traditionally, the channel coefficients in the Lth-order Rice fading model are assumed to be independent and identically distributed Gaussian random variables. In this paper, such an assumption has been relaxed to allow the coefficients in model (3) to be random variables obeying any probabilistic distribution on the interval $[0, 1]$. Note that the stochastic Rice fading model (3) could simultaneously characterize the phenomena of channel fadings and random time-delays.

The traditional output feedback controller is $u_k = K\tilde{y}_k$, where $K \in \mathbb{R}^{n_u \times n_y}$ is the controller gain to be designed. In this paper, as discussed in the introduction, the practical controller within a networked environment is sometimes subject to randomly occurring gain variation. In such a case, the *actual* controller can be described by:

$$u_k = (K + \alpha_k \Delta K)\tilde{y}_k, \tag{4}$$

where the stochastic variable α_k has the expectation $\bar{\alpha}$ and variance $\tilde{\alpha}^2$ and is uncorrelated with β_k^s $(s = 0, 1, \cdots \ell)$. ΔK quantifies the controller gain variation satisfying the following norm-bounded multiplicative form [39]:

$$\Delta K = K H_m F_m E_m \tag{5}$$

where H_m and E_m are known matrices with appropriate dimension and F_m is an unknown matrix satisfying $F_m^T F_m \leq I$.

Remark 3: In real-world systems, the controller gain variations are inevitable due to the actuator degradations and the requirements for re-adjustment of controller gains during the controller implementation process [34]. The models of such uncertain gain variations can be classified into two types: the additive uncertainties proposed by Keel et al. [29] and the multiplicative uncertainties with the form of (5). In addition, for networked systems, the actual values of the component parameters in control devices may experience random yet abrupt changes due mainly to the random fluctuations of the network loads with impact on the controller parameter implementation. In (4), the random variable α_k is exploited to govern the probabilistic appearance of such a controller gain variation.

Denote $\tilde{\beta}_k^s := \beta_k^s - \bar{\beta}_s$, $\tilde{\alpha}_k := \alpha_k - \bar{\alpha}$, and $\zeta_k := [v_k^T \quad \xi_k^T]^T$ with $\{\zeta_k\}_{k\in[-\ell,\ -1]} = 0$. Applying the controller (4) with both (5) and (3) to the system (1), we obtain the closed-loop system as follows:

$$\begin{cases} x_{k+1} = (A + \bar{\beta}_0 B_k C)x_k + \sum_{s=1}^{\ell} \bar{\beta}_s B_k C x_{k-s} + A_d \sum_{d=1}^{\infty} \mu_d x_{k-d} + \sum_{s=0}^{\ell} \tilde{\beta}_k^s B_k C x_{k-s} \\ \qquad + \sum_{s=0}^{\ell} \bar{\beta}_s B_k E \tilde{I}_1 \zeta_{k-s} + \sum_{s=0}^{\ell} \tilde{\beta}_k^s B_k E \tilde{I}_1 \zeta_{k-s} + (B_k \tilde{I}_2 + D\tilde{I}_1)\zeta_k + \tilde{\alpha}_k \bar{\beta}_0 \tilde{B}_k C x_k \\ \qquad + \tilde{\alpha}_k \tilde{\beta}_k^0 \tilde{B}_k C x_k + \tilde{\alpha}_k \sum_{s=1}^{\ell} \bar{\beta}_s \tilde{B}_k C x_{k-s} + \tilde{\alpha}_k \sum_{s=1}^{\ell} \tilde{\beta}_k^s \tilde{B}_k C x_{k-s} \\ \qquad + \tilde{\alpha}_k \sum_{s=0}^{\ell} \bar{\beta}_s \tilde{B}_k E \tilde{I}_1 \zeta_{k-s} + \tilde{\alpha}_k \sum_{s=0}^{\ell} \tilde{\beta}_k^s \tilde{B}_k E \tilde{I}_1 \zeta_{k-s} + \tilde{\alpha}_k \tilde{B}_k \tilde{I}_2 \zeta_k, \\ z_k = L x_k, \\ x_k = \phi_k, \quad \forall\ k \in \mathbb{Z}^-, \end{cases} \tag{6}$$

where $B_k := B(K + \bar{\alpha}\Delta K)$, $\tilde{B}_k := B\Delta K$, $\tilde{I}_1 := [I \quad 0]$ and $\tilde{I}_2 := [0 \quad E_y]$.

The objective of this paper is to design a non-fragile H_∞ controller (4) for the discrete-time systems (1) with both channel fadings (3) and infinitely distributed delays. More specifically, we are interested in looking for the parameter K such that the following requirements are met simultaneously:

a) for $\zeta_k = 0$, the closed-loop system (6) is stochastically stable;

b) under the zero-initial condition, for a given disturbance attenuation level $\gamma > 0$ and all nonzero ζ_k, the output z_k satisfies

$$\sum_{k=0}^{\infty} \mathbb{E}\{||z_k||^2\} \leq \gamma^2 \sum_{k=0}^{\infty} ||\zeta_k||^2. \tag{7}$$

III. Main Results

In this section, by resorting to the Lyapunov functional method and the stochastic analysis technique, sufficient conditions are provided to guarantee the stability and the H_∞ performance for the closed-loop systems (6).

Before proceeding, we introduce the following lemmas that will be used in deriving our main results.

Lemma 1: (Liu et al. [16]) Let $M \in \mathbb{R}^{n_x \times n_x}$ be a positive semidefinite matrix, $x_i \in \mathbb{R}^{n_x}$, and constants $a_i > 0$ $(i = 1, 2, \cdots)$. If the series $\{a_i\}_{i\geq 1}$ is convergent, then we have

$$\Big(\sum_{i=1}^{\infty} a_i x_i\Big)^T M \Big(\sum_{i=1}^{\infty} a_i x_i\Big) \leq \Big(\sum_{i=1}^{\infty} a_i\Big) \sum_{i=1}^{\infty} a_i x_i^T M x_i.$$

Lemma 2: (Boyd et al. [40]) Let $M = M^T$, U and W be real matrices of appropriate dimensions with V satisfying $V^T V \leq I$, then

$$M + UVW + W^T V^T U^T \ < \ 0.$$

if and only if there exists a positive scalar ε such that

$$M + \varepsilon U U^T + \varepsilon^{-1} W^T W \ < \ 0$$

or, equivalently,

$$\Pi = \begin{bmatrix} M & \varepsilon U & W^T \\ \varepsilon U^T & -\varepsilon I & 0 \\ W & 0 & -\varepsilon I \end{bmatrix} < 0.$$

First of all, we provide the following analysis result which serves as a theoretical basis for the subsequent design issue.

Theorem 1: Consider the discrete-time system (1) subject to infinite-distributed delays, channel fadings and ROGVs. Let the prescribed H_∞ performance index $\gamma > 0$ and the controller parameter K be given. The closed-loop system (9) is stochastically stable while achieving the performance constraint (7) if there exist symmetric positive definite matrices P, R_i $(i = 1, 2, \cdots, \ell)$ and Q satisfying the following linear matrix inequality:

