Uncertainty modelling in power system state estimation

A.K. Al-Othman and M.R. Irving

Abstract: A method for uncertainty analysis in power system state estimation is proposed. The two-step method uses static weighted least-squares analysis to compute 'point' state estimates. Linear programming is then employed to obtain the upper and lower bounds of the uncertainty interval. It is shown that the method can provide useful additional information for both metered and nonmetered elements of the system. The effects of network parameter errors are also studied. For illustrative purposed, the proposed method is tested using the six-bus and IEEE 30-bus standard systems. Results show that the proposed method is an accurate and reliable tool for estimating the uncertainty bounds in power system state estimation.

1 Introduction

The availability of an accurate picture of the system-state is an important aspect of power system operation. While a supervisory control and data acquisition (SCADA) system is capable of providing operators with measured information, a state estimator has the ability to filter the available information creating a more accurate and complete picture of the system conditions. The traditional objective of state estimation is to reduce the effect of measurement errors by utilising the redundancy available in the measurement system. In particular, the objective is to reduce the variance of the estimates and improve their overall accuracy. The other major objectives of state estimation methods include: detection of gross errors, detection of invalid topological information and detection of model parameter errors.

If the errors in the measurements follow a known probability distribution, then the set of feasible estimates can also be modelled by a probability distribution function. Unfortunately, the statistics of the observation errors are difficult to characterise in practice. In such circumstances, it is desirable to provide not just a single 'optimal' estimate of each state variable but also an uncertainty range within which we can be assured that the 'true' state variable must lie. The idea of an uncertainty range is recognisable in engineering practice, where the accuracy of a particular measurement is often described as (for example) plus or minus 2%, rather than by quantifying the standard deviation or variance.

Schweppe [1] introduced the concepts of uncertainty in the general context of engineering analysis, estimation and optimisation. These concepts have been extended and developed and have been applied in a number of areas. Uncertainty modelling in state estimation has been considered in the context of water distribution networks. Bargiela and Hainsworth [2] introduced bounds on the measurements, with an intention to increase the robustness of the estimation. The approach has been developed by Brdys and Chen [3], who introduced the term set bounded state estimation (SBSE). Nagar and Powell [4] have applied concepts from robust control theory and allowed for uncertainty in both the parameters and the measurements. The uncertainty is isolated with the use of a linear fractional transformation, and the problem is formulated as a convex semidefinite programming problem. A linear matrix inequalities approach is then used to solve the semidefinite programming problem.

In conventional state estimation techniques, the accurate knowledge of error statistics of transducers and metering equipments is a prerequisite. However, such information may not be precisely known, leading to less accurate estimates. Providing estimated bounds together with the point estimates gives additional information that can improve the overall quality of the estimation. Knowledge of the limiting values or bounds that apply to measured quantities, facilitates a problem formulation that enables the computation of bounds on state estimates. Thus, the main theme of this paper is to model the uncertainties associated with the measured quantities in a way that defines an interval (range) with respect to their nominal values. The range is governed by the tolerance of the measuring instrument (a quantification of accuracy usually provided by the manufacturer). By utilising appropriate mathematical programming techniques, the confidence interval (or bounds) of the state variables can be computed.

A two-step method is proposed for estimating the uncertainty interval around the system state variables. The first step uses weighted least systems (WLS) as a point estimator to compute the expected values of the state variables. A linear programming formulation is then utilised to find the tightest possible upper and lower bounds on these estimates. It is also shown that the method can provide useful additional information for metered and nonmetered elements of the network.

2 Uncertainty and state estimation

The uncertainty is a parameter associated with the measurement that describes the dispersion of the values

IEE Proceedings online no. 20041167

doi:10.1049/ip-gtd:20041167

Paper first received 16th September 2004. Originally published online: 28th January 2005

The authors are with the Brunel Institute of Power Systems, Brunel University, Uxbridge, Middlesex UB8 3PH, UK

E-mail: abdul.al-othman@brunel.ac.uk, malcolm.irving@brunel.ac.uk

that could reasonably be attributed to the measured quantity [5]. This uncertainty reflects the lack of complete knowledge of the exact value of the quantity being measured. Theoretically, availability of complete knowledge about the measured quantity requires an infinite amount of information, which is naturally impossible. Phenomena that contribute to the uncertainty are called sources of uncertainty. According to [5], there are various possible sources of uncertainty in a measurement, among which:

- Incomplete definition of the measured quantity
- Inadequate realisation of the definition of the measured quantity
- Nonrepresentative sampling (sample measured may not fully represent the measured quantity)
- Incomplete knowledge of environmental conditions
- Human error in reading analogue instruments
- Approximations incorporated in the measurement procedure
- Finite resolution of instrumentation

The uncertainty in power system state estimation is primarily due to either measurement inaccuracy or inaccuracies related to the network mathematical model used. For example meter inaccuracies and calibration errors are major sources of measurements uncertainty. Parameter approximations in modelling of the π-equivalent (line resistance, reactance, shunt capacitance) and the time-skew between metered values are further sources of uncertainty in state estimation. Unfortunately, the magnitudes of such errors and approximations are not known, which in turn leads to uncertainty in the estimates obtained in state estimation.

