Mathematical Analysis of the Turbine Coefficient of Performance for Tidal Stream Turbines

Shady Hossam E. Abdel Aleem
15th May Higher Institute of Engineering, 15th of May City Helwan, Cairo, Egypt
engyshady@ieee.org

Ahmed Faheem Zobaa
School of Engineering and Design, Brunel University, Uxbridge, UB8 3PH, U.K.
azobaa@ieee.org

Ahmed Mohamed Ibrahim
Department of Electrical Power and Machines Engineering, Cairo University, Giza, Egypt
dr-ah-ibr@hotmail.com

Abstract- **Unregulated water currents such as tides and ocean currents include energy that could be utilized for electricity production. These currents can be seen as dead bodies of water with potential energy, driven by gravity or alive moving with a kinetic energy (KE). Tidal stream turbines are a relatively new technology for extracting *KE* from tidal currents, which is currently in progress from development stage to industrial execution. One of the most important factors in tidal power analysis is the rotor efficiency coefficient or turbine coefficient of performance (λ). It depends on the rotor blade geometry and water velocity. This article presents a mathematical description of good interpolating functions which describe this coefficient analytically, for tidal stream turbines. Nonlinear curve-fitting solver in least-squares sense has been used in this study. Various interpolation functions have been proposed. The proposed mathematical descriptions can be very helpful for tidal power analysis and output power estimation.**

***Index Terms*— Ocean energy, rotor efficiency coefficient, tidal currents, tidal stream turbines.**

I. NOMENCLATURE

A	Swept area (m^2)
D	Turbine's rotor diameter (m)
F	Axial thrust or force generated (N)
n	Number of response values
m	Number of fitted coefficients
P_m	Mechanical power (watts)
R	Turbine's rotor radius (m)
R^2	Proportion of variance coefficient
RMSE	Root-mean-squared error
SS_E	Sum of squares due to error
T	Torque generated by the flow (Nm)
TSR	Dimensionless Tip-Speed-Ratio
U	Uniform flow velocity (m/s)
U_U	Upstream water velocity at the entrance of the rotor blades (m/s)
U_D	Downstream water velocity at the exit of the rotor blades (m/s)
x	Coefficients that best fit the proposed equations
λ	Turbine coefficient of performance
λ_T	Torque coefficient of performance
λ_F	Thrust coefficient of performance
ν	Residual degrees of freedom
ρ	Water density (kg/m^3)
ω	Angular velocity (rad/s)

II. INTRODUCTION

Currently, the responsiveness towards the green house gas emissions has arisen. Worldwide organisations are aware that different emissions such as carbon dioxide (CO_2) and other risky gases are harmful towards the atmosphere which resulting climate change. These emissions are mainly emitted from fossil fuels combustion where it is used as main source of energy. Also, the significant increment of global fuel price is also lead many researchers to start looking for alternative renewable energy source [1] – [3].

The development of offshore renewable energy can be seen as environmentally desirable, especially for the amelioration of climate change, including meeting international CO_2 reduction targets. Since the early 2000s, a number of large-scale wave and tidal current prototypes have been demonstrated around the world, but marine renewable energy technology is still 10–15 years behind that of wind energy. However, having started later, the developing technology can make use of more advanced science and engineering, and it is therefore reasonable to expect rapid progress. Tide power paddle wheels were used in *Egypt* 1100 AD, and about the same time tidal mills were used in *UK* and *France*. Tidal stream systems make use of the *KE* of moving water to power turbines, in a similar way to windmills that use moving air. This method is gaining in popularity because of the lower cost and lower ecological impact compared to barrages, which make use of the potential energy in the difference in height (or head) between high and low tides [2].

Ocean energy (marine energy) refers to the energy carried by ocean waves, tides, salinity, and ocean temperature differences. The conversion of tidal energy into electricity has been widely investigated and can be compared to the technology used in hydroelectric power plants [2], [4].

During the last three years, much attention has been focused on tidal stream technology. It is now developing apace, different turbine designs are being proposed, and experimental performance testing is being carried out at small scale. However, there is the issue of how to convert the results from the experimental to the full scale [4], [5].

Tidal energy is very similar to the wind energy; both of them depend on a moving fluid, also the way of extracting kinetic energy from their moving fluids is very similar.