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ * & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ * & * & \Pi_{33} & \Pi_{34} \\ * & * & * & \Pi_{44} \end{bmatrix} < 0 \tag{8}$$

where

$$\begin{aligned}
\Lambda_1 &= [\bar{\beta}_1 I, \bar{\beta}_2 I, \cdots, \bar{\beta}_l I], \quad \Lambda_0 = [\bar{\beta}_0 I, \bar{\beta}_1 I, \cdots, \bar{\beta}_l I], \\
\mathcal{I}_{1v} &= \mathrm{diag}\{\tilde{\beta}_1^2, \tilde{\beta}_2^2, \cdots, \tilde{\beta}_l^2\}, \quad \mathcal{I}_{0v} = \mathrm{diag}\{\tilde{\beta}_0^2, \tilde{\beta}_1^2, \cdots, \tilde{\beta}_l^2\}, \\
\Pi_{11} &= (A + \bar{\beta}_0 B_k C)^T P (A + \bar{\beta}_0 B_k C) + \tilde{\beta}_0^2 C^T B_k^T P B_k C \\
&\quad + (\tilde{\alpha}^2 \tilde{\beta}_0^2 + \tilde{\alpha}^2 \bar{\beta}_0^2) C^T \tilde{B}_k^T P \tilde{B}_k C + \sum\nolimits_{j=1}^{\ell} R_j - P + \bar{\mu} Q + L^T L, \\
\Pi_{12} &= (A + \bar{\beta}_0 B_k C)^T P B_k C \Lambda_1 + \tilde{\alpha}^2 \bar{\beta}_0 C^T \tilde{B}_k^T P \tilde{B}_k C \Lambda_1, \quad \Pi_{13} = (A + \bar{\beta}_0 B_k C)^T P A_d, \\
\Pi_{14} &= (A + \bar{\beta}_0 B_k C)^T P (B_k E \tilde{I}_1 \Lambda_0 + B_k \tilde{I}_2 \mathbf{I}_D + D \tilde{I}_1 \mathbf{I}_D) + \tilde{\beta}_0^2 C^T B_k^T P B_k E \tilde{I}_1 \mathbf{I}_D \\
&\quad + \tilde{\alpha}^2 \bar{\beta}_0 C^T \tilde{B}_k^T P \tilde{B}_k E \tilde{I}_1 \Lambda_0 + \tilde{\alpha}^2 \bar{\beta}_0 C^T \tilde{B}_k^T P \tilde{B}_k \tilde{I}_2 \mathbf{I}_D + \tilde{\alpha}^2 \tilde{\beta}_0^2 C^T \tilde{B}_k^T P \tilde{B}_k E \tilde{I}_1 \mathbf{I}_D, \\
\Pi_{22} &= \Lambda_1^T C^T B_k^T P B_k C \Lambda_1 + \mathcal{I}_{1v} \otimes (C^T B_k^T P B_k C) + \tilde{\alpha}^2 \Lambda_1^T C^T \tilde{B}_k^T P \tilde{B}_k C \Lambda_1 \\
&\quad + \tilde{\alpha}^2 \big(\mathcal{I}_{1v} \otimes (C^T \tilde{B}_k^T P \tilde{B}_k C)\big) - \mathrm{diag}\{R_1, R_2, \cdots, R_\ell\}, \\
\Pi_{23} &= \Lambda_1^T C^T B_k^T P A_d, \quad \Pi_{33} = A_d^T P A_d - \bar{\mu}^{-1} Q, \quad \mathbf{I}_D = [I \ 0....0], \\
\Pi_{24} &= \Lambda_1^T C^T B_k^T P (B_k E \tilde{I}_1 \Lambda_0 + B_k \tilde{I}_2 \mathbf{I}_D + D \tilde{I}_1 \mathbf{I}_D) + \big[0 \quad \mathcal{I}_{1v} \otimes (C^T B_k^T P B_k E \tilde{I}_1)\big] \\
&\quad + \tilde{\alpha}^2 \Lambda_1^T C^T \tilde{B}_k^T P (\tilde{B}_k E \tilde{I}_1 \Lambda_0 + \tilde{B}_k \tilde{I}_2 \mathbf{I}_D) + \tilde{\alpha}^2 \big[0 \quad \mathcal{I}_{1v} \otimes (C^T \tilde{B}_k^T P \tilde{B}_k E \tilde{I}_1)\big], \\
\Pi_{34} &= A_d^T P (B_k E \tilde{I}_1 \Lambda_0 + B_k \tilde{I}_2 \mathbf{I}_D + D \tilde{I}_1 \mathbf{I}_D), \\
\Pi_{44} &= (B_k E \tilde{I}_1 \Lambda_0 + B_k \tilde{I}_2 \mathbf{I}_D + D \tilde{I}_1 \mathbf{I}_D)^T P (B_k E \tilde{I}_1 \Lambda_0 + B_k \tilde{I}_2 \mathbf{I}_D + D \tilde{I}_1 \mathbf{I}_D) \\
&\quad + \mathcal{I}_{0v} \otimes (\tilde{I}_1^T E^T B_k^T P B_k E \tilde{I}_1) + \tilde{\alpha}^2 \big(\mathcal{I}_{0v} \otimes (\tilde{I}_1^T E^T \tilde{B}_k^T P \tilde{B}_k E \tilde{I}_1)\big) \\
&\quad + \tilde{\alpha}^2 (\tilde{B}_k E \tilde{I}_1 \Lambda_0 + \tilde{B}_k \tilde{I}_2 \mathbf{I}_D)^T P (\tilde{B}_k E \tilde{I}_1 \Lambda_0 + \tilde{B}_k \tilde{I}_2 \mathbf{I}_D) - \frac{\gamma^2}{\ell + 1} I.
\end{aligned}$$

Proof: For notational simplicity, we denote the following variables:

$$x_k^* := [x_{k-1}^T \ x_{k-2}^T \ \cdots \ x_{k-\ell}^T]^T, \quad \zeta_k^* := [\zeta_k^T \ \zeta_{k-1}^T \ \zeta_{k-2}^T \ \cdots \ \zeta_{k-\ell}^T]^T,$$

$$\chi_{xd} := \sum\nolimits_{d=1}^{\infty} \mu_d x_{k-d}, \quad \tilde{x}_k = \big[x_k^T \ x_k^{*T} \ \chi_{xd}^T\big]^T, \quad \tilde{\eta}_k := \big[x_k^T \ x_k^{*T} \ \chi_{xd}^T \ \zeta_k^{*T}\big]^T.$$

Then, define the following Lyapunov-Krasovskii functional candidate:

$$V_k := V_{1,k} + V_{2,k} + V_{3,k}, \tag{9}$$

where

$$V_{1,k} = x_k^T P x_k, \quad V_{2,k} = \sum_{j=1}^{\ell} \sum_{i=k-j}^{k-1} x_i^T R_j x_i, \quad V_{3,k} = \sum_{d=1}^{\infty} \mu_d \sum_{\tau=k-d}^{k-1} x_\tau^T Q x_\tau. \tag{10}$$

Calculating the difference of $V_{1,k}$ along the trajectory of system (6) and taking the mathematical expectation, we have