Traditionally, the uncertainty is handled in estimation by using probability theory. Problems arise, however, due to the possibly invalid underlying assumptions concerning the probabilistic model of uncertainty and nonlinearities in the network model. As a result, the power system operator can be faced with estimates whose accuracy cannot be robustly assessed.

In power system state estimation, inequality constraints have been applied in optimisation to deal with uncertainties. In [6], inequality constraints are employed in a least absolute values (LAV) estimator for handling uncertainty in pseudomeasurements, since they are not measured but are know to vary within bounded intervals. An inequality constrained LAV estimator based on penalty functions, was formulated in [7] to estimate states of external systems. A parameter-bounding model derived from bounded noise measurements was used in [8] with a reformulated constrianed WLS, to handle unmeasured loads in the system. Schweppe [1] was the first to introduce the concept of unknown-but-bounded errors for modelling uncertainty in estimation problems. Measurements are assumed to be inexact and have errors that are unknown but fall within a bounded range. Hitherto, no research seems to have been conducted on uncertainty interval analysis for power system state estimation. This study introduces a double-sided inequality contrained formulation to estimate the uncertainity interval of the state variables. The uncertainty is modelled via deterministic upper and lower bounds on measurement errors, which take into account known meter accuracies.

3 Problem formulation

3.1 Weighted least squares

WLS is the most popular method of point estimation. For a set of measurement equations.

$$z = h(x) + \varepsilon \tag{1}$$

where: z: is the $(m \times 1)$ measurement vector; h: is a vector of nonlinear functions that relate the states to the measurements; x: is an $(n \times 1)$ state vector to be estimated; and ε is an $(m \times 1)$ measurement error vector.

The measurements are usually obtained from transducers in the electrical network. For observability, it is necessary that $m \geq n$ and that the m measurements are in locations such that the resulting Jacobian (sensitivity matrix with respect to the state variables) has rank n. The measurement error vector ε is assumed to be zero mean, normally distributed, with known convariance

$$E(\varepsilon) = 0 \tag{2}$$

$$E(\varepsilon^T) = R \tag{3}$$

where E denotes the expected value, and R is the measurement covariance matrix. It is also assumed that the measurement errors are uncorrelated, so that R is a diagonal matrix. Therefore $[R]_{ij} = \delta_{ij}\sigma_j^2$, where σ_j is the standard deviation of the jth measurement and δ_{ij} is the Kronecker delta.

The optimal state estimate vector x may be determined by minimising the sum of weighted squares of residuals

$$\min F(x) = [z - h(x)]^T R^{-1} [z - h(x)] \tag{4}$$

(4) is linearised using a Taylor series expansion, retaining the first two terms and ignoring higher-order terms. This leads to a linear WLS problem having the solution.

$$\Delta x = (J^T R^{-1} J)^{-1} J^T R^{-1} \Delta z \tag{5}$$

where J is the Jacobian of $h(x)$.

Repeated linearisation and solution of (5) then solves the nonlinear problem via the Newton–Raphson approach. The dependence on the iteration index is implicitly assumed for Δx, J and Δz, where the current state vector is updated at each iteration until a stopping criterion is reached. Further details of the WLS formulation are available in [9–15].

3.2 Uncertainty interval estimation via linear programming (UILP)

Uncertainty intervals can be determined by the solution of a series of appropriately formulated optimisation problems. Each measurement, with its associated uncertainty, can be represented by upper and lower limits. These constraint limits define the tolerances on the measurements (i.e. the range of values within which the true value of the measured quantity must lie). Minimising a particular state variable of interest, subject to all the measurement inequality constraints, provides the lower bound on that state variable. Similarly, maximising the state variable, again subject to all the measurement inequalities, provides the upper bound for that state. In mathematical form

$$\begin{gathered} \min_x x_i \\ \text{subject to } z^l \leq h(x) \leq z^u \end{gathered} \tag{6}$$

where z^l is the lower bound of the measurement vector and z^u is the upper bound, with

