Cross-Layer Design for OFDMA Wireless Networks with Finite Queue Length Based on Game Theory

A thesis submitted for the degree of Doctor of Philosophy

by

Ilias G. Nikolaros

Department of Electronic and Computer Engineering

College of Engineering, Design & Physical Science,

Brunel University, London, UK

November 2014

ABSTRACT

In next generation wireless networks such as 4G- LTE and WiMax, the demand for high data rates, the scarcity of wireless resources and the time varying channel conditions has led to the adoption of more sophisticated and robust techniques in PHY such as orthogonal frequency division multiplexing (OFDM) and the corresponding access technique known as orthogonal frequency division multiplexing access (OFDMA). Cross-layer schedulers have been developed in order to describe the procedure of resource allocation in OFDMA wireless networks.

The resource allocation in OFDMA wireless networks has received great attention in research, by proposing many different ways for frequency diversity exploitation and system's optimization. Many cross-layer proposals for dynamic resource allocation have been investigated in literature approaching the optimization problem from different viewpoints i.e. maximizing total data rate, minimizing total transmit power, satisfying minimum users' requirements or providing fairness amongst users.

The design of a cross-layer scheduler for OFDMA wireless networks is the topic of this research. The scheduler utilizes game theory in order to make decisions for subcarrier and power allocation to the users with the main concern being to maintain fairness as well as to maximize overall system's performance. A very well known theorem in cooperative game theory, the Nash Bargaining Solution (NBS), is employed and solved in a close form way, resulting in a Pareto optimal solution. Two different cases are proposed. The first one is the symmetric NBS (S-NBS) where all users have the same weight and therefore all users have the same opportunity for resources and the second one, is the asymmetric NBS (A-NBS), where users have different weights, hence different priorities where the scheduler favours users with higher priorities at expense of lower priority users.

As MAC layer is vital for cross-layer, the scheduler is combined with a queuing model based on Markov chain in order to describe more realistically the incoming procedure from the higher layers.

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1. **Nikolaros, I.G**.; Zarakovitis, C.C.; Skordoulis, D.; Hadjinicolaou, M.G.; Qiang Ni; , "Cross-layer Design for Multiuser OFDMA Systems with Cooperative Game and MMPP Queuing Considerations," *Computer and Information Technology (CIT), 2010 IEEE 10th International Conference on* , vol., no., pp.2649-2654, June 29 2010-July 1 2010
2. Zarakovitis, C.C.; **Nikolaros**, **I.G**.; Qiang, Ni., "A Game Theoretic Cross-Layer Design for Resource Allocation in Heterogeneous OFDMA Networks," IEEK Transactions on Smart Processing and Computing, vol. 1, no. 1, pp. 50-64, August 2012
3. Zarakovitis, C.C.; **Nikolaros, I.G**.; Skordoulis, D.; Hadjinicolaou, M.G.; Qiang Ni; , "A Comparative Study on Iterative Methods Regarding Cross-Layer Optimization for OFDMA Systems," *Computer and Information Technology (CIT), 2010 IEEE 10th International Conference on* , vol., no., pp.420-425, June 29 2010-July 1 2010
4. Zarakovitis, C.C.; Qiang Ni; **Nikolaros, I**.; Tyce, O.; , "A Novel Game-Theoretic Cross-Layer Design for OFDMA Broadband Wireless Networks," *Communications (ICC), 2010 IEEE International Conference on*, vol., no., pp.1-6, 23-27 May 2010 doi: 10.1109/ICC.2010.5502203

LIST OF ABBREVIATIONS

Abrreviation	Description
A/D	Analog to Digital
AMC	Adaptive Modulation and Coding
A-NBS	Asymmetric Nash Bargaining Solution
APA	Adaptive Power Allocation
AWGN	Adaptive White Gaussian Noise
BE	Best Effort
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
BS	Base station
CAC	Connection Admission Control
CDMA	Code Division Multiple Access
CP	Cyclic Prefix
CSI	Channel State Information
CSIT	Channel State Information at the Transmitter
CSMA/CD	Carrier Sense Multiple Access with Collision Detection
D/A	Digital to Analog
DL	Down Link
dMMPP	discrete Markov Modulated Poisson Process
DSA	Dynamic Subcarrier Allocation
FDD	Frequency Division Duplexing
FDMA	Frequency Division Multiple Access
FFT	Fast Fourier Transform
ICI	Inter Carrier Interference
IFFT	Inverse Fast Fourier Transform
ISI	Inter Symbol Interference
KKT	Karush Kuhn Tucker
MAC	Medium Access Control
MDP	Markov Decision Problem
MCM	Multi-Carrier Modulation
M-QAM	M-symbol Quadrature Amplitude Modulation
NBS	Nash Bargaining Solution
NE	Nash Equilibrium
NLOS	Non Line Of Sight
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access

PAPR	Peak to Average Power Ratio
PHYL	Physical Layer
QAM	Quadrature Amplitude Modulation
QoS	Quality of Service
QPSK	Quadrature Phase Shift Keying
RT	Real Time
S-NBS	Symmetric Nash Bargaining Solution
SNR	Signal to Noise Ratio
TDD	Time Division Duplexing
TDMA	Time Division Multiple Access
UL	Up link
WF	Water Filling

ACKNOWLEDGEMENTS

Through these lines I would like to thank and express my gratitude to all those who have guided me, helped me and supported me during this long journey until completing my doctoral thesis.

First and foremost I am grateful to my supervisor Professor John Cosmas for his support, guidance and encouragement which allowed me to finalize my research and complete this work. I also want to sincerely acknowledge the valuable help and guidance of my second supervisor Dr. Rajagopal Nilavalan.

Furthermore, I would like to thank my family for their unending support which was a paramount factor in the continuation of my work and essential in pursuing my effort until the end. A special thank to my cousin Dr. Agis Ioannis Kitsikis who offered his advice and assistance whenever asked.

Finally I would like to express my gratitude to my original supervisor Dr. Marios Hadjinicolaou who passed away a few years ago. Dr. Hadjinicolaou was the one who inspired me and introduced me to scientific research. I would not have begun my Ph.D degree without his continuous encouragements and thus dedicate this thesis to his memory.

Ilias G. Nikolaros
Uxbridge, London, UK
November 2014

CHAPTER 1

Introduction

Overview

Wireless networks have been designed to exploit every single point of available resources in order to provide the necessary services to end users. Specifically, 4G networks, like 3GPP-LTE or WiMax, have adopted the orthogonal frequency division multiplexing (OFDM) and orthogonal frequency division multiple access (OFDMA) as the most robust and effective modulation and medium access techniques [1].

The most critical element for the performance of a wireless network is the way that resources are allocated to the users. The cross-layer scheduler is responsible to determine the amount of resources which are assigned to each user separately. Following that, the design of the scheduler is a vital issue for user satisfaction and overall system performance. The most common scenario comprises of one base station (BS) which serves a number of users distributed around it. The subcarriers and the power are assigned to the users based on the scheduler's algorithm. Since, such networks are employed mostly in urban areas the biggest problem is the channel fade for some users. Additionally, a cross-layer includes the medium access control (MAC) layer description which is defined by a queuing model.

Motivation

The increasing demand of high performance wireless networks that handle resources fairly and efficiently in order all users be benefited, became the motivation for developing a game theory based cross-layer scheduler.

The scheduler design is challenging as many aspects must be considered and combined, increasing the complexity of the system. Many proposals have been introduced in the literature each one aiming to improve the system's performance in different terms. However, most of these studies distinguish a few major proposal cases. The most extensive researched case, are the proposals that aim to optimize overall system performance in terms of data rate [2]. This kind of approach is characterized as opportunistic as its main drawback is that users who are in deep fade experience starving of available capacity. Additionally, minimization of the total transmit power is also a case, which attain research interest, where authors try to satisfy users resources demands by consuming the least power but this fails to exploit all the available resources in the system's favour [3]. As in the abovementioned proposals the fairness issue has been totally neglected by some schemes like Max-Min and proportional fairness [4] [5] [6] [7], which provide fairness amongst the users at the expense of lower overall performance comparing with the previous ones.

In order to eliminate the drawback of fairness and the system's downgrade performance, the authors propose solutions which are based on cooperative game theory and in particular on the Nash Bargaining Solution (NBS). These efforts are efficient as they achieve Pareto optimal solution and retain fairness [8] [9]. However, these investigations do not consider the queue part of the cross-layer design and conclude in numerical or algorithmic methods giving always sub-optimal solutions.

Scope of this thesis

The scope of this thesis, which originates from the above argument, is to develop a cross-layer scheduler which will optimize system performance on several aspects. The cross-layer scheduler aims to enhance the system's capability to deliver services to the users in terms of fairness, to satisfy users' QoS demands and simultaneously to maximize the overall system's performance.

The first objective is to build the constraint optimization problem bearing in mind the MAC layer characteristics. MAC layer is described by a discrete Markov Modulated Poisson Process (dMMPP) which can represent the real incoming data traffic from the higher layers. In order for users to benefit from fairness characteristics, cooperative game theory is utilized and two problems are formulated based on symmetric NBS (S-NBS) and asymmetric NBS (A-NBS) respectively. The following objective is the analytical solution of the constraint optimization problems being derived and finally, the last objective is to carry out simulations results to validate the proposed theoretical models.

Contribution to knowledge

In this thesis a cross-layer scheduler for OFDMA wireless networks is presented. The scheduler is formulated by employing the NBS from the cooperative game theory which is associated with a dMMPP queuing model with finite queue length.

The contributions of this work are:

1. The innovation of this work is the formulation of the cross-layer problems. In section 4.4 problem formulation based on symmetric NBS is shown on page 43, and section 4.5 where the cross-layer optimization problem based on asymmetric NBS is presented, page 61. Also innovation of this work is the combination these two optimization problems with finite queue model which is presented in section 4.3.
2. In this work the NBS is presented in both symmetric (S-NBS) and asymmetric (A-NBS) cases whereas only the symmetric case is proposed by other authors.
3. The utility function that is used in this proposal does not utilize only data rate, which participates and in many other proposals, but takes into account user's queue length and normalized delay.
4. The solution of the both constraint optimization problems is derived through a low complexity closed-form analytical way instead of sub-optimal numerical or algorithmic procedures.
5. Through simulation results it is shown that the proposed cross-layer schemes are superior to the other fair-considered schemes and is a tradeoff between optimal performance and fairness.

Thesis outline

The thesis is organized as follows.

Chapter 2, is a presentation of the background knowledge about OFDM, OFDMA and game theory. The main advantages of the OFDMA technique in the physical (PHY) layer are presented. Furthermore, game theory basic theorems are mentioned in order to be familiar with games structure. Nash equilibria and Nash Bargaining Solution are mentioned since they are extensively used in this thesis.

Chapter 1

A comprehensive literature review, which includes the work that has been done, is presented in Chapter 3. All the OFDMA cross-layer schemes are mentioned with their advantages and their drawbacks.

In Chapter 4 takes place the cross-layer constraint problem formulation for both schedulers. The dMMPP queuing model for the MAC layer is first presented. The formulation of the cross-layer constraint optimization problems, based on S-NBS and A-NBS are presented subsequently. Problems, through relaxation, are transformed into convex optimization problems over a convex set are solved by utilizing the Lagrangian method.

Finally, Chapter 5 draws on the simulation results to present conclusions and give future directions. Simulations results are compared with other cross-layer schemes in order to validate the superiority of this work.

The work contained within this thesis is that of the author's unless otherwise stated. To the best of my knowledge none of the work which is presented here has been published by anyone else except what is acknowledged.

Ilias G. Nikolaros

November 2014

CHAPTER 2

Overview of OFDM-OFDMA Technique and Game Theory

Introduction

This chapter is an overview of Orthogonal Frequency Division Multiplexing which is a modern transmission technique and Orthogonal Frequency Division Multiple Access which is a multiple channel access technique. In addition, a presentation of Game Theory is presented in this chapter.

Orthogonal Frequency Division Multiplexing (OFDM)

OFDM is one of the most promising techniques for wireless 3G (3rd generation) and 4G (4th generation) networks. Its philosophy is based on the principle of dividing the spectrum into more than one base frequency (subcarriers) and by converting the serial data into parallel, transmitting them in parallel instead of serial way exploiting the number of subcarriers. This technique is known as Multi-Carrier Modulation (MCM) due to the fact that many subcarriers have to be modulated according to parallel data streams which result in a much lower bit rate per subcarrier [10].

The bandwidth of each subcarrier is considered to be $\Delta f = B / N$, where B represents the system's total bandwidth and N denotes the number of subcarriers. Assuming that the total data rate is R before spectrum is divided into N subcarriers the data rate of each subcarrier will be R / N. In high data rate systems the symbol duration is small therefore; the intersymbol interference (ISI) effect is more probable. In OFDM, by splitting the data stream into many parallel streams increases the symbol duration so that the channel's delay spread is only a small fraction of the symbol duration. If T_s denotes the symbol duration and τ the channel's delay spread, the following inequality must be valid $T_s >> \tau$ in order to support non line of sight (NLOS) transmissions [11].

In addition, a guard interval between OFDM symbols is used to keep each OFDM symbol independent of the others, maintaining the orthogonality among them. Today, cyclic prefix (CP) is adopted instead of the empty guard interval. In order to create CP, the last ν symbols are copied and pasted in the front of an OFDM symbol as it can be seen in the Figure 1.

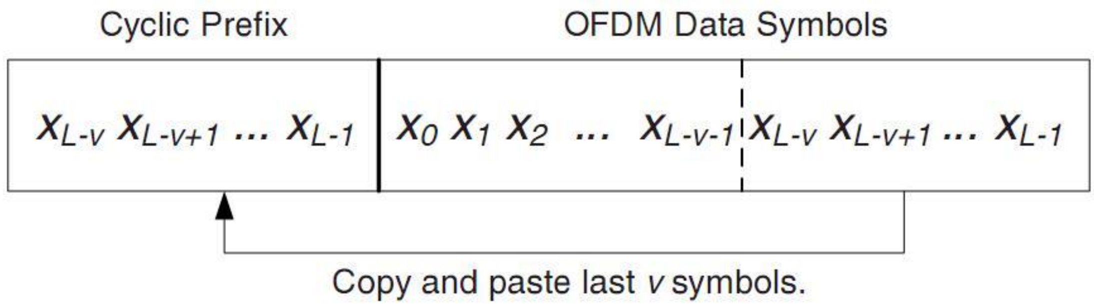

Figure 1 The OFDM cyclic prefix

By adding CP, the channel provides circular convolution and IFFT/FFT can be used to eliminate the inter-carrier interference (ICI) and ISI within each OFDM symbol with a penalty of more energy consumption which is acceptable [10]. The following Figure 2 shows an OFDM system with four subcarriers.

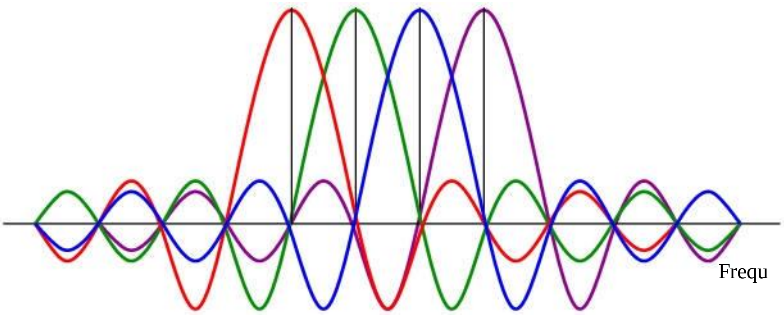

Figure 2 OFDM system

The receiver, in order to demodulate the signal, should choose the right frequency and the optimal time which means it must be in frequency and timing synchronization with the transmitter. In any other case any synchronization error in frequency or in time result in sampling error and wrong received signal. Figure 3 depicts such a case.

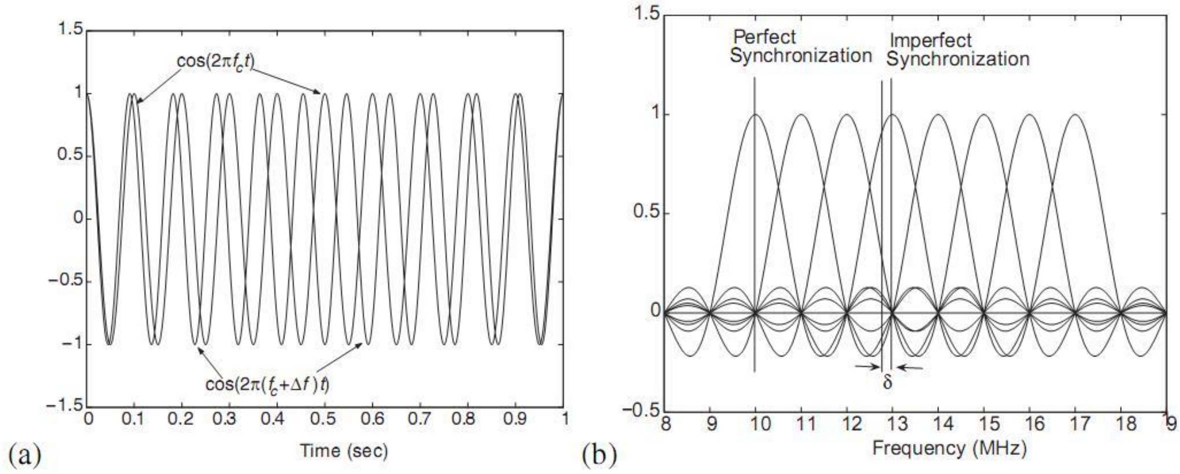

Figure 3 (a) two subcarriers in time domain (b) eight subcarriers in frequency domain

The advantages of using OFDM are significant. The reduced computational complexity is one of the main advantages as OFDM can be easily implemented using IFFT/FFT and reduce the complexity especially at the receiver. The exploitation of frequency diversity is another advantage of this technique. Additionally, OFDM uses adaptive modulation and coding which depends on subcarrier's fade making the system more robust against frequency selective (multipath) fading, resulting in reducing the errors. Finally, it offers robustness against narrow band interference since it affects only a portion of subcarriers [10] [12].

OFDM technique has also some disadvantages. As mentioned before, time and frequency synchronization are very critical factors. Timing offset is not as critical as frequency because we can deal with it using CP. On the other hand, the tightly packed subcarriers lead to frequency offset due to noise which is very critical for the system. This affects frequency synchronization and results in errors. Another drawback is the high peak-to-average power ratio (PAPR) of the OFDM signals. This causes nonlinearities and stringent requirements on the A/Ds and D/As [10] [12].

Orthogonal Frequency Division Multiple Access (OFDMA)

As mentioned above, OFDM is a technique which forms many independent subcarriers for data transmission instead of transmitting only one stream. OFDM systems follow the one transmitter and one receiver model in which all subcarriers are used by only one user [12]. Instead, OFDMA is a MAC layer protocol for wireless networks using OFDM in the PHY layer. OFDMA applies in wireless networks where one BS and multiple users are considered.

This can be expressed in terms of subcarriers where each terminal occupies a number of subcarriers with the limitation that each subcarrier is exclusively assigned to only one user [12].

Multiuser protocols in wireless networks try to address the problem of accessing the medium. In a wireless network all users must be served based on a contention or a non-contention way. When contention protocols are used each station transmits when it decides. Many times, when more than one station decides to transmit simultaneously leads to collision and all packets are destroyed. All the previous transmissions should be repeated after a random time interval which is determined by their back-off algorithms. Such protocols are ALOHA and Carrier Sense Multiple Access with collision Avoidance (CSMA/CA). Both of them have been adopted in many wireless networks such as GSM and IEEE 802.11 [12].

On the other hand, in non-contention protocols a central decision point is assumed. Normally, it is the base station which decides which user will transmit at each timeslot. The controller's duty is to coordinate the users' transmissions and inform them when they are able to transmit in order collisions be avoided. In such protocols we can divide the spectrum in more than one narrow sub-channels (frequency division) and each user transmits in one sub-channel, or we can divide the channel in terms of time, giving the ability to each user to transmit using the whole spectrum for a small time interval (time division). A combination of the above techniques can be adopted as well. Time division multiple access (TDMA), frequency division multiple access (FDMA) and code division multiple access (CDMA) are such techniques [10].

In TDMA technique, time is divided into frames and then into time-slots. Each frame consists of many timeslots and each timeslot is occupied by one user. Time-slots can be assigned in a static or dynamic way based on the user's demands. On the other hand FDMA is based on the same pattern but instead of time, frequency is divided into several subcarriers with smaller bandwidth. The system assigns one or more frequency to each user in order to send its data. These two techniques can be combined by implementing TDMA in each produced frequency by FDMA in order more users being accommodated. CDMA is the dominant multiple access technique for current cellular systems. According to this technique all users may use all the available bandwidth continually. Different channels are defined by the users using spreading codes, which are different for each pair (base station-terminal). The codes should be orthogonal amongst terminals in order to be feasible for each terminal to distinguish its data addressed to it [11] [12].

All the above multiple access protocols provide orthogonality which means one user's transmission does not affect other users' transmissions. A drawback of those techniques is that lack of users' data led to considerably reduced system's efficiency due to the fact that a portion of available resources is assigned to those users.

OFDMA can be considered as a hybrid technique combining both TDMA and FDMA techniques. Multiple subcarriers can be assigned to one user in a static or dynamic way maintaining orthogonality amongst them. Further, matching subcarriers in groups form brunches of subcarriers, called subchannels. Each subcarrier is assigned to one user for a small time interval called time-slot. A time-slot consists of one subchannel over one, two or three OFDMA symbols depending on the particular sub-channelization scheme which is used. ODFMA also supports time division duplexing (TDD) and frequency division duplexing (FDD). In TDD mode the same frequency is used for downlink (DL) and uplink (UL) but the OFDM symbol is divided into the downlink and the uplink part with a guard interval in between. On the FDD mode different frequencies are used for downlink and uplink respectively. Figure 4 shows an OFDM symbol in TDD mode. The downlink to uplink ratio may vary in order different traffic profiles being supported.

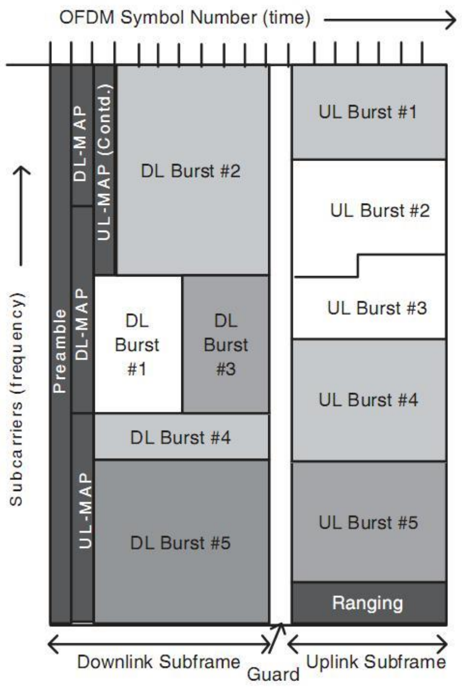

Figure 4 A sample of TDD frame structure

The OFDMA implementation gives the network a number of important advantages. By using the entire spectrum to transmit, the effectiveness is lower than dividing the spectrum into subcarriers and a different data stream is transmitted on each subcarrier. That happens due to the fact that the possible existence of deep fading in a point of spectrum degrades the entire spectrum's performance. The subcarrier technique reduces that effect, because subcarriers which could be in deep fade for one user could be favorable to another user. Since each user has a different channel response for each subcarrier, the spectrum exploitation is more efficient. This is known as multiuser diversity. In addition, the users' ability to occupy as many subcarriers as they need, improves system's performance instead of occupying the whole spectrum for a time period (TDMA) or portion of spectrum (FDMA), which sometimes could be more than enough for their needs. Hence, the right exploitation of scarce resources in a wireless network is a major factor which must be taken into account and OFDMA gives that opportunity. As mentioned above in OFDM systems the PAPR is high, which is not the case in OFDMA since each user has only a subset of subcarriers which are transmitted with low total power in comparison to transmitting over the entire bandwidth [10] [12].

Game Theory

Game theory is an area of applied mathematics and was implemented in the field of economics at the beginning. Pioneers in the evolution of game theory was John von Neumann with his article in 1928 and later on with a book titled "*Theory of Games & Economic Behavior*" that he wrote with Oskar Morgenstern in 1944 [13] [14]. To analyze more game theory let's define first what situation can be characterized as a game. A game is a situation in which:

a. At least two players exist. As a player could be defined as any person, creature or entity (i.e. could be a nation).
b. Each player has a number of available actions, strategies, and has to choose one of them to follow.
c. The game outcome is determined by the strategies chosen by the players.
d. The game outcome is determined by the users' payoffs. These payoffs represent the users' satisfaction from the game.

Game theory studies how players could play rationally and reach an outcome. Each player's strategy has a specific payoff, based on which the player chooses that particular strategy. However, the game outcome is determined by all players' strategies due to the fact that some players could be in conflict. A game is characterized by interactions between players. How does one player's strategy affect others players in choosing their strategies and how rational is this? The game theory tries to answer these questions [15].

The player's payoff as a result of a strategy is not always very clear. This means that when a player wants to choose a strategy he might be a little confused. For example, let's imagine a player with three strategies *a, b, c*. Let's assume that the options are "I like car (*a)* because it is faster than car (*b)*", "I like car (*b)* more than car (*c)* because it is more comfortable" and finally "I like car (*c)* better than car (*a)* because it is cheaper". In this case we can see that strategy (*a)* is better than (*b)*, strategy (*b)* is better than (*c)* but (*c)* is better than *(a)*. Von Neumann and Morgenstern in their book [13] define the term utility function. Utility function gives the ability to assign a real number as a payoff of each strategy. The strategy with the biggest utility function is the best of all. Utility function of strategy (*c)* for example, is denoted as $u(c)$ and is a function which maps the strategy *(c)* to a real number $\left(u(\cdot)\rightarrow\Re\right)$. When $u(a)>u(b)$, this means that strategy *(a)* dominates strategy *(b)* [14]. Since a monotone function could be used as a utility function, multiple consistent utility functions are in existence. For a player the dominant strategy, if any, is a strategy which gives the better payoff to that particular player regardless which strategy the other players will choose [15].

There are many types of games. Considering the communication between players as criterion there are the cooperative and non-cooperative games, static and dynamic considering time impact on the game. In static games all the players choose a strategy simultaneously, however, in dynamic games a player chooses his strategy based on what has happened to the game up to that time. Another type of games is the complete or incomplete information games considering the amount of knowledge which is available to the player about other players' strategies and payoffs. Perfect or imperfect information games regarding the knowledge of individual players about the game, is also a category. Perfect and complete information games are not exactly the same. In perfect information games you know the strategies and payoffs of others but you also know their actions inside the game [16]. Zero sum games are also very widely known where the players' payoffs add to zero. On the other hand, in non-zero sum games because players' interests are not strictly opposed, their payoffs summation is not zero. This kind of game combines cooperation with competitive strategies. In all games rational players are assumed. At a game against nature only one rational payer is considered because nature's strategies are illogical and affect a player's payoff whereas nature has no awareness or interest in, the outcome of the game [15].

A way to represent a game is the strategic form and the extensive form. The strategic form consists of the list of players, the available strategies for each player and the payoffs associated with any strategy combination. For example let's assume a game with two players, *play1* and *play2*, with two strategies, *A,* and *B,* for each of them. A matrix representation of the strategic form of the above game is a 2×2 matrix.

Play1 \ Play2	A	B
A	$\pi_1(A,A),\pi_2(A,A)$	$\pi_1(A,B),\pi_2(A,B)$
B	$\pi_1(B,A),\pi_2(B,A)$	$\pi_1(B,B),\pi_2(B,B)$

Table 1 Matrix representing game's payoffs

Where $\pi_1(A,A),\pi_2(A,A)$ denotes the payoffs π_1, π_2 of players *play1* and *play2* respectively, when strategy *A* has been chosen by both players. Most of the times player *i's* strategies are denoted as s_i whereas others' players strategies except for *i* are denoted as s_{-i}. With π_i or u_i is denoted, the utility function's payoff of player *i* when a strategy has been chosen [14].

Most of the time, matrix representation is used when players choose a strategy without knowledge of what other players have chosen. Extensive form, which is a graphical representation, as a game tree, is a representation which is considered in dynamic games when the players' decisions are sequential like a game of poker [15]. Considering the previous example and assuming that *play1* chooses his strategy first and then *play2*, the corresponding tree of that game will be:

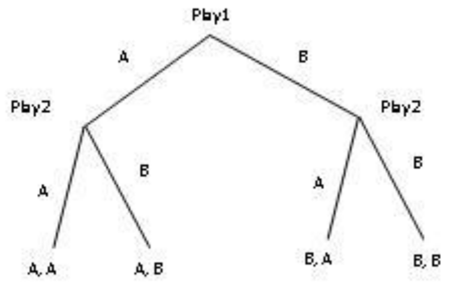

Figure 5 Game tree, payoffs to (*play1*, *play2*)

Two of the most widely known theorems in game theory are the Nash Equilibrium (NE) [17] and the Nash Bargaining Solution (NBS) [18] which, have been formulated by the winner of Nobel Prize in Economics of 1994, John Nash.

Nash Equilibrium

Let's assume a non-cooperative non-zero sum game. As mentioned above, in zero-sum games each player chooses the strategy which dominates all the others in order to get a better payoff.