$$
\begin{aligned}
&\mathbb{E}\{\Delta V_{1,k}\} := \mathbb{E}\{V_{1,k+1} - V_{1,k}\} \\
= \mathbb{E}\Big\{ & x_k^T (A+\bar{\beta}_0 B_k C)^T P (A+\bar{\beta}_0 B_k C) x_k + 2x_k^T (A+\bar{\beta}_0 B_k C)^T P \Big(\sum_{s=1}^{\ell} \bar{\beta}_s B_k C x_{k-s}\Big) \\
&+ 2x_k^T (A+\bar{\beta}_0 B_k C)^T P \Big(A_d \sum_{d=1}^{\infty} \mu_d x_{k-d}\Big) + 2x_k^T (A+\bar{\beta}_0 B_k C)^T P \Big(\sum_{s=0}^{\ell} \bar{\beta}_s B_k E \tilde{I}_1 \zeta_{k-s}\Big) \\
&+ 2x_k^T (A+\bar{\beta}_0 B_k C)^T P (B_k \tilde{I}_2 + D\tilde{I}_1)\zeta_k + \Big(\sum_{s=1}^{\ell} \bar{\beta}_s B_k C x_{k-s}\Big)^T P \Big(\sum_{s=1}^{\ell} \bar{\beta}_s B_k C x_{k-s}\Big) \\
&+ 2\Big(\sum_{s=1}^{\ell} \bar{\beta}_s B_k C x_{k-s}\Big)^T P \Big(A_d \sum_{d=1}^{\infty} \mu_d x_{k-d}\Big) + 2\Big(\sum_{s=1}^{\ell} \bar{\beta}_s B_k C x_{k-s}\Big)^T P \Big(\sum_{s=0}^{\ell} \bar{\beta}_s B_k E \tilde{I}_1 \zeta_{k-s}\Big) \\
&+ 2\Big(\sum_{s=1}^{\ell} \bar{\beta}_s B_k C x_{k-s}\Big)^T P (B_k \tilde{I}_2 + D\tilde{I}_1)\zeta_k + \Big(A_d \sum_{d=1}^{\infty} \mu_d x_{k-d}\Big)^T P \Big(A_d \sum_{d=1}^{\infty} \mu_d x_{k-d}\Big) \\
&+ 2\Big(A_d \sum_{d=1}^{\infty} \mu_d x_{k-d}\Big)^T P \Big(\sum_{s=0}^{\ell} \bar{\beta}_s B_k E \tilde{I}_1 \zeta_{k-s}\Big) + 2\Big(A_d \sum_{d=1}^{\infty} \mu_d x_{k-d}\Big)^T P (B_k \tilde{I}_2 + D\tilde{I}_1)\zeta_k \\
&+ \sum_{s=0}^{\ell} \tilde{\beta}_s^2 x_{k-s}^T (B_k C)^T P B_k C x_{k-s} + 2\sum_{s=0}^{\ell} \tilde{\beta}_s^2 x_{k-s}^T (B_k C)^T P B_k E \tilde{I}_1 \zeta_{k-s} \\
&+ \Big(\sum_{s=0}^{\ell} \bar{\beta}_s B_k E \tilde{I}_1 \zeta_{k-s}\Big)^T P \Big(\sum_{s=0}^{\ell} \bar{\beta}_s B_k E \tilde{I}_1 \zeta_{k-s}\Big) + 2\Big(\sum_{s=0}^{\ell} \bar{\beta}_s B_k E \tilde{I}_1 \zeta_{k-s}\Big)^T P (B_k \tilde{I}_2 + D\tilde{I}_1)\zeta_k \\
&+ \sum_{s=0}^{\ell} \tilde{\beta}_s^2 \zeta_{k-s}^T (B_k E \tilde{I}_1)^T P B_k E \tilde{I}_1 \zeta_{k-s} + \zeta_k^T (B_k \tilde{I}_2 + D\tilde{I}_1)^T P (B_k \tilde{I}_2 + D\tilde{I}_1)\zeta_k \\
&+ x_k^T \tilde{\alpha}^2 (\bar{\beta}_0 \tilde{B}_k C)^T P (\bar{\beta}_0 \tilde{B}_k C) x_k + 2\tilde{\alpha}^2 x_k^T (\bar{\beta}_0 \tilde{B}_k C)^T P \Big(\sum_{s=1}^{\ell} \bar{\beta}_s \tilde{B}_k C x_{k-s}\Big) \\
&+ 2\tilde{\alpha}^2 x_k^T (\bar{\beta}_0 \tilde{B}_k C)^T P \Big(\sum_{s=0}^{\ell} \bar{\beta}_s \tilde{B}_k E \tilde{I}_1\Big)\zeta_{k-s} + 2\tilde{\alpha}^2 x_k^T (\bar{\beta}_0 \tilde{B}_k C)^T P \tilde{B}_k \tilde{I}_2 \zeta_k \\
&+ x_k^T \tilde{\alpha}^2 \tilde{\beta}_0^2 (\tilde{B}_k C)^T P \tilde{B}_k C x_k + 2\tilde{\alpha}^2 \tilde{\beta}_0^2 x_k^T (\tilde{B}_k C)^T P \tilde{B}_k E \tilde{I}_1 \zeta_k \\
&+ \tilde{\alpha}^2 \Big(\sum_{s=1}^{\ell} \bar{\beta}_s \tilde{B}_k C x_{k-s}\Big)^T P \Big(\sum_{s=1}^{\ell} \bar{\beta}_s \tilde{B}_k C x_{k-s}\Big) + 2\tilde{\alpha}^2 \Big(\sum_{s=1}^{\ell} \bar{\beta}_s \tilde{B}_k C x_{k-s}\Big)^T P \Big(\sum_{s=0}^{\ell} \bar{\beta}_s \tilde{B}_k E \tilde{I}_1 \zeta_{k-s}\Big) \\
&+ 2\tilde{\alpha}^2 \sum_{s=1}^{\ell} \bar{\beta}_s x_{k-s}^T (\tilde{B}_k C)^T P \tilde{B}_k \tilde{I}_2 \zeta_k + \tilde{\alpha}^2 \sum_{s=1}^{\ell} \tilde{\beta}_s^2 x_{k-s}^T (\tilde{B}_k C)^T P \tilde{B}_k C x_{k-s} \\
&+ 2\tilde{\alpha}^2 \sum_{s=1}^{\ell} \tilde{\beta}_s^2 x_{k-s}^T (\tilde{B}_k C)^T P \tilde{B}_k E \tilde{I}_1 \zeta_{k-s} + \tilde{\alpha}^2 \Big(\sum_{s=0}^{\ell} \bar{\beta}_s \tilde{B}_k E \tilde{I}_1 \zeta_{k-s}\Big)^T \\
&\times P \Big(\sum_{s=0}^{\ell} \bar{\beta}_s \tilde{B}_k E \tilde{I}_1 \zeta_{k-s}\Big) + 2\tilde{\alpha}^2 \Big(\sum_{s=0}^{\ell} \bar{\beta}_s \tilde{B}_k E \tilde{I}_1 \zeta_{k-s}\Big)^T P (\tilde{B}_k \tilde{I}_2)\zeta_k \\
&+ \tilde{\alpha}^2 \sum_{s=0}^{\ell} \tilde{\beta}_s^2 \zeta_{k-s}^T (\tilde{B}_k E \tilde{I}_1)^T P \tilde{B}_k E \tilde{I}_1 \zeta_{k-s} + \zeta_k^T \tilde{\alpha}^2 (\tilde{B}_k \tilde{I}_2)^T P (\tilde{B}_k \tilde{I}_2)\zeta_k - x_k^T P x_k \Big\}.
\end{aligned} \tag{11}
$$

By using the Kronecker product operation, it follows from (11) that

$$\begin{aligned}\mathbb{E}\{\Delta V_{1,k}\} = \mathbb{E}\Big\{& x_k^T(A+\bar{\beta}_0 B_k C)^T P(A+\bar{\beta}_0 B_k C)x_k + 2x_k^T(A+\bar{\beta}_0 B_k C)^T P B_k C\Lambda_1 x_k^* \\ &+ 2x_k^T(A+\bar{\beta}_0 B_k C)^T P A_d\chi_{xd} + 2x_k^T(A+\bar{\beta}_0 B_k C)^T P\bar{B}_k\zeta_k^* + x_k^{*T}\Lambda_1^T C^T B_k^T P B_k C\Lambda_1 x_k^* \\ &+ 2x_k^{*T}\Lambda_1^T C^T B_k^T P\bar{B}_k\zeta_k^* + 2x_k^{*T}\Lambda_1^T C^T B_k^T P A_d\chi_{xd} + \chi_{xd}^T A_d^T P A_d\chi_{xd} + 2\chi_{xd}^T A_d^T P\bar{B}_k\zeta_k^* \\ &+ \zeta_k^{*T}\bar{B}_k^T P\bar{B}_k\zeta_k^* + \tilde{\beta}_0^2 x_k^T C^T B_k^T P B_k C x_k + 2\tilde{\beta}_0^2 x_k^T C^T B_k^T P B_k E\tilde{I}_1\mathbf{I}_D\zeta_k^* + x_k^{*T}\tilde{C}_1 x_k^* \\ &+ 2x_k^{*T}\tilde{C}_2\zeta_k^* + \zeta_k^{*T}\tilde{E}_1\zeta_k^* + (\tilde{\alpha}\bar{\beta}_0)^2 x_k^T C^T\tilde{B}_k^T P\tilde{B}_k C x_k + 2\tilde{\alpha}^2\bar{\beta}_0 x_k^T C^T\tilde{B}_k^T P\tilde{B}_k C\Lambda_1 x_k^* \\ &+ 2\tilde{\alpha}^2\bar{\beta}_0 x_k^T C^T\tilde{B}_k^T P\tilde{B}_k E\tilde{I}_1\Lambda_0\zeta_k^* + 2\tilde{\alpha}^2\bar{\beta}_0 x_k^T C^T\tilde{B}_k^T P\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^* + \tilde{\alpha}^2\tilde{\beta}_0^2 x_k^T C^T\tilde{B}_k^T P\tilde{B}_k C x_k \\ &+ 2\tilde{\alpha}^2\tilde{\beta}_0^2 x_k^T C^T\tilde{B}_k^T P\tilde{B}_k E\tilde{I}_1\mathbf{I}_D\zeta_k^* + \tilde{\alpha}^2 x_k^{*T}\Lambda_1^T C^T\tilde{B}_k^T P\tilde{B}_k C\Lambda_1 x_k^* + 2\tilde{\alpha}^2 x_k^{*T}\Lambda_1^T C^T\tilde{B}_k^T P\tilde{B}_k E\tilde{I}_1\Lambda_0\zeta_k^* \\ &+ \tilde{\alpha}^2\zeta_k^{*T}\Lambda_0^T\tilde{I}_1^T E^T\tilde{B}_k^T P\tilde{B}_k E\tilde{I}_1\Lambda_0\zeta_k^* + 2\tilde{\alpha}^2 x_k^{*T}\Lambda_1^T C^T\tilde{B}_k^T P\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^* + \tilde{\alpha}^2\zeta_k^{*T}I_D^T\tilde{I}_2^T\tilde{B}_k^T P\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^* \\ &+ 2\tilde{\alpha}^2\zeta_k^{*T}\Lambda_0^T\tilde{I}_1^T E^T\tilde{B}_k^T P\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^* + \tilde{\alpha}^2 x_k^{*T}\tilde{C}_3 x_k^* + 2\tilde{\alpha}^2 x_k^{*T}\tilde{C}_4\zeta_k^* + \tilde{\alpha}^2\zeta_k^{*T}\tilde{E}_2\zeta_k^* - x_k^T P x_k\Big\}.\end{aligned} \tag{12}$$