$$z^l = z - \tau^- \tag{7}$$

$$z^u = z - \tau^+ \tag{8}$$

where τ^+ and τ^- are the transducer tolerances. The tolerance describes the deterministic uncertainty of each measurement. They represent the overall accuracy of the meter and can usually be provided by the manufacturer. Different values for the elements of positive and negative tolerances are permissible so that a transducer can be specified to have asymmetric accuracy if required (e.g. an accuracy of -3% to $+5\%$ of the nominal value). However, without loss of generality, we will usually assume that $\tau^+ = \tau^- = \tau$, giving a symmetric tolerance around the nominal value. It is assumed that the transducer tolerances τ are known and fixed. In reality, the instrument inaccuracies will increase as the instruments age under the action of various processes and as the instruments may not be recalibrated. It should be noted that measurement recalibration is rarely carried out in a systematic manner by utilities [16], mainly due to the fact that large numbers of measurements exist in a power network and the time and expertise required to check each individual transducer would be expensive.

Equation (6) defines a nonlinear constrained optimisation problem, which can be solved directly by a suitable nonlinear programming algorithm such as sequential quadratic programming [17]. However, it is known that power system models are amenable to solution using the Newton–Raphson approach. Consequently, an alternative approach is to linearise (6) about a suitable point $\hat{x}$ (which in this case can be provided by the WLS estimate) and then a series of linear programmes are solved to obtain updates dx_i to the uncertainty bounds on the state variables. For example, the incremental change to the lower bound for the ith state can be computed by solving the following LP problem:

$$\begin{aligned} &\min_{\Delta x} dx_i \\ &\text{subject to} \quad \Delta z^l \leq J\Delta x \leq \Delta z^u \end{aligned} \tag{9}$$

Similarly, the incremental change to the upper bound on the ith state can be found by solving the LP problem

$$\begin{aligned} &\min_{\Delta x} dx_i \\ &\text{subject to} \quad \Delta z^l \leq J\Delta x \leq \Delta z^u \end{aligned} \tag{10}$$

where J is the Jacobian of $h(x)$ evaluated at $\hat{x}$, and Δz^l and Δz^u are vectors of the incremental changes to measurement lower and upper bounds, respectively, computed in the following form:

$$\Delta z^l = z^l - h(\hat{x}) \tag{11}$$

$$\Delta z^u = z^u - h(\hat{x}) \tag{12}$$

Therefore by performing $2n$ linear programming solutions, all the elements of the vectors dx^+ and dx^- can be calculated. Once dx^+ and dx^- are known, the bounds on $\hat{x}$ are simply found as

$$x^+ = \hat{x} + dx^+ \tag{13}$$

$$x^- = \hat{x} + dx^- \tag{14}$$

where $\hat{x}$ is the point estimate obtained by WLS.

The computational burden of the process arises from the need to perform two LP solutions for every uncertainty interval sought. Nevertheless, with the measurement redundancy level available in power systems, the computational time is reasonable using modern hardware and software. For large networks it is possible that the dual LP formulation could be applied to reduce the execution time [18–20].

3.3 Uncertainty interval calculation for other quantities

In addition to solving for the uncertainty ranges of the state variables (voltage magnitudes and phase angles), it is possible to compute the uncertainty range of other estimated quantities (such as power flows and injections), whether these quantities are measured or not. For example, with a change in the objective functions of (9) and (10), the incremental change in the lower bound of the ith measurement could be found by solving the following LP problem:

$$\begin{aligned} &\min_{\Delta x} j_i\Delta x \\ &\text{subject to} \quad \Delta z^l \leq J\Delta x \leq \Delta z^u \end{aligned} \tag{15}$$

where j_i is the ith row of the Jacobian corresponding to the ith measurement. The solution of (15) is a set of incremental changes for all system state variables. Evaluating the objective function at this solution, i.e. $j_i[\Delta x]_{\min}$, provides the incremental change in the lower bound for the ith measurement.

Similarly, the incremental change in the upper bound of the ith measurement is constructed from

$$\begin{aligned} &\max_{\Delta x} j_i\Delta x \\ &\text{subject to} \quad \Delta z^l \leq J\Delta x \leq \Delta z^u \end{aligned} \tag{16}$$

Ultimately, the uncertainty bounds are computed as

$$Z_i^u = Z_i + j_i[\Delta x]_{\max} \tag{17}$$

$$Z_i^l = Z_i + j_i[\Delta x]_{\min} \tag{18}$$

Uncertainty estimation is a 'worst-case' analysis in the sense that the LPs are seeking the extreme limits of uncertainty for the quantity of interest. This property is illustrated in Fig. 1, based on a simple example described in [21, 22]. The sequence of LPs calculates the bounding polytope due to uncertainty, i.e. the interior diamond shape in the Figure. WLS estimation, on the other hand, produces an 'average-case' or maximum likelihood estimate of x.