Rose	Colin	
A	(2,3)	(3,2)
B	(1,0)	(0,1)

Table 2 Game 1 Payoffs to (Rose, Colin)

In Game 1 it can be seen from table 2 that from Rose's point of view Rose prefer Rose A to Rose B because Rose A gives a better payoff to Rose regardless of what Colin will choose. Hence Rose A dominates Rose B. On the other hand, Colin should choose Colin A because his payoff is better than choosing Colin B, knowing that Rose will choose Rose A. Trying to predict the outcome of the above game, the (2,3) would be the choice. Domination strategies could be applied in non-zero sum games as in zero-sum games. However, this is not always the case.

In non-cooperative non-zero sum games typically players do not know how the other players will act and which strategy they will choose. However, a guess based on rational factors could always be a case. The guess about other players' strategies, determines a player's strategy due to the fact that player's strategy should be the best response to other players' strategies. The same happens with all players. They try to choose the best response knowing that the others try to guess each other's choice. Certainly, the guess will not be always accurate. In case that all players have guessed correctly and they are happy with their strategies, the game is in equilibrium and is called Nash Equilibrium in honor of John Nash [14] [15].

Definition: In a game with N players a strategy vector $s^* = s_1^*, s_2^*, \ldots, s_N^*$ is a NE if $\pi_i\left(s_i^*, s_{-i}^*\right) \geq \pi_i\left(s_i, s_{-i}^*\right)$, $\forall i \text{ and } \forall s_i$, where s_i^* denotes the best response of player i to a strategy vector s_{-i}^* [14].

The s_i^* strategy represents the best response strategy and not the dominant strategy in terms of how the dominant strategy has been defined. It can be proved that in a game with a finite number of players and finite strategies there is always a NE. Many times more than one NE point exists in a game. The question which arises is: which is the best? In case where there are more than one NE points acceptable, then it should be that point which is Pareto optimal at the same time. An outcome of a game is said to be Pareto optimal if there is no other strategy which could lead to a better payoff for one player without affecting other players' payoffs. If no one is Pareto optimal any arbitrary NE point is acceptable [15].

Until now only pure strategies have been considered. Many times a combination of strategies has led to an acceptable game outcome or even more to NE. In that case, a player chooses his strategy based on probabilities assigned to them. Suppose a player has M strategies, $s^1, s^2 \ldots s^M$. A mixed strategy for this player is the probability distribution over its pure strategies with probability of each strategy $p^k \geq 0, \forall k$, $k \in [1, M]$ and $\sum_{k=1}^{M} p^k = 1$. Hence, in those cases each strategy is chosen with probability p^k. According to the above mixed strategies, the expected payoff of player i will be:

$$p^1 \times \pi_i\left(s^1, s_{-i}\right) + p^2 \times \pi_i\left(s^2, s_{-i}\right) + \ldots + p^M \times \pi_i\left(s^M, s_{-i}\right)$$

, where s_{-i} is assumed to be other players' pure strategies [14].

Prisoner's Dilemma

Prisoner's dilemma is a very well-known problem in game theory which was formulated in 1950 by Albert W. Tucker. Is a non-cooperative

Two suspects are arrested by the police for a joint crime. The police keep them in separate cells in prison. Because the police do not have sufficient evidence to convict them, they need at least the confession of one of them. The district attorney visits them separately giving them the following options. If one confesses and testifies against the other when the other remains silent, the betrayer goes free and the other (silent) will be convicted to 10 years sentence. If both confess the crime, they will be convicted to 5 years sentence each. Finally, if neither confesses, they will be convicted to only 6 months sentence each. Table 3 shows the payoff matrix of this game.

Prisoner 1	Prinoner 2 confess	 Non confess
Confess	(5 years, 5 years)	(0, 10 years)
Non confess	(10 years, 0)	(6 mon, 6 mon)

Table 3 Prisoner's dilemma payoff matrix

Each of the prisoners has to choose to betray or not. Both of them will find out what the other did after the end of the investigation. The dilemma is obvious. Probably, both prisoners based on their rationality will choose to confess, because they have a better payoff no matter what the other prisoner will do, which leads to 5 years sentence for each as payoff. This NE point is not Pareto optimal due to the fact that both prisoners can achieve better payoff by choosing not to confess. The conflict between individual rationality and group rationality is responsible for the game's outcome. Because the game is non-cooperative each prisoner chooses the strategy which is the best response to the strategy that he assumes the other will choose. In case the prisoner's dilemma plays repeatedly, at the end, both prisoners will choose not to confess having mutual benefit [15].

Nash Bargaining Solution

John Nash formulated the Nash Bargaining Solution in 1950. Bargaining theory is an important topic in game theory and is included in cooperative games. Bargaining situation is a situation, in which two or more players that have conflicting interests try to cooperate in an outcome which will be mutually beneficial. Also, bargaining is a process through which the players, on their own, try to reach an agreement. Bargaining process includes the offers each player makes and the counter-offers from the other players in their effort to reach an agreement. Many bargaining strategies have been developed [19].

Bargaining process is time consuming and aims at an outcome which will be efficient and fair for all the payers. It is very important that NBS is efficient because it is Pareto optimal by definition. NBS is presented in the following paragraphs.

Let's assume a game which consists of two players A, and B bargain over a partition of cake size π where $\pi > 0$. The set of possible agreements between two players is denoted by $X = \{(x_A, x_B) : 0 \le x_A \le \pi \text{ and } x_B = \pi - x_A\}$, where x_i is the player's i share with $i \in \{A, B\}$. If U_i is player's i utility function for each $x_i \in [0, \pi]$, $U_i(x_i)$ is player's payoff from obtaining a share x_i, the utility function $U_i : [0, \pi] \to \Re$ is strictly increasing and concave. In case those players fail to reach an agreement, then each of them obtains a utility d_i which is called disagreement point and is the share that any player could achieve without cooperation. It is stated that $d_i \ge U_i(0)$. However, there is an agreement point $x \in X$ such that $U_A(x) > d_A$ and $U_B(x) > d_B$ which ensures that there exist a mutually beneficial agreement point. Let's define as Ω the possible utility pairs that could be obtained through agreement as $\Omega = \{(u_A, u_B) :$ there exists $x \in X$ such that $U_A(x_A) = u_A$ and $U_B(x_B) = u_B\}$. For an arbitrary utility of player A, $u_A \in [U_A(0), U_A(\pi)]$ and from the state that U_i is strictly monotonic, there is unique share $x_A \in [0, \pi]$ such that $U_A(x_A) = u_A$. If player A obtains a share x_A, then player B will obtain $x_B = \pi - x_A$ and utility function of player B will be $U_B(\pi - x_A)$. However, because $x_A = U_A^{-1}(u_A)$, where U_A^{-1} is the inverse of U_A, player's B utility function can be written in relation with player A as $g(u_A) \equiv U_B(\pi - U_A^{-1}(u_A))$ when player A obtains share x_A. Hence, now the feasible utility space Ω can be denoted as $\Omega = \{(u_A, u_B) : U_A(0) \le u_A \le U_A(\pi) \text{ and } u_B = g(u_A)\}$. The NBS of the above described bargaining process is a unique pair of utilities (point) which maximizes the Nash product and solves the following maximization problem:

$$\max_{(u_A, u_B) \in \Theta} (u_A - d_A)(u_B - d_B) \tag{2.1}$$

, where $\Theta \equiv \{(u_A, u_B) \in \Omega : u_A \ge d_A \text{ and } u_B \ge d_B\} \Rightarrow$

$\Theta \equiv \{(u_A, u_B) : U_A(0) \le u_A \le U_A(\pi) \text{and } u_B = g(u_A), u_A \ge d_A \text{ and } u_B \ge d_B\}$. The problem (2.1) has a unique solution as the Nash product $(u_A - d_A)(u_B - d_B)$ is continuous and strictly quasiconcave, g is strictly decreasing and concave and Θ is non empty. That pair is called NBS and denoted as (u_A^N, u_B^N) [19] [15].

In a general definition a bargaining problem with K players is a pair of (Ω, d), where $\Omega \subset \Re^K$ and $d \in \Re^K$. The definitions of Ω and d have been stated above. The NBS is a function $f : \Omega \to \Re^K$ such that:

$$f(\Omega, d) \arg\max_{U_i \in \Omega} \prod_{i=1}^{K} (u_i - d_i) \qquad (2.2)$$

and satisfies the following axioms.

Axiom 1: independence of linear transformations. For any linear transformation μ is $\mu(f(\Omega, d)) = f(\mu(\Omega), \mu(d))$.

Axiom 2: Pareto optimality. There is no other utility vector u_i' such that $u_i' \ge f_i(\Omega, d), \forall i$ and $u_i' > f_i(\Omega, d), \exists i$, $i \in K$.

Axiom 3: Symmetry. If Ω is invariant under all exchanges of players then $f_i(\Omega, d) = f_j(\Omega, d), \forall i, j \in K$.

Axiom 4: Independence of irrelevant alternatives. For any closed convex $G \subset \Omega$ and $f(\Omega, d) \in G$ then the NBS point of G is the same as before $f(G, d) = f(\Omega, d)$.

There is another case for NBS named asymmetric NBS case in which each player has its particular weight w_j giving the ability to assign priorities to the users. The summation of users' weights must be equal to one. In case of two players, following the same definitions as the NBS, for each $w \in (0,1)$ the Nash product becomes $(u_A - d_A)^w (u_B - d_B)^{1-w}$ and the maximization problem is defined as:

$$\max_{(u_A,u_B)\in\Theta}\left(u_A-d_A\right)^{w}\left(u_B-d_B\right)^{1-w} \tag{2.3}$$

Similarly to NBS case the asymmetric NBS solution is a function $X:\Omega\rightarrow\Re^{K}$

$$X\left(\Omega,d\right)\arg\max_{U_i\in\Omega}\prod_{i=1}^{K}\left(u_i-d_i\right)^{w_i} \tag{2.4}$$

All the axioms stand also for asymmetric NBS case but the axiom 4. Because of the different players' weights there is no symmetry and payoffs are distributed to them according to their weights [19].

Summary

Efficient resource allocation is the main aspect for next generation wireless networks. A critical element for wireless networks is the profitable medium exploitation, and OFDMA has been adopted as the most sophisticated and robust technique. The first part of this chapter makes an introduction to OFDM and OFDMA providing the main characteristics of these techniques in order to ensure the definition of the physical layer of 4G wireless networks, which is based on these techniques. Also, the NBS has established which can be used to optimize resource allocation which is the main aim of the thesis. The next chapter provides a literature review on cross-layer schedulers with respect to OFDM-OFDMA

CHAPTER 3

Literature Review on Resource Allocation in OFDMA Systems

Introduction

In this chapter a detailed literature review is provided according to published articles and books. This includes the research that has been done in cross-layer design in wireless OFDM/OFDMA systems in recent years as well as all the different approaches which are based on numerical, algorithmic, analytical methods or on game theory.

Work has been done

OFDM technique was adopted in commercial products in 1999 in IEEE 802.11a wireless standard which is described in [20]. According to [20], all subcarriers are assigned to only one user per time-slot which means only one user can use the physical medium each time. In this standard OFDM provides a data rate from 6Mbits/s to 54Mbits/s in the frequency band of 5GHz. The number of subcarriers is 52 which are modulated by BPSK, QPSK and M-QAM modulation techniques. Later on, OFDMA was adopted by the new wireless protocol for metropolitan wireless networks as the physical layer in IEEE 802.16 standard [21] [1]. In [21] and [1] subcarriers are assigned to more than one user in a static manner, hence many users can transmit or receive data simultaneously. However, the static manner of allocation cannot exploit the frequency diversity, leading to low performances and making the necessity for dynamic resource allocation, which includes subcarrier and power allocation, being the only way in order for high performances to be achieved. Thus, a large research field on that topic is created. Dynamic resource allocation has been investigated by many authors using different approaches. The goal each time may differ (maximize total throughput, minimize power, satisfying users' needs, fair resource allocation), but the resources are allocated following a scheme which provides better result than static allocation.

In [22], a heuristic algorithm for subcarrier allocation, which minimizes the total transmit power in a multiuser scenario is presented by authors, considering the downlink case. Perfect channel knowledge at the transmitter is assumed and the subcarrier assignment is done regarding channel gain. Corresponding amount of power is allocated to each subcarrier to overcome channel's noise while satisfying users' transmission requirements. Scheduler algorithm achieves the optimal solution by an iteration process.

A very widely used way to solve such constraint optimization problems is to use convex optimization [23] by creating a convex objective function. In [3], authors construct a convex optimization problem to minimize the power assuming a Rayleigh channel and perfect channel knowledge at the base station. However, due to the fact that the solution of convex objective function demands so much time, they also proposed a suboptimal solution with equally divided power for each subcarrier which results in a quicker allocation without complicated calculations and is very close to the optimal solution.

In [24] a proportionally fair power allocation scheme is proposed. Subcarrier allocation algorithm is based on channel gain and each user's weight factor. Power allocation is based on water-filling (WF) [25] method which gives advantage to the users with good channel conditions and which leads to unfair assignment amongst users. The proportional fair power allocation algorithm in [24] grants identical increment throughput in each subcarrier, providing fairness among users. In the first step, the above algorithm allocates the power among users and afterwards among subcarriers which are assigned to a particular user.

In [26], authors present a dynamic resource allocation approach which tries to satisfy users' Quality of Service (QoS) constraints whilst maximizes total data rate. The subcarrier allocation is based on a cost function which includes delay constraints and throughput requirements on the one hand, and channel's gain for each subcarrier for each user on the other hand. The next step of the proposed algorithm refers to the bit and power allocation to each subcarrier aimed at maximizing the total rate given the subcarrier allocation which is based on the previous constraints. The power cost function increases the rate of subcarrier in which the power demands are the least among the others of the same user.

A cross-layer problem formulation for resource allocation in OFDM systems is presented in [27]. An algorithm, which manages the resources according to MAC layer of each user and regarding users QoS constraints, is proposed. MAC layer does not include a proper queuing model but instead a packet analysis has been done in relation to queue length and transmission rate. The problem is formulated into a constrained optimization problem with constraints regarding QoS and users' queue length in order to maintain fairness amongst users and maximize power efficiency. Integer constraint relaxation is used to simplify the main problem in terms of subcarrier and power allocation and finally solve it in polynomial time.

In [28] [29], as well as in [22], authors investigate the subcarrier, bit and power allocation in a multiuser OFDM system aiming to minimize total power for downlink communication. The optimization problem which is formulated on the basis of power minimization is a convex minimization problem and it is solved by using the Lagrangian method. Once subcarriers are determined for each user then a bit loading algorithm is applied, using adaptive modulation and coding (AMC) to allocate bits to subcarriers considering the channel gain and the bit error rate levels for each of them.

In [30], a novel loading for OFDM systems, algorithm is proposed, which aims to maximize the total system's throughput while satisfying total power and users' rate constraints. Firstly, the number of subcarriers and power for each user is determined and then, the particular subcarriers and power for each of them is determined using the Hungarian algorithm [31]. Also, a sub-optimal solution for subcarrier allocation for OFDMA networks based on Hungarian algorithm is proposed in [32].

In [4] as well, authors aim, by the formulated an optimization problem, to maximize overall user's capacity, and at the same time maintain proportional fairness amongst users under the total power constraint. Subcarrier and power distribution are carried out separately in an effort to reduce the complexity. Solution is derived by a sub-optimal algorithm.

Also in [33], a method which maximizes system's overall data rate, developing a transmission power adaptation technique is investigated. Subcarriers are assigned based on channel gains and each subcarrier is occupied by the user who has the best channel gain for that particular subcarrier. In this way, each subcarrier can achieve the maximum rate, resulting in increasing the system's overall rate. The adaptive power allocation is determined by the water-filling algorithm giving more power to subcarriers which are in fade and less to subcarriers which have high channel gain. Authors, also prove that maximum data rate can be achieved only with exclusive subcarrier assignment to only one user instead to allow users to share subcarriers. However, due to the fact that power allocation, according to the water-filling algorithm, is computationally demanding, they equally distribute power among subcarriers regardless of channel gains and show that the outcome is slightly different in comparison with the water-filling method.

Trying to solve the above mentioned problem of computationally inefficient proposals and algorithms, authors in [34] propose simple algorithms with good performances. The proposed algorithm minimizes the total power consumption whereas satisfying users' transmission rates. The average signal to noise ratio (SNR) and rate requirements are the criteria for the algorithm to decide the number of subcarriers each user needs and then two different ways give the exact subcarrier assignment. The first, rate-craving, a greedy algorithm estimates users' transmission rate on each subcarrier and allocation is done by meeting their rate constraints and maximizing system's throughput. The second, amplitude-craving, a greedy algorithm allocates each subcarrier to its best user without user having the right to ask for more subcarriers. However, because channel gains are normalized, the users who are in deep fade have more opportunities for more resources giving to that algorithm a kind of fairness. Both of the solution proposals are numerical with low complexity.

Also in [35], a reduced complexity algorithm is proposed for resource allocation in OFDMA systems. This scheme proposes a solution which aims to maximize total throughput retaining proportionality amongst users and constraints such as total power and bit error rate. Non-iterative methods result in a low complexity algorithm with higher rates as outcome compared with root finding algorithm, where many iterations are needed.

In [36] authors propose a resource allocation algorithm in order to support multimedia services without considering queue characteristics. The system can support real-time (RT) and best-effort (BE) users. Authors, aim to maximize system throughput whereas, QoS requirements of both type of users, RT and BE, are satisfied. For simplicity in this proposal the total available power is equally distributed among subcarriers. The optimization problem is transformed into a dual optimization problem. Similarly, in [37] authors propose a cross-layer optimization algorithm for video transmission over 4G LTE network under application rate constraint. A proportional fair cross-layer framework is considered in [38] which apply three different algorithms for multimedia services. Results show that the three algorithms achieve good performance whereas fairness is maintained.

The resource allocation in OFDMA wireless networks for fair scalable video transmission is investigated in [39]. Authors, address the resource allocation problem by formulating an optimization problem which is decomposed into two sub-problems. Even though, the main scheduler's goal is to maximize the system's aggregate rate, fairness is also considered in the constraint optimization problem. An optimal solution is achieved by the iterative local approximation algorithm, which is proved to converge to the unique optimal solution. A sub-optimal low-complexity algorithm, based on the first-step iterative approximation algorithm, is designed, since the overall complexity of the first algorithm leads to delays. Simulations show that the first-step algorithm achieves also good performance providing good video quality in comparison with the original one.

A cross-layer framework for elastic and delay sensitive traffic is proposed in [40]. Priority for the scheduler is the delay sensitive users and resources are allocated in such a way that elastic users will occupy any resources if sensitive users do not need them. The constraint optimization problem is considering imperfect channel knowledge and is solved utilizing the dual decomposition method.

Authors in [41], address the dynamic resource allocation problem in OFDMA wireless networks whilst retaining QoS constraints in long term way. That means QoS requirements are met in long time duration and not within a time-slot. The delay requirements are expressed in virtual queue lengths and the scheduler aims to minimize the virtual queue length for each user. The instantaneous optimization problem is modelled as a convex optimization problem and solved by utilizing Lagrangian method and Karush Kuhn Tucker (KKT) conditions. Simulations show that, this scheme has better performance by providing smaller delays and additionally has better fairness than other opportunistic schemes.

In [42], resource allocation in OFDMA systems regarding users' queuing status has been investigated. The objective function in the optimization problem is formulated regarding queue length and channel gains of each user. User satisfaction is measured using a utility function for mean queue delay, which can offer QoS and a kind of fairness among users. Subcarrier allocation is done by two algorithms. In "weighted max algorithm without frugality constraint", subcarriers are allocated to the users which have better channel conditions or long average delay. However, in "weighted max algorithm with greedy reassignment", the scheduler allocates the subcarriers to the users according the previous algorithm and consequently subcarriers which belong to empty queue users, are reassigned to the other users. Simulations show low delays in relation to the proportional fair algorithm, which does not take into account queue status.

A maximization cross-layer optimization problem with proportional fairness is addressed in [43]. The MAC layer has been totally ignored and the proportional fairness is achieved by factors which are applied to each user. In case that all fairness factors are identical and equal to one then the optimization problem is transformed to a clear max-min problem. Authors consider equally distributed power along subcarriers for subcarrier allocation and following that in the next step power is optimally assigned to each subcarrier. Thus, the proposed solution is suboptimal in terms of subcarrier allocation.

In [44], a low complexity cross-layer design is presented. The scheduler aims to maximize the weighted aggregate capacity considering heterogeneous traffic in the MAC layer. The weight of each user is calculated on the basis of its selection for transmission packets. Therefore, each packet has its weight and can affect user priority. Also, authors here propose a suboptimal solution. In this case the resource allocation is based on packet weights and not on users' QoS requirements.

An investigation for cross-layer optimization is presented in [45] and [46]. The authors have used a utility function in order to describe users' needs, which is a function of data rate. Physical and MAC layers are related through this utility function. Optimization problem is proven to be convex, and then global maximum represents optimal solution. A dynamic subcarrier allocation (DSA) with equal power allocation and an adaptive power allocation (APA) with static subcarrier allocation are analysed, as well as a joint DSA and APA method. Also, efficiency and fairness is provided by this scheme. No user can increase its utility without harming another one, which proves the system's efficiency. In addition, proportional fairness is granted in this proposal.

Moreover authors in [47], propose a utility function based scheduling algorithm with fairness considerations. The optimization problem is a maximization problem of the system's total utility function. MAC layer is taken into account in the scheduler design as well as the channel conditions for each user. The algorithm performs subcarrier allocation regarding each user's QoS requirements and queue delay. Fairness is applied through different utility functions, to the users with longer waiting time in the queue. Simulation results show that the scheduler performs better in comparison with conventional approaches whereas it provides good fairness amongst users. Also in [48], the dynamic resource allocation is performed by utilizing a utility function scheduler which provides improved fairness in comparison with greedy schemes.

Furthermore, maximization of the aggregate utility is investigated in [49] [50] [51] [52] [53] where subcarrier and power allocation are performed regarding users' QoS constraint as well as the maximization of the system's overall performance.

A work for resource allocation in OFDMA wireless networks for heterogeneous users, capable to support real time users and proportional fairness is presented in [54]. Authors here, consider a flat-fading channel, meaning that all subcarriers are equal with respect to a user, leading to a decrease the algorithm's complexity, as adjusted subcarriers are assigned to each user. The proposed technique aims to maximize the long term received rates providing long term proportional fairness. The constraint optimization problem is formulated including the real-time users' constraints as well as power optimization. Simulations of the proposed scheme have shown that it outperforms in terms of proportional fairness and real-time users, in comparison with other schemes.

In [9], a low complexity algorithm for subcarrier allocation in OFDMA systems based on NBS is investigated. The authors, trying to provide fairness in resource allocation among users while maximizing total throughput, propose a solution based on game theory and particularly on NBS. NBS properties through Pareto optimality ensure an optimal solution in the constraint optimization problem. Minimum users' requirements are fulfilled in order for a user to join the game for benefit more resources. A two user bargaining algorithm is developed and then is applied to the all users, who are divided in random coalitions of two or by the Hungarian method. Through iterations, the algorithm finds the optimal solution. Negotiations between users aim to improve users' payoff, to provide fairness among them and to maximize system's payoff as well. Simulations proved that the total rate is close to the greedy opportunistic schemes whereas fairness amongst users is achieved with low algorithm complexity.

Authors in [55], propose a low complexity algorithm for channel allocation which is based on NBS. The basic idea, in this investigation, is about forming coalitions comprising of two users, who negotiate the usage of the subcarriers meeting their rate constraints. The algorithm is applied continuously and new coalitions are formed every new iteration until no more improvement can be achieved. The last solution is algorithmic, using the Hungarian method with overall complexity of $\mathcal{O}\left(N \cdot \log N\right)$.

A non-cooperative game is utilized for power and bit allocation in [56]. Authors, propose a non-cooperative game for power allocation through a water-filling algorithm. In the first step of the algorithm, each user takes a portion of the power and then a bit allocation procedure is performed. In the second step, the rest of the available power, if there is any, is assigned to the users by the greedy algorithm.

In [57] as well, a cooperative game approach for resource allocation is provided. Fairness among same and different classes of users and efficiency are granted. However, no coalitions are formed in this scheme in contrast with [9], and no power allocation is considered. The problem is analyzed in a Taylor series and is solved in an analytical way having as utility function the users' mean data rate. It has to be mentioned that in this particular case the users' initial requirements are zero.

Another cooperative game approach for OFDMA resource allocation is presented in [8]. The proposed solution, considering users' requirements and the total power constraint, is based on NBS and maximizes system's throughput retaining fairness amongst users as it can be seen from simulations results. Users' satisfaction is expressed as function of average throughput. In order to reduce the algorithm's complexity, subcarrier allocation is assumed with equal power distribution in the beginning. Moreover, an optimal power allocation algorithm is developed defining the water-filling levels for each subcarrier resulting in an optimal solution.

A very interesting investigation is presented in [58]. A game model for power allocation among subcarriers is introduced with channel uncertainty considerations. Since the overall concept is about choosing the appropriate power level for each subcarrier without previous knowledge of channel gain, the game is classified as a game with one non-rational player (channel) and is called game against nature [15]. Transmission rate and power amounts define the game's payoff for each user and the resulting utility function is a trade-off between power consumption and throughput maximization. Simulations show that good results can be achieved, which are close to the optimal water-filling solution.

In [59], is proposed a downlink scheme for resource allocation in OFDM systems regarding different allocation procedures for subcarrier and power. A gradient based scheduling algorithm optimizes system's performances subject to different subcarrier allocation schemes and SNRs constraints, allocating each subcarrier to one or more users per timeslot. The formula of subchannels is adopted. A subchannel consists of more than one subcarrier with resembling attributes and is assigned to each user. Three subchannelization modes are presented in [59]. These are: i) adjacent subchannelization, where adjacent subcarriers are grouped together, exploiting frequency diversity; ii) interleaved subchannelization, where subcarriers are perfectly interleaved and iii) random subchannelization, where subcarriers are randomly assigned to the users. Except for optimal algorithm, two reduced complexity heuristic algorithms are presented with the first to allocate subcarriers based on product of rate and user's weight with equal power distribution and the other, following the same procedure as the above with power allocation to be performed as well. Simulations showed that heuristic algorithms have performed very closely to the optimal solution.

Almost, in all previously mentioned works, perfect channel state information is assumed at the transmitter. In [60], authors present a rate maximization scheme by investigating imperfect (partial) channel knowledge at the transmitter. A power allocation algorithm based on partial channel is implemented aiming to maximize average data rate subject to the constraint of channel outage probability. By modelling channel uncertainty as ergodic process and as quasi-static model, power optimization algorithms are introduced. Their results are compared with water-filling in perfect channel state information at the transmitter (CSIT) and equally distributed power among subcarriers. Simulations show that in partial CSIT with optimal power allocation, the outage rate is very close to water-filling with perfect CSIT and also that the joint loading with optimal power allocation scheme performs better than individual loading because of the exploitation of frequency diversity.

In [2], a cross-layer scheduler is proposed considering heterogeneous delay requirements for users and assuming a perfect CSIT. A queue analysis is presented regarding users' delay requirements. Afterwards, a cross-layer design is performed by expressing delay requirements in physical layer units and by fitting that constraint into the convex optimization problem which maximizes the total system's throughput subject to users' constraints and total power. Subcarrier and power, water-filling, allocation arises as the solution of the abovementioned optimization problem satisfying users' QoS requirements, while maximizing the total system throughput. Simulations results show that the scheduler is able to provide the desirable QoS level for each user and maximize the overall system's performance.

An expansion of work in [2] is [61], in which a cross-layer scheduler is presented as in [2], but the effect of outdated channel state information was investigated. In the queue model, packet errors are taken into account resulting in retransmissions and reducing goodput. The scheduler provides power and subcarrier allocation as the solution of the convex combinatorial optimization formulated problem regarding all the constraints. Simulations showed that the system is robust enough even under high channel uncertainty.

In [62], a cross-layer design is also presented. The proposed scheme is considering an M/G/1 queuing model in MAC layer and partial channel state information (CSI) at the transmitter. A convex optimization problem is formulated under the constraints of heterogeneous users' QoS, which is based on queue delay expressed in rate units, total available power and channel errors. The problem solution results in subcarrier and power allocation aiming to maximize the total system performance under the above mentioned constraints. Power consumption is proven by simulations to be lower in comparison with [2] and [61], which is a crucial factor as well as the novel offline algorithm which has been introduced to calculate Lagrangian multipliers for DSA and APA. Also, an energy efficient proposal for OFDMA systems considering imperfect channel is proposed in [63].

In [64], an adaptive fair resource allocation scheme for OFDMA systems is investigated. The queuing model which is considered in that scheme is based on discrete Markov Modulated Poisson process (dMMPP) and its performance like packet dropping probability, average packet transmission rate and average packet delay was measured. Fairness is achieved according to Generalized Processor Sharing (GPS) scheme [65] and the optimization problem aims at maximizing the total throughput maintaining fairness among users simultaneously in terms of data rate. Subcarrier allocation is treated as an assignment problem and the Hungarian method [31] is used for solving that problem in an optimal approach. However, a suboptimal less complex approach, which is named iterative approach, is adopted giving almost the same results as the optimal solution.