where

$$\begin{aligned}\bar{B}_k &= B_k E\tilde{I}_1\Lambda_0 + B_k\tilde{I}_2\mathbf{I}_D + D\tilde{I}_1\mathbf{I}_D, \quad \tilde{C}_1 = \mathcal{I}_{1v}\otimes(C^T B_k^T P B_k C), \\ \tilde{C}_2 &= \big[0 \quad \mathcal{I}_{1v}\otimes(C^T B_k^T P B_k E\tilde{I}_1)\big], \quad \tilde{E}_1 = \mathcal{I}_{0v}\otimes(\tilde{I}_1^T E^T B_k^T P B_k E\tilde{I}_1), \\ \tilde{C}_3 &= \mathcal{I}_{1v}\otimes(C^T\tilde{B}_k^T P\tilde{B}_k C), \quad \tilde{C}_4 = \big[0 \quad \mathcal{I}_{1v}\otimes(C^T\tilde{B}_k^T P\tilde{B}_k E\tilde{I}_1)\big], \\ \tilde{E}_2 &= \mathcal{I}_{0v}\otimes(\tilde{I}_1^T E^T\tilde{B}_k^T P\tilde{B}_k E\tilde{I}_1)\end{aligned}$$

Next, it can be derived that

$$\begin{aligned}\mathbb{E}\{\Delta V_{2,k}\} := \mathbb{E}\{V_{2,k+1} - V_{2,k}\} &= \sum_{j=1}^{\ell}\mathbb{E}\Big\{x_k^T R_j x_k - x_{k-j}^T R_j x_{k-j}\Big\} \\ &= \mathbb{E}\Big\{\sum\nolimits_{j=1}^{\ell} x_k^T R_j x_k - x_k^{*T}\mathrm{diag}\{R_1, R_2, \cdots, R_\ell\}x_k^*\Big\}\end{aligned} \tag{13}$$

and

$$\begin{aligned}\mathbb{E}\{\Delta V_{3,k}\} &:= \mathbb{E}\{V_{3,k+1} - V_{3,k}\} \\ &= \mathbb{E}\Big\{\sum_{d=1}^{\infty}\mu_d\sum_{\tau=k+1-d}^{k} x_\tau^T Q x_\tau - \sum_{d=1}^{\infty}\mu_d\sum_{\tau=k-d}^{k-1} x_\tau^T Q x_\tau\Big\} \\ &= \mathbb{E}\Big\{\bar{\mu}x_k^T Q x_k - \sum_{d=1}^{\infty}\mu_d x_{k-d}^T Q x_{k-d}\Big\}.\end{aligned} \tag{14}$$

From Lemma 1, it can be easily seen that

$$-\sum_{d=1}^{\infty}\mu_d x_{k-d}^T Q x_{k-d} \le -\frac{1}{\bar{\mu}}\Big(\sum_{d=1}^{\infty}\mu_d x_{k-d}\Big)^T Q\sum_{d=1}^{\infty}\mu_d x_{k-d} = -\bar{\mu}^{-1}\chi_{xd}^T Q\chi_{xd} \tag{15}$$

where $\bar{\mu}$ is defined in (2).

Substituting (15) into (14) results in

$$\mathbb{E}\{\Delta V_{3k}\} \le \mathbb{E}\Big\{\bar{\mu}x_k^T Q x_k - \bar{\mu}^{-1}\chi_{xd}^T Q\chi_{xd}\Big\}. \tag{16}$$

Obviously, for $\zeta_k^*=0$, the combination of (12), (13) and (16) shows that

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k|\zeta_k^*=0\}=\mathbb{E}\{V_{k+1}|\zeta_k^*=0\}-\mathbb{E}\{V_k|\zeta_k^*=0\}\\
:=&\ \mathbb{E}\{\Delta V_{1,k}|\zeta_k^*=0\}+\mathbb{E}\{\Delta V_{2,k}|\zeta_k^*=0\}+\mathbb{E}\{\Delta V_{3,k}|\zeta_k^*=0\}\\
\le&\ \mathbb{E}\Big\{x_k^T(A+\bar{\beta}_0B_kC)^TP(A+\bar{\beta}_0B_kC)x_k+2x_k^T(A+\bar{\beta}_0B_kC)^TPB_kC\Lambda_1x_k^*\\
&+2x_k^T(A+\bar{\beta}_0B_kC)^TPA_d\chi_{xd}+x_k^{*T}\Lambda_1^TC^TB_k^TPB_kC\Lambda_1x_k^*+\chi_{xd}^TA_d^TPA_d\chi_{xd}\\
&+2x_k^{*T}\Lambda_1^TC^TB_k^TPA_d\chi_{xd}+\tilde{\beta}_0^2x_k^TC^TB_k^TPB_kCx_k+x_k^{*T}\tilde{C}_1x_k^*\\
&+(\tilde{\alpha}\bar{\beta}_0)^2x_k^TC^T\tilde{B}_k^TP\tilde{B}_kCx_k+2\tilde{\alpha}^2\bar{\beta}_0x_k^TC^T\tilde{B}_k^TP\tilde{B}_kC\Lambda_1x_k^*+\tilde{\alpha}^2\tilde{\beta}_0^2x_k^TC^T\tilde{B}_k^TP\tilde{B}_kCx_k\\
&+\tilde{\alpha}^2x_k^{*T}\Lambda_1^TC^T\tilde{B}_k^TP\tilde{B}_kC\Lambda_1x_k^*+\tilde{\alpha}^2x_k^{*T}\tilde{C}_3x_k^*+\sum\nolimits_{j=1}^{\ell}x_k^TR_jx_k\\
&-x_k^{*T}\mathrm{diag}\{R_1,R_2,\cdots,R_\ell\}x_k^*+\bar{\mu}x_k^TQx_k-\bar{\mu}^{-1}\chi_{xd}^TQ\chi_{xd}-x_k^TPx_k\Big\}\\
=&\ \mathbb{E}\{\tilde{x}_k^T\tilde{\Pi}\tilde{x}_k\}
\end{aligned}\tag{17}$$

where

$$\tilde{\Pi}=\begin{bmatrix}\tilde{\Pi}_{11}&\Pi_{12}&\Pi_{13}\\ *&\Pi_{22}&\Pi_{23}\\ *&*&\Pi_{33}\end{bmatrix}\tag{18}$$

with

$$\begin{aligned}
\tilde{\Pi}_{11}\ =\ &(A+\bar{\beta}_0B_kC)^TP(A+\bar{\beta}_0B_kC)+\tilde{\beta}_0^2C^TB_k^TPB_kC\\
&+(\tilde{\alpha}\bar{\beta}_0)^2C^T\tilde{B}_k^TP\tilde{B}_kC+\tilde{\alpha}^2\tilde{\beta}_0^2C^T\tilde{B}_k^TP\tilde{B}_kC+\sum\nolimits_{j=1}^{\ell}R_j-P+\bar{\mu}Q.
\end{aligned}$$

According to the Schur Complement Lemma, it follows from (8) that $\tilde{\Pi}<0$ holds. Therefore, the system (6) is stochastically stable.