However, despite their obvious similarities; there are many different points in their environment. First of all, density of air is different than that of water. Specifically, water is over eight hundred times denser than air. Accordingly, tidal turbine would have a radius smaller than the wind turbine. Additionally, the average velocities of tides are less than the average velocities of the wind, thus, diameter of the rotor of a tidal turbine is three times smaller than diameter of the rotor of a wind turbine, assuming they have equal power coefficients of performance [6]. In a good site, water-flow velocities are around 3 m/s, but there are places with velocities as high as 5 or 6 m/s. The world's first underwater turbine servicing consumers on a regular basis uses a 2.5 m/s flow. It is operated by *Hammerfest Stroem* in northern *Norway*, way beyond the Arctic Circle. Compared with wind, these are modest velocities; however, the high density of water causes these small velocities to lead to attractive power densities [7]. Furthermore, rotor inertia, working environment, cleanliness and velocity variation effects are important factors differentiate between tidal and wind turbines. Reader could refer to [6] for more details about differences in fundamental design for wind and tidal turbines.

It must be mentioned that it is interesting to carry on some measurements on water-flow velocities in some good sites in *Egypt* such as *Alexandria* and *Port-Said*, in order to capture their modest water-flow velocities, especially under the influence of their common storms.

Generally, tidal stream turbines can be classified according to:

- ***Rotor configuration:*** axial- or cross-flow, open or ducted.
- ***Drive train configuration:*** indirect drive, when a rotor is connected to a generator via a gearbox, or direct drive, when a rotor is directly connected to a generator.
- ***Type of supporting structure:*** fixed to the seabed, gravity based or floating.
- ***Blades way of connection:*** the blades can be fixed (fixed pitch) or being made rotatable about their axes (variable pitch).

In the variable pitch case, the blades' point of reference towards the current flow direction can be changed thus the power take-off can be controlled (pitch controlled turbine). However; they are mechanically high complex than fixed pitch turbines [8].

Additional classifications based on other parameters are also possible. Fig. 1 shows a neat classification of turbines based on axis direction [2].

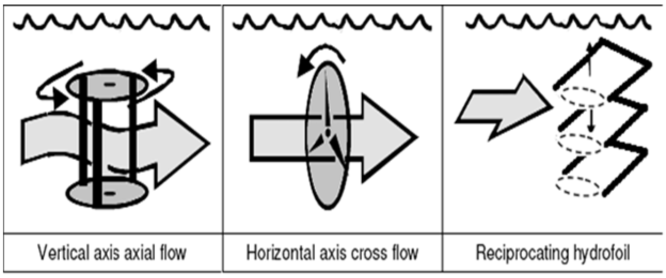

Fig.1 Classification of turbines based on axis direction [2]

III. ENERGY CALCULATIONS OF TIDAL CURRENT

The kinetic energy *KE* in joules for water of mass *m* (given in kilograms) moving with speed *U* (given in meters per second) is given as

$$KE = \frac{1}{2} m U^2 \qquad (1)$$

The focus of electric power output can be maintained by focusing on mechanical power output. The mechanical power in a moving fluid can be given as the rate of flow of kinetic energy, as following,

$$P_m = \frac{1}{2} \dot{m} \left(U_U - U_D \right)^2 \qquad (2)$$

where: U_U represents the upstream water velocity at the entrance of the rotor blades, and U_D represents the downstream water velocity at the exit of the rotor blades. Also, $\dot{m}$ represents the mass flow rate of water through the rotating blades (given in kilograms per second), and is given as

$$\dot{m} = \rho A \left(\frac{U_U + U_D}{2} \right) \qquad (3)$$

where: ρ and A denote the density of seawater and turbine frontal area (area swept by the rotor blades in square meters). Thus, the mechanical power P_m extracted by the rotor, which is driving the electrical generator, is

$$P_m = \frac{1}{2} \rho A \left(\frac{U_U + U_D}{2} \right) \left(U_U - U_D \right)^2 \qquad (4)$$

After some mathematical manipulations and simplifying for (4), the mechanical power is given as

$$P_m = \frac{1}{4} \rho A U_U^3 \left(\left(1 - \frac{U_D}{U_U} \right)^2 \left(1 + \frac{U_D}{U_U} \right) \right) \qquad (5)$$

Commonly, it is given as a fraction of upstream water velocity as following

$$P_m = \frac{1}{2} \rho A U_U^3 \left(\lambda \right) \qquad (6)$$

where λ represents the rotor efficiency coefficient or turbine coefficient of performance. Different values of λ depend on the ratio (U_D/U_U) or the downstream to the upstream water velocity ratio, so that

$$\lambda = \frac{1}{2}\left(1-\frac{U_D}{U_U}\right)^2\left(1+\frac{U_D}{U_U}\right) \tag{7}$$

or

$$\lambda = \frac{1}{2}\left(1-\frac{\omega R}{U_U}\right)^2\left(1+\frac{\omega R}{U_U}\right) \tag{8}$$