In [66], a cross-layer scheduler is presented with finite queue length in the MAC layer. Its goal is to maximize aggregate system's throughput, whereas the QoS constraints of users are guaranteed. The necessary bandwidth per user is denoted by queuing analysis. Subcarrier assignment is done by each user occupying subcarrier/s, the rate of which is very close to their needs instead of occupying subcarriers with higher data rate regarding their transmission needs, which leads to an increase of system's performance. Performances from simulations are shown to be very close to optimal throughput without QoS constraints.

A cross-layer optimization scheme with finite state Markov chain in MAC layer is considered in [67]. Authors, examine a multiple-input multiple-output OFDMA scheme which aiming to meet users' QoS requirements under the constraints of data loss and buffer overflow. The solution of the optimization problem is derived through Markov decision process by minimizing the probability of data loss and providing the optimal transmission rate for every state of buffer.

In [68], an adaptive resource allocation and connection admission control (CAC) scheme is investigated. Resource allocation and CAC are formed in a non-cooperative game in which each side (base station and new connection) tries to maximize their payoffs. A queuing model is used to determine QoS constraints for real and non-real time services, in terms of delay and throughput which have to be satisfied by the base station. When a new connection is requested, the Nash equilibrium is found between base station and new connection in which the best response strategies are selected in order for both players' payoff to be maximized. The acceptance by the base station of a new connection means that new connection's QoS requirements are fulfilled without degradation below an acceptable level of the already existing connections. Simulations showed that user's QoS requirements are fulfilled and are above the acceptance level each time a new connection is requested and admitted.

In [69], a resource allocation scheme based on utility function which considers transmission rate, normalized user's delay and a prioritization factor for each user is presented by the authors. The summation of users' utilities is the total system's utility and expresses the total system's performance. The proposed utility function proved to keep balance between efficiency and fairness amongst users, while it maximizes the total system's payoff at the same time and satisfies the users' requirements in terms of delay. In the formulated optimization problem for DSA and APA, constraints regarding total power and power per subcarrier are considered and the problem is solved by a heuristic algorithm using Lagrange multipliers theorem. Simulations showed that delay constraints and fairness can be guaranteed with a cost of a lower total system's throughput.

In [70], an optimal resource allocation considering imperfect CSIT is investigated. The partial CSIT is used in the maximization of both, continuous and discrete weighted sum rate under the limitations of total power and BER providing a more close to reality approach. Low complexity algorithms are proposed due to the dual optimization framework which is adopted for both problems. Simulations showed that the proposed resource allocation algorithm, based on imperfect CSIT, performs very close to perfect CSIT but errors and packet retransmissions can be avoided by adopting a less aggressive strategy, especially when high level channel error is sensed.

A very interesting work in resource allocation and admission control for OFDMA systems is presented in [71]. The system is considered to be heterogeneous consisting of high priority and best effort users in analogy 50% of each and resource allocation aims to maximize best effort users' total utility, whereas high priority users' QoS requirements are fulfilled under the restriction that a new high priority user is accepted by the system only when its requirements can be met by the network. A number of adjacent subcarriers form subchannels and instead of subcarriers the subchannels are used in allocation. This tactic reduces the feedback from mobile users while only one carrier to noise ratio is demanded per subchannel and this is the carrier to noise ratio of the worst subcarrier. Similarly, the complexity is degraded due to the fact that the number of subchannels is much smaller than subcarriers. Two separate non-linear mixed integer problems for cluster and power allocation are formed where optimal and suboptimal for cluster assignment and optimal for power allocation solutions are provided. Suboptimal algorithms demand less computational power however, resulting in almost the same outcome with optimal algorithm as is proven by simulations.

In [72] also, an admission control resource allocation scheme is presented. The system use resource utility functions for allocation or reallocating resources to connections. Factors which are considered for resource allocation are the age of the connection, the penalty that considered when a connection is dropped and the sensitiveness to reallocation frequency. Algorithm aims to maximize the system's overall utility.

Two algorithms with low computational complexity for resource allocation, in comparison with other investigations, are proposed in [73], aiming to minimize total transmit power whereas through power and subcarrier assignment, user's requirements are met. The first algorithm, which is near optimal, is based on dynamic programming. In the beginning, all subcarriers belong to all users and then a removal technique with as many levels as the subcarriers, removes at each level a subcarrier from all users but one. Hence, at the end each subcarrier is assigned to only one user. This dynamic programming resource allocation algorithm is represented by a tree with the number of levels being equal to the number of subcarriers and each level includes branches for all the users. The second algorithm, which is based on linear programming technique, is called branch-and-bound and achieves optimal solution. The branch and bound technique is based on defining and initial upper bound and lower bound in terms of required power, which leads to the removal of sub-trees that are not searched. The second algorithm has more complexity than the first one, but in both cases once the subcarrier allocation has been done, an optimal mono-rate power allocation algorithm determines each subcarrier's power share.

A hierarchical resource allocation scheme for OFDMA distributed wireless systems is investigated in [74]. In this research an architecture consisting of one central unit, which is wired connected with access points where users are connected wirelessly to access points forming sub-cells, is considered in an effort to reduce the complexity of centralized systems. A two steps scheduler is developed where subcarriers are allocated to access points in the first step and subcarrier and power are allocated to the users in the second step. In order for both efficiency and fairness to be achieved, the NBS is adopted in both steps providing good performances as is shown in simulation results.

In [75], a cross-layer design scheme based on cooperative game theory is investigated. Nash and Raiffa-Kalai-Smorondisky [76] bargaining solutions are used so that Pareto optimal solutions are achieved. It has been shown that Raiffa-Kalai-Smorondisky solution can achieve better performance than NBS in terms of maximal rate because both minimum and maximum rate are considered instead of NBS, where only minimum rate is considered. Simulations have shown that, NBS performs better when the distance between minimum and maximum rate is relatively close, whereas, Raiffa-Kalai-Smorondisky performs better than NBS in case of large a difference between the rate edge values.

A cross-layer scheduler considering not only channel's condition but queue status as well is presented in [77]. Authors aim to maximize the total system's throughput whereas proportionality lies amongst users. Reducing the probability of resources being allocated to the users without enough data to transmit, system saves resources which can be assigned proportionally to the starving users. In the first step, a suboptimal iterative algorithm allocates subcarriers to the users assuming equal power distribution among subcarriers, under the constraint that each user's rate must be equal to or less than its queue length. Continuously, a bit loading algorithm determines the power for each bit, by allocating bits to the user's subcarrier with the least power demand. Simulations showed high total throughput and low power consumption.

Another way for resource allocation in OFDMA systems is based on subchannels which are groups of subcarriers assigned to a user, while in most cases each subcarrier is assigned separately. Such a scheme is investigated in [78], where groups of adjacent subcarriers are allocated to the users based on BER constraint and is compared with the same scheme allocation is based on subcarrier's SNR. BER is calculated for each subchannel as long as the best affordable modulation level is chosen in order that efficiency is maximized. BER's chunk allocation performances proved to be better than SNR's chunk allocation because the former way has smaller outage probability.

In [79], authors formed the cross-layer optimization problem as a Markov Decision Problem (MDP) [80]. Channel state and queue state information are the factors which affect MDP's actions (regarding power and subcarrier allocation). Delay minimization is the major constraint in this scheduler considering heterogeneity of the users and the arrival/departure process. Next, the minimization problem is transformed into a reduced state Bellman equation [81], and a delay optimal allocation results are obtained by an online stochastic value iteration solution. Iterations are proven to converge to the optimal solution with probability almost equal to one (1) providing an affective suboptimal low-complexity power and subcarrier allocation.

Summary

With no doubt, cross-layer optimization for wireless networks is a very challenging and crucial topic for their performance. The scarcity of the available resources combining with the channel's fluctuations makes the design of sophisticated schedulers a necessity. In this chapter a detailed literature review is presented including all the available work that has been done in this topic. Subcarrier and power allocation, in many different ways and algorithms, are presented, aiming to different optimization factor each time.

Most of the efforts are about maximization of the overall system's capacity using greedy methods leading to an unfair distribution of the available resources. Yet, all the optimization problems have been formulated as constraint optimization problems, which means, that users demands, QoS restrictions, real-time users delay requirements, channel state information and power restrictions are some of the constraints that have been taken into account. Power minimization is also a favourite topic amongst the proposed schedulers. Furthermore, fairness is introduced in some schemes with the most popular being the proportional fairness algorithm which utilizes the idea of the utility function. In addition, cross-layer schedulers based on game theory, mostly on NBS, reported, a trade-off between efficiency and fairness being achieved.

Finally, all the research in cross-layer design, which has been done so far, is presented in chapter 3. MAC layer constraints, QoS requirements and many other restrictions are considered in the authors' presented proposals and each one optimizes the system in a specific way.

CHAPTER 4

Resource allocation based on Nash Bargaining Solution

Introduction

As it has been mentioned in Chapter 2, the essential characteristic for next generation networks, such as 4G, LTE, and WiMax is to provide high data rates as well as low delays and thus satisfy the users' growing need for QoS applications. Fairness must also be considered in modern scheduling algorithms, as greedy algorithms work in favour of those who are in good channel conditions. The lack of unlimited available resources, in wireless networks, forces us to use an efficient and high reliability modulation technique in the PHYL such as orthogonal frequency division multiplexing (OFDM). The aforementioned modulation technique due to orthogonality offers high data rate at the PHYL whilst minimising the ISI [21]. Based on this robust technique the orthogonal frequency division multiple access has been introduced and adopted as the basic multiple access method for the new generation wireless networks [1]

Efficient resource allocation amongst users comprises the most crucial issue in OFDMA systems, which represent the way, in which power and subcarriers are dynamically assigned to the users by the BS. Cross-layer design includes the implementation of a scheme for resource allocation in the PHYL according to users' QoS requirements as well as the queue status from MAC layer. In cross-layer optimization we can differentiate schemes to designs which are aiming to minimize the total power required [22] [3], schemes which are aiming to maximize system's aggregate throughput [45] [2] and schemes which are providing proportional fairness among users [5] [35]. Some OFDMA cross-layer designs do not take into account the queue status ignoring MAC's layer influence. However, most of the cross-layer designs have totally ignored the fairness issue in resource allocation, leading to totally unfair approaches; furthermore, due to scarcity of resources some users who are in deep fade may be starved of high quality transmission resources.

Cooperative game theory has been in great attention as a decision tool for resource allocation in wireless networks providing fairness amongst users, whilst maximizing the aggregate system's performance [9] [82] [83]. NBS is the most well-known theorem in such cases and has been employed in many proposals [9] [8] [84]. NBS uses utility functions, which have been introduced in the area of economics, to evaluate users' satisfaction (payoff of the game) in a real number in order to be comparable and understandable [15].

System model

Our system, which is illustrated in Figure 6, comprises of a single OFDMA cell. The cell accommodates K users who are served by N subcarriers of total bandwidth BW Hz, which is cell's bandwidth. Hence, each subcarrier has a bandwidth of BW/N Hz. We assume that channel state information for each subcarrier is known to the transmitter (perfect CSI) by exploitation of pilot subcarriers, whereas channel gain remains the same during an OFDM symbol due to slow fading.

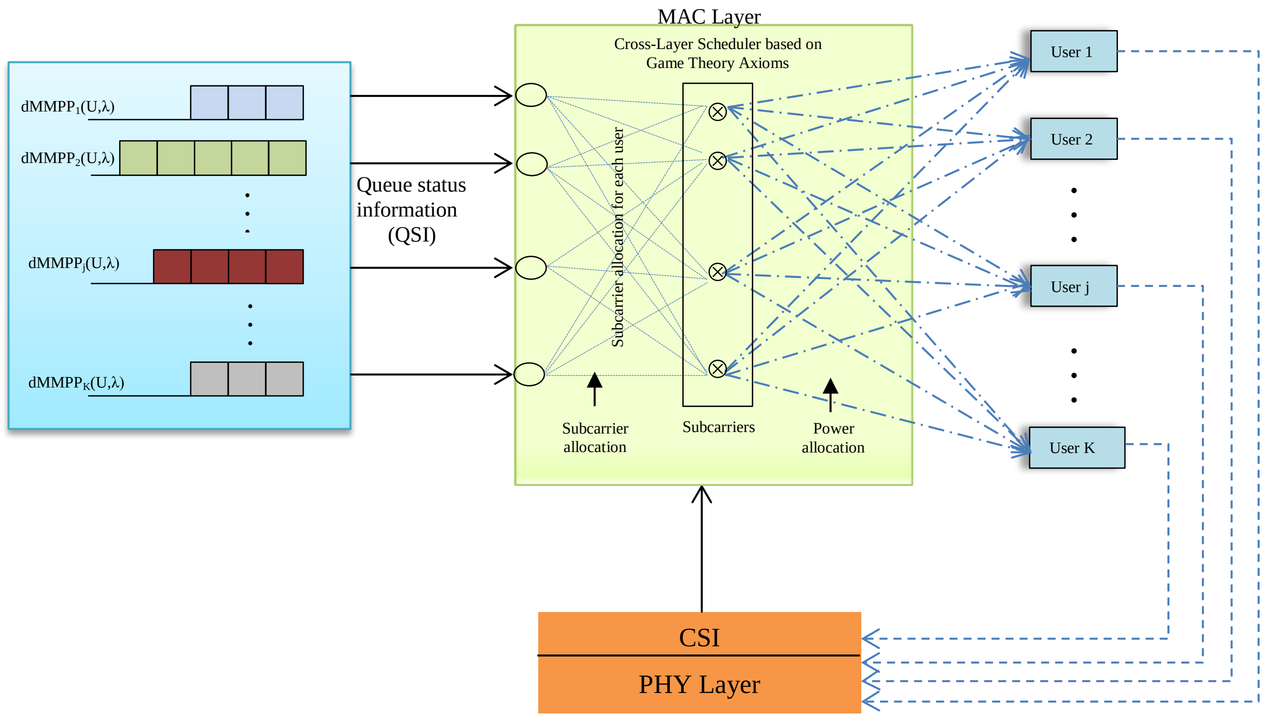

Figure 6 Cross-Layer OFDMA System Model

The users' QoS requirements and channel response for each subcarrier for each user are collected by the cross-layer scheduler. Then the resource allocation is performed by the BS and a number of subcarriers are allocated to each user servicing each user's queue. Following that action, the corresponding power is given to each subcarrier and all the information is passed to the OFDMA transmitter in order to perform the transmission.

Adaptive modulation and coding is used in order for better performance to be achieved per subcarrier regarding instantaneous channel conditions. M-ary quadrature amplitude modulation (M-QAM) is utilized in order each subcarrier achieves its best performance according to its instantaneous channel conditions. Transmitter performs inverse fast Fourier transform (IFFT) and appends the CP. The received OFDM symbol, after the extraction of the CP and performing fast Fourier transform (FFT), on i^{th} subcarrier of user j is denoted as:

$$y_{ij} = x_{ij} h_{ij} + z_{ij} \tag{4.1}$$

, where x_{ij} is the transmitted data symbol from base station to the j^{th} user on the i^{th} subcarrier, h_{ij} is the independent identically distributed (i.i.d) zero mean complex Gaussian with the unit variance complex channel gain of the j^{th} user on i^{th} subcarrier and z_{ij} is the zero mean complex Gaussian noise with unit variance $z \sim CN\left(0, \sigma_z^2\right)$.

Each user's rate is denoted as $r_j = \sum_{i=1}^{N} s_{ij} r_{ij}$, where s_{ij} is the element of the subcarrier allocation matrix $S_{N\times K} = \left[s_{ij} \right]$ in which $s_{ij} = 1$ when the subcarrier i is allocated to the j^{th} user otherwise $s_{ij} = 0$ and r_{ij} is the data rate (bits/sec/Hz) on i^{th} subcarrier of j^{th} user. The aggregate system's data rate is denoted by:

$$r = \sum_{j=1}^{K} r_j = \sum_{j=1}^{K} \sum_{i=1}^{N} s_{ij} r_{ij} \tag{4.2}$$

The power which is allocated to user j on subcarrier i is denoted by p_{ij} from the corresponding power allocation matrix $P_{N\times K} = \left[p_{ij} \right]$ whereas the rate r_{ij} forms the data rate matrix $R_{N\times K} = \left[r_{ij} \right]$. If P_{total} is the total base station's power then the following inequality (4.3) must be valid:

$$\sum_{j=1}^{K}\sum_{i=1}^{N} s_{ij}\, p_{ij} \le P_{total}\,,\;\; P_{total} \ge 0 \tag{4.3}$$

In addition, each subcarrier can be occupied by only one user which means equation (4.4) must be stand.

$$\sum_{j=1}^{K} s_{ij} = 1,\; \forall i \tag{4.4}$$

By assuming perfect CSI at the transmitter and quadrature amplitude modulation (QAM) modulation technique, case we consider, the BER for the j^{th} user on the i^{th} subcarrier based on [85] would be $BER \approx 0.2\exp\left[\frac{-1.5\gamma_{ij}}{2^{r_{ij}}-1}\right]$, where $\gamma_{ij} = \frac{\left|h_{ij}\right|^2 p_{ij}}{\sigma_z^2}$, h_{ij} and p_{ij} have already been defined and σ_z^2 is the variance of adaptive white Gaussian noise (AWGN) and denoted as $\sigma_z^2 = N_0 BW / N$, where N_0 is the density of noise[1]. Hence the rate of j^{th} user on i^{th} subcarrier using AMC will be:

$$r_{ij} = \log_2\left(1 + \frac{p_{ij}\left|h_{ij}\right|^2 c_3}{\sigma_z^2}\right) = \log_2\left(1 + c_3\gamma_{ij}\right) \tag{4.5}$$

, where $c_3 = \frac{-1.5}{\ln(5 \cdot BER)}$.

Queuing model

Packet arrival process

The packets that each user has to send through the PHYL arrive from the higher layers. A queue model should be adopted in order to describe that process. In most cases an infinite queue with arrival rate following the Poisson process is used which is not realistic as the queues are not infinite and the arrival rate varies. Since the queue model has an important role in the cross-layer scheme a finite length queue with variable arrival rate is adopted in this

[1] It can be assumed without loss of reality that the channel response h_{ij}, hence $\gamma_{ij} = \left|h_{ij}\right|^2 p_{ij} / \sigma_z^2$ remains constant over an OFDM symbol (since the OFDM symbol time is usually much less than channel's coherence time).

work based on discrete Markov Modulated Poisson Process providing time-varying realistic scenario. The packet arrival rate it depends on the state of Markov chain and packets are dropped when there is no available space in the queue.

Let us assume that the packet arrival process follows a discrete Markov Modulated Poisson Process model [86] [87] [88]. The packet arrival rate λ_s is determined by the phase s of Markov chain, which has total phases *(i.e.,s=1,2,…,S)*. Hence, we have a dMMPP(**U**,**λ**) process where **U** is the transition probability matrix of modulating Markov chain and **λ** represents the matrix of Poisson arrival rate of each state s.

$$\mathbf{U} = \begin{bmatrix} u_{1,1} & u_{1,2} & \cdots & u_{1,S} \\ u_{2,1} & u_{2,2} & \cdots & u_{2,S} \\ \vdots & \vdots & \ddots & \vdots \\ u_{S,1} & u_{S,2} & \cdots & u_{S,S} \end{bmatrix}, \quad \boldsymbol{\lambda} = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_S \end{bmatrix}$$

The probability that *m (m=0,1,2,..,M)* Poisson arrivals will occur in one time-slot with mean rate λ_s is given by the diagonal probability matrix $\mathbf{U_m}$:

$$\mathbf{U}_m = \begin{bmatrix} f_m(\lambda_1,T) & & & \\ & f_m(\lambda_2,T) & & \\ & & \ddots & \\ & & & f_m(\lambda_S,T) \end{bmatrix}$$

, where $f_m\left(\lambda_s,T\right) = \dfrac{e^{-\lambda_s T}\left(\lambda_s T\right)^m}{m!}$ and denotes the probability that *m* packets with mean rate λ_s will arrive in time interval *T*. We truncated the maximum number of arrival packets during one time-slot at *M* such that $\sum_{m=M+1}^{+\infty} \dfrac{e^{-\lambda_s T}\left(\lambda_s T\right)^m}{m!} < er \quad \forall s$, where “*er*” is a very small number.

If **z** is the stationary probability of matrix **U** and **e** is the column matrix of ones of size $(1xS)^T$, then **zU**=**z** and **ze**=**1**. Following that, the average arrival rate $\bar{\lambda}$ (packets/time-slot) of the dMMPP is obtained as:

$$\bar{\lambda} = \mathbf{z}\sum_{m=0}^{M} m\mathbf{U_m}\mathbf{e} \tag{4.6}$$

State Space and Transition Probability Matrix

Knowing that the length of queue is L and the maximum number of packets that can arrived in a time-slot is based on the S state Markov chain is M, the state space of the queue for a particular user will be $\psi = \{(Q, A), 0 \le Q \le L, 1 \le A \le S\}$, where Q symbolizes the number of packets that lie in the queue and A denotes the phase of Markov chain, which defines the arrival process. Following that, the state transition probability matrix of this state space can be expressed as matrix $\mathbf{P}$.

We assume that the queue state is observed at the end of each time-slot. The scheduler firstly removes the transmitted packets from the queue based on the transmission rate of each user, and secondly, the arriving packets during the time-slot are placed at the end of the queue and will be transmitted at the next time-slot at the earliest.

$$\mathbf{P} = \begin{bmatrix} a_{0,0} & a_{0,1} & \cdots & a_{0,M} & \cdots & a_{0,L} \\ a_{1,0} & a_{1,1} & \cdots & a_{1,M} & \cdots & a_{1,L} \\ \vdots & & \ddots & & \ddots & \vdots \\ & & & \ddots & \ddots & \\ a_{L,1} & a_{L,2} & & \cdots & & a_{L,L} \end{bmatrix}$$

The rows of matrix $\mathbf{P}$ $(1...L)$ denote the number of packets in the queue, and sub-matrix $\alpha_{l,l'}$ denotes the queue's transition probability that, at the end of time-slot *t-1* there are l packets and become l' at the end of the next time-slot t. Due to the fact that L is greater than the maximum number of incoming packets M ($M<L$), some elements of table $\mathbf{P}$ will be null. In addition, as long as the transmission rate is greater than the arrival rate, no packet will be dropped. In each time-slot more than one packet can have arrived or be transmitted without error, hence there are cases in which the space in the queue will be enough for incoming and cases in which some packets will be dropped due to lack of space. From row 0 to row $R-1$ (where R is the maximum total transmission rate in packet/time-slot for a user) is the case that no packet is dropped because the transmission rate is greater than the number of packets in the queue. The second part is from row R to $L-M$ and represents the case in which there are enough packets for transmission but no packet is dropped. The last part of matrix $\mathbf{P}$ is from row $L-M+1$ to L and illustrates the case when some of the incoming packets are dropped due to insufficient space in the queue.

In order to calculate state transition probability matrix **P,** we have to calculate the rate probabilities. Because this is impossible by measuring the real rate values, a probabilistic method should be adopted. The following table shows the available rates in IEEE 802.16 according to the instantaneous SNR levels.

Rate ID(*n*)	Modulation & Coding	Bits / symbol	SNR(dB)
0	BPSK(1/2)	0.5	6.4
1	QPSK(1/2)	1	9.4
2	QPSK(3/4)	1.5	11.2
3	16-QAM(1/2)	2	16.4
4	16-QAM(3/4)	3	18.2
5	64-QAM(1/2)	4	22.7
6	64-QAM(3/4)	4.5	24.4

Table 4 IEEE 802.16 profiles

As it can be seen from table 4 the available rates are $N_{mod}=7$ and hence, the instantaneous rate (4.5) will be $r_{ij} \in \{0.5, 1, 1.5, 2, 3, 4, 4.5\}$ (*bits/OFDM symbol*). The SNR is divided into $N_{mod}+1$ discrete and non-overlapping intervals while rate ID n ($n=0,1,\ldots N_{mod}$) is selected when the instantaneous SNR is $\gamma \in [\gamma_n, \gamma_{n+1})$ [2] (i.e., $\gamma_0 \leq \gamma_{ij} < \gamma_1$ then ID0 rate is selected). In order to avoid errors which lead to retransmissions we do not transmit any packet when $\gamma < \gamma_0$. Considering the *Nakagami-m* [10] [89] channel and by stetting m=1 (Rayleigh channel) the probability for rate ID n to be used in subcarrier i for user j is given as:

$$\Pr(n) = \int_{\gamma_n}^{\gamma_{n+1}} p_\gamma(\gamma_{ij})\, d\gamma_{ij} = \exp\left(-\frac{\gamma_n}{\overline{\gamma}_{ij}}\right) - \exp\left(-\frac{\gamma_{n+1}}{\overline{\gamma}_{ij}}\right) \qquad (4.7)$$

, where $\overline{\gamma}_{ij}$ indicates the average SNR lever for the user j over the given subcarrier. Hence the rate probabilities will be given as $r_{ij} = [\Pr(0), \Pr(1), \Pr(2), \Pr(3), \Pr(4), \Pr(5), \Pr(6)]$ and the user's overall rate is based on the number of subcarriers that it has been assigned which

[2] Where $\gamma_{N_{mod}+1} = +\infty$.

will be obtained by discrete convolution among the subcarriers' transmission probability matrices as:

$$r_j = \odot r_{ij},\ \forall i \text{ which is assigned to user } j \tag{4.8}$$

Because AMC is used as it was mentioned before, we assume that we can reach up to maximum $r_{ij} = N_{\text{mod}}$ transmission rate which is given by(4.5). The interval points can be obtained by (4.5) as $\gamma_{r_{ij}} = \dfrac{\left(2^{r_{ij}}-1\right)\ln\left(0.2/BER\right)}{-1.5}$, $r_{ij} \in \left(0,1,2,\ldots R\right)$. Then the rate in a subcarrier will be r (bits/sec/Hz) as long as the SNR is $\gamma_r \le \gamma_{ij} \le \gamma_{r+1}$. According to Rayleigh fading channel and based on SNR's probabilities the corresponding probability choosing modulation level r for subcarrier i for user j is: $P\left(r_{ij}\right) = \int_{\gamma_r}^{\gamma_{r+1}} p_\Gamma\left(\gamma_{ij}\right) d\gamma_{ij}$. Then the probability mass function (pmf) for the transmission rate in the time interval of a timeslot for that subcarrier for that user will be $c_{ij} = \left[P\left(0\right), P\left(1\right), \ldots, P\left(R\right)\right]$. Respectively the pmf for j user's total transmission rate could be obtained by discrete convolution of the c_{ij} subcarriers' matrices which have been assigned to that user $r_j = \odot c_{ij}$ ($\forall i$ which is assigned to j^{th} user).

The number of packets that has been transmitted during one time-slot has to be calculated. Let d be the number of packets that have been successfully transmitted during one time-slot with $d \le r\left(r \in \left\{0,1,2,\ldots,R\right\}\right)$ where r indicates the users' total transmission rate in packets per time-slot. Then the corresponding probability matrix is denoted as $\mathbf{E}_d$ and is obtained as:

$$\mathbf{E}_d = \mathbf{I}\sum_{r=d}^{D}\left(\binom{r}{d}\Pr(r)(P_{per})^d\ (1-P_{per})^{r-d}\right) \tag{4.9}$$

where $\Pr(r)$ is the probability of having total transmission rate r, P_{per} is the probability that a packet is in error, D is the maximum number of packets that can be transmitted which depends from the number of packets in the queue and the maximum transmission rate D=min($\bar{x}$,R) where $\bar{x}$ denotes the number of packets that reside in the queue. Finally, matrix **I** indicates the identity matrix of size *S*x*S*. To calculate P_{per} we assume that errors are i.i.d hence the formula $P_{per} = 1-(1-BER)^F$ can be used, denoting by F the packet length in bits.

Hence, in the first and second part of matrix $\mathbf{P}$, in which belong the rows *v=0,1,...,L-M* based on $\mathbf{E}_d$ $\mathbf{U}$ and $\mathbf{U}_m$ the elements could be calculated as

$$a_{v,v-n} = \mathbf{U} \sum_{d-m=n} \mathbf{U}_m \mathbf{E}_d \tag{4.10}$$

$$a_{v,v+k} = \mathbf{U} \sum_{m-d=k} \mathbf{U}_m \mathbf{E}_d \tag{4.11}$$

$$a_{v,v} = \mathbf{U} \sum_{m=d} \mathbf{U}_m \mathbf{E}_d \tag{4.12}$$

, where $\mathrm{d} \in \{0,1,2\ldots,\mathrm{D}\}$, $\mathrm{m} \in \{0,1,2,\ldots,\mathrm{M}\}$ represent the number of packets which have been successfully transmitted and the number of arrived packets respectively. The third part of matrix $\mathbf{P}$ within rows $\mathrm{v} = \mathrm{L}-\mathrm{M}+1,\ \mathrm{L}-\mathrm{M}+2,\ldots,\ \mathrm{L}$ includes the phenomenon in which packets are dropped due to the lack of space in the queue. For the case that incoming packets exceed the available space in the queue, $v+k \geq L$ and the above equation becomes:

$$a_{v,v+k} = \sum_{i=k}^{M} \hat{a}_{v,v+i} \ , \qquad v+k \geq L \tag{4.13}$$

and for the case that queue is already full, $\mathrm{v} = \mathrm{L}$ the last equation above becomes

$$a_{v,v} = \hat{a}_{v,v} + \sum_{i=1}^{M} \hat{a}_{v,v+i} \ , \qquad v = L \tag{4.14}$$

, where $\hat{a}_{v,v}$ is obtained from the case where no packet is dropped.