Let us now move to the analysis of the H_∞ performance for the system (6). For this purpose, we establish a cost function

$$\mathcal{J}(n):=\sum_{k=0}^{n}\mathbb{E}\{||z_k||^2\}-\gamma^2\sum_{k=0}^{n}||\zeta_k||^2.\tag{19}$$

In terms of (12), (13), (16) and (17), we obtain

$$\begin{aligned}
&\mathbb{E}\{\Delta V_k\}\\
\le&\ \mathbb{E}\Big\{\tilde{x}_k^T\tilde{\Pi}\tilde{x}_k+2x_k^T(A+\bar{\beta}_0B_kC)^TP\bar{B}_k\zeta_k^*+2x_k^{*T}\Lambda_1^TC^TB_k^TP\bar{B}_k\zeta_k^*\\
&+2\chi_{xd}^TA_d^TP\bar{B}_k\zeta_k^*+2\tilde{\beta}_0^2x_k^TC^TB_k^TPB_kE\tilde{I}_1\mathbf{I}_D\zeta_k^*+\zeta_k^{*T}\bar{B}_k^TP\bar{B}_k\zeta_k^*\\
&+2x_k^{*T}\tilde{C}_2\zeta_k^*+\zeta_k^{*T}\tilde{E}_1\zeta_k^*+2\tilde{\alpha}^2\bar{\beta}_0x_k^TC^T\tilde{B}_k^TP\tilde{B}_kE\tilde{I}_1\Lambda_0\zeta_k^*\\
&+2\tilde{\alpha}^2\bar{\beta}_0x_k^TC^T\tilde{B}_k^TP\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^*+2\tilde{\alpha}^2\tilde{\beta}_0^2x_k^TC^T\tilde{B}_k^TP\tilde{B}_kE\tilde{I}_1\mathbf{I}_D\zeta_k^*\\
&+2\tilde{\alpha}^2x_k^{*T}\Lambda_1^TC^T\tilde{B}_k^TP\tilde{B}_kE\tilde{I}_1\Lambda_0\zeta_k^*+2\tilde{\alpha}^2x_k^{*T}\Lambda_1^TC^T\tilde{B}_k^TP\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^*\\
&+2\tilde{\alpha}^2x_k^{*T}\tilde{C}_4\zeta_k^*+\tilde{\alpha}^2\zeta_k^{*T}\Lambda_0^T\tilde{I}_1^TE^T\tilde{B}_k^TP\tilde{B}_kE\tilde{I}_1\Lambda_0\zeta_k^*+\tilde{\alpha}^2\zeta_k^{*T}\tilde{E}_2\zeta_k^*\\
&+2\tilde{\alpha}^2\zeta_k^{*T}\Lambda_0^T\tilde{I}_1^TE^T\tilde{B}_k^TP\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^*+\tilde{\alpha}^2\zeta_k^{*T}I_D^T\tilde{I}_2^T\tilde{B}_k^TP\tilde{B}_k\tilde{I}_2\mathbf{I}_D\zeta_k^*\Big\}.
\end{aligned}\tag{20}$$

Under the zero-initial condition and the hypothesis of $\{\zeta_k\}_{k\in[-\ell,\ -1]} = 0$, it can be shown from (8) that

$$\begin{aligned}
\mathcal{J}(n) = & \sum_{k=0}^{n}\mathbb{E}||z_k||^2 - \frac{\gamma^2}{\ell+1}\sum_{k=0}^{n}\sum_{s=0}^{\ell}||\zeta_{k-s}||^2 \\
& + \gamma^2\sum_{k=0}^{n}\Big\{\frac{1}{\ell+1}\sum_{s=0}^{\ell}||\zeta_{k-s}||^2 - ||\zeta_k||^2\Big\} \\
\leq & \sum_{k=0}^{n}\mathbb{E}||z_k||^2 - \frac{\gamma^2}{\ell+1}\sum_{k=0}^{n}\sum_{s=0}^{\ell}||\zeta_{k-s}||^2 \\
\leq & \sum_{k=0}^{n}\mathbb{E}\Big\{||z_k||^2 - \frac{\gamma^2}{\ell+1}\sum_{s=0}^{\ell}||\zeta_{k-s}||^2 + \Delta V(k)\Big\} - \mathbb{E}V(n+1) \\
\leq & \sum_{k=0}^{n}\mathbb{E}\Big\{\tilde{\eta}_k^T\Pi\tilde{\eta}_k\Big\} \\
< & \ 0.
\end{aligned} \tag{21}$$

Letting $n \to \infty$, it follows immediately from the above inequality that

$$\sum_{k=0}^{\infty}\mathbb{E}\{||z_k||^2\} \leq \gamma^2\sum_{k=0}^{\infty}||\zeta_k||^2,$$

which completes the proof. ■

Next, in terms of the obtained results of Theorem 1, we aim at designing a controller in the form of (4), i.e., we are interested in determining the controller parameters such that the closed-loop system in (6) is stochastically stable with a guaranteed H_∞ performance. The following theorem provides sufficient conditions for the existence of such non-fragile H_∞ controller for system (6).

Theorem 2: Consider the discrete-time system (1) with non-fragile controller, infinite-distributed delays as well as channel fadings. For the given prescribed H_∞ performance index $\gamma > 0$, the close-loop system (6) is stochastically stable while achieving the performance constraint (7) if there exist symmetric positive definite matrices P, R_i $(i = 1, 2, \cdots, \ell)$ and Q, four matrices Ψ_1, Ψ_2, Ψ_3 and $\bar{K}$, and a positive constant scalar ε satisfying

$$\Upsilon = \begin{bmatrix} \Gamma & * & * & * \\ \Upsilon_{21} & \Upsilon_{22} & * & * \\ 0 & \Upsilon_{32}^T & -\varepsilon I & * \\ \varepsilon\mathcal{N} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{22}$$

where

$$\hat{\mathcal{I}}_{1v} = \mathrm{diag}\Big\{\tilde{\beta}_1, \tilde{\beta}_2, \cdots, \tilde{\beta}_l\Big\}, \quad \hat{\mathcal{I}}_{0v} = \mathrm{diag}\Big\{\tilde{\beta}_0, \tilde{\beta}_1, \cdots, \tilde{\beta}_l\Big\},$$

$$\Omega = [B((B^T B)^{-1})^T \;\; B^{\perp}]^T, \quad \Upsilon_{22} = I \otimes (P - \Psi\Omega - \Omega^T \Psi^T), \quad \tilde{K} = [\bar{K}^T 0]^T,$$

$$\Gamma = \mathrm{diag}\Big\{\sum_{j=1}^{\ell} R_j - P + \bar{\mu}Q + L^T L, \; -\mathrm{diag}\{R_1, R_2, \cdots, R_\ell\}, \; -\bar{\mu}^{-1}Q, \; -\frac{\gamma^2}{\ell+1} I\Big\},$$

$$\Upsilon_{21} = \begin{bmatrix} \Psi\Omega A + \bar{\beta}_0 \tilde{K} C & \tilde{K} C \Lambda_1 & \Psi\Omega A_d & \tilde{K} E \tilde{I}_1 \Lambda_0 + \tilde{K} \tilde{I}_2 \mathbf{I}_D + \Psi\Omega D \tilde{I}_1 \mathbf{I}_D \\ \tilde{\beta}_0 \tilde{K} C & 0 & 0 & \tilde{K} E \tilde{I}_1 \mathbf{I}_D \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \hat{\mathcal{I}}_{1v} \otimes (\tilde{K} C) & 0 & [0 \;\; \hat{\mathcal{I}}_{1v}] \otimes (\tilde{K} E \tilde{I}_1) \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{32} = \begin{bmatrix} 0 & 0 & 0 & \bar{\alpha}\tilde{K} H_m \\ 0 & \bar{\alpha}\tilde{K} H_m & 0 & 0 \\ 0 & \tilde{\alpha}\tilde{K} H_m & 0 & 0 \\ 0 & 0 & 0 & \tilde{\alpha}\tilde{K} H_m \\ 0 & 0 & \bar{\alpha}\hat{\mathcal{I}}_{1v} \otimes (\tilde{K} H_m) & 0 \\ 0 & 0 & \tilde{\alpha}\hat{\mathcal{I}}_{1v} \otimes (\tilde{K} H_m) & 0 \end{bmatrix}, \quad \Psi = \begin{bmatrix} \Psi_1 & \Psi_3 \\ 0 & \Psi_2 \end{bmatrix},$$