TSR is the dimensionless *Tip-Speed-Ratio,* and it is given as a function of turbine geometry, where *R* is the turbine's rotor radius rotating with an angular velocity ω (in radians per second). It is defined as

$$TSR = \frac{U_D}{U_U} = \frac{\omega R}{U_U} \tag{9}$$

It must be mentioned that turbines are now being designed to 20 m diameters [9]. For a given design current speed, the turbine coefficient of performance λ varies with *TSR*. Beside, for a specific *TSR*; λ is maximized. Practically, $\lambda_{maximum}$ is less than 0.5[5], [8].

A similar analysis for the torque, T, and the axial thrust on the turbine, F, will lead to torque and thrust coefficients, λ_T and λ_F respectively, where

$$T = \frac{1}{2}\rho A U_U^2\left(\lambda_T\right) \tag{10}$$

$$F = \frac{1}{2}\rho A U_U^2\left(\lambda_F\right) \tag{11}$$

The tip speed ratio is one of the major parameters to be selected. The turbine's performance is usually efficient when *TSR* is in the range of 3 to 7 (i.e. $\lambda > 0.4$). However, too large a *TSR* will cause ωR to exceed rapidly. At the same time, lower values of λ_T correspond to lower bending moments in the blades, from this point of view, lower *TSR* is better. *TSR* must be sufficiently low to avoid undue noise and undue stresses. Large tip speed ratios require good hydrofoils. One reason for repeated iterations is to focus on the value of *TSR* that optimizes the performance. Many modern turbines can operate at the selected tip speed ratio [8]. Plot of ideal turbine coefficient of performance and thrust coefficient versus the tip speed ratio *TSR* shows that both λ and λ_T are single maximum-value functions (unimodal functions), as shown in Fig. 2.

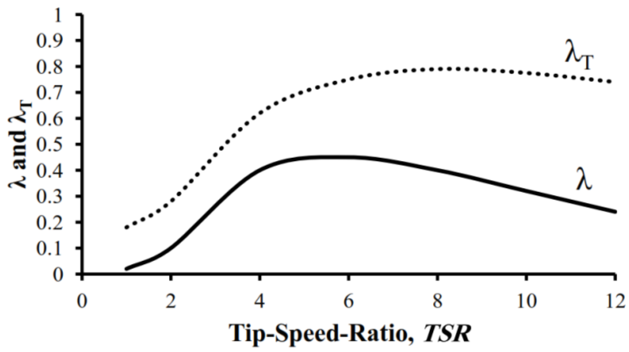

Fig. 2 (λ and λ_T) versus *TSR*

IV. Problem under Study

In this paper, the relation between the turbine coefficient of performance (λ) and *TSR* has been studied for a test marine turbine (R=10 m) directly taken form [4], rotating in a sea water has a uniform flow velocity of 3.08 m/s.

Fig. 3 shows the experimental dependence between them. It is obvious that the dependence does not have the form of the ideal turbine coefficient of performance characteristics.

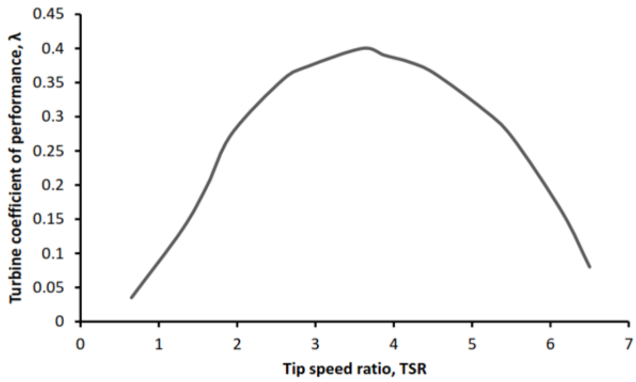

Fig. 3 Experimental λ versus *TSR* [4]

Using the nonlinear curve-fitting (data-fitting) problems in least-squares sense (lsqcurvefit); various interpolation functions have been proposed [10].

The *lsqcurvefit* solves nonlinear data-fitting problems. It requires a user-defined function to compute the vector-valued function F (*x*, *TSR*), where *x* are the coefficients that best fit the following equation [11].