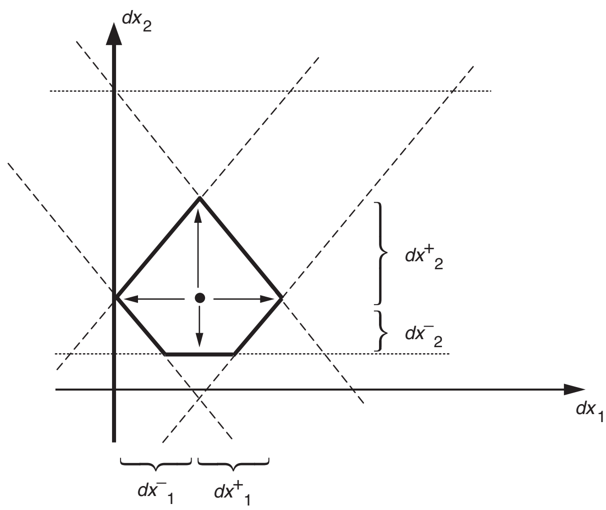

Fig. 1 *Two-dimensional example of LP uncertainty estimation*

The geometry of the uncertainty estimation problem leads to the question of whether an infeasible problem might arise (i.e. no feasible polytope exists). This cannot occur if all the measurement uncertainties are correctly specified. However, if any gross measurement errors have not been eliminated, an infeasible problem is likely to arise. To avoid this in practice, it would be recommended that gross error detection and elimination be performed prior to the point estimation and uncertainty interval estimation procedures. Alternatively, a robust estimator, such as least median of squares or least trimmed squares [23, 24], may be used instead of WLS for more accurate estimation of the centre point.

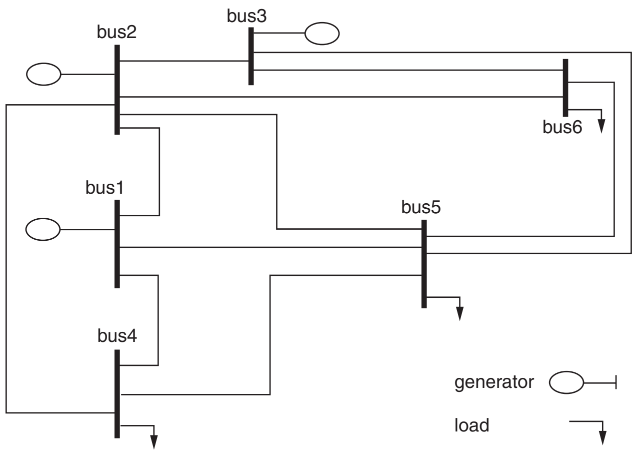

Fig. 2 *Online diagram of six-bus system*

4 Implementation of case studies and results analysis

This Section presents some typical results obtained by applying the proposed algorithms to the six-bus test system from [25] (shown in Fig. 2), IEEE 30-bus and IEEE 118-bus test network data. The computation of all state variables and some measurements will be shown to illustrate the concepts. However, for improved computational efficiency, only the variables of present interest to the power system operator would need to be computed. The LP problems have been solved by the function *linprog* incorporated in the MATLAB™ optimisation toolbox [17].

4.1 Confidence bounds analysis with UILP

Table 1 shows a comparison of simulated (from a load flow solution) and estimated states for the six-bus network. The measurement uncertainty has been represented as a uniform distribution over the interval [−3%, 3%] of the nominal value of the measurements. A WLS estimator was used to compute the (centre point) estimated states. In the tests presented here and in further tests the Newton–Raphson process was found to perform reliably, with convergence occurring within three to four iterations. This is consistent with the behaviour of the Newton–Raphson process in solving other types of power system state estimation problems.