Stationary Probability

As long as the state transition matrix **P** has been obtained then we are able to derive queuing measures via the steady state probability matrix **P**. Due to the fact that the queue's length is finite, the steady state probability matrix $\boldsymbol{\pi}$ can be obtained by the equations $\boldsymbol{\pi}\mathbf{P}=\boldsymbol{\pi}$ and $\boldsymbol{\pi}\mathbf{e}=1$. The $\boldsymbol{\pi}$ matrix contains the steady-states probabilities for the number of packets in the queue, $\boldsymbol{\pi}=[\boldsymbol{\pi}_\mathbf{0}, \boldsymbol{\pi}_\mathbf{1}, ..., \boldsymbol{\pi}_\mathbf{l}, ..., \boldsymbol{\pi}_\mathbf{L}]$ Each element of matrix $\boldsymbol{\pi}$ is another matrix $\boldsymbol{\pi}_\mathbf{l}$ with size $1 \times S$ and its elements represent the probability that queue has *l* packets when the phase of the underlying Markov process is *s*.

Packet loss probability**:**

In order to calculate P_{drop} firstly, the number of dropping packets per timeslot must be obtained. Assuming that v packets are in the queue and during a time another m packets arrive. The queue size will be increased by m packets and no one packet is dropped in case there is enough space for m packets. In case that $m > L - v$ the dropped packets are expressed as $v + m - L$. Based on matrices $\boldsymbol{\pi}$ and $\mathbf{P}$ the average number of dropped packets is expressed as:

$$\bar{x}_{drop} = E\left[x_{drop}\right] = \sum_{s=1}^{S}\sum_{v=0}^{L}\sum_{m=L-v+1}^{M} \boldsymbol{\pi}_{v,s}\left(\sum_{j=1}^{S}\left[a_{v,v+m}\right]_{s,j}\right)\times\left(v+m-L\right) \tag{4.15}$$

, where $\left[a_{v,v+m}\right]$ is a sub matrix of matrix $\mathbf{P}$ and expresses the probability to arrive m packets in the queue when already v packets reside in the queue and the sum factor $\sum_{j=1}^{S}\left[a_{v,v+m}\right]_{s,j}$ expresses the aggregate probability that the queue length will be increased by m packets according to all phases of Markov process. Hence the probability having dropped packets in the queue is:

$$P_{drop} = \frac{\bar{x}_{drop}}{\bar{\lambda}} \tag{4.16}$$

, where $\bar{\lambda}$ is the user's mean arrival rate. A packet will arrive at the receiver with probability $\left(1 - P_{drop}\right)\left(1 - P_{Per}\right)$ so the packet loss probability can be expressed as:

$$\xi = 1 - \left(1 - P_{drop}\right)\left(1 - P_{Per}\right) \tag{4.17}$$

, where P_{drop} is denoted the probability of a packet dropped in the queue and P_{per} is denoted the probability a packet received in error.

Average rate per user (throughput):

$$\varphi = \bar{\lambda}\left(1 - P_{drop}\right) \tag{4.18}$$

Furthermore, in order to calculate the average throughput, we do not consider the packets which have been received in error and the average throughput is expressed in packets/time-slot.

Average delay: The mean number of packets in the queue will be

$$\overline{x} = E\left[x\right] = \sum_{x=1}^{L} x\left(\sum_{s=1}^{S} \pi_{x,s}\right) \tag{4.19}$$

Where L is the queue finite length, and M is the maximum number of packets which can have arrived in a time-slot.

Hence the mean delay (time-slots) can be expressed as:

$$\tau = \frac{\overline{x}}{\varphi} = \frac{\overline{x}}{\overline{\lambda}\left(1-P_{drop}\right)} \tag{4.20}$$

Problem formulation based on symmetric NBS

Let $K\left(j=1,\ldots,K\right)$ be the set of players. Let Ω be a closed and convex subset of $\Re^K$, representing the set of all feasible payoff allocations that the players can get if they all work together. Let f_j defined in Ω be the utility function of each user and $u^0 = \left(u_1^0, u_2^0, \ldots, u_K^0\right)$ be the disagreement payoff allocation that the players would expect if they fail to cooperate and reach an agreement. Suppose that exists $\left\{y \in \Omega \mid f\left(y\right) \geq u^0\right\}$ where $f\left(y\right) = \left(f_1, \ldots, f_K\right)$. Let the set of the achievable utilities be denoted as $U = \left\{f\left(y\right) \middle| y \in \Omega\right\}$ and the sets of utility measures that satisfy the minimum utility bounds u^0 be defined as $G = \left[U, u^0 \middle| U \subset \Re^K\right]$. Then the bargaining solution is a function $S_{NBS}\left|G \to \Re^K\right.$ and satisfies the following axioms [90]:

Axiom 1: $S_{NBS}\left(U, u^0\right)$ is Pareto Optimal. There is no other solution which benefits more at least one user without degrading any other user's payoff.

Axiom 2: Guarantees the minimum required utility $S\left(U, u^0\right) \in U^0$, where $U^0 = \left\{u \in U \middle| u \geq u^0\right\}$.

Axiom 3: Independence of irrelevant alternatives. If the feasible solution set shrinks but still the NBS solution point remains in the new set then the solution of the new set is the same

NBS point. Let $V \subset U$, $\left(V, u^0\right) \in G$ and $S_{NBS}\left(U, u^0\right) \in G$ the solution of the new set is $S_{NBS}\left(U, u^0\right) = S_{NBS}\left(V, u^0\right)$.

Axiom 4: Provides symmetry, meaning that all users have same priority. Assuming two users the feasible set U is symmetric when for $u_j^0 = u_{j'}^0$ then $S_{NBS}\left(U, u^0\right)_j = S_{NBS}\left(U, u^0\right)_{j'}$

If the utility function f_j is concave upper bounded and is defined in Ω, which is convex and $\Omega \subset \Re^K$, and J ($J \subseteq K$) be the set of those users who can reach strictly better performance than their initial performance, then a symmetric Nash bargaining point exists y. The vector y verifies $f_j\left(y\right) \geq u_j^0, \forall j \in J$ and solves uniquely the following maximization problem that satisfies all the above axioms.

$$\max \prod_{j \in J} \left(f_j\left(y\right) - u_j^0 \right) \tag{4.21}$$

As utility function quantifies the profit earned by a user who follows a particular strategy, more than one factor can be utilized. The utility function that we consider here except data rate, takes into account the mean queue length as well as the normalized delay of each user. The proposed utility function is a monotonically increasing function of the wireless link quality, meaning that if more resources are allocated to the user leads to an increase of its performance.

$$f_j = r_j(t) * \overline{x}_j(t) * \theta_j(t) \tag{4.22}$$

Where $r_j(t)$ denotes the rate of user j, $\overline{x}_j(t)$ denotes the mean queue length of user j at time t and $\theta_j(t)$ denotes the normalized delay of user j at time t. The normalized delay is defined as $\theta_j\left(t\right) = \dfrac{\tau_j\left(t\right) - \overline{\tau}(t)}{\overline{\tau}(t)}$, where $\tau_j\left(t\right)$ express the mean delay of user j and $\overline{\tau}\left(t\right)$ denotes the average delay of all users [86].

The maximization problem based on Nash Bargaining Solution can be written as

$$\max_{S,P} \prod_{j=1}^{K} \left(f_j - u_j^0 \right) \tag{4.23}$$

By substituting the utility function from (4.22) this is transformed as:

$$\max_{S,P}\prod_{j=1}^{K}\left(f_j-u_j^0\right)\Rightarrow\max_{S,P}\sum_{j=1}^{K}\ln\left(f_j-u_j^0\right)\Rightarrow\max_{S,P}\sum_{j=1}^{K}\ln\left(r_j\overline{x}_j\theta_j-u_j^0\right) \tag{4.24}$$

Furthermore, the cross-layer optimization problem is about to derive the optimal allocation strategies in terms of subcarrier, power and data under physical constraints and is formulated as:

$$\begin{aligned}
&\max_{S,P}\ \sum_{j=1}^{K}\sum_{i=1}^{N}s_{ij}\ln\left(\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)\\
&s.t\ \left(A1\right)s_{ij}=\left\{0,1\right\}\\
&\quad\left(A2\right)\sum_{j=1}^{K}s_{ij}=1,\forall i\\
&\quad\left(A3\right)p_{ij}\geq 0\\
&\quad\left(A4\right)\sum_{j=1}^{K}\sum_{i=1}^{N}p_{ij}s_{ij}\leq P_{total}\\
&\quad\left(A5\right)\sum_{i=1}^{N}s_{ij}\ln\left(\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)\geq q_j
\end{aligned} \tag{4.25}$$

Constraints (A1) and (A2) ensure that each subcarrier is assigned to only one user, constraint (A3) assures that power takes only positive values and that the overall power limit is the system total power as appointed in constraint (A4). The minimum user demand is expressed in constraint A(5) where $q_j=\varphi_jF\frac{N}{BW}\frac{1}{T_s}bits/\sec/Hz$, where F is the packet length in bits, and defines the queue rate as it has been previously expressed.

From the optimization problem in (4.25) we have two constraints due to the fact that we are dealing with logarithms. These are:

$\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0>1$ and $1+\frac{p_{ij}\left|h_{ij}\right|^2c_3}{\sigma_z^2}>1$ which is obviously that both of them are satisfied.

The Lagrange function according to the optimization problem in (4.25) will be [23]:

$$L\left(\left\{p_{ij}\right\},\left\{s_{ij}\right\},\mu,\rho,\nu\right)=\sum_{j=1}^{K}\sum_{i=1}^{N}s_{ij}\ln\left(\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)-\mu\left(\sum_{j=1}^{K}\sum_{i=1}^{N}s_{ij}p_{ij}-P_{total}\right)$$
$$-\nu_i\left(\sum_{i=1}^{N}s_{ij}-1\right)+\rho_j\left(\sum_{i=1}^{N}s_{ij}\ln\left(\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)-q_j\right)$$
(4.26)

where ρ_j, μ, and ν_i are the Lagrangian multipliers related to constraints A(5), A(4) and A(2) respectively. The above problem is a mixed integer and continue variable problem since s_{ij} is discrete and p_{ij} continuous variable and is hard to solve. That means there are K^N possible subcarrier assignments and in case of a large number of subcarriers, i.e. $N=2048$ and $K=200$, leads to an unrealistic complexity. In order to avoid the above complexity we transform the optimization problem in (4.25) to convex using the technique presented [28] [91]. We relax the integer constraint $s_{ij}=\{0,1\}$ into $\tilde{s}_{ij}\in(0,1]$, which is a continuous variable and represent the time-sharing of a subcarrier amongst more than one user. Next because the $\tilde{s}_{ij}\in(0,1]$ is not convex over $\left(\tilde{s}_{ij},p_{ij}\right)$, we let $\tilde{p}_{ij}=p_{ij}\tilde{s}_{ij}$ which is a continuous variable and the problem is reformulated in terms of $\tilde{s}_{ij}$ and $\tilde{p}_{ij}$ as:

$$\max_{\mathcal{S}:\left\{\tilde{s}_{ij}\in(0,1],\sum_{j=1}^{K}\tilde{s}_{ij}=1\right\},\mathcal{P}:\left\{\tilde{p}_{ij}\geq 0\right\}}\sum_{j=1}^{K}\sum_{i=1}^{N_F}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)$$
$$\text{s.t. (A6)}\ \sum_{i=1}^{N_F}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)\geq q_j \qquad (4.27)$$
$$\text{(A7)}\ \sum_{j=1}^{K}\sum_{i=1}^{N_F}\tilde{p}_{ij}\leq P_{TOTAL}$$

Proposition 1: The cross-layer optimization problem in (4.27) is convex over a feasible convex set $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$.

Proof: The optimization problem in (4.27) is a summation of positive values and has the form of $f\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)=\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}E}{\tilde{s}_{ij}}\right)A-Y\right)$, where *A*, *E*, and *Y* are positive values. The

Hessian matrix can be found to be semi-definite which means that $f\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$ is concave function over $K\cdot N+K\cdot N$ dimensional space $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$. Thus, the maximization function $\max\limits_{\mathcal{S}:\left\{\tilde{s}_{ij}\in(0,1],\sum_{j=1}^{K}\tilde{s}_{ij}=1\right\},\mathcal{P}:\left\{\tilde{p}_{ij}\geq 0\right\}} \sum_{j=1}^{K}\sum_{i=1}^{N_F}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)$ is concave function as it is a linear combination of positive concave functions. Moreover, the inequality constraints (A6) in (4.27) are convex and (A7) are all affine. Regarding the fact that constraints (A6) and (A7) lied in convex sets as well as the main optimization problem, the intersection of convex sets is also convex. Hence, a unique optimal global solution exists.

Thus, by setting the user's initial demands, $u_j^0=\varphi_j$, the convex set over $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$ is non-empty as, i.e. for $\tilde{p}_{ij}=0$ and $\tilde{s}_{ij}=1$, and the constraints (A1), (A2), (A3) (A6) and (A7) are satisfied. Let us denote by S_1 the feasible set over $\tilde{s}_{ij}$ which satisfies the constraints (A1), (A2) and (A6) referring to subcarrier allocation along with the constraint $\tilde{s}_{ij}\in(0,1]$. Moreover by S_2 is denoted the feasible set over $\tilde{p}_{ij}$ that satisfies the power allocation constraints as they described in (A3) and (A7). Following that, in the $K\cdot N+K\cdot N$ dimensional space $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$, the constraints (A1), (A2), (A6) and $\tilde{s}_{ij}\in(0,1]$ of $\tilde{s}_{ij}$ define a cylinder with base S_1 whereas another cylinder with base S_2 is defined according the constraints (A3) and (A7) of $\tilde{p}_{ij}$. Hence, the intersection of the two cylinders defines a convex set, which is non-empty. This completes the proof of proposition 1.■

The Lagrange based on optimization problem of (4.27) will be:

$$\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\mu,\rho,\nu\right)=\sum_{j=1}^{K}\sum_{i=1}^{N}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)-\mu\left(\sum_{j=1}^{K}\sum_{i=1}^{N}\tilde{p}_{ij}-P_{total}\right)-$$
$$-\nu_i\left(\sum_{i=1}^{N}\tilde{s}_{ij}-1\right)+\rho_j\left(\sum_{i=1}^{N}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)-q_j\right)$$

(4.28)

Optimal power allocation policies for S-NBS

The optimization problem in (4.27) is convex over a set in respect of $\tilde{p}_{ij}$ and $\tilde{s}_{ij}$ hence, the KKT conditions are sufficient to locate the global maxima. The optimal power allocation policies are calculated through the first derivative of the Lagrangian in (4.28) over $\tilde{p}_{ij}$.

Proposition 2: The optimal power allocation matrix for the S-NBS case $P^*_{N\times K} = \left[\tilde{p}^*_{ij}\right]$ has individual elements the $\tilde{p}^*_{ij}$ which are derived from:

$$\tilde{p}^*_{ij} = \begin{cases} \dfrac{\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\dfrac{\left|h_{ij}\right|^2 c_3\left(\rho^*_j+1\right)}{\mu^*\sigma_z^2 W\left[\dfrac{\left|h_{ij}\right|^2 c_3\left(\rho^*_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu^*\sigma_z^2}\right]}-1\right)^+, & \tilde{s}^*_{ij}=1 \\ 0, & 0\le \tilde{s}^*_{ij}<1 \end{cases} \tag{4.29}$$

where ρ^*_j, and μ^*, are the optimal Lagrangian multipliers. By the notation $(x)^+$ we denote $\max(0,x)$ and where $W(\cdot)$ denotes the Lambert-*W* [92] function. Power only exists if allocation exists otherwise no power is allocated to that particular subcarrier.

Proof: The proof of *Proposition 2* is presented in Section A of the Appendix.■

Optimal subcarrier allocation policies for S-NBS

Consequently, for the corresponding optimal subcarrier allocation policy we calculate the first derivative of the Lagrangian $\tilde{L}$ in (4.28) over $\tilde{s}_{ij}$ based to the KKT conditions.

Proposition 3: The optimal subcarrier allocation matrix for the S-NBS case $S^*_{N\times K} = \left[\tilde{s}^*_{ij}\right]$ has individual elements $\tilde{s}^*_{ij}$ which are derived by:

$$\tilde{s}^*_{ij} = \begin{cases} 0, \text{ if } v^*_i > H_{ij}\left(\rho^*_j,\mu^*\right) \\ 1, \text{ if } v^*_i < H_{ij}\left(\rho^*_j,\mu^*\right) \end{cases} \tag{4.30}$$

where ρ^*_j, μ^*, and v^*_i are the optimal Lagrangian multipliers and $H_{ij}\left(\rho^*_j,\mu^*\right)$ is:

$$H_{ij}\left(\rho_j^*,\mu^*\right)=\left(\rho_j^*+1\right)\left(\ln\left(\log_2\left(\beta\right)^+\bar{x}_j\theta_j-u_j^0\right)-\frac{\left(\beta-1\right)^+}{\ln\left(2\right)\beta\left(\log_2\left(\beta\right)^+\bar{x}_j\theta_j-u_j^0\right)}\right)-v_i^*$$

Definition of β is presented in the Section B of the Appendix. Hence, each subcarrier is assigned to the user with the best $H_{ij}\left(\rho_j^*,\mu^*\right)$ and that will be the optimal user for that subcarrier. The iteration process which is followed for the definition of the optimal user for a particular subcarrier is:

$$\text{For } i=1 \text{ to } N,\; j^*=\arg\max_{j\in K} H_{ij}\left(\rho_j^*,\mu^*\right),\; \text{for } \tilde{s}_{ij}^*=\begin{cases}1, & j=j^*\\ 0, & j^* \text{ does not exists}\end{cases} \tag{4.31}$$

Clearly the search for the optimal subcarrier allocation $\left\{\tilde{s}_{ij}^*\right\}$ has linear complexity $\mathcal{O}(N\text{x}K)$ which is computationally efficient.

Proof: The proof of *Proposition 3* is presented in Section B of Appendix. ■

Calculation of the Lagrange Multipliers for S-NBS case

Lagrangian multipliers μ,ρ_j must be defined first in order to find optimal power $P^*=\left[p_{ij}^*\right]$ and subcarrier allocation $S^*=\left[s_{ij}^*\right]$ policies. Assuming that $P_{TOTAL}\geq P_{\min}$, where $P_{\min}$ is the minimum power which is needed in order minimum users' QoS constraints are satisfied, the Lagrange multipliers ρ_j^*,μ^* are calculated by an iterative procedure, which is based on the following system of equations:

$$\begin{cases} P\left(\mu^*,\rho_j^*\right)=P_{total}-\sum_{j=1}^{K}\sum_{i=1}^{N}\tilde{s}_{ij}^*\frac{\sigma_z^2}{\left|h_{ij}\right|^2c_3}\left(\dfrac{\left|h_{ij}\right|^2c_3\left(\rho_j^*+1\right)}{\mu^*\sigma_z^2W\left[\dfrac{\left|h_{ij}\right|^2c_3\left(\rho_j^*+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu^*\sigma_z^2}\right]}-1\right)^+=0\\ f_j\left(\mu^*,\rho_j^*\right)=\rho_j^*\left(\tilde{r}_j^*-q_j\right)=0, \quad \forall j\end{cases}$$

By substituting r_j^* with $\tilde{r}_j^* = \sum_{i=1}^{N} \tilde{s}_{ij}^* \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}^*\sigma_z^2}\right)\bar{x}_j\theta_j - u_j^0 \right)$

$$= \sum_{i=1}^{N} \tilde{s}_{ij}^* \ln\left(\log_2\left(1+\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j^*+1\right)}{\mu^*\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j^*+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu^*\sigma_z^2}\right]}\right)\bar{x}_j\theta_j - u_j^0 \right)$$

Hence the above equations are transformed into:

$$\begin{cases} P\left(\mu^*,\rho_j^*\right) = P_{total} - \sum_{j=1}^{K}\sum_{i=1}^{N} \tilde{s}_{ij}^* \frac{\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j^*+1\right)}{\mu^*\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j^*+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu^*\sigma_z^2}\right]} - 1\right)^+ = 0 \\ f_j\left(\mu^*,\rho_j^*\right) = \rho_j^*\left(\sum_{i=1}^{N} \tilde{s}_{ij}^* \ln\left(\log_2\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j^*+1\right)}{\mu^*\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j^*+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu^*\sigma_z^2}\right]}\right)\bar{x}_j\theta_j - u_j^0\right) - q_j\right) = 0 \end{cases} \Rightarrow$$

$$\begin{cases} P\left(\mu^*,\rho_j^*\right) = P_{total} - \sum_{j=1}^{K}\sum_{i=1}^{N} \tilde{s}_{ij}^* \frac{\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\beta-1\right)^+ = 0 \\ f_j\left(\mu^*,\rho_j^*\right) = \rho_j^*\left(\sum_{i=1}^{N} \tilde{s}_{ij}^* \ln\left(\log_2\left(\beta\right)^+ \bar{x}_j\theta_j - u_j^0\right) - q_j\right) = 0 \end{cases} \tag{4.32}$$

In order to calculate the optimal solutions $\tilde{s}_{ij}^*$ and $\tilde{p}_{ij}^*$ from equations (4.31) and (4.29) respectively, we have first to obtain the optimal Lagrangian multipliers $\{\mu^*\}$ and $\{\rho_j^*\}$. Initially $\{\mu^*\}$ takes a positive arbitrary value and algorithm search for the set of $\{\rho_j^*\}$ such that the QoS demands in (A6) (4.27) are met. After $\{\rho_j^*\}$ are found then $\{\mu^*\}$ changes value until both of the following equations are fulfilled.

$$\begin{cases} P_{\min} = \sum_{j=1}^{K}\sum_{i=1}^{N} \tilde{s}_{ij}^* \frac{\sigma_z^2}{|h_{ij}|^2 c_3} (\beta - 1)^+ \\ \sum_{i=1}^{N} \tilde{s}_{ij}^* \ln\left(\log_2\left(1 + \frac{\tilde{p}_{ij}^* |h_{ij}|^2 c_3}{\tilde{s}_{ij}^* \sigma_z^2}\right) \overline{x}_j \theta_j - u_j^0 \right) = q_j \end{cases}$$

Continuously the algorithm updates $\{\mu^*\}$ and $\{\rho_j^*\}$ in a way that each user's QoS requirements are met hence, from (4.32) must stands that $f\left(\{\mu^*\},\{\rho_j^*\}\right) = 0, \forall \rho_j^* \in \{\rho_j^*\}$. Additionally, Lagrangian multipliers update continuously until all available power is exploited which, means that from (4.32) must stands that $P\left(\{\mu^*\},\{\rho_j^*\}\right) \approx 0$. The system of equations in (4.32) is solved using the Semi-Implicit-Root (SIR) finding approach [93] [94]. As it can be conceived, the Lagrangian multipliers $\{\mu^*\}$ and $\{\rho_j^*\}$ adjust the power and subcarrier allocation respectively.

During the iteration process for a particular user j the Lagrangian multiplier ρ_j^* increases and the function $H_{ij}\left(\rho_j^*, \mu^*\right)$ decreases for all the subcarriers. Consequently, as it concludes by (4.30) more $\tilde{s}_{ij}^*$s become one for that particular user and the overall data rate increases. Simultaneously, for the other users some of the $\tilde{s}_{ij}^*$s change from one to zero and their corresponding data rate decrease as well as the overall data rate. Furthermore, when $\{\mu^*\}$ increases for all system users consequently, the optimal power $\tilde{p}_{ij}^*$ for each subcarrier increases. As long as the aggregate data rate does not exceed the maximum number bits/OFDM symbol the algorithm will converge to a unique solution which satisfies all the constraints of (4.27). Moreover, the above unique solution is also optimal and satisfies all constraints since the problem in (4.27) is a convex optimization problem over a convex set.

Following the above procedure the convergence of the algorithm is guaranteed and always a unique optimal solution which satisfies all the constraints exists.

The initial complexity of optimization problem in (4.25) is $\mathcal{O}\left(K^N\right)$ because this is a mixed combinatorial search exponential affected by the number of users and subcarriers. By utilizing the time sharing method, the optimization problem transformed in (4.27), has reduced complexity to $\mathcal{O}\left(K \times N\right)$, which is linear to the number of users and subcarriers. The proposed scheme has lower complexity compared with other solutions. Moreover, in case that the initial optimization problem solved through the dual decomposition method has higher complexity because the complexity of the utilized ellipsoid method, which is a part of the overall solution, has complexity of $\mathcal{O}\left(\left(K+1\right)^2\right)$ [90]. In [22] the proposed scheme adopted the Hungarian method, which has complexity of $\mathcal{O}\left(N^4\right)$, and which is significantly higher than the proposed solution. Furthermore, the complexity in scheme proposed in [9] has complexity of $\mathcal{O}\left(N \times K^2 \times \log_2\left(N\right) + K^4\right)$, when more than two users are considered, and $\mathcal{O}\left(N \times K \times \log_2\left(N\right)\right)$ in two user case. Following the above mentioned comparisons, the theoretical complexity of the proposed solution has the lower complexity amongst all.

Problem formulation based on Asymmetric NBS

In case of Asymmetric Nash Bargaining Solution a weight factor w_j is introduced for each user which defines the priority of that individual user. In contrast with Symmetric NBS, Asymmetric NBS fairness is applied amongst users who have the same weight factor. Expressed more analytical there is no fairness amongst all system's users but only amongst users who belong in the same class (i.e. have the same weight factor). Weights are useful to assign priorities to the users based and their QoS profiles. High weight corresponds to high priority users whereas low weight to low priority users. Based on S-NBS properties which are presented in page 44 the asymmetric NBS problem is defined as follows.

For each $w_j \in \left(0,1\right)$, an A-NBS solution is the function $f_j : \Omega \rightarrow \Re^K$ and there is $y \in \Omega$ such that $f_j\left(y\right) \geq u_j^0, \forall j$ which is the unique solution to the following maximization problem

$$\max_{S,P} \prod_{j\in J} \left(f_j\left(y\right) - u_j^0 \right)^{w_j} \tag{4.33}$$

After further formulations the above formula can be written as:

$$\max_{S,P} \sum_{j=1}^{K} \ln\left(f_j - u_j^0 \right)^{w_j} \Rightarrow \max_{S,P} \sum_{j=1}^{K} w_j \sum_{i=1}^{N} s_{ij} \ln\left(r_{ij} x_j \theta_j - u_j^0 \right) \tag{4.34}$$

By utilizing the same utility function as in S-NBS case as it states in (4.22) we formulate the A-NBS cross-layer constraint optimization problem. In A-NBS case must be mentioned that, the symmetry axiom (Axiom 4) from the NBS axioms does not apply in this case, as users have different priorities factors. Hence, the cross-layer constraint optimization problem for A-NBS scheme aims to derive the optimal allocation policies for users in terms of subcarrier, power and data, whereas satisfies the physical constraints and affirms users' different requirement based on their weights is formulated as:

$$\begin{aligned}
&\max_{S,P} \; \sum_{j=1}^{K} w_j \sum_{i=1}^{N} s_{ij} \ln\left(\log_2\left(1 + \frac{p_{ij}\left|h_{ij}\right|^2 c_3}{\sigma_z^2} \right) \overline{x}_j \theta_j - u_j^0 \right) \\
&s.t \;\; \left(A8\right) \mathrm{s}_{ij} = \left\{0,1\right\} \\
&\quad \left(A9\right) \sum_{j=1}^{K} s_{ij} = 1, \forall i \\
&\quad \left(A10\right) p_{ij} \geq 0 \\
&\quad \left(A11\right) \sum_{j=1}^{K} \sum_{i=1}^{N} p_{ij} s_{ij} \leq P_{total} \\
&\quad (A12) w_j \sum_{i=1}^{N} s_{ij} \ln\left(\log_2\left(1 + \frac{p_{ij}\left|h_{ij}\right|^2 c_3}{\sigma_z^2} \right) \overline{x}_j \theta_j - u_j^0 \right) \geq q_j \\
&\quad (A13) \sum_{j=1}^{K} w_j = 1
\end{aligned} \tag{4.35}$$

Constraints (A8) and (A9) ensures that each subcarrier is assigned only to one user, constraint (A10) makes sure that power takes only positive values whereas constraint (A11) defines that the total power must be less or equal to the available power. The constraint (A12) expresses the minimum user demands having the same meaning as (A5) in (4.25), whereas the (A13) constraint is to ensure that users' weight summation must be equal to one.

In the optimization problem in (4.35) there are two constraints due to the fact that we are dealing with logarithms. These are:

$\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2 c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0 > 1$ and $1+\frac{p_{ij}\left|h_{ij}\right|^2 c_3}{\sigma_z^2} > 1$ which both of them are stand.