The size of the vector returned by the user-defined function must be the same as the size of the vectors λ and *TSR*, so that *i* has a starting value of 1.

$$\underset{x}{\text{Minimize}} \sum_i \left(F\left(x_i, TSR_i\right) - \lambda_i\right)^2 \tag{12}$$

The mathematical description of this relation is very important for marine power analysis, as done before in early stages for wind turbine analysis.

V. The Indices describe Goodness-of-Fit

The following widely known statistical indices are proposed for the goodness-of-fit comparison:

1. *The sum of squares due to error (SS_E):* the summed square of residuals. A value closer to 0 indicates that the model has a smaller random error component, and that the fit will be more useful for prediction. SS_E expression is given below; where *n* is the total number of responses. λ_i's represent the response value of λ, and $\hat{\lambda}_i$ represent the predicted response value from fitting.

$$SS_E=\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}\left(\lambda_i - \hat{\lambda}_i\right)^2 \qquad (13)$$

2. *R^2:* the ratio of the sum of squares of the regression and the total sum of squares. A large value of R^2 suggests that the model has been successful in explaining the variability in the response. When R^2 is small, it may be an indication that we need to find an alternative model, such as a multiple regression mode. R^2 is defines as

$$R^2=1\text{-}SS_E\left(\frac{1}{\sum_{i=1}^{n}\lambda_i^2-\left(\sum_{i=1}^{n}(\lambda_i)^2/n\right)}\right) \qquad (14)$$

3. *Root-mean-squared error (RMSE):* an estimate of the standard deviation of the random component in the data, and is defined as

$$RMSE=\sqrt{\frac{SS_E}{\nu}} \qquad (15)$$

where ν represents the residual degrees of freedom, which is defined as the number of response values n minus the number of fitted coefficients m estimated from the response values (*i. e.* $\nu= n\text{-}m$). An *RMSE* value closer to 0 indicates a fit that is more useful for prediction.

Curve Fitting Toolbox software supports these goodness-of-fit statistics for parametric models. Reader should refer to [10], [12] about detailed clarification of these indices.

VI. Simulated Results and Their Discussion

Table I shows the results of trials of nonlinear curve-fitting problem under study. Figs. 4–8 show the graphical representation of each fit compared to the original experimental data, respectively. Comparison of the results given in Table I show that the proposed method is acceptable, providing acceptable goodness-of-fit indices for the non linear curve-fitting problem under study. Table I shows the most accepted trials arranged in ascending order according to their goodness. Trial 5 shown in Table I and Fig. 8 is the most convenient fit compared with the other trials, because the proposed fit-equation results in-lowest SS_E value, lowest *RMSE* and highest R^2 value than the others shown in Table I [13].

TABLE I

Simulation Results: S represents *TSR*

Trial	Fit	Coefficients (95% confidence)	SS_E	R^2	RMSE
1	$\lambda_1= a_0 + a_1\cos(w*S) + b_1\sin(w*S) + a_2\cos(2w*S) + b_2\sin(2w*S) + a_3\cos(3w*S) + b_3\sin(3w*S)$	$a_0 = 1.089*10^7$, $a_1 = -1.624*10^7$, $b_1 = -1.881*10^6$, $a_2 = 6.373*10^6$, $b_2 = 1.497*10^6$, $a_3 = -1.028*10^6$, $b_3 = -3.708*10^5$, and $w = 0.02793$.	0.0001969	0.9997	0.005729
2	$\lambda_2= p_1S^7 + p_2S^6 + p_3S^5 + p_4S^4 + p_5S^3 + p_6S^2 + p_7S + p_8$	$p_1 = -9.396*10^{-5}$, $p_2 = 0.002663$, $p_3 = -0.03141$, $p_4 = 0.1978$, $p_5 = -0.7065$, $p_6 = 1.357$, $p_7 = -1.065$, and $p_8 = 0.3159$.	0.0001529	0.9992	0.005048
3	$\lambda_3= (p_1S^3 + p_2S^2 + p_3S + p_4)/(S^4 + q_1S^3 + q_2S^2 + q_3S + q_4)$	$p_1 = -11.01$, $p_2 = 87.49$, $p_3 = -92$, $p_4 = 36.32$, $q_1 = -25.7$, $q_2 = 193.9$, $q_3 = -298$, and $q_4 = 416.6$.	0.0001404	0.9993	0.004838
4	$\lambda_4= a_0\text{-}a_1\cos(S)\text{-}a_2\cos(2S)\text{-}a_3\cos(3S)\text{-}a_4\cos(4S)\text{-}b_1\sin(S)\text{-}b_2\sin(2S)\text{-}b_3\sin(3S)\text{-}b_4\sin(4S)–b_5\sin(5S)$	$a_0 = 0.2631$, $a_1 = 0.1374$, $a_2 = 0.007213$, $a_3 = -.006686$, $a_4= -0.002296$, $b_1 = 0.08926$, $b_2 = 0.04355$, $b_3 = 0.01387$, $b_4 = 0.002402$, and $b_5 = 0.004312$.	$6.221*10^{-5}$	0.9997	0.003944
5	$\lambda_5= a_0\text{-}a_1\cos(S)\text{-}a_2\cos(2S)\text{-}a_3\cos(3S)\text{-}a_4\cos(4S)\text{-}a_5\cos(5S)\text{-}a_6\cos(6S)\text{-}b_1\sin(S)\text{-}b_2\sin(2S)–b_3\sin(3S)\text{-}b_4\sin(4S)\text{-}b_5\sin(5S)–b_6\sin(6S)$	$a_0 = 0.2635$, $a_1 = 0.139$, $a_2 = 0.007062$, $a_3 = -0.006336$, $a_4 = -0.001343$, $a_5 = -0.001123$, $a_6= -0.003596$, $b_1 = 0.08746$, $b_2 = 0.04316$, $b_3 = 0.01339$, $b_4 = 0.001731$, $b_5 = 0.005092$, and $b_6 = -0.002068$.	$3.423*10^{-7}$	0.9999	0.000585