Table 1: Estimated state variables and uncertainty bounds for the six-bus network

Bus	True states		$LP_{(lower\ bound)}$		$WLS_{(centre\ point)}$		$LP^{+}_{(upper\ bound)}$	
	|V|(pu)	δrad	|V|(pu)	δrad	|V|(pu)	δrad	|V|(pu)	δrad
1	1.0500	0	1.0417	0	1.0738	0	1.1018	0
2	1.0500	−0.0650	1.0265	−0.0908	1.0678	−0.0612	1.0908	−0.0440
3	1.0700	−0.0756	1.0248	−0.1027	1.0606	−0.0683	1.0891	−0.0560
4	0.9864	−0.0729	0.9678	−0.0865	1.0058	−0.0693	1.0321	−0.0541
5	0.9797	−0.0799	0.9395	−0.0891	0.9614	−0.0845	1.0038	−0.0664
6	1.0014	−0.0860	0.9768	−0.0900	1.0005	−0.0840	1.0411	−0.0257

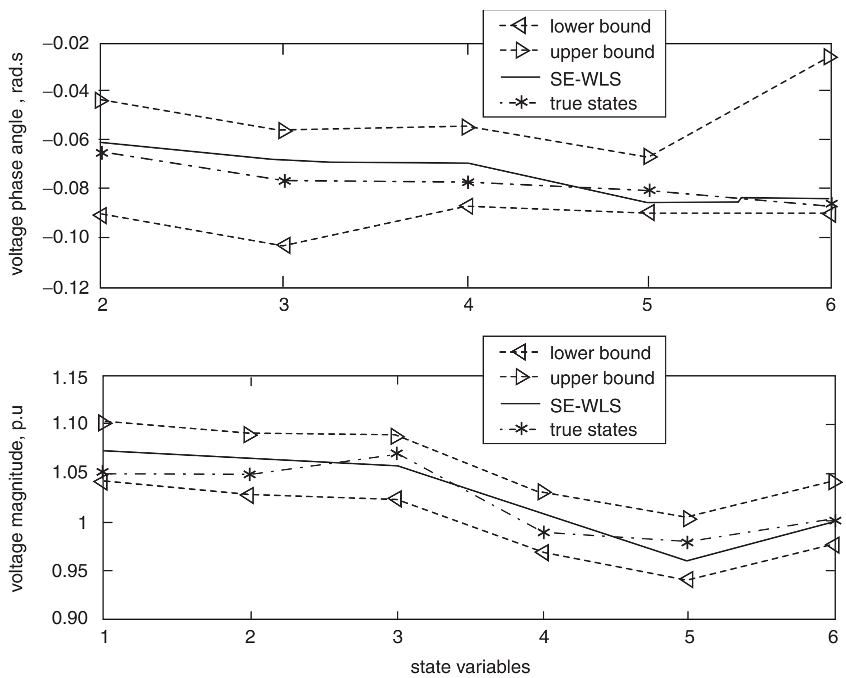

Fig. 3 *Estimated states and uncertainty bounds for six-bus network*

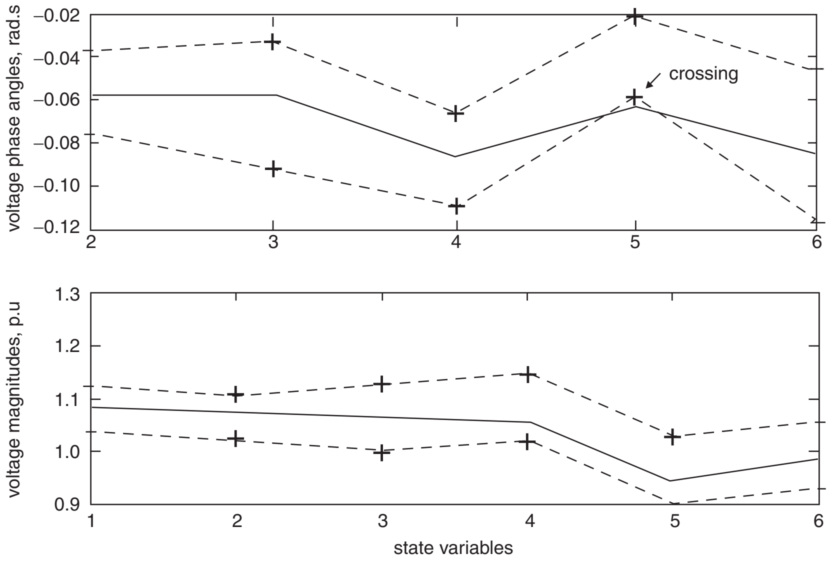

Fig. 4 *Estimated states and uncertainty bounds for six-bus network with parameter uncertainty*

Table 2: Estimated state variables and uncertainty bounds for the IEEE 30-bus network