$$\mathcal{M} = \begin{bmatrix} 0 & 0 & 0 & \bar{\alpha} B K H_m \\ 0 & \bar{\alpha} B K H_m & 0 & 0 \\ 0 & \tilde{\alpha} B K H_m & 0 & 0 \\ 0 & 0 & 0 & \tilde{\alpha} B K H_m \\ 0 & 0 & \bar{\alpha}\hat{\mathcal{I}}_{1v} \otimes (B K H_m) & 0 \\ 0 & 0 & \tilde{\alpha}\hat{\mathcal{I}}_{1v} \otimes (B K H_m) & 0 \end{bmatrix},$$

$$\mathcal{N} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \tilde{\beta}_0 E_m C & 0 & 0 & \tilde{\beta}_0 E_m E \tilde{I}_1 \mathbf{I}_D \\ 0 & I \otimes (E_m C) & 0 & [0 \;\; I] \otimes (E_m E \tilde{I}_1) \\ \bar{\beta}_0 E_m C & E_m C \Lambda_1 & 0 & E_m (E \tilde{I}_1 \Lambda_0 + \tilde{I}_2 \mathbf{I}_D) \end{bmatrix}.$$

Furthermore, if (22) holds, then the controller gain matrix is given by $K = \Psi_1^{-1}\bar{K}$.

Proof: To begin with, the inequality (8) can be rewritten as

$$\Pi \;=\; \Gamma + \Gamma_1^T P \Gamma_1 + \Gamma_2^T P \Gamma_2 + \Gamma_3^T P \Gamma_3 + \Gamma_4^T (I \otimes P) \Gamma_4 + \Gamma_5^T (I \otimes P) \Gamma_5 + \Gamma_6^T (I \otimes P) \Gamma_6 < 0 \qquad (23)$$

where

$$\Gamma_1 = [\; A + \bar{\beta}_0 B_k C \;\; B_k C \Lambda_1 \;\; A_d \;\; B_k E \tilde{I}_1 \Lambda_0 + B_k \tilde{I}_2 \mathbf{I}_D + D \tilde{I}_1 \mathbf{I}_D \;],$$

$$\Gamma_2 = [\; \tilde{\beta}_0 B_k C \;\; 0 \;\; 0 \;\; \tilde{\beta}_0 B_k E \tilde{I}_1 \mathbf{I}_D \;], \; \Gamma_3 = [\; \tilde{\alpha}\tilde{\beta}_0 \tilde{B}_k C \;\; 0 \;\; 0 \;\; \tilde{\alpha}\tilde{\beta}_0 \tilde{B}_k E \tilde{I}_1 \mathbf{I}_D \;],$$

$$\Gamma_4 = [\; \tilde{\alpha}\bar{\beta}_0 \tilde{B}_k C \;\; \tilde{\alpha}\tilde{B}_k C \Lambda_1 \;\; 0 \;\; \tilde{\alpha}\tilde{B}_k (E \tilde{I}_1 \Lambda_0 + \tilde{I}_2 \mathbf{I}_D) \;],$$

$$\Gamma_5 = [\; 0 \;\; \hat{\mathcal{I}}_{1v} \otimes (B_k C) \;\; 0 \;\; [0 \;\; \hat{\mathcal{I}}_{1v}] \otimes (B_k E \tilde{I}_1) \;],$$

$$\Gamma_6 = [\; 0 \;\; \tilde{\alpha}\hat{\mathcal{I}}_{1v} \otimes (\tilde{B}_k C) \;\; 0 \;\; \tilde{\alpha}[0 \;\; \hat{\mathcal{I}}_{1v}] \otimes (\tilde{B}_k E \tilde{I}_1) \;].$$

Then, applying the Schur Complement Lemma, the above inequality is equivalent to

$$\Xi_k := \begin{bmatrix} \Gamma & \tilde{\Gamma}_{1k}^T \\ * & -I \otimes P^{-1} \end{bmatrix} < 0 \tag{24}$$

where

$$\tilde{\Gamma}_{1k} = \begin{bmatrix} A + \bar{\beta}_0 B_k C & B_k C \Lambda_1 & A_d & B_k E \tilde{I}_1 \Lambda_0 + B_k \tilde{I}_2 \mathbf{I}_D + D \tilde{I}_1 \mathbf{I}_D \\ \tilde{\beta}_0 B_k C & 0 & 0 & \tilde{\beta}_0 B_k E \tilde{I}_1 \mathbf{I}_D \\ \tilde{\alpha} \tilde{\beta}_0 \tilde{B}_k C & 0 & 0 & \tilde{\alpha} \tilde{\beta}_0 \tilde{B}_k E \tilde{I}_1 \mathbf{I}_D \\ \tilde{\alpha} \bar{\beta}_0 \tilde{B}_k C & \tilde{\alpha} \tilde{B}_k C \Lambda_1 & 0 & \tilde{\alpha} \tilde{B}_k (E \tilde{I}_1 \Lambda_0 + \tilde{I}_2 \mathbf{I}_D) \\ 0 & \hat{\mathcal{I}}_{1v} \otimes (B_k C) & 0 & [0 \quad \hat{\mathcal{I}}_{1v}] \otimes (B_k E \tilde{I}_1)\ ] \\ 0 & \tilde{\alpha} \hat{\mathcal{I}}_{1v} \otimes (\tilde{B}_k C) & 0 & \tilde{\alpha} [0 \quad \hat{\mathcal{I}}_{1v}] \otimes (\tilde{B}_k E \tilde{I}_1) \end{bmatrix}.$$

Furthermore, note that Ξ_k can be decomposed as follows:

$$\Xi_k = \Xi + \Delta \Xi_k \tag{25}$$

with

$$\Xi = \begin{bmatrix} \Gamma & \bar{\Gamma}^T \\ * & -I \otimes P^{-1} \end{bmatrix}, \quad \Delta \Xi_k = \begin{bmatrix} 0 & \tilde{\Gamma}_{2k}^T \\ * & 0 \end{bmatrix},$$

$$\bar{\Gamma} = \begin{bmatrix} A + \bar{\beta}_0 BKC & BKC\Lambda_1 & A_d & BKE\tilde{I}_1 \Lambda_0 + BK\tilde{I}_2 \mathbf{I}_D + D\tilde{I}_1 \mathbf{I}_D \\ \tilde{\beta}_0 BKC & 0 & 0 & BKE\tilde{I}_1 \mathbf{I}_D \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \hat{\mathcal{I}}_{1v} \otimes (BKC) & 0 & [0 \quad \hat{\mathcal{I}}_{1v}] \otimes (BKE\tilde{I}_1) \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_{2k} = \begin{bmatrix} \bar{\beta}_0 \bar{\alpha} B\Delta KC & \bar{\alpha} B\Delta KC\Lambda_1 & 0 & \bar{\alpha} B\Delta KE\tilde{I}_1 \Lambda_0 + \bar{\alpha} B\Delta K\tilde{I}_2 \mathbf{I}_D \\ \tilde{\beta}_0 \bar{\alpha} B\Delta KC & 0 & 0 & \bar{\alpha} \tilde{\beta}_0 B\Delta KE\tilde{I}_1 \mathbf{I}_D \\ \tilde{\alpha} \tilde{\beta}_0 B\Delta KC & 0 & 0 & \tilde{\alpha} \tilde{\beta}_0 B\Delta KE\tilde{I}_1 \mathbf{I}_D \\ \tilde{\alpha} \bar{\beta}_0 B\Delta KC & \tilde{\alpha} B\Delta KC\Lambda_1 & 0 & \tilde{\alpha} B\Delta K(E\tilde{I}_1 \Lambda_0 + \tilde{I}_2 \mathbf{I}_D) \\ 0 & \bar{\alpha} \hat{\mathcal{I}}_{1v} \otimes (B\Delta KC) & 0 & \bar{\alpha} [0 \quad \hat{\mathcal{I}}_{1v}] \otimes (B\Delta KE\tilde{I}_1) \\ 0 & \tilde{\alpha} \hat{\mathcal{I}}_{1v} \otimes (B\Delta KC) & 0 & \tilde{\alpha} [0 \quad \hat{\mathcal{I}}_{1v}] \otimes (B\Delta KE\tilde{I}_1) \end{bmatrix}$$

$$= \mathcal{M}(I \otimes F_m)\mathcal{N}.$$

Denoting $\tilde{\mathcal{M}} = [0 \quad \mathcal{M}^T]^T$, $\tilde{\mathcal{N}} = [\mathcal{N} \quad 0]$, we have

$$\Xi_k = \Xi + \Delta \Xi_k = \Xi + \tilde{\mathcal{M}}(I \otimes F_m)\tilde{\mathcal{N}} + \tilde{\mathcal{N}}^T (I \otimes F_m)^T \tilde{\mathcal{M}}^T < 0. \tag{26}$$