The Lagrange function which arises from the above optimization problem is [23]:

$$L\left(\left\{p_{ij}\right\},\left\{s_{ij}\right\},\mu,\rho,\nu,\zeta\right)=\sum_{j=1}^{K} w_j \sum_{i=1}^{N} s_{ij}\ln\left(\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2 c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)-\mu\left(\sum_{j=1}^{K}\sum_{i=1}^{N} p_{ij}s_{ij} - P_{total}\right)$$
$$-\nu_i\left(\sum_{j=1}^{K} s_{ij}-1\right)+\rho_j\left(w_j\sum_{i=1}^{N} s_{ij}\ln\left(\log_2\left(1+\frac{p_{ij}\left|h_{ij}\right|^2 c_3}{\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)-q_j\right)-\zeta\left(\sum_{j=1}^{K} w_j - 1\right)$$
(4.36)

The above problem (4.35) is a mixed, integer and continuous variable problem since s_{ij} and p_{ij} are discrete and continue variables respectively. Such problems are hard to solve as i.e. in case of $K=200$ and $N=2048$ there are K^N different subcarrier allocation policies which is hard to calculate. According to the technique which has been used in S-NBS case, we relax the integer constraint $s_{ij}=\{0,1\}$ into $\tilde{s}_{ij}\in(0,1]$, which is a continuous variable and represent the time-sharing of a subcarrier amongst more than one user. It follows that because the $\tilde{s}_{ij}\in(0,1]$ is not convex over $\left(\tilde{s}_{ij}, p_{ij}\right)$ we let $\tilde{p}_{ij}=p_{ij}\tilde{s}_{ij}$ which is continuous variable and the problem in (4.35) is reformulated in terms of $\tilde{s}_{ij}$ and $\tilde{p}_{ij}$ as:

$$\max_{\mathcal{S}:\left\{\tilde{s}_{ij}\in(0,1],\sum_{j=1}^{K}\tilde{s}_{ij}=1\right\},\mathcal{P}:\left\{\tilde{p}_{ij}\geq 0\right\}} \sum_{j=1}^{K} w_j \sum_{i=1}^{N_F}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)$$
$$\text{s.t. (A14) } w_j\sum_{i=1}^{N_F}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\geq q_j \qquad (4.37)$$
$$\text{(A15) } \sum_{j=1}^{K}\sum_{i=1}^{N_F}\tilde{p}_{ij}\leq P_{TOTAL}$$
$$(A16)\sum_{j=1}^{K} w_j = 1$$

Proposition 4: The cross-layer optimization problem in (4.37) is convex over a feasible convex set $\left(\tilde{s}_{ij}, \tilde{p}_{ij}\right)$.

Proof: Similarly to the previous problem in (4.27) the new A-NBS optimization problem as described in (4.37) is a summation of positive elements following the form $f\left(\tilde{s}_{ij}, \tilde{p}_{ij}\right) = w_j \tilde{s}_{ij} \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}E}{\tilde{s}_{ij}}\right)A - Y\right)$, where A, E and Y are positive values. By calculating the Hessian matrix it can be seen that it is semi-definite hence, the $f\left(\tilde{s}_{ij}, \tilde{p}_{ij}\right)$ is concave function over $K \cdot N + K \cdot N$ dimensional space $\left(\tilde{s}_{ij}, \tilde{p}_{ij}\right)$. Subsequently the maximization function $\max\limits_{\mathcal{S}:\left\{\tilde{s}_{ij}\in(0,1],\sum_{j=1}^{K}\tilde{s}_{ij}=1\right\},\mathcal{P}:\left\{\tilde{p}_{ij}\geq 0\right\}} \sum_{j=1}^{K} w_j \sum_{i=1}^{N_F} \tilde{s}_{ij} \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)$ is concave as a linear combination of positive concave functions. Moreover, the inequality constraints in (A14) are convex and (A15) and (A16) are all affine. As the constraints (A14)-(A16) and the main optimization problem reside in convex sets, the intersection of convex sets is also convex. Therefore, a unique global optimal solution exists.

Furthermore, by defining user's initial demands as $u_j^0 = \varphi_j$ the convex set over $\left(\tilde{s}_{ij}, \tilde{p}_{ij}\right)$ is non-empty as, i.e. for $\tilde{p}_{ij} = 0$ and $\tilde{s}_{ij} = 1$ the constraints (A8)-(A10) and (A14)-(A16) are satisfied. Let us denote by S_1 the feasible set over $\tilde{s}_{ij}$ which satisfies constraints (A8), (A9), (A14) and (A16) referring to subcarrier allocation along with the constraint $\tilde{s}_{ij} \in (0,1]$. Likewise, with the notation S_2 is denoted the feasible set over $\tilde{p}_{ij}$ which satisfies the power allocation constraints, referring in (A10) and (A15). Consequently, in the $K \cdot N + K \cdot N$ dimensional space $\left(\tilde{s}_{ij}, \tilde{p}_{ij}\right)$, the constraints (A8), (A9), (A14), (A16) and $\tilde{s}_{ij} \in (0,1]$ of $\tilde{s}_{ij}$ define a cylinder with base S_1. Similarly the constraints (A10) and (A15) of $\tilde{p}_{ij}$ define also a cylinder with base S_2. Thus, the intersection of the cylinders defines a convex set which is non-empty. This completes the proof of proposition 2. ■

Continuously the new Lagrangian based on (4.37) optimization problem is reformulated in:

$$\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\mu,\rho,\nu,\zeta\right)=\sum_{j=1}^{K}w_j\sum_{i=1}^{N}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)-\mu\left(\sum_{j=1}^{K}\sum_{i=1}^{N}\tilde{p}_{ij}-P_{total}\right)-$$

$$-\nu_i\left(\sum_{j=1}^{K}\tilde{s}_{ij}-1\right)+\rho_j\left(w_j\sum_{i=1}^{N}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)-q_j\right)-\zeta\left(\sum_{j=1}^{K}w_j-1\right)$$

(4.38)

Optimal power allocation policies for A-NBS

The optimization problem in (4.37) is convex over a set in respect to $\tilde{p}_{ij}$ and $\tilde{s}_{ij}$ hence, the KKT conditions are sufficient to locate the global maxima. In order to define the optimal power allocation policies we differentiate the Lagrangian $\tilde{L}$ (4.38) over $\tilde{p}_{ij}$.

Proposition 5: The optimal power allocation matrix for the A-NBS case $P_{N\times K}^{*}=\left[\tilde{p}_{ij}^{*}\right]$ has individual elements the $\tilde{p}_{ij}^{*}$ which are derived from:

$$\tilde{p}_{ij}^{*}=\begin{cases}\dfrac{\sigma_z^2}{\left|h_{ij}\right|^2c_3}\left(\dfrac{\left(\rho_j^*+1\right)\left|h_{ij}\right|^2c_3w_j}{\mu^*\sigma_z^2W\left[\dfrac{\left(\rho_j^*+1\right)\left|h_{ij}\right|^2c_3w_j}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu^*\sigma_z^2}\right]}-1\right)^{+} & ,\ \tilde{s}_{ij}^{*}=1\\ 0 & ,\ 0\le\tilde{s}_{ij}^{*}<1\end{cases}\qquad(4.39)$$

where ρ_j^*, and μ^*, are the optimal Lagrangian multipliers, where $(x)^+$ it denotes the $\max(0,x)$ and the $W(\cdot)$ indicates the Lambert-*W* function. Only in the case that the subcarrier has been allocated to a particular user the BS allocates power.

Proof: The proof of *Proposition 5* is presented in Section C of the Appendix. ■

Optimal subcarrier allocation policies for A-NBS

Accordingly, for the optimal subcarrier allocation policy we differentiate the Lagrangian $\tilde{L}$ in (4.38) over $\tilde{s}_{ij}$ according to KKT conditions.

Proposition 6: The optimal subcarrier allocation matrix for the A-NBS case $S^*_{N\times K} = \left[\tilde{s}^*_{ij}\right]$ has individual elements $\tilde{s}^*_{ij}$ which are derived from:

$$\tilde{s}^*_{ij} = \begin{cases} 0, \text{ if } v^*_i > H_{ij}\left(\rho^*_j, \mu^*\right) \\ 1, \text{ if } v^*_i < H_{ij}\left(\rho^*_j, \mu^*\right) \end{cases} \tag{4.40}$$

where ρ^*_j, μ^*, and v^*_i are the optimal Lagrangian multipliers and $H_{ij}\left(\rho^*_j, \mu^*\right)$ is:

$$H_{ij}\left(\rho^*_j, \mu^*\right) = \left(\rho^*_j + 1\right) w_j \left(\ln\left(\log_2\left(\upsilon\right)\bar{x}_j\theta_j - u^0_j\right) - \frac{\left(\upsilon - 1\right)\bar{x}_j\theta_j}{\ln\left(2\right)\left(\upsilon\right)\left(\log_2\left(\upsilon\right)\bar{x}_j\theta_j - u^0_j\right)} \right) - v^*_i$$

Definition of υ is presented in Section D of the Appendix. Afterwards, subcarriers are assigned to the users according to the best $H_{ij}\left(\rho^*_j, \mu^*\right)$. The optimal user for each subcarrier is derived by the following iteration process:

$$\text{For } i = 1 \text{ to } N,\ \ j^* = \arg\max_{j\in K} H_{ij}\left(\rho^*_{ij}, \mu^*\right),\ \text{for } \tilde{s}^*_{ij} = \begin{cases} 1, j = j^* \\ 0, j^* \text{ does not exists} \end{cases} \tag{4.41}$$

It is obviously that the search for the optimal subcarrier allocation $\left\{\tilde{s}^*_{ij}\right\}$ has linear complexity $\mathcal{O}(N\text{x}K)$ which is computationally efficient.

Proof: The proof of *Proposition 6* is presented in Section D of the Appendix.■

Calculation of Lagrangian multipliers for A-NBS case

Next step for the problem solution is to calculate the optimal Lagrangian multipliers $\left\{\mu^*\right\}$, $\left\{\rho^*_j\right\}$ and continuously calculate optimal, power and subcarrier allocation matrices. From Lagrangian, regarding total power and delay constraints, we have:

$$\begin{cases} P\left(\mu^*,\rho_j^*\right)=P_{total}-\sum_{j=1}^{K}\sum_{i=1}^{N}\tilde{s}_{ij}^*\frac{\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\frac{\left(\rho_j^*+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu^*\sigma_z^2 W\left[\frac{\left(\rho_j^*+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu^*\sigma_z^2}\right]}-1\right)^+=0 \\ f\left(\mu^*,\rho_j^*\right)=\rho_j^*\left(r_j^*-q_j\right)=0, \quad \forall j \end{cases}$$

By substituting optimal rate $r_j^*=w_j\sum_{i=1}^{N}\tilde{s}_{ij}^*\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}^*\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}^*\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)\Rightarrow$

$$r_j^*=w_j\sum_{i=1}^{N}\tilde{s}_{ij}^*\ln\left(\log_2\left(\frac{\left(\rho_j^*+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu^*\sigma_z^2 W\left[\frac{\left(\rho_j^*+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu^*\sigma_z^2}\right]}\right)\overline{x}_j\theta_j-u_j^0\right)\Rightarrow$$

$$r_j^*=w_j\sum_{i=1}^{N}\tilde{s}_{ij}^*\ln\left(\log_2\left(\upsilon\right)\overline{x}_j\theta_j-u_j^0\right)$$

The system's equations for calculation of Lagrangian multipliers become:

$$\begin{cases} P\left(\mu^*,\rho_j^*\right)=P_{total}-\sum_{j=1}^{K}\sum_{i=1}^{N}\tilde{s}_{ij}^*\frac{\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\upsilon-1\right)^+=0 \\ f\left(\mu^*,\rho_j^*\right)=\rho_j^*\left(\sum_{i=1}^{N}\tilde{s}_{ij}^* w_j\ln\left(\log_2\left(\upsilon\right)\overline{x}_j\theta_j-u_j^0\right)-q_j\right)=0, \quad \forall j \end{cases} \tag{4.42}$$

The optimal solutions for subcarrier $\tilde{s}_{ij}^*$ and power $\tilde{p}_{ij}^*$ allocations are derived from equations (4.41) and (4.39) respectively, and have as predecessor the calculation of Lagrangian multipliers $\left\{\mu^*\right\}$, $\left\{\rho_j^*\right\}$. As have been mentioned in S-NBS case, $\left\{\mu^*\right\}$ takes initially a positive arbitrary value and then, the algorithm finds the set of $\left\{\rho_j^*\right\}$ such that the constraint (A14) of (4.37) is fulfilled for all users. In the next step, $\left\{\mu^*\right\}$ and $\left\{\rho_j^*\right\}$ are updated repeatedly until the solution system of (4.42) is met.

Similarly to the S-NBS case, the convergence of the iteration process is granted and the complexity is $\mathcal{O}\left(K\times N\right)$ which is linear to the number of users and subcarriers. The complexity of the A-NBS scheme compared with other proposals [22] [9] is the lowest amongst all making this scheme more attractive.

Summary

The limited available resources in wireless networks led to the adoption of sophisticated techniques and complicated resource allocation by the designers. Much research has been done and many solutions have been proposed by researchers for resource allocation in wireless OFDMA networks. The increase of the system's aggregate throughput, the lower power consumption or the satisfaction of the users QoS demands are the main target by most of the proposals. Fairness amongst system users has been totally ignored by many proposed cross-layer schemes. The proposed game theory based cross-layer schemes, target to maintain users' QoS demands, to optimise aggregate system throughput and, mainly, provide fairness amongst users. The utilization of the Nash bargaining solution ensures that, the solution of both constraint optimization problems is Pareto optimal. Additionally, the packet arrival from MAC layer is described by the a discrete Markov Modulated Poisson Process queuing model, giving a more realistic approach as, multiple arrival rates can be adopted, case that is close to reality.

The two proposed cross-layer constraint optimization problems are formulated based on symmetric NBS and asymmetric NBS axioms respectively. Both problems are recognised as mixed combinatorial problem which are suffered from high complexity. Both constraint cross-layer problems are transformed into lower complexity problems through relaxation technique. Consequently, is proved that these problems are convex optimization problems over a convex set hence, KKT conditions are mandatory and enough to locate to global optimal solution. The optimal subcarrier allocation policy and the optimal power allocation policy, in both fair cross-layer schemes, are derived by analytical mathematical solution.

Considering the complexity of our proposed fair cross-layer algorithms $\mathcal{O}(N\text{x}K)$, must be mentioned that is the lower amongst other schemes. As long as the Lagrangian multipliers are calculated offline, the overall complexity is proportional to the number of users and the number of subcarriers. Furthermore, in asymmetric NBS case each user has its individual weight giving the opportunity to the BS to prioritize users who are in different QoS policy than the other users. This is very important if we consider users who deal with real time applications or have low delay constraint. In asymmetric NBS case the fairness is applied to the users who are in the same priority class and not amongst all users. Other schemes do not have a mechanism to give priorities to the users whereas fairness is maintained at the same time.

Both solutions are considered fair and Pareto optimal by definition. Also, it has to be mentioned, that the solution of both constraint optimization problems is derived through analytical way instead of algorithmic or numerical approach.

Appendix

A. Optimal power for S-NBS case

The optimization problem in (4.28) is convex over a set in respect of $\tilde{p}_{ij}$ and $\tilde{s}_{ij}$ hence, the KKT conditions are sufficient to locate the global maxima. The KKT conditions are:

$$1)\ \frac{\partial\tilde{L}}{\partial\tilde{p}_{ij}}=0,\ 2)\ \frac{\partial\tilde{L}}{\partial\tilde{s}_{ij}}=0,\ 3)\ \mu\frac{\tilde{p}_{ij}}{\tilde{s}_{ij}}=0,\ 4)\ \nu_i\left(1-\frac{\tilde{p}_{ij}}{\tilde{s}_{ij}}\right)=0,\ 5)\ \mu\left(P_{TOTAL}-\sum_{i=1}^{N_F}\sum_{j=1}^{K}\tilde{p}_{ij}\right)=0,$$

$$6)\ \tilde{p}_{ij}\geq 0,\ 7)\ \rho_j\geq 0,\ 8)\ \mu\geq 0,\ 9)\ \nu_i\geq 0,\ 10)\ 1-\frac{\tilde{p}_{ij}}{\tilde{s}_{ij}}\geq 0,\ 11)\ P_{TOTAL}-\sum_{i=1}^{N_F}\sum_{j=1}^{K}\tilde{p}_{ij}\geq 0,$$

$$12)\ \rho_j\cdot\sum_{i=1}^{N_F}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2\eta}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)=0,\ 13)\sum_{i=1}^{N_F}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2\eta}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)-q_j\geq 0,$$

In order to calculate the optimal power for each subcarrier we calculate the partial derivative of $\tilde{L}$ in respect of power $\left(\tilde{p}_{ij}\right)$.

If $\tilde{s}_{ij}\neq 0$ then $\left.\frac{\partial\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\rho,\mu,\nu\right)}{\partial\tilde{p}_{ij}}\right|_{(\tilde{p}_{ij},\tilde{s}_{ij},\rho_j,\mu,\nu)=(\tilde{p}_{ij}^*,\tilde{s}_{ij}^*,\rho_j^*,\mu^*,\nu^*)}=0\Rightarrow$

$$\tilde{s}_{ij}\frac{\partial}{\partial\tilde{p}_{ij}}\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)\right)-\mu+\tilde{s}_{ij}\rho_j\frac{\partial}{\partial\tilde{p}_{ij}}\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)-q_j\right)=0\Rightarrow$$

$$\tilde{s}_{ij}\frac{\frac{\partial}{\partial\tilde{p}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)}{\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0}+\tilde{s}_{ij}\rho_j\frac{\frac{\partial}{\partial\tilde{p}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)}-\mu=0\Rightarrow$$

$$\tilde{s}_{ij}\frac{\frac{\partial}{\partial\tilde{p}_{ij}}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\left(\log_{2}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}-u_{j}^{0}\right)}+$$

$$+\tilde{s}_{ij}\rho_{j}\frac{\frac{\partial}{\partial\tilde{p}_{ij}}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\left(\log_{2}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}-u_{j}^{0}\right)}-\mu=0\Rightarrow$$

$$\tilde{s}_{ij}\frac{\left(\frac{\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\left(\log_{2}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}-u_{j}^{0}\right)}+$$

$$+\tilde{s}_{ij}\rho_{j}\frac{\left(\frac{\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\left(\log_{2}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}-u_{j}^{0}\right)}-\mu=0\Rightarrow$$

$$\tilde{s}_{ij}\frac{\left(\frac{\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\left(\log_{2}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}-u_{j}^{0}\right)}+$$

$$+\tilde{s}_{ij}\rho_{j}\frac{\left(\frac{\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\left(\log_{2}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^{2}c_{3}}{\tilde{s}_{ij}\sigma_{z}^{2}}\right)\overline{x}_{j}\theta_{j}-u_{j}^{0}\right)}=\mu\Rightarrow$$

$$\frac{1}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}+$$

$$+\rho_j\frac{1}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}=\frac{\mu\sigma_z^2}{\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3}\Rightarrow$$

$$\left(\rho_j+1\right)\frac{1}{\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}=\frac{\mu\cdot\sigma_z^2\ln(2)}{\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3}\Rightarrow$$

If we denote $\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)=g$ then we have

$$\left(\rho_j+1\right)\frac{1}{g\left(\log_2 g\cdot\overline{x}_j\theta_j-u_j^0\right)}=\frac{\mu\cdot\sigma_z^2\ln(2)}{\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3}\Rightarrow\left(\rho_j+1\right)\frac{1}{g\left(\log_2 g\cdot\overline{x}_j\theta_j-u_j^0\right)}=\frac{\mu\cdot\sigma_z^2\ln(2)}{\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3}\Rightarrow$$

$$\frac{1}{g\left(\log_2 g\cdot\overline{x}_j\theta_j-u_j^0\right)}=\frac{\mu\cdot\sigma_z^2\ln(2)}{\left(\rho_j+1\right)\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3}\Rightarrow g\left(\log_2 g\cdot\overline{x}_j\theta_j-u_j^0\right)=\frac{\left(\rho_j+1\right)\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3}{\mu\cdot\sigma_z^2\ln(2)}$$

If we denote as $a=\frac{\left(\rho_j+1\right)\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3}{\mu\cdot\sigma_z^2\ln(2)}$ then the above formula is written as:

$$g\left(\log_2 g\cdot\overline{x}_j\theta_j-u_j^0\right)=a\Rightarrow g\cdot\log_2 g\cdot\overline{x}_j\theta_j-g\cdot u_j^0=a\Rightarrow g\cdot\log_2 g-\frac{g\cdot u_j^0}{\overline{x}_j\theta_j}=\frac{a}{\overline{x}_j\theta_j}$$

Furthermore, by setting $\omega=\frac{u_j^0}{\overline{x}_j\theta_j}$, $\varepsilon=\frac{a}{\overline{x}_j\theta_j}$, and we can simplify the formula as:

$$g\cdot\log_2 g-g\cdot\omega=\varepsilon.$$

Because ω is a positive real number there exist such a number β which satisfies the equation $\omega=\log_2\beta$. Therefore, the equation is transformed in:

$$g\cdot\log_2 g-g\cdot\log_2\beta=\varepsilon\Rightarrow\log_2 g^g-\log_2\beta^g=\varepsilon\Rightarrow\log_2\left(\frac{g}{\beta}\right)^g=\varepsilon$$

Multiply both sides with $1/\beta$ which is positive value:

$$\frac{1}{\beta}\log_2\left(\frac{g}{\beta}\right)^{g}=\frac{\varepsilon}{\beta}\Rightarrow\log_2\left(\frac{g}{\beta}\right)^{\frac{g}{\beta}}=\frac{\varepsilon}{\beta}$$

By Setting $\phi=\frac{g}{\beta}$ then equation is written $\log_2\phi^{\phi}=\frac{\varepsilon}{\beta}\Rightarrow\phi^{\phi}=2^{\frac{\varepsilon}{\beta}}$. The final equation can be solved based on Lambert W function [92] according to which the solution of $x^x=z$ is given from $x=\frac{\ln z}{W(\ln z)}$, where $W(\cdot)$ is the Lambert W function.

In our case the solution will be:

$$\phi=\frac{\ln\left(2^{\frac{\varepsilon}{\beta}}\right)}{W\left[\ln\left(2^{\frac{\varepsilon}{\beta}}\right)\right]}\Rightarrow\frac{g}{\beta}=\frac{\ln\left(2^{\frac{\varepsilon}{\beta}}\right)}{W\left[\ln\left(2^{\frac{\varepsilon}{\beta}}\right)\right]}\Rightarrow g=\beta\frac{\ln\left(2^{\frac{\varepsilon}{\beta}}\right)}{W\left[\ln\left(2^{\frac{\varepsilon}{\beta}}\right)\right]}$$

by substituting g, ε ω, β, and, α we have

$$1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2c_3}{\tilde{s}_{ij}\sigma_z^2}=2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\frac{\ln\left(2^{\frac{(\rho_j+1)\frac{\left|h_{ij}\right|^2c_3}{\mu\sigma_z^2\ln(2)}}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}}}\right)}{W\left[\ln\left(2^{\frac{(\rho_j+1)\frac{\left|h_{ij}\right|^2c_3}{\mu\sigma_z^2\ln(2)}}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}}}\right)\right]}\Rightarrow\tilde{p}_{ij}=\frac{\tilde{s}_{ij}\sigma_z^2}{\left|h_{ij}\right|^2c_3}\left(\frac{\left|h_{ij}\right|^2c_3\left(\rho_j+1\right)}{\mu\sigma_z^2W\left[\frac{\left|h_{ij}\right|^2c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}-1\right)$$

Hence the optimal power allocation policy is given by:

$$\tilde{p}_{ij}^{*}=\begin{cases}\dfrac{\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\dfrac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\dfrac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\overline{x}_j\theta_j}}\mu\sigma_z^2}\right]}-1\right), & \text{if } \tilde{s}_{ij}^{*}=1\\ 0 & \text{, if } 0<\tilde{s}_{ij}^{*}<1\end{cases} \tag{4.43}$$

More clear when a subcarrier has been allocated to a user then the base station allocates power to that particular subcarrier which means that $\tilde{s}_{ij}^{*}=1$. This completes the proof of *Proposition 2*.■

B. Optimal subcarrier allocation policy for S-NBS case

Additionally, for the corresponding optimal subcarrier allocation policy we derivate the Lagrangian $\tilde{L}$ (4.28) over $\tilde{s}_{ij}$ and according to the KKT conditions, which have been mentioned in Section A of the Appendix, should be:

$$\left.\frac{\partial\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\rho,\mu,\nu\right)}{\partial\tilde{s}_{ij}}\right|_{(\tilde{p}_{ij},\tilde{s}_{ij},\rho_j,\mu,\nu)=(\tilde{p}_{ij}^{*},\tilde{s}_{ij}^{*},\rho_j^{*},\mu^{*},\nu^{*})}=0\Rightarrow$$

$$\frac{\partial}{\partial\tilde{s}_{ij}}\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)-\frac{\partial}{\partial\tilde{s}_{ij}}\mu\left(\tilde{p}_{ij}-P_{total}\right)-\frac{\partial}{\partial\tilde{s}_{ij}}\nu_i\left(\tilde{s}_{ij}-1\right)+$$

$$\frac{\partial}{\partial\tilde{s}_{ij}}\rho_j\left(\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)-q_j\right)=0\Rightarrow$$

$$\frac{\partial}{\partial\tilde{s}_{ij}}\left(\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)\right)-0-\nu_i+\rho_j\frac{\partial}{\partial\tilde{s}_{ij}}\left(\tilde{s}_{ij}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j\right)-q_j\right)=0\Rightarrow$$

$$\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)+\tilde{s}_{ij}\frac{\partial}{\partial\tilde{s}_{ij}}\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)\right)+$$

$$+\rho_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)+\tilde{s}_{ij}\frac{\partial}{\partial\tilde{s}_{ij}}\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)\right)\right)-\nu_i=0\Rightarrow$$

$$\left(\rho_j+1\right)\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)+\tilde{s}_{ij}\frac{\frac{\partial}{\partial\tilde{s}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}\right)-v_i=0\Rightarrow$$

$$\left(\rho_j+1\right)\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)+\tilde{s}_{ij}\frac{\frac{\partial}{\partial\tilde{s}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j\right)}{\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0}\right)-v_i=0\Rightarrow$$

$$\left(\rho_j+1\right)\left(\begin{array}{l}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)+\\ +s_{ij}\frac{\frac{\partial}{\partial s_{ij}}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j}{\ln\left(2\right)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}\end{array}\right)-v_i=0\Rightarrow$$

$$\left(\rho_j+1\right)\left(\begin{array}{l}\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)+\\ +s_{ij}\frac{\left(-\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3\sigma_z^2}{\left(\tilde{s}_{ij}\sigma_z^2\right)^2}\right)}{\ln\left(2\right)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}\end{array}\right)-v_i=0\Rightarrow$$

$$\left(\rho_j+1\right)\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)+s_{ij}\frac{\left(-\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}^2\sigma_z^2}\right)}{\ln\left(2\right)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)}\right)-v_i=0\Rightarrow$$

$$\left(\rho_j+1\right)\left(\begin{array}{l}\ln\left(\log_2\left(1+\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)- \\ -\cancel{\tilde{s}_{ij}}\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}^{\cancel{2}}\sigma_z^2\ln\left(2\right)\left(1+\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}\end{array}\right)-v_i=0\Rightarrow$$

$$\left(\rho_j+1\right)\left(\begin{array}{l}\ln\left(\log_2\left(1+\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)- \\ -\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2\ln\left(2\right)\left(1+\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\dfrac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j-u_j^0\right)}\end{array}\right)-v_i=0$$

By substituting the optimal power $\tilde{p}_{ij}^*=\dfrac{\tilde{s}_{ij}^*\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\dfrac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\dfrac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\overline{x}_j\theta_j}}\mu\sigma_z^2}\right]}-1\right)^+$ in the above equation we get:

$$\left(\rho_j+1\right)\ln\left(\left(\log_2\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}\right)\right)^+\bar{x}_j\theta_j-u_j^0\right)$$

$$-\left(\rho_j+1\right)\frac{\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}-1\right)^+}{\ln(2)\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}\right)^+\left(\left(\log_2\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}\right)\right)^+\bar{x}_j\theta_j-u_j^0\right)}-v_i=0$$

$$\left(\rho_j+1\right)\left(\begin{array}{l}\ln\left(\log_2\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}\right)^+ \bar{x}_j\theta_j-u_j^0\right)- \\ -\dfrac{\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}-1\right)^+}{\ln(2)\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}\right)\left(\log_2\left(\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}\right)^+ \bar{x}_j\theta_j-u_j^0\right)}\end{array}\right)-\nu_i=0$$

By defining as β the following part $\beta=\dfrac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{\mu\sigma_z^2 W\left[\frac{\left|h_{ij}\right|^2 c_3\left(\rho_j+1\right)}{2^{\frac{u_j^0}{\bar{x}_j\theta_j}}\mu\sigma_z^2}\right]}$ we have

$$\left(\rho_j+1\right)\left(\ln\left(\log_2\left(\beta\right)^+\bar{x}_j\theta_j-u_j^0\right)-\frac{\left(\beta-1\right)^+}{\ln(2)\beta\left(\log_2\left(\beta\right)^+\bar{x}_j\theta_j-u_j^0\right)}\right)-\nu_i=0 \qquad (4.44)$$

We can see from (4.44) that $\left.\dfrac{\partial\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\rho,\mu,\nu\right)}{\partial\tilde{s}_{ij}}\right|_{\left(\tilde{p}_{ij},\tilde{s}_{ij},\rho_j,\mu,\nu\right)=\left(\tilde{p}_{ij}^*,\tilde{s}_{ij}^*,\rho_j^*,\mu^*,\nu^*\right)}=\begin{cases}=0, \text{ if } 0<\tilde{s}_{ij}^*<1\\ >0, \text{ if } \tilde{s}_{ij}^*=1\\ <0, \text{ if } \tilde{s}_{ij}^*=0\end{cases}$

which means that optimal subcarrier is:

$$\tilde{s}_{ij}^* = \begin{cases} 0, \text{ if } v_i^* > H_{ij}\left(\rho_j^*, \mu^*\right) \\ 1, \text{ if } v_i^* < H_{ij}\left(\rho_j^*, \mu^*\right) \end{cases} \tag{4.45}$$