Bus	True states		$LP_{(lower\ bound)}$		$WLS_{(centre\ point)}$		$LP^{+}_{(upper\ bound)}$	
	$\|V\|$(pu)	δrad	$\|V\|$(pu)	δrad	$\|V\|$(pu)	δrad	$\|V\|$(pu)	δrad
1	1.0600	0	1.0261	0	1.0783	0	1.1410	0
2	1.0430	−0.0932	1.0124	−0.1053	1.0640	−0.0926	1.1265	−0.0817
3	1.0269	−0.1328	0.9994	−0.1494	1.0472	−0.1305	1.1129	−0.1010
4	1.0194	−0.1635	0.9998	−0.1782	1.0414	−0.1611	1.1016	−0.1293
5	1.0100	−0.2467	0.9785	−0.2738	1.0361	−0.2378	1.0903	−0.2011
6	1.0138	−0.1937	0.9884	−0.2077	1.0360	−0.1910	1.0909	−0.1581
7	1.0045	−0.2247	0.9739	−0.2495	1.0311	−0.2210	1.0940	−0.1854
8	1.0100	−0.2057	0.9769	−0.2270	1.0289	−0.2051	1.0879	−0.1685
9	1.0364	−0.2507	0.9960	−0.2500	1.0566	−0.2400	1.1111	−0.2041
10	1.0256	−0.2804	0.9899	−0.2777	1.0459	−0.2737	1.1119	−0.2481
11	1.0820	−0.2507	1.0322	−0.2627	1.1135	−0.2292	1.1873	−0.1810
12	1.0340	−0.2681	0.9771	−0.2806	1.0553	−0.2577	1.1220	−0.1922
13	1.0710	−0.2681	1.0076	−0.2787	1.0943	−0.2563	1.1743	−0.1862
14	1.0191	−0.2841	0.9466	−0.3297	1.0415	−0.2825	1.0883	−0.2011
15	1.0148	−0.2856	0.9647	−0.3191	1.0437	−0.2785	1.1061	−0.2016
16	1.0228	−0.2782	0.9762	−0.2928	1.0449	−0.2691	1.1360	−0.2261
17	1.0196	−0.2836	0.9886	−0.2679	1.0443	−0.2734	1.1183	−0.2383
18	1.0062	−0.2964	0.9352	−0.3039	1.0345	−0.2809	1.1302	−0.1994
19	1.0043	−0.2994	0.9323	−0.2893	1.0220	−0.2851	1.1211	−0.2141
20	1.0089	−0.2957	0.9373	−0.2914	1.0270	−0.2791	1.1172	−0.2150
21	1.0125	−0.2884	0.9564	−0.3013	1.0264	−0.2841	1.1051	−0.2707
22	1.0128	−0.2882	0.9573	−0.3062	1.0285	−0.2860	1.1102	−0.2755
23	1.0042	−0.2921	0.9224	−0.3324	1.0289	−0.2800	1.1075	−0.2059
24	0.9987	−0.2945	0.9156	−0.3609	1.0168	−0.3049	1.1226	−0.2692
25	0.9914	−0.2855	0.9161	−0.3593	1.0213	−0.2963	1.1273	−0.2554
26	0.9732	−0.2932	0.8981	−0.4242	1.0302	−0.3264	1.1943	−0.2393
27	0.9956	−0.2752	0.9172	−0.3246	1.0269	−0.2781	1.1145	−0.2154
28	1.0099	−0.2044	0.9801	−0.2289	1.0357	−0.2045	1.0993	−0.1619
29	0.9752	−0.2979	0.9386	−0.3091	1.0384	−0.3005	1.1784	−0.1997
30	0.9633	−0.3142	0.8544	−0.3818	0.9892	−0.3184	1.0882	−0.1967

Discrepancies between the simulated and the estimated centre point are fairly large, due to the significant noise level (i.e. $\pm 3\%$, uniformly distributed). The upper and lower uncertainty bounds of the state variables are found using (9)–(12), with $\tau \equiv 3\%$. It is apparent that the centre point estimates are within the upper and lower uncertainty bounds, in this case as illustrated by Fig. 3.

Table 3:

Type (pu)	$Z_{(lower bound)}$	$Z_{(centre point)}$	$Z_{(upper bound)}$
$P_{1(injection)}$	1.0439	1.0824	1.0835
$P_{2(injection)}$	0.4536	0.4843	0.5187
$P_{5(injection)}$	−0.7283	−0.6851	−0.6632
$Q_{1(injection)}$	−0.9623	−0.9295	−0.8972
$P_{line(bus1 to bus2)}$	0.2924	0.2924	0.3140
$P_{line(bus2 to bus1)}$	−0.2990	−0.2822	−0.2765
$P_{line(bus1 to bus4)}$	0.4262	0.4441	0.4683
$P_{line(bus4 to bus2)}$	−0.4581	−0.4334	−0.4160

Further tests were undertaken to examine the effects of parameter uncertainty in the power network parameters. An error of $+3\%$ was introduced into the resistance, inductance and capacitance parameters of the most heavily loaded line (connecting bus 1 and bus 4). Figure 4 shows the estimated states with their bounds (which are designated by + signs).