In terms of Lemma 2, one has

$$\Xi + \varepsilon^{-1} \tilde{\mathcal{M}} \tilde{\mathcal{M}}^T + \varepsilon^{-1} (\varepsilon \tilde{\mathcal{N}})^T (\varepsilon \tilde{\mathcal{N}}) < 0 \tag{27}$$

Combining (25) and (27) with the use of Schur Complement Lemma, we can see that the inequality (24) is true if

$$\begin{bmatrix} \Gamma & * & * & * \\ \bar{\Gamma} & -I \otimes P^{-1} & * & * \\ 0 & \mathcal{M}^T & -\varepsilon I & * \\ \varepsilon \mathcal{N} & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{28}$$

holds.

On the other hand, it follows from (22) that $\Psi\Omega$ is invertible. As such, pre- and post-multiplying the inequality (28) by $\mathrm{diag}\{I, I\otimes(\Psi\Omega), I, I\}$ and $\mathrm{diag}\{I, I\otimes(\Psi\Omega)^T, I, I\}$, and letting $\tilde{K} := [\bar{K}^T 0]^T = \Psi\Omega BK$, we can obtain

$$\begin{aligned}&\begin{bmatrix}\Gamma & * & * & * \\ (I\otimes(\Psi\Omega))\bar{\Gamma} & -I\otimes(\Psi\Omega P^{-1}\Omega^T\Psi^T) & * & * \\ 0 & (I\otimes(\Psi\Omega)\mathcal{M})^T & -\varepsilon I & * \\ \varepsilon\mathcal{N} & 0 & 0 & -\varepsilon I\end{bmatrix} \\ &= \Upsilon + \mathrm{diag}\Big\{0, I\otimes(\Psi\Omega+\Omega^T\Psi^T-\Psi\Omega P^{-1}\Omega^T\Psi^T-P), 0, 0\Big\} < 0\end{aligned} \tag{29}$$

Because of

$$\Psi\Omega+\Omega^T\Psi^T-\Psi\Omega P^{-1}\Omega^T\Psi^T-P = -(P-\Psi\Omega)P^{-1}(P-\Psi\Omega)^T \le 0,$$

we have the conclusion that the inequality (29) can be satisfied if $\Upsilon < 0$. Therefore, according to Theorem 1, the close-loop system (6) is stochastically stable while achieving the performance constraints (7), which completes the proof. ■

Remark 4: Note that, similar to the analysis in [3], for the standard linear matrix inequality system, the algorithm has a polynomial-time complexity. That is, the number $\mathcal{N}(\varepsilon)$ of flops needed to compute an ε-accurate solution is bounded by $O(\mathcal{M}\mathcal{N}^3\log(\mathcal{V}/\varepsilon))$, where $\mathcal{M}$ is the total row size of the linear matrix inequality system, $\mathcal{N}$ is the total number of scalar decision variables, $\mathcal{V}$ is a data-dependent scaling factor, and ε is relative accuracy set for algorithm. Let us look at the discrete-time stochastic system (1), where the variable dimensions can be seen from $x(k)\in\mathbb{R}^{n_x}$, $y(k)\in\mathbb{R}^{n_y}$, $v(k)\in\mathbb{R}^q$, $u(k)\in\mathbb{R}^u$, $H_m\in\mathbb{R}^{n_h}$ and $E_m\in\mathbb{R}^{n_e}$. ℓ is the order of the Rice fading model. From Theorem 2, we have $\mathcal{M} = 6n_x+3\ell n_x+(\ell+1)(q+n_y)+4n_h+4n_e$ and $\mathcal{N} = 0.5(n_x+1)n_x(\ell+2)+n_u^2+n_x^2-n_xn_u+n_un_y+1$. Therefore, the computational complexity of the algorithms developed can be represented as $O(n_x^7)$. Obviously, the computational complexity of the algorithm is dependent on the variable dimensions, which means that the overall computational burden is mainly caused by the time complexity of performing computations on common mathematical operations. Fortunately, research on the computational complexity of mathematical operations is a very active area in the computational mathematics, optimization and the operations research community, and substantial speed-ups can be expected in the future.

Remark 5: In this paper, the main result established in Theorem 2 contains all the information about the system parameters, the occurring probability of the randomly occurring gain variations (ROGVs) and the statistical information of channel coefficients. The main novelty is twofold: 1) the phenomenon of ROGVs, which is governed by a sequence of random variables with a known conditional probability, is introduced to reflect a more realistic controller characteristic; 2) intensive stochastic analysis is conducted to enforce the non-fragile H_∞ control problem for discrete-time stochastic systems subject to ROGVs, channel fadings, as well as infinite-distributed time-delays within the same framework.

IV. Numerical Example

In this section, we aim to demonstrate the effectiveness of the proposed non-fragile H_∞ controller design scheme. Consider the system (1), where the nominal system matrix A and the measurement output matrix

C are taken from the geared DC motor, which is a component of the MS150 Modular Servo system [42]:

$$A = \begin{bmatrix} 1 & 0.0098 \\ 0 & 0.9653 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

Suppose that, when modeling the MS150 Modular Servo system, there exist control inputs with randomly occurring gain variations (ROGVs), infinite-distributed delays and disturbances. Furthermore, the measurement information could suffer from fadings when it is transmitted through the wireless communication networks. Accordingly, in addition to the main system parameters A and C, we set other parameters as follows:

$$A_d = \begin{bmatrix} 0.06 & -0.036 \\ 0 & 0.03 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ D = \begin{bmatrix} 0.08 \\ -0.04 \end{bmatrix}, \ L = \begin{bmatrix} 0.02 \\ -0.02 \end{bmatrix}^T, \ E = 0.01, \ E_y = 0.25.$$

Let the constant sequence $\{\mu_d\}_{d \in [0,\infty)}$ be chosen as $\mu_d = 2^{-3-d}$. It is easy to check that $\bar{\mu} = \sum_{d=1}^{\infty} \mu_d = 2^{-3} < \sum_{d=1}^{\infty} d\mu_d = 2 < +\infty$, which satisfies the convergence condition (2).

Suppose that the order of the fading model is $\ell = 1$ and the probability density functions of channel coefficients are

$$\begin{aligned} f(\beta_0) &= 0.0005(e^{9.89\beta_0} - 1), \quad && 0 \le \beta_0 \le 1, \\ f(\beta_1) &= 8.5017e^{-8.5\beta_1}, && 0 \le \beta_1 \le 1. \end{aligned}$$

According to the given probability density functions, we can easily obtain that the mathematical expectation $\bar{\beta}_s$ $(s = 0, 1)$ are 0.8991 and 0.1174, and variance $\tilde{\beta}_s$ $(s = 0, 1)$ are 0.0133 and 0.01364.