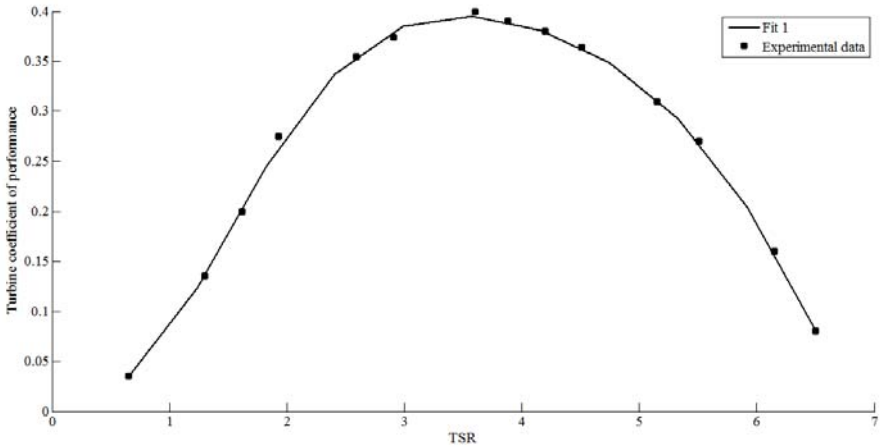

Fig. 4 λ_1 versus *TSR*, fit 1

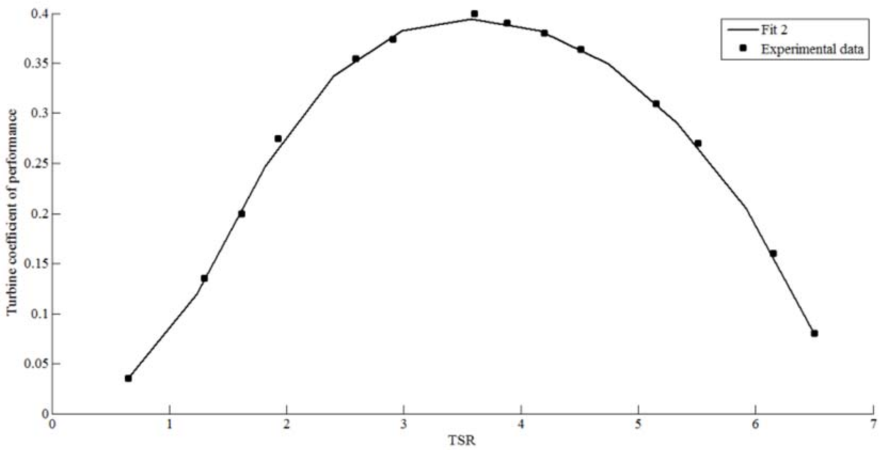

Fig. 5 λ_2 versus *TSR*, fit 2

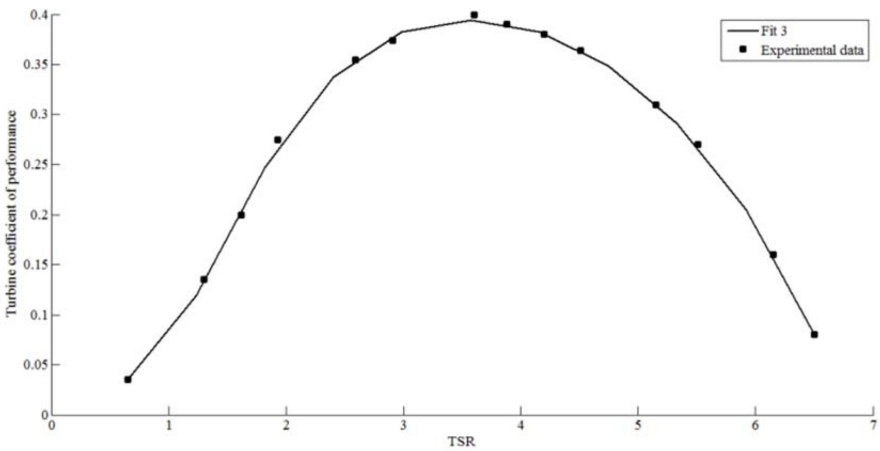

Fig. 6 λ_3 versus *TSR*, fit 3

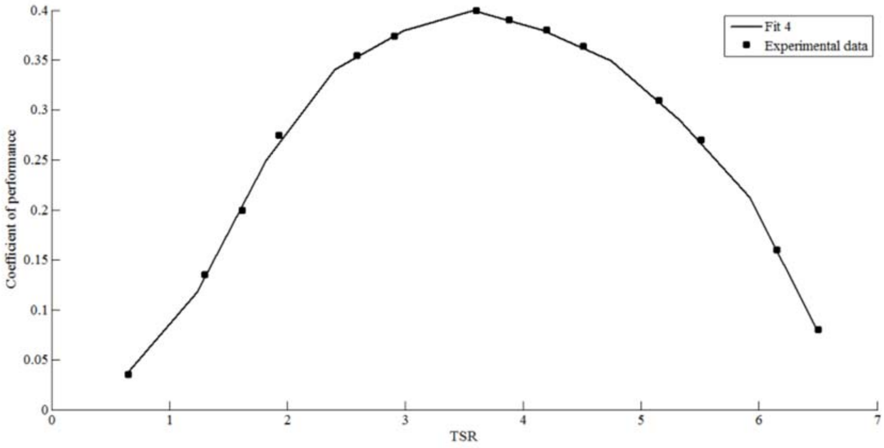

Fig. 7 λ_4 versus *TSR*, fit 4

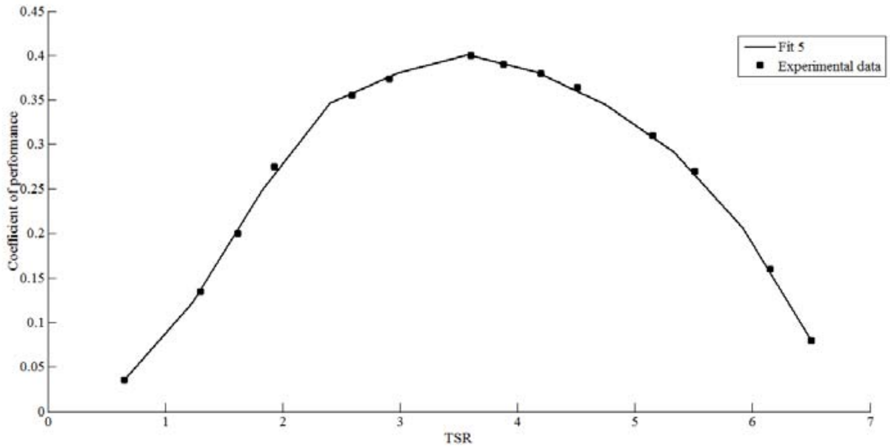

Fig. 8 λ_5 versus *TSR*, fit 5

VII. Conclusion

In this paper, the relation between the turbine coefficient of performance and tip speed ratio has been studied for a test marine turbine using the nonlinear curve-fitting (data-fitting) problems in least-squares sense. It has been demonstrated that the performance characteristic of a tidal stream turbines can be distinctively quantified by such a coefficient. The mathematical analysis of the proposed dependencies is important for tidal power analysis. Acceptable goodness-of-fit indices are obtained between experimental data and different fitting functions used.

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