In the phase angle results, it is interesting to note that a 'crossing' occurs. The estimated state δ_5 does not lie within the calculated bounds. This crossing suggests that the estimated centre point is inaccurate. The WLS process assumes normally distributed errors (where errors of any magnitude are considered to be possible) and can therefore produced estimates that fall outside the uncertainty bounds. In general, the width of the uncertainty interval, the location of the point estimate within the uncertainty range, and the occurrence of 'crossing' are examples of the useful additional information generated by UILP.

Tests were also conducted on the IEEE 30-bus network. In this example, $+10\%$ parameter errors are introduced for the most heavily loaded line. With $\tau \equiv 6\%$, the estimated states and bounds are shown in Table 2. A 'crossing' is again apparent. The state δ_{17} does not lie within its bounds.

4.2 Confidence bounds analysis of other quantities

Equations (15)–(18) may be used to find uncertainty bounds for measured and unmeasured quantities, in addition to the state variables. Selected estimated measurements, for the six-bus system, are presented, with their bounds, in Table 3.

Figure 5 shows the estimated measurement, with bounds (which are designated by + signs) for the six-bus network. The results are sorted in descending order for clarity.

The upper and lower bounds of all measurement changes from (11) and (12) are plotted in Fig. 6 along with the incremental changes of measurement obtained from the solution of the optimisation problems of (15) and (16). As expected, the estimated incremental bounds of all measurements, (i.e. solution from the optimisation problem), either lie within or on the allowed bounds.

4.3 Implications for practical use

The availability of the upper and lower bounds on state estimates, and other quantities of interest, can have practical advantages for the power system operator. For critical quantities, such as a power flow which is close to its thermal, stability or contractual limit, the operator can gain

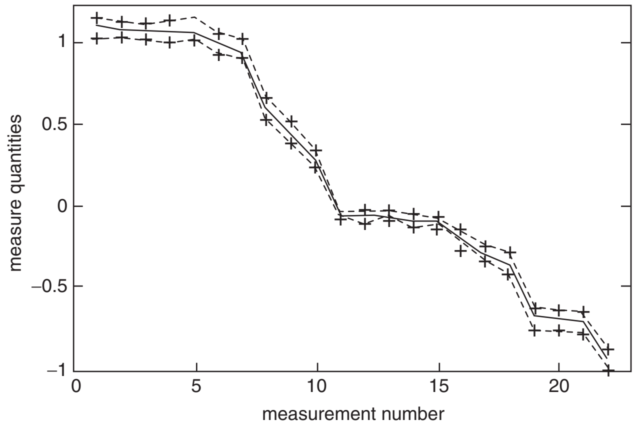

Fig. 5 *Estimated measurements with uncertainty bounds for six-bus system*

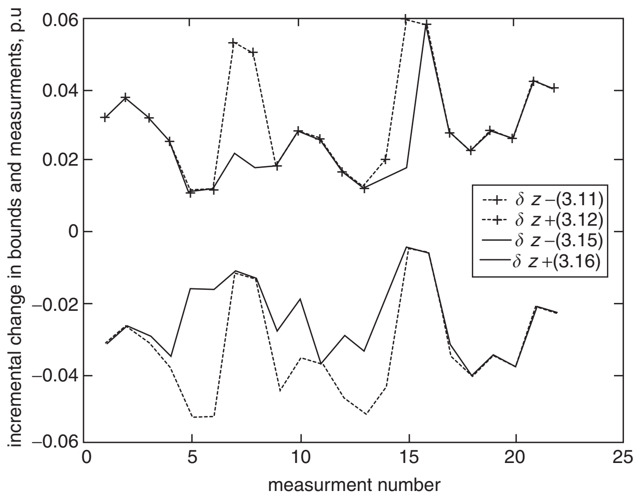

Fig. 6 *Measurement bounds*

confidence that the true value is not exceeding the constraint provided that the state estimate and both bounds are all within the limit. The uncertainty range on the estimate also gives a useful indication of the quality of the metering configuration for the relevant part of the power system. For example, where a voltage level often has a wide estimated uncertainty range, this would suggest that the metering in that area is insufficient. In rare cases in which the point state estimate is found to lie outside the uncertainty range, a deficiency in the system model or its parameter is clearly indicated. This type of additional information could be very useful during the installation or upgrading of an online state estimator.