Where $H_{ij}\left(\rho_j^*, \mu^*\right) = \left(\rho_j^* + 1\right)\left(\ln\left(\log_2\left(\beta\right)^+ \overline{x}_j\theta_j - u_j^0\right) - \frac{\left(\beta - 1\right)^+}{\ln\left(2\right)\beta\left(\log_2\left(\beta\right)^+ \overline{x}_j\theta_j - u_j^0\right)} \right)$

Hence each subcarrier is assigned to the user with the best $H_{ij}\left(\rho_j^*, \mu^*\right)$ and that user will be the optimal for that subcarrier. The iteration process is described as:

$$\text{For } i = 1 \text{ to } N,\ j^* = \arg\max_{j\in K} H_{ij}\left(\rho_j^*, \mu^*\right),\ \text{for } \tilde{s}_{ij}^* = \begin{cases} 1,\ j = j^* \\ 0,\ j^* \text{ does not exists} \end{cases} \tag{4.46}$$

This completes the proof of *Proposition 3*.■

C. Optimal power allocation for A-NBS case

The Lagrangian in (4.38) describes the cross layer problem in (4.37) which is convex over a set in respect to $\tilde{p}_{ij}$ and $\tilde{s}_{ij}$ hence, the KKT conditions are sufficient to locate the global maxima. The KKT conditions are:

$$1)\ \frac{\partial\tilde{L}}{\partial\tilde{p}_{ij}} = 0,\ 2)\ \frac{\partial\tilde{L}}{\partial\tilde{s}_{ij}} = 0,\ 3)\ \mu\frac{\tilde{p}_{ij}}{\tilde{s}_{ij}} = 0,\ 4)\ v_i\left(1 - \frac{\tilde{p}_{ij}}{\tilde{s}_{ij}}\right) = 0,\ 5)\ \mu\left(P_{TOTAL} - \sum_{i=1}^{N_F}\sum_{j=1}^{K}\tilde{p}_{ij}\right) = 0,\ 6)\ \tilde{p}_{ij} \geq 0,$$

$$7)\ \rho_j \geq 0,\ 8)\ \mu \geq 0,\ 9)\ v_i \geq 0, 10)\ w_j > 0,\ 11)\ \zeta \geq 0,\ 12) \sum_{j=1}^{K} w_j - 1 = 0,\ 13)\ 1 - \frac{\tilde{p}_{ij}}{\tilde{s}_{ij}} \geq 0,$$

$$14)\ \rho_j w_j \geq 0,\ 15)\ P_{TOTAL} - \sum_{i=1}^{N_F}\sum_{j=1}^{K}\tilde{p}_{ij} \geq 0,\ 16)\ \rho_j w_j \sum_{i=1}^{N_F}\tilde{s}_{ij} \ln\left(\log_2\left(1 + \frac{\tilde{p}_{ij}\left|h_{ij}\right|^2\eta}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) = 0,$$

$$17) w_j \sum_{i=1}^{N_F}\tilde{s}_{ij} \ln\left(\log_2\left(1 + \frac{\tilde{p}_{ij}\left|h_{ij}\right|^2\eta}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) - q_j \geq 0$$

Hence if $\tilde{s}_{ij} \neq 0$ the derivate of $\tilde{L}\left(\left\{\tilde{p}_{ij}\right\}, \left\{\tilde{s}_{ij}\right\}, \mu, \rho, v, \zeta\right)$ over $\tilde{p}_{ij}$ must be equal to zero

$$\left.\frac{\partial\tilde{L}\left(\left\{\tilde{p}_{ij}\right\}, \left\{\tilde{s}_{ij}\right\}, \mu, \rho, v, \zeta\right)}{\partial\tilde{p}_{ij}}\right|_{\left(\tilde{p}_{ij}, \tilde{s}_{ij}, \rho_j, \mu, v, \zeta\right) = \left(\tilde{p}_{ij}^*, \tilde{s}_{ij}^*, \rho_j^*, \mu^*, v^*, \zeta^*\right)} = 0 \Rightarrow$$

$$\frac{\partial}{\partial \tilde{p}_{ij}}\left(\tilde{s}_{ij} w_j \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\right) - \frac{\partial}{\partial \tilde{p}_{ij}}\mu\left(\tilde{p}_{ij} - P_{total}\right) +$$

$$\frac{\partial}{\partial \tilde{p}_{ij}}\rho_j\left(\tilde{s}_{ij} w_j \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) - q_j\right) = 0$$

$$\tilde{s}_{ij} w_j \frac{\frac{\partial}{\partial \tilde{p}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)} + \rho_j\left(\tilde{s}_{ij} w_j \frac{\frac{\partial}{\partial \tilde{p}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)}\right) - \mu = 0$$

$$\tilde{s}_{ij} w_j \frac{\left(\frac{\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)} +$$

$$\rho_j\left(\tilde{s}_{ij} w_j \frac{\left(\frac{\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)}\right) - \mu = 0$$

$$\left(\rho_j + 1\right)\left(\tilde{s}_{ij} w_j \frac{\left(\frac{\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)}\right) - \mu = 0 \Rightarrow$$

$$\frac{w_j}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)} = \frac{\mu}{\left(\rho_j + 1\right)\overline{x}_j\theta_j\left(\frac{\left|h_{ij}\right|^2 c_3}{\sigma_z^2}\right)} \Rightarrow$$

$$\frac{1}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)} = \frac{\mu\sigma_z^2 \ln(2)}{(\rho_j+1)\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3 w_j}$$

If we set as $g = 1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}$ then we get:

$$\frac{1}{\left(\log_2(g)\overline{x}_j\theta_j - u_j^0\right)g} = \frac{\mu\sigma_z^2 \ln(2)}{(\rho_j+1)\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3 w_j} \Rightarrow \left(\log_2(g)\overline{x}_j\theta_j - u_j^0\right)g = \frac{(\rho_j+1)\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 \ln(2)}$$

By setting $a = \frac{(\rho_j+1)\overline{x}_j\theta_j\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 \ln(2)}$ the above equation is transformed as:

$g\log_2(g) - \frac{u_j^0}{\overline{x}_j\theta_j} g = \frac{a}{\overline{x}_j\theta_j}$. Moreover we define $\omega = \frac{u_j^0}{\overline{x}_j\theta_j}$, $\varepsilon = \frac{a}{\overline{x}_j\theta_j}$. Thereafter equation becomes $g\log_2(g) - \omega g = \varepsilon$. Following the same procedure as in S-NBS case the ω as a positive value can be expressed as $\omega = \log_2(\beta)$ and the transformed formula is:

$$g\log_2(g) - \log_2(\beta) g = \varepsilon \Rightarrow \log_2\left(g^g\right) - \log_2\left(\beta^g\right) = \varepsilon \Rightarrow \log_2\left(\frac{g}{\beta}\right)^g = \varepsilon$$

By multiplying both side of the last equation with the positive value $1/\beta$ we get

$$\frac{1}{\beta}\log_2\left(\frac{g}{\beta}\right)^g = \frac{\varepsilon}{\beta} \Rightarrow$$

$$\log_2\left(\frac{g}{\beta}\right)^{\frac{g}{\beta}} = \frac{\varepsilon}{\beta}$$

Setting $\phi = \frac{g}{\beta}$ we get that $\log_2\left(\phi^\phi\right) = \frac{\varepsilon}{\beta} \Rightarrow \phi^\phi = 2^{\frac{\varepsilon}{\beta}}$. The last equation is solved based on Lambert W function similarly the S-NBS case.

$$\phi = \frac{\ln\left(2^{\frac{\varepsilon}{\beta}}\right)}{W\left[\ln\left(2^{\frac{\varepsilon}{\beta}}\right)\right]} \Rightarrow \frac{g}{\beta} = \frac{\ln\left(2^{\frac{\varepsilon}{\beta}}\right)}{W\left[\ln\left(2^{\frac{\varepsilon}{\beta}}\right)\right]} \Rightarrow g = \beta \frac{\ln\left(2^{\frac{\varepsilon}{\beta}}\right)}{W\left[\ln\left(2^{\frac{\varepsilon}{\beta}}\right)\right]} \Rightarrow$$

By substituting *g*, *ε*, *ω*, *β*, and, *α* and solving over $\tilde{p}_{ij}$ we have:

$$\tilde{p}_{ij} = \frac{\tilde{s}_{ij}\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\frac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 W\left[\frac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu\sigma_z^2}\right]} - 1\right) \tag{4.47}$$

Hence, the optimal power allocation matrix $P_{N\times K}^{*} = \left[\tilde{p}_{ij}^{*}\right]$ has individual matrix elements, which are given by:

$$\tilde{p}_{ij}^{*} = \begin{cases} \frac{\sigma_z^2}{\left|h_{ij}\right|^2 c_3}\left(\frac{\left(\rho_j^{*}+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu^{*}\sigma_z^2 W\left[\frac{\left(\rho_j^{*}+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu^{*}\sigma_z^2}\right]} - 1\right)^{+} & \text{,if } \tilde{s}_{ij}^{*} = 1 \\ 0 & \text{,if } 0 < \tilde{s}_{ij}^{*} < 1 \end{cases} \tag{4.48}$$

Where $(x)^{+}$ it denotes the $\max(0,x)$. This completes the proof for *Proposition 5*.■

D. Optimal subcarrier allocation policy for A-NBS case

Subsequently, for the optimal subcarrier allocation we differentiate (4.38) over $\tilde{s}_{ij}$ and according to KKT conditions must stand that

$$\left.\frac{\partial \tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\mu,\rho,\nu,\zeta\right)}{\partial \tilde{s}_{ij}}\right|_{\left(\tilde{p}_{ij},\tilde{s}_{ij},\rho_j,\mu,\nu,\zeta\right)=\left(\tilde{p}_{ij}^{*},\tilde{s}_{ij}^{*},\rho_j^{*},\mu^{*},\nu^{*},\zeta^{*}\right)} = 0 \Rightarrow$$

$$\frac{\partial}{\partial \tilde{s}_{ij}}\left(\tilde{s}_{ij} w_j \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\right) - \mu\frac{\partial}{\partial \tilde{s}_{ij}}\left(\tilde{p}_{ij} - P_{total}\right) - v_i\frac{\partial}{\partial \tilde{s}_{ij}}\left(\tilde{s}_{ij} - 1\right) +$$

$$+\rho_j\frac{\partial}{\partial \tilde{s}_{ij}}\left(\tilde{s}_{ij} w_j \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) - q_j\right) - \zeta\frac{\partial}{\partial \tilde{s}_{ij}}\left(w_j - 1\right) = 0 \Rightarrow$$

$$w_j \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) + \tilde{s}_{ij} w_j \frac{\partial}{\partial \tilde{s}_{ij}}\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\right) - 0 - v_i +$$

$$\rho_j\left(w_j \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) + \tilde{s}_{ij} w_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\right) - 0\right) - 0 = 0 \Rightarrow$$

$$\left(\rho_j + 1\right) w_j \ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) + \left(\rho_j + 1\right)\tilde{s}_{ij} w_j \frac{\partial}{\partial \tilde{s}_{ij}}\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)\right) - v_i = 0 \Rightarrow$$

$$\left(\rho_j + 1\right) w_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) + \tilde{s}_{ij}\frac{\frac{\partial}{\partial \tilde{s}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)}\right) - v_i = 0 \Rightarrow$$

$$\left(\rho_j + 1\right) w_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right) + \tilde{s}_{ij}\frac{\frac{\partial}{\partial \tilde{s}_{ij}}\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{s_{ij}\sigma_z^2}\right)\right)\overline{x}_j\theta_j}{\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\overline{x}_j\theta_j - u_j^0\right)}\right) - v_i = 0 \Rightarrow$$

$$(\rho_j+1)w_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)+\tilde{s}_{ij}\frac{\frac{\partial}{\partial\tilde{s}_{ij}}\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)}\right)-v_i=0\Rightarrow$$

$$(\rho_j+1)w_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)+s_{ij}\frac{\left(-\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3\sigma_z^2}{\left(\tilde{s}_{ij}\sigma_z^2\right)^2}\right)\bar{x}_j\theta_j}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)}\right)-v_i=0\Rightarrow$$

$$(\rho_j+1)w_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)+\tilde{s}_{ij}\frac{\left(-\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}^2\sigma_z^2}\right)\bar{x}_j\theta_j}{\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)}\right)-v_i=0\Rightarrow$$

$$(\rho_j+1)w_j\left(\ln\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)-\cancel{\tilde{s}_{ij}}\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3\bar{x}_j\theta_j}{\tilde{s}_{ij}^{\cancel{2}}\sigma_z^2\ln(2)\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\left(\log_2\left(1+\frac{\tilde{p}_{ij}\left|h_{ij}\right|^2 c_3}{\tilde{s}_{ij}\sigma_z^2}\right)\bar{x}_j\theta_j-u_j^0\right)}\right)-v_i=0\Rightarrow$$

Now by substituting the power $\tilde{p}_{ij}^*$ by representation of (4.47) we have the following formula:

$$\left(\rho_j+1\right)w_j\left(\begin{array}{l}\ln\left(\log_2\left(\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 W\left[\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu\sigma_z^2}\right]}\right)\overline{x}_j\theta_j-u_j^0\right)- \\ -\dfrac{\left(\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 W\left[\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu\sigma_z^2}\right]}-1\right)\overline{x}_j\theta_j}{\ln(2)\left(\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 W\left[\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu\sigma_z^2}\right]}\right)\left(\log_2\left(\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 W\left[\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu\sigma_z^2}\right]}\right)\overline{x}_j\theta_j-u_j^0\right)}\end{array}\right)-v_i=0\Rightarrow$$

Let us denote as $\upsilon=\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{\mu\sigma_z^2 W\left[\dfrac{\left(\rho_j+1\right)\left|h_{ij}\right|^2 c_3 w_j}{2^{\omega}\mu\sigma_z^2}\right]}$ then equation is written as:

$$\left(\rho_j+1\right)w_j\left(\ln\left(\log_2(\upsilon)\overline{x}_j\theta_j-u_j^0\right)-\frac{(\upsilon-1)\overline{x}_j\theta_j}{\ln(2)(\upsilon)\left(\log_2(\upsilon)\overline{x}_j\theta_j-u_j^0\right)}\right)-v_i=0 \qquad (4.49)$$

From (4.49) it can be seen that

$$\left.\frac{\partial\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\rho,\mu,\nu,\zeta\right)}{\partial\tilde{s}_{ij}}\right|_{(\tilde{p}_{ij},\tilde{s}_{ij},\rho,\mu,\nu,\zeta)=(\tilde{p}_{ij}^*,\tilde{s}_{ij}^*,\rho^*,\mu^*,\nu^*,\zeta^*)}\begin{cases}=0, \text{ if } 0<\tilde{s}_{ij}^*<1\\ >0, \text{ if } \tilde{s}_{ij}^*=1\\ <0, \text{ if } \tilde{s}_{ij}^*=0\end{cases}$$

Let is denote:

$$H_{ij}\left(\rho_j^*,\mu^*\right)=\left(\rho_j^*+1\right)w_j\left(\ln\left(\log_2(\upsilon)\overline{x}_j\theta_j-u_j^0\right)-\frac{(\upsilon-1)\overline{x}_j\theta_j}{\ln(2)(\upsilon)\left(\log_2(\upsilon)\overline{x}_j\theta_j-u_j^0\right)}\right)-v_i$$

Then for optimal subcarrier regarding $H_{ij}\left(\rho_j^*,\mu^*\right)$ we have that:

$$\tilde{s}_{ij}^{*} = \begin{cases} 0, \text{ if } v_i^{*} > H_{ij}\left(\rho_j^{*}, \mu^{*}\right) \\ 1, \text{ if } v_i^{*} < H_{ij}\left(\rho_j^{*}, \mu^{*}\right) \end{cases} \tag{4.50}$$

Afterwards subcarriers are assigned to the users according the best $H_{ij}\left(\rho_j^{*}, \mu^{*}\right)$ following the iteration process:

$$\text{For } i = 1 \text{ to } N,\ j^{*} = \arg\max_{j \in K} H_{ij}\left(\rho_{ij}^{*}, \mu^{*}\right),\ \text{for } \tilde{s}_{ij}^{*} = \begin{cases} 1, j = j^{*} \\ 0, j^{*} \text{ does not exists} \end{cases} \tag{4.51}$$

This completes the proof of *Proposition 6*.■

CHAPTER 5

Comparative studies

A comparative study amongst the proposed cross-layer schemes and other schemes is presented during this chapter. Simulations aim to perform an evaluation of the mathematical model, which has been presented in the Chapter 4. The proposed allocation strategies are validating in terms of data rate, power consumption, fairness and queue performance.

The simulation model

A single OFDMA cell which is considered to consists of a total bandwidth $BW = 80KHz$ equally divided to 64 subcarriers $(N = 64)$. Hence, each subcarrier has a bandwidth of $\Delta f = 1,25KHz$. The channel is assumed as frequency selective Rayleigh fading channel with Doppler shift 10Hz. Additionally, the channel between different users and different subcarriers has been modeled as i.i.d. complex Gaussian process with unit noise variance $\sigma_z^2 = 1$. Also it is assumed that channel's conditions remain invariable during each time-slot which is defined to be $T_s = 0.053\sec$. The average BER is determined to be $BER = 10^{-6}$ and the packet size is selected to be $F = 80bits$. Furthermore, heterogeneous users are divided into two different classes, class 1 and class 2, with parameters, which are defined in the following paragraphs. Class 1 users are considered as delay sensitive users with higher QoS demands than the 2 users who have lower QoS requirements.

Simulation results

In order to carry out simulations for queue, the model consists by 6 users $(K = 6)$ with two-states, $S = 2$, dMMPP for each user. Queue length is determined to be $L = 20$ packets. Two classes of users are introduced. Class 1 users are considered as multimedia users with the following parameters: $\mathbf{U}_{class1} = \begin{bmatrix} 0.3 & 0.7 \\ 0.3 & 0.7 \end{bmatrix}$, $\lambda_{class1} = \begin{bmatrix} \lambda_1 & \\ & 5 \end{bmatrix}$ with the arrival rate λ_1

varying. On the contrary class 2 users are data users with parameters set as: $\mathbf{U}_{class2} = \begin{bmatrix} 0.3 & 0.7 \\ 0.3 & 0.7 \end{bmatrix}$, $\lambda_{class2} = \begin{bmatrix} \lambda_2 & \\ & 2 \end{bmatrix}$ with the arrival rate λ_2 varying. The maximum number of packets which may arrive within a time-slot is truncated to $M = 16$ in both cases whereas the stationary probability matrix $\mathbf{z}$ is calculated as $\mathbf{z} = \begin{bmatrix} 0.3 & 0.7 \end{bmatrix}$.

Throughput comparison

During the following figures is depicted the system's performance when two class 1 users and one class 2 user are considered in case the incoming arrival rate of λ_2 for class 2 user varies for different scheduling schemes. By setting $\lambda_1 = 4$ for class 1 users the mean arrival rate is $\bar{\lambda}_{class1} = 5.9$ packets/time-slot and the throughput is $\varphi_{class1} = 5.9$ packets/time-slot as the P_{per} is very small $\left(P_{per} = 0.00079\right)$. Similarly, the average arrival rate for class 2 users will be $\bar{\lambda}_{class2} = 2.6$ packets/time-slot when $\lambda_2 = 4$ and the throughput is $\varphi_{class2} = 2.6$ packets/time-slot. Based on these parameters users' initial demands will be $u^0_{class1} = 7.12$ bits/sec/Hz for class 1 users and $u^0_{class2} = 3.14$ bits/sec/Hz for class 2 users.

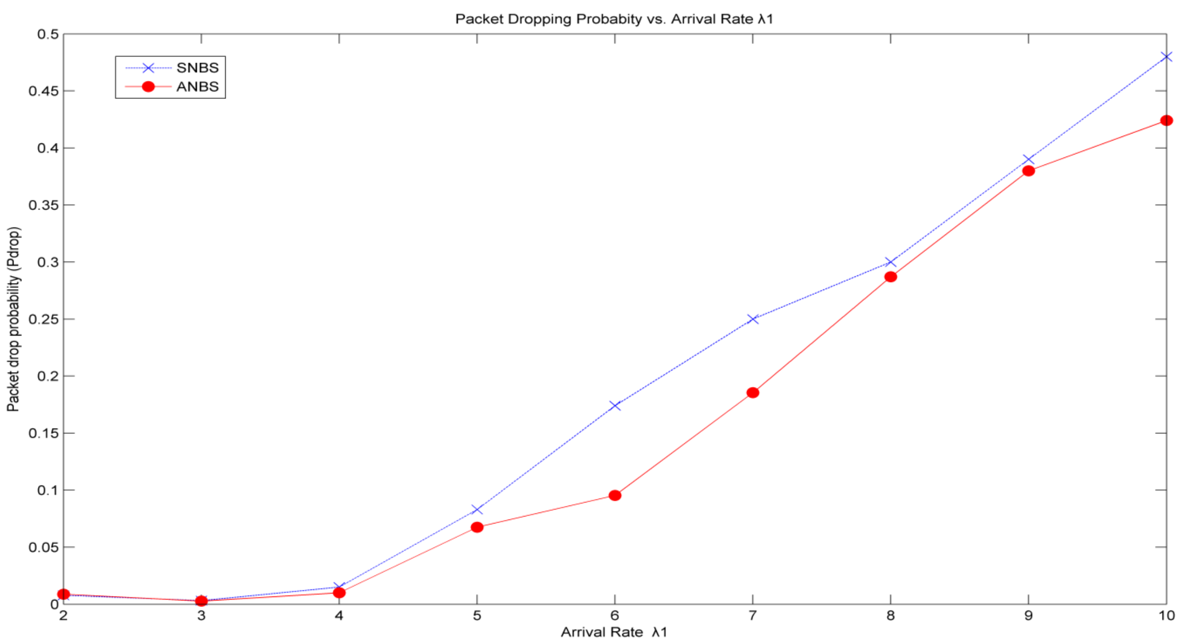

Figure 7 Packet dropping probability vs. arrival rate λ_1

Figure 7 depicts the packet dropping probability packet for S-NBS and A-NBS case as the arrival rate of class 1 user increase. The total available power is $P_{total} = 12dB$ and two class 1 users are taken into account. It can be seen that when $\lambda_1 < 4$ the drop probability is very low for both schemes, but after that threshold increases rapidly. For each case two users are considered but in A-NBS case one user has weight 0.3 and the other 0.7. In A-NBS case the performance of high priority users is measured (weight 0.7) so performs better than S-NBS case where all users are equal in resource allocation. In case when λ_1 is 10 packets /time-slot, the packet dropping probability is almost $P_{drop} = 0.5$ meaning that half of the incoming packets will be dropped. In our simulations we do not consider the packet probability as the main goal is to study the cross-layer performance amongst different proposals and not to exam the queue model.

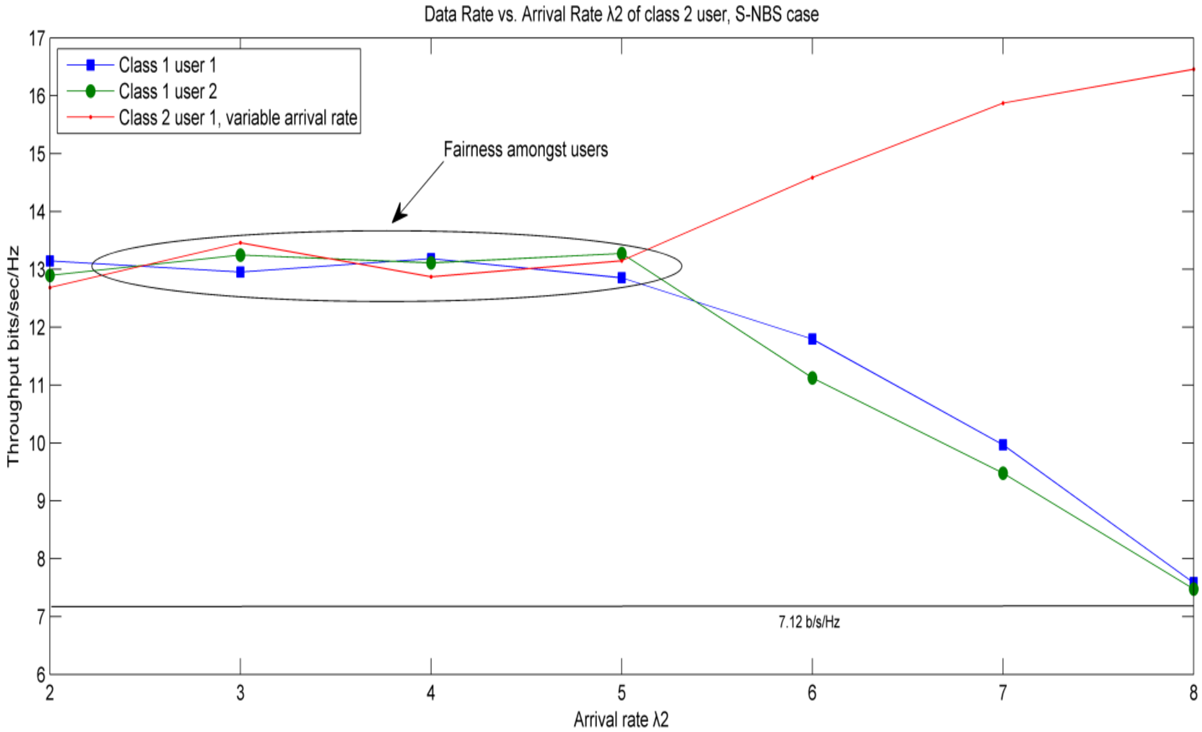

Figure 8 Throughput vs. arrival rate of class 2 user in S-NBS case

Figure 8 depicts the throughput of two class 1 users and one class 2 user as the arrival rate λ_2 changes. The total available power is $P_{total} = 16dB$ which, is enough to meet users' QoS constraints. As long as $\lambda_2 = \{2,3,4,5\}$ the QoS requirements for each user are similar and it can be seen how fairness is provided through the S-NBS scheduler. Subsequently, for $\lambda_2 > 5$ class 2 user has higher demands from other two users because, the increasing arrival rate increases the number of the packets in the queue hence, scheduler based on the utility function, allocates more resources to class 2 user in order to ensure that his demands are met.

Additionally, as long as the number of packets in the queue increases the time that a packet spend in the queue waiting increases as well therefore, higher utility payoff is applied and more resources are allocated to that user. Simultaneously, class 1 user 1 and user 2 have been downgraded in terms of data rate as less resources are been allocated to them whereas their minimum requirements are always satisfied. When $\lambda_2 = 8$ is can be seen that even though supplementary data rate is allocated to user 3 of up to 16.47bits/sec/Hz, users 1 and 2 are not starving and system can guarantee their resources demands. That proves the efficiency of the proposed scheduler.

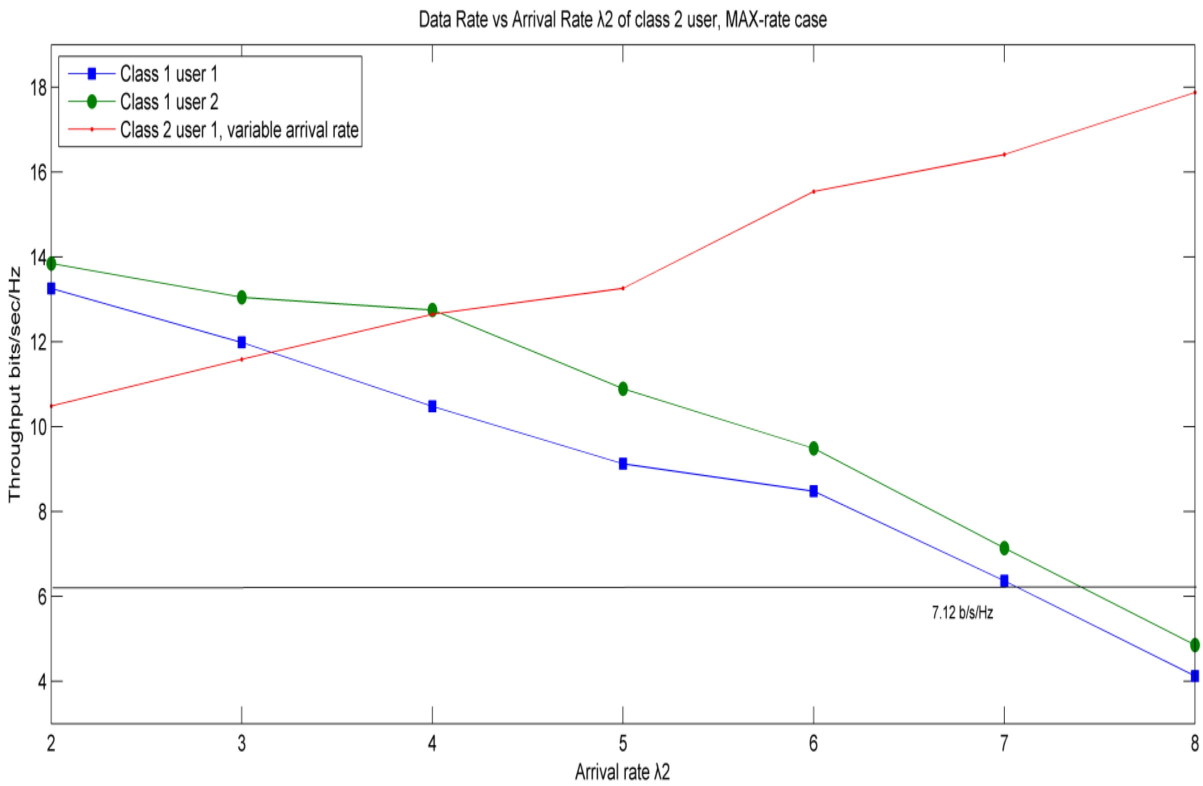

Figure 9 Throughput vs. arrival rate of class 2 user in Max-Rate case

On the contrary Figure 9 shows the opportunistic Max-Rate performance in which when the arrival rate is $\lambda_2 > 7$ the scheduler is not capable to satisfy the QoS constraints of class 1 users, as their performance are clearly lower than 7.12 bits/sec/Hz. The class 2 user increases its performance up to 17.87 bits/sec/Hz which is higher than the S-NBS case as expected. This opportunistic allocation leads to a marginally better throughput for class 2 user than S-NBS however, no fairness is considered at all and minimum requirements are not guaranteed.