In this example, the H_∞ performance level γ is taken as 1.2. The stochastic variable α_k obeys a Gaussian distribution with the expectation 0.65 and variance 0.25, and the controller gain perturbation parameters are assumed as

$$E_m = 1.0, \ H_m = 0.5.$$

Using Matlab software with YALMIP 3.0, we can obtain the set of solutions to the non-fragile H_∞ control problem as follows

$$P = \begin{bmatrix} 1.6122 & 0.0357 \\ 0.0357 & 2.0118 \end{bmatrix}, \ \Psi = \begin{bmatrix} 1.5795 & 0.1876 \\ 0 & 4.0432 \end{bmatrix}, \ \tilde{K} = -1.6026,$$

$$R_1 = \begin{bmatrix} 0.6740 & 0.0212 \\ 0.0212 & 0.0036 \end{bmatrix}, \ Q = \begin{bmatrix} 0.1717 & 0.0100 \\ 0.0100 & 0.0691 \end{bmatrix}, \ \varepsilon = 0.3837.$$

Furthermore, we can obtain the controller gain matrix $K = -1.0147$.

To further demonstrate the effectiveness of the designed non-fragile H_∞ controller, assume that the disturbances are given by

$$v_k = 0.5e^{-0.2k}\sin(k), \quad \xi_k = a_k/(k+1),$$

where a_k is generated that obeys uniform distribution over [0 1]. The simulation results are shown in Figs. 1-3. Fig. 1 plots the measurement outputs by sensors and the received signal by the controller, while Fig. 2 depicts the uncontrolled outputs of the open-loop system and the controlled outputs of the closed-loop system, respectively. We can easily find that the closed-loop system has a perfect convergence performance. In Fig. 3, it is further shown the outputs for three different cases of controller gain perturbations, that is $\Delta K = 0$, $\Delta K = KH_mF_mE_m$ and $\Delta K = 2KH_mF_mE_m$ (the perturbations have not been sufficient considered). We can find that the system performance is seriously degraded for the third case.

V. Conclusions

This paper has been concerned with the non-fragile H_∞ control problem for a class of discrete-time systems with ROGVs, channel fadings as well as infinite-distributed delays. The phenomenon of ROGVs has been introduced to account for the random nature of the controller parameter drifts/fluctuations during the implementation in a networked environment, and such ROGVs have been assumed to obey a certain probabilistic distribution. In addition, a modified stochastic Rice fadings model has been considered to cater for the phenomenon of channel fadings, where the probabilistic law for the random channel coefficients is not restricted to Gaussian. By introducing a parameter-independent slack variable with lower-triangular structure and employing the stochastic analysis approach, sufficient conditions have been obtained for the existence of admissible controllers. Finally, an illustrative simulation example has been given to illustrate the effectiveness of the proposed design method. One of the future research topics would be to extend the present results to more complex systems such as descriptor systems and Markovian jump systems [41].

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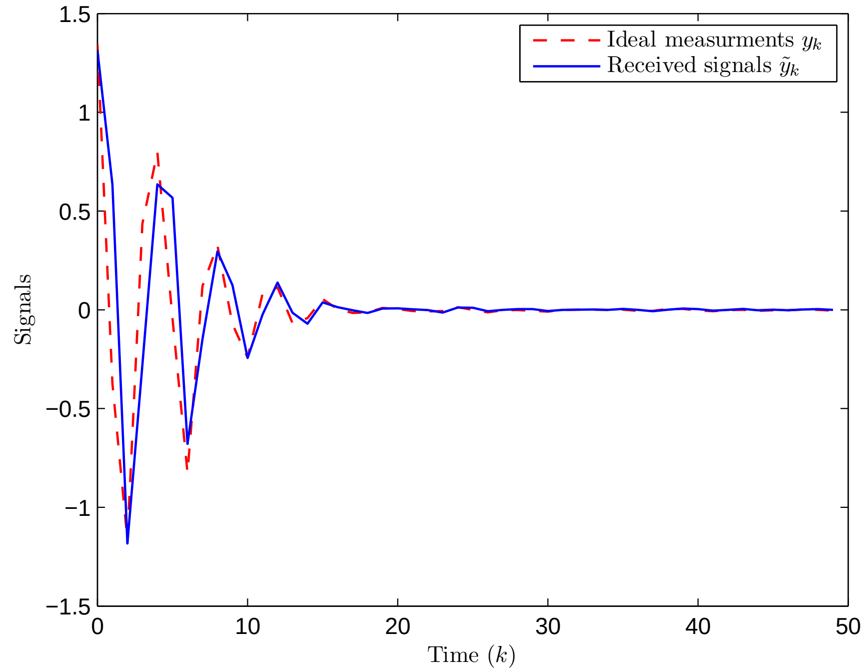

Fig. 1. Measurements/Received signals.

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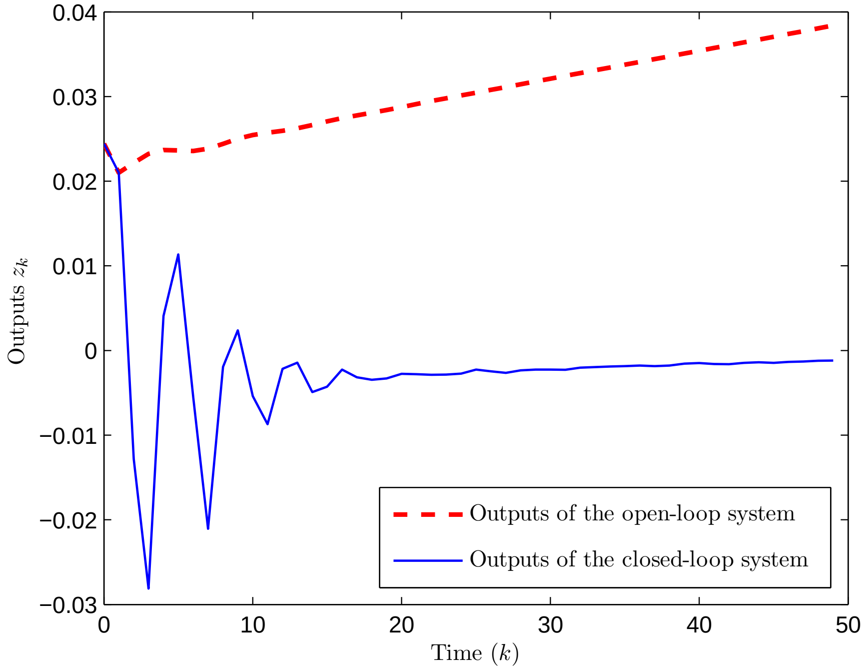

Fig. 2. Outputs z_k.

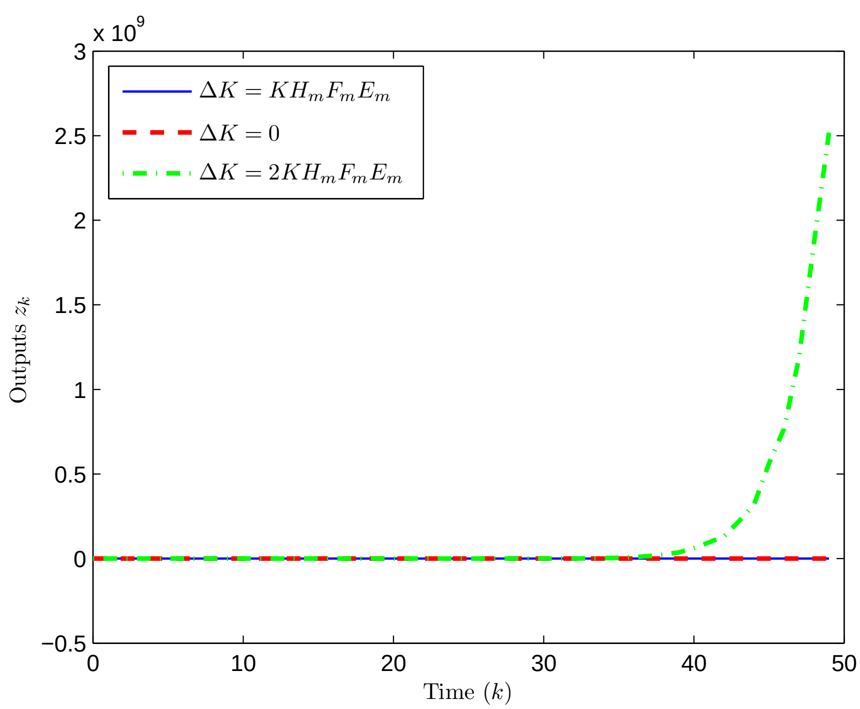

Fig. 3. Outputs z_k with the insufficient estimation of controller gain perturbations.