5 Conclusions

An analysis of uncertainty in power system state estimation has been presented in this paper. The uncertainty is modelled via deterministic upper and lower bounds on measurement errors, which take into account known meter accuracies. A conventional WLS estimator is used to obtain point estimates of the states, and then a series of LP solutions is used to compute the tightest possible bounds on the states and other quantities of interest. The method offers useful additional information to the power system operator. By estimating bounds on the estimates one can infer the quality of the metering configuration and determined the proximity of estimated quantities to voltage and flow limits with greater confidence.

6 References

1 Schweppe, F.C.: 'Uncertain dynamic systems', (Printice-Hall, Engelewood Cliffs, 1973)
2 Bargiela, A., and Hainsworth, G.: 'Pressure and flow uncertainty in water system', *ASC J Water Resourc. Planning Mand.*, 1989, **115**, (2), pp. 212–229
3 Brdys, M., and Chen, K.: 'Joint state and parameter estimation of dynamic water supply systems under bounded uncertainty'. Proc. 1993 Integrated Computer Application for water supply and distribution, Leicester, UK, 7–9 September 1993, pp. 335–353
4 Nagar, A., and Powell, R.: 'LFT/SDP approach to the uncertainty analysis for state estimation of water distribution systems', *IEE Proc. Control Theory Appl.*, 2002, **149**, pp. 137–142
5 "Expression of the uncertainty of measurement in calibration", EAL-4/02, 1999
6 Abur, A., and Celik, M.K.: 'Least absolute value state estimation with equality and inequality constraints', *IEEE Trans. Power Syst.*, 1993, **8**, pp. 680–686
7 Singh, H., Alvarado, F.L., and Liu, W.-H.E.: 'Constrained LAV state estimation using penalty functions', *IEEE Trans. Power Systems*, 1997, **12**, pp. 383–388
8 Celements, K.A., Davis, P.W., and Frey, K.D.: 'Treatment of inequality constraints in power system state estimation', *IEEE Trans. Power Syst.*, 1995, **10**, pp. 567–574
9 Schweppe, F.C., and Wildes, J.: 'Power system static-state estimation, Part I: Exact model', *IEEE Trans. Power Appar. Syst.*, 1970, **89**, pp. 120–125
10 Bjorck, A.: 'Numerical methods for least squares problems', (SIAM, Philadelphia, 1996)
11 Monticelli, A.: 'State estimation in electric power systems: a generalized approach', (Kluwen, Boston, 1999)
12 Schweppe, F.C., and Rom, D.B.: 'Power system static-state estimation, Part II: Approximate model', *IEEE Trans. Power Appar. Syst.*, 1970, **89**, pp. 125–130
13 Schweppe, F.C.: 'Power system static-state estimation, Part III: Implementation', *IEEE Trans Power Appar. Syst.*, 1970, **89**, pp. 130–135
14 Larson, R.E., Tinney, W.F., and Peschon, J.: 'State estimation in power systems, Part I: Theory and feasibility', *IEEE Trans. Power Appar. Syst.*, 1970, **89**, pp. 345–352
15 Larson, R.E., Tinney, W.F., Hajdu, L., and Piercy, D.: 'State estimation in power systems, Part II: Implementation and application', *Appar. Syst.*, 1970, **89**, pp. 352–362
16 Adibi, M.M., and Thorne, D.K.: 'Remote measurement calibration', *IEEE Trans. Power Syst.*, 1986, **1**, pp. 194–202
17 'Optimization toolbox for use with Matlab user's guide, (Maths Works Inc., 2nd edn)
18 El-Kei, A., Singh, H.: 'Fast linear programming state estimation using the dual formulation', *IEEE Trans. Power Syst.*, 1992, **7**, pp. 620–628
19 Chvatal, V.: 'Linear programming', (W. H. Freeman, 1983)
20 Vanderbei, R.: 'Linear programming: foundations and extensions', (Kluwer, Boston, 2002)
21 Pedrycz, W., and Bargiela, A.: 'Granular computing: an introduction', (Kluwer, Boston, 2002)
22 Bargiela, A., Pedrycz, W., and Tanaka, M.: 'A study of uncertain state estimation', *IEEE Trans. Syst., Man Cybern.*, 2002, **33**, pp. 288–301
23 Rousseeuw, P.J., and Leroy, A.M.: 'Robust regression and outlier detection', (Wiley, Chichester, 1987)
24 Mili, L., Phaniraj, V., and Rousseeuw, P.J.: 'Least median of squares estimation in power systems', *IEEE Trans. Power Syst.*, 1991, **6**, pp. 511–523
25 Wood, A.J., and Wollenberg, B.F.: 'Power generation, operation, and control', (Wiley, Chichester, 1996, 2nd edn)