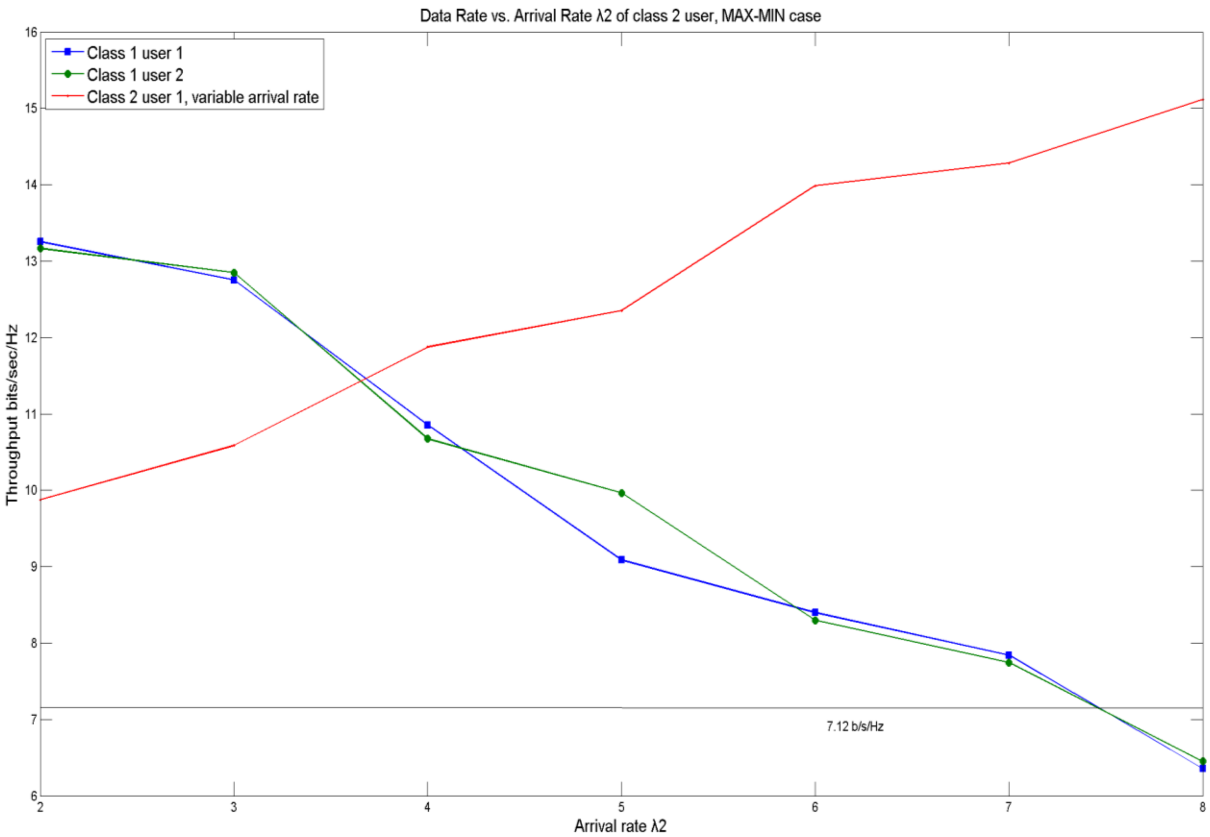

Figure 10 Throughput vs. arrival rate of class 2 user in Max-Min case

The Max-Min scheme provides the minimum rate requirements to the users and afterwards each user takes a portion of the available resources to boost its performance. As it can be seen from Figure 10 as the arrival rate of class 2 user increases the performance of class 1 users decrease. Even though that fairness is considered at the end very low performance is achieved which is lower than class 1 requirements. The two class 1 users when $\lambda_2 = 8$ have 6.45 bits/sec/Hz and 6.35 bits/sec/Hz each of them which is clearly below the threshold of 7.12 bits/sec/Hz. The scheduler fail to maintain the minimum requirements under heavy load and the overall performance is poor as class 2 user reach a maximum performance of 15.11 bits/sec/Hz, which is lower than fairness considering S-NBS scheme and A-NBS schemes.

For the Asymmetric Nash Bargaining Solution (A-NBS) case the two class 1 users have both the same weight $w_{class1} = 0.2$ when class 2 user has weight $w_{class2} = 0.6$. Hence, the summation of the weights is equal to one with class 2 user having higher priority than class 1 users. What is expected in that case is that the scheduler will retain fairness amongst class 1 users whereas class 2 user with higher priority will exploit the additional resources and will increase its performance. Figure 11 validates that scenario and shows that as long as λ_2

increases, the class 1 users are always above their QoS level and resources are fairly distributed to them. Compared with the S-NBS case, class 2 user in A-NBS scheme has better performance as it reaches 17.24 bits/sec/Hz instead of the 16.47 bits/sec/Hz.

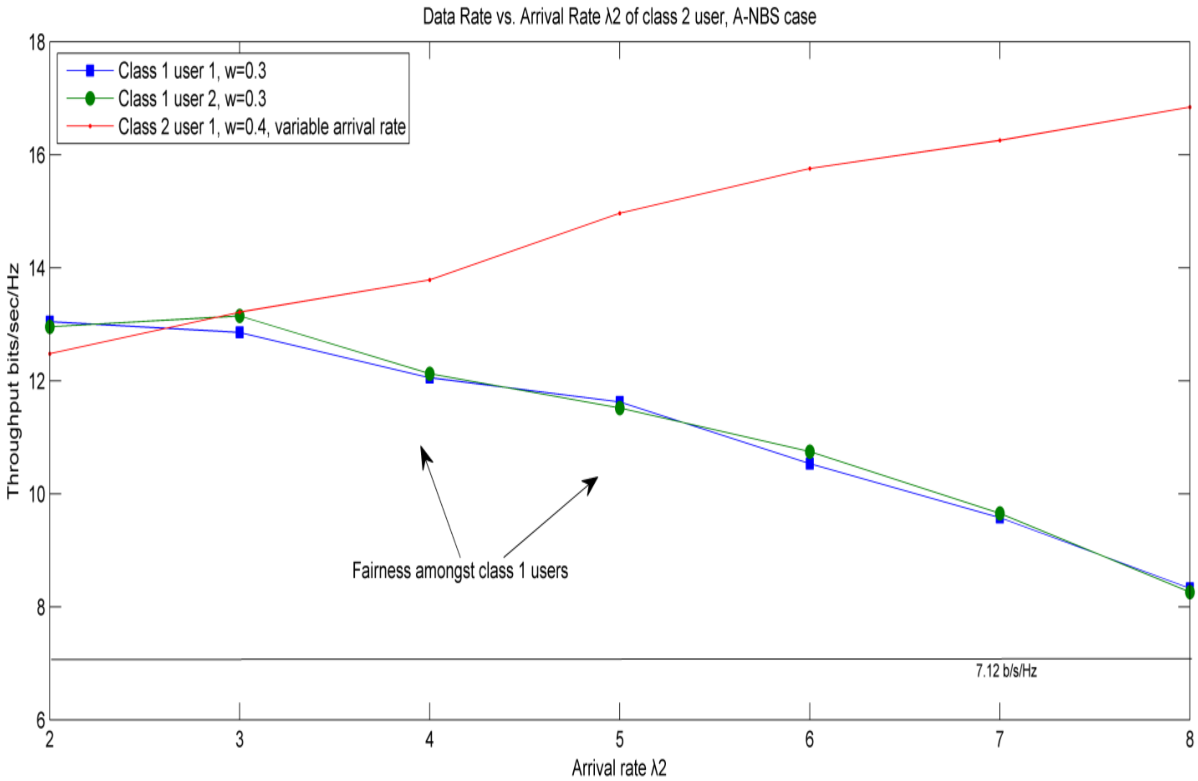

Figure 11 Throughput vs. arrival rate of class 2 user in A-NBS case

In S-NBS scheme when λ_2 varies from 2 to 5, fairness is applied to all users as they have same priorities. In A-NBS case as it can be seen from Figure 11 that class 2 user does not participate into fairness procedure with class 1 users and start to increase its performance from the beginning. The extra priority which is given to class 2 user, acts jointly with the increase of the queue length, occupying more resources from the beginning. Also Figure 12 presents a comparison throughout all schemes.

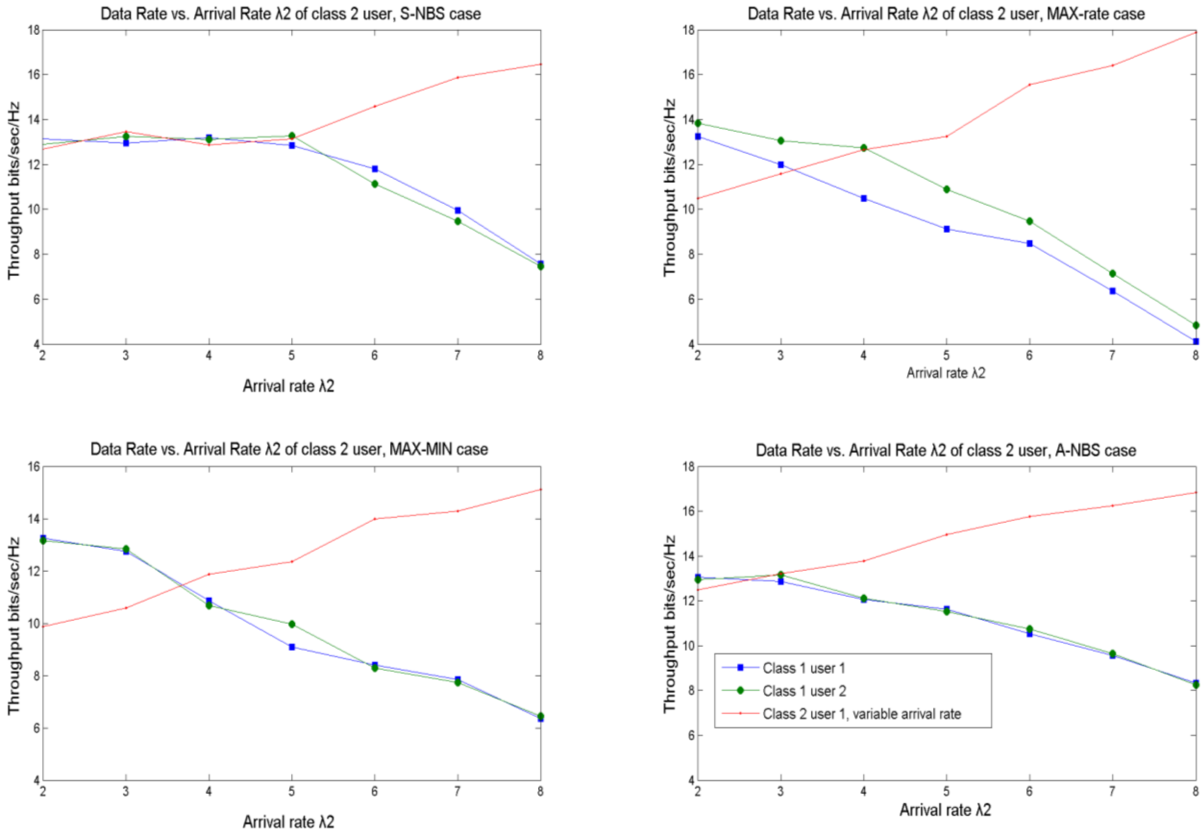

Figure 12 Comparison among all schemes

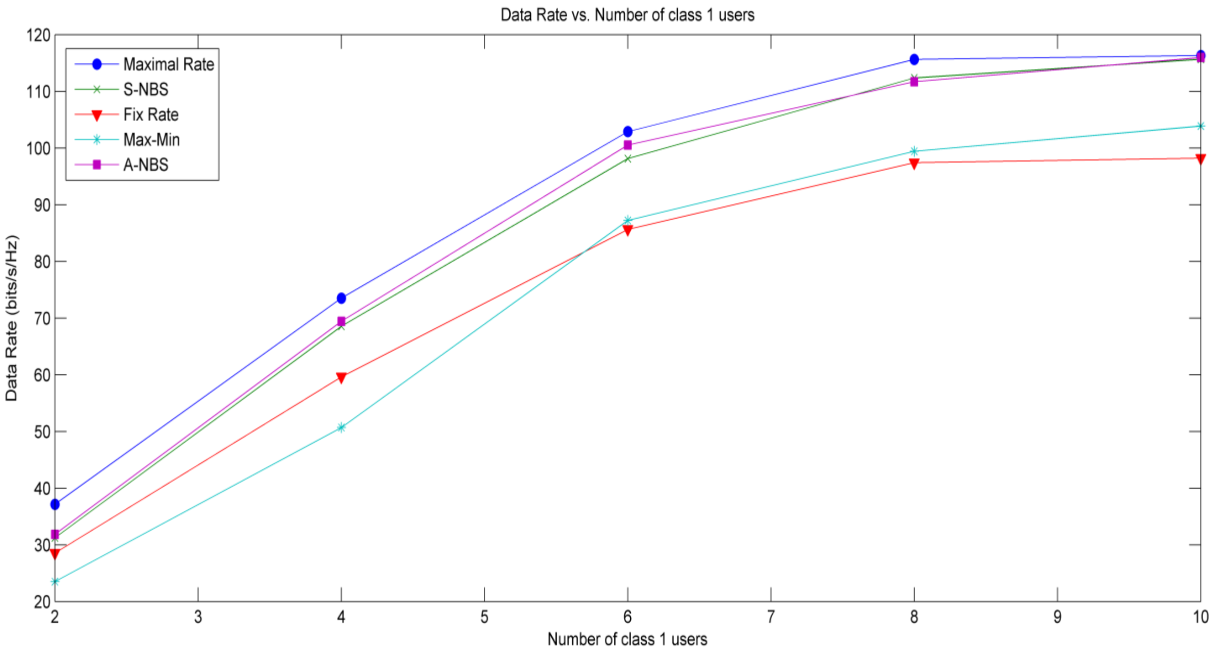

Figure 13 Data Rate vs. Number of users

Figure 13 depicts system's aggregate throughput vs. the number of users. All users are class 1 users and their number is increasing by step of two starting from two users to ten. As we can see from Figure 13, the overall data rate is increasing as long as the number of users is increasing as well. This is an expected phenomenon, as more users exploit better the channel due to multiuser diversity since the channel is varying independently for each user. That appears to have happened up to a certain point as we can see that after the 8 users the aggregate data rate does not have big fluctuations and tends to be stabilized. The higher performance is achieved by the opportunistic Max-Rate scheme, which allocates more resources to the users who are in good channel conditions ignoring fairness. The S-NBS and A-NBS schemes have slightly lower performance than Max-Rate however, as the number of users increases the gap is reduced and finally they converge to the same outcome in case of 10 users. Furthermore the two proposed schedulers are more effective than fairness considering Max-Min and Fix-Rate proposals, as the channel exploitation is more productive in S-NBS and A-NBS schemes.

Fairness index comparison

Figure 14 shows the fairness efficiency for each scheme. The fairness expresses the equality of resource distribution amongst the users in terms of subcarrier or power and the data rate that is represented. Fairness index is calculated from the formula [95] $FR = \left(\sum_{j=1}^{K} \frac{U_j}{U_j^{\min}} \right)^2 \Big/ K \sum_{j=1}^{K} \left(\frac{U_j}{U_j^{\min}} \right)^2$, where U_j and $U_j^{\min}$ are the utility and the initial utility respectively, of user j. For Max-Rate, Fix-Rate and Max-Min scheme the utility function is user’s data rate $\left(R_j\right)$ as these schemes don not use utility function and only the data rate expresses each user performance.

It is obvious that the best performance is achieved by the Max-Min and Fix-Rate schemes for which the fairness index is 1. This means perfect fairness as $FR \in \left(0,1\right]$. The result is prospective since the Fix-Rate assigns a predefined data rate to each user and Max-Min assigns the minimum requirements to each user and continuously allocates proportionally the remaining resources. For the S-NBS scheme fairness is not perfect but is always very close to 1 as the fairness index varies from 0.99 to 1 whereas in the A-NBS case the fairness index lies to a range from 0.98452 to 0.99856, which is slightly lower than S-NBS scheme because users have different weights. The unfair Max-Rate scheme has the lower performance in a decreasing path as the number of users increases. This happens because Max-Rate, without any fairness concerns, assigns resources to the users who have good channel conditions. As it can be understood the S-NBS and A-NBS schedulers are those which can provide high level of fairness and high data rate performance simultaneously to the users and therefore to the whole system.

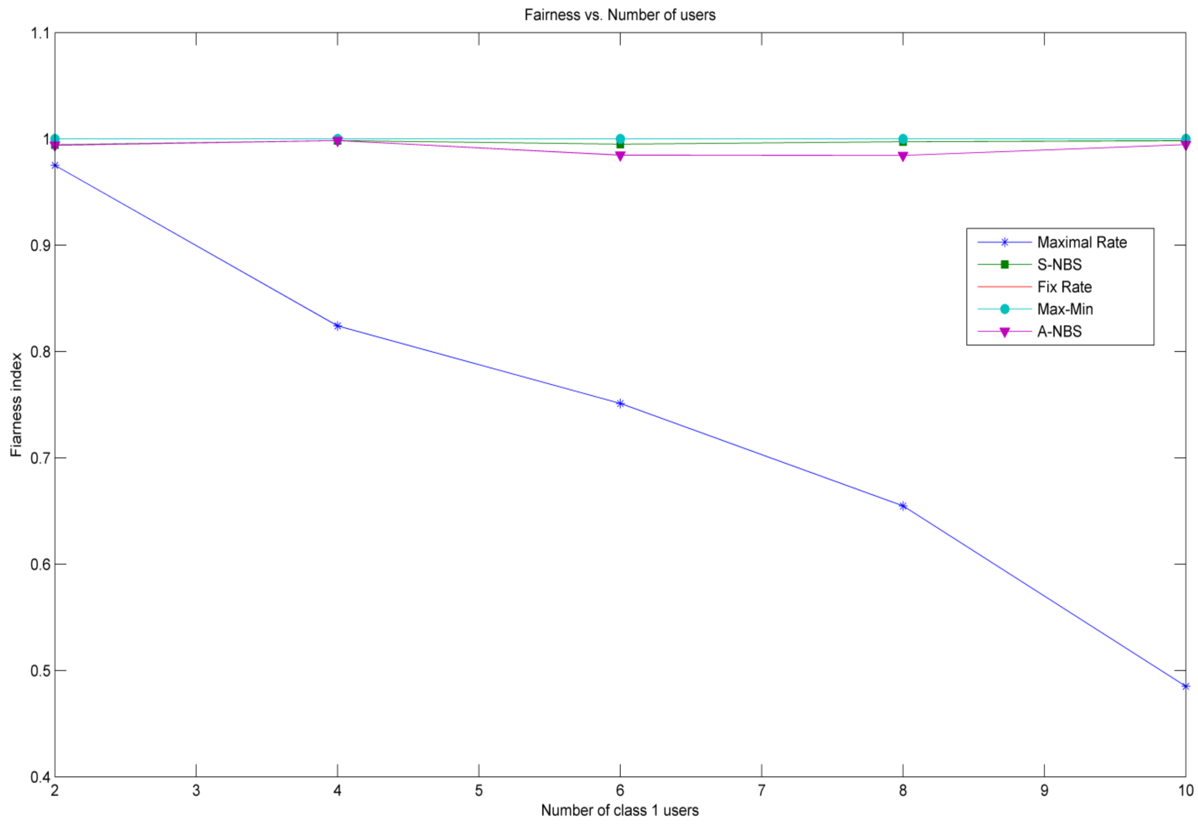

Figure 14 Fairness vs. Number of class 1 users

In a more detailed view of fairness in Figure 15, it is clear that the S-NBS scheme is very close to the ideal fairness with values over the 0.99. Lower achievements are noticed for the A-NBS scheduler as the different users' weights affects systems fairness. Here, we must notice that the fairness index amongst same class users in A-NBS case lies at a higher level which is almost the same as S-NBS, as it can be seen in Figure 15. Next table shows the users' weights for each case for A-NBS scheme.

Number of users	Weights		Number of users	Weights
2	$w_1 = 0.3$		8	4 users $w_1 = 0.1$
	$w_2 = 0.7$			4 users $w_2 = 0.15$
4	2 users $w_1 = 0.2$		10	5 users $w_1 = 0.125$
	2 users $w_2 = 0.3$			5 users $w_2 = 0.075$
6	4 users $w_1 = 0.2$			
	2 users $w_2 = 0.1$			

Table 5 Class 1 users' weights for fairness calculations

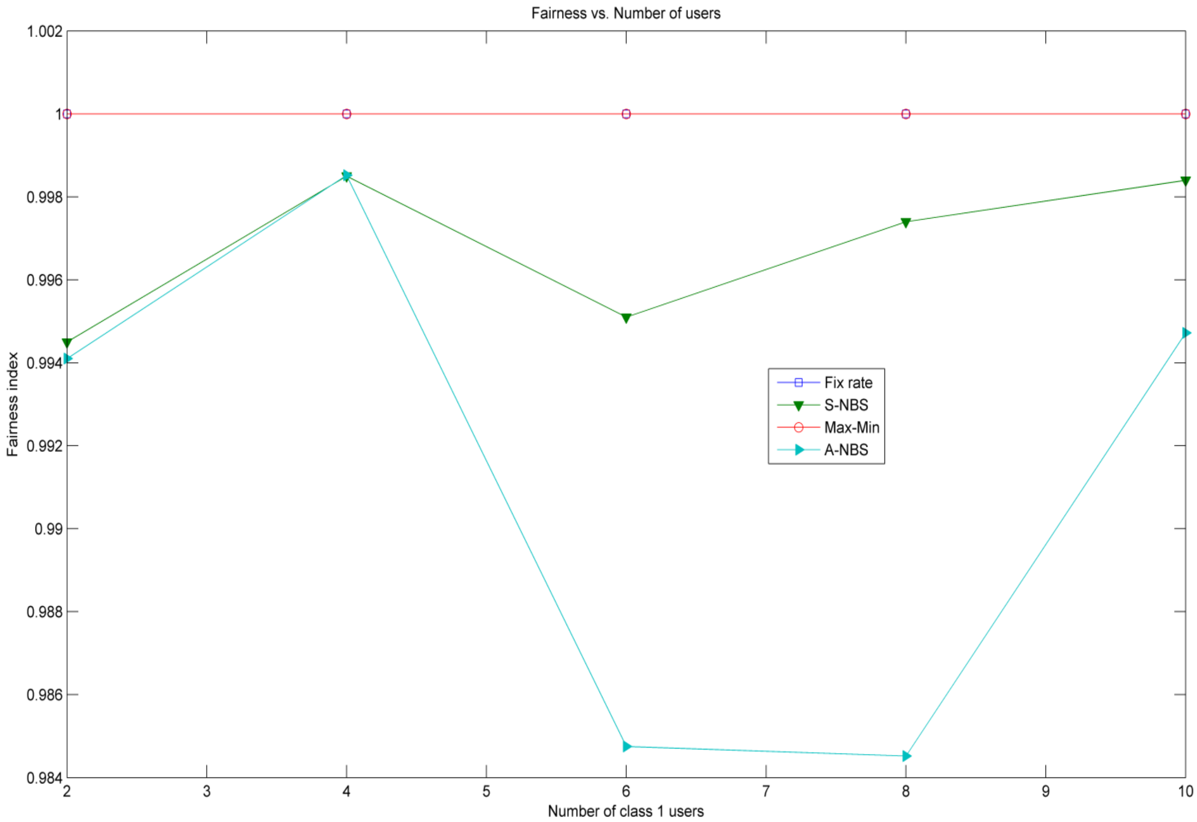

Figure 15 Fairness vs. Number of class 1 users (detailed)

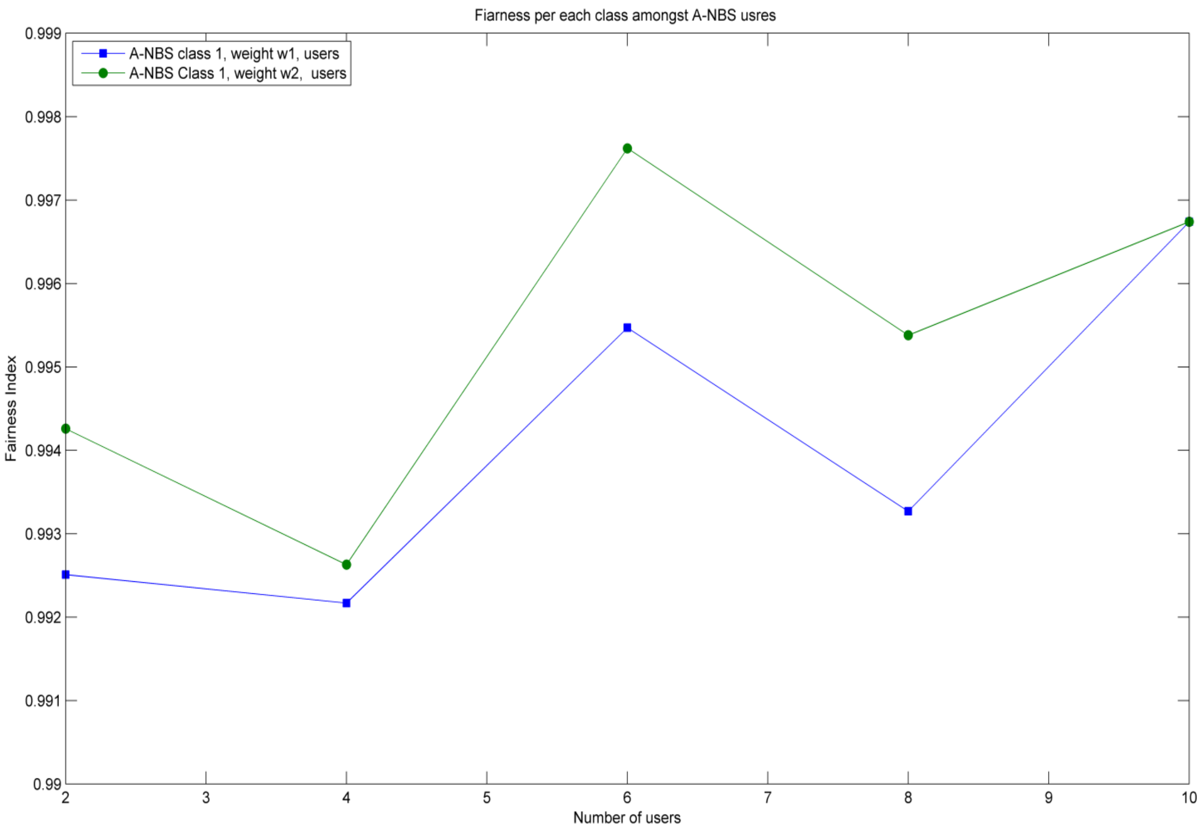

Figure 16 Fiarness vs. Number of users A-NBS case

Power comparison

As power consumption is a critical factor for wireless networks, during the next figures the power performance of each scheme is presented. The scenario includes two heterogeneous users, one of each class. The minimum data rate requirement for class 1 user is 7.12 bits/sec/Hz while class 2 user data rate requirement is 3.14 bits/sec/Hz. Figure 17, illustrates the average transmit rate vs. power for the S-NBS scheme. As it can be seen, both users are satisfied when total available power is $P_{total} \simeq 5.95dB$. Figure 18 depicts the A-NBS scheme where class 1 user has weight $w_{class1} = 0.3$ while class 2 user has weight $w_{class2} = 0.7$. Class 2 user, because of its high priority increases its performance quicker than class 1 user. Also, the required power for the system in order to provide users' initial demands, is almost the same as in S-NBS which is $P_{total} \simeq 5.9dB$. It has to be mentioned here that A-NBS case is the only one for which weights are recognized and system performs accordingly. If weights are applied in any other scheme, from those that are examined, the system will totally ignore them and will continue to perform allocation without any weight consideration. Hence, only A-NBS can be used in cases where weights need to be applied. Moreover, the performance of the greedy Max-Rate scheme is illustrated in Figure 19. It can be seen that the minimum power in order users' initial demands are satisfied is $P_{total} \simeq 5.5dB$, which is the lowest amongst all schemes, and achieves the best overall data rate without providing any kind of fairness. The performance of the Fix-Rate scheme is depicted in Figure 20. The minimum required power is $P_{total} = 8dB$ which is 2dB more than the fairness considered S-NBS and A-NBS schemes. Also a poor data rate is achieved which is the same as Max-Min scheduler in Figure 21. However, in Max-Min scheme the minimum power is significantly lower by 2dB compared to Fix-Rate scheme. In overall, the S-NBS and A-NBS schemes perform better amongst the fairness considering schemes and are very close to the fairness inconsiderate Max-Rate scheme. S-NBS and A-NBS proposals are an effective trade-off between fairness and performance. The Max-Rate scheme demands 0.4dB less power for users' minimum requirements and achieves on average 1.2 bits/sec/Hz and 2.3 bits/sec/Hz data rate per user than the S-NBS and A-NBS respectively. In contrast, fairness as well as the inability of Max-Rate to support user priorities, through weights, are its main drawbacks.

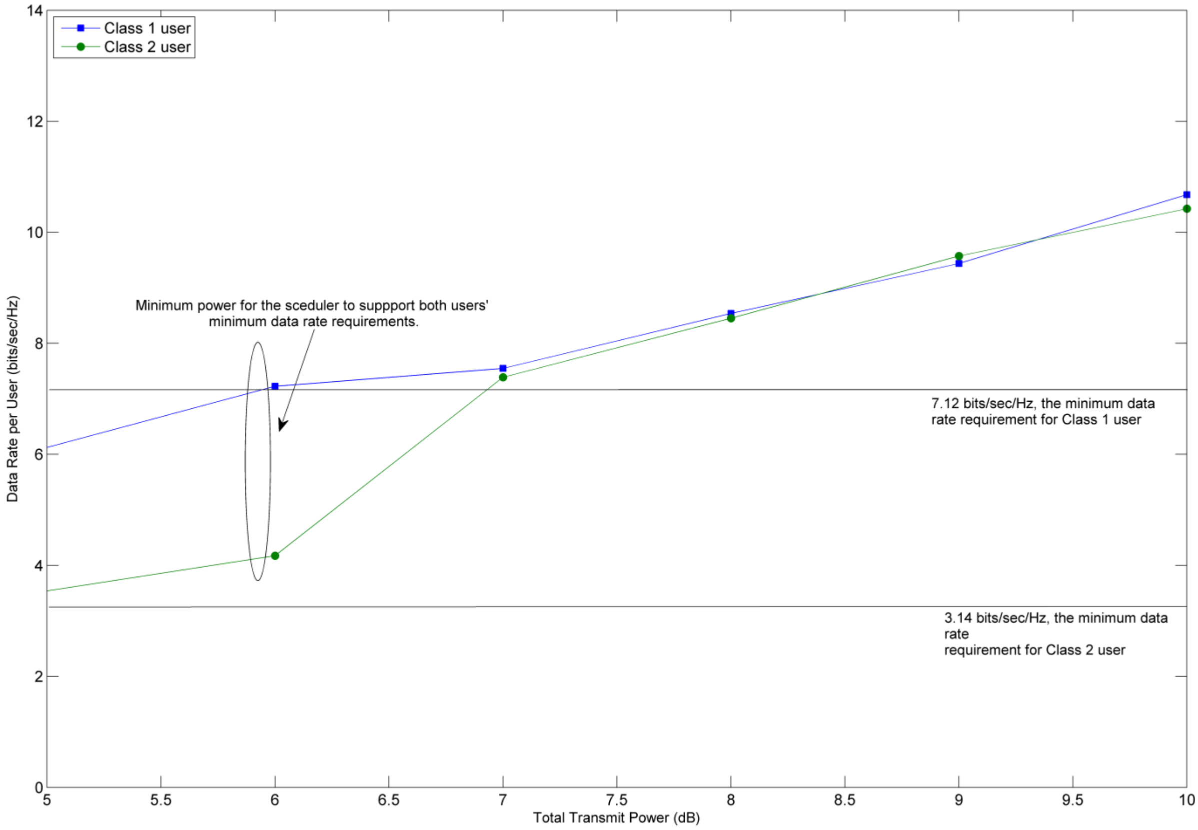

Figure 17 Average data rate vs. total transmit power for the S-NBS scheme

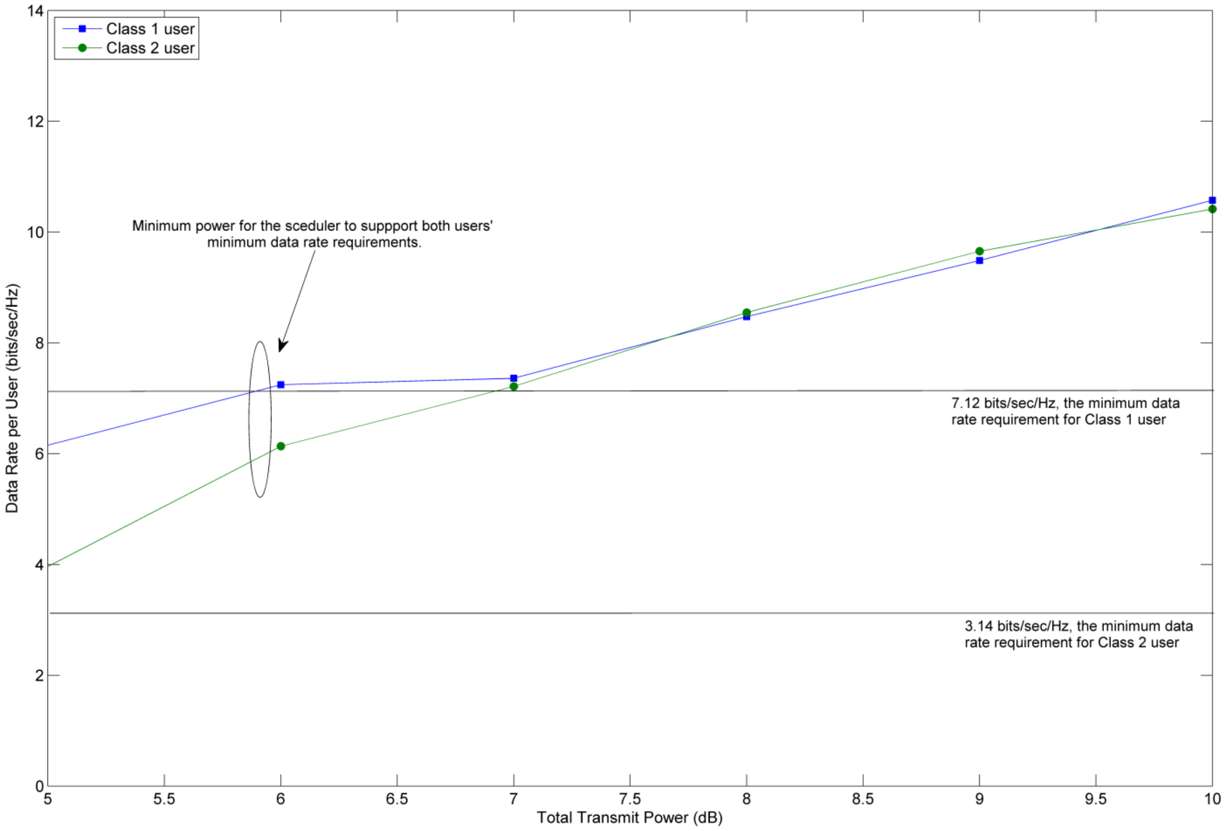

Figure 18 Average data rate vs. total transmit power for the A-NBS scheme

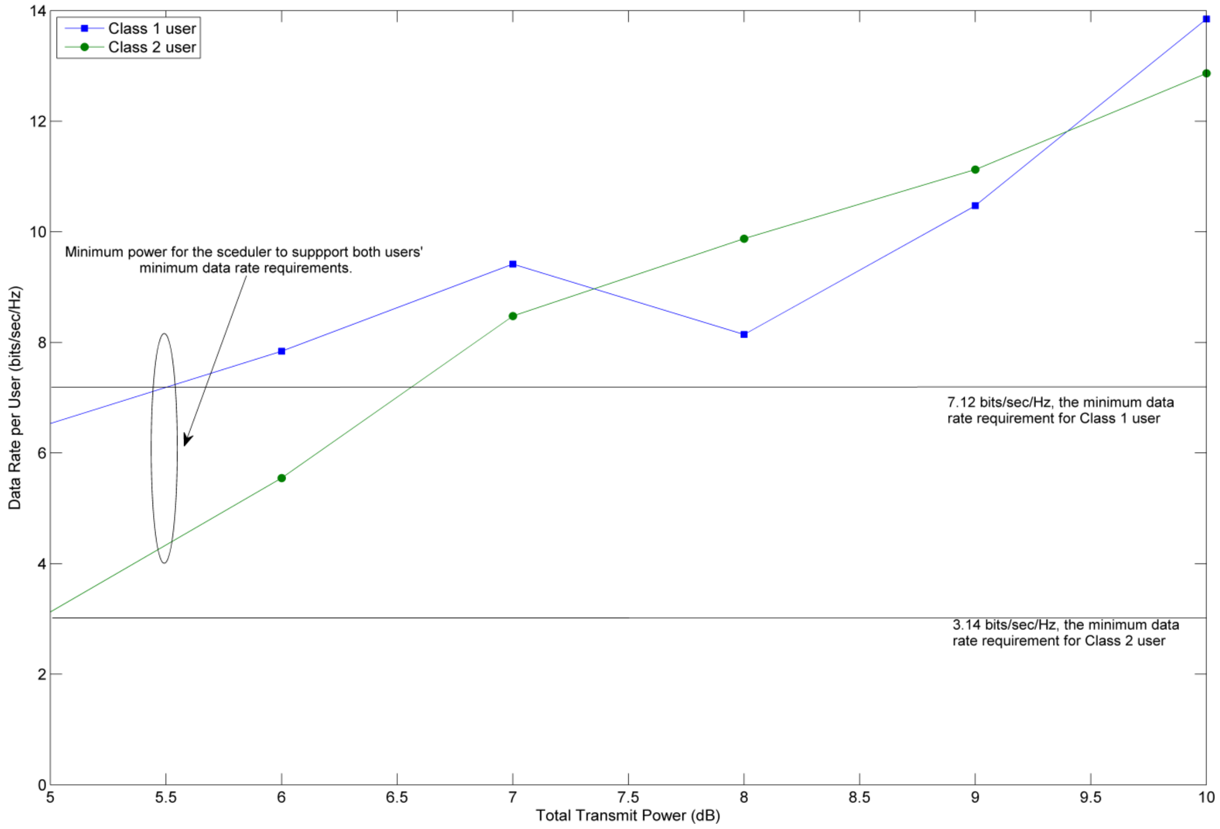

Figure 19 Average data rate vs. total transmit power for the Max-Rate scheme

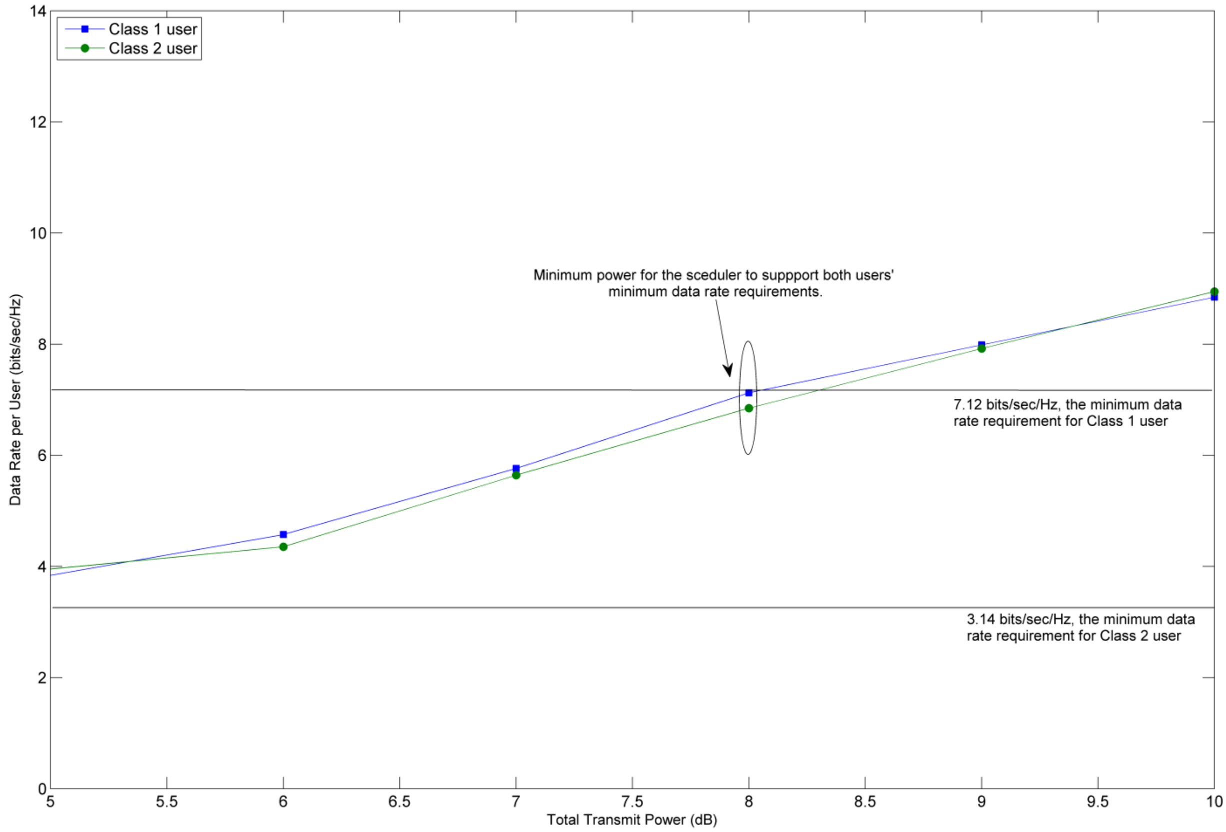

Figure 20 Average data rate vs. total transmit power for the Fix-Rate scheme

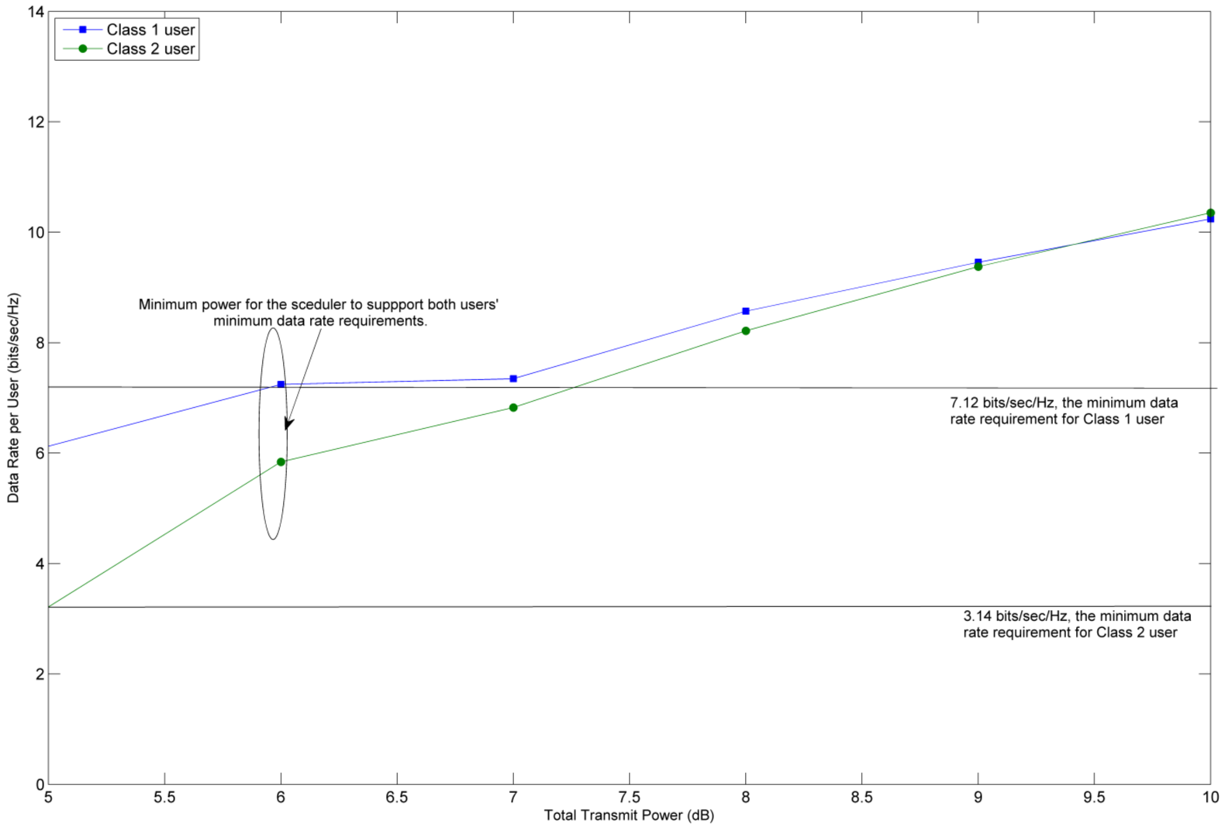

Figure 21 Average data rate vs. total transmit power for the Max-Min scheme

Figure 22 depicts the power performance gap amongst all schemes. The scenario takes into account 10 class 1 users and the available total power varies from 13.8 dB to 26dB. As it can be seen the Max-Rate has the best performance amongst all in throughput, with total data rate of 108.62 bits/sec/Hz, as well as in power consumption due to the fact that Max-Rate needs the less power amongst all schemes to satisfy users' QoS demands. The NBS schedulers compared with the greedy Max-Rate, need additional about 2.2dB power to reach the same initial data rate. When the power is 26dB the gap between S-NBS, which has total rate of 97.13 bits/sec/Hz, and Max-Rate proposal is 11.76 bits/sec/Hz which is smaller than the gap between the S-NBS and Max-Min scheme which is to 21.41 bits/sec/Hz as the overall Max-Min data rate is 75.72 bits/sec/Hz. Furthermore, Fix-Rate and Max-Min proposals, demand more power to reach to the data rate level of the other schemes hence, are less power effective. Also, the overall data rate is 72.67 bits/sec/Hz and 75.72 bits/sec/Hz for Fix-Rate and Max-Min respectively that reflects the ineffective power and channel exploitation for these schemes. On the other hand S-NBS and A-NBS schemes are a tradeoff between data rate and fairness as fairness is maintained at the price of the overall performance which at the end is close to the opportunistic Max-Rate proposal.

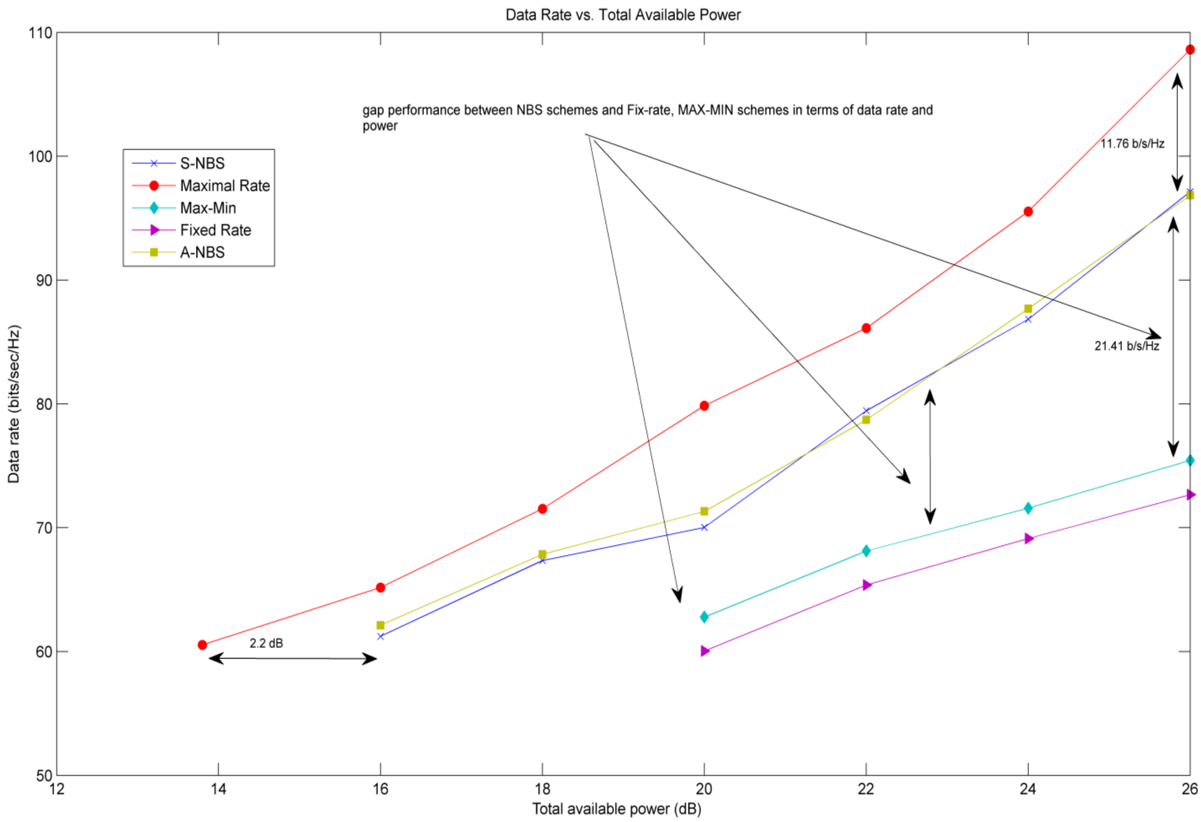

Figure 22 Data Rate vs. total available power

Summary

In this chapter a performance comparative study of the proposed cross-layer schemes is presented.

The S-NBS and A-NBNS cross-layer schemes perform very well during simulations. Both of the fair proposed cross-layer schemes in terms of throughput perform very well. They manage to satisfy user's QoS demands even under heavy load when other proposals fail to do that. The overall performance of our fair schemes is close; in any case, to greedy Max-Rate scheme which performs best of all as it is expected without provide any kind of fairness to the users. The A-NBS scheduler achieves similarly performance to the S-NBS whilst different weights are applied to the users in that case, with class 2 user be more benefited. Fix-Rate and Max-Min schemes have poor overall performance in terms of data rate in comparison with S-NBS and A-NBS.

Although, Fix-Rate and Max-Min distributes fairly the resources amongst users providing fairness index equal to one at the expense of the overall performance, the proposed schemes are capable to provide same level of fairness realizing significant better overall performance. Furthermore, in the overall system's data rate proposed schemes prove their superiority in comparison with Fix-Rate and Max-Min and only Max-Rate achieve marginally better performance. Consequently, S-NBS and A-NBS comprise a trade-off between efficiency and fair resource allocation.

In terms of power consumption the proposed S-NBS and A-NBS schemes perform better than Fix-Rate and Max-Min which are also fair. Max-Rate needs the least power to satisfy users' requirements with S-NBS and A-NBS being next and very close to Max-Rate. Hence, fair resource allocation is achieved at the expense of power consumption which is expected as users with worse channel conditions than others will demand more power in order to achieve same performance with the users with good channel conditions.

Key role in the performance of the proposed schemes plays the utility function which is utilized. Data rate, queue length as well as normalized delay are factors which contribute for user satisfaction and affect the distribution of the resources amongst users. In the case of A-NBS, weights can be adopted giving priorities to the users based on the user's QoS demands (i.e. more resources would be allocated to the multimedia users) or to the user's price policy (some users would be willing to pay in order to buy bandwidth). Even though heterogeneous users are considered in our simulations only A-NBS scheme can apply weights whereas other schemes are not aware of users' weights. Thus, A-NBS can be used in cases where different services are defined and cross-layer design must support them.

CHAPTER 6

Conclusions and Future Work

Thesis Summary

This thesis presents the design of two novel cross-layer schedulers for OFDMA wireless networks with finite queue considerations, which are based on game theory and in particular on NBS. First cross-layer optimization problem is formulated as a symmetric NBS case whereas, the second optimization problem which is based on asymmetric NBS, expresses the ability of the scheduler to provide different priorities to the users by assigning different weights to them. In the introduction in Chapter 1, the motivation, the scope of this thesis as well as the contributions are declared.

In Chapter 2 a brief presentation of OFDM, OFDMA and game theory are presented. The main attributes of OFDM and OFDMA are mentioned while more focus is given in robust characteristics of this sophisticated technology. Following that, an introduction to game theory is presented. Types of games and their unique aspects are included in this chapter with additional paradigms. This chapter has focused on Nash's theorems (Nash Equilibrium and Nash Bargaining Solution) as the two proposed solutions are relying on them.

An extensive literature review across a variety of cross-layer schemes can be found in Chapter 3. The research which has been done so far in OFDMA cross-layer optimization is included in this chapter. Many ways for optimization, have been investigated each of them aiming to optimize the performance of the wireless network. Some proposals aim to maximize system's total capacity using greedy methods. Other schemes aim to minimize power consumption whereas some schemes try to provide fairness and user satisfaction. The fairness element in general is missing from the constraints in the optimization problem which is a basic feature for modern wireless networks.

Chapter 5

Chapter 4 presents the formulation of the two cross-layer schedulers. The queueing model for the MAC layer is mentioned based on which we derive the users' QoS requirements. Furthermore, the two constraint optimization problems based on S-NBS and A-NBS, with their closed-form solutions are presented.

Finally, in Chapter 5 the evaluation of the two proposed schemes is mentioned. Simulation results in comparison with other schemes are depicted. Moreover, a discussion about results gives the opportunity to argue about the superiority of the proposed finite-queue cross-layer schedulers.

Thesis Conclusion

The thesis objective is to propose a cross-layer scheduler for wireless OFDMA networks with finite queue, which will optimize the system overall performance, meeting users' QoS requirements when at the same time all the available resources are fairly distributed amongst them. Simulation results confirm that all these requirements are met.

More specifically, both optimization problems are solved via analytical methods when most game theoretic based solutions are algorithmic or numerical leading to suboptimal solutions. The constraint optimization problem, in both cases, is transformed into a convex optimization problem over a convex set. In addition, the proposed schemes have complexity $\mathcal{O}\left(K \times N\right)$ which is the lowest amongst all the schemes. The utility function is taking into account the queue length of the users as well as the delay so that users with large queues would benefit more from the scheduler. The proposed schemes succeed to maintain data rate for the users above their initial demands when the compared algorithms fail to do that. Additionally, S-NBS and A-NBS maximize system's aggregate data rate and perform very close to the greedy, fairness-inconsiderate Max-Rate scheduler which is very important as fairness is achieved at the expense of slightly lower performance in comparison with the opportunistic schemes. Furthermore, compared to fairness-considered schemes such as, Fix-Rate and Max-Min; the proposed schemes achieve superior performance. Moreover, the ability in A-NBS case to give priority to the users, by assigning different weights, is also very important giving the ability to the BS to support multimedia, real-time users or price-based services. In terms of power consumption the proposed schemes behave very close to the best performance which is the greedy scheme. S-NBS and A-NBS consumes less power than fairness-considered schemes while the extra demanded power regarding the greedy scheme, is the cost for providing higher rate to the users with bad channel conditions. Finally, the fairness index in S-NBS case is always very close to 1 as well as in A-NBS case when same classes of users are compared.

Future Work

The outline of this thesis is the optimization of the cross-layer of a wireless OFDMA networks with finite queue using game theory. Basic motivation for this work was the lack of fairness in the proposed schemes. During this thesis many thoughts arose about how this work can be extended and which would be the future steps for cross-layer designs in wireless OFDMA networks.

The further improvement of the power management in BS could be a possible future effort because, in wireless networks most of the times we refer to battery supplied BSs and mobile devices thus, power is a very critical aspect for the overall system performance. Hence, further investigation leading in “green systems” with better power exploitation and improved performance should be considered as a future research topic.

All this study is about downlink traffic. As long as the complexity of subcarrier and power allocation in the BS is relatively high a way to reducing that complexity should be investigated. The implementation of a game for uplink case which will be amongst users in order to choose the subcarriers that they will prefer based on their QoS requirements and channel condition could be formulated. Then, users through pilot subcarriers would inform BS for the results and BS will implement only the power allocation to each subcarrier reducing the decision time and the complexity in BS.

The presented work is about a single cell of an OFDMA wireless network. As in the wireless networks smaller cells within a cell are used for better coverage and reception this work could be applied in a wireless network with relay nodes or in cognitive radio networks where the same frequencies are used by primary and secondary users.

Additionally, further research could be the study of the dMMPP queue model in such type of wireless networks. As long as our goal is to describe the packet traffic in realistic way research has shown that Markov Modulated Processes do not behave so accurate. Authors in [96] show experimentally that, given the same measures of count correlation and interval correlation, the Markov Modulated Processes are likely to give optimistic performance values for loss rate, mean delay etc. For a stable MMBBAP/D/1 queue with arrival process represented by a Markov Modulated batch Bernoulli arrival process (MMBBAP) in comparison to those values obtained for a stable BRAP/D/1 queue with Batch Renewal Arrival Processes (BRAP). Hence, the adoption of BRAP queue model instead of dMMPP could be a future work. The selection of the appropriate queue model for computer network in general is not always easy choice [97].

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